

# Development of a robotics lab for control theory on the example of sloshing free liquid transport

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Autonomous Lab  
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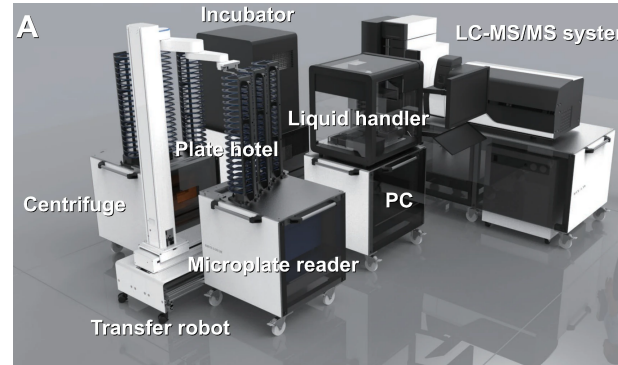
Perception and State estimation  
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Task planning  
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Sloshing Problem  
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# Autonomous Lab

- automation of various (repetitive) tasks
  - provide high precision and repeatability
  - act in a collaborative and dynamic environment
  - adaptability to the environment
- **Assignment of a complex task in a preexisting lab environment**



Robot centric autonomous lab (Fushimi, K.)

Autonomous Lab



Perception and State estimation



Task planning



Sloshing Problem



# Perception and State estimation

- "Global"tracking system
- Known lab geometry
- Autonomous exploration and mapping (SLAM)
- Local sensors (imaging, depth,...)
- Odometry
- Cross validation of data

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# Control of 6 DoF Manipulators

- Task planning
- Path planning (global)
  - path = series of waypoints (poses)
  - shortest path
  - obstacle avoidance
  - exponentially increasing complexity with respect to degrees of freedom
- Trajectory (local) planning and optimization
  - speed
  - energy consumption
  - mechanical stress
  - smoothness
  - external requirements eg anti-sloshing
  - trajectory = connects waypoints
- Motion control system

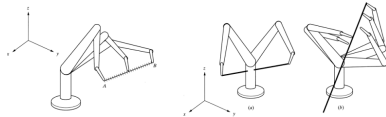
# Trajectory planning and optimization

- Optimization methods
  - mathematical optimization
  - evolutionary algorithms
  - stochastic optimization
  - reinforcement learning
- multi objective optimization while minimizing:
  - energy consumption
  - mechanical stress
  - time

# Trajectory planning

## Joint space

- motion described by joint values
- motion is unpredictable
- sequential motions follow a straight line
- IK solution from initial to final point (once)
- discretize individual joint trajectories
- difficult to deal with obstacles



## Operational space

- path and motion is known
- easy to visualize
- prone to singularities such as self-collision and sudden change in joint angles
- calculate complete path
- discretize path
- solve IK for each point
- can deal with obstacles
- Computationally expensive



- Tracking system
- Robot
  - Mobile platform
  - Two 6 or 7 DoF manipulators
  - Endeffector (gripper)
  - Camera system for visual feedback
- Laboratory equipment
  - measurements units
  - heater/cooler
  - centrifuge
  - etc.

# Task planning

- tasks are a confined series of goals
- task planning needs feedback about the environment and status to react to disturbances
- a task can be defined and planned offline, but most likely needs to be adapted online
- In the context of an autonomous biolab most tasks involve fluid handling in some sense leading to the basis problem of sloshing

# Sloshing Problem

- occurs in partially filled containers subjected to external forces
- can cause spillage or force slow movements
- suppression by feedforward control already tested by Reinhold et al. (2019)
- Obtaining measurements for feedback control is challenging

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# Modeling of liquid dynamics

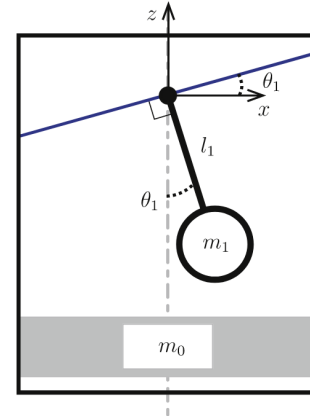
- linearized Navier-Stokes equations
- superposition of j different modes
- each mode is described by a natural frequency  $\omega_j$
- damping ratio is described by empirical relationships

$$\omega_{nj} = \sqrt{\frac{g\xi_j}{R} \tanh\left(\frac{h\xi_j}{R}\right)} \quad (1)$$

$$\delta_j = \frac{2.89}{\pi} \sqrt{\frac{\nu}{R^3 g}} \left[ 1 + \frac{0.318}{\sinh\left(\frac{1.84h}{R}\right)} \frac{1 - \frac{h}{R}}{\cosh\left(\frac{1.84h}{R}\right)} \right] \quad (2)$$

# Simplifications

- equivalent pendulum modeling
- relations between physical parameters and mode characteristics yield
  - pivot points near surface
  - first asymmetric mode is dominant
  - pendulum orthogonal to surface
  - planar surface
- excitation happens at the pivot point



# Modeling as pendulum on a (mobile) plane

- earth bound spherical coordinate system
- pendulum attachment point distance to plates  
COG is constant
- forces acting on the attachment point are directly  
correlated to the forces acting on the plate via the  
lever arms
- sloshing dynamics are described in the body  
frame around the pivot point
- liquid behaviour is approximated as a pendulum

# Modeling of sloshing

- unforced dampened pendulum:

$$\ddot{\theta} = -\delta - \frac{g}{l} \sin(\theta) \quad (3)$$

$$\dot{\theta}_1 = \theta_2 \quad (4)$$

$$\dot{\theta}_2 = -\delta_3 \dot{\theta}_1^3 - \delta \dot{\theta}_1 + \frac{1}{l} [-gR(\theta_1) + A(t)] - [a \left( \frac{\theta}{\theta_{crit}} \right)^b + c \left( \frac{\theta}{\theta_{crit}} \right)^d] \quad (5)$$

$$A(t) = -x_0 \omega^2 \sin(\omega t) \quad (6)$$