Saving, Capital Accumulation, and Output

Intermediate Macroeconomics

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Outline

- Interactions between Output and Capital
- The Implications of Alternative Saving Rates
- Example

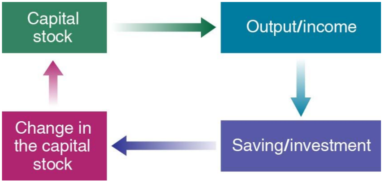
Motivation

- Since 1970, in the U.S. the ratio of saving to gross domestic product has averaged only 17%, compared to 23% in Germany and 29% in Japan.
- How is saving rate connected with capital accumulation given that capital accumulation can drive the economic growth.
- examine the effects of the saving rate on the level and growth rate of output.

Interactions between Output and Capital

Output in the long run depends on two relations:

- The amount of capital determines the amount of output being produced
- The amount of output produced determines the amount of saving, which in turn determines the amount of capital being accumulated over time



The Effects of Capital on Output

- Recall that

$$\frac{Y}{N} = F(\frac{K}{N}, 1) \equiv f(\frac{K}{N})$$

- Assume that N is constant, and there is no technological progress, so f does not change over time:

$$\frac{Y_t}{N} = f(\frac{K_t}{N})$$

- Higher capital per worker leads to higher output per worker.

The Effects of Output on Capital Accumulation

- Assume
  - Closed economy: $I = S + (T - G)$
  - Public saving $(T - G)$ is 0: $I = S$
  - Assume a constant and exogenous saving rate: $S = sY$
- So the relation between output and investment:

$$I_t = sY_t$$

- Investment is proportional to output.

Physical Capital Accumulation(I)

- The evolution of the capital stock is:

$$K_{t+1} = (1-\delta)K_t + I_t$$

- Replace investment by the above expression and divide both sides by N:

$$\frac{K_{t+1}}{N} = (1-\delta)\frac{K_t}{N} + s\frac{Y_t}{N}$$

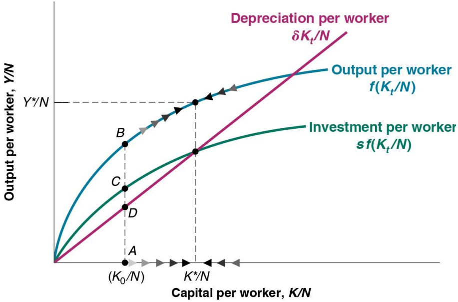
or

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N} = sf(\frac{K_t}{N}) - \delta\frac{K_t}{N}$$

- The change in the capital stock per worker is equal to saving per worker minus depreciation.

Physical Capital Accumulation(II)

When capital and output are low(high), investment per worker exceeds (is less than) depreciation per worker, the change in capital per worker is positive (negative), capital increases(decreases).



Steady State

- The state in which output per worker and capital per worker are no longer changing is called the **steady state** of the economy.

$$sf(\frac{K^*}{N}) = \delta \frac{K^*}{N}$$

- The steady-state value of capital per worker is such that the amount of saving per worker is sufficient to cover depreciation of the capital stock per worker.

$$\frac{Y^*}{N} = f(\frac{K^*}{N})$$

Example

- Assume the production function f takes form:

$$Y = K^{1/2}N^{1/2}$$

- The capital accumulation equation can thus be expressed as:

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s(\frac{K_t}{N})^{1/2} - \delta\frac{K_t}{N},$$

which describes the evolution of capital over time.

Example Cont'd

- This implies capital per worker in the steady state(K^*/N) becomes:

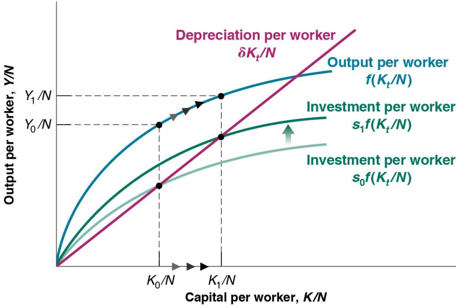
$$\frac{K^*}{N} = (\frac{s}{\delta})^2$$

- The steady state output per worker is:

$$\frac{Y^*}{N} = (\frac{K^*}{N})^{1/2} = \frac{s}{\delta}$$

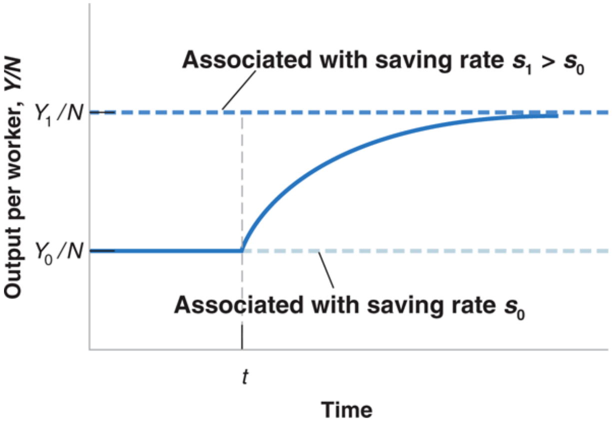
The Saving Rate and Output

- The saving rate has no effect on the long-run growth rate of output per worker, which is equal to zero.
- The saving rate determines the level of output per worker in the long run.



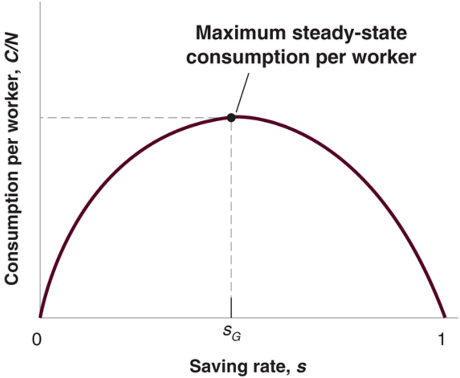
The Saving Rate and Output

An increase in the saving rate will lead to higher growth of output per worker for some time, but not forever.



Golden Rule

- What matters to people is not how many is produced, but how much they consume.
- **Golden-rule level of capital**: The level of capital associated with the saving rate that yields the highest level of consumption in steady state.



Saving Rates and Consumption

- For a saving rate between zero and the golden-rule level, a higher saving rate leads to higher capital per worker, higher output per worker and higher consumption per worker.
- For a saving rate greater than the golden-rule level, a higher saving rate still leads to higher capital per worker and output per worker, but lower consumption per worker.
- An increase in the saving rate leads to lower consumption for some time but higher consumption later.

Previous Example Cont'd

- In the steady state of the previous example, consumption per worker is:

$$\frac{C}{N} = \frac{Y}{N} - \delta \frac{K}{N}$$

- The steady-state consumption per worker is:

$$\frac{C}{N} = \frac{s}{\delta} - \delta(\frac{s}{\delta})^2 = \frac{s(1-s)}{\delta}$$

The above is maximized when $s = 1/2$.

Example Cont'd

Saving rate S	$\frac{K}{N}(\%)$	$\frac{Y}{N}(\%)$	$\frac{C}{N}(\%)$
0.0	0.0	0.0	0.0
0.1	1.0	1.0	0.9
0.2	4.0	2.0	1.6
0.3	9.0	3.0	2.1
0.4	16.0	4.0	2.4
0.5	25.0	5.0	2.5
0.6	36.0	6.0	2.4
1.0	100.0	10.0	0.0

Table: Steady-state Levels of Capital, Output and Consumption per Worker with respect to Different Saving Rate