Cubic Splines

Octave Project 2

# Description of the problem:

This project’s goal is to code routines to construct cubic spline curves in both 2D and 3D. By definition a cubic spline curve is defined by given pointswith , there is a unique cubic spline through the given points satisfying . The concept is to select points in or , then using a regular mesh , finally construct and evaluate a cubic spline whose output will be plotted.

# Explanation of the method:

A cubic spline is a piecewise polynomial . Let

Where Moreover we assume that p is continuous with first and second continuous in . A basis for the space

Therefore can be expressed as a linear combination of elements in *B* in the following way.

Given points and with with , The cubic spline is determined by the solution in of the system of equations

This system of equation has unknowns and n equations, to fore a unique solution we must impose two more conditions .

Additionally, we have to remove all the shifted elements that do not satisfy the conditions in each of the equations. The final system is.

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We can solve the system of equations by doing a RREF

After the RREF the values we get in the last two columns will be the values for , the value of the column represent the values of *a* for and the values of column the ones of .

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# Examples:

Cubic spline though

With a regular mesh

Now the polynomial for x would be the values for from the first columns and the one for y the values from the second column.

# Observations:

Figure 6: Newton

Figure 5: Lagrange

Figure 4: Gauss-Jordan

Figure : Newton Method

Figure 2: Lagrange Method

Figure 1: Gauss-Jordan

As we can see in figures 1, 2 and 3, previous methods using Chebyshev meshes still work but they start doing strange things in the endpoints, as expected. However, once we change the mesh to a regular mesh to make a fair comparison with cubic splines, we can see in figures 4, 5 and 6 that Gauss-Jordan completely breaks and the other two methods create very strange curves. Therefore, we can conclude that cubic spline is a much more reliable interpolation method.

# Bibliography:

MAT300 Lecture Notes: lecture9.