Annex II: Simulation

P. Adrian Garlati B.

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Abstract

I document the different steps followed to obtain the simulation results from the labor market model.

1 Model setup

The Tax reform introduced not only a reduction in payroll taxes but also a change in the funding of contributory social benefits using a new tax on profits. To investigate the final effect of all these changes on informality I propose a labor market model that internalizes them. The following is a modified version of the model proposed by Antón (2014). It's main characteristics are: a) static, b) single good sold in a perfectly competitive market, c) individuals characterized by different managerial ability (Lucas, 1978), d) taxes on payroll and profits, e) imperfect enforcement of payroll tax, f) employees formal/informal status defined by employers' compliance with payroll tax, g) informal employees, self-employed and employers receive Non-Contributory Social Benefits (NCSB), h) formal employees receive Contributory Social Benefits (CSB) and i) Social Benefits are not fully valued (Levy, 2008) and j) Government collects taxes and makes social benefits transfers keeping a balanced budget.

The model has one time period with a single representative household composed by a continuum of individuals of mass one. Each individual is endowed with a managerial ability z that has a probability density function $g(z)$ with support $[\underline{z}, \overline{z}]$; $G(\cdot)$ denotes the cumulative distribution function. Individuals can choose to be employees, self-employed or employees according to their managerial ability. How individuals make this choice is explained below.

2 Profits

Employers produce a single product y using labor l and managerial ability z. The product is sold in a perfectly competitive market at price p, which is used as numeraire. Technology is characterized by a Cobb-

Douglas production function $y = z^{1-\gamma}(l_I + l_F)^{\gamma}$. Employers are required to pay τ_l for each unit of labor hired but imperfect enforcement of payroll tax allows hiring formal employees, paying τ_l and a wage w_F, or informal employees, paying only wage w_I. Employers face a probability of audit θ_l and, if audited, they pay a fine $\sigma_l \tau_l w_I l_I$ were $\sigma_l \in [0,1]$ measures the severeness of the fine and l_I is the level of informal employment used by the firm. Government also raises a fraction τ_π of firms' profits[1]. Employer's problem is

$$\pi(w_I, w_F, z, \tau_l, \tau_\pi) =$$

$$\max_{l_I, l_F} (1-\tau_\pi)\left\{z^{1-\gamma}(l_I + l_F)^{\gamma} - (1+\tau_l) w_F l_F - (1+\theta_l\sigma_l\tau_l) w_I l_I\right\} \tag{1}$$

There will be payroll tax evasion as long as $(1+\theta_l\sigma_l\tau_l) w_I \leq (1+\tau_l) w_F$. To obtain a non-degenerated distribution of informal labor across firms in equilibrium, it is assumed that θ_l is not constant but an increasing function of l_I and z, this is, $\theta_l(l_I, z)$ with $\frac{\partial \theta_l}{\partial l_I} > 0$ and $\frac{\partial \theta_l}{\partial z} > 0$. From $\pi(w_I, w_F, z, \tau_l, \tau_\pi)$ we obtain factor demands $l_I(w_I, w_F, z, \tau_l)$ and $l_F(w_I, w_F, z, \tau_l)$. Notice how τ_π play no direct role in firms' labor demands. For any level of τ_π the level of labor employment chosen by firms does not change. Also, observe how formal and informal workers imply the same productivity for the firm and firms will hire informal workers as long as they are cheaper than formal ones.

Self-employed, on the other hand, are informal employees and part-time entrepreneurs whose profits are given by

$$\pi^{SE}(w_I, z) = \max_{l_o}\left\{z^{1-\gamma} l_o^{\gamma} + w_I(\kappa - l_o)\right\} \tag{2}$$

Where $(1-\kappa)$ is the fraction of labor self-employed lose as consecuence of commitment to managerial activities (Gollin, 2008). With $0 \leq l_o \leq \kappa \leq 1$, $(\kappa - l_o)$ is the fraction of time offered to the market. As with employers, we obtain labor demands $l_o(w_I, z)$.

3 Households

The representative household has a concave utility function $u(C)$ where C is the total consumption of goods. Household income come from labor, if employee, and profits from firms, if employer. The non-decreasing nature of the distribution $g(z)$ guarantees two critical levels of managerial ability z_1 and z_2 such that for $z < z_1$ the individual chooses to be an employee, for $z_1 \leq z < z_2$ the individual chooses to be self-employed

[1] For simplicity it is assumed that enforcement on this tax is perfect.

and for $z \geq z_2$ the individual chooses to be an employer. This means that a mass $[\underline{z}, z_1)$ will be employees, a mass $[z_1, z_2)$ will be self-employed and a mass $[z_2, \overline{z}]$ will be employers. The household also has to choose the fraction η of formal employees. Informal employees and employers receive a NCSB transfer T_I and formal employees receive a CSB transfer T_F. The household problem is given by

$$\underset{C}{max}\, u(C) = \underset{\eta, z_1, z_2}{max}\ u\left(\int_{\underline{z}}^{z_1} WNg(z)\, dz + \int_{z_1}^{z_2} \left[\pi^{SE}(\cdot, z) + T_I\right] g(z)\, dz + \int_{z_2}^{\overline{z}} \left[\pi(\cdot, z) + T_I\right] g(z)\, dz\right)$$

where $WN = \eta(w_F + T_F) + (1-\eta)(w_I + T_I)$. Household's first order condition for η is given by

$$w_F + T_F = w_I + T_I \tag{3}$$

Eq. (3) can be interpreted as indifference between formal and informal employment. At z_1 individual has to be indifferent between being employee or self-employed then

$$\eta(w_F + T_F) + (1-\eta)(w_I + T_I) = \pi^{SE}(\cdot, z_1) + T_I$$

And given (3) we have the final condition for z_1

$$w_I = \pi^{SE}(\cdot, z_1) \tag{4}$$

which, as π^{SE} is strictly increasing in z, it uniquely determines z_1.

Individuals at z_2 will also me indifferent between self-employment and being employer then

$$\pi^{SE}(\cdot, z_2) = \pi(\cdot, z_2) \tag{5}$$

In addition, Goverment must keep a balanced budget, this is, total revenues must equal total expenses. Before the reform it equates the total revenues from payroll tax to CSB expenses

$$\tau_l w_F \eta \int_{\underline{z}}^{z_1} g(z)\, dz = \tau_{CSB}\eta \int_{\underline{z}}^{z_1} g(z)\, dz \tag{6}$$

And after the reform it equates the total revenues from payroll tax and profit tax to CSB expenses

$$\tau_l w_F \eta \int_{\underline{z}}^{z_1} g(z)\, dz + \tau_\pi^{TR} \int_{z_2}^{\overline{z}} \pi(\cdot, z)\, g(z)\, dz = \tau_{CSB}\eta \int_{\underline{z}}^{z_1} g(z)\, dz \tag{7}$$

and before and after reform total revenues from profit tax equal NCSB expenses

$$\tau_\pi \int_{z_2}^{\bar{z}} \pi(\cdot, z) g(z) dz = \tau_{NCSB} \left[(1-\eta) \int_{\underline{z}}^{z_1} g(z) dz + \kappa \int_{z_1}^{z_2} g(z) dz + \int_{z_2}^{\overline{z}} g(z) dz \right] \tag{8}$$

where $\tau_\pi = \tau_\pi^0$ before the reform while $\tau_\pi = \tau_\pi^0 + \tau_\pi^{TR}$ after the reform where τ_π^{TR} is the part of the tax on profit used to fund CSB. On the other hand, every formal employee receives w_F and CSB transfers T_F. From (6) we have before the reform $\tau_{CSB} = \tau_l w_F$ while after $\tau_{CSB} = \tau_l w_F + \tau_\pi^{TR} \frac{\int_{z_2}^{\bar{z}} \pi(\cdot,z) g(z) dz}{\eta \int_{\underline{z}}^{z_1} g(z) dz}$. Following Levy (2008), formal workers valuate these benefits by a fraction $\beta_F \in [0,1]$ then $T_F = \beta_F \tau_l w_F$ before the reform and $T_F = \beta_F \left(\tau_l w_F + \tau_\pi^{TR} \frac{\int_{z_2}^{\bar{z}} \pi(\cdot,z) g(z) dz}{\eta \int_{\underline{z}}^{z_1} g(z) dz} \right)$ after the reform. Likewise, informal employees receive w_F and NCSB transfers[2] by τ_{NCSB} but value these benefits by $\beta_I \in [0,1]$. These assumptions mean that the first order condition for η before the reform can be written as

$$w_F + \beta_F \tau_l w_F = w_I + \beta_I \tau_{NCSB} \tag{9}$$

and after the reform as

$$w_F + \beta_F \left(\tau_l w_F + \tau_\pi^{TR} \frac{\int_{z_2}^{\overline{z}} \pi(\cdot, z) g(z) dz}{\eta \int_{\underline{z}}^{z_1} g(z) dz} \right) = w_I + \beta_I \tau_{NCSB} \tag{10}$$

Eq. (9) mean that for any given β_F, β_I, τ_l and τ_{NCSB} then w_I is determined by w_F before the Tax reform and also τ_π after (Eq. (10)).

Eqs. (9) and (10) represent the fact that the reform changed the funding of CSB transfers using the new tax on profits. This means that even if τ_π does not affect firms' labor demand directly, the new amount of CSB transfers could be different from before the reform and household choose a different level of η. This is a departure from Antón (2014) who only considers the change in level of payroll taxes and not the change in funding of CSB.

4 Equilibrium

All individuals such that $z \in [\underline{z}, z_1)$ are employees and $z \in [z_1, z_2)$ are self-employed then supply of labor in the economy is

$$N \equiv G(z_1) + \kappa \int_{z_1}^{z_2} g(z) dz \tag{11}$$

On the other hand, individuals with $z \in [z_1, z_2)$ are self-employed with equilibrium labor demands given by $l_o(w_F^*, z)$ and those with $z \in [z_2, \overline{z}]$ are employers with equilibrium labor demands $l(w_F^*, z, \tau_l) =$

[2] For simplicity I assume these benefits do not change after the Tax reform.

$l_I(w_F^*, z, \tau_l) + l_F(w_F^*, z, \tau_l)$. The equilibrium in the labor market is given by

$$N = \kappa \int_{z_1}^{z_2} l_o(w_F^*, z)\, g(z)\, dz + \int_{z_2}^{\overline{z}} l(w_F^*, z, \tau_l)\, g(z)\, dz \tag{12}$$

Finally, the resource constraint yields the equilibrium in the goods market

$$C = Y(\tau_l, \tau_\pi, w_F^*) = \int_{z_1}^{z_2} y_o(\cdot, z)\, g(z)\, dz + \int_{z_2}^{\overline{z}} y(\cdot, z)\, g(z)\, dz \tag{13}$$

In equilibrium we obtain (w_F^*) given $(\tau_l, \tau_\pi, T_F, T_I)$. Equilibrium informal wage w_I^* is obtained from eq. (9) before the Tax reform and Eq. (10) after the Tax reform.

4.1 Simulation

As the Tax reform introduced a structural change in the model, represented by Eqs. (9) and (10), straightforward comparative statics are not possible. Lower payroll taxes increases labor demand from employers and pushes z_1 up (Eq. (12)) but increased firms' profits attracts more self-employed to become employer, this is, a decrease in z_2. So final directions of z_1 and z_2 are uncertain. On the other hand, the increased labor demand and lower labor supply will lead to an increase in w_F, alongside w_I as indicated by Eq. (9). Finally, comparing Eqs. (9) and (10), CSB transfers can go up or down after the Tax reform leading to an uncertain new level of η chosen by households. These effects in conjunction lead to an uncertain final effect on the informality rate in the economy.

In order to measure the effect of the Tax reform on informality, I carry out a simulation exercise. For this, I obtain main conditions of equilibrium, specific density function and respective parameters calibrations. Assume $\theta_l(l_I, z) = \lambda_l l_I z$ where λ_l is a positive parameter. Firms profits are given by

$$\pi = (1 - \tau_\pi)\left\{z^{1-\gamma}(l_I + l_F)^\gamma - (1 + \tau_l)\, w_F l_F - (1 + \lambda_l l_I z \sigma_l \tau_l)\, w_I l_I\right\}$$

Then the first order conditions for l_I and l_F are given by

$$z^{1-\gamma}\gamma(l_I + l_F)^{\gamma-1} - w_I - 2\lambda_l \sigma_l \tau_l w_I z l_I = 0 \tag{14}$$

$$z^{1-\gamma}\gamma(l_I + l_F)^{\gamma-1} - (1 + \tau_l)\, w_F = 0 \tag{15}$$

From (14) and (15)

$$w_I + 2\lambda_l z\sigma_l\tau_l w_I l_I = (1+\tau_l)\, w_F$$

$$l_I = \frac{(1+\tau_l)(w_F/w_I) - 1}{2\lambda_l z\sigma_l\tau_l} \tag{16}$$

Eq. (16) represents the *demand for informal labor by a firm of managerial talent* z.

$$z^{1-\gamma}\gamma\,(l_I + l_F)^{\gamma-1} = (1+\tau_l)\, w_F$$

$$l_F + l_I = l = \left[\frac{\gamma}{(1+\tau_l)\, w_F}\right]^{\frac{1}{1-\gamma}} z \tag{17}$$

Eq. (17) represents the *demand for total labor by a firm of managerial talent* z.

Now, the first order condition for self-employed is

$$\gamma z^{1-\gamma} l_o^{\gamma-1} - w_I = 0 \tag{18}$$

$$l_o^{\gamma-1} = \frac{w_I}{\gamma z^{1-\gamma}}$$

$$l_o = \left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}} z$$

In order to solve for the general equilibrium we need to assume specific functions. For utility I assume $u(C) = C$. For distribution for z, following Antón (2014), I assume a truncated Pareto distribution of the form

$$G(z) = \frac{1 - (\underline{z}/z)^S}{1 - (\underline{z}/\bar{z})^S}$$

where $S \in (0,1)$ is a shape parameter and $\underline{z}$ and $\bar{z}$ are the minimum and maximum values for z respectively. Replacing this distribution in Eq. (12) we obtain

$$\frac{\underline{z}^S S}{1-S}\left\{\kappa\left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}}\left(z_2^{1-S} - z_1^{1-S}\right) + \left[\frac{\gamma}{(1+\tau_l)\, w_F}\right]^{\frac{1}{1-\gamma}}\left(\bar{z}^{1-S} - z_2^{1-S}\right)\right\} =$$
$$1 - \kappa\left(\underline{z}/z_2\right)^S - (1-\kappa)\left(\underline{z}/z_1\right)^S \tag{19}$$

and Eq. (13)

$$\int_{\underline{z}}^{z_1} WNg(z)\,dz + \int_{z_1}^{z_2} \left[\pi^{SE}(\cdot, z) + T_I\right] g(z)\,dz + \int_{z_2}^{\bar{z}} \left[\pi(\cdot, z) + T_I\right] g(z)\,dz = \int_{z_1}^{z_2} y_o(\cdot, z)\, g(z)\,dz + \int_{z_2}^{\bar{z}} y(\cdot, z)\, g(z)\,dz$$

$$\tau_{NCSB}\left[\frac{1-(\underline{z}/\bar{z})^S}{\underline{z}^S}\right] + w_I\left(\underline{z}^{-S} - \kappa z_2^{-S}\right) - (1-\kappa)\, w_I z_1^{-S} - \left(\frac{S}{1+S}\right)\left[\frac{\left[(1+\tau_l)\, w_F\right]^2 - w_I^2}{2\lambda_l \sigma_l \tau_l w_I}\right]\left(\bar{z}^{-S-1} - z_2^{-S-1}\right) = \gamma^{\frac{1}{1-\gamma}}\left(\frac{S}{1-S}\right)\left[\left(\frac{1}{w_I}\right)^{\frac{\gamma}{1-\gamma}}\left(z_2^{1-S} - z_1^{1-S}\right) + \left[\frac{1}{(1+\tau_l)\, w_F}\right]^{\frac{\gamma}{1-\gamma}}\left(\bar{z}^{1-S} - z_2^{1-S}\right)\right] \tag{20}$$

On the other hand, the critical cut-off z_1 is the result of eq. (4) which can be written as

$$w_I = z_1^{1-\gamma} l_o(\cdot, z_1)^\gamma + w_I\left(\kappa - l_o(\cdot, z_1)\right) \tag{21}$$

And the critical cut-off z_2 is the result of eq. (5) which can be written as

$$z_1^{1-\gamma} l_o(\cdot, z_2)^\gamma + w_I\left(\kappa - l_o(\cdot, z_2)\right) = \tag{22}$$

$$(1-\tau_\pi)\left\{z_2^{1-\gamma} l(\cdot, z_2)^\gamma - (1+\tau_l)\, w_F l_F(\cdot, z_2) - (1+\lambda_l l_I(\cdot, z_2)\, z_2 \sigma_l \tau_l)\, w_I l_I(\cdot, z_2)\right\} \tag{23}$$

In addition, from Goverment budget constraint (6) we have $\tau_l w_F = \tau_{CSB}$ and from (7) we have

$$\tau_l w_F + \tau_\pi^{TR} \frac{\int_{z_2}^{\bar{z}} \left\{z^{1-\gamma}(l_I + l_F)^\gamma - (1+\tau_l)\, w_F l_F - (1+\lambda_l l_I z \sigma_l \tau_l)\, w_I l_I\right\} z^{-S-1} dz}{\eta \int_{\underline{z}}^{z_1} z^{-S-1} dz} = \tau_{CSB}$$

$$\tau_l w_F - \frac{\tau_\pi^{TR} S}{\eta\left(z_1^{-S} - \underline{z}^{-S}\right)}\left\{\left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right)\left[\frac{1}{(1+\tau_l)\, w_F}\right]^{\frac{\gamma}{1-\gamma}}\left(\frac{\bar{z}^{1-S} - z_2^{1-S}}{1-S}\right) - \left[\frac{(1+\tau_l)^2 w_F^2/w_I - 2(1+\tau_l)\, w_F + w_I}{4\lambda_l \sigma_l \tau_l}\right]\left(\frac{\bar{z}^{-S-}}{}\right. \tag{24}$$

And from (8)

$$\tau_\pi \int_{z_2}^{\bar{z}} \left\{ z^{1-\gamma} \left(l_I + l_F\right)^\gamma - \left(1+\tau_l\right) w_F l_F - \left(1 + \lambda_l l_I z \sigma_l \tau_l\right) w_I l_I \right\} z^{-S-1} dz = \tau_{NCSB} \left[(1-\eta) \int_{\underline{z}}^{z_1} z^{-S-1} dz + \kappa \int_{z_1}^{z_2} z^{-S-1} dz + \int\right.$$

$$\left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right) \left[\frac{1}{\left(1+\tau_l\right) w_F}\right]^{\frac{\gamma}{1-\gamma}} \left(\frac{\bar{z}^{1-S} - z_2^{1-S}}{1-S}\right) - \left[\frac{\left(1+\tau_l\right)^2 w_F^2 / w_I - 2\left(1+\tau_l\right) w_F + w_I}{4 \lambda_l \sigma_l \tau_l}\right] \left(\frac{\bar{z}^{-S-1} - z_2^{-S-1}}{1+S}\right) = -\frac{\tau_{NCSB}}{\tau_\pi S} \left[(1-\eta)\left(z_1^{-S} - \underline{z}^{-S}\right) + \kappa \left(z_2^{-S} - z_1^{-S}\right) + \bar{z}^{-S} - z_2^{-S}\right] \tag{25}$$

4.2 Final equations for simulation

Before reform

$$w_F \left(1 + \beta_F \tau_l\right) = w_I + \beta_I \tau_{NCSB} \tag{26}$$

$$l_o\left(\cdot, z_1\right) = \left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}} z_1 \tag{27}$$

$$w_I \left(1 + l_o\left(\cdot, z_1\right) - \kappa\right) = z_1^{1-\gamma} l_o\left(\cdot, z_1\right)^\gamma \tag{28}$$

$$l_F\left(\cdot, z_2\right) + \frac{\left(1+\tau_l\right)\left(w_F/w_I\right) - 1}{2 \lambda_l \sigma_l \tau_l z_2} = l\left(\cdot, z_2\right) = \left[\frac{\gamma}{\left(1+\tau_l\right) w_F}\right]^{\frac{1}{1-\gamma}} z_2 \tag{29}$$

$$l_o\left(\cdot, z_2\right) = \left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}} z_2 \tag{30}$$

$$\frac{\underline{z}^S S}{1-S} \left\{ \kappa \left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}} \left(z_2^{1-S} - z_1^{1-S}\right) + \left[\frac{\gamma}{\left(1+\tau_l\right) w_F}\right]^{\frac{1}{1-\gamma}} \left(\bar{z}^{1-S} - z_2^{1-S}\right) \right\} = 1 - \kappa \left(\underline{z}/z_2\right)^S - (1-\kappa)\left(\underline{z}/z_1\right)^S \tag{31}$$

$$\tau_{NCSB}\left[\frac{1-\left(\underline{z}/\bar{z}\right)^{S}}{\underline{z}^{S}}\right]+w_I\left(\underline{z}^{-S}-\kappa z_2^{-S}\right)-\left(1-\kappa\right)w_I z_1^{-S}-\left(\frac{S}{1+S}\right)\left[\frac{\left[\left(1+\tau_l\right)w_F\right]^2-w_I^2}{2\lambda_l\sigma_l\tau_l w_I}\right]\left(\bar{z}^{-S-1}-z_2^{-S-1}\right)\gamma^{\frac{1}{1-\gamma}}=$$

$$\left(\frac{S}{1-S}\right)\left[\left(\frac{1}{w_I}\right)^{\frac{\gamma}{1-\gamma}}\left(z_2^{1-S}-z_1^{1-S}\right)+\left[\frac{1}{\left(1+\tau_l\right)w_F}\right]^{\frac{\gamma}{1-\gamma}}\left(\bar{z}^{1-S}-z_2^{1-S}\right)\right] \tag{32}$$

$$z_1^{1-\gamma}l_o\left(\cdot,z_2\right)^{\gamma}+w_I\left(\kappa-l_o\left(\cdot,z_2\right)\right)=$$

$$\left(1-\tau_\pi\right)\left\{z_2^{1-\gamma}l\left(\cdot,z_2\right)^{\gamma}-\left(1+\tau_l\right)w_F l_F\left(\cdot,z_2\right)-\left(1+\lambda_l l_I\left(\cdot,z_2\right)z_2\sigma_l\tau_l\right)w_I l_I\left(\cdot,z_2\right)\right\} \tag{33}$$

$$\left(\gamma^{\frac{\gamma}{1-\gamma}}-\gamma^{\frac{1}{1-\gamma}}\right)\left[\frac{1}{\left(1+\tau_l\right)w_F}\right]^{\frac{\gamma}{1-\gamma}}\left(\frac{\bar{z}^{1-S}-z_2^{1-S}}{1-S}\right)-\left[\frac{\left(1+\tau_l\right)^2w_F^2/w_I-2\left(1+\tau_l\right)w_F+w_I}{4\lambda_l\sigma_l\tau_l}\right]\left(\frac{\bar{z}^{-S-1}-z_2^{-S-1}}{1+S}\right)=$$

$$-\frac{\tau_{NCSB}}{\tau_\pi S}\left[\left(1-\eta\right)\left(z_1^{-S}-\underline{z}^{-S}\right)+\kappa\left(z_2^{-S}-z_1^{-S}\right)+\bar{z}^{-S}-z_2^{-S}\right] \tag{34}$$

After reform

$$\left(w_F+\beta_F\tau_{CSB}\right)=w_I+\beta_I\tau_{NCSB}$$

$$\tau_{CSB}=\tau_l w_F-\frac{\tau_\pi^{TR}S}{\eta\left(z_1^{-S}-\underline{z}^{-S}\right)}\left\{\left(\gamma^{\frac{\gamma}{1-\gamma}}-\gamma^{\frac{1}{1-\gamma}}\right)\left[\frac{1}{\left(1+\tau_l\right)w_F}\right]^{\frac{\gamma}{1-\gamma}}\left(\frac{\bar{z}^{1-S}-z_2^{1-S}}{1-S}\right)-\left[\frac{\left(1+\tau_l\right)^2w_F^2/w_I-2\left(1+\tau_l\right)w_F+w_I}{4\lambda_l\sigma_l\tau_l}\right.$$

$$l_o\left(\cdot,z_1\right)=\left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}}z_1 \tag{35}$$

$$w_I\left(1+l_o\left(\cdot,z_1\right)-\kappa\right)=z_1^{1-\gamma}l_o\left(\cdot,z_1\right)^{\gamma} \tag{36}$$

$$l_F\left(\cdot,z_2\right)+\frac{\left(1+\tau_l\right)\left(w_F/w_I\right)-1}{2\lambda_l\sigma_l\tau_l z_2}=l\left(\cdot,z_2\right)=\left[\frac{\gamma}{\left(1+\tau_l\right)w_F}\right]^{\frac{1}{1-\gamma}}z_2 \tag{37}$$

$$l_o(\cdot, z_2) = \left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}} z_2 \tag{38}$$

$$\frac{\underline{z}^S S}{1-S}\left\{\kappa\left(\frac{\gamma}{w_I}\right)^{\frac{1}{1-\gamma}}\left(z_2^{1-S} - z_1^{1-S}\right) + \left[\frac{\gamma}{(1+\tau_l)\,w_F}\right]^{\frac{1}{1-\gamma}}\left(\bar{z}^{1-S} - z_2^{1-S}\right)\right\} = \tag{39}$$

$$1 - \kappa\left(\underline{z}/z_2\right)^S - (1-\kappa)\left(\underline{z}/z_1\right)^S \tag{40}$$

$$\tau_{NCSB}\left[\frac{1-\left(\underline{z}/\bar{z}\right)^S}{\underline{z}^S}\right] + w_I\left(\underline{z}^{-S} - \kappa z_2^{-S}\right) - (1-\kappa)\,w_I z_1^{-S} - \left(\frac{S}{1+S}\right)\left[\frac{\left[(1+\tau_l)\,w_F\right]^2 - w_I^2}{2\lambda_l\sigma_l\tau_l w_I}\right]\left(\bar{z}^{-S-1} - z_2^{-S-1}\right)\gamma^{\frac{1}{1-\gamma}} = \tag{41}$$

$$\left(\frac{S}{1-S}\right)\left[\left(\frac{1}{w_I}\right)^{\frac{\gamma}{1-\gamma}}\left(z_2^{1-S} - z_1^{1-S}\right) + \left[\frac{1}{(1+\tau_l)\,w_F}\right]^{\frac{\gamma}{1-\gamma}}\left(\bar{z}^{1-S} - z_2^{1-S}\right)\right] \tag{42}$$

$$z_1^{1-\gamma} l_o(\cdot, z_2)^\gamma + w_I\left(\kappa - l_o(\cdot, z_2)\right) = (1-\tau_\pi)\left\{z_2^{1-\gamma} l(\cdot, z_2)^\gamma - (1+\tau_l)\,w_F l_F(\cdot, z_2) - (1+\lambda_l l_I(\cdot, z_2)\, z_2\sigma_l\tau_l)\, w_I l_I(\cdot, z_2)\right\} \tag{43}$$

$$\left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right)\left[\frac{1}{(1+\tau_l)\,w_F}\right]^{\frac{\gamma}{1-\gamma}}\left(\frac{\bar{z}^{1-S} - z_2^{1-S}}{1-S}\right) - \left[\frac{(1+\tau_l)^2 w_F^2/w_I - 2(1+\tau_l)\,w_F + w_I}{4\lambda_l\sigma_l\tau_l}\right]\left(\frac{\bar{z}^{-S-1} - z_2^{-S-1}}{1+S}\right) =$$
$$-\frac{\tau_{NCSB}}{\tau_\pi S}\left[(1-\eta)\left(z_1^{-S} - \underline{z}^{-S}\right) + \kappa\left(z_2^{-S} - z_1^{-S}\right) + \bar{z}^{-S} - z_2^{-S}\right] \tag{44}$$

For calibration, some parameters' values are chosen using previous values extracted from the literature; others are set to match Colombian data by the time the Tax reform was passed. The corresponding values for the first group are: $\gamma = 2/3$ (González et al., 2012), $\beta_F = 0.48$,$\beta_I = 0.525$(Olivera and Cuesta, 2010), $\tau_l = 0.603$, $\tau_\pi = 0.33$ and $\sigma_l = 1.5$ (Antón, 2014). The parameters left $S, \bar{z}, \underline{z}, \tau_{NCSB}$ and λ_l are set to match the informal rate of the Treated group (34.8% by December 2012, model 31.6%), the formal/informal wage rate (1.50 by December 2012, model 1.50) and the public expenditure on NCSB / GDP (1.8% by 2013, model 3.75%). For the model with the tax reform $\tau_l = 0.468$, $\tau_\pi^0 = 0.25$ and $\tau_\pi^{TR} = 0.09$. I calibrate Eqs. 192021 and **??** for simulation using Matlab R2017b by Mathworks. This simulation exercise indicates that the Tax reform should reduce informality rate by 2.6 pps.

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