1 Java

ArrayList
A resizable array.

LinkedList
A list of elements where ea. element points to the next.

ArrayDeque
A resizable array that acts like a double ended queue, which means that you can enqueue and dequeue elements from both ends of the queue. This can be used as either a stack or a normal queue.

TreeSet
An ordered set of unique values.

TreeMap
Ordered keys mapped to values.

HashSet
Unordered keys mapped to values.

HashMap
An unordered set of unique values.

Priority Queue
min or max, not both.

```java
// Enqueue/Dequeue: O(log(n))
pQueue.remove();
pQueue.add();
pQueue.poll();

// Retrieval: O(1)
pQueue.peek();
pQueue.element();
pQueue.size();
```

Custom Sorting

```java
import java.util.Arrays;
import java.util.Comparator;

Arrays.sort(ToSort,
    new Comparator<ToSortClass>() {
        @Override
        public int compare(
            ToSortClass o1, ToSortClass o2
        ) {
            return Integer.compare(o1.value,
                                   o2.value);
        }
    }
);
```

2 Backtracking

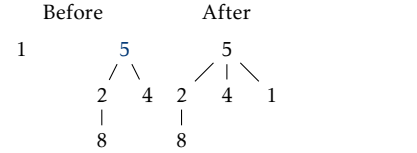Recursion, try ea. possibility and go next, if no work go back

```java
// ex: Sudoku
for (int i=1;i<=9;i++) { // place 1-9 for sudoku
  if (check(r,c,i)) { // if can place value i
    grid[r][c]=i; // place it
    if (solve(r,c+1)) { // solve w/ board next pos
      return true; // it's solved
    }
    // backtrack, rm val for later calls
    grid[r][c]=0;
  }
}
```
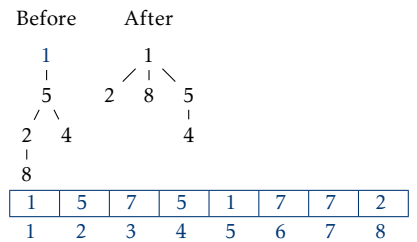
3 Data Structures

Disjoint Sets
A set of sets. Each set has a marked "leader" element. Two sets A and B are **disjoint** if $A \cap B = \emptyset$

Array Representation:
Value in any given index corresponds to its direct parent. Value will be the same as the index if it is the "root" of a tree set or is just an individual value.
Find: Returns the marked "leader" element of a set.

Union Operator:

(prioritize smaller tree height)
Merges two disjoint sets together. If visualizing as a tree set, take the tree with the smaller height, and merge it into the taller tree.

Before   After

```
1        5        5
        / \      / \ \
       2   4    2  4  1
       |        |
       8        8
```

Path Compression:

(findset on 8)

Before   After

```
  1           1
  |          /|\
  5        2 8 5
 / \          |
2   4         4
|
8
```

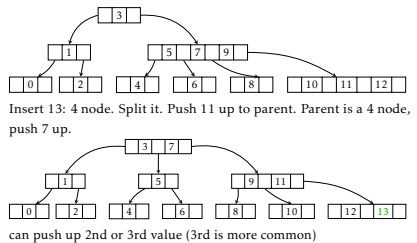| 1	5	7	5	1	7	7	2
1	2	3	4	5	6	7	8

First, you find the root of this tree which is 1. Then you go through the path again, starting at 8, changing the parent of each of the nodes on that path to 1.

1	5	7	5	1	7	7	1
1	2	3	4	5	6	7	8
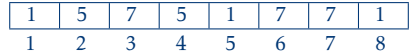
Then, you take the 2 that was previously stored in index 8, and then change the value in that index to 1.

2-4 Trees
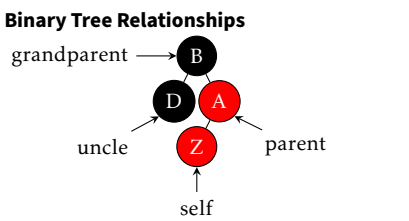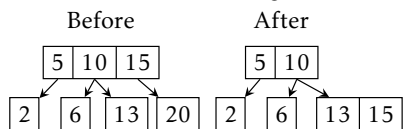num of children is equal to entries + 1 || 0

Insertion:

Insert 13: 4 node. Split it. Push 11 up to parent. Parent is a 4 node, push 7 up.

can push up 2nd or 3rd value (3rd is more common)

Deletion:

1. Find key to remove and replace w/ next higher key.
2. If sibling > 1 key, steal an adjacent key, make taht the parent and bring down the current parent.
3. If no adjacent sibling has greater than one key, steal a key from a parent.
4. If parent is the root and contains only one key and sibling has only one key, fuse it into a key node and make it the new root.

Delete 20 from the following 2-4 tree.

Before   After

5	10	15		5	10

| 2 | 6 | 13 | 20 | | 2 | 6 | 13 | 15 |

Binary Tree Relationships

grandparent → B, D, A, uncle, Z (self), parent

Red-Black Trees
1. A node is either red or black.
2. The root and leaves (NIL) are black.
3. If a node is red, then its children are black.
4. All paths from a node to its NIL descendants contain the same number of black nodes.
5. The longest path (root to farthest NIL) is no more than twice the length of the shortest path (root to nearest NIL).
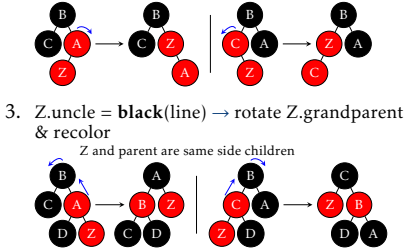
   • Shortest path: all black nodes
   • Longest path: alternating red and black

Insertion:

Strategy:
1. Insert Z and color it red
2. Recolor and rotate nodes to fix violation

Z is illegal scenarios
0. Z = root → color black
1. Z.uncle = red → recolor
2. Z.uncle = **black**(triangle) → rotate Z.parent

Z right child, parent left child & vice-versa

3. Z.uncle = **black**(line) → rotate Z.grandparent & recolor
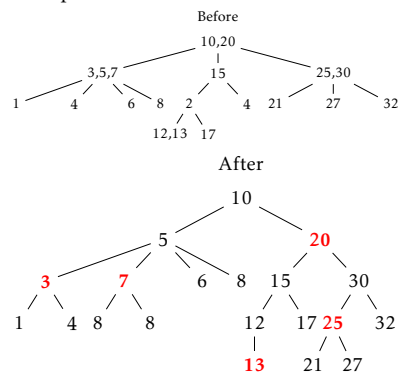
Z and parent are same side children

Deletion:

DB node has...

1. **black sibling with at least one red child.** This fixes the problem structurally. No extra work is required after this case completes. This corresponds to a transfer operation in a 2-4 Tree.
2. **black sibling with two black children.** This uses recoloring and no structural change. It may solve the problem, but may ALSO propagate the DB node to the parent of the current DB node. This corresponds to a fusion and drop operation in a 2-4 Tree.
3. **red sibling.** A structural change here puts you in case 1 or case 2. At this point, a single application of either case is sufficient. This corresponds to a fusion where you have enough values in the parent node to drop one into the fused child.

Convert from a 2-4 Tree:

Strategy:

1. Each **2-Node** should be colored **black**.
2. Each **3-Node** should be converted into 2 nodes, with its parent node being **black** and child being **red**.
3. Each **4-Node** should be converted into 3 nodes, with its parent node being **black** and 2 children being **red**.

Example:

Before

```
              10,20
      3,5,7     15      25,30
    1  4  6 8  2  4 21  27  32
           12,13 17
```

After

```
            10
         5       20
      3    7    6  8 15  30
    1  4 8  8      12 17 25  32
                      13 21 27
```

Skip Lists
Stacks of linked lists.

Insertion:
Insert at bottom level. 50% chance it's added up, again & again. Stop if it is only one at top level.

Deletion:
Delete value and update all pointers.

4 Algorithm Analysis

$\lg n = \log_2 n$

Order Notation

Big-Oh (O):

Big-Oh is an upper bound. It simply guarantees that a function is no larger than a constant times a function $g(n)$, for $O(g(n))$.
$f(n) = O(g(n))$ iff $\forall n \geq n_0$ (const n_0)
$f(n) \leq cg(n)$ for some const c

Big-Omega (Ω):

$f(n) = \Omega(g(n))$ iff $\forall n \geq n_0$ (const n_0)
$f(n) \geq cg(n)$ for some const c

Big-Theta (Θ):

$f(n) = \Theta(g(n))$ iff $f(n) = O(g(n))$
and $f(n) = \Omega(g(n))$
$f(n) = \Theta(g(n))$
iff $\lim_{n \to \infty} \frac{f(n)}{g(n)} > 0$, where const c and $c > 0$

Master Theorem

of sub probs in recursion → A

cost of work done out of recursive call → $O(n^k)$

$$T(n) = A \; T\left(\frac{n}{B}\right) + O(n^k)$$

size of ea. sub problem → $\frac{n}{B}$

$$T(n) = \begin{cases} B^k < A \Rightarrow O(n^{\log_B A}) \\ B^k = A \Rightarrow O(n^k \lg n) \\ B^k > A \Rightarrow O(n^k) \end{cases}$$

Expectation Definition

$E(x) = \sum_{x \in X} x \cdot p(x)$

Binomial Theorem

$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$

# of ords of k suc $n-k$ fail — same for fails

$\binom{n}{k} \; p^k \; (1-p)^{n-k}$

prob of k suc in their slots

Binary Search Average Case Run Time
$O(\log n)$

Make Heap Worst Case Run Time
$O(n \log n)$

Quick Select Average Case Run Time
$O(n)$

5 Sorting

Lower Bounds

Comparison Sorts:

Given an input of n numbers to sort, they can be arranged in $n!$ different orders. With k cols, each with 2 possible answers, there are 2^k possible distinct rows. $\Omega(n \log n)$

Input	$a[1] > a[2]$	$a[1] > a[3]$	$a[2] > a[3]$,...	$a[n-2] > a[n-1]$
2,1,4,3	T	F	F	T
2,3,1,4	F	T	T	F
4,3,2,1	T	T	T	T

Adjacent Element Swap Sorts:

An inversion in a list of numbers is a pair of numbers that are out of order relative to each other.
The average number of inversions in a random list of distinct numbers is:
$\frac{1}{2}\binom{n}{2} = \frac{n(n-1)}{2} = \Omega(n^2)$
The average case run-time of all of these algorithms is $\Omega(n^2)$.

Bucket Sort
Inputs randomly distributed in range $[x, N)$, for n amount of values, create different buckets to hold the values. $\frac{N}{n}$ will give the new ranges. $O(n)$
Consider sorting a list of 10 numbers known to be in between 0 and 2, not including 2 itself. Thus, each bucket will store values in a range of $\frac{2}{10} = .2$ In particular, we have the following list:

Bucket	Range of Values
0	$[0, .2)$
1	$[.2, .4)$
2	$[.4, .6)$
3	$[.6, .8)$
4	$[.8, 1)$
5	$[1, 1.2)$
6	$[1.2, 1.4)$
7	$[1.4, 1.6)$
8	$[1.6, 1.8)$
9	$[1.8, 2)$

Counting Sort
In counting sort, each of the values being sorted are in the range from 0 to m, inclusive. Here is the algorithm for sorting an array a[0],...,a[n-1]:

1. Create an aux c, indexed from $c[0]$ to $c[m]$ and init each value in the array to 0.
2. Run through the input array a, tabulating the number of occurrences of each value 0 through m by adding 1 to the

value stored in the appropriate index in c. (Thus, c is a freq array.)

3. Run through the array c, a 2nd time so that the value stored in each array slot represents the number of elements $\leq$ the index value in the original array a.

4. Now, run through the original input array a, and for each value in a, use the aux array c to tell you the proper placement of that value in the sorted input, which will be stored in a new array $b[0]..b[n-1]$.

5. Copy the sorted array from b to a.

Radix Sort
input: non-neg ints, k digits long, $O(nk)$.

1. Sort the values using a $O(n)$ stable sort on the kth most sig. digit.

2. Decrement k by 1

3. Repeat step 1. (Unless $k = 0$, then you're done.)

unsorted	v_1	v_2	v_3
235	162	628	162
162	734	734	175
734	674	235	235
175	235	237	237
237	175	162	628
674	237	674	674
628	628	175	734

v_1: sorted by units digit
v_2: sorted by tens digit
v_3: sorted by hundreds digit

6 Greedy Algorithms
Fractional Knapsack
Goal is to maximize the value of a knapsack that can hold at most W units worth of goods from a list of items $I_1, I_2, \ldots I_n$. Each item has 2 attrs:

1. A value/weight; let this be v_i for item I_i.

2. Weight available; let this be w_i for item I_i.

The algorithm is as follows:

1. Sort the items by value/unit.

2. Take as much as you can of the most expensive item left, moving down the sorted list. You may end up taking a fractional portion of the "last" item you take.

Single Room Scheduling
Given a single room to schedule, and a list of requests, the goal of this problem is to maximize the total number of events scheduled. Each request simply consists of the group, a start time and an end time during the day.
Here's the greedy solution:

1. Sort the requests by finish time.

2. Go through the requests in order of finish time, scheduling them in the room if the room is unoccupied at its start time.

Multiple Room Scheduling
Given a set of requests with start and end times, the goal here is to schedule all events using the minimal number of rooms. Once again, a greedy algorithm will suffice:

1. Sort all the requests by start time.

2. Schedule each event in any available empty room. If no room is available, schedule the event in a new room.

Change
The goal here is to give change with the minimal number of coins possible for a certain number of cents using 1 cent, 5 cent, 10 cent, and 25 cent coins.
The greedy algorithm is to keep on giving as many coins of the largest denomination until you the value that remains to be given is less than the value of that denomination. Then you continue to the lower denomination and repeat until you've given out the correct change.

## Huffman Coding
For the following character frequencies:

Character	Frequency
a	12
b	2
c	7
d	13
e	14
f	85

Create a binary tree for each char that also stores the frequency w/ which it occurs. The algorithm is as follows:

1. Find the two bin trees in the list that store min freqs at their nodes.

2. Connect these two nodes at a newly created common node that will store NO character but will store the sum of the freqs of all nodes connected below it.

Repeat until only one tree is left:
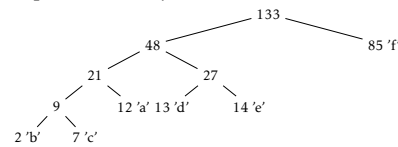
One the tree is built, each leaf node corresponds to a letter w/ a code. To determine the code for a node, walk a std search path from the root to the leaf node.
For every step to the left, append a 0 to the code and for every step right, append a 1.
For the ex. tree we get the codes:

Character	Code
a	001
b	0000
c	0001
d	010
e	011
f	1

Calculating Bits Saved:
total bits = $\sum \text{char}_{\text{freq}} \cdot \text{char}_{\text{\# bits}} =$
$(12 \cdot 3)_a + \cdots + (85 \cdot 1)_f = 238$

Assuming the original file is storing each of the 6 chars with a 3-bit code. Since there are 133 such characters, the total num of bits used before huffman is $3 \cdot 133 = 399$.
$\therefore$ we saved $399 - 238 = 161$ bits.

7 Unweighted Graphs
Types

undirected & unsigned   directed   weighted
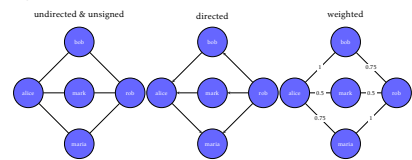
Depth First Search
Search down a path from a vertex as far as you can go. Then backtrack to the last vertex from which a different path could have been taken. $O(V + E)$

Breadth First Search
Search all the paths at a uniform depth from the source before moving into deeper paths.

Topological Sort
Can find a top. ord. in $O(V + E)$ time.

Topological Ordering:
An ordering of the nodes in a directed graph where for each directed edge from node A to node B, node A appears before node B in the ordering.
Top. ords. are NOT unique.
A graph which contains a cycle cannot have a valid ordering:
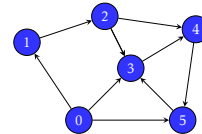
The only type of graph which has a valid top. ordering is a **Directed Acyclic Graph (DAG)**. These are graphs with directed edges and no cycles.
ie: Program dependency graph.
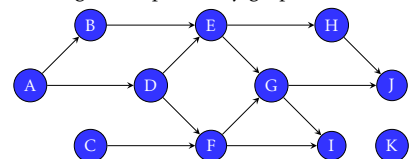
Topological Algorithm:
1. Pick an unvisited node

2. Beginning w/ the selected node, do a DFS exploring only unvisited nodes.

3. On the recursive callback of the DFS, add the current node to the top. ordering in rev. order.

8 Weighted Graphs
Dijkstra's
Finds the shortest path from a src vertex to all other vertices in a weighted directed graph w/out negative edge weights. (uses BFS)

Dijkstra's Trace:
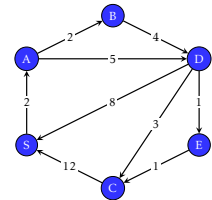
			Distances			
processed	S	A	B	C	D	E
	0	∞	∞	∞	∞	∞
S (0)	0	2	∞	12	8	∞
A (2)	0	2	4	12	7	∞
B (4)	0	2	4	12	7	∞
D (7)	0	2	4	10	7	8
E (8)	0	2	4	9	7	8
C (9)	0	2	4	9	7	8

Minimum Spanning Trees
tree: A connected graph w/out cycles.
Spanning Tree: A subtree of a graph that includes each vertex of the graph. A subtree of a given graph as a subset of the components of that given graph.
Minimum Spanning Tree: Only def for weighted graphs. This is the spanning tree of a given graph whose $\sum \text{edge weights}$ is min, compared to all other spanning trees.
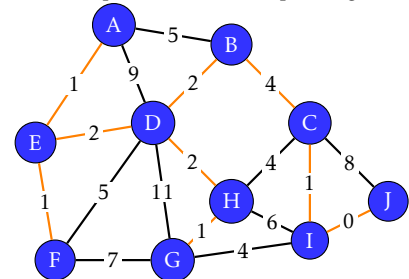
(orange) Minimum spanning tree with weight 14

Kruskal's
1. Sort edges by ascending edge weight.

2. Walk through the sorted edges and look at the two nodes the edge belongs to, if the nodes are already unified we don't include this edge, otherwise we include it and unify the nodes.

3. The algorithm terminates when every edge has been processe or all the vertices have been unified.

Prim's
A greedy MST algorithm that works well on dense graphs. However, when finding the minimum spanning forest on a disconnected graph, Prim's must be run on each connected component individually.
The lazy version of Prim's has a runtime of $O(E \log E)$, and the eager version has a better runtime of $O(E \log V)$.

Lazy Prim's:
1. Maintain a min Priority Queue that sorts edged based on min edge cost.

2. Start the algorithm on any node **s**. Mark **s** as visited and iterate over all edges of **s**, adding them to the PQ.

3. While the PQ is not empty and a MST has not been formed, dequeue the next cheapest edge from the PQ. If the dequeued edge has already been visited, skip it and poll again. Otherwise, marked the current node as visited and add the selected edge to the MST.

4. Iterate over the new current node's edges and all all its edges to the PQ. Do not add edges to the PQ which point to already visited nodes.

```java
// edge implements Comparable
public ArrayList<edge> prims(
    ArrayList<edge>[] graph
) {
    int n = graph.length;
    // shortened to PQ for spacing
    PQ<edge> pq = new PriorityQueue<edge>();
    boolean[] used = new boolean[n];
    used[0] = true;
    for (edge e: graph[0])
        pq.offer(e);

    ArrayList<edge> mst = new ArrayList<edge>();
    while (pq.size() > 0 && mst.size() < n-1) {
        edge cur = pq.poll();
        if (used[cur.u] && used[cur.v]) continue;
        int newV = !used[cur.u] ? cur.u : cur.v;
        mst.add(cur);

        used[newV] = true;
        for (edge e: graph[newV])
            pq.offer(e);
    }
    if (mst.size() < n-1) return null;
    return mst;
}
```

Network Flow
Max-Flow, Min-Cut Theorem: The value of the maximal flow in a flow network equals the value of the minimum cut.

Ford Fulkerson:
While there exists an augmenting path: Add the appropriate flow to that augmenting path.
Typically, DFS is used to check for the existence of an augmenting path.
worse-case, the algorithm takes $O(|f|E)$ time, where $|f|$ is the maximal flow of the network.

Edmonds Karp Algorithm:
A variation on the Ford-Fulkerson method. Idea is to stry to choose good augmenting paths. In this algorithm, the augmenting path suggested is the augmenting path with the mininal number of edges. (Can be found using BFS) The total number of iters is $O(VE)$. Thus, total run time with the graph stored as an adj. matrix is $O(V^3E)$.

Dinic's Algorithm:
A strongly polynomial max flow algorithm with a runtime of $O(V^2E)$.
The strongly polynomial means that the runtime does not depend on the capacity values of the flow graph.
Extremely fast and works even better on bipartite graphs, with a time complexity of $O(\sqrt{V}E)$ due to the algorithm's reduction to Hopcroft-Karp.
The main idea is to guide augmenting paths from $s \to t$ using a level graph.

Level Graph:

An edge is only part of the level graph if it makes progress towards the sink. That is, the edge must go from a node at Level L to another at level $L + 1$.

Algorithm Steps:

1. Construct a level graph by doing a BFS from the src to label all the levels of the current flow graph.

2. If the sink was never reached while building the level graph, then stop and return the max flow.

3. Using only valid edges in the level graph, do multiple DFSs from $s \rightarrow t$ until a blocking flow is reached, and sum over the bottleneck values of all the augmenting paths found to calculate the max flow.

4. Repeat Steps $1 \rightarrow 3$

9 Divide and Conquer

Integer Multiplication

Imagine multiplying an n-bit number by another n-bit number, where n is a perfect power of 2. (This will make the analysis easier.) We can split up each of these numbers into two halves.

$I \times J = [(I_h \times 2^{\frac{n}{2}} + I_l)] \times [(J_h \times 2^{\frac{n}{2}} + J_l)] =$
$I_h \times J_h \times 2^n + (I_l \times J_h + I_h \times J_l) \times 2^{\frac{n}{2}} + I_l \times J_l$

This way, we have broken down the problem of 2 n-bit nums into 4 mults of n/2-bit nums plus 3 addtions. Thus the run-time

$T(n) = 4T(n/2) + \theta(n)$

This has the solution of $T(n) = \theta(n^2)$ by the Master Theorem.
To optimize this:
$P_1 = (I_h + I_l) \times (J_h + J_l) = I_h \times J_h + I_x \times J_l + I_l \times J_h + I_l \times J_l$
$P_2 = I_h \times J_h$
$P_3 = I_l \times J_l$
$P_1 - P_2 - P_3 = I_h \times J_1 + I_l \times J_h$

$I \times J = P_2 \times 2^n + [P_1 - P_2 - P_3] \times 2^{\frac{n}{2}} + P_3$

Tromino "Tiling"

A tromino is a figure composed of three 1x1 squares in the shape of an L. Given a $2^n \times 2^n$ checkerboard with 1 missing square, we can recursively tile that square with trominoes.

1. Split the board into four equal sized squares.

2. The missing square is in one of these four squares. Recursively tile this square since it is a proper recursive case.

---

3. Although the three other squares aren't missing squares, we can "create" these recursive cases by tiling one tronimo in the center of the board, where appropriate:

1. Add this tromino
2. Recursively tile all
4 quadrents, each has "1 missing sqare"

Skyline Problem

You are to design a program to assist an architect in drawing the skyline of a city goven the locations of the buildings in the city. To make the problem tractable, all buildings are rect in shape and they share a common bottom.

$T(n) = O(n \lg n)$

You need to merge the buildings.

```
public static int[] mergeSky(
    int[] skyA, int[] skyB
) {
    int[] res = new int[skyA.length+skyB.length
        +1];
    int i = 0, j = 0, k = 0;
    int curA = 0, curB = 0;
    while (i<skyA.length || j<skyB.length) {
        if (j>=skyB.length || (i<skyA.length &&
    skyA[i]<skyB[j])){
            res[k++] = skyA[i++];
            curA = i<skyA.length ? skyA[i++] : 0;
        }
        else {
            res[k++] = skyB[j++];
            curB = j<skyB.length ? skyB[j++] : 0;
        }
        if (k<res.length) res[k++] = Math.max(curA
    , curB);
    }
    return res;
}
```

Closest Pair of Points

The Closest Pair of Points Algorithm is an $O(n \log n)$ solution to the problem of finding the closest pair of points when given a set of points with (x, y) coordinates. It has a base case of 2 points and a second base case of 3 points. Base case size 3 uses a brute force algorithm to return the closest pair. Base case size 2 returns that the distance between the two points.

(a) Idea of the divide-and-conquer algorithm for the closest-pair problem.
(b) Rectangle that may contain points closer than d_{min} to point p.

1. Sort the points based on their x coordinates (Use merge or quick)

2. Recursively split the sorted array in halves (like a merge sort does) until the base case of 3 or 2 is met.

3. To merge the halves, find the minimum distance out of the two halves, and label it delta.

4. Create an Array of all of the points within 'delta' distance of the halfway line.

5. Sort this array based on the y coordinates (use merge or quick).

6. Brute force search for the smallest distance within this array (Max size of array is 8)

7. This smallest distance is the true smallest distance for this section. Repeat the process.

Strassen's Algorithm

Recurrence relation is $T(n) = 7T(\frac{n}{2}) + O(n^2)$ for a matrix C=AB where A and B are matrixes are n x n.
matrix multiplication can be written as:
$q1 = (a11 + a22) * (b11 + b22)$
$q2 = (a21 + a22) * b11$
$q3 = a11 * (b12 - b22)$
$q4 = a22 * (b21 - b11)$
$q5 = (a11 + a12) * b22$
$q6 = (a21 - a11) * (b11 + b12)$
$q7 = (a12 - a22) * (b21 + b22)$
-which makes C:
$c11 = q1 + q4 - q5 + q7$
$c12 = q3 + q5$
$c21 = q2 + q4$
$c22 = q1 + q3 - q2 + q6$
This speeds up the process for bigger matrices which is $T(n) = O(n^3)$.

10 Dynamic Programming

Fibonacci

The key idea here is that at any given point, we only need the previous two values in the sequence to calculate the next one. So, toggle back and forth with the term we overwrite. (Initially, to calculate F(2), we can add F(0) and F(1), and then discard F(0). To calculate F(3), we add F(2) and F(1) and then discard F(1). And so on.) If this function looks complex, tracing through a few iterations might clarify what's going on.

```
int ultraFancyFib(int n)
{
    int[] f = new int[2];

    f[0] = 0;
    f[1] = 1;

    for (int i = 2; i <= n; i++)
        f[i%2] = f[0] + f[1];

    return f[n%2];
}
```

Combinations

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Longest Common Subsequence (LCS DP)

The trick here is to realize that at any given time, as we fill our row from left to right, there's only one value from the previous row that we might have to refer to that has already been overwritten. We can store that in a temporary variable at each iteration of the inner for-loop.

```
int lcss(String a, String b)
{
    if (a.length() < b.length())
    {
        String temp = a; a = b; b = temp;
    }

    // Note: This is counting on Java to initialize
    matrix[0] to 0.
    // In another language, we would have to do
    that manually in
    // order for this solution to work.

    int[] matrix = new int[b.length()+1];
    int diagonal;

    for (int i = 1; i <= a.length(); i++)
    {
        diagonal = 0;

        for (int j = 1; j <= b.length(); j++)
        {
            int weAreAboutToLoseThisValue = matrix[j
    ];

            if (a.charAt(i-1) == b.charAt(j-1))
                matrix[j] = 1 + diagonal;
            else
                matrix[j] = Math.max(matrix[j-1],
    matrix[j]);

            diagonal = weAreAboutToLoseThisValue;
        }
    }

    return matrix[b.length()];
}
```

Number of Ways to Make Change

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Fewest Number of Coins to Make Change

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0-1 Knapsack Problem

```
public static int knapsack(Treasure[] t, int c)
{
    int[][] matrix = new int[t.length + 1][c + 1];

    for (int i = 1; i <= t.length; i++)
        for (int j = 1; j <= c; j++)
            if (t[i-1].weight <= j)
                matrix[i][j] = Math.max(
                    matrix[i-1][j],
                    matrix[i-1][j-t[i-1].weight] + t[i
    -1].value
                );
            else
                matrix[i][j] = matrix[i-1][j];

    return matrix[t.length][c];
}
```

Floyd-Warshall's Algorithm and path reconstruction

```
// Floyd-Warshall's all shortest paths algorithm.
private void allShortestPaths()
{
    int n = adj.length - 1;
    int[][] sp = new int[n+1][n+1];

    // Initialize base cases: sp(i,i,0) = 0, sp(i,j
        ,0) = weight(i,j)
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= n; j++)
            sp[i][j] = (i == j) ? 0 : adj[i][j];

    // Bottom-up construction. Oh my gosh. So
        intense! So amazing!
    for (int k = 1; k <= n; k++)
        for (int i = 1; i <= n; i++)
            for (int j = 1; j <= n; j++)
                sp[i][j] = Math.min(sp[i][j], sp[i][k]
    + sp[k][j]);

    // Check for negative cycles.
    for (int i = 1; i <= n; i++)
        if (sp[i][i] < 0)
            return;

    // Assuming there are no negative cycles in the
        graph,
    // the shortest path between each (i, j) pair
        now lies in
    // the array at position sp[i][j]. You can do
        whatever you
    // want with that here:
    // ...

    // I'm printing the array just for confirmation
        that the method works.
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= n; j++)
            System.out.printf("%5d%s", sp[i][j], (j
    == n) ? "\n" : "");
}
```

Matrix Chain Multiplication

Consider computing A(BC):
multiplications for (BC) = 10x50x20 = 10000, creating a 10x20 answer matrix
multiplications for A(BC) = 2x10x20 = 400,
Total multiplications = 10000 + 400 = 10400.
Consider computing (AB)C:
multiplications for (AB) = 2x10x50 = 1000, creating a 2x50 answer matrix
multiplications for (AB)C = 2x50x20 = 2000,
Total multiplications = 1000 + 2000 = 3000, a significant difference.
Thus, the goal of the problem is given a chain of matrices to multiply, determine the fewest number of multiplications necessary to compute the product. We will formally define the problem below:
Let $A = A_0 \cdot A_1 \cdots A_{n-1}$
Let Ni,j denote the minimal number of multiplications necessary to find the product $A_i \cdot A_{i+1} \cdots A_j$.
And let $d_i \times d_{i+1}$ denote the dimensions of matrix Ai.
We must attempt to determine the minimal number of multiplications necessary($N_{0,n-1}$) to find A, assuming that we simply do each single matrix multiplication in the standard method.

```
int solveRec(int start, int end) {
    if (start == end) return 0;
    if (memo[start][end] != -1) return memo[start][
        end];
    int res = solveRec(start, end-1) + d[start
        ][0]*d[end][0]*d[end][0];
    for (int i=start; i<end; i++) {
        int tmp = solveRec(start, i) + solveRec(i+1,
        end) +
            d[start][0]*d[i][1]*d[end][1];
        res = Math.min(res, tmp);
```

```
    }
    return memo[start][end] = res;
}
```

Edit Distance

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Road Optimization Problem Idea

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11 Probabilistic Algorithms

Fermat's Theorem

$\forall$ prime numbers p and positive ints a such that gcd(a, p) = 1.

$$a^{p-1} \equiv (\quad \mod p)$$

Miller-Rabin Primality Test

```
public static boolean isPrime(int n) {
    if (n < 2) return false;
    for (int i=2; i*i<=n; i++)
        if (n%i == 0)
            return false;
    return true;
}
```

Even if a modulus operation happened in constant time (which for large integers this isn't the case),
the run time of this algorithm is $O(\sqrt{n})$. For large values of n (anything much beyond 1018),
this isn't feasible in a short amount of time.

Rolling Hash Function and String Matching

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