#part3 .**Prove the statement.**

1. **(Theorem 3.2){Pn} is in X.**

**(d) If E ⊂ X and if p is a limit point of E, then there is a sequence {Pn} in E such that p = limn→∞Pn.**

**Proof:**

∃ε>0 : B(p,ε)∩E=∅,

where B(x,r)={y∈X∣d(x,y)<r} (here, we let d denote the metric of the metric space X).

Notice that your approach is weird (and unclear).

If p is a limit point of E, (I denote Ep:=E∖{p})

Then

∀ε>0,B(p,ε)∩Ep≠∅,

in other words,

∀n∈N∗, ∃xn∈Ep∩B(p,1n),

therefore (xn) is a sequence of E that converges to p.

**b) Suppose {sn}, {tn} are sequences in C, and limn→∞ sn = s, limn→∞ tn = t. Then**

**(a) limn→∞ (sn + tn) = s + t**

**Proof:**

Let Ꜫ > 0. Since limn→∞sn = s and limn→∞ tn = t,

there exists N1, N2 ∈ N such that if n ≥ N1, and if n ≥ N2 then |sn − s| < Ꜫ / 2 and |tn − s| < Ꜫ/ 2 , respectively.

This imples that if n ≥ max (N1, N2), then

|(sn + tn) − (s + t)| = |(sn + tn) − (s + t)|

= |sn − s| + |tn − t| < Ꜫ.

**c)(Theorem 3.6)**

**(b) Every bounded sequence in Rk contains a convergent subsequence.**

**Proof:**

If E is finite, then the sequence {pn} is placing infinitely many things into finitely many bins.

Suppose the range E of the sequence {pn} is E={p,q,r}. Since {pn} is infinite, at least one of p,q, and rr(say p) must be hit infinitely many times by the sequence.

Then we can take a subsequence pn1,pn2,…pn1,pn2,… which is just p,p,…p,p,….

Certainly this subsequence converges to p.

**d) 3.7 Theorem .**

**The subsequential limits of a sequence {pn} in a metric space X form a closed subset of X.**

**Proof:**

Let E be the set of all p ∈ X with the property that there exists a subsequence of {Pn} which converges to p. Let q ∈ E ‘ .

We need to show that q ∈ E. Let n1 = 1. Let k >=2 and suppose that we have chosen positive integers n1 < n2 < … < nk-1.

Since q ∈ E ‘ , there exists qk ∈ E such that d (qk, q) < 1/ k .

Since qk ∈ E, there exists a subsequence of {Pn} which converges to qk.

This implies there exists an integer nk > nk-1 such that d (pnk , qk) < 1/ k .Then d (pnk , q) <=d (pnk , qk) + d (qk, q) < 2/ k

(We have defined the increasing sequence of positive integers {nk}k=1∞ by induction.)

Since the subsequence {pnk}k=1∞ has the property that d (pnk , q) < 2/ k for k >= 2 and since limk=∞ 2/ k = 0, we conclude by the lemma that {pnk}k=1∞ converges to q. Hence q∈ E.