NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM II (CA2)

MH1812 – Discrete Mathematics

March 2017 TIME ALLOWED: 40 minutes

Name:

Matric. no.: Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **FOUR (4)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1. **(25 marks)**

Solve the following linear recurrences:

(a) $a_n = 7a_{n-1}$, with initial condition $a_1 = 5$;

(b) $b_n = 20b_{n-1} - 51b_{n-2}$, with initial conditions $b_0 = 5$ and $b_1 = 6$.

QUESTION 2. **(20 marks)**

Prove that

$$\sum_{j=0}^{n} j!j = (n+1)! - 1 \qquad \forall n \in \mathbb{N}.$$

QUESTION 3. **(25 marks)**

Let $S = \{1, \ldots, n\}$ be a finite set and let $\mathcal{P}(S)$ denote the power set of S. Set $A = \{s \in \mathcal{P}(S) : |s| \text{ is even } \}$ and $B = \{s \in \mathcal{P}(S) : |s| \text{ is odd } \}$. Using the binomial theorem, or otherwise, prove that the cardinalities of A and B are equal, that is, prove that $|A| = |B|$.

QUESTION 4. **(30 marks)**

(a) Prove, for the sets A, B, C, D, that

$$(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D).$$

(b) Does equality hold? Justify your answer with either a proof or a counterexample.