5730.26 Tráðleyst samskifti V26
Notes

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1 Lecture 1

1.1 Signal and Noise

1.1.1 Material Reference

The material in this lecture is primarily based on:

TSE Book

- Chapter 1 (Sections 1.1 and 1.2): Overview of wireless communication
- Appendix A.1: Gaussian random variables
- Appendix A.2: Signal detection in Gaussian noise

Haykin

- Chapter 2.2 (pp. 8–14)
- Chapter 2.8 (pp. 47–55)

This lecture establishes the physical and mathematical foundations required for analyzing wireless communication systems.

1.2 Fundamental Quantities and Signal Representation

Wireless communication systems operate by transmitting electrical signals that carry information. Two fundamental physical quantities describe these signals:

Voltage is the electrical potential difference (measured in volts), while **Power** is the rate at which energy is transferred (measured in watts).

In a resistive system with resistance R, instantaneous power is

$$P(t) = \frac{v^2(t)}{R}.$$

Because power depends on the square of voltage, small voltage variations can produce large power differences. This quadratic relationship is one reason why wireless systems are analyzed primarily in terms of power rather than voltage.

Wireless signals operate at high frequencies, ranging from kilohertz to gigahertz. Higher frequencies allow:

- Higher data rates
- Smaller antenna sizes
- Greater available bandwidth

However, high frequencies also suffer from:

- Increased path loss
- Stronger attenuation

- Greater susceptibility to noise

Therefore, the **signal-to-noise ratio (SNR)** becomes a central performance metric:

$$\mathrm{SNR} = \frac{P_s}{P_n}.$$

1.3 Noise and the AWGN Model

In any realistic wireless system, the received signal is corrupted by noise. The most important noise source is thermal noise, generated by random electron motion in conductors.

Thermal noise is accurately modeled as **Additive White Gaussian Noise (AWGN)**.

It is:

- **Additive** – it adds to the signal
- **White** – constant power spectral density
- **Gaussian** – normally distributed amplitude

The received signal model becomes:

$$y(t) = x(t) + n(t).$$

This simple model underpins almost all digital communication theory.

1.4 Power in dBm and Logarithmic Representation

Because wireless power levels span many orders of magnitude, logarithmic units are used.

Absolute power in dBm:

$$P_{\mathrm{dBm}} = 10 \log_{10} \left(\frac{P}{1\,\mathrm{mW}} \right).$$

Power ratios are expressed in dB:

$$G_{\mathrm{dB}} = 10 \log_{10} \left(\frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} \right).$$

Important distinction:

- dB $\rightarrow$ ratio
- dBm $\rightarrow$ absolute power

Logarithmic representation simplifies link budget calculations because gains and losses can be added directly.

1.5 Logarithmic Identities

Wireless analysis frequently requires converting between linear and logarithmic domains.

$$\log_{10}(ab) = \log_{10}(a) + \log_{10}(b)$$

$$\log_{10}\left(\frac{a}{b}\right) = \log_{10}(a) - \log_{10}(b)$$

$$\log_{10}(a^n) = n \log_{10}(a)$$

For information theory:

$$\log_2(ab) = \log_2(a) + \log_2(b)$$

Base-2 logarithms naturally arise when measuring information in bits.

1.6 Signal Composition and Spectral Interpretation

Signals may consist of multiple sinusoidal components. For example:

$$x(t) = 2\sin(t) + \sin(5t).$$

This represents a superposition of two frequency components. According to Fourier theory, any physically realizable signal can be decomposed into sinusoids.

This decomposition allows:

- Frequency-domain analysis
- Filter design
- Bandwidth estimation

1.7 Filters and Frequency Selectivity

Filters shape signals in the frequency domain.

A low-pass filter allows frequencies below a cutoff frequency to pass.

A high-pass filter allows frequencies above the cutoff frequency.

Band-pass and notch filters similarly allow or suppress selected frequency regions.

1.8 Link Budget Analysis

A link budget accounts for all gains and losses between transmitter and receiver:

$$P_{\text{rx}} = P_{\text{tx}} + G_{\text{tx}} + G_{\text{rx}} - L_{\text{path}} - L_{\text{other}}.$$

All terms must be expressed consistently in dB or dBm.

This equation allows prediction of whether a communication link will close.

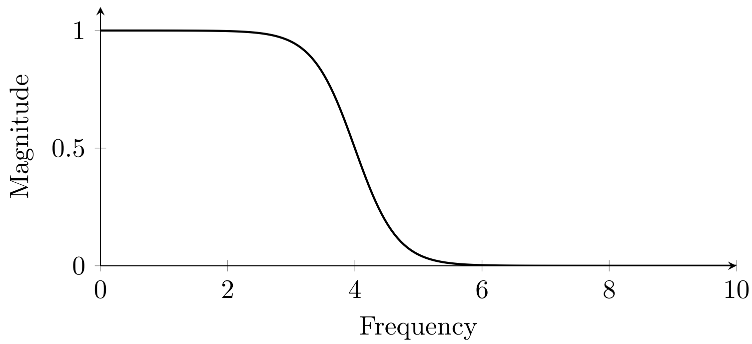


Figure 1: Low-pass filter magnitude response

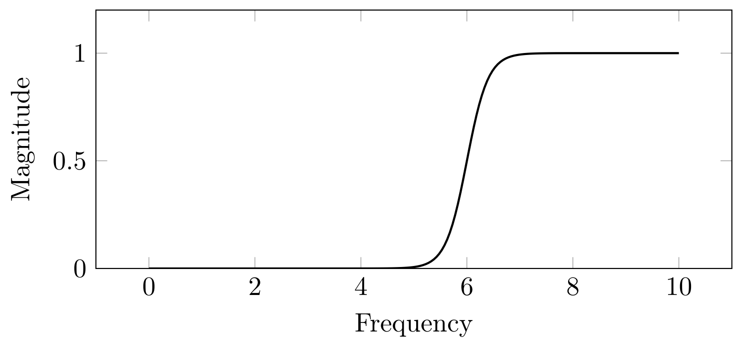


Figure 2: High-pass filter magnitude response

1.9 Thermal Noise, Temperature, and Bandwidth

Thermal noise power in linear units:

$$P_n = k_B T B$$

where:

- k_B = Boltzmann constant
- T = absolute temperature
- B = bandwidth

At room temperature:

$$N_0 \approx -174 \text{ dBm/Hz}.$$

Total noise power:

$$P_n = -174 + 10 \log_{10}(B).$$

Increasing bandwidth increases noise power linearly.

1.10 Shannon Capacity and Fundamental Limits

The maximum achievable data rate is:

$$C = B \log_2(1 + \gamma),$$

where

$$\gamma = \frac{P_s}{P_n}.$$

Key insight:
Capacity increases logarithmically with SNR. Doubling power does not double capacity.

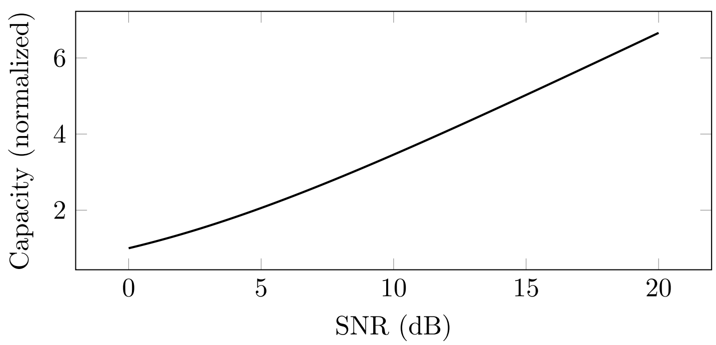


Figure 3: Logarithmic growth of capacity with SNR

1.11 Energy per Bit and BER

Digital systems are commonly analyzed using:

$$\frac{E_b}{N_0}$$

where:

- E_b = energy per bit
- N_0 = noise spectral density

The **Bit Error Rate (BER)** is the probability that a transmitted bit is incorrectly detected.
Higher E_b/N_0 reduces BER.
This metric allows comparison of modulation schemes independent of bandwidth.

2 Lecture 2

2.1 Modulation

In wireless communication, information cannot be transmitted efficiently at baseband over long distances. Antennas are physically inefficient at very low frequencies, and multiple users require frequency separation to share spectrum.

To enable radiation and spectrum allocation, a baseband message signal $m(t)$ is shifted to a higher carrier frequency f_c. This process is called modulation.

The simplest modulation is:

$$s(t) = m(t)\cos(2\pi f_c t).$$

Here, f_c determines where the signal appears in the spectrum. Modulation enables efficient transmission and multiplexing.

2.2 Baseband and Bandpass Signals

A baseband signal has spectrum centered at 0 Hz. Its bandwidth extends from $-B$ to B.

After modulation, the spectrum is shifted to $\pm f_c$, forming a bandpass signal.

Bandwidth is defined as:

$$B = f_H - f_L.$$

Spectral efficiency measures how efficiently bandwidth is used:

$$\eta = \frac{R_b}{B}.$$

2.3 Amplitude Modulation (AM)

In AM, the carrier amplitude varies with the message:

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t).$$

The modulation index is:

$$\mu = k_a \max |m(t)|.$$

If $\mu > 1$, overmodulation occurs and distortion appears.

AM is simple but inefficient in power because energy is transmitted in both the carrier and sidebands.

2.4 Double Sideband Suppressed Carrier (DSB-SC)

Removing the carrier produces:

$$s(t) = m(t)\cos(2\pi f_c t).$$

The bandwidth is:

$$B_{DSB} = 2B.$$

This improves power efficiency but still occupies twice the baseband bandwidth.

2.5 Angle Modulation: FM and PM

Frequency modulation (FM):

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)dt\right).$$

Peak deviation:

$$\Delta f = k_f A_m.$$

Carson's rule:

$$B_{FM} = 2(\Delta f + B).$$

Phase modulation (PM):

$$s(t) = A_c \cos(2\pi f_c t + k_p m(t)).$$

FM and PM are collectively called angle modulation and offer better noise immunity than AM.

2.6 Complex Baseband Representation

Bandpass signals can be expressed using complex notation:

$$s(t) = I(t) + jQ(t) = Ae^{j\theta}.$$

Symbols are transmitted over finite time intervals:

$$R_s = \frac{1}{T_s}.$$

Bit rate:

$$R_b = R_s \log_2 M.$$

Increasing M increases data rate but reduces noise tolerance.

2.7 Electromagnetic Wave Propagation

In free space, the electric field of a propagating wave is:

$$E(f,t,r) = \frac{E_0}{r} e^{-j(2\pi f t - kr)}, \quad k = \frac{2\pi}{\lambda}.$$

Amplitude decays as $\frac{1}{r}$ and power decays as $\frac{1}{r^2}$.
Free-space received power:

$$P_r = P_t \left(\frac{\lambda}{4\pi d}\right)^2.$$

Since $\lambda = \frac{c}{f_c}$, higher carrier frequencies suffer greater path loss.

More generally:

$$PL(d) = PL(d_0) + 10n \log_{10}\left(\frac{d}{d_0}\right).$$

2.8 Multipath Propagation

In realistic environments, signals reach the receiver via multiple paths due to reflection, diffraction, and scattering.

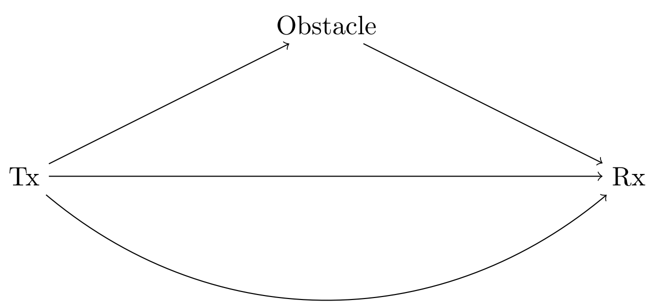
Each path has:

$$\text{Amplitude } \alpha_i, \quad \text{Delay } \tau_i, \quad \text{Phase } \phi_i.$$

Phase depends on distance:

$$\phi = \frac{2\pi d}{\lambda}.$$

Even small path differences create large phase shifts, producing constructive and destructive interference.



This causes fading and inter-symbol interference (ISI).

2.9 Time Variation and Doppler

Wireless channels vary over time due to motion and environmental changes.

Slow fading results from distance and shadowing.

Fast fading results from multipath interference.

Motion causes Doppler shift:

$$f_D = \frac{v}{\lambda}\cos(\theta).$$

Doppler introduces frequency spreading and limits coherence time.

2.10 Linear Time-Varying Channel Model

The wireless channel is modeled as:

$$y(t) = \int h(t,\tau)x(t-\tau)d\tau + n(t).$$

The impulse response is:

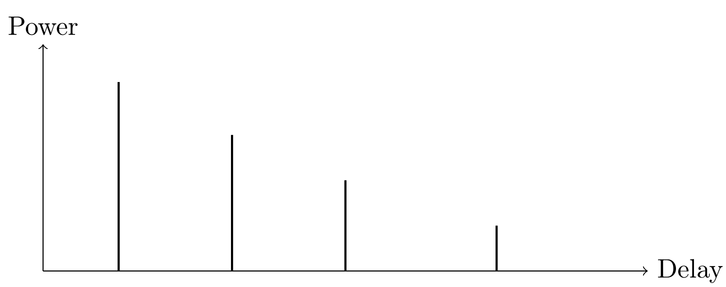
$$h(t,\tau) = \sum_{i=1}^{N} \alpha_i(t)\delta(\tau - \tau_i).$$

If only a Line-of-Sight (LOS) path exists:

$$h(\tau) = \alpha_0\delta(\tau - \tau_0), \quad \tau_0 = \frac{D}{c}.$$

2.11 Power Delay Profile (PDP)

The Power Delay Profile describes received power versus delay.



Delay spread T_d determines coherence bandwidth:

$$W_c \approx \frac{1}{T_d}.$$

If:

$$B \ll W_c \Rightarrow \text{Flat fading.}$$

If:

$$B > W_c \Rightarrow \text{Frequency-selective fading.}$$

2.12 Four-Domain Channel Representation

The wireless channel can be represented in four equivalent domains:

Time–Delay:

$$h(t,\tau)$$

Time–Frequency:

$$H(f,t)$$

Delay–Doppler:

$$S(\tau,\nu)$$

Frequency–Doppler:

$$H(f,\nu)$$

$$\begin{tikzcd}
h(t,\tau) \arrow[rr, leftrightarrow, "\text{Fourier in } \tau"] \arrow[d, leftrightarrow, "\text{Fourier in } t"'] & & H(f,t) \arrow[d, leftrightarrow, "\text{Fourier in } t"] \\
S(\tau,\nu) \arrow[rr, leftrightarrow, "\text{Fourier in } \tau"'] & & H(f,\nu)
\end{tikzcd}$$

Each representation emphasizes different physical aspects: delay spread, Doppler spread, frequency selectivity, and time variation.

2.13 Noise and SNR

Wireless systems are affected by Additive White Gaussian Noise (AWGN):

$$y(t) = x(t) + n(t).$$

Noise power over bandwidth B:

$$P_n = N_0 B.$$

Signal-to-noise ratio:

$$\text{SNR} = \frac{P_s}{P_n}.$$

2.14 Multiple Access and Duplexing

Spectrum is shared using:

FDMA (frequency separation), TDMA (time separation), CDMA (code separation), OFDM/OFDMA (subcarrier allocation).

Communication modes include simplex, half-duplex, and full-duplex.