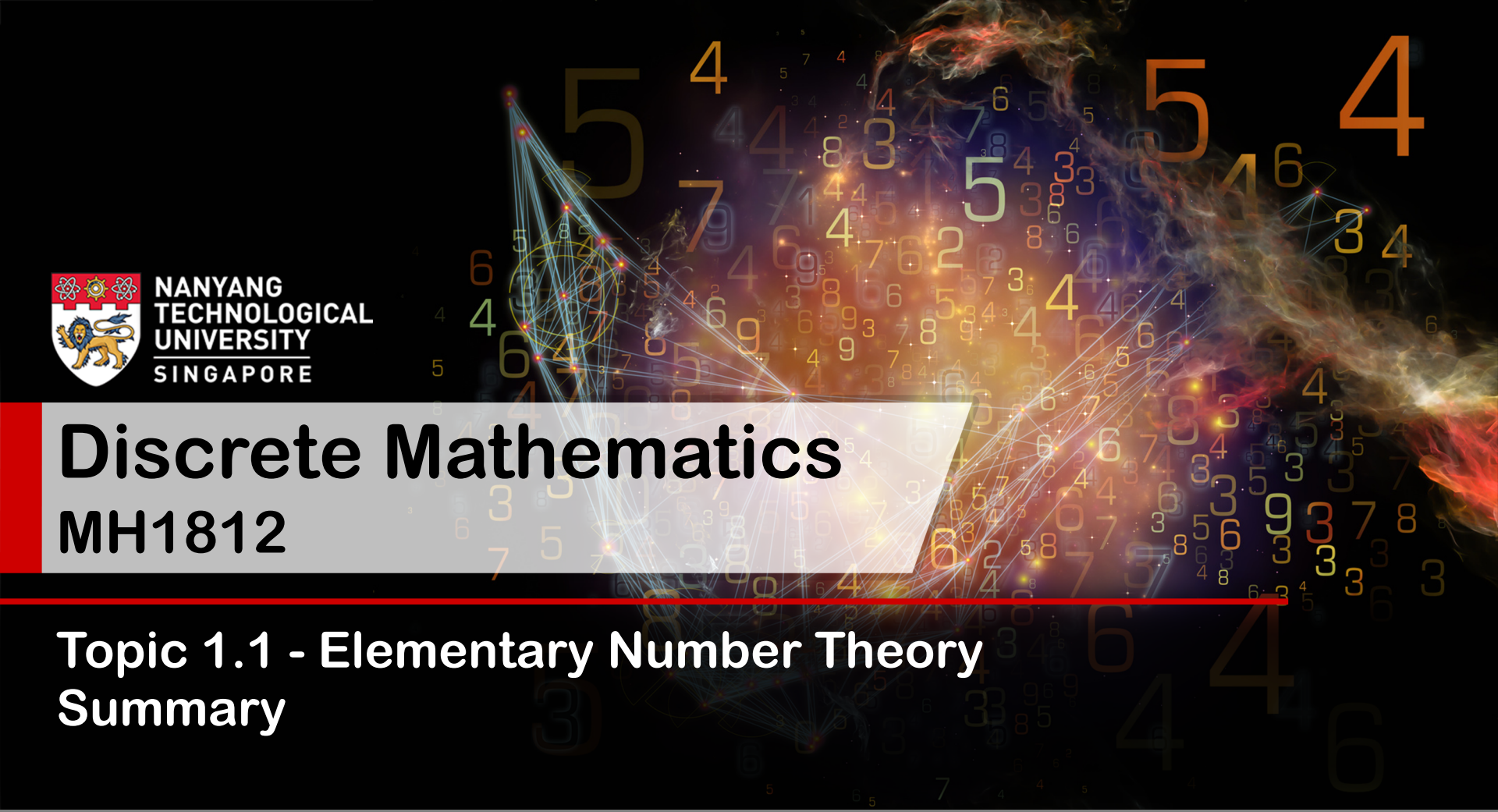


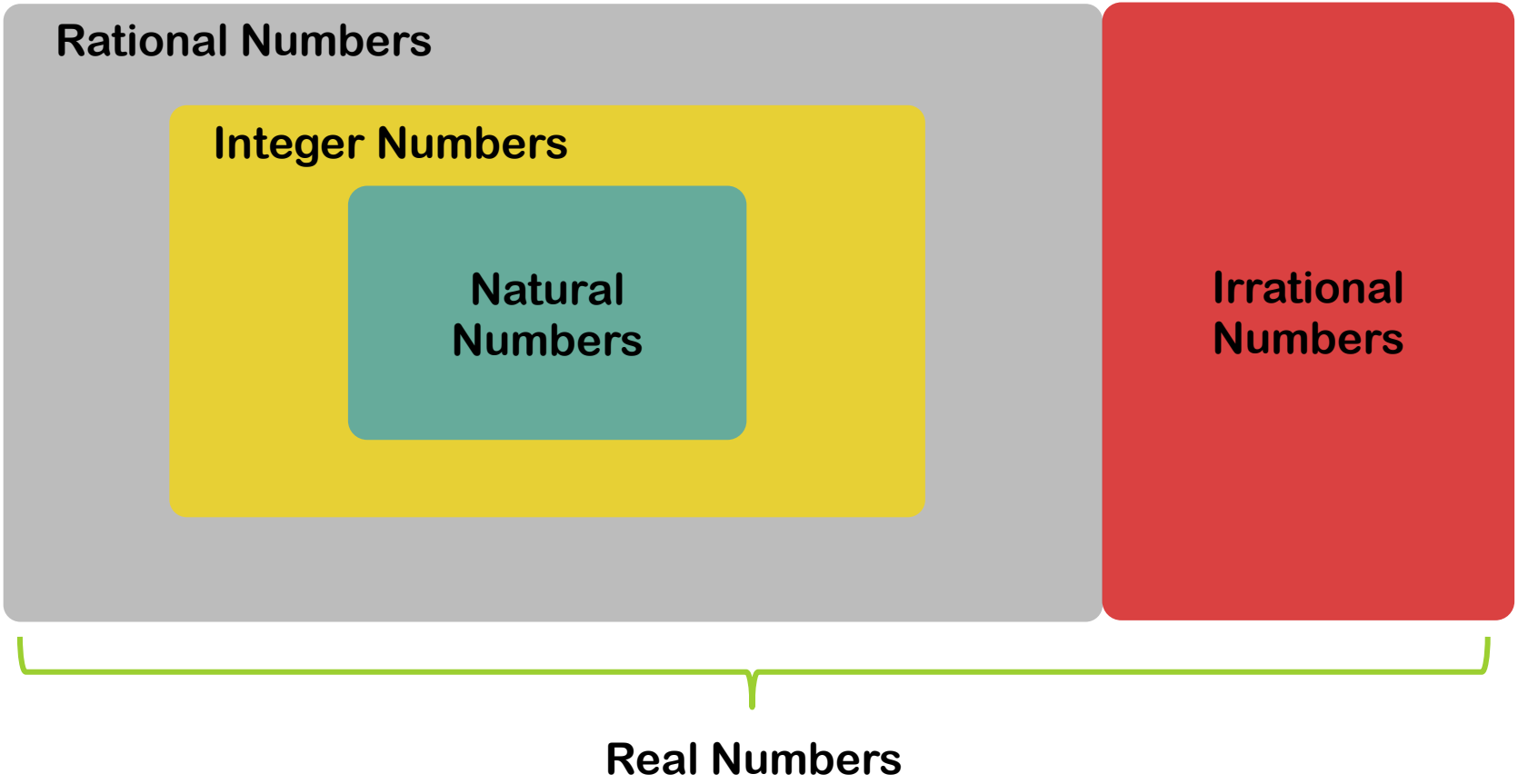
NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Discrete Mathematics

MH1812

Topic 1.1 - Elementary Number Theory Summary

Numbers: In a nutshell…



Euclidean Division: Modulo n

For a positive integer n, two integers a and b are said to be congruent modulo n, if $a - b$ is an integer multiple of n.

We write: $a \equiv b \pmod{n}$

If $a \equiv b \pmod{n}$**, then** $a - b = qn$ **and** $a = qn + b$.

Euclidean Division: Modular Arithmetic

$$a \equiv b \pmod{n} \leftrightarrow a = qn + b$$

Integers mod n can be represented as elements between 0 and $n - 1$: $(0, 1, 2, \ldots, n - 1)$

Addition mod n

$(a \bmod n) + (b \bmod n) \equiv (a + b) \bmod n$

Multiplication mod n

$(a \bmod n) * (b \bmod n) \equiv (a * b) \bmod n$

Euclidean Division: Modular Arithmetic

University Challenge Question:

What day of the week will it be 100 days after Monday?

Euclidean Division: Modular Arithmetic

Divisibility by 9:

Let N be a positive integer and let s be the sum of the digits of N.

Then $N \equiv s \pmod{9}$

Euclidean Division: Modular Arithmetic

Testing for squares:

Is $1234567 = x^2$ where x is an integer?

Euclidean Division: Modular Arithmetic

Testing for sums of squares:

Is $1234567 = x^2 + y^2$ where x and y are both integers?

Operator Closure

Consider a set S with an operator Δ.

Then S is closed under Δ if the result of the operation Δ on any two elements of S results in an element of S.

- To show S is *not* closed with respect to operator Δ:

  Just need to find (two) elements x and y in S such that $x \Delta y$ is not in S

- To show S *is* closed with respect to operator Δ:

  Need to show that $x \Delta y$ is in S for all x and y in S

Operator Closure

- *Is the set of even integers closed with respect to division?*

- *Is the set of odd integers closed with respect to multiplication?*

- *Is the set of prime numbers closed with respect to addition?*