Exercises for Chapter 8

Exercise 70. Consider the sets $A = \{1, 2\}$, $B = \{1, 2, 3\}$ and the relation $(x, y) \in R \iff (x-y)$ is even. Compute the inverse relation R^{-1}. Compute its matrix representation.

Exercise 71. Consider the sets $A = \{2, 3, 4\}$, $B = \{2, 6, 8\}$ and the relation $(x, y) \in R \iff x \mid y$. Compute the matrix of the inverse relation R^{-1}.

Exercise 72. Let R be a relation from $\mathbb{Z}$ to $\mathbb{Z}$ defined by $xRy \leftrightarrow 2|(x - y)$. Show that if n is odd, then n is related to 1.

Exercise 73. This exercise is about composing relations.

1. Consider the sets $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$, $C = \{c_1, c_2, c_3\}$ with the following relations R from A to B, and S from B to C:

$$R = \{(a_1, b_1), (a_1, b_2)\},\ S = \{(b_1, c_1), (b_2, c_1), (b_1, c_3), (b_2, c_2)\}.$$

   What is the matrix of $R \circ S$?

2. In general, what is the matrix of $R \circ S$?

Exercise 74. Consider the relation R on $\mathbb{Z}$, given by $aRb \iff a - b$ divisible by n. Is it symmetric?

Exercise 75. Consider a relation R on any set A. Show that R symmetric if and only if $R = R^{-1}$.

Exercise 76. Consider the set $A = \{a, b, c, d\}$ and the relation

$$R = \{(a, a), (a, b), (a, d), (b, a), (b, b), (c, c), (d, a), (d, d)\}.$$

Is this relation reflexive? symmetric? transitive?

Exercise 77. Consider the set $A = \{0, 1, 2\}$ and the relation $R = \{(0, 2), (1, 2), (2, 0)\}$. Is R antisymmetric?

Exercise 78. Are symmetry and antisymmetry mutually exclusive?

Exercise 79. Consider the relation R given by divisibility on positive integers, that is $xRy \leftrightarrow x|y$. Is this relation reflexive? symmetric? antisymmetric? transitive? What if the relation R is now defined over non-zero integers instead?

Exercise 80. Consider the set $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. Show that the relation $xRy \leftrightarrow 2|(x - y)$ is an equivalence relation.

Exercise 81. Show that given a set A and an equivalence relation R on A, then the equivalence classes of R partition A.

Exercise 82. Consider the set $A = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and the relation

$$xRy \leftrightarrow \exists c \in \mathbb{Z},\ y = cx.$$

Is R an equivalence relation? is R a partial order?