Nanyang Technological University
HE2001 Microeconomics II

Solution 4

Question 1

(a) The present value of consumption should be equal to the present value of income:

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

(b) Set $c_1 = 0$ and we have $c_2 = (1+r)m_1 + m_2$, i.e., the income earned in period 1 can be all saved for later consumption. This is the future value of his total endowment.

(c) Set $c_2 = 0$ and we have $c_1 = m_1 + \frac{m_2}{1+r}$, i.e., he borrows money to consume in period 1 and use all income in period 2 to repay. This is the present value of his total endowment.

Rewrite the budget constraint as

$$c_2 = -(1+r)c_1 + (1+r)m_1 + m_2$$

The slope is $-(1+r)$.

Question 2

(a) $PV = \$2{,}000 + \$1{,}100/(1+0.1) = \$3{,}000$
$FV = (\$2{,}000)(1+0.1) + \$1{,}100 = \$3{,}300$

Denote (C_1, C_2) as his two-year consumption bundle. His budget constraint must satisfy PV of comsuption = PV of endowment or budget:

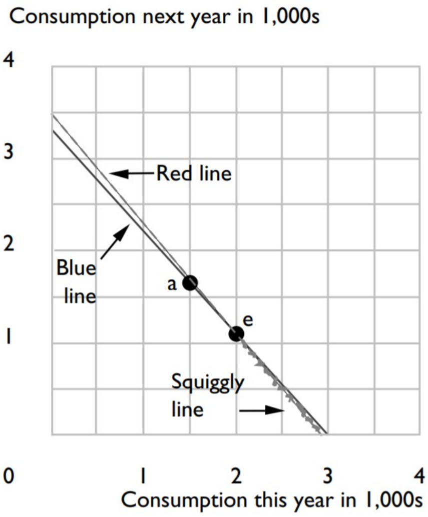
$$C_1 + \frac{C_2}{1+0.1} = 3{,}000 \text{ or } C_2 = -1.1C_1 + 3{,}300$$

drawn as the blue line in the graph.
The endowment ($2,000, $1,100) is labeled with letter E.

(b) The marginal rate of substitution between consumption of the two years is given by

$$MRS = -\frac{MU_1}{MU_2} = -\frac{C_2}{C_1}$$

(c) From the budget constraint equation $C_1 + \frac{C_2}{1.1} = 3{,}000$ in part (a), the slope is -1.1.



(Blue line: the budget constraint $C_2 = -1.1C_1 + 3{,}300$, *when the interest rate is 10%;*
Red line: the budget constraint $C_2 = -1.2C_1 + 3{,}500$, *when the interest rate is 20%;)*

The function of the indifference curve is $MU_1 dc_1 + MU_2 dc_2 = 0$. The slope of the indifference curve is the marginal rate of substitution. Hence we can write the equation as

$$\frac{C_2}{C_1} = 1.1$$

(d) The two equations above give the point of tangency of the budget line and the indifference curve. That is the optimal consumption bundle that Nickleby would choose.
Solving the two equations, we have $(C_1, C_2) = (1{,}500, 1{,}650)$, labeled with point A in the graph.

(e) Compare his endowment (2,000, 1,100) and his consumption (1,500, 1,650). He will save \$500 in the first year, which will become \$550 in the second year.

(f) At a higher interest rate, the present value of Nickleby's income is

$$PV = \$2{,}000 + \$1{,}100/(1 + 0.2) = \$2{,}916.7$$

The budget equation will become

$$C_1 + \frac{C_2}{1 + 0.2} = 2{,}916.7 \text{ or } C_2 = -1.2C_1 + 3{,}500$$

Increasing the interest rate pivots the budget line around the endowment to a steeper position (from the blue line to the red line).
By WARP, $A \succ$ all points within the old budget set. By montonicity, there exists a point in new budget set $\succ A$. Hence, all points on new budget line which lie within old budget set will not be chosen.

(g) Solve the new budget equation and the MRS equation:

$$C_1 + \frac{C_2}{1.2} = 2{,}916.7 \text{ and } \frac{C_2}{C_1} = 1.2$$

We have $(C_1, C_2) = (1{,}458.3, 1{,}750)$.

(h) Compare the endowment to the consumption value again, he will save \$541.7 in the first year now, which will become \$650 in the second year. The rising interest rate motivates consumers to save more for a higher return in the future.

Question 3

(a) With different interest rates for borrowing and lending, Laertes has different slopes of budget line depending on whether he is a borrower or a lender.

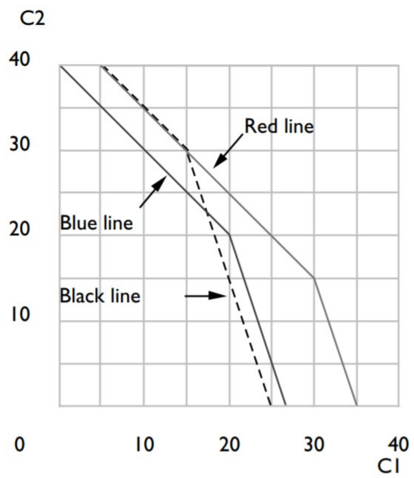
Note that for borrower: $C_1 > E_1, C_2 < E_2,$ and $C_2 = E_2 - (C_1 - E_1)(1 + r)$
For lender: $C_1 < E_1, C_2 > E_2,$ and $C_2 = E_2 + (E_1 - C_1)(1 + r)$

In this case, if Laertes is a borrower, his budget constraint is

$$C_1 + \frac{C_2}{3} = 20 + \frac{20}{3} = 26.7, \text{for } C_1 \geq 20;$$

If he is a lender, the budget constraint is $C_1 + C_2 = 20 + 20 = 40, \text{for } C_1 < 20.$

Draw this budget set with blue lines in the graph. The kink will be at his endowment point (20, 20).



(Blue line: the budget constraint when the endowment is (\$20, \$20);
Red line: the budget constraint when the endowment is (\$30, \$15);
Black line: the budget constraint when the endowment is (\$15, \$30))

(b) With the new endowment (\$30, \$15), the two budget constraints are given by

$$\text{Borrower: } C_1 + \frac{C_2}{3} = 30 + \frac{15}{3} = 35, \text{for } C_1 \geq 30;$$

$$\text{Lender: } C_1 + C_2 = 30 + 15 = 45, \text{for } C_1 < 30.$$

Draw this budget set with red lines in the graph. It has a kink at the point (30, 15).

Laertes would become better off. Observe the relative position of the two budget sets in the graph. For any bundle that he could afford without investing (points on the blue line), he can always obtain some better bundle (on the red line) that allows higher consumption through investing in this project.
Basically when the endowment changes, the slope as a borrower or lender will stay the same, but the kink point will change.

(c) With the new endowment ($15, $30), the two budget constraints are given by

$$\text{Borrower: } C_1 + \frac{C_2}{3} = 15 + \frac{30}{3} = 25, \text{for } C_1 \geq 15;$$

$$\text{Lender: } C_1 + C_2 = 15 + 30 = 45, \text{for } C_1 < 15.$$

Draw this budget set with black in the graph. It has a kink at the point (15, 30).

Now we cannot tell. Observe the relative position of the blue line and the black line which intersect now. If he invests in this project, he can afford some things he couldn't afford before. But some things he could afford before will become unavailable now. We need more information about preferences to come to a conclusion.

Question 4

(a) The endowment is (1,000, 150). Represent c_2 as a function of c_1.

$$c_2 = (1 - 0.25)(1000 - c_1) + 150 = -0.75c_1 + 900$$

where $c_1 \leq 1000$, since villagers can save but cannot borrow.
It is like a negative interest rate- the saved crop will be reduced in the future:

$$c_1 + \frac{c_2}{0.75} = 1{,}000 + \frac{150}{0.75} = 1{,}200$$

This budget line is drawn with the red line in the graph. Numbers are given on the axis (1,000 and 900).

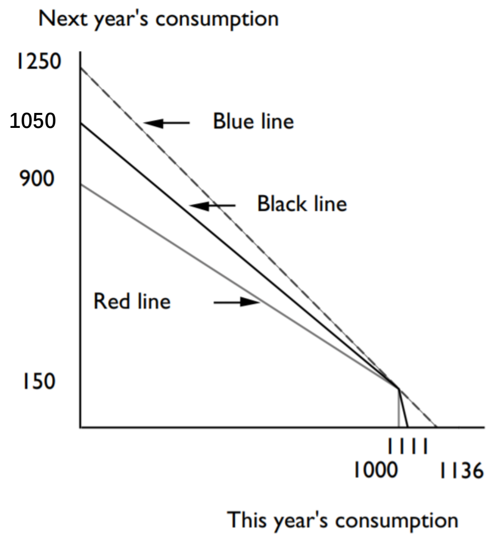
(b) The utility maximization problem is given by

$$\max_{c_1, c_2} U(c_1, c_2) = c_1 c_2 \, , s.t. \, c_2 = -0.75c_1 + 900 \text{ and } c_1 \leq 1000$$

or equivalently $\max_{c_1} U(c_1) = -0.75c_1^2 + 900c_1$, where $c_1 \leq 1000$

(Alternatively, we set MRS = price ratio: $\frac{c_1}{p_2} = \frac{c_2}{p_1}$, $c_1 = \frac{c_2}{0.75}$)

The maximum is obtained at $c_1 = 600 < 1{,}100$. Thus the villagers will consume 600 bushels this year, and store 400 bushels for the next year. With 100 bushels eaten by rats, the remaining 300 bushels plus the harvest of the next year, 150 bushels, will be the future consumption. The equation also yields $c_2 = 450$.



(Red line: the budget constraint without trade $0.75c_1 + c_2 = 900$*;*
Blue line: the budget constraint with trade $1.1c_1 + c_2 = 1250$*;*
Black line: the budget constraint with trade and transportation cost)

(c) Villagers can now consume more than 1,000 bushels this year by borrowing, and they would prefer to sell the unconsumed corn rather than storing it as there is no loss. The only constraint is that PV of consumption = PV of harvest:

$$c_1 + \frac{c_2}{1.1} = 1000 + \frac{150}{1.1} \text{ or } c_2 = -1.1c_1 + 1250$$

The new budget constraint is drawn with the blue line in the graph. The two intercepts are 1,250 and 1,136. The opportunity to trade makes higher consumption available.

The utility maximization problem is given by

$$\max_{c_1, c_2} U(c_1, c_2) = c_1 c_2 \,, s.t. \, c_2 = -1.1c_1 + 1250$$

or equivalently $\max_{c_1} U(c_1) = -1.1c_1^2 + 1250c_1$

(Alternatively, we set MRS = price ratio: $\frac{c_1}{p_2} = \frac{c_2}{p_1}$, $c_1 = \frac{c_2}{1.1}$)

The maximum is obtained at $c_1 = 568$. Thus the villagers will consume 568 bushels this year, and sell the remaining 432 bushels to earn \$432. In the next year, the interest increases the amount of money to \$475, so they can buy 475 bushels, in addition to the harvest of 150 bushels. The equation also yields $c_2 = 625$.

(d) With the transportation cost, the difference between the endowment and consumption each year will generate utility loss.

If villagers sell corn this year, they will need to haul out $(1000 - c_1)$ this year, and haul in $(c_2 - 150)$ next year. Taking the PV of two transportation costs into account, the budget constraint becomes

$$c_1 + \frac{c_2}{1.1} = 1000 + \frac{150}{1.1} - (1000 - c_1)(0.1) - \frac{(c_2 - 150)(0.1)}{1.1}$$

Similarly, if the villagers buy corn this year, they will need to haul in $(c_1 - 1000)$ this year, and haul out $(150 - c_2)$ next year. The budget constraint becomes

$$c_1 + \frac{c_2}{1.1} = 1000 + \frac{150}{1.1} - (c_1 - 1000)(0.1) - \frac{(150 - c_2)(0.1)}{1.1}$$

Taken together, we get the budget constraint with the transportation cost:

$$0.9c_1 + c_2 = 1050, if\ c_1 \leq 1000$$

$$1.21c_1 + 0.9c_2 = 1345, if\ c_1 > 1000$$

Draw these two line segments in the graph with black. The transportation cost is like an additional cost to consuming away from bundle. We can see that the new budget line now lies lower than when there is no transport costs.

(Note: Approximation may lead to slightly different intercepts compared to the numbers given in the graph. Just make sure the constraint equation is in the correct form.)

Question 5

(a) Her endowment point E is (20, 40). If she doesn't buy the cookie jar, the constraint for two periods is given by

$$c_1 \leq 20\ for\ period\ 1;$$

$$c_2 \leq 40 + (20 - c_1) = 60 - c_1\ for\ period\ 2$$

drawn as the two line segments in blue in the graph.
This is what happens when you have an indivisible good for saving. Note that she cannot transfer money from the future (borrow).
These are perfect substitutes and have the same price, so she will choose any point on the sloped portion, all giving a total utility of 60.

If she buys the cookie jar, the available money becomes \$8 in period 1 and \$60 in period 2. Label the point (8, 60) as point A, which gives a total utility of 68.

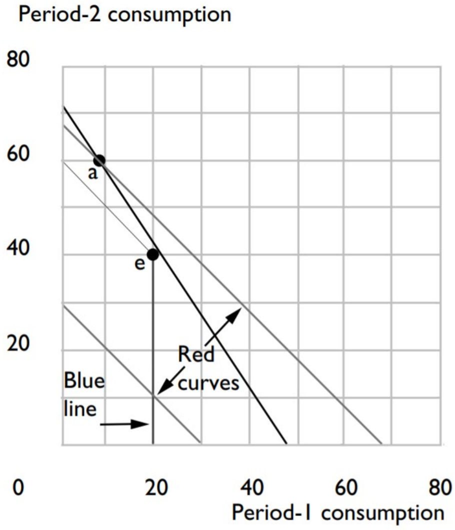
Marsha would prefer a higher utility of 68 and invest in the cookie jar.

(b) If she can borrow or lend now, the budget constraint becomes

$$c_1 + \frac{c_2}{1.5} = 8 + \frac{60}{1.5} \text{ or } 1.5c_1 + c_2 = 72$$

Drawn as the black line in the graph.
The utility function is in the form of a summation (the consumption of two periods are

perfect substitutes of each other), so the indifference curves are given by the lines with a slope of -1 and different intercepts, like the two red curves in the graph.



(Blue line: the budget constraint without borrowing, lending, or buying the jar;
Red curves: indifference curves for two-period consumption;
Black line: the budget constraint with borrowing and lending)

(c) If she cannot borrow or lend, her utility is 20 if she doesn't buy the jar, and only 8 if she buys the jar. So she should not buy it.

If she can borrow or lend and she chooses not to buy the jar, the budget constraint is

$$c_1 + \frac{c_2}{1.5} = 20 + \frac{40}{1.5} \text{ or } 1.5c_1 + c_2 = 70$$

With a utility function of perfect complements, the maximum is obtained when the consumptions of two periods are equal.

$$1.5c_1 + c_2 = 70 \text{ and } c_1 = c_2$$

So we can solve for $c_1 = c_2 = 28$. The optimal utility is 28.

If she buys the jar, the budget constraint is $1.5c_1 + c_2 = 72$. Obviously, it gives a higher budget line. We similarly solve for equal consumptions $c_1 = c_2 = 28.8$. With a higher utility obtained at 28.8, she should buy the jar.

If the interest rate rises to 100%, the budget constraint is

$$c_1 + \frac{c_2}{2} = 20 + \frac{40}{2} \text{ or } 2c_1 + c_2 = 80 \text{ if she doesn't buy the jar;}$$

$$c_1 + \frac{c_2}{2} = 8 + \frac{60}{2} \text{ or } 2c_1 + c_2 = 76 \text{ if she buys the jar.}$$

Now the constant terms change in the equation. We can solve this again (If do not invest, utility is 80/3. If invest, utility is 76/3) and determine that Marshaw should not invest in the jar. Intuitively, this is because current income is now worth more (the increasing rate of the value of jar is around 67%. Now the 100% interest rate is higher than this).