SC1004/CX1104: Quiz 1 **Name:** ____________________

23/09/2022

Sem 1 AY 22-23 **Tutorial Group:** ____________________

50 Minutes

Max marks: 40

Answer all questions. Write the correct choice of answer in the box provided. You can use your OWN blank papers for working, but they will not be collected.

1. (2 points) After applying a series of elementary row operations on the augmented matrix of a linear system of equations, a student obtained the following matrix:

$$\begin{bmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & 2 & 1 \end{bmatrix}$$

Determine if the linear system is consistent.

A. Consistent. B. Inconsistent. C. Cannot be determined

2. (2 points) Find the value of t for which the following system has a solution:

$$\begin{array}{rcrcrcrcl} -x_1 & & & + & x_3 & - & x_4 & = & 3 \\ 2x_1 & + & 2x_2 & - & x_3 & - & 7x_4 & = & 1 \\ 4x_1 & - & x_2 & - & 9x_3 & - & 5x_4 & = & t \\ 3x_1 & - & x_2 & - & 8x_3 & - & 6x_4 & = & 1 \end{array}$$

A. -2. B. -3. C. 1 D. 3

3. (2 points) The following system of equations

$$\begin{array}{rcrcl} ax & + & by & = & c \\ dx & + & ey & = & f, \end{array}$$

where $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ has

A. no solution. B. one solution. C. infinite solutions D. 3 solutions

4. (2 points) The augmented matrix of a linear system of equations whose solution set is

$$\left\{ \begin{bmatrix} 2 & - & 2x_3 \\ & - & 4x_3 \\ & & x_3 \end{bmatrix} : x_3 \in \mathbb{R} \right\}$$

is

A. $\begin{bmatrix} 2 & 0 & -2 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ B. $\begin{bmatrix} -2 & 0 & 2 & 1 \\ -4 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 4 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 2 & 2 & 0 & 1 \\ 1 & 0 & 0 & 4 \end{bmatrix}$

5. (2 points) A homogeneous system of 5 equations in 4 unknowns has 4 pivots. The number of solutions for such a system is

A. 0 B. 1 C. infinite D. cannot be determined

6. (2 points) Let $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ be an independent set of vectors. Which of the following statements is true?

A. $\mathbf{u}$ is a linear combination of $\mathbf{v}$ and $\mathbf{w}$

B. $\{\mathbf{u}, \mathbf{v}, \mathbf{u} + \mathbf{v}\}$ is independent

C. $a\mathbf{u} + b\mathbf{u} + c\mathbf{w} = \mathbf{0}$ for some non zero scalars a, b, c

D. $\{\mathbf{u}, \mathbf{v}, \mathbf{u} + \mathbf{v} + \mathbf{w}\}$ is independent

7. (2 points) The general solution of $10x_1 - 3x_2 - 2x_3 = 7$ can be written in parametric form as $x = \mathbf{p} + t_1\mathbf{v_1} + t_2\mathbf{v_2}$, where $\mathbf{p}$ is the particular solution. The first element of $\mathbf{p}$, $\mathbf{v_1}$ and $\mathbf{v_2}$ are, respectively,

A. $10, -3, -2$

B. $-3, -2, 7$

C. $3, 2, -7$

D. $0.7, 0.3, 0.2$

8. (2 points) Let A be the 3×3 matrix

$$\begin{bmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{bmatrix}$$

where x, y, z are real numbers. Determine whether the given matrix is invertible.

A. Invertible B. Not invertible C. Cannot be determined

9. (2 points) State whether the following statement is TRUE or FALSE:

For a matrix $B = \begin{bmatrix} -1 & 1 & -1 \\ 0 & -1 & 0 \\ 1 & 2 & -2 \end{bmatrix}$, there exists a non-singular matrix A such that $A^2 = AB + 2A$.

10. (2 points) State whether the following statement is TRUE or FALSE:

The transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ xy \end{bmatrix}$$

is linear.

11. (2 points) Which of the following vectors belong to the kernel space of a linear transformation defined by the matrix $M = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$?

A. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ B. $\begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$ C. $\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

12. (2 points) If B is a symmetric matrix, then $(A^T - B)^T + C(B^{-1}C)^{-1}$ can be simplified to
A. $A + 2B$ B. $A - 2B$ C. A D. $A + C^{-1}$

13. (2 points) The set of skew-symmetric matrices W of the form $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is a subspace of the vector space of all 3×3 matrices. A spanning set for W contains

A. $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$ D. All of the above

14. (2 points) State whether the following statement is TRUE or FALSE:

Let W be the subset of $\mathbb{R}^3$ defined by $W = \{\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 = 3x_2 \text{ and } x_3 = 0\}$.

The set W is closed under addition.

15. (2 points) Find the spanning set for the null space $\mathbf{N}(B)$ of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ -2 & -2 & -4. \end{bmatrix}$

A. $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ C. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$ D. None of the above

16. (2 points) State whether the following statement is TRUE or FALSE:

The set $S = \{A \in M_{2\times 2} : |A| \neq 0\}$ is a subspace of $M_{2\times 2}$, the set of all 2×2 matrices.

17. (2 points) For a matrix A, if $\mathbf{N}(A)$ is a subspace of $\mathbb{R}^4$ and $\mathbf{C}(A)$ is a subspace of $\mathbb{R}^{10}$, then the size of the matrix is
A. 10×4 B. 5×2 C. 2×5 D. 4×10

18. (2 points) State whether the following statement is TRUE or FALSE:

$\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

19. (2 points) State whether the following statement is TRUE or FALSE:

$$\begin{vmatrix} 2 & 4 & 3 \\ 3 & 5 & 4 \\ 2.89 & 5 & 1.67 \end{vmatrix} = \begin{vmatrix} 3 & 5 & 4 \\ 2.89 & 5 & 1.67 \\ 2 & 4 & 3 \end{vmatrix}.$$

20. (2 points) State whether the following statement is TRUE or FALSE:

The intersection of two subspaces can never be a subspace.

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