Comparison between the Panelling Method and the Jorgenson Method

Launch System Flipped Class Report

Panelling Project Team - PANAIR

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1 Introduction

This report conducts a concise analysis and comparison of results derived from two distinct methodologies employed to estimate static aerodynamic characteristics for slender bodies with circular cross-sections and angle of attack set at 10°. The focus is on studying the evolution of C_P (pressure coefficient) and C_N (normal force coefficient) under these conditions. The selected methods for comparison are the Paneling Method 2 and Jorgensen's Method 3, both specifically applicable to supersonic Mach number scenarios. The investigation employs the initial conditions outlined in Table 1, and the analysis is conducted for two varying cone lengths.

Parameter	Value
Angle of Attack	10°
Diameter	1 m
Altitude	0 m
Length	5
Cone lengths	0.5 , 1 m
Mach Number	4

Table 1: Initial Condition

2 Paneling Method

It's important to notice that, while the Paneling method demonstrates suitability for high Mach numbers, it lacks appropriateness when dealing with low-profile Mach numbers. The mesh surfaces (with a square configuration) of both the body and the cone were plotted based on the initial data, as illustrated in Figure 1. The center point of each panel was identified, serving as reference for establishing the normal vector $\hat{n}$ for each panel, the incoming flux is shown in blue. Subsequently, an assessment was conducted for each panel to determine whether the surface was wetted or not, taking into account the angle ϕ, Equation 1. Afterwards, the local angle of attack θ was computed, as expressed in Equation 2.

$$\phi = \arccos \frac{\mathbf{v} \cdot \hat{n}}{|\mathbf{v}|} \tag{1}$$

$$\theta = \arcsin \frac{\mathbf{v} \cdot \hat{n}}{|\mathbf{v}|} \tag{2}$$

Where $\mathbf{v}$ is the upstream velocity. The value of pressure coefficient (C_P) is obtained from Newton method 3 and modified Newton method 4:

$$C_{P_{Newton}} = 2 * \sin^2 \theta \tag{3}$$

$$C_{P_{Newton-mod}} = C_{P,max} * \sin^2 \theta \tag{4}$$

$$C_{P,max} = \frac{2}{M^2\gamma}\left\{\left[\frac{(\gamma+1)^2M^2}{4\gamma M^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \times \left[\frac{1-\gamma+2\gamma M^2}{\gamma+1}\right] - 1\right\} \tag{5}$$

In the Equation 5 M is the Mach number in the upper stream flow, while the specific heat ratio was considered $\gamma = 1.4$, taken as constant. However, this should be reevaluated if changes in the atmospheric parameters should be considered. In the case where $\phi < 90°$ the pressure coefficient $C_P = 0$, otherwise the above formulas are used.

The C_N for the panelling method was calculated using:

$$d\overline{F} = -C_P q A \hat{n} \tag{6}$$

where A is the surface area, q is the dynamic pressure and A_{ref} is the reference area of cross section. To compute the normal force N, the normal component of the force F (in magnitude), was taken; more specifically, F was obtained by summing up all the $d\overline{F}$ contributions of each panel.

$$C_N = \frac{N}{qA_{ref}} \tag{7}$$

Once the overall normal force has been computed, the normal coefficient is considered as shown in Equation 7.

3 Jorgensen's Method

For the validation of the paneling, Jorgensen's method is used to compute the normal force N and the normal force coefficient C_N.

$$C_N = \frac{A_{\text{base}}}{A_{\text{ref}}} \sin(2\alpha)\cos(\alpha/2) + \eta C_{D,N} \frac{A_p}{A_{\text{ref}}} \sin^2(\alpha) \tag{8}$$

where α is the angle of attack, A_{ref} is the reference area considered for the cross-sectional area, A_{base} is the base area, and $A_p = (L_{\text{nose}}/2 + L_{\text{cylinder}}) \times D$ is the planform area. Additionally, η is the crossflow drag proportionality factor, and $C_{D,N}$ is the drag coefficient of the cylinder section. For this case, η has been taken as 0.6 based on L/d as define in [2]. The $C_{D,N}$ value is considered to be around 1.3, as taken from the literature [2]. The $C_{D,N}$ value also depends on the Reynolds number (Re), and the corresponding Re value was selected based on the initial parameters.

The obtained normal coefficient was utilized in the formula for normal force (Equation 9).

Parameter	**Value**
$C_{D,N}$	1.3
η	0.6
$\rho[kg/m^3]$	1.225
$A_{base}[m^2]$	0.785

Table 2: Data for Jorgensen Method

$$N = \frac{1}{2} C_N \rho (MV_s)^2 A_{ref} \tag{9}$$

with M the mach number and V_s the speed of sound. As it can be noticed from the Equation 8, in the Jorgensen method, the differences in values of C_N and therefore of N, from one case to the other, consists in the change of value of A_p with respect to the length of the nose and of the cylinder.

4 Results and Comparison

C_P values were computed for both the nose and the cylindrical body. To enhance the precision of the normal force coefficient calculation and facilitate mesh simplification, a square mesh was employed instead of a triangular one as shown in Figure 1. Table 3 show that an increase of the nose length corresponds to a decrease of C_P subsequently influencing the C_N as discussed in the conclusion.

Nose Length $[m]$	**0.5**	**1**
Newton	0.0302	0.0302
Modified Newton	0.0270	0.0270

Table 3: C_p cylinder

Nose Length $[m]$	**0.5**	**1**
Newton	0.3755	0.13
Modified Newton	0.3364	0.1165

Table 4: C_p cone

Nose Length $[m]$	**0.5**	**1**
Newton $[MN]$	0.17380	0.12325
Modified Newton $[MN]$	0.1557	0.11042

Table 5: Normal Force[MN] values from paneling method

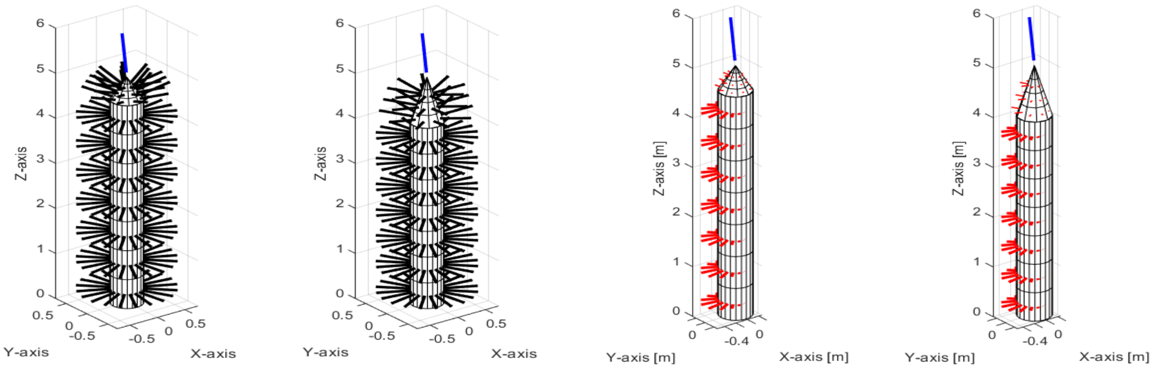
Nose Length $[m]$	**0.5**	**1**
Newton	0.7275	0.5526
Modified Newton	0.6518	0.4951

Table 6: C_N values from paneling method

Nose Length $[m]$	**0.5**	**1**
C_N	0.592	0.578

Table 7: C_N values from Jorgensen method

It should be mentioned that the differences in the results from the two methods can be attributed to the free variables. The choices made include the values of $C_{D,N}$ and η, which were obtained from the literature [2] as shown in Table 2. The Panelling Method introduces additional choices such as the number of elements and mesh size. In Table 6 and Table 7 are shown the results for both methods, a higher C_N is obtained for the case of shorter cone, this can be expected due to a reduction of the body's section maintaining a constant total length (for the case of $1m$ cone). Higher error is obtained for the short cone case, indicating that a finer mesh accuracy should be applied to the cylinder. It was considered that the obtained error was sufficiently small, resulting in a reasonable computation time. However, for those interested in achieving a more refined solution and minimizing errors further, these parameters should be adjusted. The obtained values are also influenced by the atmospheric model used [5].



(a) Mesh for cone of $0.5m$ (b) Mesh for cone of $1m$ (c) Surface Forces for cone of $0.5m$ (d) Surface Forces for cone of $1m$

Figure 1: Meshes and Surfaces for both nose cases

Bibliography

[1] Joseph A. Schetz., *Hypersonic and High-Temperature Gas Dynamics*, Second. Ed. American Institute of Aeronautics and Astronautics, 2006, pp. 53-62.

[2] Leland H. Jorgensen., *Prediction of static aerodynamic characteristics for Space-Shuttle-like and other bodies at angle of attack from* 0° *to* 180°, Ames Research Center (NASA), Jan. 1973.

[3] Pasquale,M. , Sforza, *Manned spacecraft design principles*, 4^{th} Elsevier Butterworth-Heinemann, pag. 252-255, 2022.

[4] Filippo Maggi, Courses in Launch System, Academic Year 2023/2024.

[5] NASA, NOAA, *U.S. Standard Atmosphere*, Oct. 1976.