SCHOOL OF MATHEMATICAL SCIENCES

BACHELOR OF SCIENCE (HONS) IN ACTUARIAL STUDIES
BACHELOR OF SCIENCE (HONS) IN INDUSTRIAL STATISTICS

ACADEMIC SESSION: AUGUST 2020 SEMESTER

MAT3034 STOCHASTIC PROCESSES

ASSIGNMENT

DUE DATE: 11th NOVEMBER 2020, 4pm

INSTRUCTIONS TO CANDIDATES

1. There are **TWO (2)** pages in this Assignment excluding the cover page.
2. This assignment will contribute **30%** to your final grade.
3. This is an **individual assignment**.
4. The assignment must be EITHER typewritten with double line spacing on A4 size paper and single sided, OR handwritten. The assignment should be submitted with a cover sheet which includes your name, your ID number, subject name, subject code, and tutor's name.
5. Marks will be allocated for correctness and clarity of presentation.
6. You are required to submit this assignment in eLearn with the file name to be your student ID.

IMPORTANT

Assignments must be submitted on their due dates. If an assignment is submitted after its due date, the following penalty will be imposed:

- One to two days late : 20% deducted from the total marks awarded.
- Three to five days late : 40% deducted from the total marks awarded.
- More than five days late : Assignment will not be marked.

Question 1 (6 marks)

In a factory, there are in total two machines. In any particular day, at least one machine must be at work. Each of the machine will have the $\frac{1}{3}$ probability of breaking down. Suppose that, if a machine breaks down during day *n*, it will return to service at the beginning of day $n+3$. If at any time both machines are in the repair facility, the facility calls a special super repairman who gets them both working by the next morning. (For example, if on day *n* there is one machine already in the repair facility and the other machine breaks down during day *n*, then both machines will be working at the beginning of day $n+1$.) Define a state space (be specific in describing in the meaning of each state) and find the corresponding single-step transition matrix *P* for a Markov chain that describes this new system.

Question 2 (4 marks)

Use Excel to draw a Markov Chain trajectory for this four-state markov chain in 3500 time units and conclude whether this is a periodic Markov Chain. If yes, provide an Excel spreadsheet (on sheet 1) to show case on how to determine its period.

State	**1**	**2**	**3**	**4**
1	0	0.4	0.6	0
2	1	0	0	0
3	0	0	0	1
4	0	0.7	0.3	0

Question 3 (5 marks)

A non-symmetric two dimensional random walk is a random walk that moves at each time unit, one step to the right, left, up or down but with unequal probability. Simulate using Excel (on sheet 2) the trajectory (for 500 time units) of a non-symmetric random walk with the following probabilities:

Movement	Probability
Right	0.4
Left	0.1
Up	0.4
Down	0.1

The initial position of the random walk is at the origin, i.e. (0, 0).

(a) Plot the trajectory in a chart for the first 100 time units.

(4 marks)

(b) Comment on the trajectory drawn from part (a).

(1 mark)

Question 4 (10 marks)

John buys a new Perodua Bezza for 40 thousand ringgit and plans to keep it for T years. The lifetime X, of a Bezza is exponentially distributed with mean $\frac{1}{\lambda}$ years. If the Bezza fails before time T, it is scrapped and John must immediately buy a new car. If the Bezza survives to time T, then John can sell it for half the original price and buy a new car.

(a) Suppose R is the time to replace a new Bezza, show that the expected time (in years) to replace a Bezza is given by $E[R]=\frac{1}{\lambda}\left(1-e^{-\lambda T}\right)$.

(4 marks)

(b) Hence, show that cost per year C (in thousand ringgit), for this strategy is given by

$$C=20\lambda\left(1+\frac{1}{1-e^{-\lambda T}}\right).$$

(3 marks)

(c) Show that the optimal strategy is to never sell the car.

(1 mark)

(d) Give an intuitive explanation for the answer in part (c).

(2 marks)

Question 5 (5 marks)

(a) Define the Geometric Brownian motion.

(2 marks)

(b) Given that $\{X(t), t\geq 0\}$ is a Geometric Brownian motion process and the cost per unit option is c. Determine how the value of c can be found so that no sure-win is possible.

(3 marks)

--END OF PAPER--