MH1812 Discrete Mathematics: Quiz (CA) 1

Name: Tutorial Group:

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There are 3 (THREE) questions, please try all of them, and justify all your answers! Best of luck!

Question 1 (40 points)

a) Compute the addition table for integers modulo 3 (10 points).

b) Compute $7 \cdot 8 \cdot 9 \cdot 10$ modulo 3 (10 points).

c) Show by direct proof that $n^3 - n$ is always divisible by 3, for n any positive integer (20 points).

a) The addition table for integers modulo 3 is:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

b) Since 3 divides 9, the result modulo 3 is 0.

c) We note that $n^3 - n = n(n^2 - 1) = n(n-1)(n+1)$. Now any positive integer n is either a multiple of 3, say $n = 3k$, or when divided by 3 there is a remainder of 1, say $n = 3k+1$, or a remainder of 2, $n = 3k+2$. If $n = 3k$, $n^3 - n = 3k(n-1)(n+1)$ is divisible by 3, if $n = 3k+1$ then $n^3 - n = n(3k)(n+1)$ is divisible by 3 and if $n = 3k+2$, then $n^3 - n = n(n-1)(3k+3)$ is divisible by 3.

This can be rewritten by considering integers modulo 3, this is the same idea. To show that 3 divides $n^3 - n$ is the same thing has $n^3 - n \equiv 0 \pmod 3$. Then once one has the idea to look at integers modulo 3, write n as $3k$, $3k+1$, or $3k+2$, and compute $n^3 - n$ for each case, for example $(3k)^3 - (3k)$ is clearly divisible by 3, the same computation can be done to show that $n^3 - n$ is a multiple of 3 for $3k+1$ and $3k+2$.

Question 2 (40 points)

a) Prove or disprove the following statement (20 points):

$$(p \wedge q) \to p \equiv T.$$

b) Decide whether the following argument is valid (20 points):

$\neg d \to h$;
$\neg h \to d$;
$\therefore \neg d \vee \neg h$

a) One should prove the statement. Using the conversion theorem, and De Morgan's Law

$$\neg(p \wedge q) \vee p \equiv (\neg p \vee \neg q) \vee p \equiv T$$

since $\neg p \vee p$ is always true. Alternatively, $r \to p$ is always true, but for r true and p false. Here this means that we would need p false and $p \wedge q$ true, which is not possible, therefore it is always true. A 3rd way is to use a truth table.

b) The argument is not valid. There are several ways to see it. One is a truth table, which shows that when $d = T$ and $h = T$, then the conclusion is false. Yet, the premises are true, since $\neg d$ and $\neg h$ are false. Another way is to notice that one premise is the contrapositive of the other, therefore both of them are equivalent, from which one finds the same counterexample.

Question 3 (20 points)

Consider the domains $X = \{2, 3\}$ and $Y = \{2, 4, 6\}$, and the predicate $P(x, y) =$ "x divides y". What are the truth values of these statements:

a) $\exists x \in X,\ \forall y \in Y,\ P(x, y)$ (10 points).

b) $\neg(\exists x \in X,\ \exists y \in Y,\ P(x, y))$ (10 points).

a) This is true, there exists an $x \in X$, namely $x = 2$, such that this x divides y no matter which y you pick in Y, that is $x = 2$ divides 2,4 and 6.

b) This is false. One way to look at it is to say that since there exists x in X, say $x = 2$, for which there exists a y in Y, say $y = 4$ for which x divides y, then what is inside the parenthesis is true, therefore its negation is false. Another way is to write

$$\forall x \in X,\ \forall y \in Y, \neg P(x, y).$$

This is also false.