TEST CATALOG FOR THE BINOMIAL DISTRIBUTION

Statistical model:

- $X \sim binomial(n, p)$
- Parameter estimate: $\hat{p} = x/n$
- Where the observation is *x* = 'number of successes out of *n* trials'

Hypothesis test (two-tailed):

- $H_0: p = p_0$
- $H_1: p \neq p_0$
- Test size: $z = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} \sim N(0,1)$
- Approximate p-value: $2 \cdot |1 - \Phi(|z|)|$

95% confidence interval:

- $p_- = \frac{1}{n+1.96^2}\left[x + \frac{1.96^2}{2} - 1.96\sqrt{\frac{x(n-x)}{n} + \frac{1.96^2}{4}}\right]$

- $p_+ = \frac{1}{n+1.96^2}\left[x + \frac{1.96^2}{2} + 1.96\sqrt{\frac{x(n-x)}{n} + \frac{1.96^2}{4}}\right]$

TEST CATALOG FOR THE POISSON DISTRIBUTION

Statistical model:

- $X \sim poisson(\lambda \cdot t)$
- Parameter estimate: $\hat{\lambda} = x/t$
- Where the observation is *x* = 'number of arrivals/events observed over a period of time *t*'

Hypothesis test (two-tailed):

- $H_0 : \lambda = \lambda_0$
- $H_1 : \lambda \neq \lambda_0$
- Test size: $z = \frac{x - \lambda \cdot t}{\sqrt{\lambda \cdot t}} \sim N(0,1)$
- Approximate p-value: $2 \cdot |1 - \Phi(|z|)|$

95% confidence interval:

- $\lambda_- = \frac{1}{t}\left[x + \frac{1.96^2}{2} - 1.96\sqrt{x + \frac{1.96^2}{4}}\right]$
- $\lambda_+ = \frac{1}{t}\left[x + \frac{1.96^2}{2} + 1.96\sqrt{x + \frac{1.96^2}{4}}\right]$

TEST CATALOG FOR THE MEAN (KNOWN VARIANCE)

Statistical model:

- $X_1, X_2, \dots, X_n$ are i.i.d. samples of a random variable *X* with mean μ and variance σ^2.
- Parameter estimate:

$$\hat{\mu} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim \mathrm{N}(\mu, \sigma^2/n)$$

- Where the observation is $\bar{x}$ = 'the average of *n* samples drawn from *X*'s distribution'.
- NOTE: The statistical model is only true if *n* is sufficiently large ($n \geq 30$) <u>or</u> if the samples are drawn from a normal population with mean μ and variance σ^2.

Hypothesis test (two-tailed):

- $H_0: \mu = \mu_0$
- $H_1: \mu \neq \mu_0$
- Test size: $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$
- Approximate p-value: $2 \cdot |1 - \Phi(|z|)|$

95% confidence interval:

- $\mu_- = \bar{x} - 1.96 \cdot \sigma/\sqrt{n}$
- $\mu_+ = \bar{x} + 1.96 \cdot \sigma/\sqrt{n}$

TEST CATALOG FOR THE MEAN (UNKNOWN VARIANCE)

Statistical model:

- $X_1, X_2, \dots, X_n$ are i.i.d. samples of a random variable X with mean μ and variance σ^2.
- Parameter estimates:

$$\hat{\mu} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N(\mu, \sigma^2/n)$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- Where the observation is $\bar{x}$ = 'the average of n samples drawn from X's distribution'.
- NOTE: The statistical model is only true if n is sufficiently large ($n \geq 30$) or if the samples are drawn from a normal population with mean μ and variance σ^2.

Hypothesis test (two-tailed):

- $H_0: \mu = \mu_0$
- $H_1: \mu \neq \mu_0$
- Test size: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim t(n-1)$
- Approximate p-value: $2 \cdot \left(1 - t_{cdf}(|t|)\right)$

95% confidence interval:

- $\mu_- = \bar{x} - t_0 \cdot s/\sqrt{n}$
- $\mu_+ = \bar{x} + t_0 \cdot s/\sqrt{n}$

where `t0 = tinv(1-0.05/2,n-1)`

Test catalog for comparing two means (known variance)

Statistical model:

- $X_{1i} \sim N\left(\mu_1, \sigma_1^2\right), i = 1,2, \dots, n_1$ and $X_{2i} \sim N\left(\mu_2, \sigma_2^2\right)$ $i = 1,2, \dots, n_2$
- Parameter estimate:
$$\hat{\delta} = \bar{x}_1 - \bar{x}_2 \sim N\left(\mu_1 - \mu_2, \sigma_1^2/n_1 + \sigma_2^2/n_2\right)$$
- Where the observation is $\bar{x}_1 - \bar{x}_2$ = 'the difference between two sample means'.

Hypothesis test (two-tailed):

- $H_0: \mu_1 = \mu_2$
- $H_1: \mu_1 \neq \mu_2$
- Test size: $z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sigma\sqrt{1/n_1 + 1/n_2}} \sim N(0,1)$
- Approximate p-value: $2 \cdot |1 - \Phi(|z|)|$

95% confidence interval:

- $\delta_- = (\bar{x}_1 - \bar{x}_2) - 1.96 \cdot \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
- $\delta_+ = (\bar{x}_1 - \bar{x}_2) + 1.96 \cdot \sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

TEST CATALOG FOR COMPARING TWO MEANS (UNKNOWN VARIANCE)

Statistical model:

- $X_{1i} \sim N\left(\mu_1, \sigma_1^2\right), i = 1,2,\ldots,n_1$ and $X_{2i} \sim N\left(\mu_2, \sigma_2^2\right)$ $i = 1,2,\ldots,n_2$
- Parameter estimate:

$$\hat{\delta} = \bar{x}_1 - \bar{x}_2 \sim N\left(\mu_1 - \mu_2, \sigma_1^2/n_1 + \sigma_2^2/n_2\right)$$
$$s^2 = \frac{1}{n_1 + n_2 - 2}\left((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right)$$

- Where the observation is $\bar{x}_1 - \bar{x}_2$ = 'the difference between two sample means'.

Hypothesis test (two-tailed):

- $H_0: \mu_1 = \mu_2$
- $H_1: \mu_1 \neq \mu_2$
- Test size: $t = \frac{(\bar{x}_1 - \bar{x}_2)}{s\sqrt{1/n_1 + 1/n_2}} = \sim t(n_1 + n_2 - 2)$
- Approximate p-value: $2 \cdot \left(1 - t_{cdf}(|t|, n_1 + n_2 - 2)\right)$

95% confidence interval:

- $\delta_- = (\bar{x}_1 - \bar{x}_2) - t_0 \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
- $\delta_+ = (\bar{x}_1 - \bar{x}_2) + t_0 \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

where `t0 = tinv(1-0.05/2,n1+n2-2)`

TEST CATALOG FOR PAIRED DATA

Statistical model:

- $d_i = X_{1i} - X_{2i}$, where $d_i \sim N(\delta, \sigma^2), i = 1,2, \dots, n$
- Parameter estimate:

$$\hat{\delta} = \bar{d} = \frac{1}{n}\sum_{i=1}^{n} X_{1i} - X_{2i}$$

$$s_d^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2$$

- Where the observation is $\bar{d}$ = 'the average of the differences between paired samples'.

Hypothesis test (two-tailed):

- $H_0: \delta = \delta_0$
- $H_1: \delta \neq \delta_0$
- Test size: $t = \frac{\bar{d}-\delta_0}{s_d/\sqrt{\mathrm{n}}} = \sim t(\mathrm{n}-1)$
- Approximate p-value: $2 \cdot \left(1 - t_{cdf}(|t|, \mathrm{n}-1)\right)$

95% confidence interval:

- $\delta_{-} = \bar{d} - t_0 \cdot \frac{s_d}{\sqrt{n}}$
- $\delta_{+} = \bar{d} + t_0 \cdot \frac{s_d}{\sqrt{n}}$

where `t0 = tinv(1-0.05/2,n-1)`

TEST CATALOG FOR THE SLOPE IN SIMPLE LINEAR REGRESSION

Statistical model:

- $y_i \sim N(\alpha + \beta x_i, \sigma^2)$ for $i = 1,2, \dots, n$ are independent samples.
- Parameter estimates:

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \cdot \bar{x}$$

$$s_r^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left(y_i - (\hat{\alpha} + \hat{\beta} x_i)\right)^2$$

Hypothesis test (two-tailed):

- $H_0: \beta = \beta_0$
- $H_1: \beta \neq \beta_0$
- Test size: $t = \frac{\hat{\beta} - \beta_0}{s_r \sqrt{1/\sum_{i=1}^{n}(x_i - \bar{x})^2}} \sim t(n-2)$
- Approximate p-value: $2 \cdot \left(1 - t_{cdf}(|t|, n-2)\right)$

95% confidence interval:

- $\beta_- = \hat{\beta} - t_0 \cdot s_r \sqrt{\frac{1}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$
- $\beta_+ = \hat{\beta} + t_0 \cdot s_r \sqrt{\frac{1}{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$

where `t0 = tinv(1-0.05/2,n-2).`

TEST CATALOG FOR THE INTERCEPT IN SIMPLE LINEAR REGRESSION

Statistical model:

- $y_i \sim N(\alpha + \beta x_i, \sigma^2)$ for $i = 1,2,\ldots,n$ are independent samples.
- Parameter estimates:

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \cdot \bar{x}$$

$$s_r^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left(y_i - (\hat{\alpha} + \hat{\beta}x_i)\right)^2$$

Hypothesis test (two-tailed):

- $H_0: \alpha = \alpha_0$
- $H_1: \alpha \neq \alpha_0$
- Test size: $t = \frac{\hat{\alpha}-\alpha_0}{s_r\sqrt{\frac{1}{n}+\frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}} \sim t(n-2)$
- Approximate p-value: $2 \cdot \left(1 - t_{cdf}(|t|, n-2)\right)$

95% confidence interval:

- $\alpha_- = \hat{\alpha} - t_0 \cdot s_r\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}$
- $\alpha_+ = \hat{\alpha} + t_0 \cdot s_r\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}$

where `t0 = tinv(1-0.05/2,n-2).`