Tæthedsfunktioner

Gyldighed

$$\int_{-\infty}^{\infty} f_X(x)\,\mathrm{d}x = 1$$

CDF til PDF

$$f_X(x) = \frac{\mathrm{d}}{\mathrm{d}x} F_x(x)$$

PMF til CDF

$$F_X(x_i) = \sum_{x=-1}^{x_i} f_X(x)$$

PDF til CDF

$$F_X(x_i) = \int_{-\infty}^{x_i} f_X(u)\,\mathrm{d}u$$

Marginale tæthedsfunktioner

$$f_X(x) = P(X = x) = \sum_y P(X = x, Y = y) = \sum_y f_{XY}(x, y)$$

$$f_Y(y) = P(Y = y) = \sum_x P(X = x, Y = y) = \sum_x f_{XY}(x, y)$$

Sandsynlighed (pdf)

$$Pr(X > x) = 1 - F_X(x)$$

$$N = N_{1k} + N_{10k} + N_{100k}$$

Betinget sandsynlighed

$$f_{X\|Y}(x\|y) = \frac{f_{XY}(x, y)}{f_Y(y)} = Pr(X = x \,\|Y = y)$$

Middelværdi

Forventningsværdi (pdf)

$$E\left[X\right] = \overline{X} = \mu_X = \int_{-\infty}^{\infty} x_i \cdot f_X(x)\,\mathrm{d}x$$

Forventningsværdi (pmf)

$$E\left[X\right] = \sum_{i=1}^{n} x_i \cdot E\left(X_i\right) = \sum_{i=1}^{n} x_i \cdot f_X\left(x = x_i\right)$$

Uniform fordeling

$$E\left[X\right] = \frac{a+b}{2}$$

Ensemble middelværdi

$$\mu_X(t) = E\left[X(t)\right] = \int_{-\infty}^{\infty} x(t) \cdot f_X(x(t))\,\mathrm{d}x\,(t)$$

$$\mu_X(n) = E\left[X(n)\right] = \int_{-\infty}^{\infty} x(n) \cdot f_X(x(n))\,\mathrm{d}x\,(n)$$

Tidslig middelværdi

$$\mu_{xi} = \left(X_i\right)_T = \lim_{T \to \infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} x_i(t)\,\mathrm{d}t$$

$$\mu_X(t) = E\left[X(t)\right] = E\left[a \cdot t\right]$$

Varians

Varians (pdf)

$$var(x) = \sigma_X{}^2 = E\left[x^2\right] - E\left[x\right]^2 = \int_{-\infty}^{\infty} x_i{}^2 \cdot f_X(x)\,\mathrm{d}x - E\left[x\right]^2$$

Varians (pmf)

$$E\left[X^2\right] = \sum_{i=1}^{n} x_i{}^2 \cdot E\left(X_i\right) = \sum_{i=1}^{n} x_i{}^2 \cdot f_X\left(x = x_i\right)$$

$$\mu_X(t) = E\left[X(t)\right] = E\left[a \cdot t\right]$$

Normal fordeling

$$\sigma_x{}^2 = E\left[x^2\right] - E\left[x\right]^2$$

$$E\left[X_n\right] = E\left[2 \cdot w(n) - 1\right] = 2 \cdot E\left[w(n)\right] - 1$$

$$E\left[w(n)\right] = \frac{a+b}{2}$$

Uniform fordeling

$$var(w(n)) = \frac{(b-a)^2}{12}$$

$$var(2 \cdot w(n) - 1) = 2^2\ \frac{(b-a+1)^2 - 1}{12}$$

$$var(X(t)) = var(a \cdot t) = t^2\ var(a)$$

Ensemble varians

$$E\left[3 \cdot x^2\right] = 3 \cdot E\left[X^2\right]$$

$$var(X(t)) = \sigma_X{}^2\ (t) = E\left[X(t) - \mu_X(t)^2\right]$$

$$var(X(n)) = \sigma_X{}^2\ (n) = E\left[X(n) - \mu_X(n)^2\right]$$

Varians af en konstant

$$Var(k) = E\left[k^2\right] - E\left[k\right]^2 = k^2 - k^2 = 0$$

Standardafvigelse

Standardafvigelse

$$\sigma_x = \sqrt{E\left[x^2\right] - E\left[x\right]^2}$$

$$\sigma_X = \sqrt{\sigma_X{}^2}$$

Hypoteser

Estimator (middelværdi)

$$\delta = \overline{x}_1 - \overline{x}_2 \sim N\left(0, \frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}\right)$$

Estimator(varians)

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left(\left(n_1 - 1\right)\ s_1{}^2 + \left(n_2 - 1\right)\ s_2{}^2\right)$$

$$\sigma = \sqrt{s^2}$$

Test size

$$t = \frac{\delta}{s \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

P value

$$p_{val} = 2 \cdot \left(1 - t_{cdf}\left(\left|t\right|, n_1 + n_2 - 2\right)\right)$$

95% konfidens interval

$$\delta_{lower} = \left(x_1 - x_2\right) - t_0 \cdot s \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\delta_{upper} = \left(x_1 - x_2\right) + t_0 \cdot s \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Korrelation

$$\rho = E\left[\frac{X-\overline{X}}{\sigma_X}\cdot\frac{Y-\overline{\overline{Y}}}{\sigma_Y}\right] = \frac{E[XY]-E[X]\,E[Y]}{\sigma_X\cdot\sigma_Y} = \frac{cov(X,Y)}{\sigma_X\cdot\sigma_Y}$$

$$\sigma_Y = \sqrt{E\left[Y^2\right]-E[Y]^2}$$

Autokorrelation

$$R_{XX}(t_1,t_2) = E\left[X(t_1)\,\overline{X(t_2)}\right]$$

$$X(n) := w(n)$$

$$R_{XX}(\tau) := E(X(n)\cdot X(n-\tau)) \rightarrow E(w(n)\cdot w(n-\tau))$$

$\tau := 0,1..3$ $\quad$ $n := 1,2..10$

$$R_{XX}(\tau) \rightarrow \begin{bmatrix} E\left(w(n)^2\right) \\ E(w(n)\cdot w(n-1)) \\ E(w(n)\cdot w(n-2)) \\ E(w(n)\cdot w(n-3)) \end{bmatrix}$$

Sandsynlighed

Prior

$Pr(A) = 0.001$

$Pr(\overline{A}) = 1 - Pr(A)$

$Pr(B)$

Likelihood

$Pr(B \| A) = 0.92$

$Pr(B \| A') = 1 - Pr(Pr(B \| A))$

$Pr(B' \| A) = 0.98$

$Pr(B' \| A') = 1 - Pr(B' \| A)$

Posterior

$$Pr(B' \| A) = \frac{Pr(A \| B') \cdot Pr(B')}{Pr(A)}$$

Total Probabilty (Sum Rule)

$$Pr(B) = Pr(B \cap A) + Pr(B \cap \overline{A}) = Pr(B \| A) \cdot Pr(A) + Pr(B \| \overline{A}) \cdot Pr(\overline{A})$$

$$Pr(A) = Pr(A \cap B) + Pr(A \cap \overline{B}) = Pr(A \| B) \cdot Pr(B) + Pr(A \| \overline{B}) \cdot Pr(\overline{B})$$

Total Probabilty (Bayes Rule)

$$Pr(A \| C) = \frac{Pr(C \| A)\ Pr(A)}{Pr(C)}$$

Joint Events

$Pr(B \cap A) = Pr(B \| A)\ Pr(A)$

Relative Frequency Approach

$N_A = 28131$

$N_B = 29785$

$N = N_A + N_B = 57916$

$28131 + 29785 = 57916$

$$Pr(A) = \frac{N_A}{N}$$

Binomial

$$Pr_n(k) = \frac{n!}{k!\ (n-k)!}\ p^k\ q^{n-k} = \begin{bmatrix} n \\ k \end{bmatrix} p^k\ q^{n-k}$$

Ensemble

Ensemble mean

$$\mu_X(x) = E[X(n)] = \frac{a+b}{2}$$

Ensemble variance

$$var(x) = E\left[X(n)^2\right] - E[X(n)]^2 = \frac{(b-a)^2}{12}$$

Statistik

Known variance

$$\mu_{est} = \overline{\overline{x}} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$z = \frac{\overline{x} - \mu_0}{\sigma \cdot \sqrt{n}}$$

$$p = 2 \cdot |1 - \Phi(|z|)|$$

$$\mu_{lower} = \overline{x} - 1.96 \cdot \frac{\sigma}{\sqrt{n}}$$

$$\mu_{upper} = \overline{x} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}$$

$$s = \sqrt{s^2}$$

$$z = \frac{\left(\overline{x_1} - \overline{x_2}\right)}{\sigma \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Unknown variance

$$\mu_{est} = \overline{x} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2$$

$$t = \frac{\overline{x} - \mu_0}{s \cdot \sqrt{n}}$$

$$p = 2 \cdot |1 - t_{cdf}(|t|)|$$

$$t_0 = tinv\left(\frac{1 - 0.05}{2}, n - 1\right)$$

$$\mu_{lower} = \overline{x} - t_0 \cdot \frac{s}{\sqrt{n}}$$

$$\mu_{upper} = \overline{x} + t_0 \cdot \frac{s}{\sqrt{n}}$$

Poisson

$H_0 \quad \lambda = \lambda_0$

$H_1 \quad \lambda ! = \lambda_0$

$$\lambda = \frac{x}{t}$$

$$z = \frac{x - t \cdot \lambda}{\sqrt{t \cdot \lambda}}$$

$$z = \frac{X - E[X]}{\sqrt{var(X)}} = \frac{X - t \cdot \lambda}{\sqrt{t \cdot \lambda}}$$ (normal approximation)

$$p = 2 \cdot |1 - \Phi(|z|)|$$

$$\lambda_{lower} = \frac{1}{t}\left[x + \frac{1.96^2}{2} - 1.96 \cdot \sqrt{x + \frac{1.96^2}{4}}\right]$$

$$\lambda_{upper} = \frac{1}{t}\left[x + \frac{1.96^2}{2} + 1.96 \cdot \sqrt{x + \frac{1.96^2}{4}}\right]$$

Binomial

$$p_{est} = \frac{x}{n}$$

$$z = \frac{x - n \cdot p_0}{\sqrt{n \cdot p_0 \left(1 - p_0\right)}}$$

$$p = 2 \cdot \left|1 - \Phi\left(\left|z\right|\right)\right|$$

$$p_{lower} = \frac{1}{n + 1.96^2} \left[x + \frac{1.96^2}{2} - 1.96 \cdot \sqrt{\frac{x\left(n - x\right)}{n} + \frac{1.96^2}{4}} \right]$$

$$p_{upper} = \frac{1}{n + 1.96^2} \left[x + \frac{1.96^2}{2} + 1.96 \cdot \sqrt{\frac{x\left(n - x\right)}{n} + \frac{1.96^2}{4}} \right]$$

Regression

$$\alpha = \overline{y} - \beta\ \overline{x} \qquad \text{(skæring)}$$

$$\beta = \frac{\sum_{i=1}^{n} \left(x_i - \overline{x}\right)\left(y_i - \overline{y}\right)}{\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2} \qquad \text{(hældning)}$$

$$\varepsilon_i = y_i - \left(\alpha + \beta x_i\right) \qquad \text{(residual)}$$

$$s_r^{\ 2} = \frac{1}{n-2} \sum_{i=1}^{n} \left(y_i - \left(\alpha + \beta x_i\right)\right)^2 = \frac{1}{n-2} \sum_{i=1}^{n} \varepsilon_i^{\ 2}$$

$$\delta_{est} = \overline{x_1} - \overline{x_2}$$

$$\delta_{lower} = \left(\overline{x_1} - \overline{x_2}\right) - 1.96 \cdot \sigma \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Konfidensinterval for hældningen β

$$\beta_{lower} = \beta - t_o \cdot \sqrt{\frac{S_r^{\ 2}}{\sum_{i=1}^{n} \left(t_i - \overline{t}\right)^2}}$$

$$\beta_{upper} = \beta + t_o \cdot \sqrt{\frac{S_r^{\ 2}}{\sum_{i=1}^{n} \left(t_i - \overline{t}\right)^2}}$$