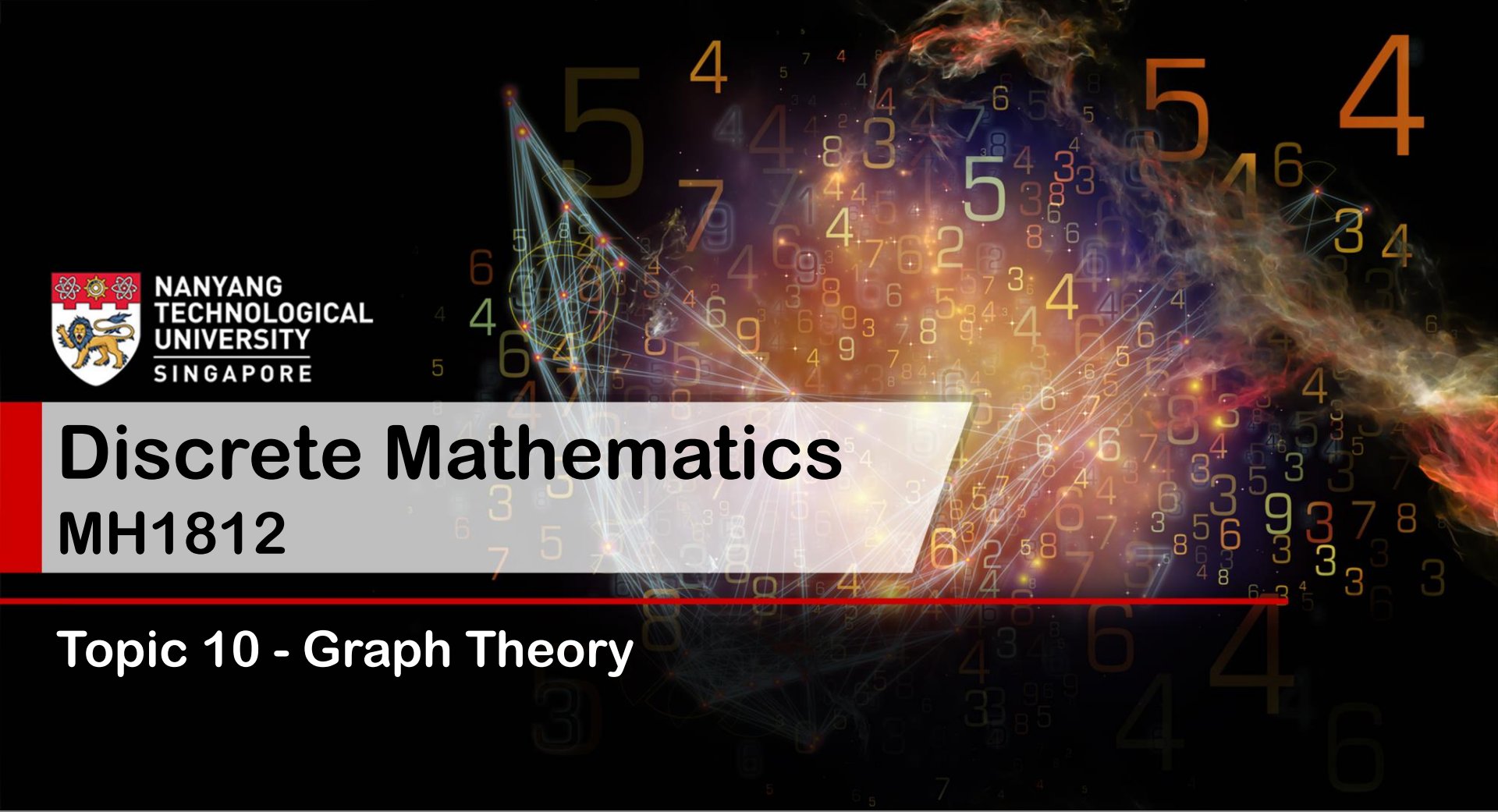


NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

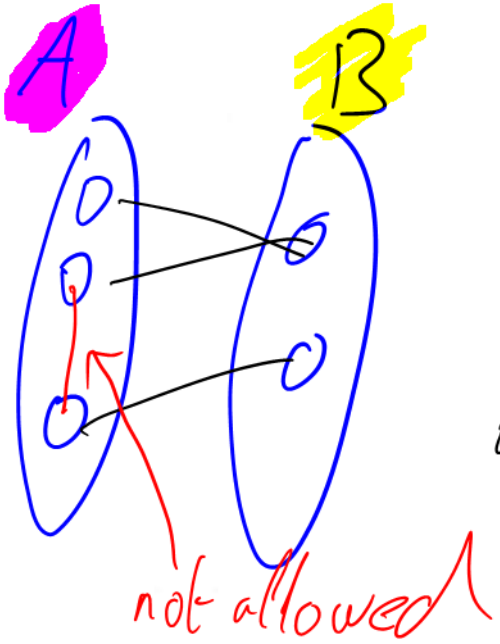
Discrete Mathematics

MH1812

Topic 10 - Graph Theory

(a) Is the following graph shown on Figure 1 bipartite? Justify your answer.

(b) Does the following graph shown on Figure 1 contain an Euler path? Justify your answer.



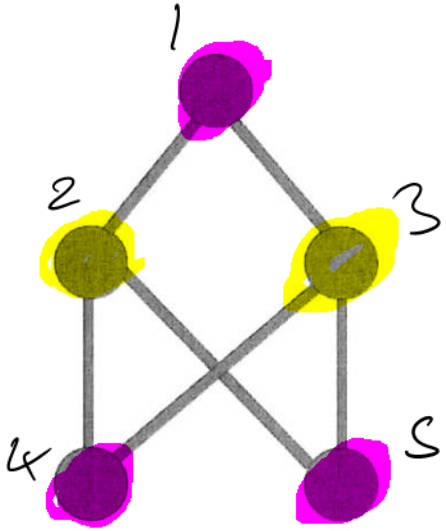
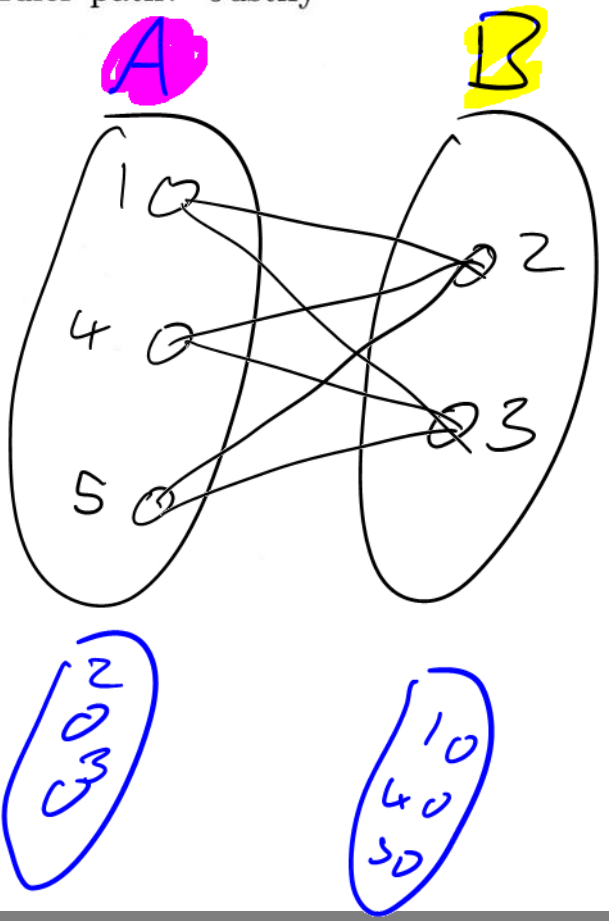
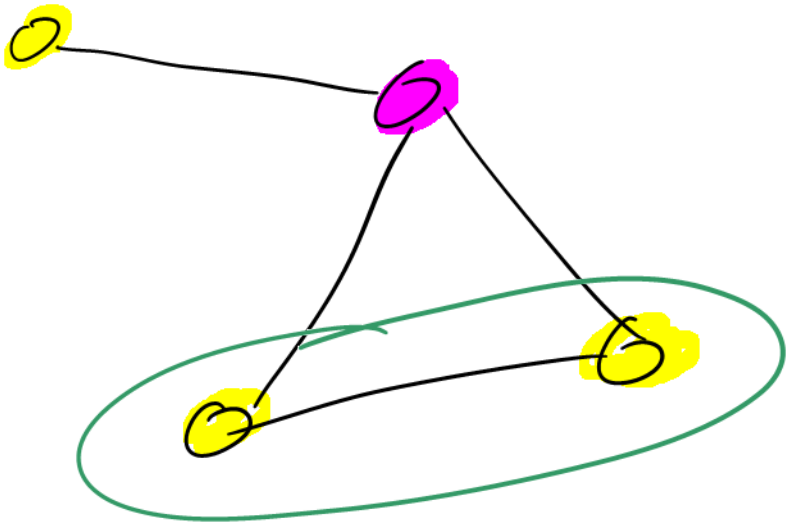


Figure 1: Graph



Bipartite?

No

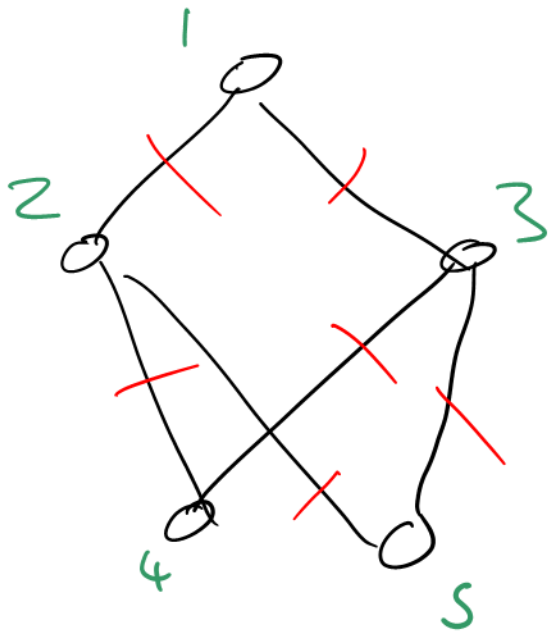


↑ violate bipartiteness condition

Euler visit every edge precisely once

vertex	degree
1	2
2	3
3	3
4	2
5	2

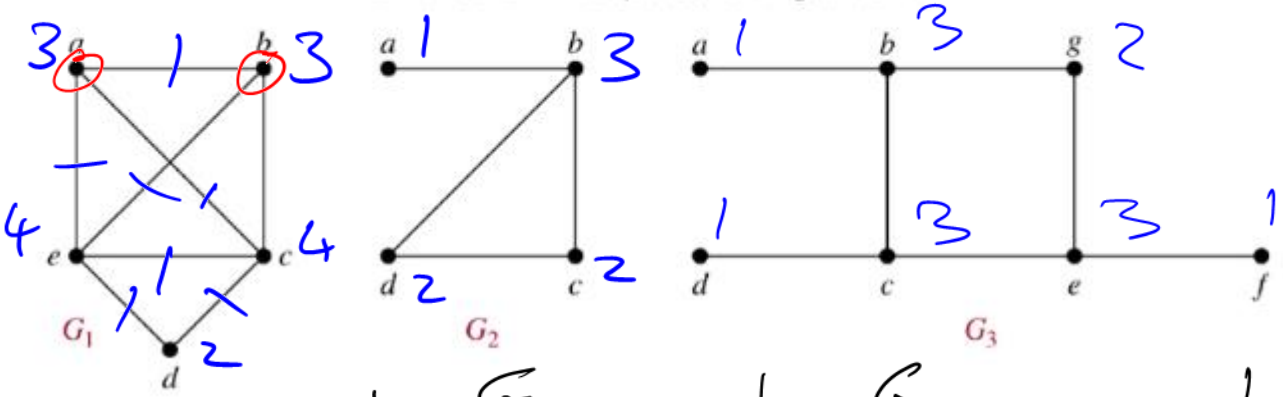
← start/end vertices of Euler path



Euler thm $\Rightarrow \exists$ Euler path

Explicit example: 2 1 3 5 2 4 3

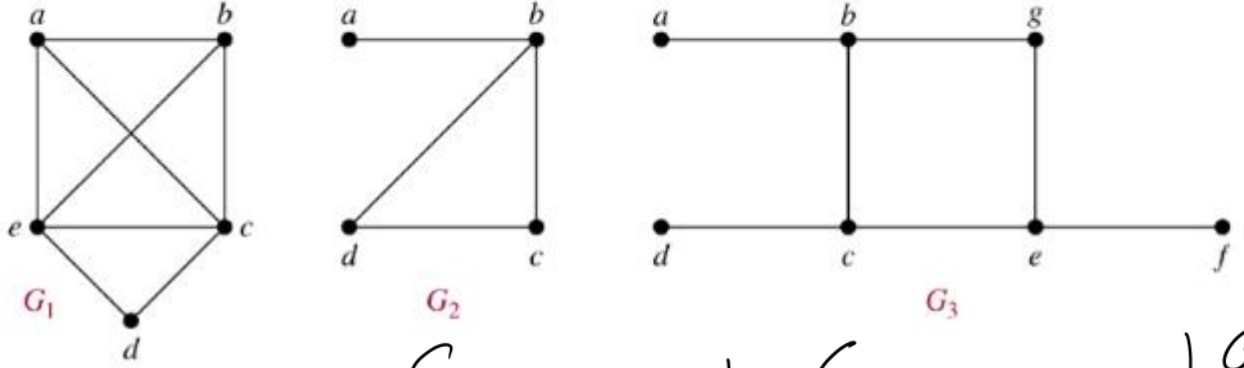
Find the Euler path, Euler circuit, Hamilton path, and Hamilton circuit of the following graphs (with a and b as starting and ending points for paths), if any.



	G_1	G_2	G_3
Euler path	aedcebacb	abdcb	DNE (> 2 odd degree)
Euler circuit	DNE (has odd deg vertices)	DNE	DNE

Find the Euler path, Euler circuit, Hamilton path, and Hamilton circuit of the following graphs (with a and b as starting and ending points for paths), if any.

(path = trail)



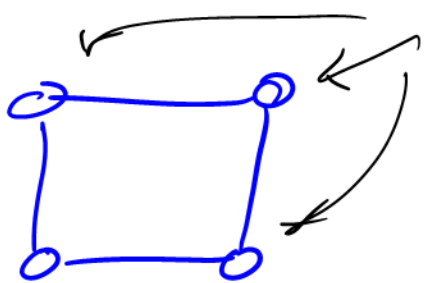
	G_1	G_2	G_3
Hamilton path	a e d c b	DNE (a is adjacent only to b)	DNE (> 2 vertices have degree 1)
Hamilton circuit	a e d c b a	DNE (has vertex w/ deg = 1)	DNE

Hamilton visit every vertex precisely once

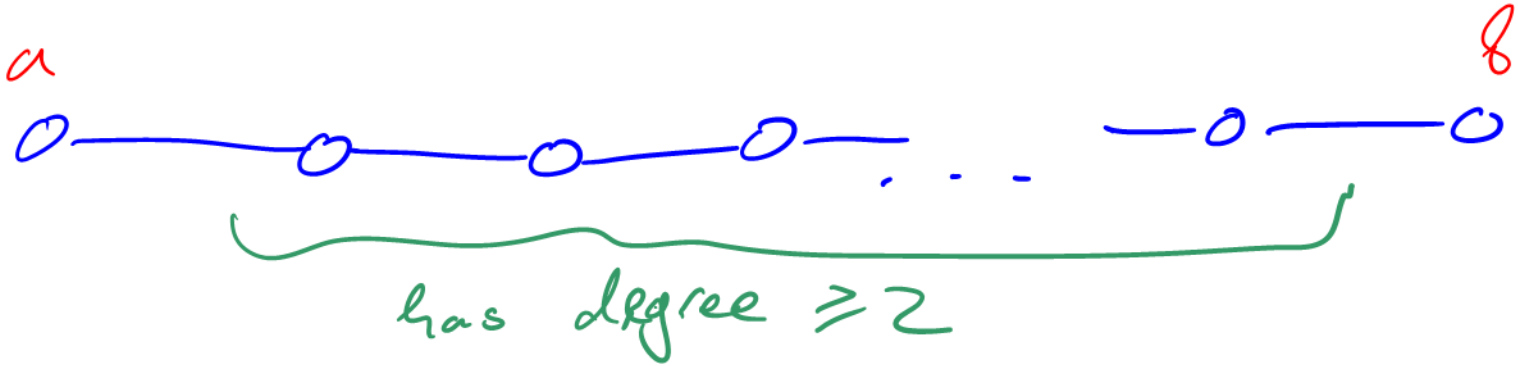
G_2 Hamilton path (from a to b)

↑ want this vertex to not be b

G_2 Hamilton circuit:



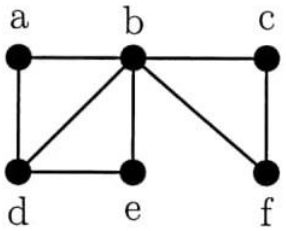
G_3 Hamilton path



QUESTION 3.

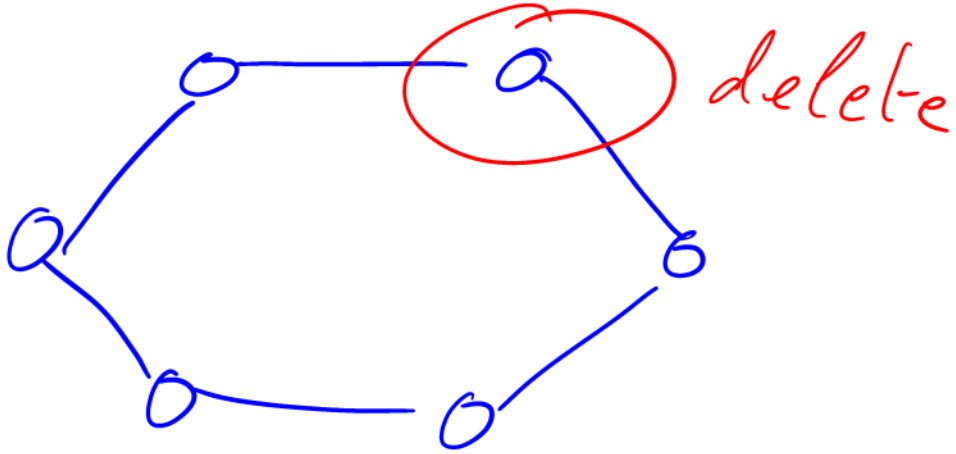
(a) Let A, B, and C be sets, show $(B - A) \cup (C - A) = (B \cup C) - A$. **(10 marks)**

(b) Refer to the graph below, find Euler Path, Euler Circuit and Hamilton Circuit if any, justify your answer if it does not exist. **(8 marks)**



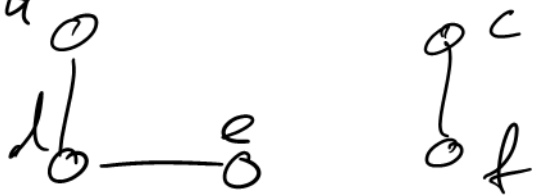
Hamilton circuit DNE

Suppose ∃ Hamilton circuit



$\Rightarrow$ if we delete any vertex
then resulting graph is still connected

But if we delete vertex b then
the resulting graph is

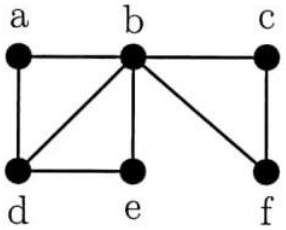


∴ $\not\exists$ Hamiltoa circuit.

QUESTION 3.

(a) Let A, B, and C be sets, show $(B - A) \cup (C - A) = (B \cup C) - A$. **(10 marks)**

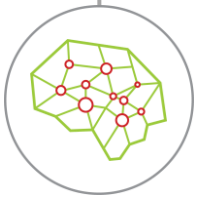
(b) Refer to the graph below, find Euler Path, Euler Circuit and Hamilton Circuit if any, justify your answer if it does not exist. **(8 marks)**



Your Learning Roadmap

1. Elementary Number Theory

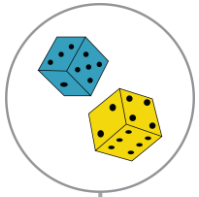
2. Propositional Logic



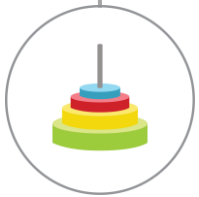
3. Predicate Logic

4. Proof Techniques

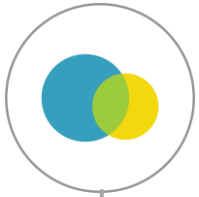
5. Combinatorics



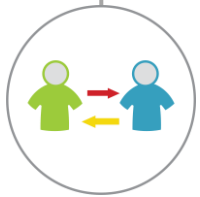
6. Linear Recurrence Theory



7. Set Theory



8. Relations



9. Functions

10. Graph Theory