

Q1 Lay5e/Ch5.1/pg273/Ex6+7

6. Is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? If so, find the eigenvalue.

7. Is $\lambda = 4$ an eigenvalue of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If so, find one corresponding eigenvector.

Q2) Lay5e/Ch5.1/pg274/

Find the eigenvector corresponding to eigenvalue = -2, for the following matrix

14. $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}, \lambda = -2$

Q3) Lay5e/Ch5/pg 274/Q20

20. Without calculation, find one eigenvalue and two linearly independent eigenvectors of $A = \begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{bmatrix}$. Justify your answer.

Q4) Lay5e/Ch5/pg274/Q21

In Exercises 21 and 22, A is an $n \times n$ matrix. Mark each statement True or False. Justify each answer.

21. a. If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector \mathbf{x} , then λ is an eigenvalue of A .
b. A matrix A is not invertible if and only if 0 is an eigenvalue of A .
c. A number c is an eigenvalue of A if and only if the equation $(A - cI)\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

- d. Finding an eigenvector of A may be difficult, but checking whether a given vector is in fact an eigenvector is easy.
- e. To find the eigenvalues of A , reduce A to echelon form.

22. a. If $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ , then \mathbf{x} is an eigenvector of A .
- b. If \mathbf{v}_1 and \mathbf{v}_2 are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

- d. The eigenvalues of a matrix are on its main diagonal.
- e. An eigenspace of A is a null space of a certain matrix.

Ans: 21) F,T,T,T,F

22) F,F,x,F,T

Q5) Lay5e/ch5.1/pg 274/Q25

25. Let λ be an eigenvalue of an invertible matrix A . Show that λ^{-1} is an eigenvalue of A^{-1} . [Hint: Suppose a nonzero \mathbf{x} satisfies $A\mathbf{x} = \lambda\mathbf{x}$.]

Q6) Lay5e/Ch5.2/pg 282/Q27

27. Let $A = \begin{bmatrix} .5 & .2 & .3 \\ .3 & .8 & .3 \\ .2 & 0 & .4 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} .3 \\ .6 \\ .1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$,
 $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

- a. Show that \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 are eigenvectors of A . [Note: A is the stochastic matrix studied in Example 3 of Section 4.9.]
- b. Let \mathbf{x}_0 be any vector in \mathbb{R}^3 with nonnegative entries whose sum is 1. (In Section 4.9, \mathbf{x}_0 was called a probability vector.) Explain why there are constants c_1 , c_2 , and c_3 such that $\mathbf{x}_0 = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3$. Compute $\mathbf{w}^T\mathbf{x}_0$, and deduce that $c_1 = 1$.
- c. For $k = 1, 2, \dots$, define $\mathbf{x}_k = A^k\mathbf{x}_0$, with \mathbf{x}_0 as in part (b). Show that $\mathbf{x}_k \rightarrow \mathbf{v}_1$ as k increases.

EXAMPLE 2 Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k , given that $A = PDP^{-1}$, where

$$P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$$

PRACTICE PROBLEMS

1. The matrix A below has eigenvalues 1 , $\frac{2}{3}$, and $\frac{1}{3}$, with corresponding eigenvectors \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 :

$$A = \frac{1}{9} \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 5 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

Find the general solution of the equation $\mathbf{x}_{k+1} = A\mathbf{x}_k$ if $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 11 \\ -2 \end{bmatrix}$.

2. What happens to the sequence $\{\mathbf{x}_k\}$ in Practice Problem 1 as $k \rightarrow \infty$?

In Exercises 21 and 22, A , B , P , and D are $n \times n$ matrices. Mark each statement True or False. Justify each answer. (Study Theorems 5 and 6 and the examples in this section carefully before you try these exercises.)

21. a. A is diagonalizable if $A = PDP^{-1}$ for some matrix D and some invertible matrix P .
b. If \mathbb{R}^n has a basis of eigenvectors of A , then A is diagonalizable.
c. A is diagonalizable if and only if A has n eigenvalues, counting multiplicities.
d. If A is diagonalizable, then A is invertible.
22. a. A is diagonalizable if A has n eigenvectors.
b. If A is diagonalizable, then A has n distinct eigenvalues.
c. If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A .
d. If A is invertible, then A is diagonalizable.
31. Construct a nonzero 2×2 matrix that is invertible but not diagonalizable.
32. Construct a nondiagonal 2×2 matrix that is diagonalizable but not invertible.

5.4 EXERCISES

1. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ and $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ be bases for vector spaces V and W , respectively. Let $T : V \rightarrow W$ be a linear transformation with the property that

$$T(\mathbf{b}_1) = 3\mathbf{d}_1 - 5\mathbf{d}_2, \quad T(\mathbf{b}_2) = -\mathbf{d}_1 + 6\mathbf{d}_2, \quad T(\mathbf{b}_3) = 4\mathbf{d}_2$$

Find the matrix for T relative to \mathcal{B} and \mathcal{D} .

Q10) Lay5e/Ch5.4/pg 295/Q3

3. Let $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for \mathbb{R}^3 , $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for a vector space V , and $T : \mathbb{R}^3 \rightarrow V$ be a linear transformation with the property that

$$T(x_1, x_2, x_3) = (x_3 - x_2)\mathbf{b}_1 - (x_1 + x_3)\mathbf{b}_2 + (x_1 - x_2)\mathbf{b}_3$$

- Compute $T(\mathbf{e}_1)$, $T(\mathbf{e}_2)$, and $T(\mathbf{e}_3)$.
- Compute $[T(\mathbf{e}_1)]_{\mathcal{B}}$, $[T(\mathbf{e}_2)]_{\mathcal{B}}$, and $[T(\mathbf{e}_3)]_{\mathcal{B}}$.
- Find the matrix for T relative to \mathcal{E} and \mathcal{B} .

Q11) Lay5e/Ch5.4/pg 295/Q7

7. Assume the mapping $T : \mathbb{P}_2 \rightarrow \mathbb{P}_2$ defined by

$$T(a_0 + a_1t + a_2t^2) = 3a_0 + (5a_0 - 2a_1)t + (4a_1 + a_2)t^2$$

is linear. Find the matrix representation of T relative to the basis $\mathcal{B} = \{1, t, t^2\}$.

===== End =====