

NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I (CA1)

MH1812 – Discrete Mathematics

February 2020

TIME ALLOWED: 50 minutes

Name:

Matric. no.:

Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1.**(30 marks)**

- (a) Which integer $a \in \{0, 1, \dots, 14\}$ is congruent to $2020 + 1010 + 550 + 225$ modulo 15? (10 marks)
- (b) Write down each integer $a \in \{0, 1, 2\}$ for which there exists an integer n such that $a \equiv n^2 + n - 1 \pmod{3}$. (10 marks)
- (c) Let $S = \{\text{integers congruent to 1 modulo 5}\}$ and Δ be multiplication. Is S closed under Δ ? Justify your answer. (10 marks)

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QUESTION 2.**(40 marks)**

(a) Prove or disprove the following logical equivalences:

(i) (10 marks)

$$p \wedge (T \rightarrow p) \equiv p$$

(ii) (10 marks)

$$(p \wedge q \wedge r) \rightarrow (p \vee s) \equiv (p \rightarrow s) \vee (q \rightarrow s) \vee (r \rightarrow s)$$

(b) Decide whether or not the following argument is valid (20 marks):

$$\begin{aligned} &\neg q \vee p; \\ &\neg q \rightarrow F; \\ &p \rightarrow (\neg r \rightarrow s); \\ &q \rightarrow \neg r \\ &\therefore s \end{aligned}$$

Briefly justify your answers.

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QUESTION 3.**(30 marks)**

- (a) Let X and Y be domains, and let $P(x)$ and $Q(y)$ be predicates. Which of the following statements is the *negation* of the statement

$\forall x \in X, \exists y \in Y, P(x) \vee \neg Q(y)$? (10 marks)

- (i) $\forall y \in Y, \exists x \in X, \neg P(x) \wedge Q(y)$;
 - (ii) $\exists x \in X, \forall y \in Y, P(x) \rightarrow \neg Q(y)$;
 - (iii) $\exists y \in Y, \forall x \in X, \neg P(x) \rightarrow \neg Q(y)$;
 - (iv) $\exists x \in X, \forall y \in Y, \neg P(x) \wedge Q(y)$;
 - (v) none of the above.
- (b) Consider the domains $A = \{3, 4\}$ and $B = \{0, 3, 6\}$ and the predicate $P(x, y) = "x^2 - y \geq 9"$.

Determine the truth value of the following statements:

- (i) $\forall x \in A, \exists y \in B, P(x, y)$; (10 marks)
- (ii) $\exists x \in A, \forall y \in B, P(x, y)$. (10 marks)

Briefly justify your answers.

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