

MH1820 Introduction to Probability and Statistical Methods

Tutorial 11 (Week 12)

Problem 1 (Confidence Intervals)

A random sample of 100 automobile owners in Tekong state shows that an automobile is driven on average 23,500 km per year with a standard deviation of 3900 km. Assume the population distribution is normal.

Construct a 99% confidence interval for the average number of km per year an automobile is driven in this state.

Problem 2 (Confidence Intervals)

A random sample of 10 chocolate energy bar of a certain brand has, on average, 230 calories per bar, with a standard deviation of 15 calories. Construct a 99% confidence interval for the true mean calorie content of this brand of energy bar. Assume that the population distribution of the calorie content is normal.

Problem 3 (Confidence Intervals)

Suppose the fat content of certain steaks follows a $N(\mu, \sigma^2)$ distribution. The following observations x_1, \dots, x_{16} for the fat content are given.

5.33, 4.25, 3.15, 3.70, 1.61, 6.39, 3.12, 6.59, 3.53, 4.74, 0.11, 1.60, 5.49, 1.72, 4.15, 2.28

Suppose that both μ and σ^2 are *unknown*.

- (i) Find 90%, 95%, and 99% confidence intervals for μ .
- (ii) Find 90%, and 95% confidence intervals for σ^2 .

Problem 4 (Hypothesis Testing)

In the journal Hypertension, researchers report that individuals who practice Transcendental Meditation (TM) lower their blood pressure significantly. If a random sample of 225 male TM practitioners meditate for 8.5 hours per week with a standard deviation of 2.5 hours, does that suggest that, on average, men who use TM meditate more than 8 hours per week? Use a significance level of $\alpha = 0.05$. State the null hypothesis, alternative hypothesis, test statistic and the conclusion.

Problem 5 (Hypothesis Testing)

An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. A random sample of

30 bulbs has an average life of 788 hours. Use a 0.01 level of significance to test the hypothesis that $\mu = 800$ hours against the alternative hypothesis, $\mu \neq 800$ hours.

Problem 6 (Hypothesis Testing)

Past experience indicates that the time required for high school seniors to complete a standardized test is a normal random variable with a mean of 35 minutes. A random sample of 20 high school seniors took an average of 33.1 minutes to complete this test with a standard deviation of 4.3 minutes. Test the hypothesis that $\mu = 35$ against the alternative hypothesis that $\mu < 35$, at the 0.05 level of significance.

Answer Keys. **Q1.** $22495.75 < \mu < 24504.25$ **Q2.** $214.58 < \mu < 245.42$ **Q3(i).** $[2.800, 4.419]$, $[2.626, 4.594]$, $[2.249, 4.971]$ **Q3(ii).** $[2.047, 7.050]$, $[1.861, 8.172]$ **Q4.** $H_0: \mu = 8$, $H_1: \mu > 8$, $T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$, p -value is 0.0013, Reject H_0 **Q5.** p -value is 0.101, Do not reject H_0 **Q6.** p -value is less than $\alpha = 0.05$, Reject H_0 .