Time-Independent Perturbation Theory Applied to a Spin-1/2 Particle

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This review examines the application of the first-order time-independent perturbation theory to a spin 1/2 particle. Energy corrections and eigenstates are derived using a perturbative Hamiltonian that includes a small perturbation in the spin direction S_y. The main goal of this paper is to present how a degenerate two-level system can be analysed systematically using principles of quantum mechanics and perturbation theory.

I. INTRODUCTION

This study examines a spin 1/2 particle in the eigenbasis of $\hat{S}_z$, where the unperturbed Hamiltonian $H^{(0)}$ consists of the spin-squared operator $\mathbf{S}^2$, while the perturbation δH introduces a term in the S_y direction. The objective is to ascertain the impact of perturbation on the eigenstates and energy levels up to the first order in perturbation theory.

II. DISCUSSION

The Hamiltonian of the system is given by:

$$H(\lambda) = H^{(0)} + \lambda \delta H = \frac{E_0}{\hbar^2}\mathbf{S}^2 + \lambda \frac{2E_0}{\hbar}\hat{S}_y \tag{1}$$

where $\mathbf{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$ and λ is a small perturbation parameter. For a spin-1/2 particle, the operator $\mathbf{S}^2$ has an eigenvalue of $\frac{3}{4}\hbar^2$, which gives the unperturbed energy:

$$E^{(0)} = \frac{E_0}{\hbar^2}\hbar^2 \frac{1}{2}(\frac{1}{2}+1) = \frac{3}{4}E_0 \tag{2}$$

We proceed by introducing the perturbation $\delta H = \lambda \frac{2E_0}{\hbar}\hat{S}_y$, employing the Pauli matrices. It can be represented in matrix format as:

$$\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{3}$$

Thus, the full Hamiltonian including the perturbation becomes:

$$H(\lambda) = \frac{3}{4}E_0 I + \lambda \frac{E_0}{\hbar}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{4}$$

To solve this system, the perturbation term is diagonalized. The eigenstates of $\hat{S}_y$, which will also diagonalize δH, are given by:

$$\psi_\pm = \frac{1}{\sqrt{2}}(\uparrow \pm i\downarrow) \tag{5}$$

These eigenstates correspond to the eigenvalues ± 1 of $\hat{S}_y$, meaning the perturbation shifts the energy levels by $\pm\lambda E_0$. Therefore, the corrected energies to first order in λ are:

$$E_\pm = \frac{3}{4}E_0 \pm \lambda E_0 \tag{6}$$

The key observation is that the perturbation does not lift the degeneracy of the system, but instead splits the energy levels symmetrically, depending on the sign of the perturbation.

III. CONCLUSION

Time-independent perturbation theory has been applied to analyze the effects of a small perturbation in the S_y direction on a spin 1/2 particle. By diagonalizing the perturbation, it is concluded that the energy levels split symmetrically around the unperturbed energy level.