NTU SSS Economics HE1001
Tutorial 4 (Week 6): Production

1. Ed's building company has the following production function

$$q = 20L - L^2$$

Where q is the number of houses built and L is the quantity of labor Ed employs.

a. Derive the MP and AP.

Answer:

MP = dq/dL = 20 – 2L; AP = 20 – L

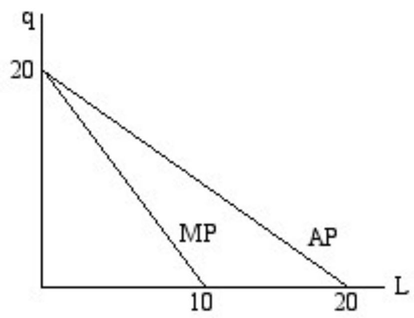
b. For what values of L is the MP > 0? For what values of L is the MP diminishing?

Answer:

MP > 0 if and only if 20 - 2L > 0, or L < 10.

c. Draw the MP and AP on a graph.

Answer:



2. John's Donut Shoppe has the production function $q = 5L^2 - 1/3\,L^3$, with L denoting the labor unit.

   a. Derive the MP_L.

      Answer: $MP_L = 10L - L^2$

   b. Derive the AP_L.

      Answer: $AP_L = \frac{q}{L} = 5L - 1/3L^2$

   c. At what level of L would the highest MP_L be?

      Answer:

$$\frac{\partial MP_L}{\partial L} = 10 - 2L = 0$$
$$L = 5$$

   d. At what level of L would the diminishing marginal returns begin?

      Answer:
      The marginal product is max at L=5 and becomes 0 when L=10.

$$\text{After } MP_L = 0$$
$$10L - L^2 = 0$$
$$L = 10$$

      So the diminishing marginal return begins after L=5.

   e. Show (without graph) that when AP_L=MP_L, then it must be that AP_L is at the maximum.

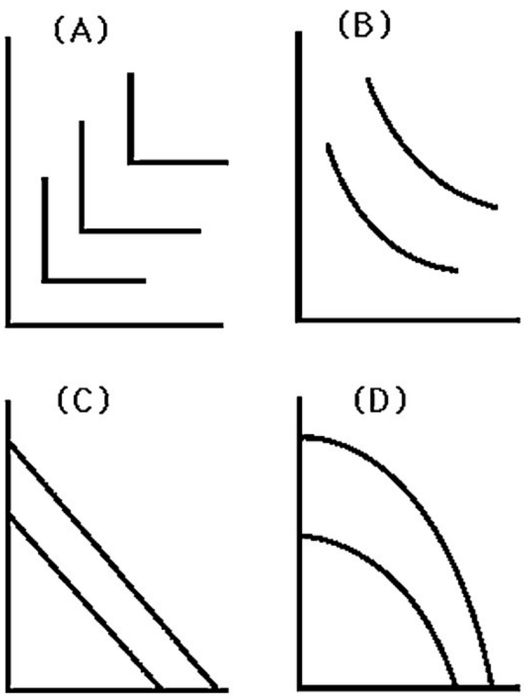
      Answer:

$$AP_L = MP_L$$
$$5L - \frac{1}{3}L^2 = 10L - L^2$$
$$\frac{2}{3}L = 5$$
$$L = 7.5$$

      AP_L is maximized when the following first order condition for maximization is satisfied:

$$\frac{\partial AP_L}{\partial L} = 5 - \frac{2}{3}L = 0$$
$$L = 7.5$$

3. Consider the following isoquant graphs of long run production with 2 variable inputs.



a. Lectures in microeconomics can be delivered either by a professor (labor) or a webcast (capital) or any combination of both. Each minute of the professor's time delivers the same amount of information as a minute of the webcast. Which graph in the above figure best represents the isoquants for lectures in microeconomics when capital per day is on the vertical axis and labor per day is on the horizontal axis?

   Answer: C

b. Which graph in the above figure represents the isoquants where, as the amount of labor used increases and the amount of capital used decreases, the marginal product of labor rises when capital per day is on the vertical axis and labor per day is on the horizontal axis?

   Answer: D

c. The production function for hamburgers can be written as $q = 0.1X + 0.1Y$, where X is Canadian ground beef and Y is U.S. beef, both measured in pounds. Which graph in the figure best represents the isoquants for the hamburger production when U.S. ground beef is on the vertical axis and Canadian ground beef is on the horizontal axis?

   Answer: C

4. Singapore Metal Company produces brass fittings. Their engineers, with the help of economists, estimate the production function represented below as relevant for their long-run capital labor decisions.

$$Q = 500L^{0.6}K^{0.8}$$

Where Q = annual output measured in pounds, L=labor measured in person hours, K=capital measured in machine hours. Singapore Metal's employees are relatively highly skilled and earn S$15 per hour. The firm estimates a rental charge of S$50 per hour on capital. Davy forecasts annual costs of S$500,000 per year.

a. Determine the firm's optimal capital labor ratio, given the information above.

Answer:

$$MP_L = 300L^{-0.4}K^{0.8} = 300\frac{K^{0.8}}{L^{0.4}}$$

$$MP_K = 400L^{0.6}K^{-0.2} = 400\frac{L^{0.6}}{K^{0.2}}$$

$$MRTS = \frac{MP_L}{MP_K} = \frac{300\frac{K^{0.8}}{L^{0.4}}}{400\frac{L^{0.6}}{K^{0.2}}} = 0.75\frac{K^{0.8}}{L^{0.4}}\frac{K^{0.2}}{L^{0.6}}$$

$$MRTS = 0.75\frac{K}{L}$$

$$\text{Equate: MRTS} = \frac{w}{r}$$

$$\frac{w}{r} = \frac{15}{50}$$

$$0.75\frac{K}{L} = \frac{15}{50}$$

$$\frac{K}{L} = 0.4$$

$$K = 0.4L$$

b. How much capital and labor should the firm employ, given the $500,000 budget? Calculate the firm's output.

Answer:

$$C = 500000$$
$$C = wL + rK$$
$$500000 = 15L + 50K$$
$$K = 0.4L$$
$$500000 = 15L + 20L$$
$$L = 14285.71 \sim 14286 \text{ hours}$$

$$L = 14286$$
$$C = wL + rK$$
$$500000 = 15(14286) + 50K$$
$$K = 5714$$

Optimal quantity:

$$Q = 500(L)^{0.6}(K)^{0.8}$$
$$Q = 500(14286)^{0.6}(5714)^{0.8}$$
$$Q = 157{,}568{,}191$$

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