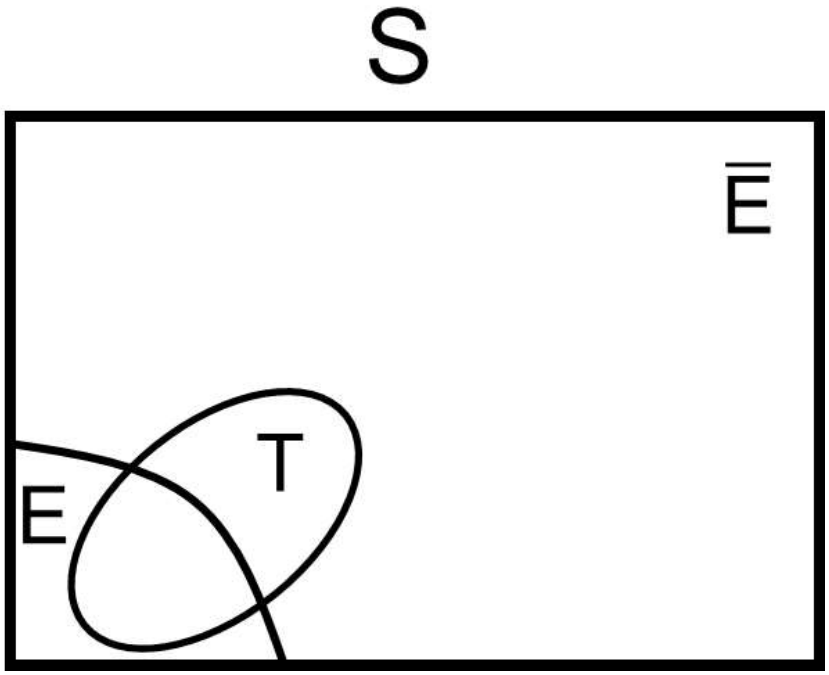
Example: Ebola Test

- Event E: Patient are infectious with Ebola.
- Event T: The Ebola test is positive.



Example: Ebola Test

- **Prior**: What are the probability of a patient having Ebola?

$$Pr(E)$$

- **Likelihood**: What are the probability of a positive test given infectious with Ebola? Or of a negative test given not infectious with Ebola?

$Pr(T|E)$ *Sensitivity* $Pr(\bar{T}|\bar{E})$ *Specificity*

- **Posterior**: What are the probability of being infectious given that a test is positive?

$$Pr(E|T)$$

Example: Ebola Test — Total Probability

- **Prior**: What are the probability of a patient having ebola?

Complement of E

$$Pr(E) = 0,01 \qquad Pr(\bar{E}) = 1 - 0,01 = 0,99$$

- **Likelihood**: What are the probabilities of the tests?

$$Pr(T|E) = 0,9$$ ← Sensitivity

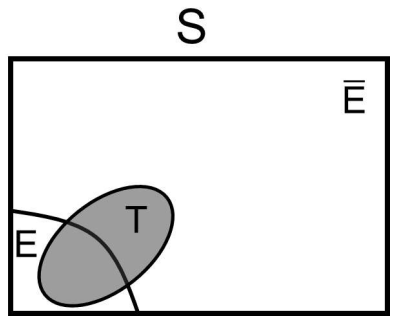
$$Pr(\bar{T}|\bar{E}) = 0,8$$ ← Specificity

- **Complement**: What are the probability of a patient having a positive test without being infectious?

$$Pr(T|\bar{E}) = 1 - Pr(\bar{T}|\bar{E}) = 0,2$$

Example: Ebola Test — Total Probability

- **Total Probability with the Sum Rule**: What are the probability of a patient having a positive test?



$$Pr(T) = Pr(T \cap E) + Pr(T \cap \bar{E})$$

- **The Product Rule**: We can with Bayes rule find

$$\begin{aligned} Pr(T) &= Pr(T|E)\,Pr(E) + Pr(T|\bar{E})\,Pr(\bar{E}) \\ &= 0{,}9 \cdot 0{,}01 + 0{,}2 \cdot 0.99 \\ &= 0{,}207 \end{aligned}$$

Ebola Example — Posterior

- **We have**: We now know the probabilities:

$$P(E) = 0,01 \quad \leftarrow \text{Prior}$$
$$P(T) = 0,207 \quad \leftarrow \text{Total probability}$$
$$P(T|E) = 0,9 \quad \leftarrow \text{Likelihood}$$

- **Product Rule**: What are the probability of being infectious given that a test is positive?

Bayes rule

$$Pr(E|T) = \frac{Pr(T|E)Pr(E)}{Pr(T)} = \frac{0,9 \cdot 0,01}{0,207} = 0,043$$

Ebola Example — Posterior

- What are the probability of being infectious given that a test is positive?

$$Pr(E|T) = \frac{Pr(T|E)Pr(E)}{Pr(T)} = \frac{0{,}9 \cdot 0{,}01}{0{,}207} = 0{,}043$$

- What are the probability of not being infectious given that a test is positive?

$$Pr(\bar{E} \,|\, T) = 1 - Pr(E|T) = 0{,}957$$

- What are the probability of not being infectious given a negative test?

$$Pr(\bar{E}|\bar{T}) = \frac{Pr(\bar{T} \,|\, \bar{E})Pr(\bar{E})}{Pr(\bar{T})} = \frac{0{,}8 \cdot 0{,}99}{0{,}793} = 0{,}999$$

- What are the probability of being infectious given that a test is negative?

$$Pr(\mathrm{E} \,|\, \bar{T}) = 1 - Pr(\bar{E}|\bar{T}) = 0{,}001$$

Ebola Example — Conclusion

- If the test is negative, it is allmost certain (99,9%) that you're not being infectious:

$$Pr(\bar{E}|\bar{T}) = 0{,}999$$

- If the test is positive, there is still only a small risk (4,3%) that you actually are being infectious:

$$Pr(E|T) = 0{,}043$$