

# SC1005 Digital Logic

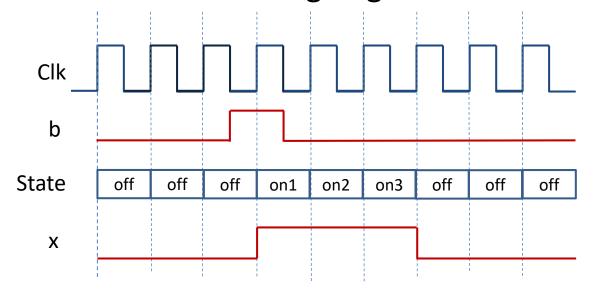
**Recap and Discussion** 

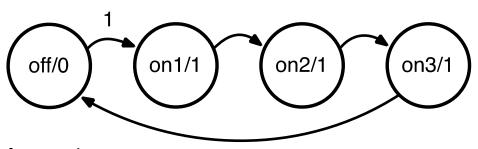
Lecture 21 and 22

Finite State Machine, FSM in Verilog

# **Recap: Finite State Machines**

- A finite state machine (FSM) describes a system using a finite number of states and the associated transitions
- In synchronous design, an FSM is in one state for the duration of each clock cycle
- At every rising clock edge, the FSM may transition to another state depending on the input values at that rising edge



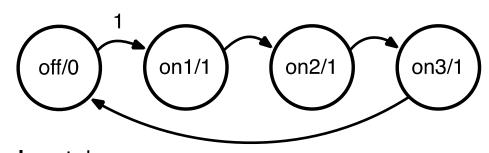


Input: b
Output: x

Current State	b	Next State	Output x
off	0	off	0
off	1	on1	0
on1	0	on2	1
on1	1	on2	1
on2	0	on3	1
on2	1	on3	1
on3	0	off	1
on3	1	off	1

# **Recap: Finite State Machines**

- A finite state machine (FSM) describes a system using a finite number of states and the associated transitions
- In synchronous design, an FSM is in one state for the duration of each clock cycle
- At every rising clock edge, the FSM may transition to another state depending on the input values at that rising edge



Input: b
Output: x

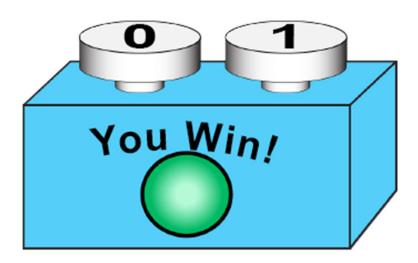
Clk									
b									
State	off	off	off	on1	on2	on3	off	off	off
x									

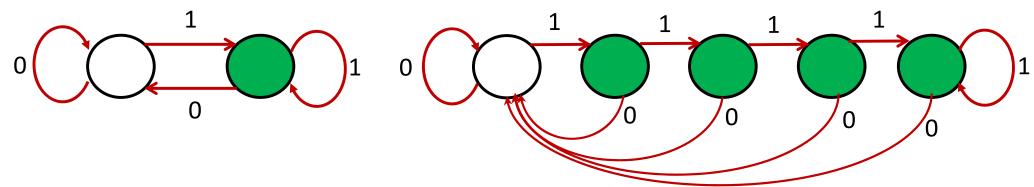
Current	Next	Output	
State	b = 0	1	$\overline{X}$
off	<u>off</u>	on1	0
on1	on2	on2	1
on2	on3	on3	1
on3	off	off	1

It is often easier to visualize when the state transition table is represented in this form

#### **Exercise 1**







#### Are these two FSMs different?

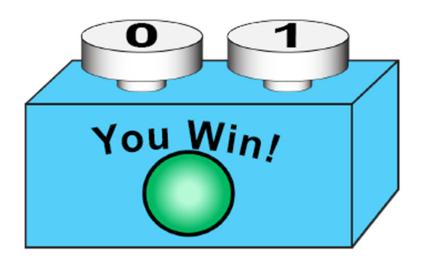
- A. Yes
- B. No

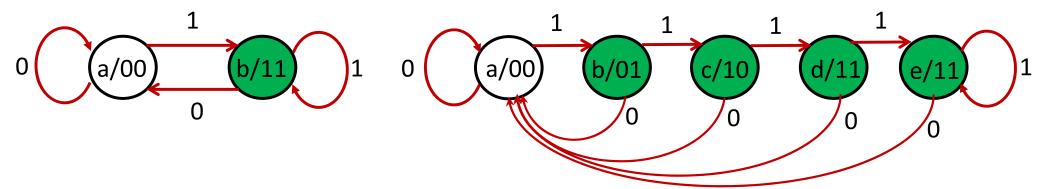
Ans: No. They are the same.

You can use state reduction techniques to reduce the RHS FSM to the LHS FSM.

#### **Exercise 1a**







#### Are these two FSMs different?

- A. Yes
- B. No

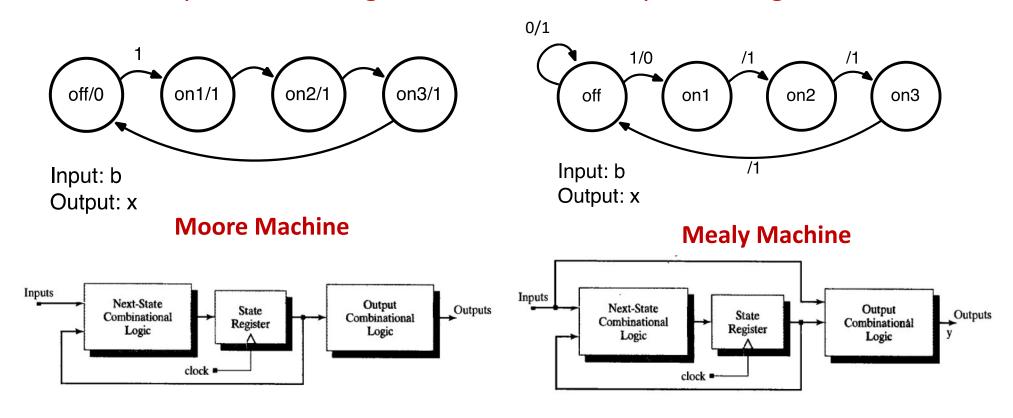
Ans: Yes, they are different.

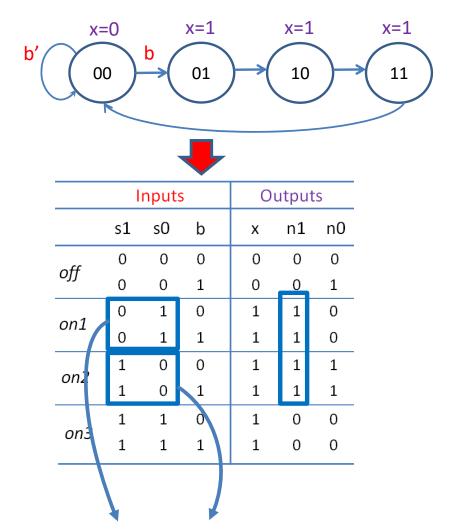
You can only use state reduction techniques to combine states d and e, due to the others having different outputs.

# **Recap: Moore and Mealy Machines**

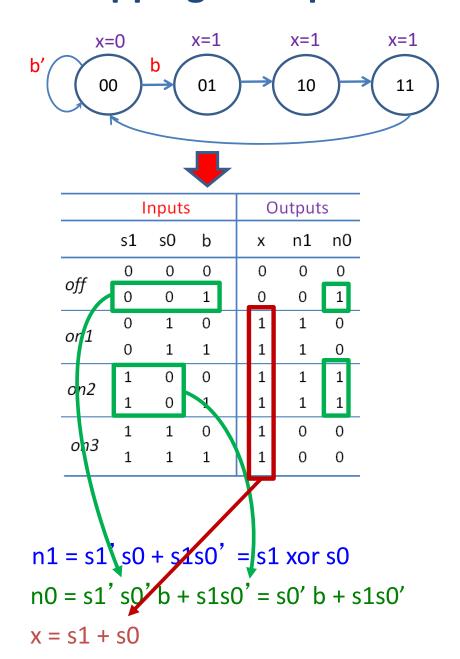


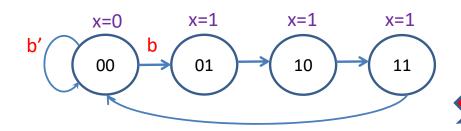
- Moore Machines: The outputs depend only on the state,
   i.e. in state x, the output is always f
- Mealy Machines: The output depends on the state and the current value of the inputs
  - Outputs can change mid-state, if the inputs change





n1 = s1's0 + s1s0' = s1 xor s0



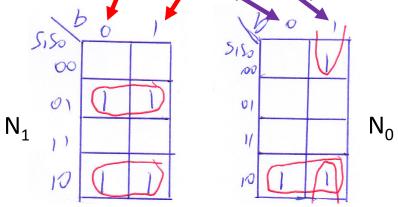


	Inputs			0	utput	S
	s1	s0	b	х	n1	n0
	0	0	0	0	0	0
off	0	0	1	0	0	1
on1	0	1	0	1	1	0
OHI	0	1	1	1	1	0
on2	1	0	0	1	1	1
0112	1	0	1	1	1	1
	1	1	0	1	0	0
on3	1	1	1	1	0	0

But, it may be easier to redraw the transition table from before (now referred to as an Excitation Table).

	State S <sub>1</sub> S <sub>0</sub>	Next sta $b = 0$	te N <sub>1</sub> N <sub>0</sub>	Output (x)
off	0 0	00	0 1	0
off on1	0 1	10	10	1
on3	11	0 0	0 0	1
on2	10	11	11	1

Then the next state & output expressions are:

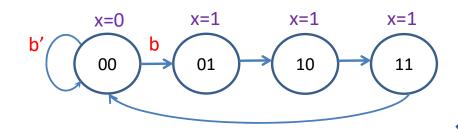


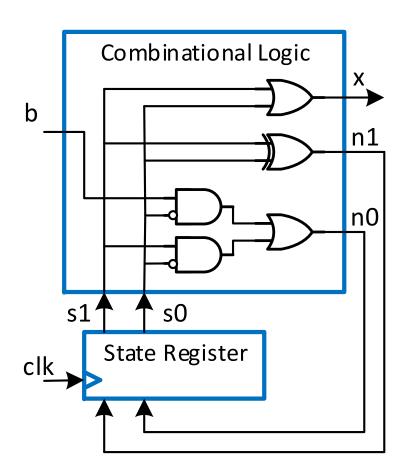
$$N_1 = S_1' S_0 + S_1 S_0' = S_1 xor S_0$$

$$N_0 = S_0' b + S_1 S_0'$$
  
  $x = S_1 + S_0$ 

#### Same as before.

Also note grey code order in Ex Table.

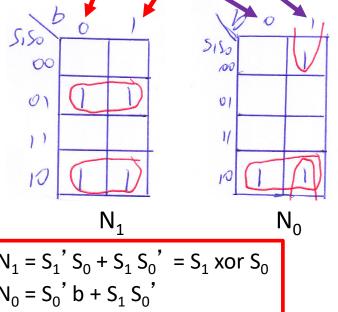




But, it may be easier to redraw the transition table in a different form.

	State S <sub>1</sub> S <sub>0</sub>	Next star $b = 0$	te $N_1N_0$	Output (x)
off	0 0	0 0	0 1	0
on1	0 1	1 0	1 0	1
on3	1 1	0 0	0 0	1
on2	1 0	1 1	1 1	1

Then the next state & output expressions are:

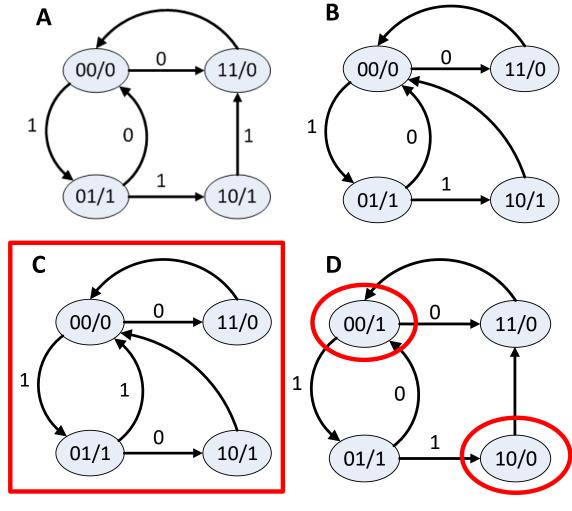




$$N_1 = S_1' S_0 + S_1 S_0' = S_1 xor S_0$$
  
 $N_0 = S_0' b + S_1 S_0'$   
 $x = S_1 + S_0$ 

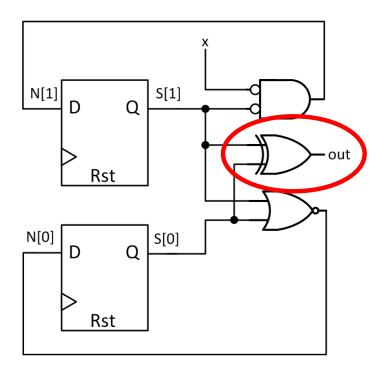
#### **Exercise 2**

Which of the following FSMs corresponds to the given circuit?



Input: x

Output: out



D is obviously wrong as the output is incorrect. When in State **01**, a 1 on x will result in:

$$N[1] = x'.S[1]' = 0$$

$$N[0] = (S[1]+S[0])' = 0$$

That is a next state of 00.

The answer can only be C

Ans: C

# **Exercise 2 (a better but longer approach)**

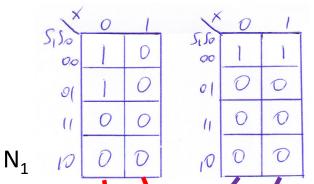
 $N_0$ 

Which FSM corresponds to the given circuit?

1. Get the next state and output expressions

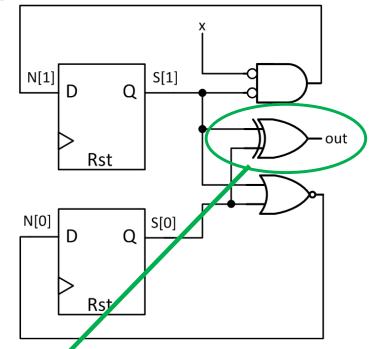
$$N_1 = S_1' x'$$
  
 $N_0 = (S_1 + S_0)' = S_1' S_0'$   
out =  $S_1 \land S_0$ 

2. Generate the next state K-maps



3. Then get the state transition table

State S <sub>1</sub> S <sub>0</sub>	Nex	t sta 0	16 N <sub>1</sub> N	[q	Output (out)
0 0 0 1 1 1 1 0		1 1 1 0 0 0 0 0	0 1 0 0 0 0 0 0		0 1 0 1



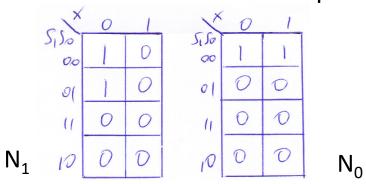
# **Exercise 2 (a better but longer approach)**

Which FSM corresponds to the given circuit?

1. Get the next state and output expressions

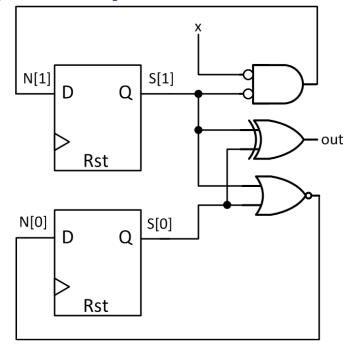
$$N_1 = S_1' x'$$
  
 $N_0 = (S_1 + S_0)' = S_1' S_0'$   
out =  $S_1 \land S_0$ 

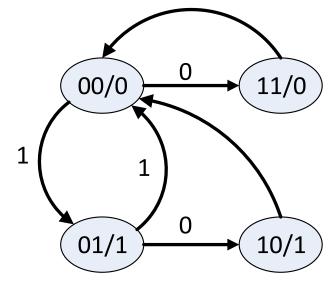
2. Generate the next state K-maps



3. Then get the state transition table

State S <sub>1</sub> S <sub>0</sub>	Next sta $x = 0$	te $N_1N_0$	Output (out)
0 0	1 1	0 1	0
0 1	1 0	0 0	1
1 1	0 0	0 0	0
1 0	0 0	0 0	1





So obviously Ans: C

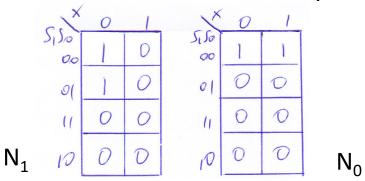
# **Exercise 2 (a better but longer approach)**

Which FSM corresponds to the given circuit?

1. Get the next state and output expressions

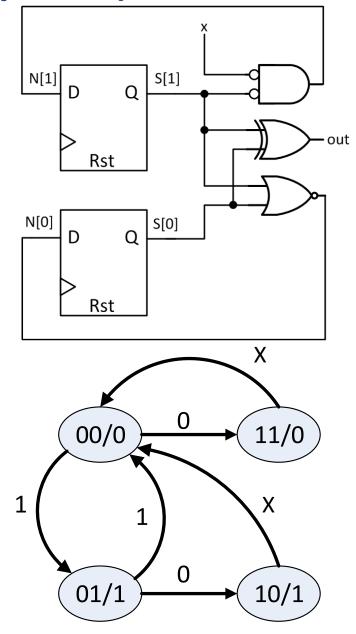
$$N_1 = S_1' x'$$
  
 $N_0 = (S_1 + S_0)' = S_1' S_0'$   
out =  $S_1 \land S_0$ 

2. Generate the next state K-maps



3. Then get the state transition table

State S <sub>1</sub> S <sub>0</sub>	Next sta $x = 0$	te $N_1N_0$ 1	Output (out)
0 0	1 1	0 1	0
0 1	1 0	0 0	1
1 1	0 0	0 0	0
1 0	0 0	0 0	1

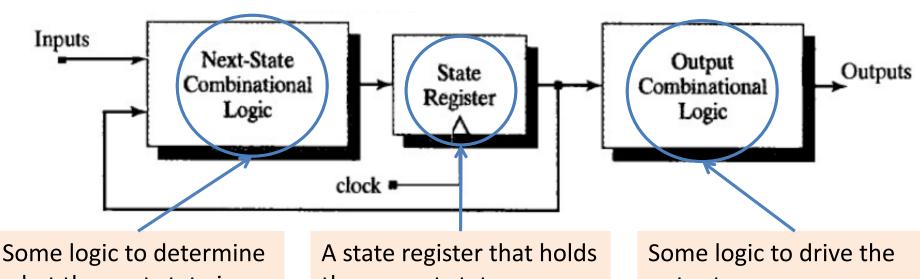


Note the unlabelled transitions are don't cares. Should probably label them as "X".

# **FSM Structure in Verilog**



- Typically Moore FSMs are easier to implement and to analyse
  - Mealy FSMs are typically more efficient (in terms of states)
- We can use this basic structure to implement any FSM



what the next state is

```
always @ *
begin
   case(st)
or continuous assign
```

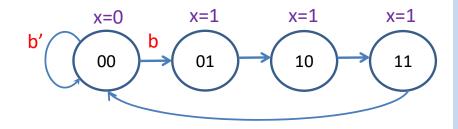
the current state

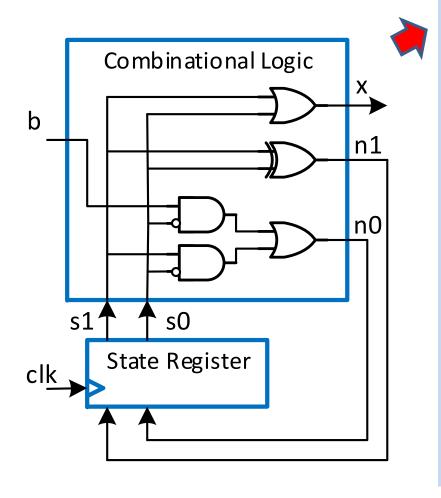
```
always @(posedge clk)
begin
```

output

Continuous assign or always @ \* block

# The previous Example 1





```
module FSM_E1 (input clk, rst, b,
                output x);
  wire n1, n0; //not really needed
  reg s1, s0;
  assign n1 = s1^s0;
  assign n0 = (\sim s0\&b) | (s1\&\sim s0);
  assign x = s1|s0;
  always @ (posedge clk)
  begin
    if(rst) begin
       s1 <= 1'b0;
       s0 <= 1'b0;
     end else begin
       s1 <= n1;
       s0 <= n0;
     end
  end
endmodule
```

# **Exercise 2 (revisited)**

Which of the following Verilog code segment is correct for the circuit shown?

```
assign N[1] = ~(x & S[1]);
assign N[0] = ~S[1] | ~S[0];
assign out = S[1] ^ S[0];

always @ (posedge clk)
begin
   if(Rst)
      N <= 2'd0;
else
   N <= S;
end</pre>
```

```
assign N[1] = ~x & ~S[1];
assign N[0] = ~(S[1] | S[0]);
assign out = S[1] ^ S[0];

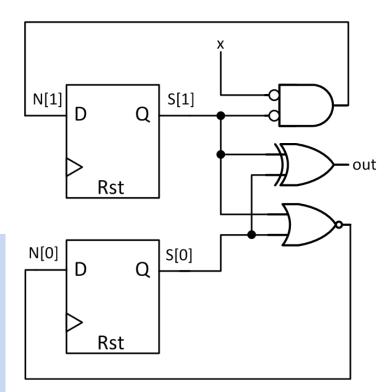
always @ (posedge clk)
begin
  if(Rst)
    N <= Z'b00;
  else
    N <= S;
and</pre>
```

```
assign N[1] = ~(x & S[1]);
assign N[0] = ~S[1] | ~S[0];
assign out = S[1] ^ S[0];

always @ (posedge clk)
begin
   if(Rst)
        S <= 2'b00;
   else
        S <= N;
end</pre>
```

```
assign N[1] = ~x & ~S[1];
assign N[0] = ~(S[1] | S[0]);
assign out = S[1] ^ S[0];

always @ (posedge clk)
begin
  if(Rst)
    S <= 2'd0;
else
    S <= N;
end</pre>
```



S is the state. So N (next state) is assigned to S on the clock edge. (Not N <= S;) Anyway, cannot assign to N using both **assign** and **always**.

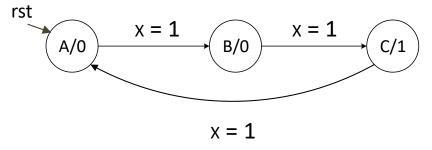
N[1] is determined by: N[1] = ~x & ~S[1]; So the only answer is D Ans: D

C

D

# Lecture 22 (Task 3)

Determine the Verilog code for the following FSM



First, is it a Moore or Mealy FSM?

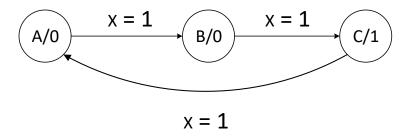
We can implement the FSM with 3 separate logic blocks

- 1. The synchronous state logic
- 2. The combinational next state logic
- 3. The combinational output logic

```
module fsm (input x, clk, rst, output reg u);
   parameter A=2'b00, B=2'b01, C=2'b10;
   reg [1:0] nst, st;
   always @ (posedge clk) // State logic
   begin
      if(rst) st <= A;
      else st <= nst;</pre>
   end
   always @ *
                             //Next state logic
   begin
                              No need for else statements
      nst = st;
                               as we have used a default
      case(st)
                                 assignment for nst.
        A: if(x) nst = B;
         B: if(x) st = C;
        C: if(x) n t = A;
      endcase
   end
   always @ *
                             // Output logic
   begin
          = 1'b0;
                         The default assignments
      case(st)
                         mean that we do not need
        C: u = 1'b1:
                         the default statement in
      endcase
   end
                         the case blocks
endmodule
```

# Lecture 22 (Task 3)

Determine the Verilog code for the following FSM



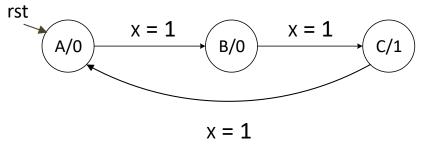
Usually, it is simpler to use just 2 separate logic blocks (even for a Moore FSM)

- 1. The synchronous state logic
- 2. The combinational next state and output logic

```
module fsm (input x, clk, rst, output reg u);
  parameter A=2'b00, B=2'b01, C=2'b10;
  reg [1:0] nst, st;
  always @ (posedge clk) // State logic
  begin
     if(rst) st <= A;</pre>
     else st <= nst;</pre>
  end
  always @ *
                             // NS & out logic
  begin
     nst = st;
          = 1'b0;
     case(st)
        A: if(x) nst = B;
        B: if(x) nst = C;
        C: begin
              if(x) nst = A;
              u = 1'b1;
           end
     endcase
  end
endmodule
```

# Lecture 22 (Task 3)

Determine the Verilog code for the following FSM



We could also start with the first implementation but use a continuous assignment for the output (u).

$$u = st[1]$$

We implement the synchronous state logic and the combinational next state logic using always blocks. Then the output logic using a continuous assignment.

```
module fsm (input x, clk, rst, output u);
  parameter A=2'b00, B=2'b01, C=2'b10;
  reg [1:0] nst, st;
  always @ (posedge clk) // State logic
  begin
     if(rst) st <= A;
     else st <= nst;</pre>
  end
  always @ *
                           //Next state logic
  begin
     nst = st;
     case(st)
        A: if(x) nst = B;
        B: if(x) nst = C;
        C: if(x) nst = A;
     endcase
  end
  assign u = st[1];
endmodule
```

This is probably the simplest and easiest way!



## CE/CZ1005 2018-2019 Sem 1 (Nov/Dec 2018)

- Q4(b) A simple vending machine dispenses healthy muesli bars. The vending machine accepts only 50 cent and one dollar coins and a muesli bar costs one dollar. If an excess amount is entered, for example 50 cents followed by one dollar, the transaction is rejected and all coins are returned.
  - (i) Sketch the Moore-style state transition diagram if the FSM has 2 inputs (*fifty and dollar*), and 3 outputs (*insert\_coin, dispense* and *reject\_money*). Use states *Ready*, *Dispense*, *Reject* and any other states necessary. A reset forces the FSM to the *Ready* state with output {*insert\_coin, dispense, reject\_money*} = {1, 0, 0}.

I assume that the FSM will return to the *Ready* state one clock cycle after *Dispense* or *Reject*. There are other assumptions that could be made.

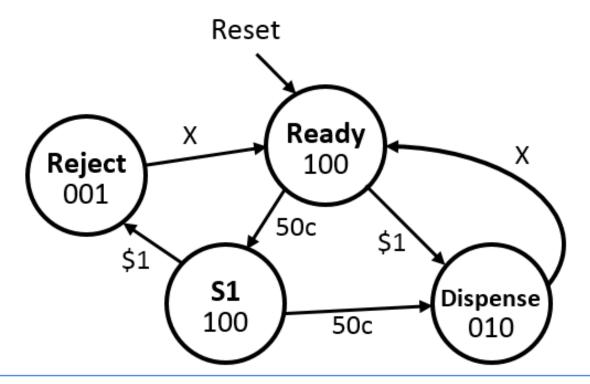
(5 marks)

(ii) Write the Verilog module for the FSM using an active low asynchronous reset. Use a separate always block for the next state and output logic.

(12 marks)



- Q4(b) A simple vending machine dispenses healthy muesli bars. The vending machine accepts only 50 cent and one dollar coins and a muesli bar costs one dollar. If an excess amount is entered, for example 50 cents followed by one dollar, the transaction is rejected and all coins are returned.
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## CE/CZ1005 2018-2019 Sem 1 (Nov/Dec 2018)

Q4(b) (ii) Write the Verilog module for the FSM using an active low asynchronous reset. Use a separate always block for the next state and output logic.

```
module vend fsm (input clk, rst, fifty, dollar, output reg insert, dispense, reject);
             //----state Constants-----
             parameter READY = 2'b00, DISPENSE = 2'b01, REJECT = 2'b10, S1 = 2'b11;
             //-----Internal Variables-----
             reg [1:0] state, next state;
             //----next state and output logic-----
             always @ * begin
                                                                  NOTE: The coin mechanism of a
                   insert = 1;
                                                                  typical vending machine only allows
Default
                   dispense = 0;
                                                                  coins to be put in one at a time.
Assignments
                   reject = 0;
                                                                   HENCE it is NOT possible to have the
                   next state = state;
                                                                  fifty and dollar inputs asserted at
                   case (state)
                                                                  the same time. Thus you do NOT
                         READY: begin
                                                                  need to use if ... else ... statements
                                if (fifty) next state = S1;
                                if (dollar) next state = DISPENSE;
                                // outputs set by default above
                         end
```



## CE/CZ1005 2018-2019 Sem 1 (Nov/Dec 2018) Q4(b) (ii) continued

```
DISPENSE: begin
                          next state = READY;
                          insert = 0;
                          dispense = 1;
                    end
                    REJECT : begin
                          next state = READY;
                          insert = 0;
                          reject = 1:
                    end
                   S1: begin
                          if (fifty) next state = DISPENSE;
                          if (dollar) next state = REJECT;
                          // outputs set by default above
                    end
             endcase
                          //default: no need as covered by default assignments above
                          // end of always block: next state and output logic
      end
      //-----Seq Logic-----
      always @ (posedge clk, negedge rst)
      begin
             if (~rst) state <= READY;</pre>
             else state <= next state;</pre>
      end
endmodule
                   // End of Module vending fsm
```



В

X=1

11

# CE/CZ1005 2018-2019 Sem 2 (April/May 2019)

Q4(b) (i) The figure shows a state transition diagram with 4 states.

Draw the state transition table

Current	Nex	Outpu	t	
State	B = 0	1	X	
A (00)	<u>A</u>	С	0	
B (01)	D	C	0	
C (10)	В	<u>C</u>	0	
D (11)	A	C	1	

01 B' 10 X=0

B'

X=0

00

Could also draw the excitation table as

Current	Nex	Output	
State	B = 0	1	$\bar{X}$
A (00)	<u>00</u>	10	0
B (01)	11	10	0
C (10)	01	<u>10</u>	0
D (11)	00	10	1

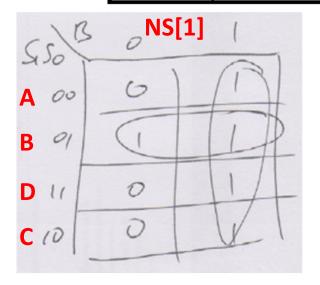
Input: B
Output: X

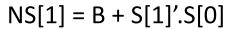


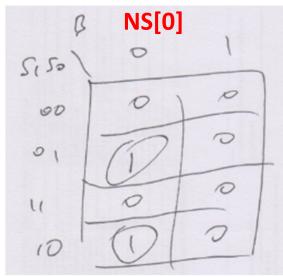
## CE/CZ1005 2018-2019 Sem 2 (April/May 2019)

(ii) Find the minimum SOP expressions for the next states and X. (current state is S[1] S[0])

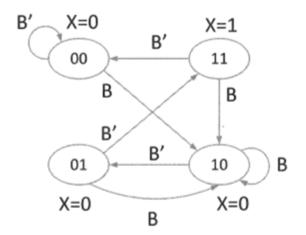
Current	Nex	Output	
State	B = 0	1	X
A (00)	<u>00</u>	10	0
B (01)	11	10	0
C (10)	01	<u>10</u>	0
D(11)	00	10	1







NS[0] = S[1]'.S[0].B' + S[1].S[0]'.B'



Input: B Output: X

$$X = S[1].S[0]$$



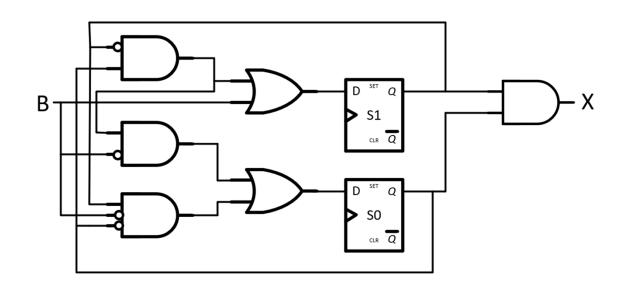
## CE/CZ1005 2018-2019 Sem 2 (April/May 2019)

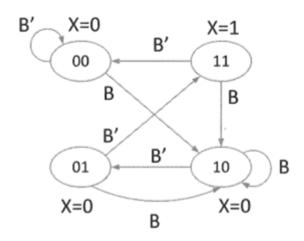
Based on the minimum SOP expressions, draw the circuit for the FSM. (iii)

Use D-FFs and gates.

$$NS[1] = B + S[1]'.S[0]$$
  $NS[0] = S[1]'.S[0].B' + S[1].S[0]'.B'$   $X = S[1].S[0]$ 

$$X = S[1].S[0]$$





Input: B Output: X



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I assume they mean "increments to" the next value, rather than counts the next value.

- Q4(b) A 3-bit odd/even up-counter has a 1-bit input *In* and a 3-bit output *Count*. When *In* is 0 it counts the next even value, and when *In* is 1 it counts the next odd value. The counter goes back to zero (initial state) only when it reaches the maximum odd count. It stays at the maximum even count state when *In* is 0.
  - (i) Draw the state transition diagram for the 3-bit odd/even upcounter.

(7 marks)

- (ii) Determine the output *Count* for the following input sequence: 1,1,0,1,0,1,0.

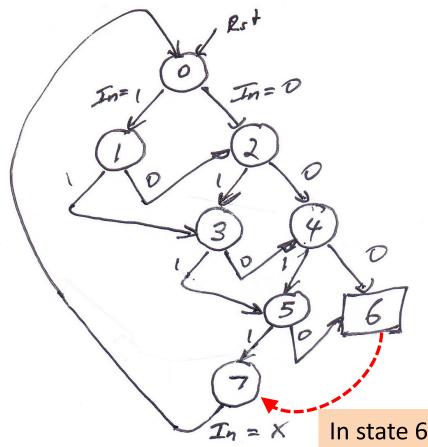
  I assume the counter starts from zero. (3 marks)
- (iii) Write the behavioral Verilog module for the odd/even up-counter discussed above using a CASE statement. Assume two additional inputs *CLK* and *RESET* for the sequential circuit implementation.

(10 marks)



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Q4(b) (i) If In=0, it should count 0, 2, 4, 6 and stop. If In=1, it should count 0, 1, 3, 5, 7, 0, 1, ...



(ii) Determine *Count* for the input seq.: 1, 1, 0, 1, 0, 1, 0 assuming we start at the reset state

Ans: Just follow the states in the state diagram

starting from S0, In=1 takes us to S1 then we go S3, S4, S5, S6, then S7 and lastly back to S0 so *Count* is 0, 1, 3, 4, 5, 6, 7, 0

In state 6, IF In=1 we SHOULD GO TO 7



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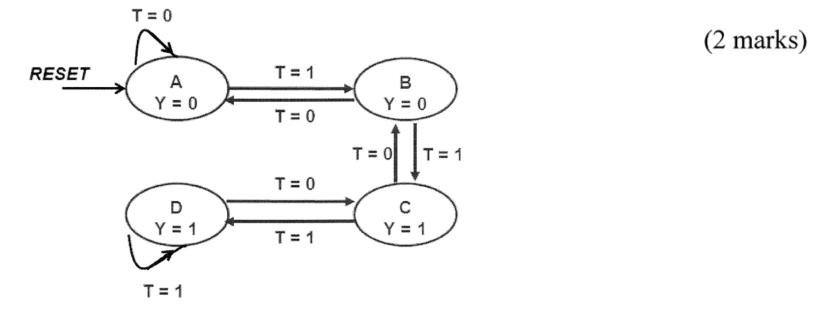
Q4(b) (iii) Write the behavioral Verilog module for the odd/even up-counter discussed above using a CASE statement. Assume two additional inputs *CLK* and *RESET* for the sequential circuit implementation.

As it says to use a case statement, I would just treat it as a FSM problem.



## CE/CZ1005 2017-2018 Semester 1 (Nov/Dec 2017)

- Q4(c) The state transition diagram of a Finite State Machine (FSM) is shown in Figure Q4b. There are three inputs RESET, CLK and T, and one output Y.
  - (i) Determine if it is a Mealy or Moore FSM. Justify your answer.



ANS: As the output Y is only determined by the current state, it is a **Moore** FSM.

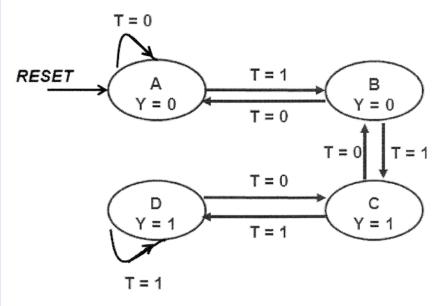


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Q4(c) (ii) Write the Verilog module for the FSM shown in Figure Q4b, using an asynchronous RESET.

```
module fsmQ4(input CLK, RESET, T, output reg Y);
   parameter A=2'b00,B=2'b01,C=2'b10,D=2'b11;
   reg [1:0] nst, st;
   always @ (posedge CLK or posedge RESET)
   begin
      if(RESET) st <= A;</pre>
      else st <= nst;</pre>
   end
   always @ * // NS & out logic
   begin
      nst = st; Y = 1'b0;
      case(st)
         A: if(T) nst = B;
         B: if(T) nst = C; else nst = A;
         C: begin if(T) nst = D; else nst = B;
               Y = 1'b1;
            end
         D: begin if(\simT) nst = C;
               Y = 1'b1;
            end
      endcase
   end
endmodule
```

(10 marks)





## CE/CZ1005 2016-2017 Semester 2 (Apr/May 2017)

Q4(a)

An autonomous vehicle developed in NTU moves students between five designated locations across the campus. The movement of the vehicle is controlled by a 1-bit input *MOVE* which is sent wirelessly from the control room. The 3-bit output *LOC* of the FSM shows the current location of the autonomous vehicle. Table Q4 shows the rules to perform a test run of this vehicle which will start from location SCSE.

Table Q4

	Next Location		
Current Location	MOVE = 0	MOVE = 1	
A SCSE	B Student Gym	D Innovation Center	
B Student Gym	A SCSE	C Auditorium	
C Auditorium	C Auditorium	E Library	
D Innovation Center	A SCSE	E Library	
E Library	D Innovation Center	<b>c</b> Auditorium	

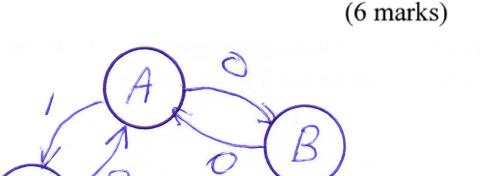
I am very lazy and do not like typing lots of text, so I will rename the states as A, B, C, D, & E.



## CE/CZ1005 2016-2017 Semester 2 (Apr/May 2017)

Q4(a) (i) Draw a Finite State Machine (FSM) for the autonomous vehicle.

Current State	Next State $M = 0$ 1		Output
State	$\mathbf{W} = 0$	1	
A	В	D	A
В	A	$\mathbf{C}$	В
C	C	Е	C
D	A	Е	D
Е	D	C	Е



(ii) What should be the input sequence such that the vehicle visits each location once and returns to SCSE? (State A)

(2 marks)

Ans: It must move A, B, C, E, D, A. There is no other solution. Thus, the input sequence is: **0 1 1 0 0**.



## CE/CZ1005 2016-2017 Semester 2 (Apr/May 2017)

Q4(a) (iii) Write the Verilog module for the FSM designed in Q4(a)(i) with inputs MOVE, RESET, CLK and output LOC. Assume the vehicle returns to SCSE when RESET is high.

```
module vfsm (input CLK, RST, M, output [2:0] LOC);
  parameter A=0, B=1, C=2, D=3, E=4;
  reg [2:0] nst, st;
  always @ (posedge CLK) // State logic
  begin
     if(RST) st <= A;</pre>
     else st <= nst;</pre>
  end
  always @ *
                            // NS logic
  begin
     nst = st;
     case(st)
        A: if(M) nst = D; else nst = B;
        B: if(M) nst = C; else nst = A;
        C: if(M) nst = E; else nst = C;
        D: if(M) nst = E; else nst = A;
        E: if(M) nst = C; else nst = D;
     endcase
  end
  assign LOC = st;  // Output Logic
endmodule
```

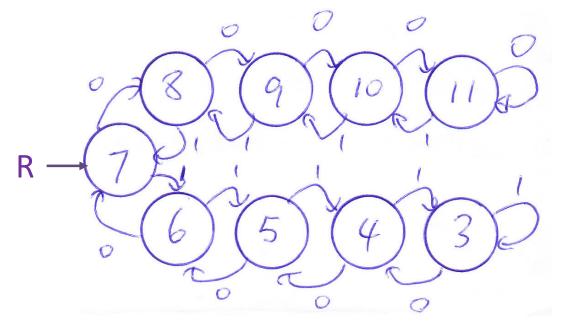
(10 marks)

Current	Next State		Output
State	$\mathbf{M} = 0$	1	_
A	В	D	A
В	A	C	В
C	C	Е	C
D	A	Е	D
Е	D	C	Е



## CE/CZ1005 2016-2017 Semester 1 (Nov/Dec 2016)

- Q4(b) You are required to design a special up-down counter. The counter never counts below 3 and above 11. The counter has a reset input (R) that sets it to 7 (that is, when R is 1), and a direction input (D) that determines whether it counts up or down (up if D is 0 and down if D is 1). If the counter is reset to 7 and D is 1, its successive states will then be 6, 5, 4, 3 and it will stay at 3 until R is 1 or D is 0. Similarly, if after reset D is 0, its successive states will then be 8, 9, 10, 11 and it will stay at 11 until R is 1 or D is 1.
  - (i) Draw the state transition diagram for the special up-down counter.



(7 marks)



## CE/CZ1005 2016-2017 Semester 1 (Nov/Dec 2016)

Q4(b) (ii) Write a Verilog module to implement the special up-down counter.

```
module spec_cnt (input clk, R, D,
               output reg [3:0] cout);
    always @(posedge clk)
    begin
        if (R) cout <= 7;
        else begin
            if (D && (cout != 3))
                cout <= cout -1;</pre>
            else if (!D && (cout != 11))
                cout <= cout +1;
        end
    end
endmodule
```

(8 marks)

While it would be possible to write this as a normal state machine using a case statement to move between states, it is probably much easier to just use a counter.

