MH1820 Introduction to Probability and Statistical Methods
Tutorial 1 (Week 2)

In this tutorial, we will compute some probabilities. We assume that all possible outcomes in the (finite) sample space have the same probability. Therefore, the probability of an event $E \subseteq \Omega$ can be found using the formula

$$\mathbb{P}(E) = \frac{|E|}{|\Omega|}.$$

Problem 1 (n-tuples, Multiplication principle)

(a) How many 7-digit numbers are there all of whose digits are odd?

(b) How many 5-digit numbers are there all of whose digits are primes? (a prime is an integer $p \geq 2$ whose only divisors are 1 and p; note that 0 and 1 are not primes)

(c) How many 5-digit numbers are there? (all digits are allowed, but the first digit must not be 0)

(d) If a 5-digit number is chosen randomly, what is the probability that all its digits are primes?

(e) How many 6-digit numbers are there with no repeated digits?

(f) If a 6-digit number is chosen randomly, what is the probability that it has no repeated digits?

Problem 2 (Permutations)

(a) How many possibilities are there to line up 5 persons in a queue?

(b) Suppose there are 2000 spectators in a soccer-stadium. During halftime, a queue of 5 spectators forms in front of a coffee shop. How many possibilities are there for such a queue to form from the spectators?

Problem 3 (Combinations)

We consider a standard poker deck with 52 cards:
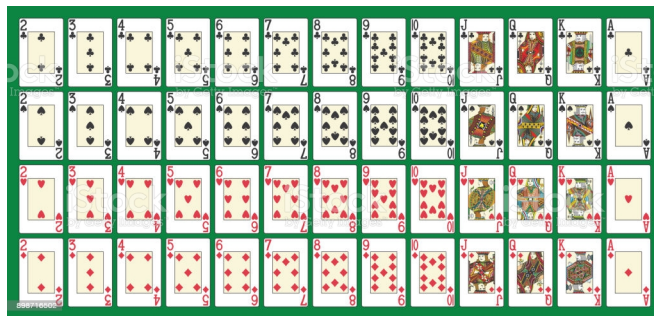
Note that there are 4 suits (clubs, spades, hearts, diamonds) and 13 ranks (ace, king queen, etc.). For the definition of poker hands (flush, straight, etc.) see
https://en.wikipedia.org/wiki/List_of_poker_hands

(a) How many ways are there to select a poker hand of 5 cards from the deck of 52 cards?

(b) What is the probability that a randomly chosen poker hand of 5 cards forms a
(i) flush (excluding straight flush)   (ii) pair,   (iii) two pairs,   (iv) full house?

(c) What is the probability that a randomly chosen hand forms a full house without any aces or kings?

Problem 4 (Permutations of multisets)

(a) How many words can be formed from the letters in `successlessness`? (here every permutation of these letters counts as a "word" even if does not make sense)

(b) In an orchid show, 11 orchids are to be placed along one side of the greenhouse. There are 5 lavender orchids, 4 white orchids, and 2 yellow orchids. Considering only the color of the orchids, find the number of different color displays?

(c) How many possibilities are there to split up 12 players into Team A, B, C with 5, 4, 3 players respectively?

Answer Keys.   1. (a) 5^7 (b) 4^5 (c) $90,000$ (d) 0.011 (e) $136,080$ (f) 0.1512 2. (a) 120 (b) 31840279800048000 3. (a) $2,598,960$ (b) (i) 0.002 (ii) 0.42 (iii) 0.048 (iv) 0.0014 (c) 0.001 4. (a) $21,621,600$ (b) $6,930$ (c) $27,720$.