Ch. 1. Measurements

MEASURING is an important part of our everyday lives, and very probably you took several measurements today. You might now be sipping a cup of coffee, or perhaps you checked the outside temperature on a street thermometer. You might be planning to bake a cake and need to use a scale and a cup to measure the flour and sugar. A cup, a thermometer, or a scale are measuring devices. It is critical to know how to accurately measure properties and, more importantly, how to transform measurements using prefixes and unit conversions. By learning how to measure and perform operations with units, you will gain experience performing basic chemistry calculations.

1.1 Units of Measurements and systems of units

You probably heard the term liter, kilogram, or meter. These are units of measurement. Units can be classified into different *systems of units*. For example, the unit *meter* belongs to a different system than the unit *mile*. In particular, here we will address three main systems: the English System, the Metric System, and the International system. The *Metric System* (MS) is used by scientists throughout the world and is the most common measuring system based on the meter. The English system is mostly used in the US. The *International System of Units* (SI) adopted the metric system in 1960 to provide additional uniformity for units used in the sciences. Table 1.1 summarizes some of the fundamental units for the three systems. This chapter will be mostly based on the SI units. In the following, we will introduce some common units.

`Length` What is your height? The length refers to distance and both the metric and SI unit of length is the meter (m). A smaller unit of length would be the centimeter (cm) that is commonly used in chemistry. The most important units of length are meters, inches, and miles.

`Mass` What is your weight? The mass of an object is a measure of the quantity of material it contains. You may be more familiar with the term weight rather than mass. However, mass and weight are not the same, as weight is a measure of the gravitational pull on an object. It differs depending on your location on the earth–in particular the height of your location. In the metric system, the unit of mass is the gram (g). The SI unit of mass, the kilogram (kg), is used for larger masses such as body weight. A pound, lb, is another unit of mass. The most important units of mass are g, kg, and lb.

`Temperature` How is the weather today? Is it cold or hot? You use a thermometer to measure temperature and for example assess how hot an object is, or how cold it is

▼Scales measure mass

▼Watches are used to measure time

▼Beakers can carry a liquid volume

▼Thermometers measure temperature

▼pipets are used in chemistry practice to add an exact volume of liquid

outside, or perhaps to determine if you have a fever. Temperature tells us how hot or cold an object is. Temperature can be measured in numerous units such as Celsius (°C), Fahrenheit (°F), or kelvins (K).

`Time` How long is your commute to work? It might take you hours to go to work, or maybe minutes. You probably think of time as years, days, minutes, or seconds. Of all these units, the International System of units (SI, abbreviated from the French Systeme international) uses seconds (s) to measure time. Still, time can be measured in s, min, or h and during this chapter, we will learn how to convert units of time.

`Volume` How much milk do you usually buy? Maybe a gallon. Volume is the amount of space that a substance occupies. A liter (L), not a fundamental but a derived SI unit, is commonly used to measure volume. The milliliter (mL) is more convenient for measuring smaller volumes of fluids in hospitals and laboratories. Gallon is still used in everyday life. L, mL, and gallon are units of volume. Units of volume are in general cubic units, so for example one liter is the same as one dm^3. We will cover cubit units further in this chapter.

Chemical laboratory work commonly requires the measurement of volume. There are two main types of glassware used to measure volume in a chemistry lab: graduated tools and volumetric tools. Volumetric pipets, flasks, and burets are the most accurate; the glassware makers calibrate these to a high level of accuracy, usually measured in terms of tolerance, which is the uncertainty in a measurement made with the glassware. Class A volumetric glassware has a lower tolerance than Class B; for class A, the tolerance can be as low as 0.08 ml for a 100 ml flask or pipet. Generally, measurements with class A volumetric glassware can be considered reliable to two places after the decimal point. Graduated cylinders, beakers, and Erlenmeyer flasks have less accuracy than volumetric glassware. Graduated cylinders can generally be considered reliable to within 1 percent. Beakers and Erlenmeyer flasks should not be used to measure volume unless you need only a very crude estimate because their accuracy for volume measurements is so poor. They can hold a much larger volume than any of the other types of glassware, however, which makes them useful for mixing solutions.

`Concentration` Even though we will devote a whole chapter to solutions and concentration, it felt important to introduce here the unit molarity. In chemistry, the unit molarity (M) refers to the concentration of a solution. That is the larger this number, the larger molarity, and the more concentrated a solution will be. In other words, there will be more substance in the solution.

Sample Problem 1

State the type of measurement indicated in each of the following:

(a) 1ft(foot) (b) 20Kg (c) 3L (d) 300K

SOLUTION

(a) length; (b) mass; (c) volume; (d) temperature;

STUDY CHECK

State the type of measurement indicated in each of the following: (a) 800°F (b) 1m^3 (c) 3m (d) 67s

▶Answer: (a) temperature; (b) volume; (c) length; (d) time;

Table 1.1 Different unit systems

Measuremts	Metric System	International System (SI)	English System
Length	Meter (m)	Meter (m)	Foot (ft)
Mass	Gram (g)	Kilogram (kg)	Pound (lb)
Time	Second (s)	Second (s)	Second (s)
Temperature	Celsius (°C)	Kelvin (K)	Fahrenheit (°F)
Volume*	Liter (L)	Cubic meter (m^3)	Gallon (gal)
Ammount of substance	Mole (mol)	Mole (mol)	Mole (mol)
Electric current	Ampere (A)	Ampere (A)	Ampere (A)

*Not a fundamental unit

1.2 Scientific notation

Numbers in science can often be very large or very tinny. For example, the mass of the earth is 5900000000000000000000000000 grams, whereas the size of an atom is of the order 0.00000000001cm. Scientific notation is a standard notation extensively used in the scientific community to simplify numbers. Numbers not expressed in scientific notation are referred to as numbers in full notation (e.g. 0.234g). Numbers expressed in scientific notation place the decimal after the first significant digit while being accompanied by a power of ten (e.g. 2.34×10^{-1}g). To express a number (e.g. 345000g) in scientific notation, we just need to leave the first digit followed by the decimal point which was moved in this case to the left 5 places, giving 3.45×10^5g. The power of ten can contain a positive or a negative exponent. Positive exponents represent large numbers, whereas negative exponents represent numbers smaller than 1. Below are two examples of full and scientific notation. On one hand a number smaller than one:

$$0.000134\text{g} \quad \text{(full notation)} \qquad 1.34 \times 10^{-4}\text{g} \;\; \text{(scientific notation)}$$

On the other hand, a number larger than one:

$$4500000\text{g} \quad \text{(full notation)} \qquad 4.5 \times 10^{6}\text{g} \;\; \text{(scientific notation)}$$

1.3 Prefixes & Conversion Factors

Let's consider the following measurements: 1 **k**m, 2 **c**m and 3 m that can be read as one kilometer, two centimeters, and three meters. The word kilo (**k**) and centi (**c**) are called prefixed whereas meter (m) is a simple unit. A kilometer is larger than a meter, whereas a centimeter is smaller than a meter. Prefixes such as kilo or centi are attached to units to make numbers more manageable. For example, the radius of the earth is 6356 km, and this number is easier to handle than 6356000m. At the same time, we can attach any prefix to different units. Hence, we can talk about a centimeter (**c**m) but also about a centisecond (**c**s) or centiliter (**c**L). All these units have the same prefix.

`How to identify prefixes?` Look for example at the measurement 2 cm. Centi (c) is the prefix and means 1×10^{-2} and meter (m) is the unit that refers to length. Another example, is 7 kg read as seven kilograms. Kilo (k) is the prefix and means 1×10^3, whereas gram (g) is the unit that refers to mass. The prefix refers to the first letter whereas the unit refers to the last letter.

A unit with a prefix can be bigger or smaller than the plain unit–this is the unit without a prefix–, depending on the prefix. The following prefixes reduce the unit: deci, centi, milli, micro, nano, pico, and femto. For example, a fs (femtosecond) is smaller than a s (second). Differently, the following prefixes increase the unit: Tera, Giga, Mega. For example, a Tb (terabyte) is larger than a b (byte). A byte is a unit used in computer science. Table 1.2 lists some of the metric prefixes, their symbols, and their decimal values.

Table 1.2 Different prefixes

Prefix	Symbol	Meaning	Value
exa	E	1000000000000000000	1×10^{18}
peta	P	1000000000000000	1×10^{15}
tera	T	1000000000000	1×10^{12}
giga	G	1000000000	1×10^{9}
mega	M	1000000	1×10^{6}
kilo	k	1000	1×10^{3}
hecto	h	100	1×10^{2}
deca	da	10	1×10^{1}
–	–	1	1×10^{0}
deci	d	0.1	1×10^{-1}
centi	c	0.01	1×10^{-2}
milli	m	0.001	1×10^{-3}
micro	μ	0.00001	1×10^{-6}
nano	n	0.000000001	1×10^{-9}
pico	p	0.000000000001	1×10^{-12}
femto	f	0.000000000000001	1×10^{-15}
atto	a	0.000000000000000001	1×10^{-18}

`How to write unit equalities and conversion factors` Unit equalities are simple expressions that relate a unit with a prefix. For example, one centimeter (cm) is 1×10^{-2}m. Hence we can write this as unit equality:

$$\boxed{1cm = 1 \times 10^{-2}m} \text{ unit equality}$$

Let's compare cm and m. The first, cm, is a unit with a prefix, whereas m is simply a unit of length without a prefix. To know how many m are there in a cm we need to write down a conversion factor. Think about prefixes as synonymous with a number. In this way, centi stands for 1×10^{-2}, so

$$\boxed{\frac{1cm}{1 \times 10^{-2}m} \text{ or } \frac{1 \times 10^{-2}m}{1cm}} \text{ conversion factor}$$

The relationship above is also called a *unit factor* as both ratios are equal to one.

Sample Problem 2

Complete each of the following equalities and conversion factors:

(a) 1dm=____m

(b) 1km=____m

(c) $\frac{1nm}{\_\_\_\_m}$

(d) $\frac{\_\_\_\_m}{1cm}$

SOLUTION

(a) 1dm=$1 \times 10^{-1}m$; (b) 1km=$1 \times 10^{3}m$; (c) $\frac{1nm}{1 \times 10^{-9}m}$; (d) $\frac{1 \times 10^{-2}m}{1cm}$;

STUDY CHECK

Second is a unit of time. Complete each of the following equalities and conversion factors involving seconds:

(a) 1cs=______s (b) $\frac{\_\_\_\_\_\_ s}{1Ts}$ (c) $\frac{\_\_\_\_\_\_ s}{1Ms}$

▶Answer: (a) 1cs=$1 \times 10^{-2}s$; (b) $\frac{1 \times 10^{12}s}{1Ts}$; (c) $\frac{1 \times 10^{6}s}{1Ms}$;

1.4 Using Conversion Factors

Unit equalities in the form of conversion factors are used to convert one unit into another. Sometimes one wants to get rid of a prefix, such as when we transform centimeter (cm) into meter (m). Sometimes, one wants to convert a prefix into another prefix. An example would be converting centimeters (cm) to millimeters (mm). Let's work on some examples.

Removing or adding prefixes Imagine that you need to remove a prefix from a unit, and convert 3 km (we will call this one the original unit) into meters (this is the final unit). First, you would need the conversion factor corresponding to the prefix (centi) from Table 1.2. Then you need to arrange the conversion factor by placing the prefix at the bottom of the fraction. This will cancel out the prefix in the original unit and the bottom part of the conversion factor, hence leaving the final unit on top of the conversion factor. The arrangement would be:

$$3\cancel{km} \times \frac{1 \times 10^{3}m}{1\cancel{km}} = 3000m$$

Imagine now that you need to add a prefix into a unit, and convert 4000 m in km. The same would apply for this case, but now you will have to arrange the conversion factor so that the prefix is on the top:

$$4000\cancel{m} \times \frac{1\text{ km}}{1 \times 10^{3}\cancel{m}} = 4km$$

Sample Problem 3

The length of a textbook page is 20cm. Convert 20cm to meters, expressing the result in scientific notation.

SOLUTION

In order to convert 20cm into meters, we need to remove the prefix (centi) leaving the unit (meter) without any prefix. We will use the conversion factor that relates m to cm: $\frac{1 \times 10^{-2}m}{1cm}$ or $\frac{1cm}{1 \times 10^{-2}m}$. We will arrange the conversion factor so that cm cancels giving m and hence we will use $\frac{1 \times 10^{-2}m}{1cm}$:

$$20\cancel{cm} \times \frac{1 \times 10^{-2}m}{1\cancel{cm}} = 2 \times 10^{-1}m$$

The original units and on the bottom of the conversion factor cancel and we get meters, the final unit.

STUDY CHECK

Convert 100m to km, expressing the result in scientific notation.

▶Answer: $100\cancel{m} \times \frac{km}{1 \times 10^{3}\cancel{m}} = 1 \times 10^{-1} km.$

`Switching prefixes` To switch a prefix into another prefix, such as transforming 30 millimeters (30 mm) into centimeters (cm), you will need two different conversion factors: the first conversion factor will remove the original unit (mm) introducing an intermediate unit, meters (m), whereas the second conversion factor will remove the intermediate meter and introduce the final unit (cm). You will get the conversion factors from Table 1.2. You will arrange the first conversion factor so that the original unit cancels out with the bottom of the first conversion factor, giving you an intermediate unit. You will arrange the second conversion factor so that the intermediate unit cancels out with the bottom of the second conversion factor giving the final unit. For this example:

$$30\cancel{mm} \times \frac{1 \times 10^{-3}\cancel{m}}{1\cancel{mm}} \times \frac{1cm}{1 \times 10^{-2}\cancel{m}} = 3cm$$

Sample Problem 4

The length of a textbook page is 20cm. How many mm correspond this length, expressing the result in scientific notation.

SOLUTION

We want to convert 20 cm into mm, that is, we are switching prefixed. In order to do this, you need two conversion factors: $\frac{1 \times 10^{-2}m}{1cm}$ and $\frac{1 \times 10^{-3}m}{1mm}$. You will have to arrange the number (20cm) and the two conversion factors in the following form:

$$20\cancel{cm} \times \frac{1 \times 10^{-2}\cancel{m}}{1\cancel{cm}} \times \frac{1mm}{1 \times 10^{-3}\cancel{m}} = 2 \times 10^{2} mm$$

STUDY CHECK

Convert 100mm to km, expressing the result in scientific notation.

▶Answer: $100\cancel{mm} \times \frac{1 \times 10^{-3}\cancel{m}}{1\cancel{mm}} \times \frac{1km}{1 \times 10^{3}\cancel{m}} = 1 \times 10^{-4} km.$

1.5 Units of volume and area

How big is your apartment? You might be living in a $750 ft^2$ loft in Brooklyn or a larger house Upstate. Often times we encounter cubic or square units such as cubic centimeters (cm^3) or square feet (ft^2). The equivalencies for cubic or square units should take into account the unit power (power of two or power of three). If $1cm = 1 \times 10^{-2}m$, for square units the relation should be squared and $1cm^2 = 1 \times (10^{-2})^2 m^2 = 1 \times 10^{-4} m^2$.

Another example, for the case of mm and mm^3:

$$\boxed{\frac{1mm}{1\times10^{-3}m} \text{ and } \frac{1mm^3}{1\times10^{-9}m^3}}$$

Let us work on an example in which we want to convert $30m^2$ into m^2:

$$30\cancel{m^2}\times\frac{1cm^2}{1\times10^{-4}\cancel{m^2}}=3\times10^5cm^2$$

Sample Problem 5

How many m^2 is $20cm^2$, expressing the result in scientific notation.

SOLUTION

In order to convert $20cm^2$ to square meters, we need to remove the centi prefix and that will give us the unit square meter without any prefix. We will use the conversion factor that relates m^2 to cm^2:$\frac{1\times10^{-4}m^2}{1cm^2}$ or $\frac{1cm^2}{1\times10^{-4}m^2}$.

$$20\cancel{cm^2}\times\frac{1\times10^{-4}m^2}{1\cancel{cm^2}}=2\times10^{-3}m^2$$

STUDY CHECK

Convert $100m^3$ to dm^3, expressing the result in scientific notation.

Answer:$100\cancel{m^3}\times\frac{1dm^3}{1\times10^{-3}\cancel{m^3}}=1\times10^5dm^3$.

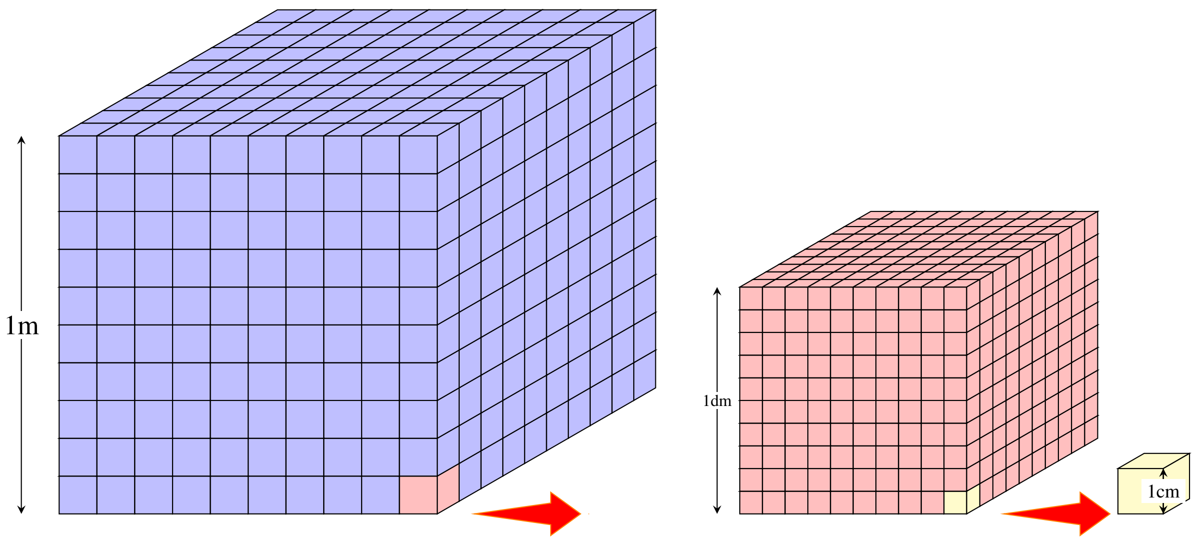


Figure 1.1 The left cube with a side of 1m has a volume of $1m^3$. The central cube with a side of 1dm has a volume of $1dm^3$, that is 1L. The right cube with a side of 1cm has a volume of $1cm^3$, that is 1mL.

`Liters and milliliters` Units such as L or mL are units of volume. As volume is a three-dimensional property, those units somehow have to be related to the units of length. One liter is the same as one dm^3 and one ml is the same as one cm^3 (See Figure 1.1). In the allied health field, the units *mL* are also written as *cc* as in cubic centiliters.

$$\boxed{1L=1dm^3 \text{ and } 1mL=1cm^3}$$

Let us work on an example in which we want to convert $30cm^3$ into L:

$$30\cancel{cm^3}\times\frac{1\cancel{mL}}{1\cancel{cm^3}}\times\frac{1\times10^{-3}L}{1\cancel{mL}}=3\times10^{-2}L$$

Sample Problem 6

Convert 30 m^3 into L, expressing the result in scientific notation.

SOLUTION

In order to convert m^3 into L we just need to remember that the L actually refers to dm^3, therefore is connected to meter. We will first convert m^3 into dm^3 and then dm^3 into L.

$$30\cancel{m^3} \times \frac{1\cancel{dm^3}}{1 \times 10^{-3}\cancel{m^3}} \times \frac{1L}{1\cancel{dm^3}} = 3 \times 10^4 L$$

STUDY CHECK

Convert 40L to cm^3, expressing the result in scientific notation.

▶Answer: $40\cancel{L} \times \frac{1\cancel{mL}}{1 \times 10^{-3}\cancel{L}} \times \frac{1cm^3}{1\cancel{mL}} = 4 \times 10^4 cm^3$.

Table 1.3 Table containing some common unit equalities

Unit	Equality
Inches (in)-centimeters (cm)	2.54 cm = 1 in
miles (mi)-meters (m)	1 mi = 1609.34m
minutes (min)-hours (h)	60 min = 1 h
minutes (min)-seconds (s)	60 s = 1 min
pound (lb)-grams (g)	454 g = 1 lb
cubic centimeter (cm^3)-mililiters (mL)	1 mL = 1cm^3
Liter (L)-cubic decimeters (dm^3)	1 L = 1dm^3
drops-mililiters* (mL)	1 mL = 15 drops

* There are several definitions of a drop

1.6 Using other equalities

How many hours are 300 minutes, or how many centimeters is 2 inches? Some of the units conversion is not based on a power of ten relationships and do not contain prefixes such as kilo or centi. Table 1.3 lists some of the common equalities that can be easily converted into conversion factors. As an example, the unit equivalency between hours and minutes is $60min = 1h$ and the conversion factor would be $\frac{60min}{1h}$ or $\frac{1h}{60min}$.

Sample Problem 7

Convert 20 in to cm, expressing the result in scientific notation.

SOLUTION

We want to convert 20 inches into centimeters. The relationship between Inch and centimeter is given in Table 1.3. In order to do this, you need the conversion factor: $\frac{1in}{2.54cm}$ or $\frac{2.54cm}{1in}$. You will have to arrange the number (20 in) and the conversion factor in the following form:

$$20\cancel{in} \times \frac{2.54cm}{1\cancel{in}} = 5.080 \times 10^1 cm$$

STUDY CHECK

Convert 200mL to drops, expressing the result in scientific notation.

▶Answer: $200\cancel{mL} \times \frac{15 drops}{1\cancel{mL}} = 3000 drops = 3 \times 10^3 drops$

1.7 Measurements and uncertainty

Most chemistry experiments require the measurement of a property (mass, volume, temperature, color...), and the validity of those experiments will depend on the reliability of each measurement. The reliability of a measurement is usually considered in terms of its *accuracy* and its *precision*. At the same time, when experimenting oftentimes, one needs to repeat measurements. The results need to take into account the average measurement as well as the standard derivation of the series of results.

`Uncertain and certain digits: how to report measured numbers`
Consider the volume as measured in the glassware on the left side of Figure 1.2. Some would say the liquid meniscus occurs at 1.52mL. Mind we need to estimate the last number by interpolating between the 0.1mL marks. As the last digit of the number associated with the volume measurement is estimated, another person could measure the volume as 1.53mL. The table below indicates the measurements of five different people.

Person	1	2	3	4
Volume(mL)	1.52	1.53	1.51	1.57

All measurements have in common the 1.5 part. The digit 5 is called a certain digit. The digit to the right of 5 is called the uncertain digit, as in a measurement it needs to be estimated. When reporting a measurement, we need to report up to the uncertain digit. These numbers on measurement are called *significant figures*. It is important to understand that all measured properties are subject to uncertainty. Uncertainty depends on one hand on the measuring device. The uncertainty on the left meniscus in Figure 1.2 is in the hundreds of the mL, whereas on the right meniscus occurs in the tenths of mL. Uncertainty also depends on the measurement process. As the uncertain digit needs to be reported in every measurement, the uncertainty on the last number on measurement is normally assumed to be ±1. For example, the right meniscus should be reported as $1.52 \ \pm 0.01$mL

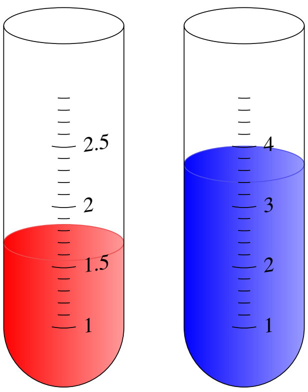


Figure 1.2 Two measuring devices. (Left side) The measuring device gives volumes to the hundredth place. (Right side)The measuring device gives volumes to the tenth place.

`Precision and accuracy` The terms precision and accuracy are two different terms often used to describe the reliability of measurement. Accuracy refers to the degree of agreement between the measured value and the true value, the real value of the property measured. Measurements that closely agree with the true value are accurate. Precision refers to the agreement between several measurements. Two measurements are accurate when they are close, independently of the true value of the property measured. Two different types of errors occur during a measurement. On one hand, random error refers to the fact that measured values can be above or below the true value. On the other hand, systematic error is an error that occurs each time in the same direction. Figure 1.3 displays accurate and inaccurate measurements as well as precise and imprecise measurements.

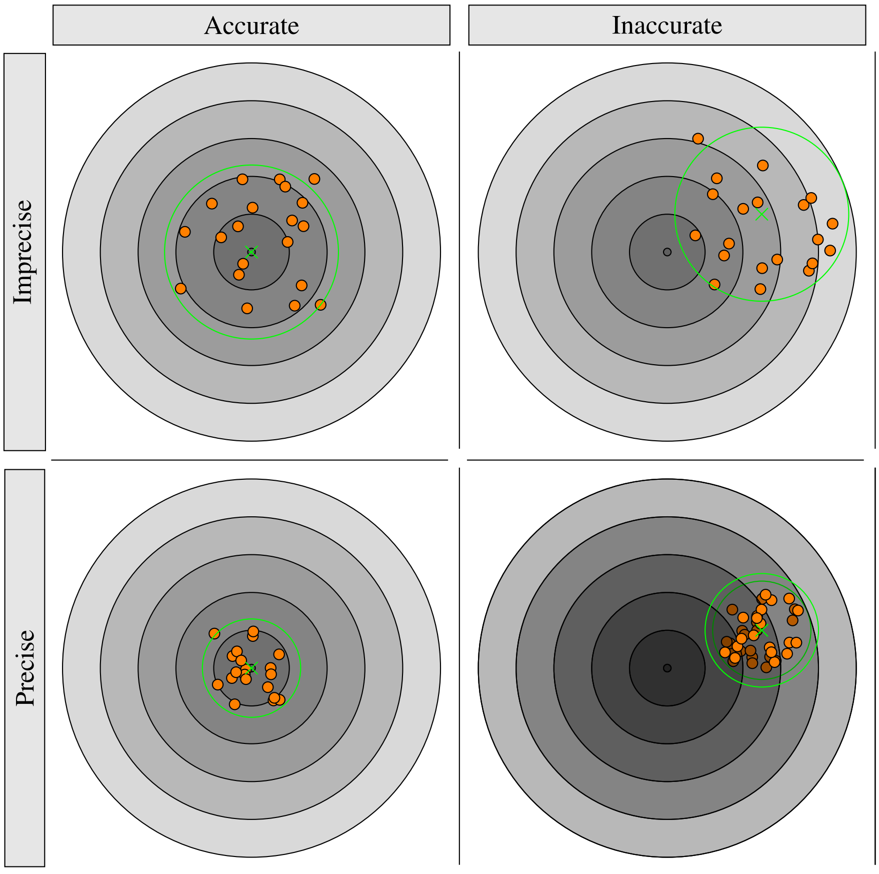


Figure 1.3 (Top Left) An accurate but imprecise measurement (Top Right) An inaccurate and imprecise measurement (Top Left) An accurate and precise measurement (Bottom Right) An inaccurate but precise measurement

`Average and standard deviation` The general approach to measuring a property such as the weight of a sample is to perform a number, n, of replicated measurements under similar conditions. Obtaining several measurements allows us to calculate the sample *average value*, $\bar{x}$, and the *standard deviation*, σ.
The sample average $\bar{x}$ is calculated using the formula:

$$\bar{x} = \frac{1}{n}\sum_{i}^{n} x_i$$

where $\sum_i^n x_i$ represents the sum of all measurements, x. The average value informs about the accuracy of the measurement. When the average is close to the true value the measurement is said to be accurate. The sample standard deviation σ provides estimates of the population values and it is calculated using the formula:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i}^{n}(x_i - \bar{x})^2}$$

The final value for measurement should be written as:

$$x = \bar{x} \pm \sigma$$

notice that the standard deviation should have the same number of decimals as the average value. The standard deviation informs about the precision or dispersion of the measurement. Very small standard deviations correspond to precise measurements. The following example demonstrates the calculation of the average and the standard deviation of a series of measurements.

Sample Problem 8

We measured the mass of the same sample several times obtaining the following values: 108.6 g, 104.2 g, 96.1 g, 99.6 g, and 102.2 g. Answer the following questions: (a) Compute the average, $\bar{x}$. (b) Compute the standard deviation of the measurement, σ. (c) Report the measured mass in the form $\bar{x} \pm \sigma$.

SOLUTION

(a) The average mass is given by:

$$\bar{m} = \frac{1}{5}(108.6\,g + 104.2\,g + 96.1\,g + 99.6\,g + 102.2\,g) = 102.1\,g.$$

(b) The standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{4}\Big((108.6\,g - 102.1\,g)^2 + (104.2\,g - 102.1\,g)^2 + (96.1\,g - 102.1\,g)^2 + (99.6\,g - 102.1\,g)^2 + (102.2\,g - 102.1\,g)^2\Big)} = 4.7\,g. \tag{1.1}$$

(c) The measured mass is given by: $102.1 \pm 4.7\,g$

STUDY CHECK

We measured the volume of the same sample several times obtaining the following values: 10.12 mL, 10.12 mL, 10.13mL, 10.14 mL, and 10.15 mL. Answer the following questions: (a) Compute the average, $\bar{x}$. (b) Compute the standard deviation of the measurement, σ. (c) Report the measured volume in the form $\bar{x} \pm \sigma$. (d) Given that he true volume is 10.50mL, describe the accuracy and precision of the measurement

Answer: (a) 10.13mL (b) 0.01mL (c) Report the measured volume in the form 10.13 ± 0.01mL. (d) precise and inaccurate

1.8 Significant Figures

Exact numbers result from counting. For example, think about how many eggs are there in your refrigerator, there might be three and this number is exact. Differently, numbers that result from a measurement are called measured values and they are subject to uncertainty– in other words error. For example, if you weigh a single egg on a scale depending on the type of scale you used and the person who carries out the measurement, you will measure 70g or 71g, or maybe 70.8g. The mass of an egg is a measured property and hence some of the digits of the measurement are uncertain. The goal of this section is, given a value, to calculate the number of significant figures of a number (we will refer to significant figures as SF, or SFs). Another goal is to estimate significant figures in the calculation to express the result with the right number of digits and significant figures.

Significant figures of numbers In general, all numbers different than zero are significant and for example, the number 123 has three significant figures. Similarly, the number 45 has two significant figures. Zeros are also significant except when:

Exception 1 *A zero is not significant when placed at the beginning of a decimal number.* For example, the number 0.123 has three significant figures, as the first zero is not significant. Similarly, the number 0.002340 has four significant figures as the first three zeros are not significant but the last zero it is. Mind the rule affects only the zeros at the beginning. A final example:

$$0.032 \ \ (2\text{SF})$$

Exception 2 *A zero is not significant when used as a placeholder in a number without a decimal point.* For example, the number 1000 has only one significant figure, and the number 3400 has two. Let us consider more examples. The number 120 has two significant figures, as according to the second rule the last zero is not significant. Differently, the number 1203 has four significant figures, as the zero in between two numbers is not affected by either the first or the second rule. A final example,

$$3200 \ \ (2\text{SF})$$

Exception 3 *A zero in a number expressed in scientific notation is significant.* For example, the zero in 3.0×10^{-2} is significant, and the number has 2SFs. A final example:

$$3.2020 \times 10^{2} \ \ (5\text{SF})$$

Sample Problem 9

Indicate the number of significant figures in the following numbers: 123, 4567, 1200, 340, 0.001, 0.023 and 0.0405.

SOLUTION

123 has three significant figures, whereas 4567 has four SF. 1200 has only 2SF as the last two zeros are not significant, and 340 has only 2SF as the last zero is not significant. 0.001 has only one significant figure as the first 3 zeros are not significant and 0.023 has only two SFs. Finally, 0.0405 has threee SFs as the first two zeros are not significant but the zero between 4 and 5 is indeed significant.

STUDY CHECK

Indicate the number of significant figures (SFs) in the following numbers: 4560, 0.123, 1000 and 0.0030.

▶Answer: 4560 has 3SF, 0.123 has 3SF, 1000 has 1SF and 0.0030 has 2SF.

`Significant figures in calculations` Two different rules allow you to express the result of calculations with the correct number of figures.

Rule 1 $(+\ -)$ *For additions or subtractions, the results has the same number of decimal places as the number with the least decimal places in the calculation.* For example:

$$34.3451 + 34.5 = 68.8\ (+\ -\text{less decimals})$$

If you add $34.3451 + 34.5$ you will obtain 68.8451, however, as 34.3451 has four decimal places (4DP) and 34.5 has one decimal place (1DP), the result of adding both numbers will have to have only one decimal place, therefore 68.8451 needs to be rounded to 68.8 (1DP). Overall, we have:

$$34.3451\ \text{(4DP)}\ + 34.5\ \text{(1DP)}\ = 68.8\ \text{(1DP)}$$

Rule 2 $(\times\ \div)$ *For multiplications and divisions, the number of significant figures of the result should be the same as the least number of significant figures involved.* For example, if you carry the following multiplication:

$$4500 \times 342 = 1500000\ (\times\ \div\ \text{less SFs})$$

the number 4500 (2SF) has two significant figures, whereas the number 342 (3SF) has three significant figures. If we multiply both numbers the results should contain just two significant figures. The result of multiplying 4500×342 is 1539000 (4SF), however, this number needs to be rounded into two significant figures into 1500000 (2SF). Overall we have:

$$4500\ \text{(2SF)}\ \times 342\ \text{(3SF)}\ = 1500000\ \text{(2SF)}$$

Sample Problem 10

Do the following calculation with the correct number of figures.

$$\frac{88.5 - 87.57}{345.13 \times 100}$$

SOLUTION

We will analyze each number indicating the number of SF and Digits (DP): 88.5(3SF, 1DP), 87.57(4SF, 2DP), 345.13(6SF, 2DP) and 100(1SF, 0DP). The result of doing the addition needs to be rounded to one single decimal place: $88.5 - 87.57 = 0.93 \simeq 0.9$. After that we have only multiplications and divisions and hence we will now focus on the number of SFs:

$$\frac{0.9\ \text{(1SF)}}{345.13\ \text{(5SF)}\ \times 100\ \text{(1SF)}}$$

The result of this operation needs to be rounded to one SF:

$$\frac{0.9}{345.13 \times 100} = 2.6077 \times 10^{-5} \simeq 3 \times 10^{5}\ \text{(1SF)}$$

STUDY CHECK

Do the following calculation with the correct number of figures: $(24.56 + 2.433) \times 0.013$

▶Answer: 0.35

Rounding The following rules indicate how to round numbers:

Rule 1 If the digit to be removed is less than 5 then the preceding digit stays the same. For example, 1.123 rounds to 1.12.

Rule 2 If the digit to be removed is more or equal to 5 then the preceding digit is increased by one. For example, 1.126 rounds to 1.13

Rule 3 When rounding to a specific number of significant figures we need to look only to the first number to the right of the last significant figure. For example, 1.126 rounds to two SF as 1.1

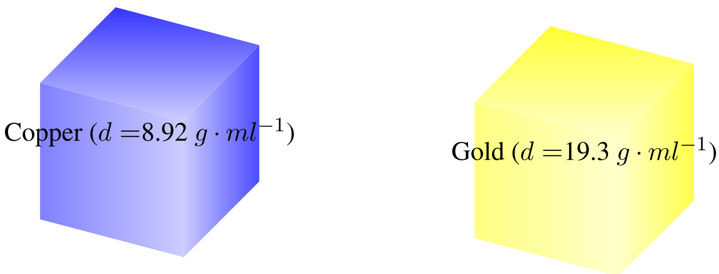
Now, let us analyze a few use cases. Imagine we need to round the number 1234cm to two SF. The results would be 1200cm. Similarly, imagine we need to round the number 0.01264cm to two SF. The results would be 0.013cm.

1.9 Density

Density refers to the mass of a substance with respect to its volume. This is an unique property for each substance. Table 1.4 reports the density of numerous substances. Indeed, density is often used as an identification tag. The formula for density is

$$\boxed{\text{Density} = \frac{\text{Mass of substance}}{\text{Volume of substance}}} \tag{1.2}$$

For example, the density for copper is 8.92 $g \cdot ml^{-1}$ and for gold is 19.3 $g \cdot ml^{-1}$. By measuring density only, you would be able to differentiate copper than gold. The larger density the more compact is an object and that means the more mass per volume it has. At the same time, for the same volume, the larger density the larger the mass of the metal.



Density and mixing A small piece of ice will float on the water. The reason for that is density: the density of ice (0.9g/mL) is smaller than the density of water (1.0g/mL) and hence ice will stay on top of the water. Objects with a density larger than 1 g/mL will sink whereas objects with a density smaller than this value will float. If you add a drop of vegetable oil to a glass of water, the drop will float. This is because the density of oil is smaller than 1g/mL.

Table 1.4 Density of some common substances at 273.15 K and 100 kPa

Substance	Density (g/mL)	Physical State
Helium	0.2	gas
Hydrogen	0.1	gas
Water	1.0	Liquid
Cooking oil	0.9	Liquid
Mercury	13.5	Liquid
Tetrachloroethene	1.6	Liquid
Gold	19.3	solid
Plastics	1.2	solid
Ice	0.916*	solid

*Ice is given at T <273.15 K

Sample Problem 11

In the figure, we mixed three liquids of density: A (0.5 g/mL), B(2 g/mL) and C(1 g/mL). Identify each liquid.

SOLUTION

The heavier the liquid, that is the larger density, the lower the liquid will arrange in the mixture. From top to bottom we have A, C and B.

STUDY CHECK

Indicate the order that the following invisible liquids will appear in a cylinder when mixed: (a) benzene(0.87 g/mL) and (b) water(1 g/mL)

▶Answer: benzene on top

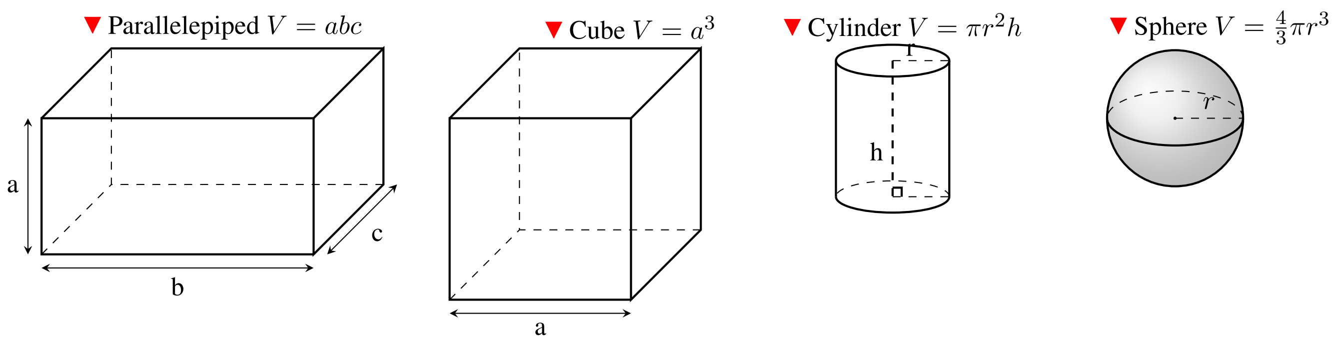


Figure 1.4 Volume of some objects

`Density and the volume of objects` Density depends on volume and in particular the larger volume the smaller density. Figure 1.4 displays the formulas to calculate the volume for some common objects, like a sphere or a cube. For example, the radius of a sphere with density d and mass m corresponds to $\sqrt[3]{3m/4d \cdot \pi}$, and the side of a cube with density d and mass m corresponds to $\sqrt[3]{m/d}$. The density of liquids results from measuring the mass of a given volume of the liquid. Differently, density is harder to obtain for solids. For metals, we can calculate density by the immersion method: when a metal is immersed in water, the water rises. This increase in volume corresponds to the volume of the solid. This way, density results from the direct measurement of mass and the measurement of volume by displacement. The following example demonstrates density calculation with the immersion method.

Sample Problem 12

After adding a 30g object into a cylinder filled of water, the level of water rises from 60mL to 90mL. Calculate the density of the object.

SOLUTION

Density is mass over volume. The mass of the object is 30g and its volume is (90-60)mL that is 30mL. Hence: $d = 30g/30mL = 1g/mL$.

STUDY CHECK

A lead weight used in the belt of a scuba diver has a mass of 226 g. When the weight is placed in a graduated cylinder containing 200.0 mL of water, the water level rises to 220.0 mL. What is the density of the lead weight (g/mL)?

Answer:11.3 g/mL.