MH1820 Introduction to Probability and Statistical Methods
Tutorial 5 (Week 6)

Note: You may use the tables provided in NTULearn > Content > TABLES.pdf for calculations using the standard normal and chi-square distribution. No interpolation is required for calculating $\Phi(z)$, i.e. you may round the number to 2 decimal places before using the table. E.g. $\Phi(0.7897)$ can be approximated by just $\Phi(0.79)$ etc.

Problem 1 (Normal distribution)

(a) If X is normally distributed with a mean of 6 and a variance of 25, find

(i) $\mathbb{P}(6 \leq X \leq 12)$

(ii) $\mathbb{P}(|X - 6| < 15)$

(iii) $\mathbb{P}(X > 21)$

(b) If $X \sim N(650, 625)$, find the constant c such that $\mathbb{P}(|X - 650| \leq c) = 0.9544$.

Problem 2 (Normal distribution) Find the distribution of $W = X^2$ when

(i) X is $N(0, 4)$

(ii) X is $N(0, \sigma^2)$

Problem 3 (Normal distribution) A candy maker produces mints that have a label weight of 20.4 grams. Assume that the distribution of the weights of these mints is $N(21.37, 0.16)$. Suppose that 15 mints are selected independently and weighted. Let Y equal the number of these mints that weigh less than 20.857 grams. Find $\mathbb{P}(Y \leq 2)$.

Problem 4 (Normal distribution) The price of an asset is such that its distribution is found by $Y = e^X$, where X is $N(10, 1)$. Find the CDF and PDF of X, and compute $\mathbb{P}(10,000 < Y < 20,000)$.
Note: $F(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(e^X \leq y) = \mathbb{P}(X \leq \ln y)$. For CDF, you can leave your answer in terms of $\Phi(\cdot)$.

Problem 5 (Chi-square distribution) If X is $\chi^2(17)$, find

(i) $\mathbb{P}(X < 7.564)$

(ii) $\mathbb{P}(X > 27.59)$

(iii) $\mathbb{P}(6.408 < X < 27.59)$

(iv) $\chi^2_{0.95}(17)$

(v) $\chi^2_{0.025}(17)$

Problem 6 (Chi-square distribution)
Cars arrive at a tollbooth at a mean rate of 5 cars every 10 minutes according to a Poisson distribution. Let X be the time (in minutes) that the toll collector will have to wait before collecting the **eighth** toll.

(i) Find $\mathbb{E}[X]$ and the standard deviation of X.

(ii) Find the probability that the toll collector will have to wait longer than 26.30 minutes before collecting the eighth toll.

Answer Keys. **1(a)(i).** 0.3849 **(ii).** 0.9974 **(iii).** 0.0013 **1(b).** 50 **2(i).** CDF: $F(w) = 2\Phi\left(\frac{\sqrt{w}}{2}\right) - 1$, PDF: $f(w) = \frac{1}{2\sqrt{2\pi}} w^{-1/2} e^{-w/8}$ **2(ii).** CDF: $F(w) = 2\Phi\left(\frac{\sqrt{w}}{\sigma}\right) - 1$, PDF: $f(w) = \frac{1}{\sigma\sqrt{2\pi}} w^{-1/2} e^{-\frac{w}{2\sigma^2}}$ **3.** 0.815 **4.** 0.25 **5(i).** 0.025 **(ii).** 0.05 **(iii).** 0.94 **(iv).** 8.672 **(v).** 30.19 **6(i).** 16, 5.66 **(ii).** 0.05