Financial Markets and Expectations

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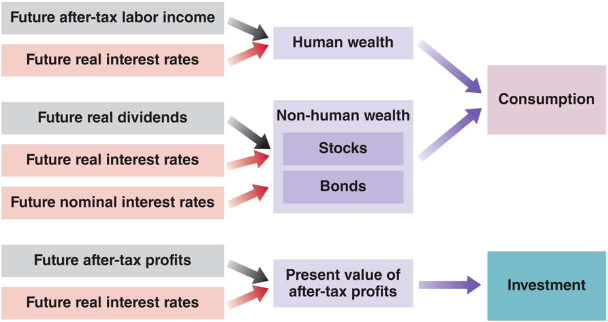
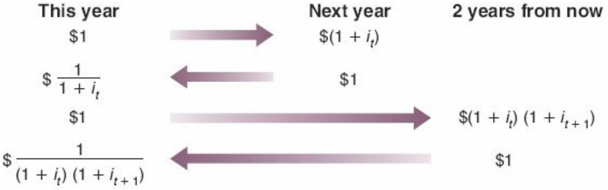
Motivation: Expectation matters!



Outline

- Expected Present Discounted Values
- Bond Prices and Bond Yields
- The Stock Market and Movements in Stock Prices

Present Discounted Values



- $\frac{1}{1+i_t}$ is the *present discounted value* of one dollar to be received next year. The rate at which you discount, in this case the nominal interest rate, i_t, is sometimes called the *discount rate*.
- The higher the nominal interest rate, the lower the value today if a dollar received next year.

A General Formula

- Consider a sequence of payment in dollars, starting today and continuing into the future: $\{\$z_t, \$z_{t+1}, \$z_{t+2}, ...\}$. The present discounted value of this sequence of payments is:

$$\$V_t = \$z_t + \frac{1}{1+i_t}\$z_{t+1} + \frac{1}{(1+i_t)(1+i_{t+1})}\$z_{t+2} + ...$$

- In practice, there are uncertainty on future payments and future interest rates. Decisions must be based upon expectations. *The expected present discounted value*(The present value) of this expected sequence of payments is given by:

$$\$V_t = \$z_t + \frac{1}{1+i_t}\$z^e_{t+1} + \frac{1}{(1+i_t)(1+i^e_{t+1})}\$z^e_{t+2} + ...$$

Special Cases

- For constant interest rates and constant payments z:

$$
\begin{aligned}
\$V_t &= \$z\left[1 + \frac{1}{1+i} + \ldots + \frac{1}{(1+i)^{n-1}}\right] \\
&= \$z\frac{1 - [1/(1+i)^n]}{[1 - 1/(1+i)]}
\end{aligned}
$$

- For constant interest rates and payment forever: $v_t = \frac{z(1+i)}{i}$.
- If $i = 0$, then the present discounted value of a sequence of expected payments is just the sum of those expected payments.

The Present Value Expressed in Nominal v.s. Real Terms

- The present value of a sequence of real payments(payments in terms of a basket of goods rather than in terms of dollars):

$$V_t = z_t + \frac{1}{1+r_t} z^e_{t+1} + \frac{1}{(1+r_t)(1+r^e_{t+1})} z^e_{t+2} + ...$$

- These two ways to compute the present value of a sequence of payments turn out to be equivalent:

$$\frac{\$V_t}{P_t} = V_t$$

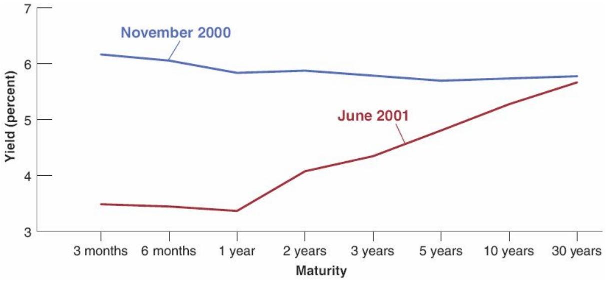
- Which way is more helpful depends on the context.

Bond Prices and Bond Yields

- **Maturity**: The length of time over which the bond promises to make payments to the holder of the bond.
- **Yield to maturity** or **yield**: The interest rates associated with bonds of different maturities
- **Short-term interest rates**: Yields on bonds with a short maturity, typically a year or less
- **Long-term interest rates**: Yields on bonds with a longer maturity than a year

Yield Curves

Term structure of interest rates or yield curve: The relation between maturity and yield



Bond Prices as Present Values

- The price of a one-year bond that promises to pay \$100 next year:

$$\$p_{1t} = \frac{\$100}{1 + i_{1t}}$$

- The price of a two-year bond that promises to pay \$100 in two years:

$$\$p_{2t} = \frac{\$100}{(1 + i_{1t})(1 + i^e_{1,t+1})}$$

Arbitrage and Bond Prices

- **Arbitrage:** The expected returns on two assets must be equal.
- **Expectations hypothesis:** Investors care only about the expected returns and do not care about risk.
- Under the expectation hypothesis, two bonds must offer the same expected one-year return:

$$1 + i_{1t} = \frac{\$p^e_{1,t+1}}{\$p_{2t}}.$$

That is,

$$\$p_{2t} = \frac{\$p^e_{1,t+1}}{1 + i_{1t}},$$

which means that the price of a two-year bond today is the present value of the expected price of the bond next year.

Arbitrage and Bond Prices

- The expected price of one-year bonds next year with a payment of \$100:

$$\$p^e_{1,t+1} = \frac{100}{(1 + i^e_{t+1})}$$

so that

$$\$p_{2t} = \frac{\$p^e_{1,t+1}}{1 + i_{1t}} = \frac{100}{(1 + i_t)(1 + i^e_{t+1})}$$

which is the same as before. Arbitrage between one- and two-year bonds implies that the price of two-year bonds is the present value of the payment in two years.

From Bond Prices to Bond Yields

- The *yield to maturity* on an n-year bond is the constant annual interest rate that makes the bond price today equal to the present value of future payments on the bond.
- The yield to maturity on a two-year bond that satisfies:

$$\$p_{2t} = \frac{\$100}{(1 + i_{2t})^2}$$

- Together with $\$p_{2t} = \frac{\$p^e_{1,t+1}}{1+i_{1t}} = \frac{\$100}{(1+i_{1t})(1+i^e_{1,t+1})}$, we can show

$$i_{2t} \approx \frac{1}{2}(i_{1t} + i^e_{1,t+1}).$$

which means that the two-year interest rate is (approximately) the average of the current one-year interest rate and next years expected one-year interest rate.

Re-introducing Risk

- Two-year bond is risky as you do not know the price at which you will sell the bond in a year. Assume a risk premium x on the two-year bond:

$$1 + i_{1t} + x = \frac{\$p^e_{1,t+1}}{\$p_{2t}}$$

This implies:

$$\begin{aligned} \$p_{2t} &= \frac{\$p^e_{1,t+1}}{(1 + i_{1t} + x)} \\ &= \frac{\$100}{(1 + i^e_{1,t+1})(1 + i_{1t} + x)} \end{aligned}$$

- the two-year yield can thus be approximated as:

$$i_{2t} \approx \frac{1}{2}(i_{1t} + i^e_{1,t+1} + x),$$

which is the average of the current and expected one-year rate plus a risk premium.

Interpreting the Yield Curve

- Typically, the yield curve should be slightly upward sloping reflecting the higher risk involved in holding longer maturity bonds.
- Downward sloping yield curve suggests that investors expect interest rates to go down slightly over time.
- At the end of Nov 2000, the US economy was sloing down. Investors expected a smooth landing from the FED slowing decreasing the policy rate.
- By June 2001, investors expect that the economy should start to recover and the FED would start increasing the policy rate.

Implication for Stock Prices

- The expected rate of return from holding stocks for one year is the same as the rate of return on one-year bonds plus the equity premium x :

$$\frac{\$D^e_{t+1} + \$Q^e_{t+1}}{\$Q_t} = 1 + i_{1t} + x,$$

in which $\$Q_t$ is the ex-divident stock price, and $\$D^e_{t+1}$ is the expected divided next year. So

$$\$Q_t = \underbrace{\frac{\$D^e_{t+1}}{1 + i_{1t} + x} + \frac{\$Q^e_{t+1}}{1 + i_{1t} + x}}_{\text{present value of future payment by holding the stock for one period}}$$

Since $\$Q^e_{t+1} = \frac{\$D^e_{t+2}}{1+i^e_{1,t+1}+x} + \frac{\$Q^e_{t+2}}{1+i^e_{1,t+1}+x}$, and thus we also have

$$\$Q_t = \underbrace{\frac{\$D^e_{t+1}}{1 + i_{1t} + x} + \frac{\$D^e_{t+2}}{(1 + i_{1t} + x)(1 + i^e_{1,t+1} + x)} + \frac{\$Q^e_{t+2}}{(1 + i_{1t} + x)(1 + i^e_{1t+1} + x)}}_{\text{present value of future payment by holding the stock for two period}}$$

Implication for Stock Prices

- Finally, the stock price should also equal to the present value of future payment by holding the stock forever:

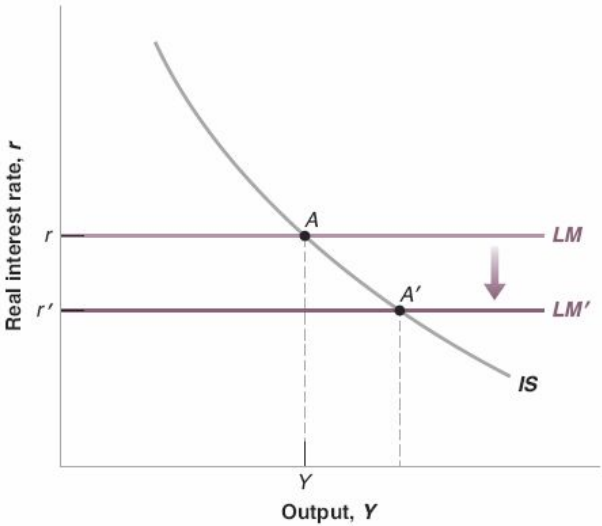
$$\$Q_t = \frac{\$D^e_{t+1}}{(1+i_{1t}+x)} + \frac{\$D^e_{t+2}}{(1+i_{1t}+x)(1+i^e_{1,t+1}+x)} + ...$$
$$+ \frac{\$D^e_{t+n}}{(1+i_{1t}+x)...(1+i^e_{1,t+n-1}+x)} + ...$$

- Replacing the nominal interest rates with the real interest rates, then the real stock price is:

$$Q_t = \frac{D^e_{t+1}}{(1+r_{1t}+x)} + \frac{D^e_{t+2}}{(1+r_{1t}+x)(1+r^e_{1,t+1}+x)} + ...$$

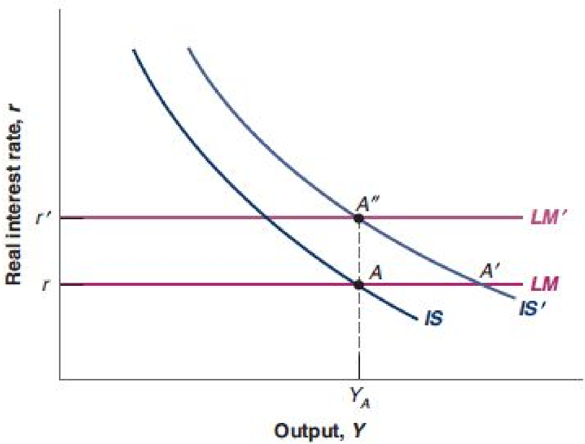
- Implications:
  - Higher expected future nominal(real) dividends lead to a higher nominal(real) stock price.
  - Higher current and expected future one-year nominal(real) interest rates lead to a lower nominal(real) stock price.
  - A higher equity premium leads to a lower stock price.

Monetary Policy and the Stock Market



What it does to the stock market depends on whether or not financial markets anticipated the monetary expansion.

An Increase in Consumer Spending and the Stock Market



What happens to the stock market depends on the reaction of the Fed.

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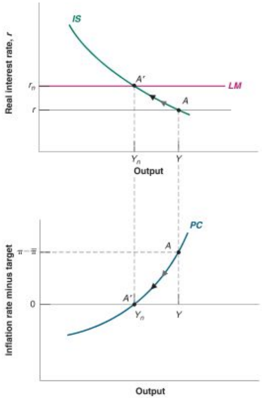
Output Gap and Inflation Gap



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