MH1820 Introduction to Probability and Statistical Methods
Tutorial 2 (Week 3)

Problem 1 (sample spaces)

For each of the following statistical experiments, write down a suitable sample space.

(i) The number of passengers on an MRT train is counted.

(ii) The height of a person is measured.

(iii) 1000 persons are selected randomly and they are asked if they have any travel plans for this year.

(iv) The maximum force (in N=Newton) is determined that a certain steel beam can withstand without breaking.

(v) At a random time, the number of seconds past the current minute is determined (for instance, if the current time is 12:13:33, the outcome is 33).

(vi) A dice is rolled 5 times.

Problem 2 (independent events)

Two integers are drawn independently at random from $1, 2, \ldots, 100$ (repetition is allowed, for instance, both numbers could be 1). Consider the following events.

E_1: both numbers are even
E_2: the first number is ≤ 50
E_3: the second number is prime

For any pair of events above, decide whether the two events are independent.

Problem 3 (conditional probability)

A researcher finds that, of 982 men who died in 2002, 221 died from some heart disease. Also, of the 982 men, 334 had at least one parent who had some heart disease. Of the latter 334 men, 111 dies from some heart disease. A man is selected from the group of 982. Given that neither of his parents had some heart disease, find the conditional probability that this man died of some heart disease.

Problem 4 (conditional probability)

(a) A dice is rolled 4 times.

(i) What is the probability that **at most** one of the rolls is a 6?

(ii) What is the probability that **at least** two of the rolls are a 6?

(iii) What is the probability that the total of the rolls is **at least** 22?

(iv) What is the probability that the total of the rolls is **at least** 22 under the condition that at least two of the rolls are a 6?

(b) A family has two children. Assume that the probability for a child to be a girl is $\frac{1}{2}$, and the probability for a child to be a boy is $\frac{1}{2}$.

(i) What is the probability that both children are girls under the condition that the first child is a girl?

(ii) What is the probability that both children are girls under the condition at least one of the children is a girl?

Problem 5 (law of total probability and Bayes' theorem)

A ball is drawn from one of 3 boxes. The boxes contain the following number of balls of colors blue (B), red (R), and yellow (Y).

	B	R	Y
Box 1	1	4	5
Box 2	6	2	2
Box 3	3	3	4

The following procedure is used to draw the ball.

- One of the boxes is chosen at random: Box 1 is chosen with probability 0.2, Box 2 is chosen with probability 0.5, and Box 3 is chosen with probability 0.3.

- A ball is drawn from the chosen box (each ball in the box is chosen with the same probability).

(i) What is the probability that a blue ball is drawn?

(ii) What is the probability that a yellow ball is drawn?

(iii) If a blue ball is drawn, what is the probability that it was drawn from Box 1?

Answer Keys. 2. $\mathbb{P}(E_1) = 0.25$, $\mathbb{P}(E_2) = 0.5$, $\mathbb{P}(E_3) = 0.25$, $\mathbb{P}(E_1 \cap E_2) = 0.125$, $\mathbb{P}(E_1 \cap E_3) = 0.005$, $\mathbb{P}(E_2 \cap E_3) = 0.125$. 3. $\frac{110}{648}$ 4. (a) (i) $\frac{1125}{1296}$ (ii) $\frac{171}{1296}$ (iii) $\frac{15}{1296}$ (iv) $\frac{15}{171}$ (b) (i) $\frac{1}{2}$, (ii) $\frac{1}{3}$. 5. (i) 0.41 (ii) 0.32 (iii) 0.049.