Exact interpolation

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Satisfying the data values

$$f(x_i) = a_1\varphi_1(x_i) + a_2\varphi_2(x_i) + \ldots + a_k\varphi_k(x_i) + \ldots + a_n\varphi_n(x_i) \qquad i = 1, 2, \ldots, n$$

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_k(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_k(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_i) & \varphi_2(x_i) & \cdots & \varphi_k(x_i) & \cdots & \varphi_n(x_i) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \varphi_2(x_n) & \cdots & \varphi_k(x_n) & \cdots & \varphi_n(x_n) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \\ \vdots \\ a_n \end{bmatrix} = \mathbf{Fa}$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Satisfying the data values

$$f(x_i) = a_1\varphi_1(x_i) + a_2\varphi_2(x_i) + \ldots + a_k\varphi_k(x_i) + \ldots + a_n\varphi_n(x_i) \qquad i = 1, 2, \ldots, n$$

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_k(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_k(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_i) & \varphi_2(x_i) & \cdots & \varphi_k(x_i) & \cdots & \varphi_n(x_i) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \varphi_2(x_n) & \cdots & \varphi_k(x_n) & \cdots & \varphi_n(x_n) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \\ \vdots \\ a_n \end{bmatrix} = \mathbf{Fa}$$

$$\underset{n\times n}{\mathbf{F}}\ \underset{n\times 1}{\mathbf{a}} = \underset{n\times 1}{\mathbf{f}} \qquad\qquad \hat{\mathbf{a}} = \mathbf{F}^{-1}\mathbf{f}$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Satisfying the data values

$$f(x_i) = a_1\varphi_1(x_i) + a_2\varphi_2(x_i) + \ldots + a_k\varphi_k(x_i) + \ldots + a_n\varphi_n(x_i) \qquad i = 1, 2, \ldots, n$$

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_k(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_k(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_i) & \varphi_2(x_i) & \cdots & \varphi_k(x_i) & \cdots & \varphi_n(x_i) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \varphi_2(x_n) & \cdots & \varphi_k(x_n) & \cdots & \varphi_n(x_n) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \\ \vdots \\ a_n \end{bmatrix} = \mathbf{Fa}$$

$$\underset{n\times n}{\mathbf{F}}\ \underset{n\times 1}{\mathbf{a}} = \underset{n\times 1}{\mathbf{f}} \qquad\qquad \hat{\mathbf{a}} = \mathbf{F}^{-1}\mathbf{f}$$

$$\hat{f}(x) = \boldsymbol{\varphi}(x)^T\hat{\mathbf{a}} = \boldsymbol{\varphi}(x)^T\mathbf{F}^{-1}\mathbf{f} = \mathbf{q}(x)^T\mathbf{f} \qquad\qquad \mathbf{q}(x) = \mathbf{F}^{-1}\boldsymbol{\varphi}(x)$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Satisfying the data values

$$f(x_i) = a_1\varphi_1(x_i) + a_2\varphi_2(x_i) + \ldots + a_k\varphi_k(x_i) + \ldots + a_n\varphi_n(x_i) \qquad i = 1, 2, \ldots, n$$

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_k(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_k(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_i) & \varphi_2(x_i) & \cdots & \varphi_k(x_i) & \cdots & \varphi_n(x_i) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \varphi_1(x_n) & \varphi_2(x_n) & \cdots & \varphi_k(x_n) & \cdots & \varphi_n(x_n) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \\ \vdots \\ a_n \end{bmatrix} = \mathbf{F}\mathbf{a}$$

$$\underset{n\times n}{\mathbf{F}}\ \underset{n\times 1}{\mathbf{a}} = \underset{n\times 1}{\mathbf{f}} \qquad \hat{\mathbf{a}} = \mathbf{F}^{-1}\mathbf{f}$$

$$\hat{f}(x) = \boldsymbol{\varphi}(x)^T \hat{\mathbf{a}} = \boldsymbol{\varphi}(x)^T \mathbf{F}^{-1}\mathbf{f} = \mathbf{q}(x)^T \mathbf{f} \qquad \mathbf{q}(x) = \mathbf{F}^{-1}\boldsymbol{\varphi}(x)$$

$$\hat{f}(x) = f_1 q_1(x) + f_2 q_2(x) + \ldots + f_k q_k(x) + \ldots + f_n q_n(x)$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

$$\varphi_1(x) = 1, \quad \varphi_2(x) = x, \quad \varphi_3(x) = x^2, \quad \ldots, \quad \varphi_k(x) = x^{k-1}, \quad \ldots, \quad \varphi_n(x) = x^{n-1}$$

$$f(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_k x^{k-1} + \ldots + a_n x^{n-1}$$

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_i \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_i & x_i^2 & \cdots & x_i^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \\ \vdots \\ a_n \end{bmatrix} = \mathbf{Fa}$$

$$\mathbf{q}(x) = \begin{bmatrix} q_1(x) \\ q_2(x) \\ \vdots \\ q_k(x) \\ \vdots \\ q_n(x) \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_i & x_i^2 & \cdots & x_i^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ x \\ \vdots \\ x^{i-1} \\ \vdots \\ x^{n-1} \end{bmatrix}$$

$$\mathbf{q}(x) = \mathbf{F}^{-1} \boldsymbol{\varphi}(x)$$

$$\hat{f}(x) = f_1 q_1(x) + f_2 q_2(x) + \ldots + f_k q_k(x) + \ldots + f_n q_n(x)$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

$$f(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_k x^{k-1} + \ldots + a_n x^{n-1}$$

$$\hat{f}(x) = f_1 q_1(x) + f_2 q_2(x) + \ldots + f_k q_k(x) + \ldots + f_n q_n(x)$$

$$\mathbf{q}(x) = \begin{bmatrix} q_1(x) \\ q_2(x) \\ \vdots \\ q_k(x) \\ \vdots \\ q_n(x) \end{bmatrix} = \mathbf{F}^{-1}\boldsymbol{\varphi}(x) = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_i & x_i^2 & \cdots & x_i^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ x \\ \vdots \\ x^{i-1} \\ \vdots \\ x^{n-1} \end{bmatrix}$$

Analytical solution

$$q_k(x) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x - x_{k-1})}{(x_k - x_{k-1})} \frac{(x - x_{k+1})}{(x_k - x_{k+1})} \cdots \frac{(x - x_n)}{(x_k - x_n)} = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)}$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

$$f(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_k x^{k-1} + \ldots + a_n x^{n-1}$$

$$\hat{f}(x) = f_1\, q_1(x) + f_2\, q_2(x) + \ldots + f_k\, q_k(x) + \ldots + f_n\, q_n(x)$$

$$\mathbf{q}(x) = \begin{bmatrix} q_1(x) \\ q_2(x) \\ \vdots \\ q_k(x) \\ \vdots \\ q_n(x) \end{bmatrix} = \mathbf{F}^{-1}\boldsymbol{\varphi}(x)$$

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ x \end{bmatrix}$$

Missing the term

$$\frac{(x - x_k)}{(x_k - x_k)} = \frac{(x - x_k)}{0} = \infty$$

Analytical … **on**

$$q_k(x) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x - x_{k-1})}{(x_k - x_{k-1})} \frac{(x - x_{k+1})}{(x_k - x_{k+1})} \cdots \frac{(x - x_n)}{(x_k - x_n)} = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)}$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

$$f(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_k x^{k-1} + \ldots + a_n x^{n-1}$$

$$\hat{f}(x) = f_1\, q_1(x) + f_2\, q_2(x) + \ldots + f_k\, q_k(x) + \ldots + f_n\, q_n(x)$$

Analytical solution

$$q_k(x) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x - x_{k-1})}{(x_k - x_{k-1})} \frac{(x - x_{k+1})}{(x_k - x_{k+1})} \cdots \frac{(x - x_n)}{(x_k - x_n)} = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)}$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

$$f(x) = a_1 + a_2 x + a_3 x^2 + \dots + a_k x^{k-1} + \dots + a_n x^{n-1}$$

$$\hat{f}(x) = f_1\, q_1(x) + f_2\, q_2(x) + \dots + f_k\, q_k(x) + \dots + f_n\, q_n(x)$$

Analytical solution

$$q_k(x) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x - x_{k-1})}{(x_k - x_{k-1})} \frac{(x - x_{k+1})}{(x_k - x_{k+1})} \cdots \frac{(x - x_n)}{(x_k - x_n)} = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)}$$

$$q_k(x_k) = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x_k - x_j)}{(x_k - x_j)} = 1 \qquad i \neq k: \quad q_k(x_i) = \frac{(x_i - x_1)}{(x_k - x_1)} \frac{(x_i - x_2)}{(x_k - x_2)} \cdots \frac{(x_i - x_i)}{(x_k - x_i)} \cdots \frac{(x_i - x_n)}{(x_k - x_n)} = 0$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

$$f(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_k x^{k-1} + \ldots + a_n x^{n-1}$$

$$\hat{f}(x) = f_1\, q_1(x) + f_2\, q_2(x) + \ldots + f_k\, q_k(x) + \ldots + f_n\, q_n(x)$$

Analytical solution

$$q_k(x) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x - x_{k-1})}{(x_k - x_{k-1})} \frac{(x - x_{k+1})}{(x_k - x_{k+1})} \cdots \frac{(x - x_n)}{(x_k - x_n)} = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)}$$

$$q_k(x_k) = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x_k - x_j)}{(x_k - x_j)} = 1 \qquad i \neq k: \quad q_k(x_i) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x_i - x_i)}{(x_k - x_i)} \cdots \frac{(x - x_n)}{(x_k - x_n)} = 0$$

$$\hat{f}(x_i) = f_1\, q_1(x_i) + f_2\, q_2(x_i) + \ldots + f_i\, q_i(x_i) + \ldots + f_n q_n(x_i) = f_i$$

Exact interpolation

Case 2: ***n* = *m*** **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

$$f(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_k x^{k-1} + \ldots + a_n x^{n-1}$$

$$\hat{f}(x) = f_1\, q_1(x) + f_2\, q_2(x) + \ldots + f_k\, q_k(x) + \ldots + f_n\, q_n(x)$$

Analytical solution

$$q_k(x) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x - x_{k-1})}{(x_k - x_{k-1})} \frac{(x - x_{k+1})}{(x_k - x_{k+1})} \cdots \frac{(x - x_n)}{(x_k - x_n)} = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x - x_j)}{(x_k - x_j)}$$

$$q_k(x_k) = \prod_{\substack{j=1 \\ j \neq k}}^{n} \frac{(x_k - x_j)}{(x_k - x_j)} = 1 \qquad i \neq k: \quad q_k(x_i) = \frac{(x - x_1)}{(x_k - x_1)} \frac{(x - x_2)}{(x_k - x_2)} \cdots \frac{(x_i - x_i)}{(x_k - x_i)} \cdots \frac{(x - x_n)}{(x_k - x_n)} = 0$$

$$\hat{f}(x_i) = f_1\, \overset{0}{q_1(x_i)} + f_2\, \overset{0}{q_2(x_i)} + \ldots + f_i\, \overset{1}{q_i(x_i)} + \ldots + f_n \overset{0}{q_n(x_i)} = f_i$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

i	1	2	3
x_i	-1.0	0.0	1.0
f_i	4.0	6.0	2.0

$$q_1(x) = \frac{(x-x_2)}{(x_1-x_2)}\frac{(x-x_3)}{(x_1-x_3)} = \frac{(x-0)(x-1)}{(-1-0)(-1-1)} = \frac{1}{2}x(x-1)$$

$$q_2(x) = \frac{(x-x_1)}{(x_2-x_1)}\frac{(x-x_3)}{(x_2-x_3)} = \frac{(x+1)}{(0+1)}\frac{(x-1)}{(0+1)} = -(x+1)(x-1)$$

$$q_3(x) = \frac{(x-x_1)}{(x_3-x_1)}\frac{(x-x_2)}{(x_3-x_2)} = \frac{(x+1)(x-0)}{(1+1)(1-0)} = \frac{1}{2}x(x+1)$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution** $\mathbf{a}$

Example: Lagrange polynomial interpolation

i	1	2	3
x_i	-1.0	0.0	1.0
f_i	4.0	6.0	2.0

$$q_1(x) = \frac{1}{2}x(x-1) = \frac{x^2 - x}{2}$$

$$q_2(x) = -(x+1)(x-1) = -x^2 + 1$$

$$q_3(x) = \frac{1}{2}x(x+1) = \frac{x^2 + x}{2}$$

Exact interpolation

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Exact interpolation

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$$\hat{f}(x) = f_1 q_1(x) + f_2 q_2(x) + f_3 q_3(x)$$

$$\hat{f}(x) = 4\frac{x^2 - x}{2} + 6(-x^2 + 1) + 2\frac{x^2 + x}{2}$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

i	1	2	3
x_i	-1.0	0.0	1.0
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$$\hat{f}(x) = f_1 q_1(x) + f_2 q_2(x) + f_3 q_3(x)$$

$$\hat{f}(x) = 4\frac{x^2 - x}{2} + 6(-x^2 + 1) + 2\frac{x^2 + x}{2}$$

$$\hat{f}(x) = 2x^2 - 2x - 6x^2 + 6 + x^2 + x$$

Exact interpolation

Case 2: $n = m$ **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

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x_i	-1.0	0.0	1.0
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$$\hat{f}(x) = 2x^2 - 2x - 6x^2 + 6 + x^2 + x$$

$$\hat{f}(x) = -3x^2 - x + 6$$

Exact interpolation

Case 2: ***n* = *m*** **(as many equations as unknowns). Unique solution a**

Example: Lagrange polynomial interpolation

i	1	2	3
x_i	-1.0	0.0	1.0
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$$\hat{f}(x) = f_1\, q_1(x) + f_2\, q_2(x) + f_3\, q_3(x)$$

$$\hat{f}(x) = 4\frac{x^2 - x}{2} + 6(-x^2 + 1) + 2\frac{x^2 + x}{2}$$

$$\hat{f}(x) = 2x^2 - 2x - 6x^2 + 6 + x^2 + x$$

$$\hat{f}(x) = -3x^2 - x + 6$$

$$f(x_1) = f(1) = -3 + 1 + 6 = 4 = f_1 \qquad f(x_2) = f(0) = 6 = f_2 \qquad f(x_3) = f(1) = -3 - 1 + 6 = 2 = f_3$$