**NTU SSS Economics HE1001**
**Tutorial 4 (Week 5): Demand**

1. Mark has the following utility function on goods x and y:

$$U(x; y) = \sqrt{x} + \sqrt{y}.$$

Prices are $p_y = 1$; $p_x = 5$:

a. Assume that Mark's income is I = 60. What is his optimal consumption of the two goods?

**Answer**:

$MU_x = \partial U(x,y)/ \partial x = (½)x^{-½}$
$MU_y = \partial U(x,y)/ \partial y = (½)y^{-½}$

$MRS = MUx/MUy = (x^{-½})/( y^{-½}) = (\sqrt{y})/(\sqrt{x})$.

if I = 60, the optimal bundle: $(\sqrt{y})/(\sqrt{x}) = 5/1$, thus:

$$y=25x$$

The budget line is:

$$5x + y =60$$

So:

$$5x + 25x =60$$
$$30x = 60$$
$$x=2$$
$$y=50$$

b. Derive Mark's demand for x, given that I = 60 and $p_y = 1$.

**Answer**:

Mark's demand for x, given that I = 60 and $p_y = 1$ also comes from the equality

$$MRS = p_x/p_y$$
$$MRS = MUx/MUy = (\sqrt{y})/(\sqrt{x}) = \sqrt{(y/x)}.$$
$$\sqrt{(y/x)} = p_x/p_y$$
$$\sqrt{(y/x)} = p_x$$
$$(y/x) = p_x^2$$
$$y=p_x^2x$$

The budget line (substitute y)

$$p_xx + y =60$$
$$p_xx + p_x^2x =60$$
$$x(p_x+ p_x^2)=60$$

Thus, the quantity demanded for good x is (solve for x as a function of and :

$$x=60/(p_x+ p_x^2)$$

$$x = 60 / [p_x(1+ p_x)]$$

When $p_x$ increases the demand for x decreases.

c. If $p_x = p_y = 1$, what is the expression for Mark's Engel curve for good x. Is it a normal good?

**Answer**:

We have

$$MRS = MUx/MUy = (\sqrt{y}/\sqrt{x})$$
$$MRS= p_x/p_y$$
$$(y/x)=1$$
$$y=x$$

The budget line:

$$x+y=I$$

Thus, the equation for the Engel Curve can be derived as:

$$x=I/2$$

Because demand for x increases when income (I) increases, good x is a **normal good**.

2. Samuel consumes two goods, pizza (X) and hamburger (Y). His utility function is as follows:

$$U(X, Y) = XY$$

Samuel has an income (I) of S$120 and the price of pizza ($P_X$) and hamburger ($P_Y$) are both S$1.

a. What is Samuel's budget line?

**Answer**:

Budget line: $120 = X + Y$

b. What quantities of X and Y will maximize Samuel's utility?

**Answer**:

$$MUx/MUy = Px/Py$$

Substituting for MUx, MUy, Px, and Py yields:

$$Y/X = 1 \text{ or } X = Y$$

Substituting into the budget line:

$$120 = X + Y = 2X$$
$$X = 60$$
$$Y = 60$$

c. Holding Samuel's income and $P_Y$ constant at $$120 and $1 respectively, what is Samuel's demand curve for pizza?

Answer:

Rewriting the budget line:

$$120 = P_xX + Y$$

$$MU_x/MU_y = P_x/P_y$$

Substituting for MUx, MUy, Px, and Py yields:

$$Y/X = P_x/1 \text{ or } Y = P_xX$$

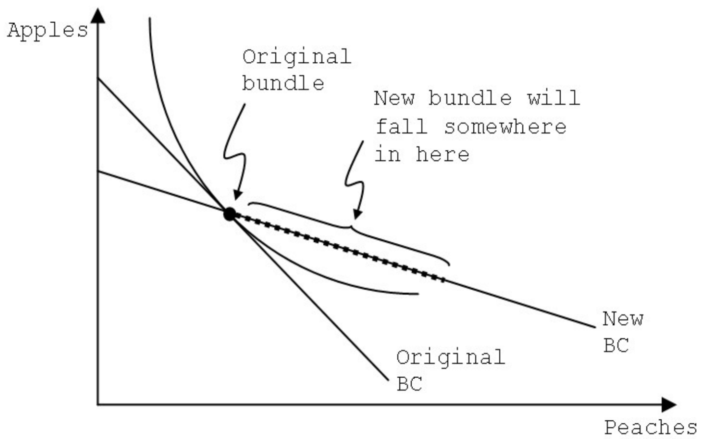
Using the budget line:

$$120 = P_xX + P_yY = P_xX + Y = 2P_xX$$

$$X = 60(1/P_x)$$

3. Suppose you consume only apples and peaches. One day the price of apples increases and the price of peaches decreases, and you find that you can still afford to buy the original bundle (initial combination of peaches and apples that you were buying before both prices change). The price changes leave you neither better nor worse off. Is this last statement true or false? Explain why.

**Answer:**




False, it leaves you better off. The relative price change implies that the original bundle is no longer optimal, and improvements can be made by consuming less apples and more peaches. This can be shown by drawing the new budget line, which MUST AT LEAST CUT THROUGH THE ORIGINAL BUNDLE (or lie above it), since "you can still afford to buy". In turn, if indifference curves are convex to the origin and follow the standard preference assumptions, then the new budget line will have a segment that lies above the original indifference curve and therefore THERE MUST BE INDIFFERENCE CURVES THAT LIE ABOVE IT. If the consumer can move to a higher indifference curve, then the person will be better off. In the question, the new budget line is flatter than the convex indifference curve that is tangent to the old budget line, and cutting through this line means there is a segment above the old indifference curve that allows one to move to a higher indifference curve tangent to the new budget line. This also apples if you draw apples on the horizontal axis and peaches on the vertical axis.

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