Ch. 1. Measurements

MEASURING is an important part of our everyday lives, and very probably you took several measurements today. You might now be sipping a cup of coffee, or perhaps you checked the outside temperature on a street thermometer. You might be planning to bake a cake and need to use a scale and a cup to measure the flour and sugar. A cup, a thermometer, or a scale are measuring devices. It is critical to know how to accurately measure properties and, more importantly, how to transform measurements using prefixes and unit conversions. By learning how to measure and perform operations with units, you will gain experience performing basic chemistry calculations.

1.1 Math skills

There are a few math skills that are critical to be able to carry out chemistry problems. Those skills ental operating with numbers, converting numbers into scientific notation, solving basic equations, and interpreting graphs.

`Place values` We can identify the place value in any number for all its digits. The place values are named based on the location concerning the decimal place. For digits to the left of the decimal place, the place values are called the ones, the tens, the hundreds, and the thousands. For example, in the value 3456s, the number 3 is the thousands place, the number 4 is the hundreds place, 5 is the tens place and 6 is the one's place. For digits to the right of the decimal place, the place values are called the tenths, the hundredths, and the thousandths, mind the suffix *ths*. For example, in the value 3.456Kg, the number 3 is the one place, the number 4 is the tenth place, 5 is the hundredth place and 6 is the thousandth place.

`Basic algebra` Positive numbers are larger than zero whereas negative numbers are smaller than zero. Negative numbers are written with a negative $(-)$ sign. The multiplication or division of two positive or two negative numbers always gives a positive result, whereas the multiplication or division of a positive and a negative number always gives a negative result. When we add two positive numbers the results are always positive, whereas when we add two negative numbers the results are always negative. When positive and negative numbers are added, the results come from substracting the smallest number from the largest number while keeping the sign of the larger number. In a calculator, there is a key $\boxed{(-)}$ (sometimes shown as $\boxed{(+/-)}$) used to switch the sign of a number.

Solving equations When solving the equation below for x

$$3x - 4 = 8$$

some basic algebra rules apply:

1. **Step one:** Place all like terms in one side
2. **Step two:** Isolate the variable you want to calculate
3. **Step three:** Check the answer

When placing like terms on the same side, you can eliminate terms by adding, subtracting, multiplying, or dividing. Make sure you apply those rules on both sides of the equation at the same time. For the example above, we will first eliminate the -4 by subtracting 4 in both sides:

$$3x - \cancel{4} + \cancel{4} = 8 + 4$$

That gives,

$$3x = 12$$

Now we will remove the 3 by diving by three on both sides:

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{12}{3}$$

Therefore we have

$$x = \frac{12}{3} = 4$$

Scientific notation Numbers in science can often be very large or very tinny. For example, the mass of the earth is 590000000000000000000000000 grams, whereas the size of an atom is of the order 0.00000000001cm. Scientific notation is a standard notation extensively used in the scientific community to simplify numbers. Numbers not expressed in scientific notation are referred to as numbers in full notation (e.g. 0.234g). Numbers expressed in scientific notation place the decimal after the first significant digit while being accompanied by a power of ten (e.g. 2.34×10^{-1}g). Numbers expressed in scientific notation have two parts: the coefficient (a number larger than one but smaller than 10) and the power of ten. For example, the number 3.45×10^{5}g has a coefficient of 3.45 and a power of ten with positive power. The power of ten can contain a positive or a negative exponent. Positive exponents represent large numbers, whereas negative exponents represent numbers smaller than 1. To express a number larger than one (e.g. 345000g) in scientific notation, we just need to leave the first digit followed by the decimal point which was moved to the left 5 places, giving 3.45×10^{5}g as indicated below

345000g (full notation) 3.45×10^{5}g (scientific notation)

To express a number smaller than one (e.g. 0.000134g) in scientific notation, we just need to leave the first digit followed by the decimal point which was moved in this case to the right 4 places, giving 1.34×10^{-4}g as indicated below

0.000134g (full notation) 1.34×10^{-4}g (scientific notation)

You can enter scientific notation numbers in a calculator using a specific key that contains the $\times$ character and the power of ten. For example, the number 1×10^{6} should be typed in a calculator as $1\boxed{\text{EE}}\,6$ or $1\boxed{\text{EXP}}\,6$ or $1\boxed{\times 10^x}\,6$, deppending on your calculator. For numbers with a negative power of ten, you should use the $\boxed{(-)}$ key (sometimes shown as $\boxed{+/-}$) to indicate the sign. For example,the number 1×10^{-5} should be typed in a calculator as $1\boxed{\text{EE}}\boxed{(-)}\,5$ or $1\boxed{\text{EXP}}\boxed{(-)}\,5$ or $1\boxed{\times 10^x}\boxed{(-)}\,5$, deppending on your calculator. The calculator display often can display a scientific notation number differently, based on the calculator brand. Below are three possible scenarios:

1.5×10^{-4}	1.5E-04
6.7×10^{-5}	6.7 -05
4.6×10^{-2}	4.6₁₀-02

You can also convert full notation numbers into scientific notation with a calculator key named $\boxed{\text{SCI}}$ often accessible through the second function key.

Sample Problem 1

Convert the following numbers from full to scientific notation or vice versa:
(a) 7462.97 (b) 0.000234 (c) 0.012

SOLUTION

The answers are 7.46297×10^3, 2.34×10^{-4}, and 1.2×10^{-2}.

STUDY CHECK

Convert the following numbers from full to scientific notation or vice versa:
(a) 12000 (b) 0.00076 (c) 45783

▶Answer: (a) 1.2×10^4 (b) 7.6×10^{-4} (c) 4.5783×10^4

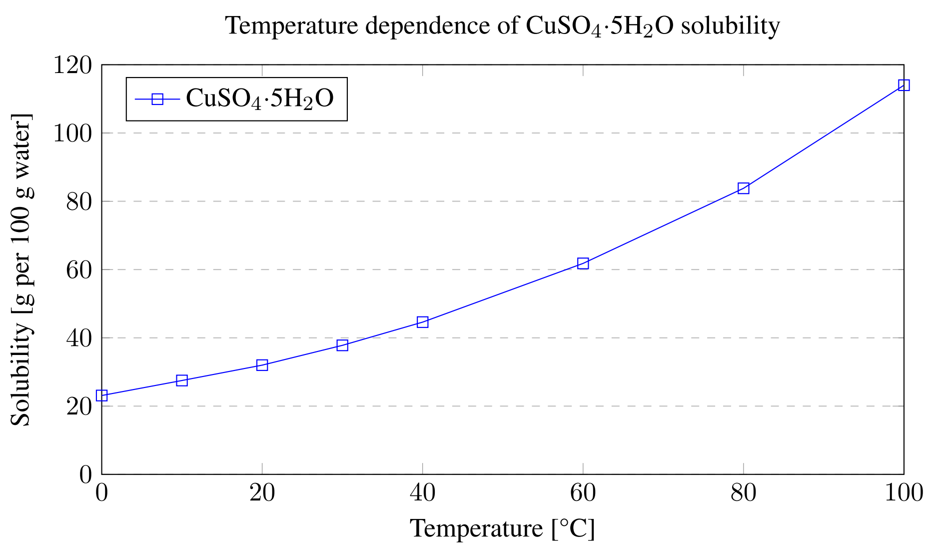

Figure 1.1 A graph plotting the change of solubility of a chemical with temperature.

Interpreting graphs A graph represents data in terms of a relationship between two variables. For example, Figure 1.1 represents the relationship between solubility and temperature. These quantities are plotted along two axes the Y or vertical axis and

▼Scales measure mass

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▼Watches are used to measure time

© www.wallpaperflare.com

▼Beakers can carry a liquid volume

© wikipedia

▼Thermometers measure temperature

© PngImg

▼pipets are used in chemistry practice to add an exact volume of liquid

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the X or horizontal axis. The title of the graph indicates what is being represented, and for example, on the graph above we have represented the change of solubility of a chemical with temperature. The vertical axis represents solubility given a range between 0 and 120 g/100g of water. The horizontal axis represents temperature with a range between 0 and 100°C. Each point in the graph represents the value of solubility for a given temperature. Based on the graph we can see that solubility slowly increases as temperature increases. We could estimate the value of solubility for any temperature within the range given and for example at a temperature of 50 °C the solubility should be close to 50 g/100g of water.

1.2 Units of Measurements and systems of units

You probably heard the term liter, kilogram, or meter. These are units of measurement. Units can be classified into different *systems of units*. For example, the unit *meter* belongs to a different system than the unit *mile*. In particular, here we will address three main systems: the English System, the Metric System, and the International system. The *Metric System* (MS) is used by scientists throughout the world and is the most common measuring system based on the meter. The English system is mostly used in the US. The *International System of Units* (SI) adopted the metric system in 1960 to provide additional uniformity for units used in the sciences. Table 1.1 summarizes some of the fundamental units for the three systems. This chapter will be mostly based on the SI units. In the following, we will introduce some common units.

`Length` What is your height? The length refers to distance and both the metric and SI unit of length is the meter (m). A smaller unit of length would be the centimeter (cm) that is commonly used in chemistry. The most important units of length are meters, inches, and miles.

`Mass` What is your weight? The mass of an object is a measure of the quantity of material it contains. You may be more familiar with the term weight rather than mass. However, mass and weight are not the same, as weight is a measure of the gravitational pull on an object. It differs depending on your location on the earth–in particular the height of your location. In the metric system, the unit of mass is the gram (g). The SI unit of mass, the kilogram (kg), is used for larger masses such as body weight. A pound, lb, is another unit of mass. The most important units of mass are g, kg, and lb.

`Temperature` How is the weather today? Is it cold or hot? You use a thermometer to measure temperature and for example assess how hot an object is, or how cold it is outside, or perhaps to determine if you have a fever. Temperature tells us how hot or cold an object is. Temperature can be measured in numerous units such as Celsius (°C), Fahrenheit (°F), or kelvins (K).

`Time` How long is your commute to work? It might take you hours to go to work, or maybe minutes. You probably think of time as years, days, minutes, or seconds. Of all these units, the International System of units (SI, abbreviated from the French Systeme international) uses seconds (s) to measure time. Still, time can be measured in s, min, or h and during this chapter, we will learn how to convert units of time.

`Volume` How much milk do you usually buy? Maybe a gallon. Volume is the amount of space that a substance occupies. A liter (L), not a fundamental but a derived SI unit, is commonly used to measure volume. The milliliter (mL) is more convenient for measuring smaller volumes of fluids in hospitals and laboratories. Gallon is still used in everyday life. L, mL, and gallon are units of volume. Units of volume are in general cubic units, so for example one liter is the same as one dm^3. We will cover cubit units further in this chapter.

Chemical laboratory work commonly requires the measurement of volume. There are two main types of glassware used to measure volume in a chemistry lab: graduated tools and volumetric tools. Volumetric pipets, flasks, and burets are the most accurate; the glassware makers calibrate these to a high level of accuracy, usually measured in terms of tolerance, which is the uncertainty in a measurement made with the glassware. Class A volumetric glassware has a lower tolerance than Class B; for class A, the tolerance can be as low as 0.08 ml for a 100 ml flask or pipet. Generally, measurements with class A volumetric glassware can be considered reliable to two places after the decimal point. Graduated cylinders, beakers, and Erlenmeyer flasks have less accuracy than volumetric glassware. Graduated cylinders can generally be considered reliable to within 1 percent. Beakers and Erlenmeyer flasks should not be used to measure volume unless you need only a very crude estimate because their accuracy for volume measurements is so poor. They can hold a much larger volume than any of the other types of glassware, however, which makes them useful for mixing solutions.

`Concentration` Even though we will devote a whole chapter to solutions and concentration, it felt important to introduce here the unit molarity. In chemistry, the unit molarity (M) refers to the concentration of a solution. That is the larger this number, the larger molarity, and the more concentrated a solution will be. In other words, there will be more substance in the solution.

Sample Problem 2

State the type of measurement indicated in each of the following:

(a) 1ft(foot) (b) 20Kg (c) 3L (d) 300K

SOLUTION

(a) length; (b) mass; (c) volume; (d) temperature;

STUDY CHECK

State the type of measurement indicated in each of the following: (a) 800°F (b) 1m^3 (c) 3m (d) 67s

▶Answer: (a) temperature; (b) volume; (c) length; (d) time;

Table 1.1 Different unit systems

Measuremts	Metric System	International System (SI)	English System
Length	Meter (m)	Meter (m)	Foot (ft)
Mass	Gram (g)	Kilogram (kg)	Pound (lb)
Time	Second (s)	Second (s)	Second (s)
Temperature	Celsius (°C)	Kelvin (K)	Fahrenheit (°F)
Volume*	Liter (L)	Cubic meter (m^3)	Gallon (gal)
Ammount of substance	Mole (mol)	Mole (mol)	Mole (mol)
Electric current	Ampere (A)	Ampere (A)	Ampere (A)

*Not a fundamental unit

1.3 Significant Figures

Exact numbers result from counting. For example, think about how many eggs are there in your refrigerator, there might be three and this number is exact. Differently, numbers that result from a measurement are called measured values and they are subject to uncertainty–in other words error. For example, if you weigh a single egg on a scale depending on the type of scale you used and the person who carries out the measurement, you will measure 70g or 71g, or maybe 70.8g. The mass of an egg is a measured property and hence some of the digits of the measurement are uncertain. The goal of this section is, given a value, to calculate the number of significant figures of a number (we will refer to significant figures as SF, or SFs). Another goal is to estimate significant figures in the calculation to express the result with the right number of digits and significant figures.

`Measured numbers` *Measured numbers* result from measuring a property such as the weight or length of an object. Those measurements result from using a measuring device such as a scale or a ruler, for example. Imagine we want to measure the length of both objects presented in Figure 1.2. The metric rules presented have a set of marked divisions which determine the number of figures given by the measurement. For example, the ruler on the left has 1cm and 0.1cm divisions, whereas the rule on the right only has 1cm divisions, hence giving fewer figures.

Let us estimate the length of the object on the right. The end of the object on the right is located between 0cm and 1cm, therefore its length is less than 1cm. Still, we can estimate an extra digit by dividing the space between the lines. Still, this last *estimated digit* might differ from person to person. The final measurement would be 0.8cm. However, some people would read the length as 0.7cm whereas others 0.9cm. Let us now estimate the length of the object on the left. The end of the object on the right is located between 3.1cm and 3.2cm, therefore its length is less than 3.2cm. We can estimate an extra digit as well, giving a final measurement of 3.15cm.

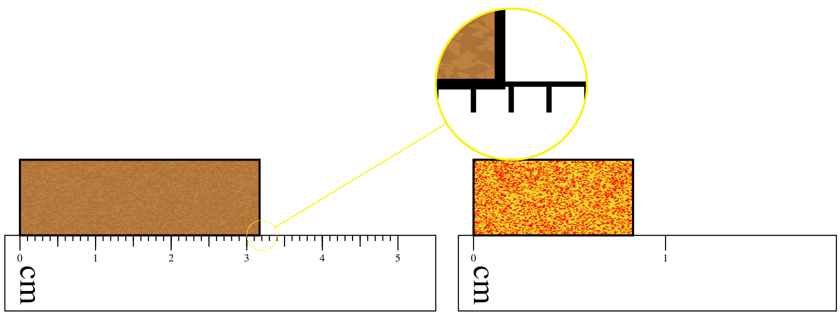


Figure 1.2 Some metric scales with two objects of different lengths. Measurements are (left) 3.15cm (right) and 0.8cm.

`Reading menisci` Reading a liquid meniscus is similar to reading any measuring scale. There are two types of menisci (see Figure 1.3). A concave meniscus, which is what you normally will see, occurs when the molecules of the liquid are attracted to those of the container. This occurs with water and a glass tube. A convex meniscus occurs when the molecules have a stronger attraction to each other than to the container, as with mercury and glass. If the meniscus is concave, read at the lowest level of the curve. If the meniscus is convex, take your measurement at the highest point of the curve. Let us read the menisci from the image below. Readings are 16.0mL (left), 8.5mL (center left), 18.0mL (center right), and 18.5mL (right).

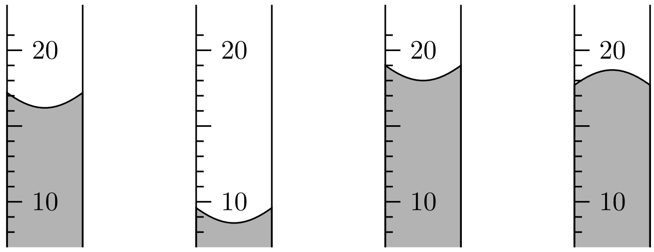


Figure 1.3 Some volumetric measurements in mL are presented in two types of meniscuses. The Left, center-right, and center-left are concave meniscuses, whereas the right image presents a convex meniscus. Readings are 16.0mL (left), 8.5mL (center left), 18.0mL (center right), and 18.5mL (right).

`Exact numbers` Exact numbers are numbers obtained by counting and not by measuring or obtained by a relationship that compared two units in the same measuring system. For example, the number of students in a class is exact as we need to count to get this number. Similarly, the number of grams in a kilogram, a thousand, is exact as the relationship between kilogram and gram is exact. Exact numbers do not have significant figures and do not limit the number of figures in a calculation.

`Significant figures of numbers` In general, all numbers different than zero are significant and for example, the number 123 has three significant figures. Similarly, the number 45 has two significant figures. Zeros are also significant except when:

Exception 1 *A zero is not significant when placed at the beginning of a decimal number.* For example, the number 0.123 has three significant figures, as the first zero is not significant. Similarly, the number 0.002340 has four significant figures as the first three zeros are not significant but the last zero it is. Mind the rule affects only the zeros at the beginning. A final example:

$$0.032 \;\; \text{(2SF)}$$

Exception 2 *A zero is not significant when used as a placeholder in a number without a decimal point.* For example, the number 1000 has only one significant figure, and the number 3400 has two. Let us consider more examples. The number 120 has two significant figures, as according to the second rule the last zero is not significant. Differently, the number 1203 has four significant figures, as the zero in between two numbers is not affected by either the first or the second rule. A final example,

$$3200 \;\; \text{(2SF)}$$

Exception 3 *A zero in a number expressed in scientific notation is significant.* For example, the zero in 3.0×10^{-2} is significant, and the number has 2SFs. A final example:

$$3.2020 \times 10^{2} \;\; \text{(5SF)}$$

Sample Problem 3

Indicate the number of significant figures in the following numbers: 123, 4567, 1200, 340, 0.001, 0.023 and 0.0405.

SOLUTION

123 has three significant figures, whereas 4567 has four SF. 1200 has only 2SF

as the last two zeros are not significant, and 340 has only 2SF as the last zero is not significant. 0.001 has only one significant figure as the first 3 zeros are not significant and 0.023 has only two SFs. Finally, 0.0405 has threee SFs as the first two zeros are not significant but the zero between 4 and 5 is indeed significant.

STUDY CHECK

Indicate the number of significant figures (SFs) in the following numbers: 4560, 0.123, 1000 and 0.0030.

Answer: 4560 has 3SF, 0.123 has 3SF, 1000 has 1SF and 0.0030 has 2SF.

`Significant figures in calculations` Two different rules allow you to express the result of calculations with the correct number of figures.

Rule 1 $(+\ -)$ *For additions or subtractions, the results has the same number of decimal places as the number with the least decimal places in the calculation.* For example:

$$34.3451 + 34.5 = 68.8\ (+\ - \text{ less decimals})$$

If you add $34.3451 + 34.5$ you will obtain 68.8451, however, as 34.3451 has four decimal places (4DP) and 34.5 has one decimal place (1DP), the result of adding both numbers will have to have only one decimal place, therefore 68.8451 needs to be rounded to 68.8 (1DP). Overall, we have:

$$34.3451 \text{ (4DP) } + 34.5 \text{ (1DP) } = 68.8 \text{ (1DP)}$$

Rule 2 $(\times\ \div)$ *For multiplications and divisions, the number of significant figures of the result should be the same as the least number of significant figures involved.* For example, if you carry the following multiplication:

$$4500 \times 342 = 1500000\ (\times\ \div \text{ less SFs})$$

the number 4500 (2SF) has two significant figures, whereas the number 342 (3SF) has three significant figures. If we multiply both numbers the results should contain just two significant figures. The result of multiplying 4500×342 is 1539000 (4SF), however, this number needs to be rounded into two significant figures into 1500000 (2SF). Overall we have:

$$4500 \text{ (2SF) } \times 342 \text{ (3SF) } = 1500000 \text{ (2SF)}$$

Sometimes we will have to add significant zeros in order to present the final result of a calculation with the correct number of digits. For example:

$$8.00 \text{ (3SF) } \div 2.00 \text{ (3SF) } = 4 \text{ (shows in calculator) } = 4.00 \text{ (3SF)}$$

Sample Problem 4

Do the following calculation with the correct number of figures.

$$\frac{88.5 - 87.57}{345.13 \times 100}$$

SOLUTION

We will analyze each number indicating the number of SF and Digits (DP):

88.5(3SF, 1DP), 87.57(4SF, 2DP), 345.13(6SF, 2DP) and 100(1SF, 0DP). The result of doing the addition needs to be rounded to one single decimal place: $88.5 - 87.57 = 0.93 \simeq 0.9$. After that we have only multiplications and divisions and hence we will now focus on the number of SFs:

$$\frac{0.9\ (1\text{SF})}{345.13\ (5\text{SF})\ \times 100\ (1\text{SF})}$$

The result of this operation needs to be rounded to one SF:

$$\frac{0.9}{345.13 \times 100} = 2.6077 \times 10^{-5} \simeq 3 \times 10^{5}\ (1\text{SF})$$

STUDY CHECK

Do the following calculation with the correct number of figures: $(24.56 + 2.433) \times 0.013$

▶Answer: 0.35

Rounding The following rules indicate how to round numbers:

Rule 1 If the digit to be removed is less than 5 then the preceding digit stays the same. For example, 1.123 rounds to 1.12.

Rule 2 If the digit to be removed is more or equal to 5 then the preceding digit is increased by one. For example, 1.126 rounds to 1.13

Rule 3 When rounding to a specific number of significant figures we need to look only to the first number to the right of the last significant figure. For example, 1.126 rounds to two SF as 1.1

Now, let us analyze a few use cases. Imagine we need to round the number 1234cm to two SF. The results would be 1200cm. Similarly, imagine we need to round the number 0.01264cm to two SF. The results would be 0.013cm.

Table 1.2 Different prefixes

Prefix	Symbol	Meaning	Value
exa	E	1000000000000000000	1×10^{18}
peta	P	1000000000000000	1×10^{15}
tera	T	1000000000000	1×10^{12}
giga	G	1000000000	1×10^{9}
mega	M	1000000	1×10^{6}
kilo	k	1000	1×10^{3}
hecto	h	100	1×10^{2}
deca	da	10	1×10^{1}
–	–	1	1×10^{0}
deci	d	0.1	1×10^{-1}
centi	c	0.01	1×10^{-2}
milli	m	0.001	1×10^{-3}
micro	μ	0.00001	1×10^{-6}
nano	n	0.000000001	1×10^{-9}
pico	p	0.000000000001	1×10^{-12}
femto	f	0.000000000000001	1×10^{-15}
atto	a	0.000000000000000001	1×10^{-18}

1.4 Prefixes & Conversion Factors

Let's consider the following measurements: 1 **k**m, 2 **c**m and 3 m that can be read as one kilometer, two centimeters, and three meters. The word kilo (**k**) and centi (**c**) are called prefixed whereas meter (m) is a simple unit. Table 1.2 lists some of the metric prefixes, their symbols, and their decimal values. A kilometer is larger than a meter, whereas a centimeter is smaller than a meter. Prefixes such as kilo or centi are attached to units to make numbers more manageable. For example, the radius of the earth is 6356 km, and this number is easier to handle than 6356000m. At the same time, we can attach any prefix to different units. Hence, we can talk about a centimeter (**c**m) but also about a centisecond (**c**s) or centiliter (**c**L). All these units have the same prefix.

How to identify prefixes? Look for example at the measurement 2 cm. Centi (c) is the prefix and means 1×10^{-2} and meter (m) is the unit that refers to length. Another example, is 7 kg read as seven kilograms. Kilo (k) is the prefix and means 1×10^{3}, whereas gram (g) is the unit that refers to mass. The prefix refers to the first letter whereas the unit refers to the last letter.
A unit with a prefix can be bigger or smaller than the plain unit–this is the unit without a prefix–, depending on the prefix. The following prefixes reduce the unit: deci, centi, milli, micro, nano, pico, and femto. For example, a fs (femtosecond) is smaller than a s (second). Differently, the following prefixes increase the unit: Tera, Giga, Mega. For example, a Tb (terabyte) is larger than a b (byte). A byte is a unit used in computer science.

How to write unit equalities and conversion factors Unit equalities are simple expressions that relate a unit with a prefix. For example, one centimeter (cm) is 1×10^{-2}m. Hence we can write this as unit equality:

$$\boxed{1cm = 1 \times 10^{-2}m} \text{ unit equality}$$

Let's compare cm and m. The first, cm, is a unit with a prefix, whereas m is simply a unit of length without a prefix. To know how many m are there in a cm we need to write down a conversion factor. Think about prefixes as synonymous with a number. In this way, centi stands for 1×10^{-2}, so

$$\boxed{\frac{1cm}{1 \times 10^{-2}m} \text{ or } \frac{1 \times 10^{-2}m}{1cm}} \text{ conversion factor}$$

The relationship above is also called a *unit factor* as both ratios are equal to one.

Sample Problem 5

Complete each of the following equalities and conversion factors:

(a) 1dm=______m

(b) 1km=______m

(c) $\frac{1nm}{\_\_\_\_m}$

(d) $\frac{\_\_\_\_m}{1cm}$

SOLUTION

(a) 1dm=$1 \times 10^{-1}m$; (b) 1km=$1 \times 10^{3}m$; (c) $\frac{1nm}{1 \times 10^{-9}m}$; (d) $\frac{1 \times 10^{-2}m}{1cm}$;

STUDY CHECK

Second is a unit of time. Complete each of the following equalities and conver-

sion factors involving seconds:
(a) 1cs=_____s (b) $\frac{\_\_\_s}{1Ts}$ (c) $\frac{\_\_\_s}{1Ms}$

▶Answer: (a) 1cs=$1\times10^{-2}s$; (b) $\frac{1\times10^{12}s}{1Ts}$; (c) $\frac{1\times10^{6}s}{1Ms}$;

1.5 Using Conversion Factors

Unit equalities in the form of conversion factors are used to convert one unit into another. Sometimes one wants to get rid of a prefix, such as when we transform centimeter (cm) into meter (m). Sometimes, one wants to convert a prefix into another prefix. An example would be converting centimeters (cm) to millimeters (mm). Let's work on some examples.

`Removing or adding prefixes` Imagine that you need to remove a prefix from a unit, and convert 3 km (we will call this one the original unit) into meters (this is the final unit). First, you would need the conversion factor corresponding to the prefix (centi) from Table 1.2. Then you need to arrange the conversion factor by placing the prefix at the bottom of the fraction. This will cancel out the prefix in the original unit and the bottom part of the conversion factor, hence leaving the final unit on top of the conversion factor. The arrangement would be:

$$3\cancel{km}\times\frac{1\times10^{3}m}{1\cancel{km}}=3000m$$

Imagine now that you need to add a prefix into a unit, and convert 4000 m in km. The same would apply for this case, but now you will have to arrange the conversion factor so that the prefix is on the top:

$$4000\cancel{m}\times\frac{1\text{ km}}{1\times10^{3}\cancel{m}}=4km$$

Sample Problem 6

The length of a textbook page is 20cm. Convert 20cm to meters, expressing the result in scientific notation.

SOLUTION

In order to convert 20cm into meters, we need to remove the prefix (centi) leaving the unit (meter) without any prefix. We will use the conversion factor that relates m to cm: $\frac{1\times10^{-2}m}{1cm}$ or $\frac{1cm}{1\times10^{-2}m}$. We will arrange the conversion factor so that cm cancels giving m and hence we will use $\frac{1\times10^{-2}m}{1cm}$:

$$20\cancel{cm}\times\frac{1\times10^{-2}m}{1\cancel{cm}}=2\times10^{-1}m$$

The original units and on the bottom of the conversion factor cancel and we get meters, the final unit.

STUDY CHECK

Convert 100m to km, expressing the result in scientific notation.

Answer: $100\cancel{m} \times \frac{km}{1 \times 10^3 \cancel{m}} = 1 \times 10^{-1} km.$

`Switching prefixes` To switch a prefix into another prefix, such as transforming 30 millimeters (30 mm) into centimeters (cm), you will need two different conversion factors: the first conversion factor will remove the original unit (mm) introducing an intermediate unit, meters (m), whereas the second conversion factor will remove the intermediate meter and introduce the final unit (cm). You will get the conversion factors from Table 1.2. You will arrange the first conversion factor so that the original unit cancels out with the bottom of the first conversion factor, giving you an intermediate unit. You will arrange the second conversion factor so that the intermediate unit cancels out with the bottom of the second conversion factor giving the final unit. For this example:

$$30\cancel{mm} \times \frac{1 \times 10^{-3} \cancel{m}}{1\cancel{mm}} \times \frac{1cm}{1 \times 10^{-2} \cancel{m}} = 3cm$$

Sample Problem 7

The length of a textbook page is 20cm. How many mm correspond this length, expressing the result in scientific notation.

SOLUTION

We want to convert 20 cm into mm, that is, we are switching prefixed. In order to do this, you need two conversion factors: $\frac{1 \times 10^{-2} m}{1cm}$ and $\frac{1 \times 10^{-3} m}{1mm}$. You will have to arrange the number (20cm) and the two conversion factors in the following form:

$$20\cancel{cm} \times \frac{1 \times 10^{-2} \cancel{m}}{1\cancel{cm}} \times \frac{1mm}{1 \times 10^{-3} \cancel{m}} = 2 \times 10^2 mm$$

STUDY CHECK

Convert 100mm to km, expressing the result in scientific notation.

Answer: $100\cancel{mm} \times \frac{1 \times 10^{-3} \cancel{m}}{1\cancel{mm}} \times \frac{1km}{1 \times 10^3 \cancel{m}} = 1 \times 10^{-4} km.$

1.6 Units of volume and area

How big is your apartment? You might be living in a $750 ft^2$ loft in Brooklyn or a larger house Upstate. Often times we encounter cubic or square units such as cubic centimeters (cm^3) or square feet (ft^2). The equivalencies for cubic or square units should take into account the unit power (power of two or power of three). If $1cm = 1 \times 10^{-2} m$, for square units the relation should be squared and $1cm^2 = 1 \times (10^{-2})^2 m^2 = 1 \times 10^{-4} m^2$. Another example, for the case of mm and mm^3:

$$\boxed{\frac{1mm}{1 \times 10^{-3} m} \quad \text{and} \quad \frac{1mm^3}{1 \times 10^{-9} m^3}}$$

Let us work on an example in which we want to convert $30m^2$ into m^2:

$$30\cancel{m^2} \times \frac{1cm^2}{1 \times 10^{-4} \cancel{m^2}} = 3 \times 10^5 cm^2$$

Sample Problem 8

How many m^2 is $20cm^2$, expressing the result in scientific notation.

SOLUTION

In order to convert $20cm^2$ to square meters, we need to remove the centi prefix and that will give us the unit square meter without any prefix. We will use the conversion factor that relates m^2 to cm^2: $\frac{1 \times 10^{-4} m^2}{1cm^2}$ or $\frac{1cm^2}{1 \times 10^{-4} m^2}$.

$$20\cancel{cm^2} \times \frac{1 \times 10^{-4} m^2}{1\cancel{cm^2}} = 2 \times 10^{-3} m^2$$

STUDY CHECK

Convert $100m^3$ to dm^3, expressing the result in scientific notation.

▶Answer: $100\cancel{m^3} \times \frac{1dm^3}{1 \times 10^{-3}\cancel{m^3}} = 1 \times 10^5 dm^3$.

Table 1.3 Table containing some common unit equalities

Unit	Equality
Inches (in)-centimeters (cm)	$2.54^{\dagger}$ cm = 1 in
miles (mi)-meters (m)	1 mi = 1609.34m
minutes (min)-hours (h)	60 min = 1 h
minutes (min)-seconds (s)	60 s = 1 min
pound (lb)-grams (g)	454 g = 1 lb
cubic centimeter (cm^3)-mililiters (mL)	1 mL = 1cm^3
Liter (L)-cubic decimeters (dm^3)	1 L = 1dm^3
drops-mililiters* (mL)	1 mL = 15 drops

* There are several definitions of a drop

† the number is exact

`Liters and milliliters` Units such as L or mL are units of volume. As volume is a three-dimensional property, those units somehow have to be related to the units of length. One liter is the same as one dm^3 and one ml is the same as one cm^3 (See Figure 1.4). In the allied health field, the units *mL* are also written as *cc* as in cubic centiliters.

$$\boxed{1L = 1dm^3 \text{ and } 1mL = 1cm^3(cc)}$$

Let us work on an example in which we want to convert $30cm^3$ into L:

$$30\cancel{cm^3} \times \frac{1\cancel{mL}}{1\cancel{cm^3}} \times \frac{1 \times 10^{-3} L}{1\cancel{mL}} = 3 \times 10^{-2} L$$

Sample Problem 9

Convert 30 m^3 into L, expressing the result in scientific notation.

SOLUTION

In order to convert m^3 into L we just need to remember that the L actually refers to dm^3, therefore is connected to meter. We will first convert m^3 into dm^3 and then dm^3 into L.

$$30\cancel{m^3} \times \frac{1\cancel{dm^3}}{1 \times 10^{-3}\cancel{m^3}} \times \frac{1L}{1\cancel{dm^3}} = 3 \times 10^4 L$$

STUDY CHECK

Convert 40L to cm^3, expressing the result in scientific notation.

▶Answer: $40\cancel{L} \times \frac{1\cancel{mL}}{1 \times 10^{-3}\cancel{L}} \times \frac{1cm^3}{1\cancel{mL}} = 4 \times 10^4 cm^3$.

1.7 Using other equalities

How many hours are 300 minutes, or how many centimeters is 2 inches? Some of the units conversion is not based on a power of ten relationships and do not contain prefixes such as kilo or centi. Table 1.3 lists some of the common equalities that can be easily converted into conversion factors. As an example, the unit equivalency between hours and minutes is $60min = 1h$ and the conversion factor would be $\frac{60min}{1h}$ or $\frac{1h}{60min}$.

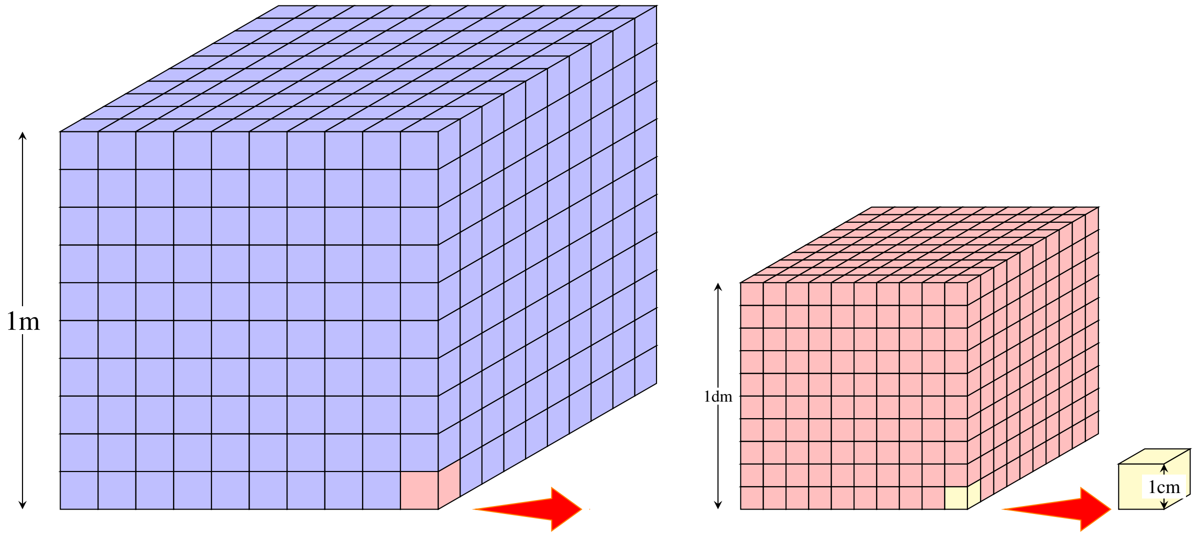


Figure 1.4 The left cube with a side of 1m has a volume of $1m^3$. The central cube with a side of 1dm has a volume of $1dm^3$, that is 1L. The right cube with a side of 1cm has a volume of $1cm^3$, that is 1mL.

Sample Problem 10

Convert 20 in to cm, expressing the result in scientific notation.

SOLUTION

We want to convert 20 inches into centimeters. The relationship between Inch and centimeter is given in Table 1.3. In order to do this, you need the conversion factor: $\frac{1in}{2.54cm}$ or $\frac{2.54cm}{1in}$. You will have to arrange the number (20 in) and the conversion factor in the following form:

$$20\cancel{in} \times \frac{2.54cm}{1\cancel{in}} = 5.080 \times 10^1 cm$$

STUDY CHECK

Convert 200mL to drops, expressing the result in scientific notation.

▶Answer: $200\cancel{mL} \times \frac{15drops}{1\cancel{mL}} = 3000drops = 3 \times 10^3 drops$

1.8 Measurements and uncertainty

Most chemistry experiments require the measurement of a property (mass, volume, temperature, color...), and the validity of those experiments will depend on the reliability of each measurement. The reliability of a measurement is usually considered in terms of its *accuracy* and its *precision*. At the same time, when experimenting oftentimes, one needs to repeat measurements. The results need to take into account the average measurement as well as the standard derivation of the series of results.

Uncertain and certain digits: how to report measured numbers Consider the volume as measured in the glassware on the left side of Figure 1.5. Some would say the liquid meniscus occurs at 1.52mL. Mind we need to estimate the last number by interpolating between the 0.1mL marks. As the last digit of the number associated with the volume measurement is estimated, another person could measure the volume as 1.53mL. The table below indicates the measurements of five different people.

Person	1	2	3	4
Volume(mL)	1.52	1.53	1.51	1.57

All measurements have in common the 1.5 part. The digit 5 is called a certain digit. The digit to the right of 5 is called the uncertain digit, as in a measurement it needs to be estimated. When reporting a measurement, we need to report up to the uncertain digit. These numbers on measurement are called *significant figures*. It is important to understand that all measured properties are subject to uncertainty. Uncertainty depends on one hand on the measuring device. The uncertainty on the left meniscus in Figure 1.5 is in the hundreds of the mL, whereas on the right meniscus occurs in the tenths of mL. Uncertainty also depends on the measurement process. As the uncertain digit needs to be reported in every measurement, the uncertainty on the last number on measurement is normally assumed to be ± 1. For example, the right meniscus should be reported as $1.52 \ \pm 0.01$mL

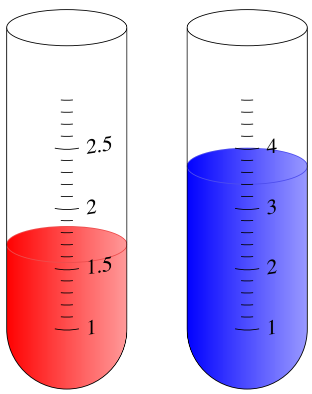


Figure 1.5 Two measuring devices. (Left side) The measuring device gives volumes to the hundredth place. (Right side)The measuring device gives volumes to the tenth place.

Precision and accuracy The terms precision and accuracy are two different terms often used to describe the reliability of measurement. Accuracy refers to the degree of agreement between the measured value and the true value, the real value of the property measured. Measurements that closely agree with the true value are accurate. Precision refers to the agreement between several measurements. Two measurements are accurate when they are close, independently of the true value of the property measured. Two different types of errors occur during a measurement. On one hand, random error refers to the fact that measured values can be above or below the true value. On the other hand, systematic error is an error that occurs each time in the same direction. Figure 1.6 displays accurate and inaccurate measurements as well as precise and imprecise measurements.

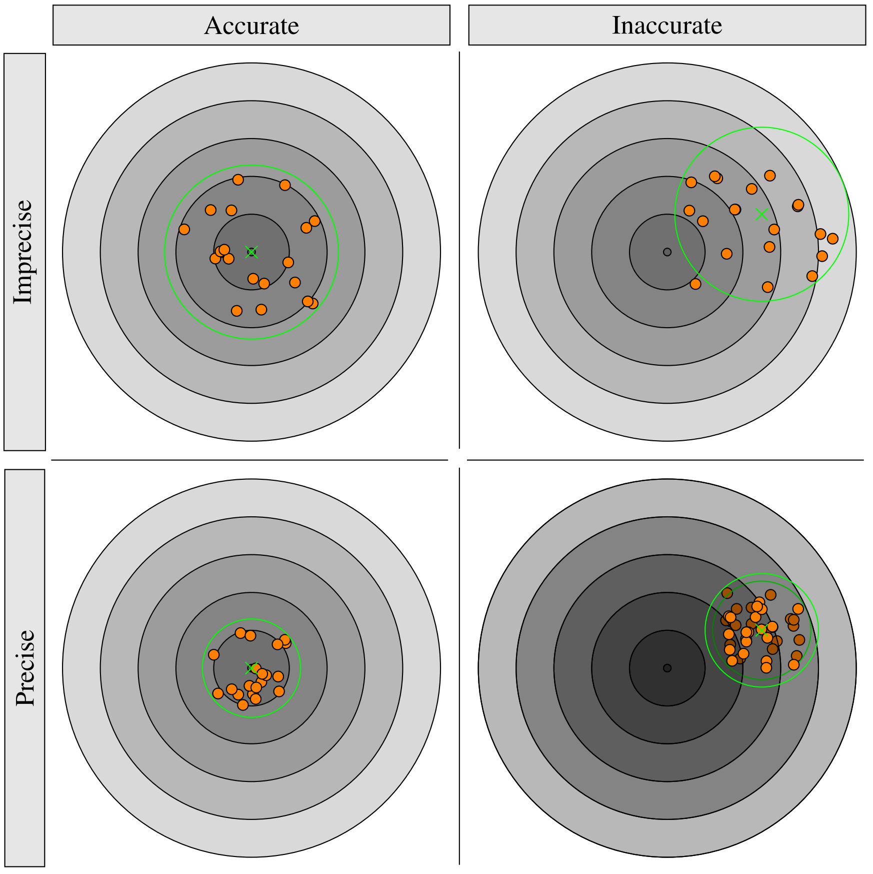


Figure 1.6 (Top Left) An accurate but imprecise measurement (Top Right) An inaccurate and imprecise measurement (Top Left) An accurate and precise measurement (Bottom Right) An inaccurate but precise measurement

`Average and standard deviation` The general approach to measuring a property such as the weight of a sample is to perform a number, n, of replicated measurements under similar conditions. Obtaining several measurements allows us to calculate the sample *average value*, $\bar{x}$, and the *standard deviation*, σ.
The sample average $\bar{x}$ is calculated using the formula:

$$\bar{x} = \frac{1}{n}\sum_{i}^{n} x_i$$

where $\sum_i^n x_i$ represents the sum of all measurements, x. The average value informs about the accuracy of the measurement. When the average is close to the true value the measurement is said to be accurate. The sample standard deviation σ provides estimates of the population values and it is calculated using the formula:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i}^{n}(x_i - \bar{x})^2}$$

The final value for measurement should be written as:

$$x = \bar{x} \pm \sigma$$

notice that the standard deviation should have the same number of decimals as the average value. The standard deviation informs about the precision or dispersion of the measurement. Very small standard deviations correspond to precise measurements. The following example demonstrates the calculation of the average and the standard deviation of a series of measurements.

Sample Problem 11

We measured the mass of the same sample several times obtaining the following values: 108.6 g, 104.2 g, 96.1 g, 99.6 g, and 102.2 g. Answer the following questions: (a) Compute the average, $\bar{x}$. (b) Compute the standard deviation of the measurement, σ. (c) Report the measured mass in the form $\bar{x} \pm \sigma$.

SOLUTION

(a) The average mass is given by:

$$\bar{m} = \frac{1}{5}(108.6\,g + 104.2\,g + 96.1\,g + 99.6\,g + 102.2\,g) = 102.1\,g.$$

(b) The standard deviation is given by:

$$\sigma = \sqrt{\frac{1}{4}\Big((108.6\,g - 102.1\,g)^2 + (104.2\,g - 102.1\,g)^2 + (96.1\,g - 102.1\,g)^2 + (99.6\,g - 102.1\,g)^2 + (102.2\,g - 102.1\,g)^2\Big)} = 4.7\,g.$$

(c) The measured mass is given by: $102.1 \pm 4.7\,g$

STUDY CHECK

We measured the volume of the same sample several times obtaining the following values: 10.12 mL, 10.12 mL, 10.13mL, 10.14 mL, and 10.15 mL. Answer the following questions: (a) Compute the average, $\bar{x}$. (b) Compute the standard deviation of the measurement, σ. (c) Report the measured volume in the form $\bar{x} \pm \sigma$. (d) Given that he true volume is 10.50mL, describe the accuracy and precision of the measurement

Answer: (a) 10.13mL (b) 0.01mL (c) 10.13 ± 0.01mL. (d) precise and inaccurate

1.9 Matter

Matter–the material of the universe–represents anything with mass that occupies space. It has different levels of organization and complexity. We can classify matter in terms of composition. Some substances are made of a single component whereas others contain multiple components. At the same time, some substances are made of many components while they appear to be made of a single component. Figure 1.7 displays the classification of matter in terms of mixture nature and composition.

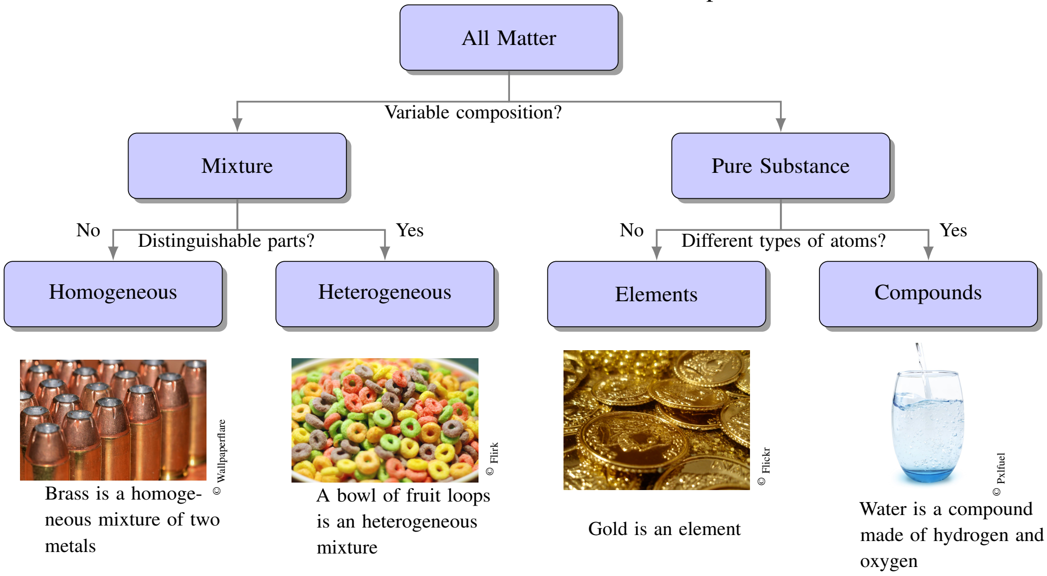
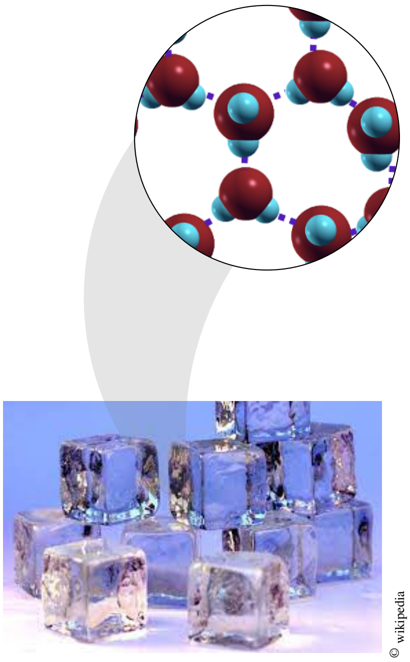


Figure 1.7 Classification of the matter

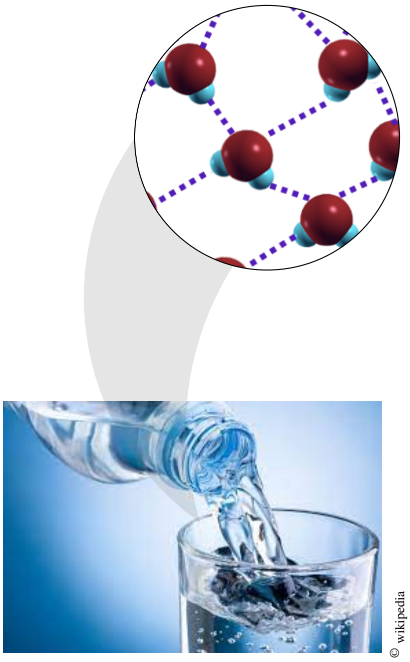
`Solids, liquids & gases` *Solid* has a well-defined shape and volume. Think about an ice cube, for example, that is made of water in the form of ice. In an ice cube, attractive forces keep the shape of the cube constant. The water molecules in the ice are arranged in rigid patterns and they can only vibrate in the solid. A *liquid*, on the other hand, has a well-defined volume. However, liquids do not have a constant shape, as their shape will depend on the shape of the container. Think about water which is a liquid. You can find many different bottle shapes. In all of them, the molecules of water will arrange to occupy the shape of the container. The volume of the liquid–the amount of space the liquid occupies–will be constant but not the shape. In a liquid, the particles move randomly but are still attached. A *gas* does not have a well-defined shape or volume. In a gas, the particles are randomly distributed and barely interact with each other, as they move at high speeds, taking the shape and volume of their container. Figure 1.8 displays the microscopic structure of the three different states of matter of water. The process of boiling and freezing represent *physical changes* and during physical changes, these substances involved do not change their composition. When water boils, both steam and water are made of the same component, H_2O. When a substance undergoes a *chemical process*, it will change its composition. For example, when burning a piece of paper, paper made of carbon, hydrogen, and hydrogen becomes ash made mostly of carbon. As such, during a chemical process, a substance (e.g. paper) becomes a new substance (e.g. carbon).

`Pure Substances and mixtures` On one hand, *pure substances* have definite composition, being only made of a single component. For example, water and gold are pure substances. However, there are two different types of pure substances: elements and compounds. *Elements* is composed of only one type of atom. Examples are silver, iron, and aluminum. They all contain one type of substance, and for example, iron is only made of iron atoms. *Compounds* are combinations of different elements. For example, water, H_2O is a compound made of a combination of hydrogen and oxygen atoms. On the other hand, *mixtures* are physical combinations of different pure substances. Mixtures have variable compositions. For example, the air is a mixture of oxygen and nitrogen. Wood, soda pop, or soil are all mixtures. Mixtures can be homogeneous or heterogeneous. In a *homogeneous mixture*–also known as solutions–the composition is uniform throughout the sample. An example of a homogeneous mixture is salty water, a solution of salt and water. *Heterogeneous mixtures* are mixtures in which the components are not uniformly distributed throughout the sample. An example would be a chocolate chip cookie in which you can differentiate the dough and the chocolate.

▼The molecular hexagonal structure of ice, solid water.



▼The molecular structure of liquid water.



▼The molecular structure of steam, gaseous water.

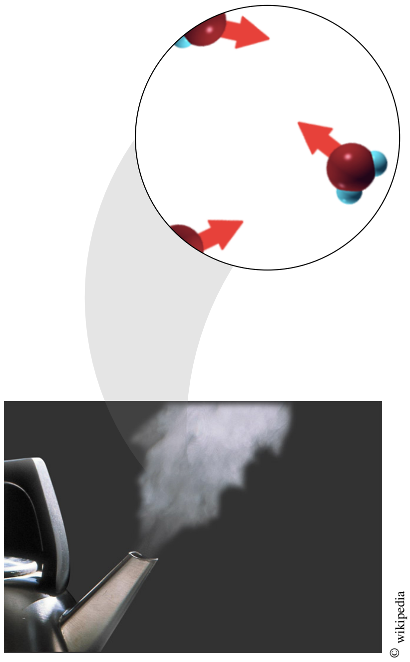


Figure 1.8 Three different states of matter, solid, liquid, and gas. Solid molecules are locked into fixed positions, while liquid molecules are together but still can move. Gas molecules are apart and move freely.

Sample Problem 12

Classify as element, compound, homogeneous mixture, heterogeneous mixture:
(a) An iron nail (b) Milk (c) Sugar (d) miso soup

SOLUTION

(a) An iron nail is an element as it is only made of iron, a single material; (b) Milk is a homogeneous mixture as it is made of water, fat, protein even though you only see a single substance; (c) Sugar is a compound made of carbon and other constituents; (d) miso soup is a mixture of water, fat and other chemicals and therefore is a mixture. As you can differentiate its constituents we call this heterogeneous mixture.

STUDY CHECK

Classify as element, compound, homogeneous mixture, heterogeneous mixture:
(a) muscle milk (b) water (c) a gold ring (d) rice & beans

▶Answer: (a) homog. mix.; (b) compound; (c) element; (d) heterog. mix.

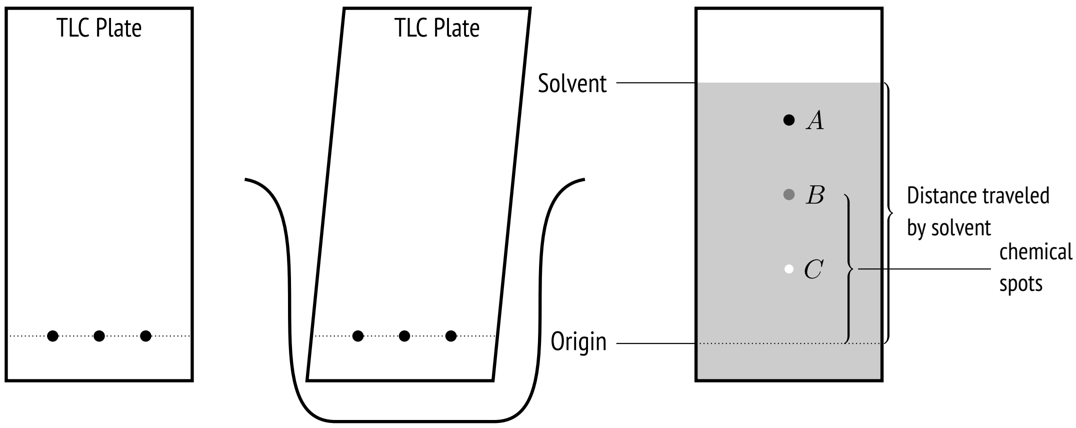


Figure 1.9 TLC set up. (Left) Solid phase with spots (Center) Solid phase inside a beaker containing the liquid phase (Right) Solid phase with separated mixture components.

`Separation Techniques` Chemists and scientists often need to separate the components of a mixture. There are many analytical techniques used to separate and identify the components of a mixture. These techniques exploit the differences in the chemical or physical properties of the components of the mixture to separate the different elements. The techniques used to separate mixtures are called *separation techniques*. Distillation is an important separation technique used to separate compounds with different volatility–different tendencies to generate vapor. For example, a mixture containing water and gasoline can be distilled as the components of the mixture largely differ in their volatility. The distillation setup consists of a distilling flask containing the mixture to be distilled, a condenser, and a receiving flask. When the mixture is heated the most volatile component will separate first and with the help of the condenser–a cooled tube–will cool down into liquid and deposit on the receiving flask. It is important to understand that only mixtures of liquids of mixtures of solids and liquids can be distilled. Another important separation technique is *filtration*. This technique is used to separate mixtures of solids and liquids. The mixture to be filtered is poured into a mesh–for example a filter–that retains the solid and lets the liquid pass. *Chromatography* is another separation technique. Indeed, the name represents a series of methods that uses two different phases, a mobile phase, and a stationary phase, to separate the components of a mixture. The principle behind this technique is the differences in affinity of the mixture components towards the mobile and stationary phases. The stationary phase is a solid whereas the mobile phase can either be a liquid (paper chromatography) or a gas (gas chromatography). Mixture components with a high affinity for the mobile phase will move faster through the chromatographic system. *Thin layer chromatography* (TLC) relies on capillarity, which is the tendency of liquid substances to rise on the surface of a material (Figure 1.9). In this technique, a drop of a liquid solution containing different substances (the sample) is deposited on a rectangular piece of filter paper, close to the bottom edge. This paper is called the stationary phase. The bottom end of the paper is immersed in a liquid called the mobile phase, to a point that is just below the spot where the sample was placed. Due to capillarity, the mobile phase will move up along the stationary phase. When the mobile phase reaches the sample, the different components of the mixture will begin to migrate, carried away by the mobile phase. The chemical compounds forming the sample will move with the mobile phase, but as different chemicals have different tendencies to stick to the mobile phase, they will cover different distances in the stationary phase. The different heights achieved by the different substances would allow you to identify those chemicals. A component of the mixture with a high affinity to the mobile phase will migrate more than a component with a higher affinity to the stationary phase. The distance traveled by a component referred to as the distance traveled by the mobile phase is a measure of the affinity between the chemical and the mobile phase. The *retention factor* R_f of a given chemical is defined as:

$$R_f = \frac{\text{distance traveled by the chemical}}{\text{total distance traveled by mobile phase}}$$

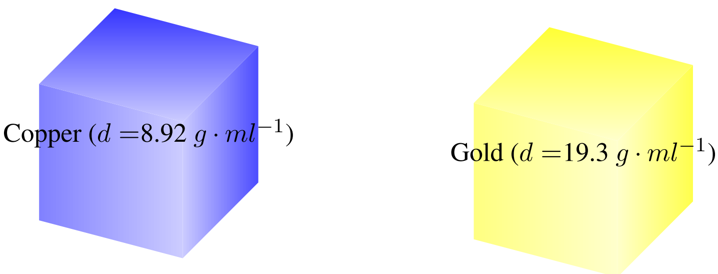
The R_f value of a substance is characteristic of that substance. When dealing with mixtures one has to calculate the R_f for each pure component separately to then compare the retention factors with the ones obtained in the mixture.

1.10 Density

Density refers to the mass of a substance with respect to its volume. This is an unique property for each substance. Table 1.4 reports the density of numerous substances. Indeed, density is often used as an identification tag. The formula for density is

$$\boxed{\text{Density} = \frac{\text{Mass of substance}}{\text{Volume of substance}}} \tag{1.1}$$

For example, the density for copper is 8.92 $g \cdot ml^{-1}$ and for gold is 19.3 $g \cdot ml^{-1}$. By measuring density only, you would be able to differentiate copper than gold. The larger density the more compact is an object and that means the more mass per volume it has. At the same time, for the same volume, the larger density the larger the mass of the metal.



`Density and mixing` A small piece of ice will float on the water. The reason for that is density: the density of ice (0.9g/mL) is smaller than the density of water (1.0g/mL) and hence ice will stay on top of the water. Objects with a density larger than 1 g/mL will sink whereas objects with a density smaller than this value will float. Figure 1.10 showcases how objects with density larger than water will sink whereas objects with a smaller density will float. If you add a drop of vegetable oil to a glass of water, the drop will float. This is because the density of oil is smaller than 1g/mL.

Table 1.4 Density of some common substances at 273.15 K and 100 kPa

Substance	Density (g/mL)	Physical State
Helium	0.2	gas
Hydrogen	0.1	gas
Water	1.0	Liquid
Cooking oil	0.9	Liquid
Mercury	13.5	Liquid
Tetrachloroethene	1.6	Liquid
Gold	19.3	solid
Plastics	1.2	solid
Ice	0.916*	solid

*Ice is given at T <273.15 K

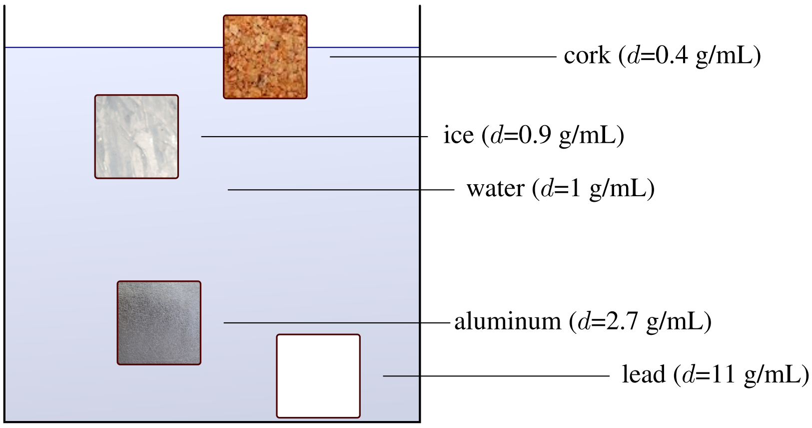


Figure 1.10 Objects with a larger density than water will sink whereas objects with a smaller density will float.

Sample Problem 13

In the figure, we mixed three liquids of density: A (0.5 g/mL), B(2 g/mL) and C(1 g/mL). Identify each liquid.

SOLUTION

The heavier the liquid, that is the larger density, the lower the liquid will arrange in the mixture. From top to bottom we have A, C and B.

STUDY CHECK

Indicate the order that the following invisible liquids will appear in a cylinder when mixed: (a) benzene(0.87 g/mL) and (b) water(1 g/mL)

▶Answer: benzene on top

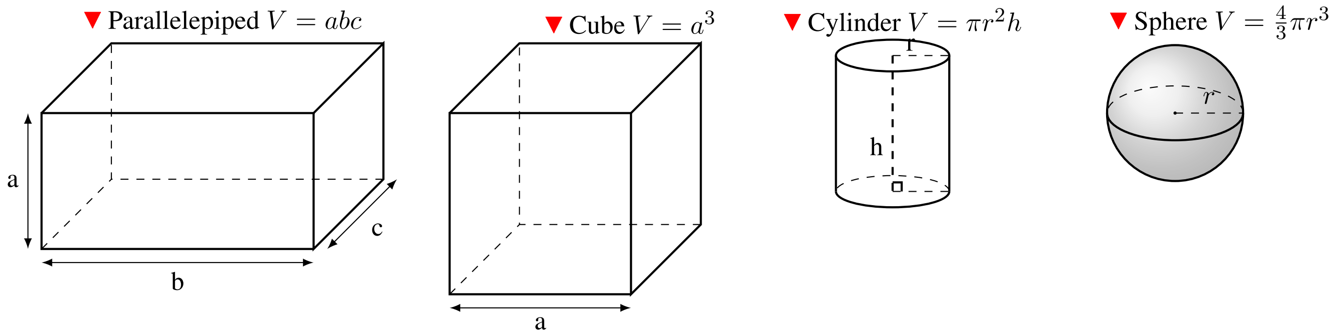


Figure 1.11 Volume of some objects

`Density and the volume of objects` Density depends on volume and in particular the larger volume the smaller density. Figure 1.11 displays the formulas to calculate the volume for some common objects, like a sphere or a cube. For example, the radius of a sphere with density d and mass m corresponds to $\sqrt[3]{3m/4d \cdot \pi}$, and the side of a cube with density d and mass m corresponds to $\sqrt[3]{m/d}$. The density of liquids results from measuring the mass of a given volume of the liquid. Differently, density is harder to obtain for solids. For metals, we can calculate density by the immersion method: when a metal is immersed in water, the water rises. This increase in volume corresponds to the volume of the solid. This way, density results from the direct measurement of mass and the measurement of volume by displacement.

The following example demonstrates density calculation with the immersion method.

Sample Problem 14

After adding a 30g object into a cylinder filled of water, the level of water rises from 60mL to 90mL. Calculate the density of the object.

SOLUTION

Density is mass over volume. The mass of the object is 30g and its volume is (90-60)mL that is 30mL. Hence: $d = 30g/30mL = 1g/mL$.

STUDY CHECK

A lead weight used in the belt of a scuba diver has a mass of 226 g. When the weight is placed in a graduated cylinder containing 200.0 mL of water, the water level rises to 220.0 mL. What is the density of the lead weight (g/mL)?

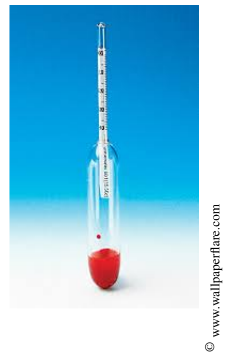
▶Answer:11.3 g/mL.

`Specific gravity` The specific gravity (ρ) of a substance is the ratio between its density and the density of a reference, normally water. It is simply calculated by dividing the density of the substance and the density of water (1g/mL at room temperature).

$$\boxed{\rho = \frac{\text{density of substance}}{\text{density of water}}} \tag{1.2}$$

A substance with a specific gravity of 1 has a density of 1g/mL. This is a unitless property that can be measured with an instrument called a hydrometer. For example, the specific density of the urine in the body is used to identify diabetes or kidney malfunctioning.

▼A hydrometer



▼Dipstick used to measure specific gravity