Smooth Operator: Modifying the Anhøj Rules to Improve Runs Analysis in Statistical Process Control

by Jacob Anhøj, Tore Wentzel-Larsen

Abstract The run charts is one form of statistical process control charts that is particularly useful for detecting minor to moderate shifts in data over time. The Anhøj rules test for shifts by looking for unusually long runs (L) of data points on the same side of the process centre (mean or median) and unusually few crossings (C) of the centre. The critical values for C and L depend on the number of available data points (N). Consequently, the diagnostic properties (e.g. sensitivity and specificity) of the Anhøj rules also depend on N and have a rugged appearance due to the discrete nature of the tests. This study demonstrates that it is possible to obtain better diagnostic properties by making minor adjustment to the critical values for C and L. We refer to these procedures as the best box and cut box adjustment procedures.

Introduction

Within statistical process control (SPC) runs analysis is being used to detect persistent shifts in process location over time (Anhøj and Wentzel-Larsen, 2018).

Runs analysis deals with the natural limits of number of runs and run lengths in random processes. A run is a series of one or more consecutive elements of the same kind, for example heads and tails, diseased and non-diseased individuals, or numbers above or below a certain value. A run chart is a point-and-line chart showing data over time with the median as reference line (Figure 1). In a random process, the data points will be randomly distributed around the median, and the number and lengths of runs will be predictable within limits. All things being equal, if the process shifts, runs tend to become longer and fewer. Consequently, runs analysis may help detect shifts in process location. Process shifts are one form of non-random variation in time series data that are of particular interest to quality control and improvement: If a process shifts, it may be the result of planned improvement or unwanted deterioration.

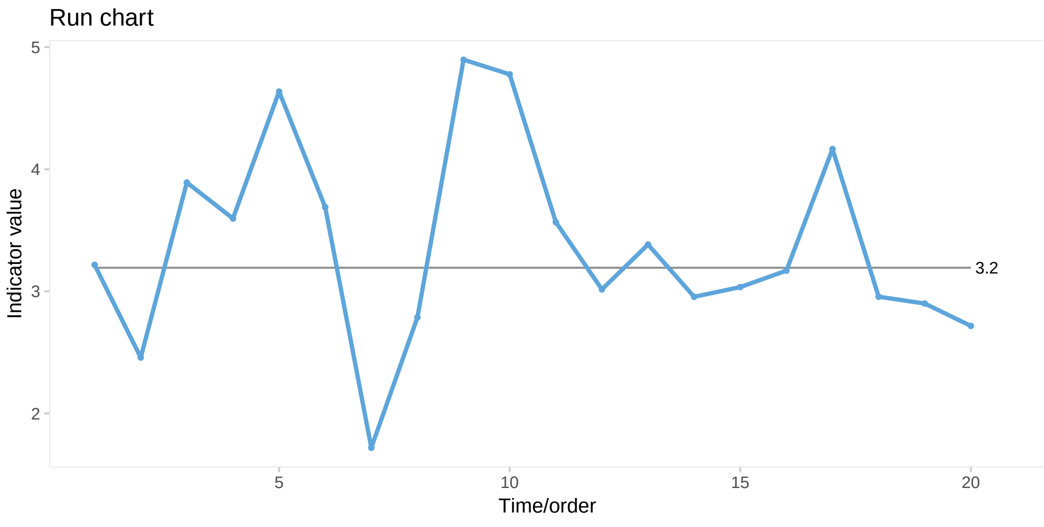

Figure 1: Run chart. Median = 3.2, longest run (L) = 4, number of crossings (C) = 9.

Several tests (or rules) based on the principles of runs analysis for detection of shifts exist. In previous papers we demonstrated, using simulated data series, that the currently best performing rules with respect to sensitivity and specificity to shifts in process location are two simple tests (Anhøj and Olesen, 2014; Anhøj, 2015; Anhøj and Wentzel-Larsen, 2018):

- Shifts test: one or more unusually long runs of data points on the same side of the centre line.
- Crossings test: the curve crosses the centre line unusually few times.

Collectively, we refer to these tests as the Anhøj rules, which are the default rules used for run and control chart analysis with the **qicharts2** package (Anhøj, 2019). For a thorough discussion of the practical use of run and control charts for quality improvement we refer to the **qicharts2** package vignette.

Critical values for run length and number of crossings depend on the total number of data points in the chart. The number of crossings follow a binomial distribution, $b(n-1, 0.5)$, where n is the number of data points and 0.5 the success probability. Thus, the lower prediction limit for number of crossings may, for example, be set to the lower 5th percentile of the corresponding cumulative binomial distribution (Chen, 2010). However, no closed form expression exists for the distribution of longest runs. Consequently, the upper prediction limit for longest runs has traditionally been either a fixed value (usually 7 or 8) (Carey, 2002) or an approximate value depending on n as with the Anhøj rules: $\log_2(n) + 3$ rounded to the nearest integer (Schilling, 2012). Figure 1 has 20 data points, the curve crosses the centre line 9 times, and the longest run (points 3-6) contains 4 data points. In a random process with 20 data points, we should expect at least 6 crossings and the longest run should include no more than 7 data points. Thus, according to the Anhøj rules, Figure 1 shows random variation.

Each of the two tests has an overall specificity (true negative proportion) around 95%. The sensitivity (true positive proportion) of a test depends on the size of the shift (signal) relative to the random variation inherent in the process (noise). When applied together, the sensitivity increases, while the specificity decreases a bit and fluctuates around 92.5% (see red line in Figure 2).

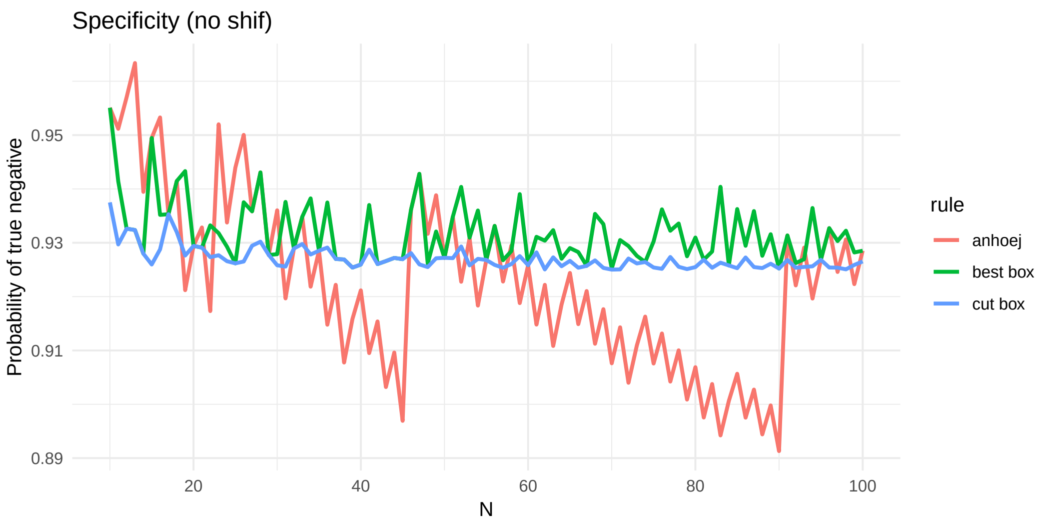

Figure 2: Specificity of the Anhøj, best box, and cut box rules. N = number of data points in run chart.

Historically, runs tests have mainly been studied individually. But what is really of interest, because the rules are linked – when one goes up, the other goes down – is the properties of the joint distribution of number of crossings (C) and longest runs (L).

We recently released an R package, **crossrun** (Wentzel-Larsen and Anhøj, 2018), that includes functions for calculating the joint probabilities of C and L in random data series of different lengths (N) and with and without shifts in process location expressed in standard deviation units (SD). Figure 3 illustrates this for a run chart with N = 11 and SD = 0 (no shift). To avoid very small numbers, the probabilities are shown using the times representation, that is, the probabilities times 2^{n-1}, which is 1024 for N = 11. The red box encloses the combinations of C and L that would indicate random variation according to the Anhøj rules (true negatives). The area outside the box represents combinations of C and L that would indicate non-random variation (false positives).

With the **crossrun** package it became feasible to calculate exact joint probabilities of C and L over a wide range of N and SD. And consequently, it became feasible to investigate the diagnostic properties of run charts using exact values for specificity and sensitivity rather than values based on time consuming, inaccurate, and complicated simulation studies.

As shown in Figure 2 the specificity of the Anhøj rules (red line) jumps up and down as N changes. This is a consequence of the discrete nature of the two tests – especially the shifts test. Although the specificity of the Anhøj rules does not decrease continuously as N increases, which is the case for other rules (Anhøj and Olesen, 2014), we hypothesised that it would be possible to improve the diagnostic value further by smoothing the specificity using minor adjustments to C and L depending on N.

The aims of this study were to provide exact values for the diagnostic properties of the Anhøj rules

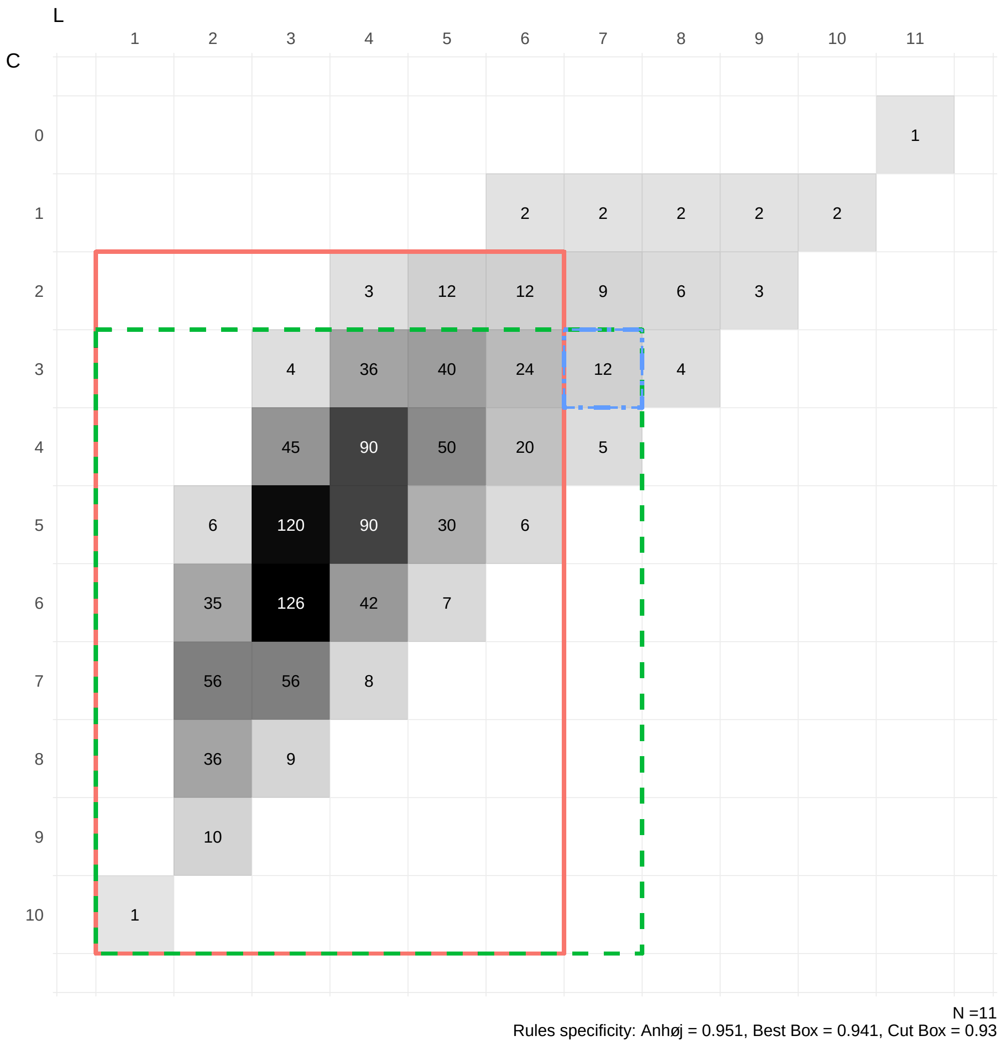

Figure 3: Borders of the Anhøj, best box, and cut box rules for N = 11 data points. C = number of crossings, L = longest run. The numbers in the cells are times representation of of the joint probabilities of longest run and number of crossings. Anhøj = red solid, best box = green dashed, cut box = blue dot-dashed.

and to suggest a "smoothing" procedure for improving the value of runs analysis.

Methods

Likelihood ratios to quantify the diagnostic value of runs rules

The value of a diagnostic test is traditionally described using terms like sensitivity and specificity. These parameters express the probability of detecting the condition being tested for when it is present and not detecting it when it is absent:

$$\text{Specificity} = \text{P(no signal} \mid \text{no shift)} = \text{P(true negative)} = 1 - \text{P(false positive)}$$
$$\text{Sensitivity} = \text{P(signal} \mid \text{shift)} = \text{P(true positive)} = 1 - \text{P(false negative)}$$

However, we usually seek to answer the opposite question: what is the likelihood that a positive or negative test actually represents the condition being tested for, which in our case is a shift in the underlying process? Likelihood ratios (LR) do this:

$$\text{LR+} = \text{TP/FP} = \text{sensitivity}/(1 - \text{specificity})$$
$$\text{LR-} = \text{FN/TN} = (1 - \text{sensitivity})/\text{specificity}$$

A likelihood ratio greater than 1 speaks in favour of the condition being tested for, while a likelihood ratio less than 1 speaks against the condition. As a rule of thumb, a positive likelihood ratio (LR+) greater than 10 is considered strong evidence that the condition is present. A negative likelihood ratio (LR-) smaller than 0.1 is considered strong evidence against the condition (Deeks and Altman, 2004). For example, if LR+ = 10 and LR- = 0.1 then a positive test means that it is 10 times *more* likely that the condition is present than not present, and a negative test means that it is 10 times *less* likely that the condition is present than not present. Thus, likelihood ratios come in pairs and are combined measures of the usefulness of a diagnostic test. Specifically, for our purpose, runs charts are diagnostic test for non-random variation in time series data (Anhøj, 2015; Anhøj and Wentzel-Larsen, 2018).

Best box and cut box adjustments to improve the Anhøj rules

To fix some terms, we define a box as a square region $C \geq c, L \leq l$. These are the square regions that may be used to define random variation. The corner of the box is its upper right square $C = c, L = l$. In Figure 3 the box $C \geq 2, L \leq 6$, marked with red, specifies the Anhøj rules for $N = 11$. The corner of this box is the square $C = 2, L = 6$.

Based on the **crossrun** package we developed two functions, `bestbox()` and `cutbox()` that automatically seek to adjust the critical values for C and L to balance between sensitivity and specificity requirements. Specifically, the `bestbox()` function finds the box with highest sensitivity for a predetermined shift (the target shift), among boxes with specificity $\geq$ a pre-determined value (the target specificity). The `cutbox()` function subsequently cuts squares from the upper and right borders of the best box, starting from the corner while keeping specificity $\geq$ its target value, and the sensitivity for the target shift as large as possible. The result of `cutbox()` is not necessarily a box, but a reasonable region for declaring random variation with one or a few squares close to the corner possibly removed.

In this study we used a target specificity of 92.5%, which is close to the actual average specificity for the Anhøj rules for N = 10-100, while the target shift was set at 0.8.

Figure 3 illustrates these principles for a run chart with 11 data points. Thus, for N = 11, the Anhøj rules would signal a shift if C < 2 or L > 6; best box would signal if C < 3 or L > 7; and cut box would signal if C < 3 or L > 7, and also when C = 3 and L = 7.

Results

We calculated the limits for the Anhøj, best box, and cut box rules together with their corresponding positive test proportions and log-likelihood ratios for N = 10-100 and SD = 0-3 (in 0.2 SD increments). We stored the `log` of likelihood ratios to preserve numerical precision. To get the actual likelihood values back, use `exp(log-likelihood)`. The limits, specificities, and sensitivities are presented in Table 1. The R code to reproduce the full results set and the figures from this article is provided in the supplementary file 'crossrunbox.R'.

Figure 2 illustrates the effect of the best box and cut box procedures on the specificity of the runs analysis. As expected, the variability in specificity with varying N is markedly reduced and kept above and closer to the specified target – more with cut box than with best box.

Figure 4 shows the probabilities of getting a signal as a function of N and SD. The upper left facet (SD = 0) contains the same data as Figure 2. As expected and shown previously in our simulation studies, the power of the runs analysis increases with increasing N and SD. The smoothing effect of best box and cut box appears to wear off as N and SD increases. Figure 5 is a blown up version of the facet with shift = 0.8 SD from Figure 4 and shows the sensitivity for the target value used in the box calculations. Exact values for shift = 0 and shift = 0.8 may be looked up in Table 1

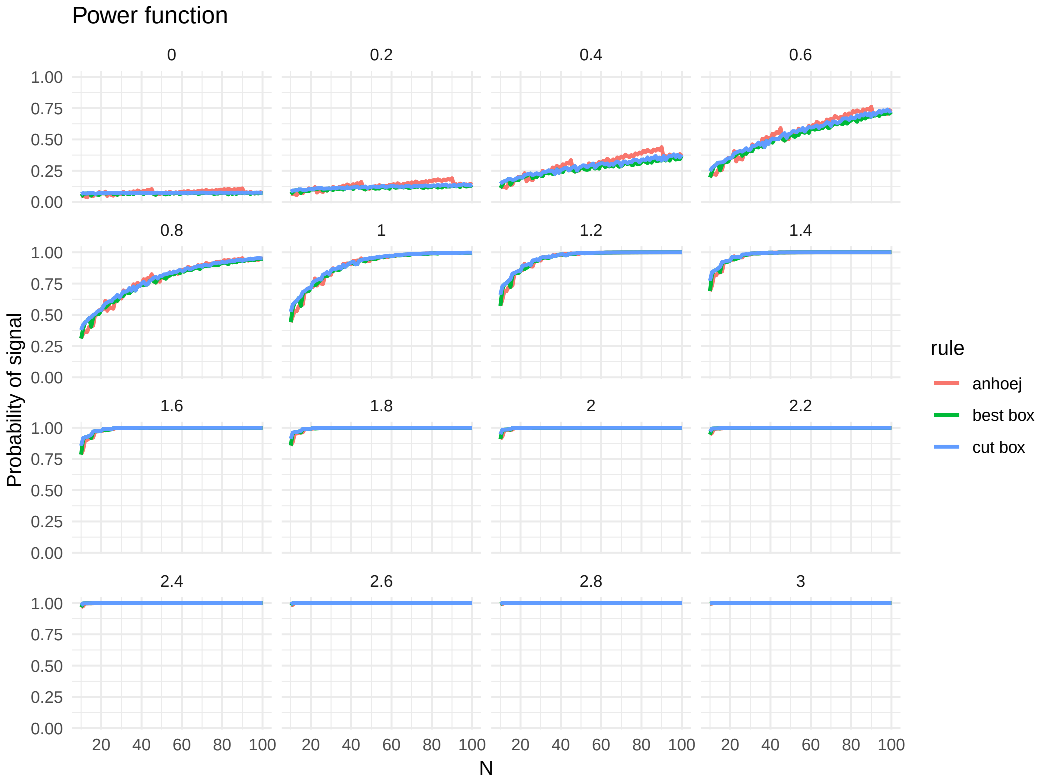

Figure 4: Power function of Anhøj, best box, and cut box rules. N = number of data points in run chart. Numbers above each facet represent the size of the shift in standard deviation units (SD) that is present in data.

Figures 6 and 7 compare the positive and negative likelihood ratios of the Anhøj rules to the box adjustments. The smoothing effect appear to be of practical value only for positive tests.

Discussion and conclusion

This study provided exact values for the diagnostic properties of the Anhøj rules for run charts with 10-100 data points including shifts up to 3 standard deviation units.

To our knowledge, and with the exception of our own **crossrun** package, the properties of the joint distribution of number of crossings and longest runs in random data series have not been studied before.

Furthermore, the study demonstrated that it is feasible to reduce the variability in run chart specificity with varying number of data points by using the best box and cut box adjustments of the Anhøj rules.

Most importantly, figures 6 and 7 confirm what we expected after years of practical experience using runs analysis, that the Anhøj rules is a useful and robust method for detection of persistent shifts only slightly larger than 1 standard deviation units and with as little as 10-12 data points. This can be seen by the fact that LR+ > 10 for SD > 1 and N $\geq$ 10. Although, the best box and cut box procedures will not change this, the box adjustments may potentially improve the practical value of runs analysis by reducing sudden shifts in sensitivity and specificity when the number of available data points changes. Whether this holds true in practice remains to be confirmed.

The study has two important limitations. First, the calculations of box probabilities require that the process centre is fixed and known in advance, for example, the median from historical data. In practice the centre line is often determined from the actual data in the run chart, in which case the calculations of box probabilities do not apply. Preliminary studies suggest that this is mostly relevant

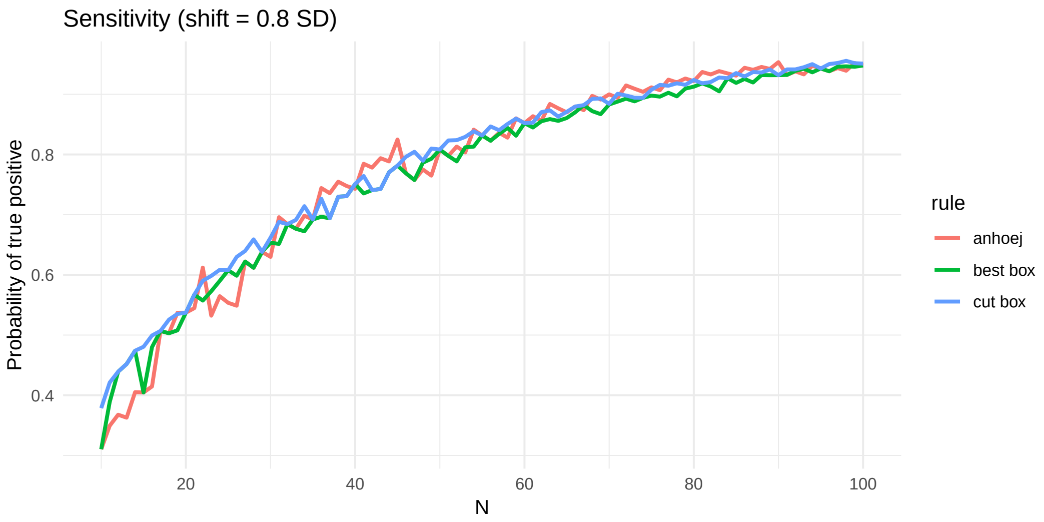

Figure 5: Sensitivity of Anhøj, best box, and cut box rules for shift = 0.8 standard deviation units. N = number of data points in run chart.

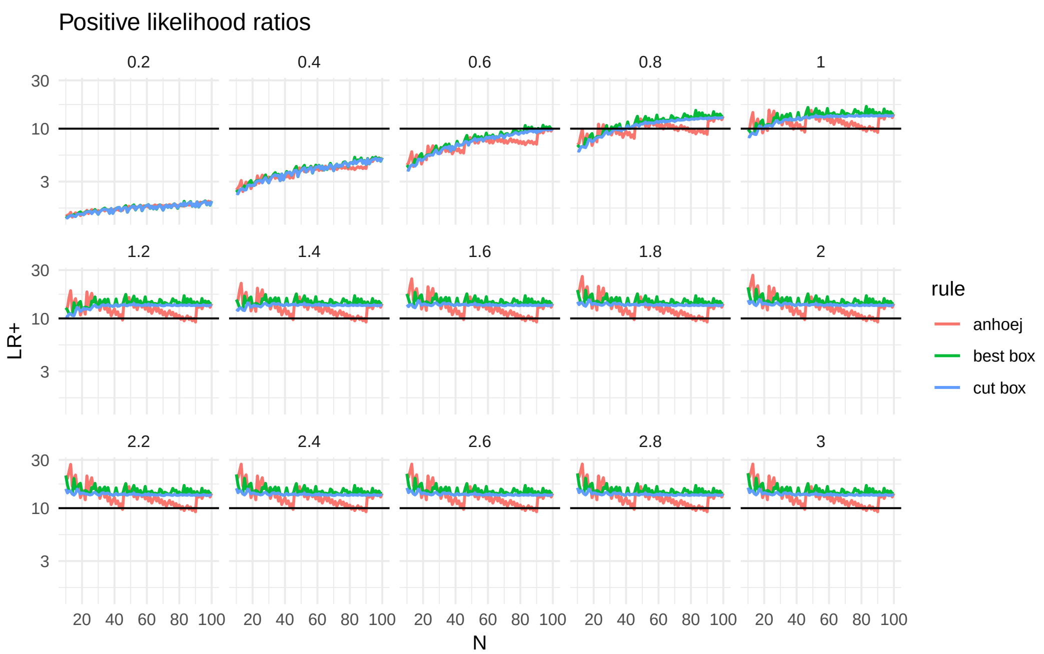

Figure 6: Positive likelihood ratio of Anhøj, best box, and cut box rules. N = number of data points in run chart. Numbers above each facet represent the size of the shift in standard deviation units that is present in data.

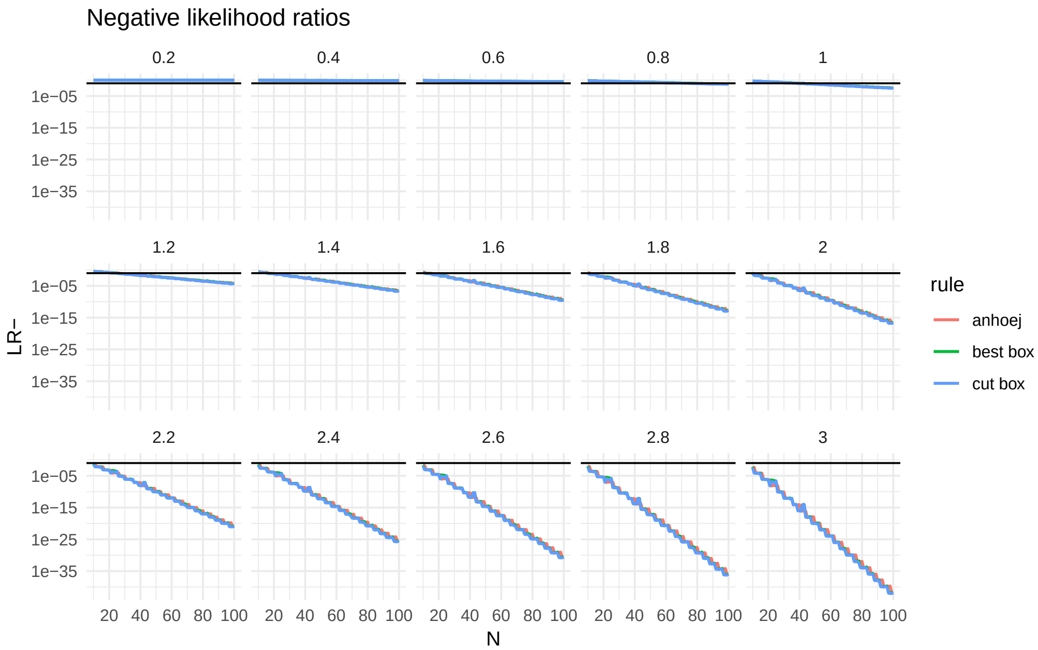

Figure 7: Negative likelihood ratio of Anhøj, best box, and cut box rules. N = number of data points in run chart. Numbers above each facet represent the size of the shift in standard deviation units (SD) that is present in data.

for short data series. We plan to include a function in a future update of **crossrun** to calculate the box probabilities with empirical centre lines.

Second, the procedures have so far only been checked for up to 100 data points. Because of the iterative procedures and use of high precision numbers using functions from the **Rmpfr** package (Maechler, 2019) to calculate the joint distributions for varying N, the computations are time consuming. On a laptop with an Intel Core i5 processor and 8 GB RAM, it takes about one hour to complete `crossrunbox.R` for N = 10-100 and SD = 0-3, and the objects created consume over 6 GB of memory. However, we have no reason to believe that the procedures are not valid for N > 100, but the application of the box procedures for larger N may be impractical at the moment.

Also, one should be aware that the value of the box procedures rely on the choice of target specificity and target shift values. Other target values will give different diagnostic properties. Preliminary studies suggest that increasing the target specificity to, say, 0.95 in fact increases the positive likelihood ratios a bit without affecting the negative likehood ratios considerably. By supplying the R code, we encourage users to adapt our findings to their own needs.

Regarding the practical application of the box adjustment of the Anhøj rules, we are in the process of testing a `method` argument for the `qic()` function from the **qicharts2** package that allows the user to choose between `"anhoej"`, `"bestbox"`, and `"cutbox"` methods to identify non-random variation in run and control charts with up to 100 data points. This will allow us and others to quickly gain practical experience with box adjustments on real life data.

In conclusion, this study provided exact values for the diagnostic properties of the Anhøj rules for run charts with 10-100 data points including shifts up to 3 standard deviation units, and demonstrated that it is feasible to reduce the variability in run chart specificity from varying numbers of data points by using the best box and cut box adjustments of the Anhøj rules.

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Appendix: Results table

Table 1: Signal limits, specificity, and sensitivity for the Anhøj and best box rules and borders for the cut box rules. N = number of data points in chart. C = lower limit for number of crossings, L = upper limit for longest run, for declaring random variation by the Anhøj and best box rules. Cbord and Lbord = Additional information for the cut box rules. When specified, parts of the border of the best box to retain to declare random variation. When not specified, cut box is identical to best box.

	Anhøj		Best box		Cut box		Specificity, shift = 0.0 SD			Sensitivity, shift = 0.8 SD		
N	C	L	C	L	Cbord	Lbord	Anhøj	Best box	Cut box	Anhøj	Best box	Cut box
10	2	6	2	6	3	5	0.9551	0.9551	0.9375	0.3103	0.3103	0.3786
11	2	6	3	7	4	6	0.9512	0.9414	0.9297	0.3493	0.3887	0.4211
12	3	7	3	6			0.9570	0.9326	0.9326	0.3677	0.4392	0.4392
13	3	7	3	6			0.9634	0.9324	0.9324	0.3628	0.4519	0.4519
14	4	7	3	6			0.9395	0.9280	0.9280	0.4051	0.4740	0.4740
15	4	7	4	7	6	6	0.9495	0.9495	0.9260	0.4046	0.4046	0.4806
16	4	7	5	8	6	7	0.9533	0.9352	0.9288	0.4146	0.4800	0.4993
17	5	7	5	7			0.9353	0.9353	0.9353	0.5069	0.5069	0.5069
18	5	7	5	7	6	6	0.9415	0.9415	0.9320	0.5030	0.5030	0.5256
19	6	7	5	7	6	5	0.9212	0.9433	0.9276	0.5370	0.5078	0.5351
20	6	7	6	7			0.9294	0.9294	0.9294	0.5372	0.5372	0.5372
21	6	7	7	8			0.9328	0.9291	0.9291	0.5447	0.5672	0.5672
22	7	7	6	7	7	6	0.9173	0.9332	0.9273	0.6121	0.5573	0.5902
23	7	8	6	7	7	6	0.9520	0.9318	0.9277	0.5322	0.5728	0.5983
24	8	8	6	7	7	6	0.9338	0.9293	0.9266	0.5646	0.5900	0.6084
25	8	8	6	7			0.9439	0.9262	0.9262	0.5536	0.6077	0.6077
26	8	8	9	9	10	7	0.9500	0.9375	0.9265	0.5488	0.5986	0.6298
27	9	8	9	8	10	7	0.9358	0.9358	0.9295	0.6221	0.6221	0.6397
28	9	8	9	8	11	7	0.9431	0.9431	0.9302	0.6118	0.6118	0.6589
29	10	8	10	8			0.9277	0.9277	0.9277	0.6382	0.6382	0.6382
30	10	8	11	10	12	9	0.9360	0.9279	0.9258	0.6299	0.6533	0.6617
31	11	8	11	9	14	8	0.9197	0.9376	0.9256	0.6958	0.6515	0.6880
32	11	8	11	8			0.9289	0.9289	0.9289	0.6843	0.6843	0.6843
33	11	8	11	8	12	7	0.9348	0.9348	0.9298	0.6766	0.6766	0.6912
34	12	8	11	8	13	7	0.9218	0.9382	0.9278	0.6982	0.6724	0.7141
35	12	8	12	8			0.9285	0.9285	0.9285	0.6920	0.6920	0.6920
36	13	8	13	9	15	8	0.9148	0.9375	0.9291	0.7442	0.6966	0.7265
37	13	8	14	10			0.9222	0.9270	0.9270	0.7356	0.6940	0.6940
38	14	8	13	8			0.9078	0.9269	0.9269	0.7548	0.7298	0.7298
39	14	8	15	11			0.9158	0.9254	0.9254	0.7475	0.7308	0.7308
40	14	8	15	9			0.9212	0.9260	0.9260	0.7430	0.7509	0.7509
41	15	8	15	9	17	8	0.9095	0.9370	0.9287	0.7846	0.7353	0.7642
42	15	8	14	8			0.9154	0.9260	0.9260	0.7782	0.7408	0.7408
43	16	8	14	8			0.9032	0.9266	0.9266	0.7938	0.7427	0.7427
44	16	8	17	10			0.9096	0.9272	0.9272	0.7884	0.7704	0.7704
45	17	8	17	9			0.8969	0.9270	0.9270	0.8249	0.7815	0.7815
46	17	9	17	9	19	8	0.9361	0.9361	0.9281	0.7687	0.7687	0.7961
47	17	9	17	9	20	7	0.9428	0.9428	0.9260	0.7576	0.7576	0.8045
48	18	9	19	12	20	11	0.9317	0.9261	0.9255	0.7750	0.7863	0.7896
49	18	9	19	10	21	9	0.9388	0.9321	0.9271	0.7648	0.7928	0.8099
50	19	9	19	9			0.9272	0.9272	0.9272	0.8082	0.8082	0.8082
51	19	9	19	9	21	8	0.9348	0.9348	0.9271	0.7976	0.7976	0.8233
52	20	9	19	9	21	7	0.9228	0.9404	0.9293	0.8131	0.7885	0.8238
53	20	9	21	11	23	9	0.9308	0.9310	0.9258	0.8034	0.8120	0.8292
54	21	9	21	10	23	8	0.9183	0.9360	0.9270	0.8413	0.8130	0.8385
55	21	9	21	9			0.9268	0.9268	0.9268	0.8315	0.8315	0.8315
56	21	9	21	9	23	8	0.9331	0.9331	0.9259	0.8228	0.8228	0.8465
57	22	9	23	12	25	11	0.9228	0.9268	0.9254	0.8360	0.8341	0.8403

Table 1: Signal limits, specificity, and sensitivity for the Anhøj and best *(continued)*

	Anhøj		Best box		Cut box		Specificity, shift = 0.0 SD			Sensitivity, shift = 0.8 SD		
N	C	L	C	L	Cbord	Lbord	Anhøj	Best box	Cut box	Anhøj	Best box	Cut box
58	22	9	23	10	24	9	0.9295	0.9285	0.9260	0.8280	0.8441	0.8506
59	23	9	23	10	26	8	0.9188	0.9390	0.9275	0.8600	0.8312	0.8595
60	23	9	23	9			0.9258	0.9258	0.9258	0.8520	0.8520	0.8520
61	24	9	23	9	24	8	0.9148	0.9311	0.9282	0.8636	0.8448	0.8529
62	24	9	25	11	27	9	0.9222	0.9304	0.9250	0.8560	0.8552	0.8703
63	25	9	25	10	27	9	0.9108	0.9323	0.9273	0.8839	0.8588	0.8732
64	25	9	26	11	27	10	0.9185	0.9270	0.9256	0.8766	0.8558	0.8628
65	25	9	26	10	27	9	0.9244	0.9290	0.9266	0.8699	0.8606	0.8709
66	26	9	27	12	29	10	0.9149	0.9283	0.9254	0.8798	0.8701	0.8796
67	26	9	27	10			0.9210	0.9257	0.9257	0.8736	0.8820	0.8820
68	27	9	27	10	29	8	0.9112	0.9354	0.9267	0.8973	0.8720	0.8923
69	27	9	28	11	29	8	0.9177	0.9335	0.9253	0.8912	0.8669	0.8931
70	28	9	29	14	30	13	0.9076	0.9252	0.9250	0.8998	0.8830	0.8841
71	28	9	29	11	31	9	0.9143	0.9305	0.9251	0.8941	0.8878	0.9008
72	29	9	29	10	30	9	0.9040	0.9294	0.9271	0.9147	0.8927	0.8979
73	29	9	30	11	31	10	0.9109	0.9276	0.9262	0.9092	0.8882	0.8943
74	29	9	30	10			0.9163	0.9264	0.9264	0.9041	0.8941	0.8941
75	30	9	31	12	32	9	0.9076	0.9302	0.9254	0.9115	0.8978	0.9076
76	30	9	31	11	34	8	0.9132	0.9362	0.9252	0.9067	0.8961	0.9155
77	31	9	31	10	33	9	0.9042	0.9322	0.9274	0.9243	0.9025	0.9142
78	31	9	32	11	33	8	0.9100	0.9336	0.9255	0.9197	0.8966	0.9182
79	32	9	33	13	37	11	0.9009	0.9275	0.9251	0.9262	0.9094	0.9157
80	32	9	33	11	35	9	0.9069	0.9310	0.9255	0.9218	0.9126	0.9238
81	33	9	33	10			0.8975	0.9269	0.9269	0.9370	0.9181	0.9181
82	33	9	34	11	36	10	0.9038	0.9284	0.9254	0.9329	0.9129	0.9203
83	34	9	33	10	36	7	0.8942	0.9404	0.9263	0.9385	0.9048	0.9279
84	34	9	35	11			0.9006	0.9258	0.9258	0.9346	0.9266	0.9266
85	34	9	35	11	38	8	0.9057	0.9363	0.9253	0.9310	0.9189	0.9352
86	35	9	35	10	36	9	0.8975	0.9294	0.9273	0.9440	0.9254	0.9295
87	35	9	35	10	38	8	0.9027	0.9359	0.9255	0.9406	0.9196	0.9369
88	36	9	37	12	38	10	0.8944	0.9276	0.9253	0.9454	0.9319	0.9362
89	36	9	37	11	39	9	0.8998	0.9316	0.9261	0.9421	0.9317	0.9411
90	37	9	38	12			0.8913	0.9252	0.9252	0.9533	0.9318	0.9318
91	37	10	37	10	39	9	0.9314	0.9314	0.9268	0.9321	0.9321	0.9413
92	38	10	39	13	41	12	0.9221	0.9262	0.9254	0.9381	0.9389	0.9413
93	38	10	39	11	40	10	0.9291	0.9270	0.9255	0.9331	0.9425	0.9449
94	39	10	39	11	42	8	0.9196	0.9365	0.9256	0.9473	0.9365	0.9500
95	39	10	39	10			0.9268	0.9268	0.9268	0.9428	0.9428	0.9428
96	39	10	39	10	41	8	0.9327	0.9327	0.9254	0.9382	0.9382	0.9502
97	40	10	41	12	42	9	0.9246	0.9303	0.9254	0.9435	0.9459	0.9520
98	40	10	41	11	44	9	0.9306	0.9322	0.9251	0.9391	0.9464	0.9556
99	41	10	42	12	43	10	0.9223	0.9282	0.9259	0.9518	0.9457	0.9516
100	41	10	41	10	42	9	0.9285	0.9285	0.9265	0.9478	0.9478	0.9510

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