MH1820 Introduction to Probability and Statistical Methods
Tutorial 6 (Week 7)

Problem 1 (MGF) (a) Let X be a discrete random variable with PMF p given by

x	-1	0	1
$p(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

and $p(x) = 0$ for $x \notin \{-1, 0, 1\}$.

Compute the MGF of X and use the properties of MGFs from the lecture to compute $\mathbb{E}[X]$ and $\mathbb{E}[X^2]$.

(b) Let X be a continuous random variable with PDF f given as follows.

$$f(x) = 2x \text{ for } 0 \leq x \leq 1 \text{ and } f(x) = 0 \text{ otherwise.}$$

Compute the MGF $\mathbb{E}[e^{tX}]$ of X for $t \neq 0$.

(c) Let X be a random variable with $M_X(t) = \dfrac{e^t}{2 - e^t}$ for all $t < \ln 2$. What is the distribution of X? Hint: Check the table for MGF of common distributions.

Problem 2 (MGF) Let $X \sim \mathit{Poisson}(\lambda)$. Using the definition of MGF, verify that the MGF of X is given by

$$e^{\lambda(e^t - 1)}.$$

Hence, find the mean and variance of X.

Hint: You may assume that $e^z = \displaystyle\sum_{x=0}^{\infty} \frac{z^x}{x!}$. This is the Maclaurin series for e^z.

Problem 3 (MGF) Suppose X and Y are independent Poisson random variables with means λ_1 and λ_2 respectively. What is the distribution of $X + Y$?

Problem 4 (Joint PMF, CDF, Marginal PMF) Let the joint PMF of X and Y be defined by

$$p(x, y) = \frac{x + y}{32},$$

for $x = 1, 2$, $y = 1, 2, 3, 4$.

(a) Find $p_X(x)$, the marginal PMF of X.

(b) Find $p_Y(y)$, the marginal PMF of Y.

(c) Find $\mathbb{P}(X > Y)$

(d) Find $\mathbb{P}(Y = 2X)$

(e) Find $\mathbb{P}(X + Y = 3)$.

(f) Find $\mathbb{P}(X \leq 3 - Y)$.

(g) Are X and Y independent or dependent? Why or why not?

Problem 5 (Joint PMF, CDF, Marginal PMF) From a standard poker deck of 52 cards, 3 cards are chosen randomly. Let X be number of clubs among the 3 cards and let Y be the number of hearts among the 3.

(a) Find the joint PMF of X and Y.

(b) Find the marginal PMFs of X and Y.

(c) Are X and Y independent?

Problem 6 (Joint PMF, CDF, Marginal PMF) There are eight similar chips in a bowl: three marked $(0,0)$, two marked $(1,0)$, two marked $(0,1)$, and one marked $(1,1)$. A player selects a chip at random and is given the sum of the two coordinates in dollars. What is the expected payoff?

Answer Keys. **1(a)** $M_X(t) = \frac{1}{2}e^{-t} + \frac{1}{4} + \frac{1}{4}e^{t}$, $-1/4$, $3/4$. **1(b)** $M_X(t) = \frac{2e^t}{t} - \frac{2e^t}{t^2} + \frac{2}{t^2}$ **1(c)** $X \sim Geom(1/2)$. **3** $X + Y$ is Poisson with mean $\lambda_1 + \lambda + 2$. **4(a)** $\frac{4x+10}{32}$ **4(b)** $\frac{3+2y}{32}$ **4(c)** $3/32$ **4(d)** $9/32$ **4(e)** $6/32$ **4(f)** $8/32$ **4(g)** Dependent. **5(a)** $p(x,y) = \frac{\binom{13}{x}\binom{13}{y}\binom{26}{3-x-y}}{\binom{52}{3}}$ **5(b)** $p_X(x) = \frac{\binom{13}{x}\binom{39}{3-x}}{\binom{52}{3}}$, $p_Y(y) = \frac{\binom{13}{y}\binom{39}{3-y}}{\binom{52}{3}}$ **5(c)** Dependent **6** \$0.75.