L9 Group Assignment

Samples of a Population - Test Data

Before software is tested on real data, it is tested with well-controlled test data. Test data is generated after a discrete stochastic process that has the following signal model:

$$X(n) = w(n)$$

where $w(n)$ is i.i.d and $w(n) = \mathrm{N}(7, 2)$

1. Are the samples Of X (n) statistically independent?

Yes they are i.i.d.

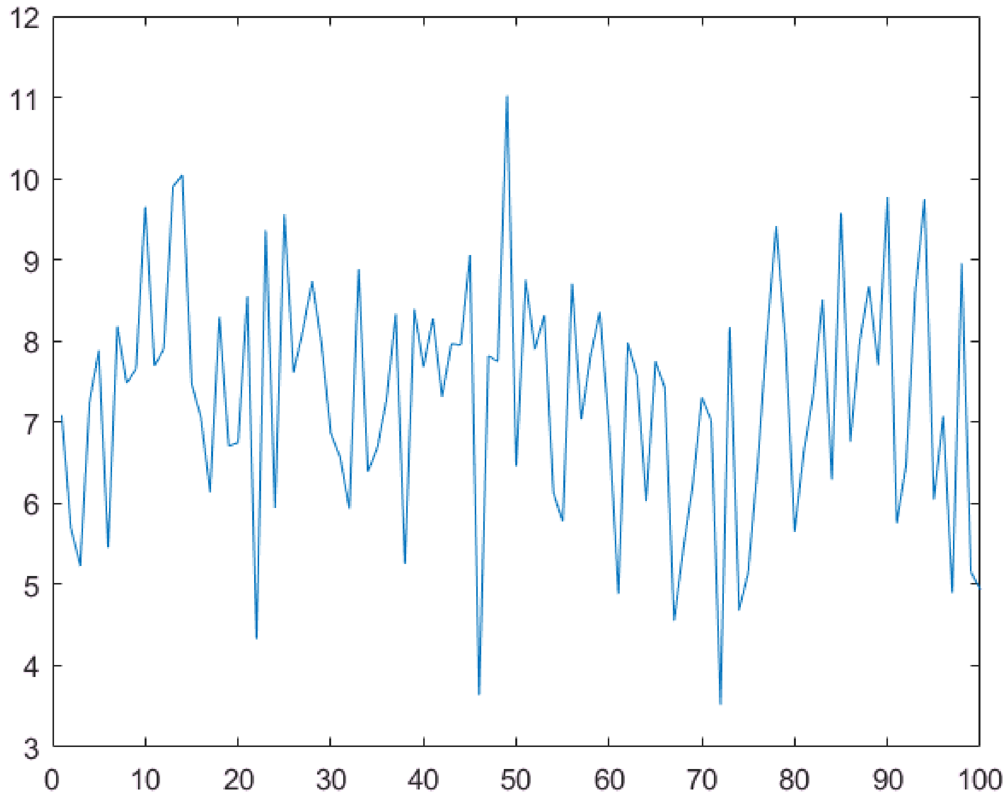
2. Is the process ergodic?

Yes it is.

3. Plot the data from one realisation of the process.

```
mu = 7; variance = 2;

X1 = sqrt(variance)*randn(1,100)+mu;
plot(X1)
```



4. What is the population in this case?

The population is all realisations of the process X(n)

5. Use the formula to calculate the sample mean.

```
X1_mean = 1/length(X1)*sum(X1)
```

```
X1_mean = 6.7970
```

6. Verify with the matlab function mean.

```
X1_mean = mean(X1)
```

```
X1_mean = 6.7970
```

7. Use the formula to calculate the sample variance.

```
X1_var = 1/(length(X1)-1).*sum((X1-X1_mean).^2)
```

```
X1_var = 2.3125
```

8. Verify with the matlab function var.

```
X1_var = var(X1)
```

```
X1_var = 2.3125
```

9. Find the z-score for the data. What does the z-score tell you?

```
n = length(X1);
z = (X1_mean-mu)/(sqrt(variance)/sqrt(n))
```

```
z = -1.4355
```

10. Find the confidence interval for the mean (lower and upper endpoint).

- Signifance level of $\alpha = 0.05$
- Confidence level of $1 - \alpha = 0.95$
- Confidence interval is $\frac{\alpha}{2}$ because it is a two-sided test

Z value is found by inverse table look-up.

```
lower_bound = X1_mean-norminv(0.975)*sqrt(2)/sqrt(length(X1))
```

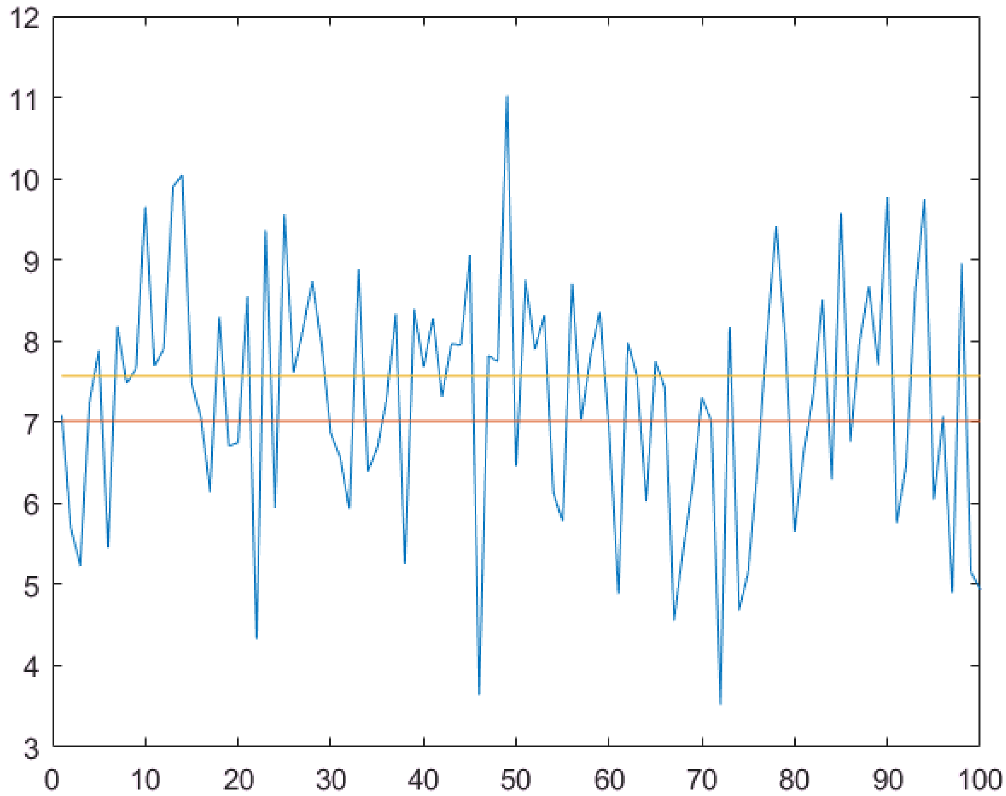
```
lower_bound = 6.5198
```

```
upper_bound = X1_mean+norminv(0.975)*sqrt(2)/sqrt(length(X1))
```

```
upper_bound = 7.0742
```

11. Draw the confidence interval on the plot together with the data. What does the confidence interval tell you?

```
plot(X1)
hold on
plot(ones(1,length(X1)).*lower_bound)
hold on
plot(ones(1,length(X1)).*upper_bound)
hold off
```



12. How large a sample size do you actually need?

- B is the desire uncertaincy
- n is the samplesize

```
B = norminv(0.975)*sqrt(2)/sqrt(length(X1))
```

```
B = 0.2772
```

```
n_min = (norminv(0.975)*sqrt(2)/B)^2
```

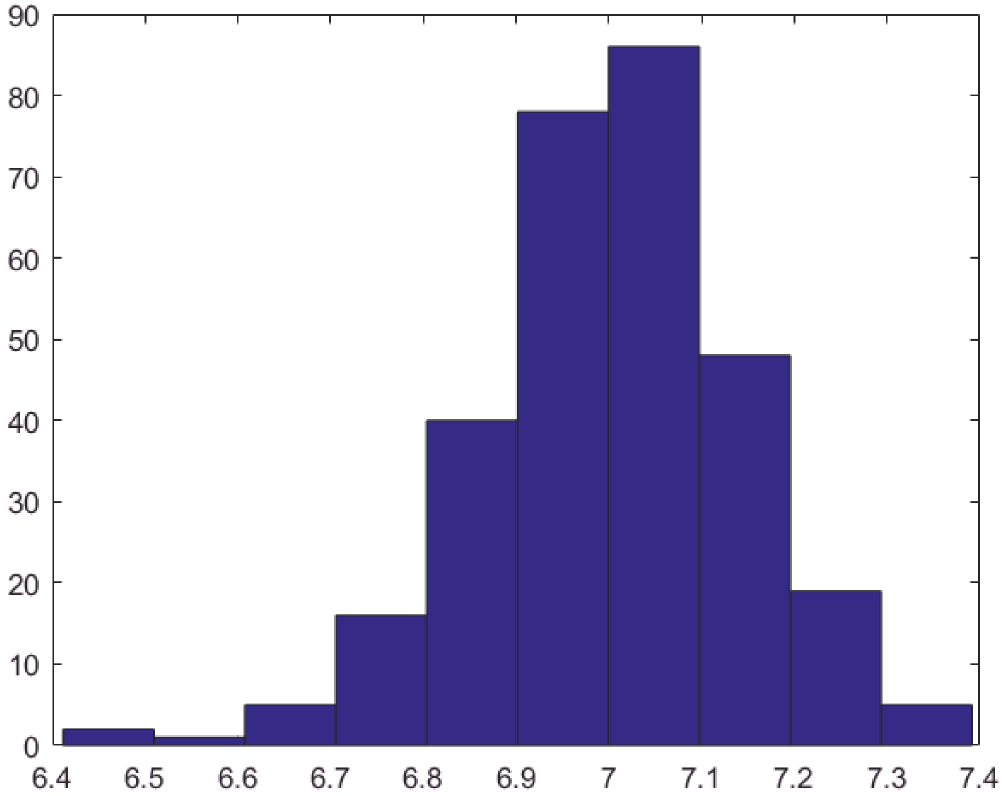
```
n_min = 100
```

13. Repeat questions 3 and 6, 30 times and make a histograms of the found sample means. What distribution do the samples have?

- Normal distribution

```
for i = 1:300
X1 = sqrt(variance)*randn(1,100)+mu;
meanX(i) = mean(X1);
end
```

```
figure(2)
hist(meanX)
```

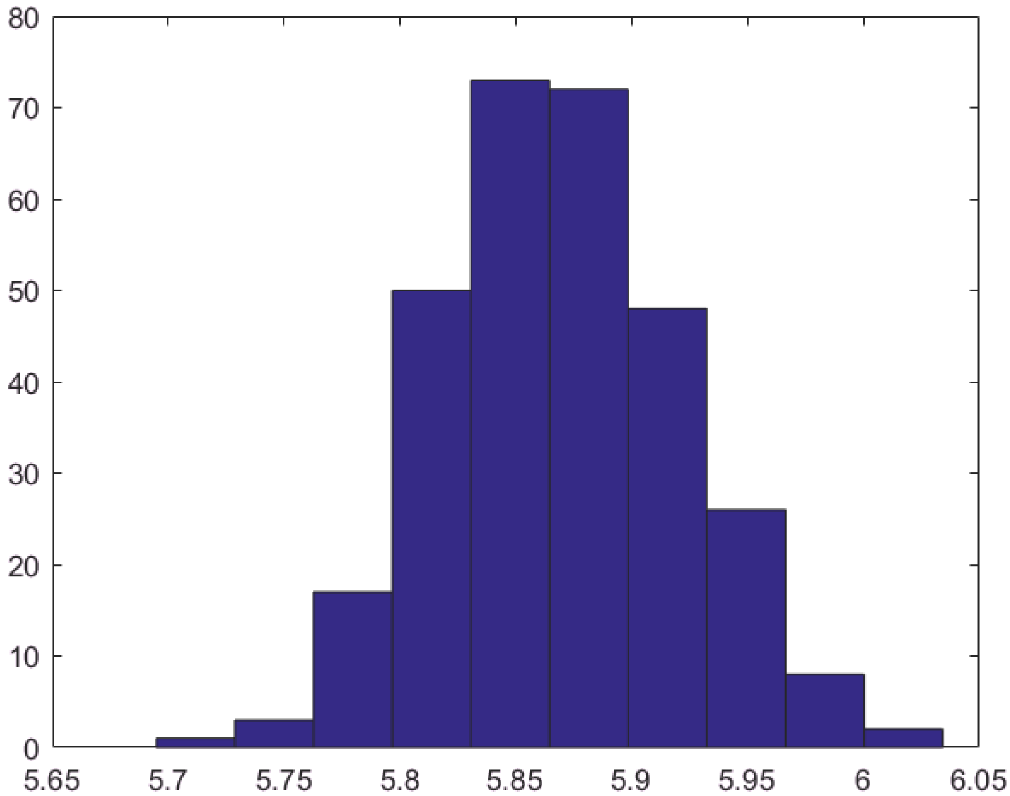


14. What would happen in question 13, if the signal model was given as: $X(n) = w(n)$ where $w(n)$ is i.i.d and $w(n) \sim U(5, 9)$

```
a = 5; b = 9;

for i = 1:300
X1 = sqrt(3)*rand(1,100)+a;
meanX(i) = mean(X1);
end

figure(3)
hist(meanX)
```



80
70
60
50
40
30
20
10
0
5.65
5.7
5.75
5.8
5.85
5.9
5.95
6
6.05