Discrete Mathematics
MH1812

Topic 7 - Set Theory
Summary

Example

elements of A

Set $A = \{1,2,3\}$ and $B = \{2\}$.

$\neq (1,2,2,3)$ $\{\quad\}$

Find:

- $A \cup B = A = \{1,2,3\} = \{1,2,2,3\} = \{1,3,2\}$
- $A \cap B = B \cap A = B = \{2\} \neq 2$ ← not set
- $A - B = \{1,3\}$
- $B - A = \{\} = \phi \neq \{\phi\}$
- $A \times B = \{(a,b) \mid a \in A, b \in B\} = \{(1,2),(2,2),(3,2)\}$
- $B \times A = \{(2,1),(2,2),(2,3)\} \neq A \times B$

Notes

- $\emptyset \neq \{\emptyset\}$; $|\emptyset| = 0$; $|\{\emptyset\}| = 1$

$\emptyset$ like empty folder
$\{\emptyset\}$ like folder containing empty folder

- $\emptyset$ is a subset of every set; $\emptyset \subseteq A \ \forall$ set A
- $\emptyset$ is an element of every power set
$\emptyset \in P(A) \ \forall$ set A

- $B \in P(A) \iff B \subseteq A$

Example

DO NOT USE MEMBERSHIP TABLE FOR CARTESIAN PRODUCT

Prove the set identity $(A - B) \cap (C - B) = (A \cap C) - B$.

A	B	C	$A-B$	$C-B$	$(A-B)\cap(C-B)$	$A\cap C$	$(A\cap C)-B$
1	1	1	0	0	0	1	0
1	1	0	0	0	0	0	0
1	0	1	1	1	1	1	1
1	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0

Example

LHS

Prove the set identity $(A - B) \cap (C - B) = (A \cap C) - B$.

LHS : $(A - B) \cap (C - B) = (A \cap \overline{B}) \cap (C \cap \overline{B})$ set diff defn

$= A \cap (\overline{B} \cap C) \cap \overline{B}$ assoc.

$= A \cap (C \cap \overline{B}) \cap \overline{B}$ comm.

$= (A \cap C) \cap (\overline{B} \cap \overline{B})$ assoc.

$= (A \cap C) \cap \overline{B}$ idempotent

$= (A \cap C) - B$. set diff defn

Example

LHS

Double inclusion.

$S = T$

$\Leftrightarrow S \subseteq T \;\&\; T \subseteq S$

Prove the set identity $(A - B) \cap (C - B) = (A \cap C) - B$.

$LHS \subseteq RHS$

Take $x \in (A-B) \cap (C-B)$

$\Rightarrow x \in A - B \;\&\; x \in C - B$

$\Rightarrow x \in A \;\&\; x \notin B \;\&\; x \in C \;\&\; \cancel{x \notin B}$

$\Rightarrow x \in A \cap C \;\&\; x \notin B$

$\Rightarrow x \in (A \cap C) - B = RHS$

$RHS \subseteq LHS$:

Take $x \in (A \cap C) - B$

$\Rightarrow \quad x \in A \cap C \ \& \ x \notin B$

$\Rightarrow \quad x \in A \ \& \ x \in C \ \& \ x \notin B$

$\Rightarrow \quad x \in A - B \ \& \ x \in C - B$

$\Rightarrow \quad x \in (A-B) \cap (C-B) = LHS$

Example

Show that $(A \times B) \cup (B \times C) \subseteq (A \cup B) \times (B \cup C)$.

Take $x \in (A \times B) \cup (B \times C)$

$\Rightarrow x \in (A \times B)$ or $x \in B \times C$

First suppose $x \in (A \times B)$

$\Rightarrow x = (x_1, x_2)$ where $x_1 \in A$ & $x_2 \in B$

$\Rightarrow x_1 \in A \cup B$ & $x_2 \in B \cup C$

$\Rightarrow x \in (A \cup B) \times (B \cup C) = RHS$

Finally suppose $x \in B \times C$

$\Rightarrow \quad x = (x_1, x_2)$ where $x_1 \in B$ & $x_2 \in C$

$\Rightarrow \quad x_1 \in A \cup B \quad$ & $\quad x_2 \in B \cup C$

$\therefore \quad x \in (A \cup B) \times (B \cup C) =$ RHS