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Bibliographic Research on Space Debris Removal

Operation Research project

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Abstract

In recent decades, the proliferation of space assets has amplified the issue of orbital debris, leading to increased risks of collisions among debris and active spacecraft, perpetuating the Kessler Syndrome. This paper reviews optimization strategies for debris removal, emphasizing methods like Branch and Bound algorithms and stochastic programming. These approaches balance computational efficiency with solution optimality, crucial for addressing the complexities of debris removal in densely populated orbital regions. The study compares methodologies across recent literature, highlighting their effectiveness in mitigating debris-related risks and optimizing mission outcomes.

Keywords: Debris removal, optimization strategies, Branch and Bound algorithms, Debris mitigation, Debris path optimization.

Acronyms

ADR Active Debris Removal

ACO Ant Colony Optimization

EP Electric Propulsion

GA Genetic Algorithms

J2 Second Zonal Harmonic

SDC Space Debris Collection

TDTSP Time Dependent Traveling Salesman Problem

TSP Traveling Salesman Problem

TCO Timeline Club Optimization

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1 | Introduction

In recent decades, the proliferation of space assets has led to an increase in orbital debris. Despite mandatory mitigation measures, orbital congestion remains a critical issue. The risk of collisions among debris and active spacecraft poses a significant threat, perpetuating a cascade effect known as the Kessler Syndrome. This phenomenon, where debris collisions generate more debris than natural causes, aggravates the debris population over time.

Efforts to mitigate this risk involve sophisticated optimization strategies incorporating gravitational perturbations and optimal control theory. These methods aim to identify efficient debris removal sequences, particularly in densely populated orbital regions. Various methodologies are explored to address the complexities of selecting optimal debris removal sequences.

This paper reviews and compares different approaches to solving the debris removal sequencing problem. It discusses methodologies ranging from combinatorial optimization techniques, such as Branch and Bound algorithms, to stochastic programming methods like Genetic Algorithms (GA) and simulated annealing. These methods offer different trade-offs between computational efficiency and solution optimality for varying problem sizes.

The complexity of the debris removal problem necessitates exploring these diverse approaches to find robust solutions that balance debris removal effectiveness with mission constraints and operational feasibility.

2 | Literature Analysis

In general, heavy debris have the potential to create significantly more debris. Conversely, debris in densely populated orbital regions with high surface area pose the highest collision risks. These regions typically have inclinations in the ranges [95,100], [70,75], or [80,85] degrees, and semi-major axes in [800,850] km, [950,1000] km, or [1450,1500] km.

In [1], transfers between debris D_i and $D_j \neq D_i$ are computed using a perturbed Lambert's problem solver. This provides osculating orbital parameters at departure time t_i to position D_j at final time t_j under perturbed Keplerian dynamics. The initial $\Delta \mathbf{V}_i^1$ and final $\Delta \mathbf{V}_j$ velocities determine the transfer cost $\Delta V^{i,j} = \Delta V_i + \Delta V_j$.

The Branch and Bound method is used for finding debris sequences, employing a global search on a multi-dimensional grid $D \times D \times T \times \tau$ (dimensions of the decision vector). Global optimization methods are suitable for large dimensional problems, aiming to find the globally best solution quickly, which is then refined using local optimization for accuracy.

The primary mission objective is to remove mass from orbits, maximizing debris mass removal while minimizing fuel usage (total ΔV cost). The objective function combines these criteria:

$$\begin{aligned} J_i &= \alpha \Delta V_i + \beta m_i + \gamma A_i \\ J &= \sum_{i \in D} J_i \end{aligned} \tag{2.1}$$

where α, β, γ are weighting factors (typically in units: α in s/m, β in 1/kg, γ in 1/m^2) and m_i and A_i are the mass and surface of the debris respectively.

To identify significant transfer features, a grid search with pruning is used for exhaustive exploration. Each debris-to-debris transfer I depends only on initial and final debris positions and velocities at $T^{0,I}$ and $T^{f,I}$, allowing for parallelizable global search using GPU or similar architectures.

Pruning constraints optimize the search by quickly discarding unlikely solutions without hindering finding good ones. Rendezvous transfers employ maneuvers such as inclination change ΔV^{inc}, altitude change ΔV^{Hoh}, and phasing correction ΔV^{ϕ}. A threshold $\Delta V^{\max}$ determines whether a transfer branch is discarded or computed via perturbed Lambert's problem.

In [1], a perturbed transfer model efficiently evaluated debris-to-debris transfers using grid search to construct sequences of 5- to 10-debris transfers. Natural dynamical perturbations and intermediate maneuvers further optimize transfer costs.

In [2], a mission targets the removal of debris from a catalog of 3629 items using the J2 perturbation model. The Branch and Bound method is employed to determine the optimal sequence for debris removal. Post-sequence establishment, a multiple homotopy method is proposed for low-thrust trajectory optimization between neighboring debris. This method involves three homotopy processes with parameters embedded in equations governing motion, thrust magnitude, and performance index.

The primary performance index aims to maximize the total score of removed debris:

$$\max J_a = \sum_{i=1}^{N} s_i \tag{2.2}$$

where s_i denotes the score of the i-th removed debris, and N is the count of removed debris. The secondary goal is to minimize mission duration:

$$\min J_b = T_f \tag{2.3}$$

with T_f as the final rendezvous time with the last debris. Thirdly, minimizing total spacecraft fuel consumption:

$$\min J_c = \sum_{i=1}^{N} \Delta m_i \tag{2.4}$$

where Δm_i is fuel consumption during the i-th rendezvous. The priority order of these indices is $J_a > J_b > J_c$, with subsequent indices considered only in case of ties.

The Branch and Bound method recursively subdivides the search space to minimize these indices, pruning suboptimal solutions using bounds on performance metrics. Each node branches into three possibilities until reaching an upper limit, at which candidate solutions are evaluated and pruned based on established bounds.

Given orbital mechanics, changing orbital planes requires significantly more ΔV than altering orbital shapes within a plane, justifying close-plane rendezvous strategies. Standard transfer time between debris is set at 50 hours, evaluating remaining candidates based on orbital plane angles relative to current debris positions at standard times. Fifteen candidates with minimal angles are initially selected, with subsequent evaluations considering phase angles and ΔV, recursively choosing three candidates per sequence iteration until mission constraints are met.

Fuel consumption estimates for each rendezvous are derived from relative orbital elements, optimizing debris removal sequence via Branch and Bound. Numerical simulations confirm the efficacy of this approach in identifying optimal debris removal sequences, supported by the robustness of the proposed multiple homotopy method in low-thrust transfer optimizations.

In [3], an evolutionary optimization approach using four-impulse transfers incorporating J_2 perturbation is applied. The object sequencing problem is analogized to a Traveling Salesman Problem (TSP), with an Ant Colony Optimization (ACO) algorithm for longer sequences analysis. The mission prioritizes minimizing total ΔV and optimizing mass and mission time.

The TSP perspective treats debris removal akin to minimizing path length across selected objects. ACO optimizes this by constructing solutions via artificial ants guided by pheromone and heuristic information. Iterative steps include solution construction, local search for refinement, and pheromone update, terminating under specific conditions.

For an Active Debris Removal (ADR) mission with chaser labeled 0 and tasked to remove M_d debris items among candidates $M = \{1, 2, \ldots, N\}$ over time horizon T, each transfer's ΔV cost and servicing times are pivotal. The integer programming formulation aims to:

$$\sum_{i \in M'} \sum_{j \in M'} \Delta V_{ij} x_{ij} - 0.4 M_d \tag{2.5}$$

subject to constraints ensuring feasible path traversal, single-object removal, time constraints, and non-negative variables.

Enhanced ACO integrates 2-Opt, insertion, and swap operators for local search, enhancing solution quality.

Computational efficiency is ensured via pruning techniques, enabling rapid evaluation of multiple mission scenarios within reasonable time frames, even for larger object sets.

In [4], the Timeline Club Optimization (TCO) algorithm is introduced to tackle the Time Dependent Traveling Salesman Problem (TDTSP) for multiple debris removal missions. Unlike traditional methods, TCO utilizes an innovative approach inspired by ACO, enhancing performance through a structured timeline-based solution exploration.

The core challenge addressed by TCO is optimizing both the sequence of debris removal (a discrete problem) and the timing of transfers (a continuous problem). Traditional ACO methods struggle with the dynamic nature of the TDTSP, often compromising between solution accuracy and computational efficiency. TCO introduces the Timeline Club, a novel data structure that replaces ACO's pheromone matrix. This club records pivotal information from elite solutions, guiding the generation of new solutions based on historical success.

Key steps in the TCO algorithm include the generation of new debris sequences guided by probabilities derived from Timeline Club data. Initially, solutions are generated randomly, and the Elitist Club retains the best solutions, ensuring uniqueness in debris sequences. The Timeline Club manages timelines associated with debris objects, influencing the probability of selecting objects based on past successful solutions.

The probability p_i of choosing the first debris object i is determined by:

$$p_i = \begin{cases} 1/N, & q \leq q_{\text{first}} \\ f(i)/\sum_{u=1}^{N} f(u), & q > q_{\text{first}} \end{cases}$$

where q is a uniformly distributed random number, and $f(i)$ depends on the optimization index of previous solutions with debris object i as the first removed.

The TCO algorithm utilizes a weight function $g(i, j)$ to determine the probability p_{ij} of transitioning from debris object i to j:

$$p_{ij} = \frac{g(i, j)}{\sum_{u \in S} g(i, u)}, \quad j \in S$$

where S is the set of unremoved debris, and $g(i, j)$ incorporates factors like transfer cost and optimization indices.

The TCO algorithm continues this process, refining solutions through iterative updates of the Timeline Club until termination conditions are met, typically after a fixed number of iterations.

Simulation results demonstrate that TCO outperforms traditional heuristic methods such as ACO and GA in terms of solution quality for multiple debris removal missions. The paper highlights the adaptability of TCO to various TDTSP constraints and recommends optimal parameter settings based on its experimental findings.

In summary, the TCO algorithm effectively integrates past experience through the Timeline Club and uses probabilistic selection mechanisms to optimize both site and time sequences in the TDTSP, offering superior performance compared to conventional methods.

In [5], a method is proposed to optimize debris selection and trajectories for space debris collection missions. The approach combines combinatorial optimization for debris selection with functional optimization for orbital maneuvers, employing a specific transfer strategy with impulsive maneuvers and a Branch and Bound algorithm.

The Space Debris Collection (SDC) problem involves non-linear, time-dependent graph optimization. It includes:

- Continuous trajectory optimization using optimal control theory between debris.
- Combinatorial debris selection akin to the TSP, managed through graph theory.

The methodology for solving the SDC problem follows these steps:

1. Formulate the problem considering mission specifics.
2. Simplify by defining a transfer strategy with impulsive maneuvers.
3. Linearize around an initial reference solution and iterate for convergence.
4. Optimize the debris selection and trajectories using the Branch and Bound algorithm.

The optimal control problem for transferring from debris i to debris j is:

$$\min_{t_i^d, t_j^a, U_{ij}(t)} C_{ij}\left(t_i^d, t_j^a, X\left(t_i^d\right), X\left(t_j^a\right), \left[U_{ij}(t), t_i^d \leq t \leq t_j^a\right]\right)$$

subject to:

$$\begin{cases} Y\left(t_i^d\right) = Y_i\left(t_i^d\right) \\ Y\left(t_j^a\right) = Y_j\left(t_j^a\right) \\ \dot{X}(t) = f\left[X(t), U_{ij}(t), t\right], \quad t_i^d \leq t \leq t_j^a \\ T_{ij} = t_j^a - t_i^d \leq T_{ij\,\max} \end{cases}$$

The TSP is extended in the SDC problem by:

- Selecting n debris from N (n-shortest path problem).
- Global duration constraint: $T \leq T_{\max}$.
- Minimum debris processing duration T_{deorb}.

In graph terms, debris are nodes, and transfers are directed edges. The mathematical formulation includes:

- Binary selection variable for edges: s_{ij}
- Arrival edges at node k: $x_k = \sum_i s_{ik} \leq 1$
- Departure edges from node k: $y_k = \sum_j s_{kj} \leq 1$
- Product of x_k and y_k: $z_k = x_k \cdot y_k$
- Binary selection variable for nodes: $s_k = x_k + y_k - z_k = \min\left(1, \sum_i s_{ik} + \sum_j s_{kj}\right)$

The Branch and Bound algorithm explores the binary tree of all combinations, using depth or breadth search strategies. It aims to find the optimal solution while minimizing computational memory usage. For the SDC mission, breadth search with constrained variable separation showed superior performance, evaluating nodes to efficiently determine the best solution.

This paper proposes an effective approach to optimize debris selection and trajectories for SDC missions, leveraging specific transfer strategies, linearization, and the Branch and Bound algorithm.

3 | Conclusion

The removal of space debris presents a significant challenge requiring sophisticated optimization strategies. This conclusion compares the methodologies and findings from several studies: [1–5].

3.1. Methodologies

Each paper adopts distinct methodologies tailored to the complexities of debris removal:

- [1] and [5] both utilize a Branch and Bound approach but apply it differently. [1] paper focuses on a grid search over a multi-dimensional decision space ($D \times D \times T \times \tau$), while [5] emphasizes a more structured approach with global optimization followed by local refinement.
- [2] explores low-thrust trajectories using a Branch and Bound method for optimal debris removal sequences. It integrates a multiple homotopy method for trajectory optimization, contrasting with the impulsive maneuver strategy of other papers.
- [3] introduces evolutionary optimization and ACO to solve debris removal problems. It emphasizes the analogy to the TSP and uses ACO for longer sequence analysis, demonstrating adaptability to different mission constraints.
- [4] proposes the (TCO) algorithm, an innovative approach inspired by ACO. It focuses on optimizing both debris sequence and timing, leveraging historical success records through a timeline-based solution exploration.

3.2. Objectives and Criteria

The primary objectives across the papers include minimizing debris mass in orbits while conserving fuel and optimizing mission time:

- [1] and [5] prioritize minimizing total ΔV cost and debris mass removal, incorporating factors such as orbital dynamics and impulsive maneuvers.
- [2] aims to maximize the total score of removed debris and minimize mission duration and fuel consumption, reflecting its focus on low-thrust trajectory optimization.
- [3] emphasizes minimizing total ΔV and optimizing mass and mission time through evolutionary and ACO-based approaches.

- [4] addresses the TDTSP, optimizing both sequence and timing of debris removal to enhance overall mission efficiency.

3.3. Performance and Computational Strategies

Computational strategies vary significantly among the papers:

- [1] and [5] employ Branch and Bound with different search strategies (grid search vs. global-local optimization), balancing between exhaustive exploration and computational efficiency.
- [2] uses a Branch and Bound method tailored for low-thrust trajectories, enhancing solution quality through multiple homotopy methods.
- [3] introduces evolutionary optimization and ACO, leveraging their ability to handle large-scale optimization problems efficiently.
- [4] innovates with the TCO algorithm, integrating historical solution data to guide probabilistic solution exploration, demonstrating superior performance in dynamic optimization scenarios.

3.4. Conclusion

In conclusion, while each paper adopts unique approaches to address the challenges of space debris removal, they collectively highlight the importance of tailored optimization strategies based on mission-specific constraints. The methodologies range from impulsive maneuver strategies with rigorous mathematical formulations to low-thrust trajectory optimizations and evolutionary algorithms. Future research could benefit from integrating these diverse approaches to develop more comprehensive and robust debris removal strategies.

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List of Symbols

Variable	**Description**
D	Set of debris
T	Set of time instances
τ	Set of transfer sequences
D_i, D_j	Debris indices
$\Delta\mathbf{V}_i^1, \Delta\mathbf{V}_j$	Initial and final velocities
$\Delta V^{i,j}$	Transfer cost
α, β, γ	Weighting factors
m_i	Mass of debris i
A_i	Surface area of debris i
J_i	Objective function for debris i
J	Total objective function
$\Delta V^{\text{inc}}, \Delta V^{\text{Hoh}}, \Delta V^{\phi}$	Maneuver types
$\Delta V^{\max}$	Threshold for transfer branch
Δm_i	Fuel consumption
J_a, J_b, J_c	Performance indices
s_i	Score of debris i
T_f	Final rendezvous time
N	Number of debris candidates
M_d	Number of debris items to remove
x_{ij}	Binary decision variable
p_i	Probability of choosing debris object i
p_{ij}	Probability of transitioning from i to j
$g(i,j)$	Weight function
S	Set of unremoved debris
C_{ij}	Cost function for transfer from debris i to j
$Y(t_i^d), Y(t_j^a)$	State variables
$U_{ij}(t)$	Control variable
T_{ij}	Transfer time
$T_{\max}, T_{\text{deorb}}$	Maximum duration and debris processing duration
$s_{ij}, x_k, y_k, z_k, s_k$	Binary selection variables