第八章作业一

一、1~4：D C D A

二、

5. $2 \cdot \vec{b} \times \vec{a}$
6. 2
7. 20
8. $(-2, 3, 0)$

1. (1)

$$\overrightarrow{AB} = (3, 5, 8)$$
$$\overrightarrow{CD} = (2, -4, -7)$$
$$\vec{m} = (5, 1, -4)$$

$$\vec{a} = 4\begin{bmatrix}3\\5\\8\end{bmatrix} + 3\begin{bmatrix}2\\-4\\-7\end{bmatrix} - \begin{bmatrix}5\\1\\-4\end{bmatrix} = \begin{bmatrix}13\\7\\7\end{bmatrix}.$$

$$\mathrm{Prj}_{\vec{\mathrm{i}}} = 13,\ \mathrm{Prj}_{\vec{\mathrm{j}}} = 7,\ \mathrm{Prj}_{\vec{\mathrm{k}}} = 7.$$

分向量分别为

$$13\vec{i},\ 7\vec{j},\ 7\vec{k}.$$

(2)

$$|\vec{a}| = \sqrt{13^2 + 7^2 + 7^2} = \sqrt{169 + 49 + 49} = \sqrt{267}$$

(3)

$$\begin{bmatrix}\cos\alpha\\\cos\beta\\\cos\gamma\end{bmatrix} = \begin{bmatrix}13/\sqrt{267}\\7/\sqrt{267}\\7/\sqrt{267}\end{bmatrix}$$

(4)

$$\begin{bmatrix}13/\sqrt{267}\\7/\sqrt{267}\\7/\sqrt{267}\end{bmatrix} \text{和} \begin{bmatrix}-13/\sqrt{267}\\-7/\sqrt{267}\\-7/\sqrt{267}\end{bmatrix}.$$

(5)

$$\overline{AC} = \sqrt{(1-0)^2 + (3-0)^2 + (2-1)^2} = \sqrt{11}$$

2.

$$\vec{a} \times \vec{b} = (-7, -5, -1)$$
$$\vec{v} = k(-7, -5, -1)$$

$$\frac{\vec{v}\cdot\vec{c}}{|\vec{c}|}=1$$

$$|\vec{c}|=\sqrt{2^2+1^2+2^2}=3$$
$$k(-14-5-2)=3$$
$$k=-\frac{1}{7}$$

$$\vec{v}=(1,\frac{5}{7},\frac{1}{7}).$$

3.

在 $\vec{a},\vec{b}$所在平面上，以$\vec{a}$为x轴建立直角坐标系。

$$\vec{a}=(4,0),\ \vec{b}=(\frac{3\sqrt{3}}{2},\frac{3}{2})$$

$$\vec{\alpha}=\vec{a}+2\vec{b}=(4+3\sqrt{3},3)$$

$$\vec{\beta}=\vec{a}-3\vec{b}=(4-\frac{9\sqrt{3}}{2},-\frac{9}{2})$$

$$S=|\alpha\times\vec{\beta}|=|-\frac{9}{2}\cdot(4+3\sqrt{3})-3\cdot(4-\frac{9\sqrt{3}}{2})|$$

$$=|-18-\frac{27\sqrt{3}}{2}-12+\frac{27\sqrt{3}}{2}|$$

$$=30.$$

4.

$$|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin(\widehat{\vec{a},\vec{b}})$$

$$\cos(\widehat{\vec{a},\vec{b}})=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\sin(\widehat{\vec{a},\vec{b}})=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

$$|\vec{a}\times\vec{b}|=2\cdot\sqrt{2}\cdot\frac{1}{\sqrt{2}}=2$$

5.

$$[(\vec{a}+\vec{b})\ \ (\vec{b}+\vec{c})\ \ (\vec{c}+\vec{a})]$$

$$(\vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c})\cdot(\vec{c}+\vec{a})$$

$$[\vec{a}\ \ \vec{b}\ \ \vec{c}]+[\vec{b}\ \ \vec{c}\ \ \vec{a}]$$

$$2[\vec{a}\ \ \vec{b}\ \ \vec{c}].$$

6.

$$\vec{\beta}\cdot\vec{\gamma}=(\lambda\vec{a}+17\vec{b})\cdot(3\vec{a}-\vec{b})$$

$$=3\lambda\vec{a}^2+(51-\lambda)\vec{a}\cdot\vec{b}-17\vec{b}^2$$

$$=12\lambda+(51-\lambda)(10\cos\frac{2\pi}{3})-425$$

$$=12\lambda-255+5\lambda-425$$

$$=17\lambda-680=0$$

$$\lambda=40.$$

7.

设$\vec{v}=(\sin\theta,\cos\theta,0)$.

$$-4\sin\theta+3\cos\theta=0$$

$$\tan\theta=\frac{3}{4}$$

$$\begin{cases}\sin\theta=\frac{3}{5}\\ \cos\theta=\frac{4}{5}\end{cases}or\begin{cases}\sin\theta=-\frac{3}{5}\\ \cos\theta=-\frac{4}{5}\end{cases}$$

$$\vec{v}=(\frac{3}{5},\frac{4}{5},0)或\vec{v}=(-\frac{3}{5},-\frac{4}{5},0).$$

8.

$$-3\vec{a}\cdot\vec{b}=(-3,6,-12)\cdot(1,5/3,-2/3)=-3+10+8=15.$$

$$\vec{a}\times3\vec{b}=(1,-2,4)\times(3,5,-2)=(-16,14,11)$$