SLAM:

Predict: $\mu^{L}_{t|t} \in \mathbb{R}^{4\times M}$ $\quad \mu^{P}_{t|t} \in \mathbb{R}^{4\times 4}$. $\quad \mu_{t|t} = [\mu^{P}_{t|t}, \mu^{L}_{t|t}] \in \mathbb{R}^{4\times(4+M)}$

$$\Sigma_{t|t} = [\Sigma^{P}_{t|t}, \Sigma^{L}_{t|t}] \in \mathbb{R}^{(6+3M)\times(6+3M)}$$

$$\mu_{t+1|t} = \begin{bmatrix} \exp(-\tau \hat{u}_t) & 0 \\ 0 & I \end{bmatrix} \mu_{t|t} \in \mathbb{R}^{4\times(4+M)}$$

$$\Sigma_{t+1|t} = \begin{bmatrix} \exp(-\tau \hat{u}_t) & 0 \\ 0 & I \end{bmatrix} \Sigma_{t|t} \begin{bmatrix} \exp(-\tau \hat{u}_t & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} \tau^2 W & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{(6+3M)\times(6+3M)}$$

$H^L \in 4\times 3M \quad H^P \in 4\times 6. \quad H = [H^P, H^L] \in 4\times \cancel{(3M+6)}\,(6+3M)$

$$H^P = M \frac{d\pi}{dq}({}_oT_1 \mu^{P}_{t+1|t} \mu^{L}_{t+1|t})\, {}_oT_1 (\mu^{P}_{t+1|t}\, \mu^{L}_{t+1|t})^{\odot}$$

$$H^L = M \frac{d\pi}{dq}({}_oT_1 \mu^{P}_{t+1|t} \mu^{L}_{t+1|t})\, {}_oT_1 \mu^{P}_{t+1|t}\, D$$

$\Bigg\} \in \mathbb{R}^{4\times(6+3M)}$ $H_{t+1|t}$

$$K_{t+1|t} = \Sigma_{t+1|t} H_{t+1|t} (H_{t+1|t} \Sigma_{t+1|t} H_{t+1|t} + I \otimes V)^{-1} \in (6+3M)\times 4$$

$$\mu^{L}_{t+1|t+1} = \mu^{L}_{t+1|t} + D K^{L}_{t+1|t} (z_{t+1} - \hat{z}_{t+1}) \qquad K^{L}_{t+1|t} \in 3M\times 4$$

$$\mu^{P}_{t+1|t+1} = \underbrace{\exp((\underbrace{K^{P}_{t+1|t}(z_{t+1} - \hat{z}_{t+1})}_{6\times 1})^{\wedge})}_{4\times 4} \mu^{P}_{t+1|t} \qquad K^{P}_{t+1|t} \in \mathbb{R}^{6\times 4}$$

$$\Sigma^{L}_{t+1|t+1} = (I - K^{L}_{t+1|t} H^{L}_{t+1|t}) \Sigma_{t+1|t}$$

$$\Sigma^{P}_{t+1|t+1} = (I - K^{P}_{t+1|t} H^{P}_{t+1|t}) \Sigma_{t+1|t}.$$

$$\Sigma_{t+1|t+1} = (I - \underbrace{K_{t+1|t}}_{(6+3M)\times 4} \underbrace{H_{t+1|t}}_{4\times(6+3M)}) \Sigma_{t+1|t} \in \mathbb{R}^{(6+3M)\times(6+3M)}$$