HE2002 Tutorial 12 Solution

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5.

(a) The present value of the consol is given as $\frac{\$z}{i} = 100/0.1 = \$1000$.

(b),(c) The formular that can be applied is:

$$\$V = \frac{\$z}{1+i}\frac{1-\frac{1}{(1+i)^n}}{1-\frac{1}{1+i}}$$

The results for different i can be shown in the following table:

i	10 years	20 years	30 years	60 years
0.01	614.46	851.36	942.69	996.72
0.02	898.25	1635.14	2239.65	3476.09
0.05	772.17	1246.22	1537.25	1892.93

Table 1: The Present Value

7.

(a) For a 3-year bond, by aritrage we should have:

$$\begin{aligned}
\frac{p^e_{2,t+1}}{p_{3,t}} &= 1+r+x \\
\frac{p^e_{1,t+2}}{p^e_{2,t+1}} &= 1+r+x \\
p^e_{1,t+2} &= \frac{60}{1+r}.
\end{aligned}$$

These together gives:

$$\begin{aligned}
p_{3,t} &= \frac{60}{(1+r+x)(1+r+x)(1+r)} \\
&= \frac{60}{(1+5\%+5\%)(1+5\%+5\%)(1+5\%)} \\
&= 47.23
\end{aligned}$$

For stocks, the present value of future dividend is[1]:

$$\begin{aligned}
&\frac{20}{(1+r+x)}+\frac{20}{(1+r+x)^2}+\frac{20}{(1+r+x)^3} \\
&\qquad=\frac{20}{(1+5\%+10\%)}+\frac{20}{(1+5\%+10\%)^2}+\frac{20}{(1+5\%+10\%)^3} \\
&\qquad=45.66
\end{aligned}$$

Investors should choose bond as it has a higher present value.

(b) For stocks, the present value becomes:

$$\begin{aligned}
&\frac{20}{(1+r+x)}+\frac{20}{(1+r+x)^2}+\frac{20}{(1+r+x)^3} \\
&\qquad=\frac{20}{(1+5\%+5\%)}+\frac{20}{(1+5\%+5\%)^2}+\frac{20}{(1+5\%+5\%)^3} \\
&\qquad=49.74
\end{aligned}$$

Investors should choose stock instead as it has a higher present value.

(c) For a 3-year bond,

$$\begin{aligned}
p_{3,t}&=\frac{60}{(1+r_1+x)(1+r_2+x)(1+r_3)} \\
&=\frac{60}{(1+5\%+5\%)(1+5\%+8\%)(1+12\%)} \\
&=43.10
\end{aligned}$$

With a slight abuse of notations, in above r_1, r_2 and r_3 denotes the interest rate in the first, second and third year, respectively. For stocks, the present value of future dividend is:

$$\begin{aligned}
&\frac{20}{(1+r_1+10\%)}+\frac{20}{(1+r_1+10\%)(1+r_2+10\%)}+\frac{20}{(1+r_1+10\%)(1+r_2+10\%)(1+r_3+10\%)} \\
&\frac{20}{(1+5\%+10\%)}+\frac{20}{(1+5\%+10\%)(1+8\%+10\%)}+\frac{20}{(1+5\%+10\%)(1+8\%+10\%)(1+12\%+10\%)} \\
&\qquad=44.21
\end{aligned}$$

The investors thus should choose stock.

[1] In this question, when we calculate the present value of the payoff received from holding the stocks, we ignore the resale-price of the stocks when they are sold 3 years later.