Stochastic debt sustainability analysis using time-varying fiscal reaction functions

An agnostic approach to fiscal forecasting

ONLINE APPENDIX

Tore Dubbert [1, 2]

tore.dubbert@wiwi.uni-muenster.de

[1]University of Göttingen, Germany

[2]University of Münster, Germany

January 19, 2023

A Data

This section provides details about the data used in this paper. Table A.1 summarizes the information for the data used for the FRF. Note that for the fiscal reaction function, for the primary balance-to-GDP ratio and the debt-to-GDP ratio, AMECO data are used as long as available. For those countries where fiscal AMECO data are not dating back all the way to the beginning of the sample - that is, to 1970 - the respective series is complemented using data from Mauro et al. (2015), using retropolation as in Berger et al. (2021). Thereby, a higher number of observations along the time dimension is obtained, with the panel dataset being balanced, ensuring that at any point in time, the degree of time variation in the time-varying parameters is driven by all countries jointly and not only by a subgroup of them. Implicit interest rates and stock-flow adjustments, obtained from AMECO as well, are used in the debt accumulation equation, as elaborated upon below.

For the VAR part, quarterly data are employed to capture correlations between the variables of interest that are more frequent than the yearly frequency for AMECO data. The variables used in the VAR and its sources are summarized in table A.2. Further note that the data limitations faced in the VAR part differ between countries. For each VAR, the longest sample available is used. Country-specific data availabilities are summarized in table A.3.

Handling of the vintages and data issues

To assess the pseudo-real time forecasting performance of primary balance and public debt projections, ten vintages with yearly data are used. The choice of vintages is motivated by reasons of consistency: The first vintage used is the AMECO dataset from autumn 2014, being the first dataset based on the European system of accounts (ESA) 2010. Using vintages before that would be problematic especially with respect to the output gap variable, as the change in accounting standards implied major revisions in the series. Thus, all vintages based on ESA 2010 standards are used for the forecasting performance evaluation to ensure a high degree of consistency between datasets. As "true values", primary balance and public debt ratios using the latest available vintage, i. e. the spring 2022 vintage, are used. This implies that the "true values" for the periods for which the latest forecasts are made (that is, 2019-2020 in the spring 2019 vintage) have all been subject to at least two revisions.

In order to realistically assess the forecasting performance of the SDSA - to avoid hindsight bias - at the moment of forecasting, only the data already available to the forecaster can be used. This implies two things:

Table A.1: Data description for the fiscal reaction function and the debt accumulation equation

Series name	Sources	Transformation
Primary balance	Mauro et al. (2015), AMECO's "Net lending (+) or net borrowing (-) excluding interest: general government :- Excessive deficit procedure"	Percentages of GDP
Public debt	Mauro et al. (2015), AMECO's "General government consolidated gross debt :- Excessive deficit procedure (based on ESA 2010) and former definitions (linked series)"	Percentages of GDP
Output gap	AMECO's "Gap between actual and potential gross domestic product at 2010 reference levels"	Percentages of potential GDP
Implicit interest rate	AMECO's "Implicit interest rate: general government :- Interest as percent of gross public debt of preceding year Excessive deficit procedure (based on ESA 2010)"	Percentage of gross public debt
Stock-flow adjustments	AMECO's "Stock-flow adjustment on general government consolidated gross debt :- Excessive deficit procedure (based on ESA 2010) "	Percentages of GDP

Table A.2: Data description for the vector autoregression

Series name	Sources	Transformation
Real Gross Domestic Product (GDP)	OECD Economic Outlook database series "Gross domestic product, nominal value, market prices", deflated by "Gross domestic product, market prices, deflator"	Δln
Real interest rate	Unweighted average of the OECD Economic Outlook database series "Long-term interest rate on government bonds" and "short-term interest rate", adjusted for year-on-year inflation using the GDP deflator	-
Inflation	OECD Economic Outlook database series "Gross domestic product, market prices, deflator"	-

Table A.3: Data availability in the VAR

Country	Earliest data availability
Austria	1970 Q1
Belgium	1960 Q1
Finland	1970 Q1
France	1970 Q1
Germany	1991 Q1
Greece	1995 Q1
Ireland	1990 Q1
Italy	1971 Q1
Japan	1969 Q1
Netherlands	1960 Q1

1. For the data taken from AMECO, at each point in time, the respective vintage publishing the EC forecasts is used.

2. For the OECD data (used for the VAR), the latest vintage available at the moment the EC vintage is published, is used.

There is one restriction to the second rule: While the AMECO vintages are always published in May (spring release) and November (autumn release) of the respective year, the OECD vintage publication date varies slightly from year to year for the respective releases. For example, most of the time, when the AMECO spring vintage is released, the OECD vintage containing information up to the *first quarter* of the respective year is available. However, in some cases, the OECD release occurs after the AMECO release date. If that is the case, technically, information (for one or two quarterly observations) is used that would not be available to the forecaster the moment the forecast is made, implying a slight information advantage for the forecasts made here. However, this advantage is small and is still a major improvement over systematically using ex-post data such as the latest data available. Given that very few observations in the sample are concerned, this circumstance is ignored for simplicity.

There is another data-related issue concerning the OECD vintages: In the "autumn" 2015 OECD vintage, both the nominal GDP series and the GDP deflator series are missing for Belgium, while in the "autumn" 2018 vintage, long-term and short-term interest rates, nominal GDP and the GDP deflator series are missing for Greece. This is dealt with in the following way: Where VAR data are missing, data from the previous vintage are included. This implies that instead of actual observations, for the last two quarterly sample observations (only), forecasts are used instead. Again, only few observations are affected.

B Stochastic debt sustainability analysis algorithm

This section lays out the complete stochastic debt simulation analysis employed in the paper. The procedure will be outlined in three subsections, dealing with the empirical FRF, the BVAR and the fiscal projection algorithm in turn. Finally, a fourth subsection is included, where a time-invariant coefficient SDSA pendant in spirit of Medeiros (2012) is presented.

B.1 Fiscal reaction function

In this section, the Gibbs sampling algorithm, used to estimate the coefficients of the time-varying panel FRF, is laid out. The full model consists of the equations (3), (4), (5) and (6) of the main paper, restated here for convenience (with slight notational differences, as elaborated upon below):

$$pb_{it} = H_{it}\beta_t + X_{it}\gamma + \epsilon_{it}, \tag{B.1}$$

$$\epsilon_{it} = \mu_t + \rho\epsilon_{i,t-1} + u_{it} \qquad u_{it} \sim N(0, \sigma^2_{u_i}), \tag{B.2}$$

$$\beta_t = \beta_0 + \sigma_\eta \tilde{\beta}_t, \tag{B.3}$$

$$\tilde{\beta}_t = \tilde{\beta}_{t-1} + \tilde{\eta}_t, \qquad \tilde{\beta}_0 = 0, \qquad \tilde{\eta}_t \sim N(0,1), \tag{B.4}$$

where $i = 1, 2, ..., N$, $t = 1, 2, ..., T$, pb_{it} is the primary balance, H_{it} is the matrix of predictors corresponding to the $m \times 1$ vector of time-varying parameters, β_t, X_{it} is the predictor matrix corresponding to the coefficients that are assumed to be fixed (γ). Note that *all variables are within-group demeaned*. For simplicity and for reasons of parsimony, the demeaning as well as the auxilliary regression to account for the endogeneity of the output gap, elaborated upon above, are conducted prior to the Markov-Chain-Monte-Carlo algorithm presented here. Following, among others, (Ghosh et al., 2013), some persistence (autocorrelation of order 1) is accounted for in the regression (measurement) error (ϵ_{it}). Additionally, time-varying unobserved components (time fixed effects) are accounted for by including μ_t.

(B.3) and (B.4) constitute a non-centered parameterization (NCP) of the time-varying parameters (see e. g. Frühwirth-Schnatter and Wagner, 2010). While a simple random walk parameterization of the time-varying parameters would “force” the parameters into a time-varying direction for any state error disturbance with variance greater zero (see e. g. Berger et al., 2021), this parameterization has the advantage that it is quite agnostic as to whether time variation is present in the data. This is the case since σ_η is assumed to be normally distributed in the NCP, with an assumed prior mean equal to zero. Thus, if the data informs β_t to be constant for $t = 1, 2, ...T$, the β_t based on the NCP will not wander off significantly from β_0.

In what follows, details on the MCMC algorithm to jointly sample the time-varying parameter vectors in β, the hyperparameters β_0, σ_η, γ, μ, ρ and σ_u^2 are provided. This section draws from Berger et al. (2021).

B.1.1 Sampling the parameters β_0, σ_η and γ

In this block, the regression parameters β_0, σ_η and γ are sampled conditionally on the time-varying parameters (β_t), the AR(1) coefficient of the autocorrelated error terms (ρ), the time fixed effects (μ_t) and the country-specific regression error variances, collected in σ_u^2. For notational convenience, define a general regression model

$$y = \chi\theta + e, \quad e \sim N(0, \Sigma), \tag{B.5}$$

where y is the dependent variable vector and χ is a predictor matrix corresponding to the parameter vector $\theta \equiv (\beta_0', \sigma_\eta', \gamma')'$. For both y and χ, observations are stacked over cross-sectional and time units, that is, over $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, with i being the slower index. The covariance matrix of the error term e is a diagonal matrix given by $\Sigma = diag\,(\sigma_u^2 \otimes \iota_T)$, where σ_u^2 is the $N \times 1$ vector of country-specific variances ($\sigma_u^2 \equiv (\sigma_{u,1}^2, \sigma_{u,2}^2, ...\sigma_{u,N}^2)'$) and ι_T is a $T \times 1$ vector of ones.

A Normal prior with $\theta \sim N(a_0, A_0)$ is assumed, where a_0 is the vector of prior means of the respective parameters and A_0 is the prior variance-covariance matrix. As this prior is conjugate, it implies a normally distributed posterior, that is, $p(\theta|\tilde{\beta}, \mu, \rho, \sigma_u^2, y, \chi) \sim N(a_T, A_T)$, where

$$a_T = A_T \left(\chi'\Sigma^{-1}y + A_0^{-1}a_0\right), \tag{B.6}$$

$$A_T = \left(\chi'\Sigma^{-1}\chi + A_0^{-1}\right)^{-1}. \tag{B.7}$$

The above can then be applied to the state-space model in equations (B.1)-(B.4): First, transform the measurement equation such that its error terms are white noise. That is, insert (B.2) into (B.1) and rewrite to obtain:

$$pb_{it}^* = H_{it}^*\beta_t + X_{it}^*\gamma + u_{it}, \tag{B.8}$$

where $pb_{it}^* = pb_{it} - \mu_t - \rho pb_{i,t-1}$ and analogously for H_{it}^* and X_{it}^*. Note that the errors in the transformed model, $u_{it} = \epsilon_{it} - \mu_t - \rho\epsilon_{i,t-1}$ are normally distributed. Next, inserting (B.3) into (B.8) yields

$$pb_{it}^* = H_{it}^*\beta_0 + H_{it}^*\sigma_\eta\tilde{\beta}_t + X_{it}^*\gamma + u_{it}, \quad u_{it} \sim N(0, \sigma_{u_i}^2), \tag{B.9}$$

which can be written as

$$\underbrace{pb_{it}^*}_{y_{it}} = \underbrace{\begin{bmatrix} H_{i,t}^* & H_{i,t}^*\tilde{\beta}_t & X_{it}^* \end{bmatrix}}_{\chi_{it}} \underbrace{\begin{bmatrix} \beta_0 \\ \sigma_\eta \\ \gamma \end{bmatrix}}_{\theta} + u_{it}. \tag{B.10}$$

θ can then be sampled from $p(\theta|\tilde{\beta}, \mu, \rho, \sigma_u^2, y, \chi) \sim N(a_T, A_T)$, where the posterior moments are given by (B.6) and (B.7).

B.1.2 Sampling the time-varying parameters

In this block, the forward-filtering backward-sampling procedure of Carter and Kohn (1994) is employed to sample the time-varying component $\tilde{\beta}$ given θ, μ, ρ and σ_u^2. The conditional linear Gaussian state-space model is given by

$$y_t = H_t s_t + e_t, \qquad\qquad e_t \sim MN(0_N, R), \tag{B.11}$$

$$s_t = F s_{t-1} + K_t v_t, \qquad s_0 \sim N(b_0, V_0), \qquad v_t \sim N(0, Q), \tag{B.12}$$

where y_t is an N x 1 vector of observations and H_t is the predictor matrix, with s_t being the corresponding time-varying parameter vector. The matrices χ, F, K, R, Q as well as the expected value and variance of the initial state s_0, that is, b_0 and P_0, are assumed to be known (conditioned upon). The disturbances e_t and v_t are assumed to be serially uncorrelated and independent of each other for $t = 1, 2, ..., T$. For details on the linear Gaussian state-space model, see Durbin and Koopman (2012).

The Kalman filter can then be employed on this linear Gaussian state-space model to filter the unknown state s_t (forward-filtering). s_t can then be sampled from its conditional distribution (backward-sampling), as described in Carter and Kohn (1994).

Rearrange terms in equation (B.9) to obtain, together with the state equation (B.4), the conditional state-space model for $\tilde{\beta}_t$:

$$\overbrace{pb^*_{it} - H^*_{it}\beta_0 - X^*_{it}\gamma}^{y_{it}} = \overbrace{H^*_{it}\sigma_\eta}^{H_t} \overbrace{\tilde{\beta}_t}^{s_t} + \overbrace{u_{it}}^{e_{it}}, \qquad u_{it} \sim N(0, \overbrace{diag(\sigma^2_u)}^{R}), \tag{B.13}$$

$$\underbrace{\tilde{\beta}_t}_{s_t} = \underbrace{I_m}_{F} \underbrace{\tilde{\beta}_{t-1}}_{s_{t-1}} + \underbrace{I_m}_{K_t} \underbrace{\tilde{\eta}_t}_{v_t}, \qquad \tilde{\eta}_t \sim N(0, \underbrace{I_m}_{Q}), \tag{B.14}$$

where I_m is the identity matrix of dimension m, m being the number of time-varying parameters in the model. Note that the $1 \times m$ vector of states, s_t, is assumed to be homogeneous across countries for each $j = 1, ..., m$. Stacking observations over $i = 1, 2, ..., N$, this can be written as

$$\overbrace{\begin{bmatrix} pb^*_{1t} - H^*_{1t}\beta_0 - X^*_{1t}\gamma \\ \vdots \\ pb^*_{Nt} - H^*_{Nt}\beta_0 - X^*_{Nt}\gamma \end{bmatrix}}^{y_t} = \overbrace{\begin{bmatrix} H^*_{1t}\sigma_\eta \\ \vdots \\ H^*_{Nt}\sigma_\eta \end{bmatrix}}^{H_t} \overbrace{\begin{bmatrix} \tilde{\beta}^1_t \\ \vdots \\ \tilde{\beta}^m_t \end{bmatrix}}^{s_t} + \overbrace{\begin{bmatrix} u_{1t} \\ \vdots \\ u_{Nt} \end{bmatrix}}^{e_t}, \tag{B.15}$$

$$\overbrace{\begin{bmatrix} u_{1t} \\ \vdots \\ u_{Nt} \end{bmatrix}}^{e_t} \sim \left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \overbrace{\begin{bmatrix} \sigma^2_{u_1} & & \\ & \ddots & \\ & & \sigma^2_{u_N} \end{bmatrix}}^{R} \right) \tag{B.16}$$

$$\underbrace{\tilde{\beta}_t}_{s_t} = \underbrace{I_m}_{F} \underbrace{\tilde{\beta}_{t-1}}_{s_{t-1}} + \underbrace{I_m}_{K_t} \underbrace{\tilde{\eta}_t}_{v_t}, \tag{B.17}$$

$$\underbrace{\tilde{\eta}_t}_{v_t} \sim N(0, \underbrace{I_m}_{Q}), \tag{B.18}$$

The time-varying component $\tilde{\beta}_t$ is initialized with mean and variance $b_0 = 0$ and $P_0 = 0.00001$. Thus, it is ensured that the time-varying parameters β_t are initialized with their starting values, collected in β_0.

The unobserved state vector $\tilde{\beta}$ is then extracted using standard forward-filtering and backward-sampling. Instead of taking the entire N x 1 observational vector y_t as the item of analysis, the approach taken here follows the univariate treatment of the multivariate series of Durbin and Koopman (2012), in which each of the elements in y_t is brought into the analysis individually. This offers significant computational gains and reduces the risk of the prediction error variance matrix becoming nonsingular during the Kalman filter procedure.

Lastly, given the components β_0, σ_η and $\tilde{\beta}$, the time-varying parameter matrix β (of dimension $T \times m$) can be constructed from (B.3).

B.1.3 Sampling the autoregressive coefficient, the unobserved component of the regression error process and the regression error variances

In this block, the autoregressive coefficient of the regression error process, ρ, the unobserved component, collected in μ, and the country-specific regression error variances, collected in σ_u^2, are drawn.

Note that, given draws of θ and β_t, ϵ_{it} and its lags are known. Thus, (B.2) breaks down to a conditional linear regression model, where ρ, μ and σ_u^2 can be obtained using a conjugate independent Normal-Inverted Gamma prior with $\rho, \mu \sim N(a_{0,\{\rho,\mu\}}, A_{0,\{\rho,\mu\}})$ and $\sigma_{u_i}^2 \sim IG(c_{0,i}, C_{0,i})$, with $c_{0,i}$ and $C_{0,i}$ being the country-specific shape and scale parameters of the prior distribution for the measurement error variance. As this prior is conjugate, it implies an (independent) Normal-Inverted Gamma posterior distribution. That is, $p(\rho, \mu|\sigma_u^2, \tilde{\beta}, \theta, y, \chi) \sim N(a_{T,\{\rho,\mu\}}, A_{T,\{\rho,\mu\}})$ and $p(\sigma_u^2|\rho, \mu, \tilde{\beta}, \theta, y, \chi) \sim IG(c_{T,i}, C_{T,i})$, $i = 1, 2, ..., N$, where $c_{T,i}$ and $C_{T,i}$ are the respective shape and scale parameters of the posterior distribution for the measurement error variance of country i. Defining ϵ as the $N \times (T-1)$ vector of stacked regressions error residuals, ϵ_{-1} as its lag, and u_i as the $(T-1) \times 1$ vector of residuals obtained from solving (B.2) for u for the respective country i, the posterior moments of the independent Normal-Inverted Gamma distribution are given by:

$$a_{T,\{\rho,\mu\}} = A_{T,\{\rho,\mu\}} \left(\chi' \Sigma^{-1} y + A_{0,\{\rho,\mu\}}^{-1} a_{0,\{\rho,\mu\}} \right) \tag{B.19}$$

$$A_{T,\{\rho,\mu\}} = \left(\chi' \Sigma^{-1} \chi + A_{0,\{\rho,\mu\}}^{-1} \right)^{-1} \tag{B.20}$$

$$c_{T,i} = c_{0,i} + (T-1)/2 \tag{B.21}$$

$$C_{T,i} = C_{0,i} + u_i' u_i / 2 \tag{B.22}$$

ρ, μ and σ_u^2 can then be sampled from $p(\rho, \mu|\sigma_u^2, \tilde{\beta}, \theta, y, \chi) \sim N(a_{T,\{\rho,\mu\}}, A_{T,\{\rho,\mu\}})$ and $p(\sigma_u^2|\rho, \mu, \tilde{\beta}, \theta, y, \chi) \sim IG(c_{T,i}, C_{T,i})$ for $i = 1, 2, ..., N$.

B.2 Bayesian Vector Autoregression

Drawing heavily from Blake and Mumtaz (2015), this section lays out the BVAR with time-varying coefficients in quarterly frequency, which is used to estimate the correlations between the macroeconomic variables to draw realizations of the primary balance and public debt in the fiscal projection exercise, elaborated upon in appendix B.3.

For each of the ten sample countries, consider a time-varying coefficient VAR(p) model in reduced form, written as

$$y_t = \phi_{1,t} y_{t-1} + \phi_{2,t} y_{t-2} + ... + \phi_{p,t} y_{t-p} + u_t, \qquad u_t \sim N(0, \Sigma), \tag{B.23}$$

$$\Phi_t = \Phi_{t-1} + e_t, \qquad e_t \sim N(0, Q), \tag{B.24}$$

$t = \{1, 2, ..., T_q\}$, where T_q is the number of quarterly observations available for the VAR. y_t is a $M \times 1$ vector of demeaned endogenous variables, $\phi_{j,t}$, $j = 1, 2, ..., p$ are $M \times M$ coefficient matrices corresponding to the respective lag matrix y_{t-j} and u_t is a $M \times 1$ vector of reduced-form shocks. The time-varying parameters are collected in $\Phi_t \equiv (vec(\phi_{1,t}), vec(\phi_{2,t}), ..., vec(\phi_{p,t}))'$ and are assumed to follow random walk processes with joint error covariance matrix Q, as outlined in (B.24). The disturbances u_t and e_t are assumed to be serially uncorrelated and independent of each other for $t = 1, 2, ..., T_q$.

With Σ and each $\phi_{j,t}$, $j = 1, 2, ...p$, $t = 1, 2, ..., T_q$ being of dimension $M \times M$ and Q being of dimension $M^2 p \times M^2 p$, the high number of parameters to be estimated motivates Bayesian estimation techniques. Following Blake and Mumtaz (2015), a Gibbs sampling algorithm to approximate the model's joint and marginal posterior distributions is employed. The following sections briefly outline this algorithm.

B.2.1 Sampling the time-varying parameters Φ

First, the time-varying parameters, collected in Φ, are sampled from their conditional posterior distributions: Express the system of equations in (B.23) and (B.24) as

$$y_t = (I_M \otimes X_t)\Phi_t + u_t, \qquad u_t \sim N(0, \Sigma), \tag{B.25}$$

$$\Phi_t = \Phi_{t-1} + e_t, \qquad e_t \sim N(0, Q), \tag{B.26}$$

where I_M is the identity matrix of dimension M and $X_t \equiv (y'_{t-1}, y'_{t-2}, ..., y'_{t-p})$. Conditionally on the data (y), Σ, Q as well as the expected value and variance of the initial state, Φ_0, the system in equations in (B.25) and (B.26) constitutes a linear Gaussian state space model.

Following Blake and Mumtaz (2015), the expected value of Φ_0, B_0, is set to $vec(\hat{\Phi})$, where

$\hat{\Phi} = (X'X)^{-1}X'y$ is the OLS estimate of the time-invariant coefficient version of (B.23). Consistently, the variance of the initial state, $V_{B_0} = \hat{\Sigma} \otimes (X'X)^{-1}$, with $\hat{\Sigma} = \frac{(y-XB_0)'(y-XB_0)}{T-K}$, where K is the number of slope coefficients in the time-invariant VAR. Then, analogously to the respective block in the FRF algorithm, the Kalman filter can be employed to filter the unknown state Φ_t (forward-filtering step) and subsequently sample Φ_t from its conditional distribution (backward-sampling step), as described in Carter and Kohn (1994).

B.2.2 Sampling the variance-covariance matrix of the state disturbances Q

Next, the variance-covariance matrix of the state disturbances, Q, is sampled from its conditional posterior distribution. Assuming that Q follows an inverted Wishart distribution a priori and given a draw of Φ_t, Q can be sampled from an inverted Wishart distribution. That is,

$$p(Q|\Phi, \Sigma, y) \sim IW(Q_1, T_1), \tag{B.27}$$

where the posterior scale and shape parameters are given by

$$\begin{aligned} Q_1 &= (\Phi_t - \Phi_{t-1})'(\Phi_t - \Phi_{t-1}) + Q_0, \\ T_1 &= T_q + T_0. \end{aligned}$$

T_0, the prior shape parameter, is the number of observations to inform the prior. It can be interpreted as the number of fictitious observations added to the model from the prior. Q_0 is the prior scale matrix.

B.2.3 Sampling the variance-covariance matrix of the VAR disturbances Σ

In this block, the variance-covariance matrix of the VAR disturbances, Σ, is sampled from its conditional posterior distribution. In particular, conditionally on Φ_t and assuming an inverted Wishart prior for Σ, it holds that

$$p(\Sigma|\Phi, Q, y) \sim IW(\Sigma_1, T_\Sigma), \tag{B.28}$$

where the posterior scale and shape parameters are given by

$$\begin{aligned} \Sigma_1 &= u'u + \Sigma_0, \\ T_\Sigma &= T_q + T_{\Sigma_0}, \end{aligned}$$

with $u \equiv (u_1, u_2, ..., u_{T_q})$, $u_t = y_t - (I_M \otimes X_t)$, $t = 1, 2, ..., T_q$. T_{Σ_0} is the prior shape parameter, that is, the number of "artificial" observations added to the sample from the prior. Σ_0 is the prior scale matrix.

B.3 Fiscal projection algorithm

Given parameter estimates for the FRF and VAR coefficients, the algorithm used to repeatedly draw realizations - thus obtaining forecast distributions - of the primary balance- and the public debt-to-GDP ratios can be laid out. The approach presented in this section largely follows Celasun et al. (2006) and Medeiros (2012) but deviates occasionally due to the usage of Bayesian estimation techniques both in the FRF and the VAR block.

More precisely, future paths of the primary balance and the public debt ratios are repeatedly drawn from the FRF and a debt accumulation function. The primary balance forecast for country i is obtained from

$$pb_{i,T+h} = \hat{\alpha}_i + H_{i,T+h}\hat{\beta}_{T+h} + X_{i,T+h}\hat{\gamma} + \epsilon_{i,T+h}, \tag{B.29}$$

with $h = 1, 2, 3$ being the respective forecast horizon and $h = 1$ being the end-of-the-year forecast ("nowcast") of the respective vintage, $h = 2$ is the forecast for the subsequent year and $h = 3$ is the two-year-ahead forecast. $\hat{\alpha}_i$ is the estimate of the country-specific constant and can be recovered from the estimated FRF from $\hat{\alpha}_i = \bar{pb}_i - \bar{H}_i\bar{\beta} - \bar{X}_i\hat{\gamma}$, where $\bar{pb}_i$, $\bar{H}_i$ and $\bar{X}_i$ are country-specific means and $\bar{\beta} = \frac{\sum_{t=1}^{T}\hat{\beta}_t}{T}$ (barring the time-varying parameters, see for example Baltagi, 2013). The forecast for $\hat{\beta}_{T+h}$ is obtained using the non-centered parameterization and thus given by $\hat{\beta}_{T+h} = \hat{\beta}_0 + \hat{\sigma}_\eta + \hat{\tilde{\beta}}_T + \sum_{j=1}^{h}\tilde{\eta}_j$.

Note that the matrices H and X contain the fitted values of the output gap (having used an auxilliary regression to account for the variable's endogeneity as elaborated upon above) and the lagged primary balance and lagged public debt ratio. To obtain a forecast for $h = 1$, the latter two are simply their end-of-sample observations, that is, pb_{iT} and $debt_{iT}$. For the output gap on the other hand, the realization in $T+1$ is unobserved and needs to be forecasted: First, the (quarterly) $ln(GDP)$ series is forecasted using the VAR and then used to get an estimate of the cycle based on the the Hodrick-Prescott filter (where a value of $\lambda = 1600$, as conventional for quarterly data, is used). The resulting output gap in quarterly frequency is then annualized for consistency with FRF data.[1]

Note that the simulation is done R times, where R is the number of retained draws from the MCMC algorithms elaborated on in B.1 and B.2. This is convenient as for each draw $r = 1, 2, ..., R$, the respective draws of the posterior distributions - that is β_T^r (to compute β_{T+h}^r), γ^r et cetera - can be used to come up with one forecasted path of the fiscal variables. Likewise, the respective set of forecast errors ϵ_{it}^r is used to come up with

[1] To avoid the end-point problem (see e. g. Everaert and Jansen, 2017), log(output) is forecasted four quarters further into the future before computing the output gap.

the realizations of $\epsilon^r_{i,T+h}$ for each respective draw: From equation (B.2), it follows that $\epsilon^r_{i,T+h} = \mu^r_{T+h} + \rho^r \epsilon^r_{i,T+h-1} + u^r_{i,T+h}$. In the benchmark specification, μ^r_{T+h} is set to μ^r_T. However, a second alternative, of setting $\mu_{T+h} = 0$, hardly changes the results.[2] $u^r_{i,T+h}$ is obtained using bootstrapping, as in Medeiros (2012). Due to the assumption of country-specific error variances σ^2_i, $i = 1, 2, ..., N$, this is done for each country separately. Lastly, note that for $h = 1$, $\epsilon_{i,T+h-1} = \epsilon_{iT}$ is observable, such that all components to compute $\epsilon_{i,T+1}$ are known. Given $\epsilon_{i,T+1}$, $\epsilon_{i,T+2}$ can then be obtained, and so can $\epsilon_{i,T+3}$.

The public debt ratio for country i is based on the following debt accumulation equation (similar to Medeiros, 2012):

$$debt_{i,T+h} = \frac{1 + iir_{i,T+h}}{1 + (\Delta y_{i,T+h} + \pi_{i,T+h})} + pb_{i,T+h} + sfa_{i,T+h}, \tag{B.30}$$

where $debt$ is the public debt-to-GDP ratio, iir is the implicit interest rate on the debt outstanding (scaled by GDP), Δy is GDP growth, π is inflation and sfa are stock-flow adjustments of the stock of public debt (scaled by GDP), that is, one-off adjustments to the level of public debt not attributable to the other components, such as the privatization of public assets. While Δy and π forecasts can be obtained directly from the VAR, iir and sfa are taken from AMECO (see data appendix).

Note that the approach outlined here means that the real interest rate as defined above is not used in the debt simulation. Nevertheless, it is included in the VAR to adequately capture the variables' correlations. Alternative debt forecasts based on the real interest rate and not the implicit interest rate (adjusted for inflation) on average perform slightly worse than the forecasts presented here.

The AMECO database contains only point forecasts. Thus, median forecasts for each variable and horizon are computed and compared to the fixed coefficient model forecast and the EC forecast, found in the AMECO vintages.

[2]Another approach would be to forecast μ^r_{T+h}, making use of its estimates given for periods $t = 1, 2, ..., T$.

B.4 The fixed coefficient model

This section briefly outlines the fixed coefficient model (the "fixed model") that is used to judge the forecast performance of the benchmark model in the main paper. First note that the fixed model uses the same set of predictors in its FRF part and the same endogenous variables in the VAR part, as elaborated upon in section 2 of the main paper. The FRF is given by

$$pb_{it} = \alpha_i + X_{it}\gamma + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_\epsilon^2) \tag{B.31}$$

$i = 1, 2, ..., N, t = 1, 2, ..., T$. Note that in the fixed model, the lagged debt ratio enters the predictor matrix X, as the corresponding slope coefficient is assumed to be time-invariant. As before, X additionally contains the lagged primary balance and the output gap. Similar to Everaert and Jansen (2018), the model is estimated using a two-stage least squares instrumental variables estimator on the within-group demeaned model to account for potential endogeneity of the output gap, which is instrumented by its first and second lag.

The VAR in this case is the time-invariant coefficient pendant of equation (7) in the main paper:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + u_t, \qquad u_t \sim N(0, \Sigma), \tag{B.32}$$

$t = \{1, 2, ..., T_q\}$, where T_q again is the number of quarterly observations in the VAR, y_t is a $M \times 1$ vector of demeaned endogenous variables, ϕ_j, $j = 1, 2, ..., p$ are $M \times M$ coefficient matrices corresponding to the respective lag matrix y_{t-j} and u_t is a $M \times 1$ vector of reduced-form shocks, and the model is estimated using equation-by-equation ordinary least squares.

The primary balance and debt projection block of the model mostly follows the approach outlined in section 2.4 in the main paper, the main difference being that, unlike for the Bayesian benchmark model, parameter uncertainty is not directly incorporated in the fiscal projection exercise.

C Further results

In this section, some results that accompany those in the main paper are presented. Section C.1 discusses the forecast model comparison results based on the Pesaran et al. (2009) test, while section C.2 analyzes the robustness of the SDSA results presented in the paper.

C.1 Forecast model comparison

This section lays out a formal test that complements the model comparison findings in the main paper. In particular, the section outlines the Pesaran et al. (2009) panel data version of the Diebold and Mariano (2002) test, which compares the forecasts of two models of interest.[3] Define the quadratic loss function of a certain variable as

$$z_{it} = \left[e(h)_{it}^{A}\right]^2 - \left[e(h)_{it}^{B}\right]^2, \tag{C.1}$$

$i = 1, 2, ..., N$, $t = 1, 2, ..., T$, where $e(h)_{it}^{A}$ is the h-period-ahead forecast error for country i in period t for the benchmark model featuring a time-varying coefficient FRF and $e(h)_{it}^{B}$ is the respective forecast error of the model of comparison, that is, either the fixed coefficient model or the EC forecast. Pesaran et al. (2009) then test the null hypothesis that $\alpha_i = 0$ for all $i = 1, 2, ..., N$ in

$$z_{it} = \alpha_i + \epsilon_{it}, \quad \epsilon_{it} \sim IID(0, \sigma_i^2), \tag{C.2}$$

the alternative hypothesis being that $\alpha_i < 0$ for some i. The test statistic is computed as

$$\overline{DM} = \frac{\bar{z}}{\sqrt{V(\bar{z})}} \sim N(0, 1), \tag{C.3}$$

with $\bar{z} \equiv \frac{1}{N}\sum_{i=1}^{N} \bar{z}_i$, $\bar{z}_i \equiv \frac{1}{T}\sum_{t=1}^{T} z_{it}$, $V(\bar{z}) \equiv \frac{1}{NT}\left[\frac{1}{N}\sum_{i=1}^{N} \hat{\sigma}_i^2\right]$, $\hat{\sigma}_i^2 \equiv \frac{\sum_{t=1}^{T}(z_{it}-\bar{z}_i)^2}{T-1}$. For the one-period-ahead and two-period-ahead forecasts ($h = 2$ and $h = 3$), the test statistic is modified to account for autocorrelation in the forecast errors by using a Newey-West type version of $Var(\bar{z}_i)$, see for example Ghysels and Marcellino (2018).

Table C.1 displays the results of this test. Values smaller than the 5% critical value of -1.645 indicate a significantly better performance of the benchmark time-varying coefficient model. The strong performance of the benchmark model is confirmed especially by the results against the fixed model, where the DM statistic provides formal evidence for the

[3] The following remarks closely follow Pesaran et al. (2009). For simplicity, whenever it does not contradict the notation used so far, their notation is used.

Table C.1: Diebold-Mariano panel test results

Model	**Primary balance**			**Public debt**		
	0p	**1p**	**2p**	**0p**	**1p**	**2p**
European Commission	1.430	2.148	1.205	-0.414	0.986	1.520
Fixed model	-1.729	1.491	1.516	-1.716	-1.093	-1.656

Notes: This table presents the results of the Pesaran et al. (2009) panel data version of the Diebold-Mariano test, where the benchmark model featuring time-varying coefficients is tested against the European Commission forecast and the forecast of the fixed coefficient model. "0p" is the nowcast, "1p" the one-year-ahead and "2p" the two-year-ahead forecasts, respectively. The test is a one-sided test with the null hypothesis that the forecasts from the two models are not significantly different, the alternative hypothesis being that the benchmark model's forecasts are significantly better. The 5% critical value is -1.645. Thus, values smaller than -1.645 indicate superiority of the benchmark model's forecasts at the respective horizon.

superiority of the benchmark model in terms of MSE for the nowcasts as well as at the two-period-ahead horizon. At the same time, the benchmark's debt forecast is not outperformed by the EC at any forecast horizon.

Additionally, the table shows that the benchmark's primary balance nowcasts are significantly better than those of the fixed model, while once again the EC forecasts do not have a clear edge over the benchmark model. However, the primary balance forecast performance at the one- and two-period horizon is worse, with the EC forecast's superiority over the benchmark model even being statistically significant for the one-period horizon. Nevertheless, the DM test results clearly show that a model averaging forecast approach that encompasses an SDSA model that features a time-varying coefficient FRF and VAR might be a helpful contributor to overall fiscal forecasting performance.

C.2 Robustness

In this section, summarized results for two alternative specifications are presented. The second specification is motivated by the fact that the time-varying parameter of the lagged primary balance displays little time variation (see figure 1 in main paper). In this specification, the coefficient for the lagged primary balance is included as a time-invariant parameter. That is, the lagged primary balance is included as a regressor in X with the corresponding coefficient being included in γ (see equation (3) in main paper). The third specification follows the baseline specification in Berger et al. (2021), who find formal evidence for time variation in the lagged debt parameter. Thus, in this specification, only the lagged debt ratio is contained in the matrix H, while the output gap as well as the lagged primary balance are included in X. Thus, their parameters are treated as fixed (in the sense of not time-varying) in this specification.

Table C.2 illustrates the forecast performance of all three specifications for all three forecast horizons. The table also repeats the results of the fixed model for reasons of comparability. Clearly, the differences in forecast performance between the benchmark specification and specification 2 are negligible. While the competitive public debt forecast performance is also visible for specification 3, its primary balance forecasts are somewhat worse, especially with respect to the nowcasts. Given the amount of time variation of the output gap coefficient in the benchmark specification, displayed in figure 1 in the main paper, the poor forecast results might be seen as a preliminary indication of model misspecification stemming from forcing the output gap coefficient to be time-invariant in specification 3. However, as indicated by the Pesaran et al. (2009) test results in table C.3, the fixed model still does not (significantly) outperform the benchmark model in terms of primary balance forecast at any horizon. Thus, even specification 3 might be worth considering in a model averaging forecast exercise, especially due to its strong public debt forecast performance.

Taken together, these findings provide some evidence that simple SDSA models featuring time-varying coefficient FRFs and VARs deserve some praise when it comes to fiscal forecasting. This finding is robust to changes in the specification, especially when it comes to the public debt forecast performance.

Table C.2: Forecast performance evaluation for the primary-balance-to-GDP and the public debt-to-GDP ratios, **all horizons, alternative specifications**

- Nowcasts -

Model	Primary balance		Public debt	
	rMSE	pval (H0: Unbiased)	rMSE	pval (H0: Unbiased)
Fixed model	1.823	0.974	0.948	0.000
Benchmark model	1.567	0.086	0.898	0.001
Specification 2	1.558	0.062	0.894	0.001
Specification 3	2.314	0.000	0.833	0.009

- One-period-ahead forecasts -

Model	Primary balance		Public debt	
	rMSE	pval (H0: Unbiased)	rMSE	pval (H0: Unbiased)
Fixed model	1.213	0.005	1.229	0.077
Benchmark model	1.279	0.000	1.102	0.380
Specification 2	1.286	0.000	1.100	0.404
Specification 3	1.358	0.000	0.988	0.820

- Two-period-ahead forecasts -

Model	Primary balance		Public debt	
	rMSE	pval (H0: Unbiased)	rMSE	pval (H0: Unbiased)
Fixed model	1.231	0.006	1.706	0.370
Benchmark model	1.293	0.000	1.361	0.880
Specification 2	1.289	0.000	1.356	0.902
Specification 3	1.589	0.000	1.275	0.391

Notes: Presented are Mean Squared Error ratios (rMSEs) of the fixed coefficient model, the benchmark model as well as two further specifications of the benchmark model (specifications 2 and 3) against the European Commission forecast. Ratios greater than one indicate that the European Commission forecast is superior. Additionally, the table contains p-values for a test of biasedness of forecast errors. That is, the null hypothesis of $\alpha = 0$ in $pb_{it} - pb_{it}^{F} = \alpha + u_{itH}$ is tested, where pb_{itH} is the actual primary balance in period t for country i and pb_{itH}^{F} is the corresponding forecast made for period t at period H (similar to An et al., 2018). The results are based on 100 forecast errors for the nowcast and one-period-ahead horizon and 50 forecast errors for the two-period-ahead horizon.

Table C.3: Diebold-Mariano panel test results

Model	**Primary balance**			**Public debt**		
	0p	**1p**	**2p**	**0p**	**1p**	**2p**
Benchmark vs. EC	1.430	2.148	1.205	-0.414	0.986	1.520
Benchmark vs. fixed model	-1.729	1.491	1.516	-1.716	-1.093	-1.656
Specification 2 vs. EC	1.365	2.168	1.319	-0.452	0.988	1.524
Specification 2 vs. fixed model	-1.717	1.520	1.447	-1.790	-1.112	-1.652
Specification 3 vs. EC	1.738	1.502	2.278	-1.505	0.672	1.662
Specification 3 vs. fixed model	-0.264	0.315	1.527	-1.907	-1.251	-1.567

Notes: This table presents the results of the Pesaran et al. (2009) panel data version of the Diebold-Mariano test, where specifications 2 and 3 are tested against the European Commission forecast and the forecast of the fixed coefficient model. "0p" is the nowcast, "1p" the one-year-ahead and "2p" the two-year-ahead forecasts, respectively. The test is a one-sided test with the null hypothesis that the forecasts from the two models are not significantly different, the alternative hypothesis being that the benchmark model's forecasts are significantly better. The 5% critical value is -1.645. Thus, values smaller than -1.645 indicate superiority of the benchmark model's forecasts at the respective horizon.

References

An, Z., Jalles, J. T., Loungani, P., and Sousa, R. M. (2018). Do imf fiscal forecasts add value? *Journal of Forecasting*, 37(6):650–665.

Baltagi, B. (2013). *Econometric analysis of panel data*. John Wiley & Sons.

Berger, T., Dubbert, T., and Schoonackers, R. (2021). Fiscal prudence: It's all in the timing–estimating time-varying fiscal policy reaction functions for core eu countries. *Available at SSRN 3810841*.

Blake, A. and Mumtaz, H. (2015). *Applied Bayesian Econometrics for central bankers*. Number 36 in Handbooks. Centre for Central Banking Studies, Bank of England.

Carter, C. K. and Kohn, R. (1994). On Gibbs sampling for state space models. *Biometrika*, 81(3):541–553.

Celasun, O., Ostry, J. D., and Debrun, X. (2006). Primary surplus behavior and risks to fiscal sustainability in emerging market countries: A "fan-chart" approach. *IMF Staff Papers*, 53(3):401–425.

Diebold, F. X. and Mariano, R. S. (2002). Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 20(1):134–144.

Durbin, J. and Koopman, S. J. (2012). *Time series analysis by state space methods*. Oxford university press.

Everaert, G. and Jansen, S. (2017). On the estimation of panel fiscal reaction functions: Heterogeneity or fiscal fatigue? national bank of belgium working paper no. 320.

Everaert, G. and Jansen, S. (2018). On the estimation of panel fiscal reaction functions: Heterogeneity or fiscal fatigue? *Economic Modelling*, 70:87 – 96.

Frühwirth-Schnatter, S. and Wagner, H. (2010). Stochastic model specification search for gaussian and partial non-gaussian state space models. *Journal of Econometrics*, 154(1):85–100.

Ghosh, A. R., Kim, J. I., Mendoza, E. G., Ostry, J. D., and Qureshi, M. S. (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *The Economic Journal*, 123(566):F4–F30.

Ghysels, E. and Marcellino, M. (2018). *Applied economic forecasting using time series methods*. Oxford University Press.

Mauro, P., Romeu, R., Binder, A., and Zaman, A. (2015). A modern history of fiscal prudence and profligacy. *Journal of Monetary Economics*, 76:55–70.

Medeiros, J. (2012). Stochastic debt simulation using VAR models and a panel fiscal reaction function – results for a selected number of countries. European Economy - Economic Papers 2008 - 2015 459, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission.

Pesaran, M. H., Schuermann, T., and Smith, L. V. (2009). Forecasting economic and financial variables with global vars. *International Journal of Forecasting*, 25(4):642–675. Special section: Decision making and planning under low levels of predictability.