AY 23/24 MH1820 Midterm Test

Name: Matriculation Number:
Tutorial Group:
Instructions
\bullet This test consists of 5 multiple choice questions and 3 computational questions.
• For each of the multiple choice questions, there is only one correct answer. Tick the correct answer.
• Answer all questions. The marks for each question are indicated.
• For the computational questions, write out the steps in your calculations clearly in the space provided after each question. Express numerical values up to 4 decimal places.
1. [2 marks] How many words can be formed from the letters in SWIFTIES? Here every permu-
tation of these letters counts as a word.
$\bigcirc 8! \qquad \bigcirc \frac{8!}{2!} \qquad \bigcirc \frac{8!}{2!2!} \qquad \bigcirc \frac{8!}{2!2!4!}$
2. [2 marks] Mr. Tan has 8 friends, of whom 5 will be chosen to attend a party. How many choices are there if 2 particular friends of Mr. Tan, say X and Y, will not attend the party together? \bigcirc 20 \bigcirc 30 \bigcirc 35 \bigcirc 36
3. [2 marks] Two fair dice (say A and B) are rolled independently. What is the conditional
probability that at least one of the dice lands on an even number given that the two dice land on different numbers ?
$\bigcirc \frac{1}{5} \qquad \bigcirc \frac{2}{5} \qquad \bigcirc \frac{3}{5} \qquad \bigcirc \frac{4}{5}$
4. [2 marks] A website receives hits at a rate of 180 per hour according to a Poisson distribution.
What is the probability that there are at least 10 hits in the first 2 minutes?
$\bigcirc \sum_{x=10}^{\infty} e^{-180} \frac{180^x}{x!} \qquad \bigcirc e^{-10} \frac{10^{-2}}{2!} \qquad \bigcirc 1 - \sum_{x=0}^{9} e^{-6} \frac{6^x}{x!} \qquad \bigcirc e^{-6} \frac{6^{10}}{10!}$
5. $[2 \text{ marks}]$ If $\mathbb{E}[X] = 1$ and $\text{Var}[X] = 5$, find $\mathbb{E}[(2+X)^2]$.
\bigcirc 12 \bigcirc 13 \bigcirc 14 \bigcirc None of the above

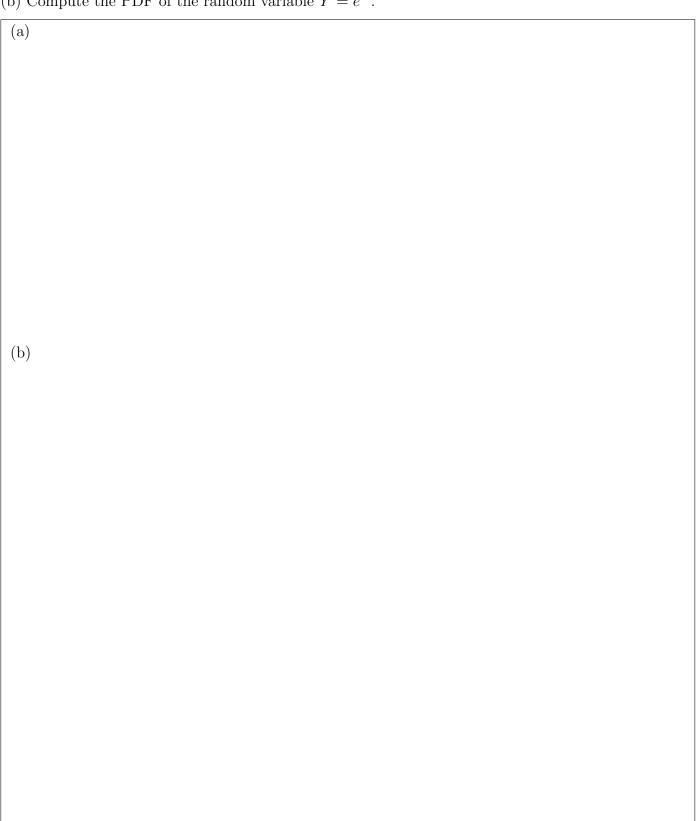
6. [6 marks] Consider the following game: Three fair dice are rolled independently. A player
bets on one of the numbers 1 through 6. If the number bet by the player appears i times, where $i=1,2,3$, then the player wins $(2 \times i)$; if the number bet by the player does not appear on any of the dice, then the player loses $(i.e. \text{ win } -\$2)$.
(a) What is the probability that the number bet by the player appears exactly two times?
(b) Let X denote the expected winning (in \$) of the player. Find $\mathbb{E}[X]$.

(a)		
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(b)		

7.	[6 marks]	Let X	be a	continuous	random	variable	with	PDF	given	by
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$$f(x) = 3x^2$$
 for $0 \le x \le 1$ and $f(x) = 0$ elsewhere.

- (a) Compute the CDF F(x) of X.
- (b) Compute the PDF of the random variable $Y = e^X$.



8. [8 marks] A student is waiting to receive notification mail telling her whether she has been accepted to a certain college. She estimates that her probability of being accepted is 0.6. She also estimates that the conditional probabilities of receiving notification mail on the following days of next week, given that she is accepted (or rejected) as follows:

Day	$\mathbb{P}(\text{mail} \text{accepted})$	$\mathbb{P}(\text{mail} \text{rejected})$
Monday	0.15	0.05
Tuesday	0.20	0.10
Wednesday	0.25	0.10

- (a) What is the probability that she receives mail on Monday?
- (b) What is the conditional probability that she receives mail on Tuesday given that she does not receive mail on Monday?
- (c) What is the conditional probability that she will be accepted given that there is no mail received on all three days (i.e. Monday, Tuesday and Wednesday)?

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