Additional practice problems :

Consider the following system of equations.

$$\begin{cases} w + x + y + z = 6 \\ w \qquad + y + z = 4 \\ w \qquad + y \qquad = 2 \end{cases} \qquad (*)$$

(a) List the leading variables ____________ .

(b) List the free variables ____________ .

(c) The general solution of $(*)$ (expressed in terms of the free variables) is

$$(\_\_\_\_\_\_\_\_\_ , \_\_\_ , \_\_\_ , \_\_\_) .$$

(d) Suppose that a fourth equation $-2w + y = 5$ is included in the system $(*)$. What is the solution of the resulting system? Answer: (___ , ___ , ___ , ___).

(e) Suppose that instead of the equation in part (d), the equation $-2w - 2y = -3$ is included in the system $(*)$. Then what can you say about the solution(s) of the resulting system? Answer: __________________________ .

Consider the following system of equations:

$$\begin{cases} -m_1 x + y = b_1 \\ -m_2 x + y = b_2 \end{cases} \qquad (*)$$

(a) Prove that if $m_1 \neq m_2$, then $(*)$ has exactly one solution. What is it?

(b) Suppose that $m_1 = m_2$. Then under what conditions will $(*)$ be consistent?

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Q1: Consider augmented matrices:

$$A = \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 2 \end{array}\right) \qquad B = \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 2 & 2 & 4 & 6 & 5 \end{array}\right)$$

Mark each statement True or False regarding each matrix.

C) System with this matrix has no solution

D) $\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ is a solution for the system with this matrix

E) $\begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$ is a solution for the system with this matrix

F) System with this matrix has infinitely many solutions

G) Matrix has pivot position in every row

H) Columns of coefficient matrix span $\mathbb{R}^3$.

Which matrix will be matrix of linear trasformation L if $L(e_1) = 3e_1 + e_3$, $L(e_2) = e_1 + 2e_2$, $L(e_3) = -e_1 - e_2 + e_3$, where e_1, e_2 and e_3 - unit vector

A) $\begin{pmatrix} 1 & 1 & -1 \\ 3 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 3 & 1 & -1 \\ 0 & 2 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ C) $\begin{pmatrix} 7 & 10 & 1 \\ 8 & 11 & 2 \\ 1 & 12 & 3 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 \\ 5 & 8 \end{pmatrix}$

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③ Consider set $\left\{ \begin{pmatrix} 2-2t \\ -4t \\ t \end{pmatrix} \middle| \, t \in \mathbb{R} \right\} = V$

For every augmented matrix of a linear system determine if A is a set of solutions:

A) $\left(\begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$ B) $\left(\begin{array}{ccc|c} -2 & 0 & 2 & 1 \\ -4 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 \end{array}\right)$ C) $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & 4 & 0 \end{array}\right)$

D) $\left(\begin{array}{ccc|c} 2 & 2 & 0 & 1 \\ 1 & 0 & 0 & 4 \end{array}\right)$

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② Given the matrix $A = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$. Which of the following matrix multiplication terms correctly represent the LU factorisation for the matrix A above

A) $\begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

E) None of the above.

Q.7 Let V be a vector space and W be a subspace of V then

(a) $u+v=v+u, \forall\, u, v \in W$

(b) $\alpha u \in W, \forall\, u \in W, \alpha$ is scalar

(c) $\alpha(\beta u)=(\alpha\beta)u, \forall\, u \in W,\ \alpha, \beta$ are scalars

(d) All of the above

Q.9 Consider the following two subsets of vector space $V_3\,(R)$

$S_1=\{(x_1, x_2, x_3) \mid x_1+x_2+x_3 \geq 0\}$

$S_2=\{(x_1, x_2, x_3) \mid {x_1}^2+{x_2}^2+{x_3}^2 \leq 1\}$

then

(a) S_1 is a subspace of $V_3\,(R)$ but not S_2

(b) S_2 is a subspace of $V_3\,(R)$ but not S_1

(c) Both S_1 and S_2 are subspaces of $V_3\,(R)$

(d) Neither S_1 nor S_2 is a subspace of $V_3\,(R)$

Q.19 Let $S=\left\{(-3,5,2),(0,\ 2,-2),\left(8,-1,\frac{1}{2}\right),(-4,1,0)\right\}$ be a subset of vector space R^3 then

(a) S is LI since it has finite vectors

(b) S is LI since it has more than three vectors

(c) S is LD since it has finite vectors

(d) S is LD since it has more than three vectors

LI : linearly independent; LD : linearly dependent

Q.23 IF vectors (p, q) and (r, s) are LD then

(a) $pq=rs$

(b) $ps=qr$

(c) $pr=qs$

(d) $pr=-qs$