Exercises for Chapter 2

Exercise 11. Decide whether the following statements are propositions. Justify your answer.

1. $2 + 2 = 5$.
2. $2 + 2 = 4$.
3. $x = 3$.
4. Every week has a Sunday.
5. Have you read "Catch 22"?

Exercise 12. Show that

$$\neg(p \vee q) \equiv \neg p \wedge \neg q.$$

This is the second law of De Morgan.

Exercise 13. Show that the second absorption law $p \wedge (p \vee q) \equiv p$ holds.

Exercise 14. These two laws are called distributivity laws. Show that they hold:

1. Show that $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$.
2. Show that $(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$.

Exercise 15. Verify $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$ by

- constructing a truth table,
- developing a series of logical equivalences.

Exercise 16. Using a truth table, show that:

$$\neg q \to \neg p \equiv p \to q.$$

Exercise 17. Show that $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$.

Exercise 18. Are $(p \to q) \vee (q \to r)$ and $p \to r$ equivalent statements?

Exercise 19. Prove or disprove the following statement:

$$p \wedge (\neg(q \to r)) \equiv (p \to r).$$

Exercise 20. Show that this argument is valid:

$$\boxed{\neg p \to F; \therefore p.}$$

Exercise 21. Show that this argument is valid, where C denotes a contradiction.

$$\boxed{\neg p \to C; \therefore p.}$$

Exercise 22. 1. Prove or disprove the following statement:

$$(p \wedge q) \to p \equiv T.$$

2. Decide whether the following argument is valid.

$\neg d \to h;$
$\neg h \to d;$
$\therefore \neg d \vee \neg h$

Exercise 23. Determine whether the following argument is valid:

$$\begin{array}{l} \neg p \to r \wedge \neg s \\ t \to s \\ u \to \neg p \\ \neg w \\ u \vee w \\ \therefore t \to w. \end{array}$$

Exercise 24. Determine whether the following argument is valid:

$$\begin{array}{l} p \\ p \vee q \\ q \to (r \to s) \\ t \to r \\ \therefore \neg s \to \neg t. \end{array}$$

Exercise 25. Decide whether the following argument is valid:

$(p \vee q) \to \neg r;$
$\neg r \to s;$
$p;$
$\therefore s$