MH1812 Discrete Mathematics: Quiz (CA) 1

Name: Tutorial Group:

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There are 3 (THREE) questions, please try all of them, and justify all your answers! Best of luck!

Question 1 (30 points)

a) Compute $1234567890 + 1234567891 + 1234567892 + 1234567893$ modulo 4 (10 points).

b) Consider the set S of multiples of 5 that is $S = \{\ldots, -10, -5, 0, 5, 10, \ldots\}$.

- Is the set S closed under addition? (10 points).
- Is the set S closed under multiplication? (10 points).

a) Note that $1234567891 = 1234567890+1$, $1234567892 = 1234567890+2$ and $1234567893 = 1234567890+3$. Then

$$1234567890 + 1234567891 + 1234567892 + 1234567893 = 4 \cdot 1234567890 + 6 \equiv 6 \mod 4 \equiv 2 \mod 4.$$

b) Let us take two multiples of 5, say $5a$ and $5b$ for some integers a, b. For addition

$$5a + 5b = 5(a + b)$$

which is a multiple of 5, and for multiplication

$$5a \cdot 5b = 5(5ab)$$

which is also a multiple of 5. So the set of multiples of 5 is closed both under addition and multiplication.

Question 2 (40 points)

a) Prove or disprove the following statement (20 points):

$$[(p \to q) \wedge \neg q] \to \neg p \equiv T.$$

b) Decide whether the following argument is valid (20 points):

$\neg p \wedge q;$
$r \to p;$
$\neg r \to s;$
$s \to t;$
$\therefore t$

a) One should prove the statement. Using the conversion theorem, and distributivity

$$(p \to q) \wedge \neg q \equiv (\neg p \vee q) \wedge \neg q \equiv (\neg p \wedge \neg q) \vee (q \wedge \neg q)$$

where $(q \wedge \neg q) \equiv F$ thus

$$(p \to q) \wedge \neg q \equiv \neg p \wedge \neg q,$$

and using again the conversion theorem

$$[(p \to q) \wedge \neg q] \to \neg p \equiv \neg(\neg p \wedge \neg q) \vee \neg p.$$

Using De Morgan's Law, we get

$$p \vee q \vee \neg p \equiv T.$$

b) The argument is valid. First

$\neg p \wedge q;$
$\therefore \neg p$

Then

$r \to p;$
$\neg p;$
$\therefore \neg r$

Next

$\neg r \to s;$
$\neg r;$
$\therefore s$

Finally

$s \to t;$
$s;$
$\therefore t$

Question 3 (20 points)

Consider the domain $\mathbb{R}$, and the predicate $P(x, y) =$ "$x^2 - y^2 > 0$". What are the truth values of these statements:

a) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ P(x, y)$ (10 points).

b) $\neg(\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ P(x, y))$ (10 points).

a) This is false. Suppose $x = 0$. Then $P(0, y) =$ "$-y^2 > 0$", and there does not exist $y \in \mathbb{R}$ such that $-y^2 > 0$.

b) This is true. One way to look at it is that the truth value of $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ P(x, y)$ is false. There cannot be a fixed x such that $x^2 - y^2 > 0$ for any choice of y. No matter how big x^2, one can always take y^2 to be larger than this given x^2. Thus the negation of false is true. Another way is to write

$$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}, \neg P(x, y),$$

or alternatively

$$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}, x^2 - y^2 \leq 0.$$

This is true. If $x \geq 0$, choose $y = x + 1$ for example. Then $x^2 - (x+1)^2 = -2x - 1 \leq 0$. If $x < 0$, then choose $y = x - 1$ for example. Then $x^2 - (x - 1)^2 = 2x - 1 \leq 0$ since x is negative.