MH1820 Introduction to Probability and Statistical Methods
Tutorial 7 (Week 8)

Problem 1 (Joint PMF, Marginal PMF, Conditional PMF)
Let W equal the weight of laundry soap in a 1-kilogram box that is distributed in Southeast Asia. Suppose that $\mathbb{P}(W < 1) = 0.02$ and $\mathbb{P}(W > 1.072) = 0.08$. Call a box of soap light, good, or heavy depending on whether $W < 1$, $1 \leq W \leq 1.072$, or $W > 1.072$, respectively. In $n = 50$ independent observations of these boxes, let X equal the number of light boxes and Y the number of good boxes.

(a) What is the joint PMF of X and Y?

(b) Give the name of the distribution of Y along with the values of the parameters of this distribution.

(c) Given that $X = 3$, how is Y distributed conditionally?

(d) Determine $\mathbb{E}[Y|X = 3]$

Problem 2 (Joint PMF, Marginal PMF, Conditional PMF)
An insurance company sells both homeowners' insurance and automobile deductible insurance. Let X be the deductible on the homeowners' insurance and Y the deductible on automobile insurance. Among those who take both types of insurance with this company, we find the following probabilities:

	$x = 100$	$x = 500$	$x = 1000$
$y = 1000$	0.05	0.10	0.15
$y = 500$	0.10	0.20	0.05
$y = 100$	0.20	0.10	0.05

(a) Compute the probabilities $\mathbb{P}(Y = 500|X = 500)$, $\mathbb{P}(Y = 100|X = 500)$.

(b) Compute the conditional means $\mathbb{E}[X|Y = 100]$, $\mathbb{E}[Y|X = 500]$.

Problem 3 (Joint PMF, Marginal PMF, Conditional PMF)
Let X and Y have a uniform distribution on the set of points with **integer** coordinates in $S = \{(x, y) : 0 \leq x \leq 7, x \leq y \leq x + 2\}$. That is, $p(x, y) = 1/24$, $(x, y) \in S$, and both x and y are integers. Find

(a) the marginal PMF $p_X(x)$ and $p_Y(y)$.

(b) the conditional PMF $h(y|x)$ of Y given $X = x$.

(c) $\mathbb{E}[Y|X = x]$.

(d) $\sigma^2_{Y|x}$.

Problem 4 (Joint PMF, Marginal PMF, Conditional PMF)
Let $p_X(x) = 1/10$, $x = 0, 1, 2, \ldots, 9$, and let the conditional PMF of Y given $X = x$ be $h(y|x) = 1/(10 - x)$, $y = x, x + 1, \ldots, 9$. Find

(a) $p(x, y)$.

(b) $p_Y(3)$.

(c) $\mathbb{E}[Y|X = 7]$.

Problem 5 (Joint PMF, Marginal PMF, Conditional PMF)
From a standard poker deck of 52 cards, 3 cards are drawn. Let X be number of clubs among the 3 cards and let Y be the number of hearts among the 3.

(a) Find the joint PMF of X and Y.

(b) Find the marginal PMFs of X and Y.

(c) Compute $P(X = 1|Y = 1)$.

(d) Let F be the joint CDF of X and Y. Compute $F(1, 1)$.

Answer Keys. **1(a).** $\frac{50!}{x!y!(50-x-y)!}(0.02)^x(0.9)^y(0.08)^{50-x-y}$ **1(b).** $Binomial(50, 0.9)$ **1(c).** $Binomial(47, 0.9/0.98)$ **1(d).** 43.1633 **2(a).** 0.5, 0.25 **2(b).** $\frac{2400}{7}$, 525 **3(a).** $p_X(x) = \frac{1}{8}$ **3(b).** $h(y|x) = \frac{1}{3}$ **3(c).** $1 + x$ **3(d).** $\frac{2}{3}$ **4(a).** $\frac{1}{10(10-x)}$ **4(b).** 0.0479 **4(c).** 8 **5(a).** $\frac{\binom{13}{x}\binom{13}{y}\binom{26}{3-x-y}}{\binom{52}{3}}$ **5(b).** $p_X(x) = \frac{\binom{13}{x}\binom{39}{3-x}}{\binom{52}{3}}$, $p_Y(y) = \frac{\binom{13}{y}\binom{39}{3-y}}{\binom{52}{3}}$ **5(c).** $\frac{338}{741}$ **5(d).** $\frac{1188}{1700}$