POLITECNICO
MILANO 1863

Orbital Mechanics

Scarpa Edoardo 220556 10664387
Torre Federico 218640 10645141
Viola Francesco 221924 10680788
Zara Andrea 225686 10698650

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1 Introduction

The interplanetary explorer mission objective is to reach the Near Earth Object number 33 with an intermediate flyby on the Earth itself. The assigned data and the time window are expressed in table 1. The optimum transfer is based on the lowest manoeuvre cost.

Departure planet	Earth
Flyby planet	Earth
Arrival NEO	33
Earliest departure	30 Jan 2033
Latest arrival	29 Jul 2067

Table 1: Assigned data, interplanetary mission

The planetary explorer mission objective is the analysis of disturbances action on the orbit evolution. The perturbations to be considered are the Earth second zonal harmonic and the solar radiation pressure. The behaviour of the Keplerian elements of the assigned orbit, when subjected to the two perturbations, is studied. At last is carried out a comparison of the adopted model results with a real mission orbit evolution.

2 Interplanetary mission

The objective of the interplanetary mission is to minimize the transfer cost from Earth to a NEO with an intermediate fly by on the Earth itself.

2.1 Time windows

2.1.1 Departure window

At first the assigned time window, with an extension of about 30 years, has been reduced to improve the analysis performances. After one Synodic period the geometry of the transfer does not repeat, due to the eccentricity of the target orbit. The synodic period is an appropriate measure for circular planet orbit because is defined as the time of two passage realtive to the Sun. Analyzing a longer time period, after 5 times the Synodic period the planets follows the same geometry again 1. This time interval, chosen for the transfer analysis, corresponds to about 8 years.

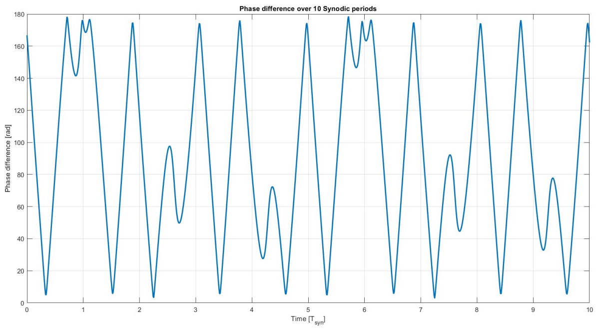


Figure 1: Phase difference over 10 Synodic periods

2.1.2 Time of flight

To identify the gravity assist and arrival time windows the time of flight of first and second Lambert problem is computed. The first Time of flight is fixed at one year because below the year the Lambert arc from Earth to Earth overlap to Earth's orbit and that condition is not compatible with the adopted approach. Instead the maximum Time of flight of first Lambert is fixed at 3 years, in this way the maximum energy of the transfer orbit is similar to Neo's orbit energy. For a longer time of flight the transfer cost is too high. For the second interplanetary leg the minimum fixed time of flight is null, because the two orbits almost intersect, while the maximum is 4 years, longer than one full period of the NEO to iniclude all the possible relative positions.

2.2 Adopted algorithm

in order to maximize the performance of the process and increase the observability of the program it is decide to use time windows that are variable for each step. The arrival time of first Lambert arc is defined as time included between the minimum and the maximum time of flight from departure. If the transfer cost of first arc is greater than the minimum of the previously computed total cost, the transfer is not considered and the program goes to the next step. After the first manoeuvre it is computed the second transfer arc with same rule and if also this Lambert's arcs is feasible the program proceeds computing the cost of Fly-by. It is added a control in Fly-by function that check if the pericenter of hyperbola is higher than the planet atmosphere. After the first analysis that uses a coarse grid more than one minima is identified. Considering only one minimum might cause the loss of the real absolute minima because the initial grid has a large time step between two possible solutions. For each found minimum a finest discretization is adopted. The interval around the minima is divided in more points and a new optimal value is chosen. This iterative process ends when the difference between two consecutive time steps is smaller than one second.

2.3 First solution

The first solution found is represented in figure 2 but it is not feasible using the patched conics method adopted for the preliminary analysis. Because the first Lambert arc is very near to the Earth orbit, the spacecraft behaviour should be modeled using a three body problem. The flyby trajectory is represented in figure 3, the pericentre of the hyperbola is located at the altitude of 100 km, limit value. Whether a larger margin from the Earth atmosphere is needed a different solution is obtained. However the total cost does not change much from the presented value. For the obtained transfer the manoeuvre costs and transfer time are expressed in table 2 and some parameters of the transfer trajectories in tables 3 and 4.

ΔV_{tot} [km/s]	$\Delta V_{departure}$ [km/s]	ΔV_{flyby}[km/s]	$\Delta V_{arrival}$ [km/s]
4.5060	6.026e-4	2.1879	2.3175
Transfer time [d]	Departure date	Flyby date	Arrival date
1099.74	25-Apr-2036 21:07:00	10-Jan-2037 13:35:52	30-Apr-2039 14:56:59

Table 2: Manoeuvre cost and dates for the designed transfer

	a [AU]	e [-]	i [°]	Ω [°]	ω [°]
First transfer arc	0.99999	0.01668	0	0	103.49
Second transfer arc	2.04744	0.52152	3.712	110.66	8.586

Table 3: Interplanetary trajectories parameters

	v_∞ [km/s]	$v_{pericentre}$ [km/s]	$h_{pericentre}$ [km]	Time inside SOI [h]
Incoming hyperbola	7.04e-4	11.099	100	181.7557
Outcoming hyperbola	7.304	13.287	100	33.4743

Table 4: Flyby hyperbolas parameters

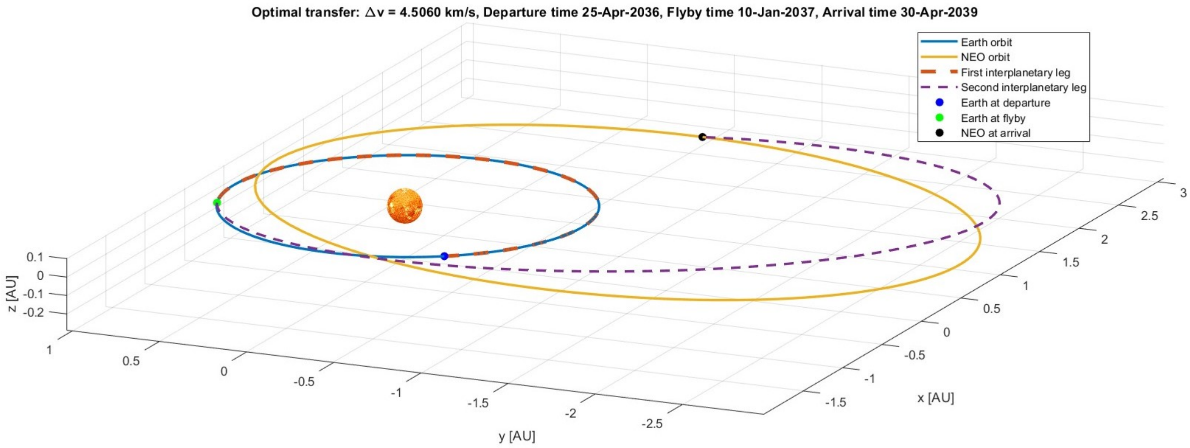


Figure 2: Interplanetary transfer

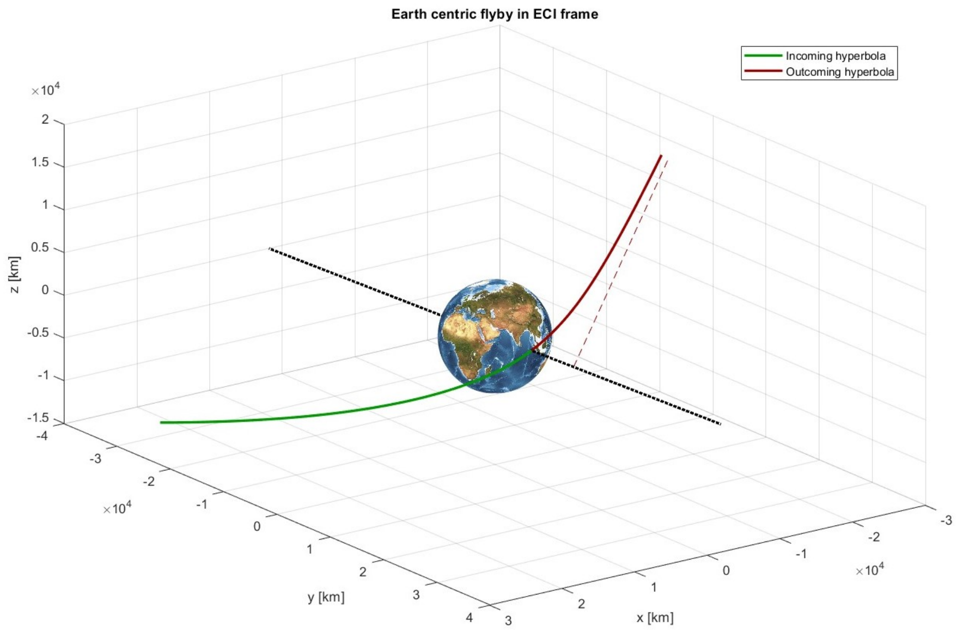


Figure 3: Flyby in ECI frame

2.4 Final solution

Imposing a limit of one year on the shortest time of flight of the first interplanetary leg the transfer trajectory leave the Earth orbit. The optimal transfer is represented in figure 4 while the transfer costs and dates in table 5 and some parameters of the transfer trajectories in table 6 and of the flyby hyperbola in table 7.

ΔV_{tot}[km/s]	$\Delta V_{departure}$[km/s]	ΔV_{flyby}[km/s]	$\Delta V_{arrival}$[km/s]
9.432	6.149	0.617	2.666
Transfer time [d]	Departure date	Flyby date	Arrival date
1538.5	12-Mar-2035 10:09:09	23-Jan-2037 13:40:43	28-May-2039 23:44:16

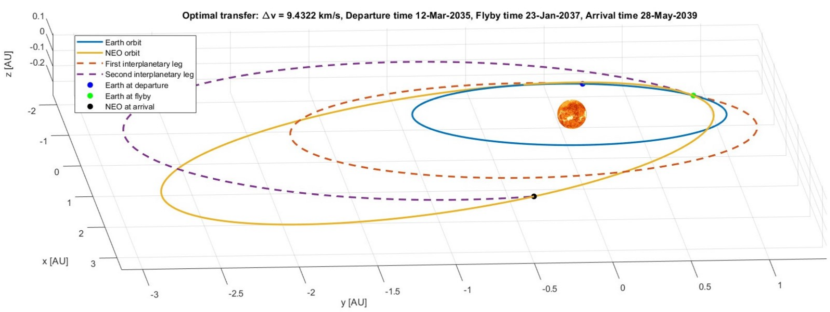
Table 5: Manoeuvre cost and dates for the optimal transfer

	a [AU]	e [-]	i [°]	Ω [°]	ω [°]
First transfer arc	1.576	0.3878	0	0	145.77
Second transfer arc	2.044	0.5191	3.442	123.90	354.33

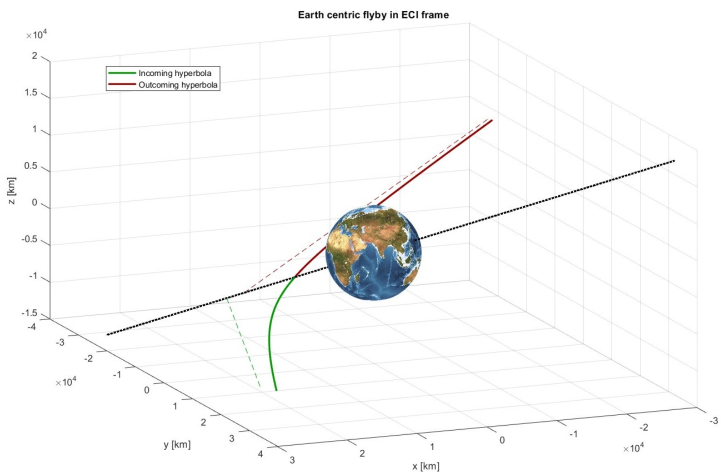
Table 6: Interplanetary trajectories parameters

	v_∞ [km/s]	$v_{pericentre}$ [km/s]	$h_{pericentre}$ [km]	Time inside SOI [h]
Incoming hyperbola	6.120	10.105	5956.89	39.6648
Outcoming hyperbola	7.092	10.722	5956.89	34.5495

Table 7: Flyby hyperbolas parameters



(a) Interplanetary transfer



(b) Flyby in ECI frame

Figure 4: Optimal solution

3 Planetary mission

The second assignment request the analysis of a planetary mission subjected to perturbations. The perturbations to be modeled are the Earth second zonal harmoinc and the solar radiation pressure. The assigned orbit Keplerian elements and the parameters of the SRP perturbation are expressed in *Table* 8. The value of Right ascension of the ascendent node, pericenter argument and initial true anomaly have been arbitrarily set to zero.

Semi-major axis [km]	Eccentricity [-]	Inclination [°]	$A/m[m^2/kg]$	c_r [-]
42164	0.0022	30.3001	7	1

Table 8: Assigned planetary mission parameter

The assigned spacecraft has a high area to mass ratio that increase the effect of the SRP disturbance. A possible application can be a deorbiting with sail passive Sun stabilization.

3.1 Perturbation model

During its motion, the spacecraft will be affected by external perturbations due to gravity (J_2 and third body perturbation), solar radiation and atmospheric drag. Since some disturbance sources have been neglected, the following discussion is referred to J_2 and SRP perturbations.

3.1.1 J$_2$ Perturbation

J_2 perturbation comes from the Earth oblateness: due to lack of simmetry, the attractive force acting on an orbiting spacecraft is not directed towards the center of the Earth. This force causes an acceleration acting on the spacecraft: by defining J_2 as the second zonal harmonic ($J_2 = 1.08 \cdot 10^{-3}$), each component can been modeled in an inertial-Earth-centered reference frame as follows

$$\underline{a}_{J_2x} = \frac{3}{2}\frac{J_2\mu R_E^2}{||r_{sc}||^5}\left(\frac{5z_{sc}^2}{||r_{sc}||^2} - 1\right)\underline{x}_{sc}$$
$$\underline{a}_{J_2y} = \frac{3}{2}\frac{J_2\mu R_E^2}{||r_{sc}||^5}\left(\frac{5z_{sc}^2}{||r_{sc}||^2} - 1\right)\underline{y}_{sc}$$
$$\underline{a}_{J_2z} = \frac{3}{2}\frac{J_2\mu R_E^2}{||r_{sc}||^5}\left(\frac{5z_{sc}^2}{||r_{sc}||^2} - 1\right)\underline{z}_{sc}$$

After a long time period, this perturbation causes a linear variation on the right ascension of the ascending node and the argument of perigee, which are known as *secular effects*:

- Nodal regression (the rotation is positive towards East):

$$\dot{\Omega} = -\frac{3}{2}\frac{nJ_2R_E^2}{a^2\left(1-e^2\right)^2}\cos i \longrightarrow per\ orbit:\ \Delta\Omega = \dot{\Omega}\frac{2\pi}{n}$$

  where $n = \sqrt{\frac{\mu}{a^3}}$ is defined as the mean motion. The nodes move eastwards if the orbit is retrograde, but they will move westwards if the orbit is prograde.

- Perigee precession (rotation is positive if it is consistent with the right-hand side rule):

$$\dot{\omega} = \frac{3nJ_2R_E^2}{2a^2\left(1-e^2\right)^2}\left(2 - \frac{5}{2}\sin^2 i\right) \longrightarrow per\ orbit:\ \Delta\omega = \dot{\omega}\frac{2\pi}{n}$$

  The perigee precession effects are comparable to a hula-hoop motion.

3.1.2 SRP Perturbation

SRP perturbation originates when the photons coming from the Sun impact the external surface of the spacecraft: for this reason, the spacecraft motion won't be disturbed when it enters the shadow of the Earth, but this case will be neglected in the following discussion. For Earth-centered orbits typical values of solar radiation pressure are between $10^{-3}N/m^2$ and $10^{-6}N/m^2$: due to the small amplitude, as close the spacecraft gets to the Earth surface as much negligible the effects of SRP become (i.e. in LEO the SRP effects are small if compared to atmospheric drag). By defining $p_{SR} = 4.5 \cdot 10^{-6}N/m^2$ the solar radiation pressure at a distance of 1 AU from the Sun, the acceleration undergone by the spacecraft can be modeled as:

$$\underline{a}_{SRP} = -p_{SR}(AU)^2 c_R \frac{\underline{r}_{SC-SUN}}{||\underline{r}_{SC-SUN}||^3}\epsilon$$

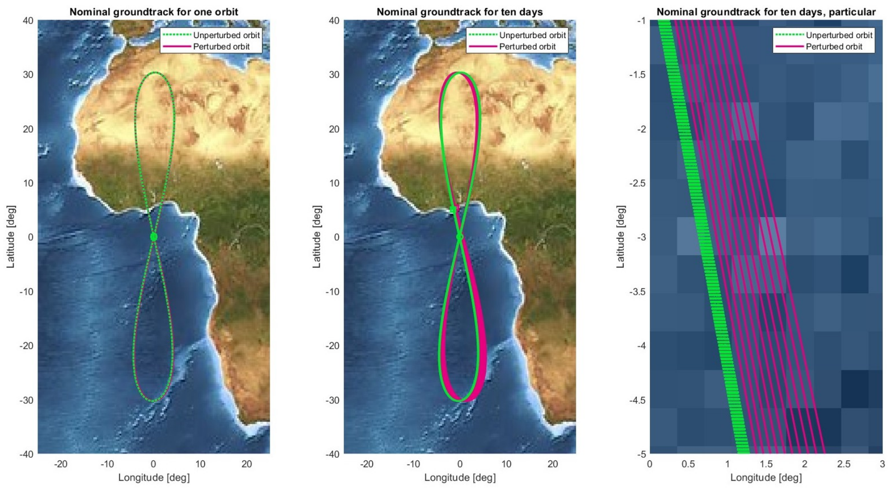
where ϵ is defined as the ratio between the spacecraft surface exposed to the sun and the spacecraft mass and c_R is a coefficient depending on the surface optical properties. Since the distance between the Sun and the satellite has been computed in an inertial-Earth-centered reference frame, the outcoming acceleration vector is reported to the same frame.

3.2 Ground track

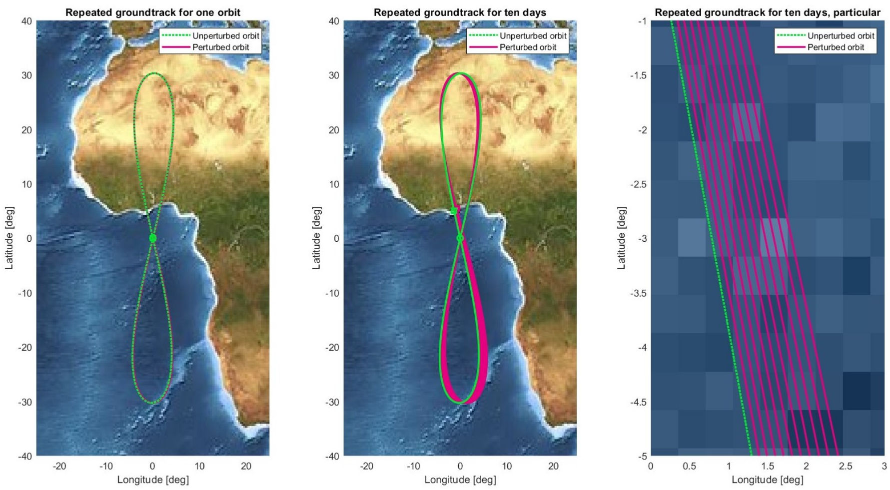
For the assigned orbit, assuming for simplicity zero initial right ascension of the Greenwich meridian, in figure 5 are displayed the nominal and repeated ground track. The repeated ground track is obtained with a modified value of semi-major axis such that the period of the new orbit is equal to the period of the earth revolution table 9.

Nominal semi-major axis [km]	Repeated ground track semi-major axis [km]
42164	42162.835

Table 9: Nominal and repeated semi-major axis value



(a) Nominal ground track



(b) Repeated ground track

Figure 5: Orbit ground track with and without perturbations for one orbit and 10 days

Only the one revolution ground track is displayed because the orbit period is almost one day, the one day ground track is equivalent. In the right plot is represented a particular of the ten days ground track to emphasise the non-repeatability of the first orbit.

3.3 Keplerian elements evolution

The effect of the considered perturbations on the nominal orbit is here studied. For each Keplerian orbital element the variations during a period of one year and 50 years are analyzed. In figure 6 is represented the evolution of the orbit subjected to the different perturbations.

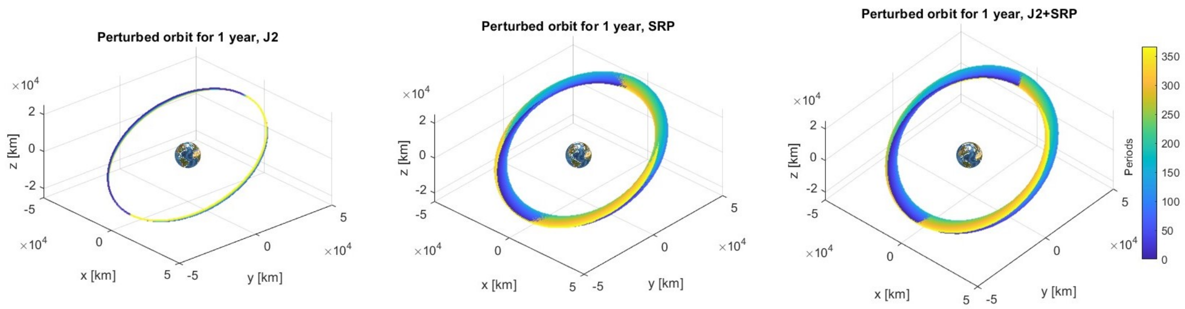


Figure 6: Evolution of the assigned orbit in 50 years

Semi-major axis

For the J2 perturbation the secular variation is zero. For SRP no eclipse condition is considered therefore the mean variation of semi-major axis is zero 8a.

Eccentricity

For the J2 perturbation the mean value is constant. SRP disturbance generate a large growth in the eccentricity of the orbit in the first part of the year that is balanced in the second part of the year where the direction of the sunlight is opposite 8b. This behaviour repeats every year 7a, a different result could have been obtained considering the eclipse phase.

Inclination

For the J2 perturbation the mean value is constant. SRP disturbance causes increasing oscillation of the inclination 8c, this behaviour is reflected in the 50 years variation where there are also decreasing oscillations 7b.

Right ascension of the ascendent node

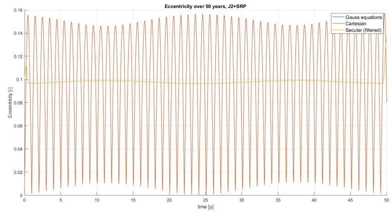
The RAAN variation is highly influenced by the J2 contribution, it follows a linear variation as expected 8d. The SRP variation introduces a smaller oscillation that, summed to J2, leave the rectilinear behaviour. The 50 year variation is an extension of the analyzed year one.

Pericenter argument

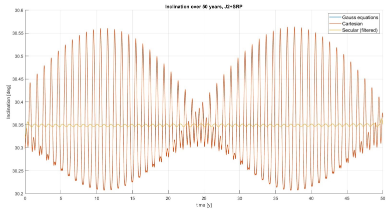
Similarly to the RAAN variation, J2 produces a linear growth of the pericenter argument, but with greater oscillation around the mean value 8e. The SRP disturbance introduces a similar linear growth, with higher slope. The initial rapid decrease in ω is caused by the arbitrary selection of the initial value of pericenter argument and departure date (that define the position with respect to the sun). During a longer period of time is evident a linear growth of the pericenter argument with large oscillation around the mean value.

True anomaly

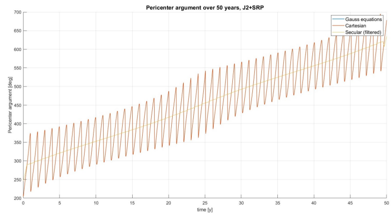
The true anomaly variation 8f due to perturbation is less recognizable than the other parameters because it is already non constant in the unperturbed case.



(a) e variation in 50 years

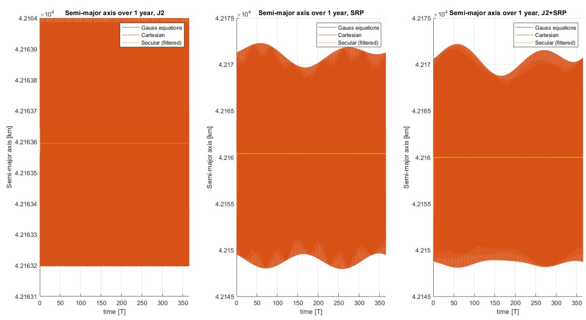


(b) i variation in 50 years

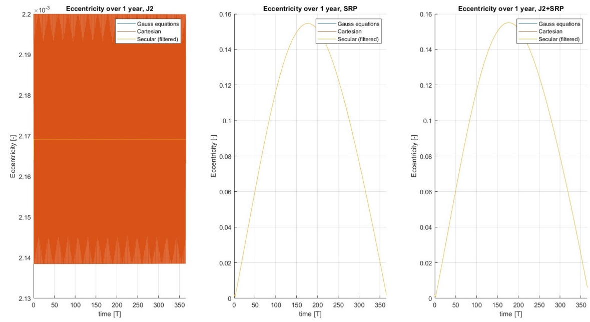


(c) ω variation in 50 years

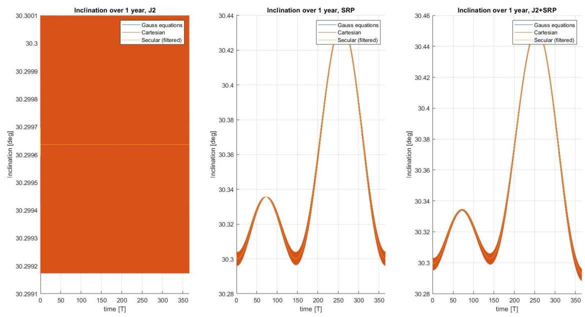
Figure 7: Keplerian elements variation in 50 years



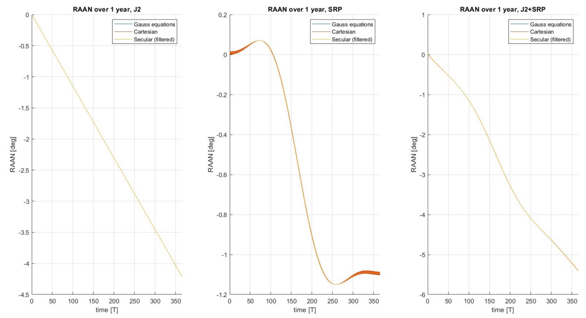
(a) Semi-major axis variation over one year



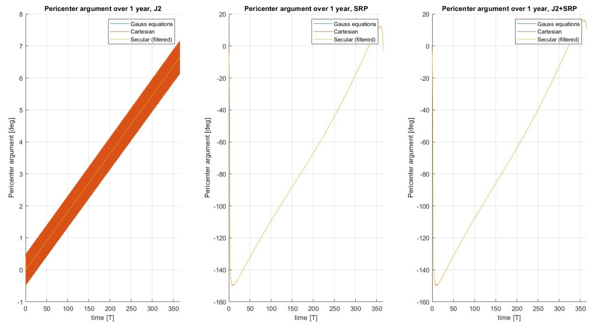
(b) Eccentricity variation over one year



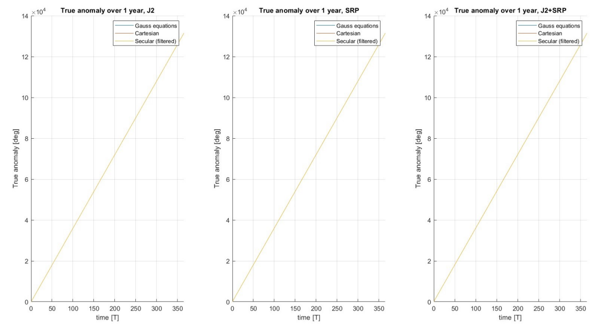
(c) Inclination variation over one year



(d) RAAN variation over one year



(e) Pericenter argument variation over one year



(f) True anomaly variation over one year

Figure 8: Keplerian elements variation in 1 year

3.4 Real mission comparison

An existing spacecraft with an orbit similar to the assigned one, in terms of semi-major axis, eccentricity and inclination, is the IRNSS 1E [1], satellite of the Indian Space Agency. In Celestrak [2] there are the TLEs of the satellite reported in table 10. To model correctly the SRP perturbation acting on the real satellite a different area to mass ratio has been considered. The main characteristic of the satellite and the used parameters are expressed in table 11, while the optical properties of the surface are unchanged. The evolution of Keplerian elements obtained using NASA horizon [3] and the perturbation model, for a period of 27 days are represented in figure 9. The J2 perturbation is the main disturbance term. The variation computed using NASA horizon for eccentricity, RAAN and pericenter argument is similar to the J2 perturbation one. Even the oscillation in the semi-major axis are similar but the secular therm is related to other non-modeled perturbations. The SRP effect, because of the smaller area to mass ratio, is

1	41241U	16003A	22365.63452234	-.00000312	00000+0	00000+0	0	9994
2	41241	30.4207	81.5679	0022158	186.7964	171.8093	1.00278935	25400

Table 10: IRNSS 1E TLE

Physical dimensions [m]	Dry mass [kg]	Lift-off mass [kg]	Area value $[m^2]$	Mass value [kg]
1.58 x 4.5 x 1.5	598	1425	6	1000

Table 11: IRNSS 1E characteristics

less significant than in the previous paragraph. In addition the NASA Horizon propagation consider the eclipse phase of the orbit, that generates a non zero mean variation. The inclination is the most different parameter, because of the constant mean value of the two considered perturbations.

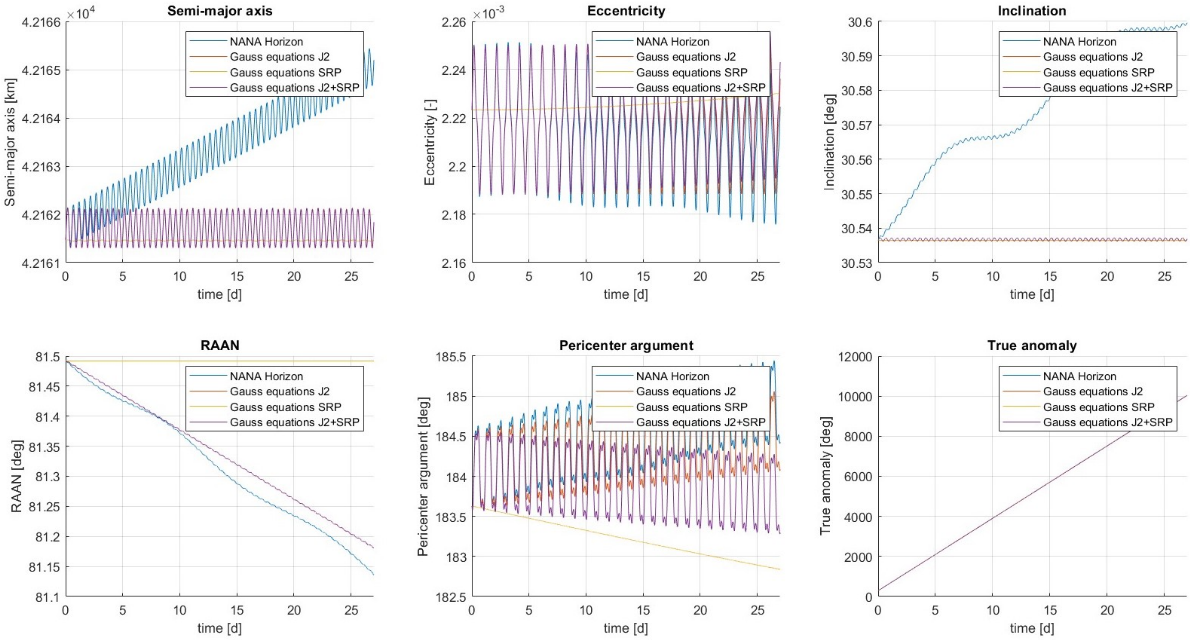


Figure 9: Evolution of Keplerian elements of IRSS 1E

4 Conclusion

Interplanetary mission - Adopted assumptions
The interplanetary explorer mission is a preliminary analysis that uses the Patched Conics Method.

- **Impulsive manoeuvre**
  The manoeuvre analyzed in the first assignment are assumed impulsive. Also the powered gravity assist is considered to happen instantaneously.
- **Sphere of Influence**
  The planets Sphere of Influence is considered infinite from the planet point of view but without extension from the Sun point of view.

The first solution found should be analyzed deeply, considering the effect of the third body gravitational field. The final solution cost of total manoeuvre is compatible with similar mission preliminary analysis, and slightly lower than the direct transfer cost without the intermediate flyby. The first Lambert arc is co-planar with the Earth orbit, only the second arc change the orbital plane and is characterized by an intermediate value of inclination between the Earth and the NEO orbit. Therefore a part of the plane change cost is given by the gravity assist.
A possible transfer optimization is the addition of multiple flybys on Venus (since its orbit is located near the NEO pericentre) or on Mars (since its orbit is near the NEO apocentre). Another improvement could be the mission analysis using multiple revolutions for the Lambert's arcs or the implementation of deep-space-manoeuvre.

Planetary mission - Adopted assumptions:
The perturbations included in the assigned model were the gravitational and the solar radiation pressure perturbation. For each of these disturbances the following assumptions has been made:

- **Gravitational perturbation**
  In order to compute a model for this disturbance source, only the second zonal harmonic has been considered, because J_2 is the dominant term in the perturbing potential approximation.
- **Solar Radiation Pressure perturbation**
  Since this kind of disturbance generates a very small perturbation in terms of magnitude, the assigned area-over-mass ratio (expressed as ϵ in section 3.1.2) has a large value. This allows the solar radiation pressure to produce an observable effect (if compared to J_2 perturbation) on the spacecraft orbit. Also, for SRP perturbation model, the orbit eclipse phases has been neglected.

Comparing the adopted model with the NASA horizon propagation for the satellite IRNSS 1E, that have a similar orbit to the assigned one, some difference emerged, in particular for the semi-major axis and the inclination variation. The missing third body perturbation effect, relevant in geostationary orbit, and the non considered eclipse phase for the SRP disturbance are the principal causes of these differences. For all the Keplerian elements variation the contribution of the second zonal harmonic is the most relevant, in particular for the RAAN and the pericenter argument where the linear variation is dominant.

References

[1] Indian Space Research Organization. "PSLV C31 IRNSS 1E". In: (2016).

[2] Dr T.S. Kelso. *Celestrak,* `https://celestrak.org`. 1985.

[3] Jet Propulsion Laboratory. *JPL Solar System Dynamics,* `https://ssd.jpl.nasa.gov`.