NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I (CA1)

MH1812 – Discrete Mathematics

February 2018 TIME ALLOWED: 40 minutes

Name:

Matric. no.: Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1. **(40 marks)**

(a) (10 marks) Which integer $a \in \{0, 1, 2, 3\}$ satisfies $a \equiv 2^{2018} \pmod 4$?

(b) (10 marks) Wednesday is two days after Monday. What day of the week is it 500 days after Tuesday?

(c) Decide whether or not the set S is closed under the operation Δ when

- $S = \{\text{odd integers}\}$ and Δ is addition. (10 marks)
- $S = \{\text{even integers}\}$ and Δ is division. (10 marks)

Briefly justify your answers.

QUESTION 2. **(40 marks)**

(a) (20 marks) Prove or disprove the following statement:

$$(p \vee r) \rightarrow (p \wedge q) \equiv (p \rightarrow q) \wedge (r \rightarrow q).$$

(b) (20 marks) Decide whether or not the following argument is valid:

$$\begin{aligned} &p \vee q; \\ &\neg p \rightarrow r; \\ &\neg q \rightarrow r; \\ &r \vee p; \\ &\therefore r \end{aligned}$$

Briefly justify your answers.

BLANK PAGE FOR WRITING

QUESTION 3. **(20 marks)**

(a) (10 marks) Consider the domain $\mathbb{Q} = \{\text{rational numbers}\}$ and the predicate $P(x, y) =$ "xy is an integer".

Determine the truth value of the statement:

$\forall x \in \mathbb{Q},\ \exists y \in \mathbb{Q},\ P(x, y).$

(b) (10 marks) Let X and Y be domains, and let $P(x)$ and $Q(y)$ be predicates. Which of the following statements is the *negation* of the statement:

$\forall y \in Y,\ \exists x \in X,\ P(x) \to Q(y)$?

(i) $\forall y \in Y,\ \exists x \in X,\ \neg P(x) \wedge Q(y)$;

(ii) $\exists y \in Y,\ \exists x \in X,\ \neg P(x) \vee Q(y)$;

(iii) $\exists y \in Y,\ \forall x \in X,\ P(x) \wedge \neg Q(y)$;

(iv) $\exists y \in Y,\ \forall x \in X,\ \neg P(x) \wedge \neg Q(y)$.

Briefly justify your answers.

BLANK PAGE FOR WRITING