Bezier Curves

Octave Project 3

# Description of the problem:

The mathematical problem we are trying to solve with these methods is interpolating 2D or 3D curves using a set of control points. Bezier curves are defined as a linear combination of Bernstein polynomials as follows. At the end we are using those control points to create the Bernstein polynomials that define the curve.

# Explanation of the method:

**Direct Evaluation:**

**De Casteljau:**

Given P0, P1 … Pn points in or the Bezier curve can be computed constructing a regular mesh of m + 1 nodes in [0, 1] and then applying linear interpolation recursively on for each node. The recursion is noted as follows:

Texto, Carta

Descripción generada automáticamenteAnd in code it looks like this

**Midpoint:**

Given P0, P1 … Pn points in or the Bezier curve can be split into two different curves, starting at P0  and ending at and starting at  and ending at Pn, where . We can construct those curves by evaluating by recursion:

**From this we can see how**  is convex and lies within the hull constructed by p0,p10, p20, p30 … and pn0. Which can be defined by the following formula:

Similarly is also convex and lies within the hull constructed by pn0 … , p31, p22, p13 and pn . Which can be defined by the following formula:

Furthermore, we can keep subdividing the curves and to approximate our curves with more precision and smoothness. In this algorithm the tₛ at which we split the curves will be ½, hence the name midpoint subdivision. Code implementation wise this algorithm has exponential growth which is something to avoid, that is why when implemented usually is restricted to a certain amount of iterations for the subdivision.

# Examples:

**Direct Evaluation:**

**De Casteljau:**

Given .

For :

For :

Gráfico, Gráfico de líneas

Descripción generada automáticamenteFor :

Gráfico, Gráfico de líneas

Descripción generada automáticamenteOutput of the program compared with the one obtained by hand.

Midpoint Subdivision:

# Bibliography:

MAT300 Lecture Notes: lecture11.