

NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM II (CA2)

**MH1812 – Discrete Mathematics**

April 2018

TIME ALLOWED: 40 minutes

Name:

Matric. no.:

Tutor group:

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INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

**QUESTION 1.****(30 marks)**

Solve the following linear recurrences, that is, write  $a_n$  and  $b_n$  in terms of  $n$ :

(a)  $a_n = 10a_{n-1} - 21a_{n-2}$  for  $n \geq 2$ , with initial conditions  $a_0 = 3$ ,  $a_1 = 5$ ;

(b)  $b_n = b_{n-1} + 2$  for  $n \geq 1$ , with initial condition  $b_0 = 2$ .

Justify your answers.

**QUESTION 2.****(30 marks)**

(a) Prove that

$$\sum_{j=1}^n j(3j-1) = n^2(n+1), \quad \forall n \in \mathbb{N}.$$

(b) Let  $A = \{0, 1\}$  and  $B = \{4, 5\}$ .(i) Write out all elements of the set  $A \times B$ .(ii) What is the cardinality of the power set of  $A \times B$ ?

**QUESTION 3.****(40 marks)**

(a) Let  $A$ ,  $B$ , and  $C$  be sets.

(i) Prove that  $\overline{(A \cap B)} \cap C = (C - A) \cup (C - B)$ ;

(ii) Is  $(C - A) \cup (C - B) = C$ ? If yes, prove it, if no, give a counterexample.

(b) Let  $S = \{3a + 6b \mid a, b \in \mathbb{Z}\}$ .

(i) Show that  $S \subseteq \mathbb{Z}$ ;

(ii) Is  $S = \mathbb{Z}$ ? If yes, prove it, if no, give a counterexample.