CITY ENGINEERING COLLEGE
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**Mathematics Assignment - 02**
**Vector Calculus**
**&**
**Vector Space and Linear Transformation**

1. Find the directional derivative of ,
   a. $\varphi = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) along vector $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$.
   b. $\varphi = xy^3 - yz^3$ at the point (2, -1, 1) in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.
2. Show that the cylindrical coordinate system is orthogonal.
3. Prove that the spherical coordinate system is orthogonal.
4. Show that the two surfaces $xz + y + z^2 = 9$ and $z = 4 - 4xy$ at (1,-1,2) are orthogonal.
5. Verify whether the vector $\vec{F} = \frac{x\hat{\imath}+y\hat{\jmath}}{x^2+y^2}$ is both solenoidal and irrotational.
6. If $\vec{F} = \nabla( x^3 + y^3 + z^3 - 3xyz)$ , find $div\, \vec{F}$and $curl\, \vec{F}$.
7. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2).
8. Define Vector space, Subspace and Linear dependent.
9. Show that the vectors (1,0,1), (1,1,0), (-1,0,-1) are linearly dependent in $V_3(R)$.
10. State Rank - Nullity Theorem. For the matrix $\begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$ Find:
    a. Rank of A b. Dim (Nul A) c. Bases
11. Prove that the subset $W = \{(x, y, z): x - 3y + 4z = 0\}$ of the vector space $R^3$ is a subspace of $R^3$.
12. Determine whether the matrix $A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$ is a linear combination of $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ in the vector space $M_{22}$ of $2 \times 2$ matrices.
13. Express the vector (3,5,2) as a linear combination of the vectors (1,1,0) ,(2,3,0),(0,0,1) of $V_3(R)$.
14. Show that the set S={(1,2,4),(1,0,0),(0,1,0),(0,0,1)} is linearly dependent.
15. Find the linear transformation $T: V_2(R) \to V_3(R)$ such that $T(1,1) = (0,1,2), T(-1,1) = (2,1,0)$.
16. State the rank-nullity theorem and verify the theorem for the linear transformation $T: R^3 \to R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.
17. Define an inner product space. Consider $f(t) = 4t + 3, g(t) = t^2$, the inner product $< f, g > = \int_0^1 f(t)g(t)dt$. Find $< f, g >$ and $\|g\|$
18. Express the matrix $M = \begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}$ is a linear combination of matrices, $P = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $R = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$.
19. Determine whether or not each of the following $x_1 = (2,2,1)$ , $x_2 = (1,3,7)$ and $x_3 = (1,2,3)$ forms a basis in $R^3$.
20. Find the dimension and basis of the subspace spanned by the vectors (2,4,2), (1,-1,0) , (1,2,1) and (0,3,1) in $V_3(R)$.