Name:

Matric. no.: Tutor group:

February 2021 **CA1** TIME ALLOWED: 50 minutes

QUESTION 1. **(15 marks)**

(a) [5 marks] Which integer $a \in \{0, 1, \ldots, 4\}$ is congruent to 2021 modulo 5?

(b) [5 marks] Which integer $a \in \{0, 1, \ldots, 9\}$ is congruent to 1812^{2021} modulo 10? Justify your answer.

(c) [5 marks] Let $S = \{\text{integers congruent to 7 modulo 6}\}$ and Δ be multiplication. Is S closed under Δ? Justify your answer.

For graders only:	Question	1(a)	1(b)	1(c)	2(a)	2(b)	2(c)	3(a)	3(b)	3(c)	Total
	Marks										

QUESTION 2. **(15 marks)**

Let $\mathbb{Q}$ denote the set of rational numbers and consider the predicate

$$P(x, y, z) = \text{“}x(y + z) = 2021\text{”}.$$

Determine the truth value of the following statements. Justify your answers.

(a) [5 marks] $\forall x \in \mathbb{Q},\ \exists y \in \mathbb{Q},\ \exists z \in \mathbb{Q},\ P(x, y, z)$;

(b) [5 marks] $\exists x \in \mathbb{Q},\ \forall y \in \mathbb{Q},\ \exists z \in \mathbb{Q},\ P(x, y, z)$;

(c) [5 marks] $\exists x \in \mathbb{Q},\ \exists y \in \mathbb{Q},\ \forall z \in \mathbb{Q},\ P(x, y, z)$.

QUESTION 3. **(20 marks)**

(a) [5 marks] Use a truth table to prove or disprove the following equivalence.

$$(p \vee (p \to F)) \wedge q \equiv p \to q$$

(b) [5 marks] Prove the following equivalence using De Morgan's law, double negation, the conversion theorem, and distributivity (noting where each is used).

$$p \vee (\neg(q \to r)) \equiv (p \vee q) \wedge (\neg p \to \neg r)$$

(c) [10 marks] Decide whether or not the following argument is valid:

$$\begin{aligned} & q \wedge r \to p; \\ & T \to p \wedge r; \\ & p \to (\neg r \to s); \\ & r \to \neg s; \\ \therefore\ & s \vee q. \end{aligned}$$

Briefly justify your answer.