Exercises for Chapter 3

Exercise 26. Consider the predicates $M(x, y) =$ "x has sent an email to y", and $T(x, y) =$ "x has called y". The predicate variables x, y take values in the domain $D = \{\text{students in the class}\}$. Express these statements using symbolic logic.

1. There are at least two students in the class such that one student has sent the other an email, and the second student has called the first student.

2. There are some students in the class who have emailed everyone.

Exercise 27. Consider the predicate $C(x, y) =$ "x is enrolled in the class y", where x takes values in the domain $S = \{\text{students}\}$, and y takes values in the domain $D = \{\text{courses}\}$. Express each statement by an English sentence.

1. $\exists\, x \in S,\ C(x, \text{MH1812})$.

2. $\exists\, y \in D,\ C(\text{Carol}, y)$.

3. $\exists\, x \in S,\ (C(x, \text{MH1812}) \wedge C(x, \text{CZ2002}))$.

4. $\exists\, x \in S,\ \exists\, x' \in S,\ \forall y \in D,\ ((x \neq x') \wedge (C(x, y) \leftrightarrow C(x', y)))$.

Exercise 28. Consider the predicate $P(x, y, z) =$ "$xyz = 1$", for $x, y, z \in \mathbb{R}$, $x, y, z > 0$. What are the truth values of these statements? Justify your answer.

1. $\forall\, x,\ \forall\, y,\ \forall\, z,\ P(x, y, z)$.

2. $\exists\, x,\ \exists\, y,\ \exists\, z,\ P(x, y, z)$.

3. $\forall\, x,\ \forall\, y,\ \exists\, z,\ P(x, y, z)$.

4. $\exists\, x,\ \forall\, y,\ \forall\, z,\ P(x, y, z)$.

Exercise 29. Consider the domains $X = \{2, 3\}$ and $Y = \{2, 4, 6\}$, and the predicate $P(x, y) =$ "x divides y". What are the truth values of these statements:

a) $\exists x \in X,\ \forall y \in Y,\ P(x, y)$.

b) $\neg(\exists x \in X,\ \exists y \in Y,\ P(x,y))$.

Exercise 30. 1. Express

$$\neg(\forall\ x,\ \forall\ y,\ P(x,y))$$

in terms of existential quantification.

2. Express

$$\neg(\exists\ x,\ \exists\ y,\ P(x,y))$$

in terms of universal quantification.

Exercise 31. Consider the predicate $C(x,y) =$ "x is enrolled in the class y", where x takes values in the domain $S = \{\text{students}\}$, and y takes values in the domain $C = \{\text{courses}\}$. Form the negation of these statements:

1. $\exists\ x,\ (C(x, \text{MH1812}) \wedge C(x, \text{CZ2002}))$.
2. $\exists\ x\ \exists\ y,\ \forall z,\ ((x \neq y) \wedge (C(x,z) \leftrightarrow C(y,z)))$.

Exercise 32. Show that $\forall x \in D,\ P(x) \rightarrow Q(x)$ is equivalent to its contrapositive.

Exercise 33. Show that

$$\neg(\forall\ x, P(x) \rightarrow Q(x)) \equiv \exists x,\ P(x) \wedge \neg Q(x).$$

Exercise 34. Let y, z be positive integers. What is the truth value of "$\exists y,\ \exists z,\ (y = 2z \wedge (y \text{ is prime}))$".

Exercise 35. Consider the domains $X = \{2,4,6\}$ and $Y = \{2,3\}$, and the predicate $P(x,y) =$ "x is a multiple of y". What are the truth values of these statements:

1. $\forall x \in X,\ \exists y \in Y,\ P(x,y)$.
2. $\neg(\forall x \in X,\ \forall y \in Y,\ P(x,y))$.

Exercise 36. Write in symbolic logic "Every SCE student studies discrete mathematics. Jackson is an SCE student. Therefore Jackson studies discrete mathematics".

Exercise 37. Here is an optional exercise about universal generalization. Consider the following two premises: (1) for any number x, if $x > 1$ then $x - 1 > 0$, (2) every number in D is greated than 1. Show that therefore, for every number x in D, $x - 1 > 0$.