Støddfrøðiligt grundarlag til tilgjørt vit - 2025

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Contents

1 Linear Algebra

1.1 Matrix

A matrix is usually written as $\mathbf{A} \in \mathbb{R}^{m \times n}$ where m means the amount of rows and n is the amount of columns. An example of a matrix:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad a_{ij} \in \mathbb{R}$$

1.2 Matrix addition

Property	**Description**
Associative	$(A+B)+C = A+(B+C)$
Commutative	$A+B = B+A$
Identity	$A+O = O+A = A$, where O is the zero matrix
Inverse	$A+(-A) = O$, where $-A$ is the additive inverse

Example

1.3 Matrix multiplication

Property	**Description**
Associative	$(AB)C = A(BC)$
Distributive over Addition	$A(B+C) = AB+AC$ $(A+B)C = AC+BC$
Identity	$AI = IA = A$, where I is the identity matrix
Non-commutative	$AB \neq BA$ (in general)

Example

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{2\times 3},$$

$$B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2},$$

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix} \in \mathbb{R}^{2\times 2},$$

$$BA = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 2 \\ -2 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}.$$

1.4 Inverse and determinant

A matrix only has a inverse if its determinant is not 0 so always calculate it before finding the inverse. If it exists the following holds:

$$\mathbf{AB} = \mathbf{BA} = \mathbf{I}_n$$

The determinant is easy to find for a 2×2 matrix:

$$Det(\mathbf{A}) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{21}a12$$

After checking that the determinant is not zero proceed to find the inverse. If you're working with a 2×2 matrix the following setup gives the inverse, fig. **??**
For a 3×3 matrix Sarrus's rule can be used.
For matrices 3×3 and larger one can use Laplace expansion.

1.5 Transposed matrix

To find the transpose

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

then

$$\mathbf{A}^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

1.6 Particular and general solution

1.7 Gauss elimination

When doing gaussian elimination you are allowed to do the following operations:

- Exchange of two equations (rows in the matrix representing the system of equations)
- Multiplication of an equation (row) with a constant $\lambda \in \mathbb{R}\{0\}$
- Addition of two equations (rows)

Definition 2.6 (Row-Echelon Form). A matrix is in row-echelon form if:

- All rows that contain only zeros are at the bottom of the matrix; correspondingly, all rows that contain at least one nonzero element are on top of rows that contain only zeros.
- Looking at nonzero rows only, the first nonzero number from the left (also called the pivot or the leading coefficient) is always strictly to the right of the pivot of the row above it.

Remark (Reduced Row Echelon Form). An equation system is in *reduced row-echelon form* (also: *row-reduced echelon form* or *row canonical form*) if

- It is in row-echelon form.
- Every pivot is 1.
- The pivot is the only nonzero entry in its column.

Minus-1 trick

1.8 Groups

Definition 2.7 (Group). Consider a set G and an operation $\otimes : G \times G \to G$ defined on G. Then $G := (G, \otimes)$ is called a *group* if the following hold:

1. **Closure** of G under $\otimes$: $\forall x, y \in G : x \otimes y \in G$
2. **Associativity**: $\forall x, y, z \in G : (x \otimes y) \otimes z = x \otimes (y \otimes z)$
3. **Neutral element**: $\exists e \in G \, \forall x \in G : x \otimes e = x$ and $e \otimes x = x$
4. **Inverse element**: $\forall x \in G \, \exists y \in G : x \otimes y = e$ and $y \otimes x = e$, where e is the neutral element. We often write x^{-1} to denote the inverse element of x.

Remark The inverse element is defined with respect to the operation $\otimes$ and does not necessarily mean $\frac{1}{x}$. ♢

1.9 Vector spaces

Definition 1.1 (Vector Space)**.** A real-valued *vector space* $V = (\mathcal{V}, +, \cdot)$ is a set $\mathcal{V}$ with two operations

$$+ : \mathcal{V} \times \mathcal{V} \to \mathcal{V} \tag{2.62}$$

$$\cdot : \mathbb{R} \times \mathcal{V} \to \mathcal{V} \tag{2.63}$$

where

1. $(\mathcal{V}, +)$ is an Abelian group.
2. Distributivity:
   (a) $\forall \lambda \in \mathbb{R},\ x, y \in \mathcal{V} : \lambda \cdot (x + y) = \lambda \cdot x + \lambda \cdot y$.
   (b) $\forall \lambda, \psi \in \mathbb{R},\ x \in \mathcal{V} : (\lambda + \psi) \cdot x = \lambda \cdot x + \psi \cdot x$.
3. Associativity (outer operation): $\forall \lambda, \psi \in \mathbb{R},\ x \in \mathcal{V} : \lambda \cdot (\psi \cdot x) = (\lambda\psi) \cdot x$.
4. Neutral element with respect to the outer operation: $\forall x \in \mathcal{V} :\ 1 \cdot x = x$.

1.10 Basis and rank

1.11 Linear mappings

1.12 Affine spaces