Discrete Mathematics

MH1812

Topic 8 - Relations
Summary

Example

$R \subseteq A \times B$

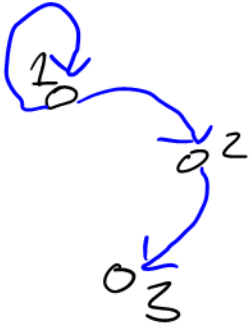
Consider the relation $R = \{(1,1), (1,2), (2,3)\}$ on the set $A = \{1,2,3\}$.

$\subseteq A \times A$

- Is it Reflexive? Symmetric? Anti-symmetric? Transitive?

Reflexive? No, $(2,2) \notin R$.

$\forall x \in A, \ (x,x) \in R$



symmetric? No, $(1,2) \in R$ but $(2,1) \notin R$

$\forall x,y \in A, \ (x,y) \in R \rightarrow (y,x) \in R$

Anti-symmetric? Yes, check everything.

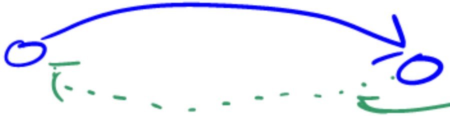
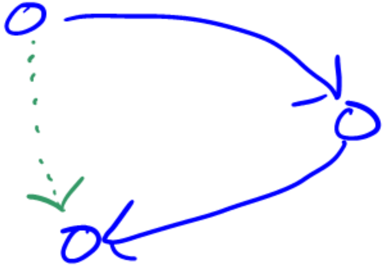
$$\forall x,y \in A,\ (x,y)\in R \wedge (y,x)\in R \rightarrow x=y$$

x	y	$(x,y)\in R \wedge (y,x)\in R$	$x=y$	$(x,y)\in R \wedge (y,x)\in R \rightarrow (x=y)$
1	1	T	T	T
1	2	F		T
1	3	F		⋮
2	1	F		
2	2	F		
2	3	F		
3	1	F		
3	2	F		
3	3	F		T

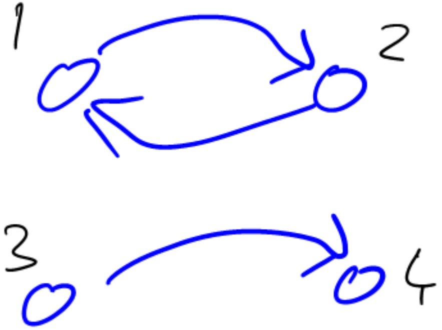
Transitive? No, $(1,2) \in R \wedge (2,3) \in R$ but $(1,3) \notin R$

$\forall x, y, z \in A.\ (x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$

if $|A| \leq 2 \Rightarrow \forall$ relation is symmetric or anti symmetric?

	$\exists$ (there exists)
Not reflexive	 no arrow
Not symmetric	 no arrow
Not anti-symmetric	
Not transitive no arrow in given direction	  no arrow

Symmetry vs anti-symmetry

	Anti symmetric	Not anti-symmetric
Symmetric		
Not symmetric		

$R = \{(1,2),(2,1),(3,4)\}$

Symmetric/anti-sym

$A = \{1,2\}$ $\quad R = \{(2,2)\}$

Symm

$$\forall\, x,y \in A,\ (x,y) \in R \rightarrow (y,x) \in R$$

x	y	$(x,y)\in R$	$(y,x)\in R$	$(x,y)\in R \rightarrow (y,x)\in R$
1	1	F	F	T
1	2	F	F	T
2	1	F	F	T
2	2	T	T	T

Example

Let relation *R* be defined on set $A=\{a, b, c, d\}$ and $R=\{(a, b),(b, a),(b, c),(c, d)\}$.

- **Find R^{-1} and R^t.**

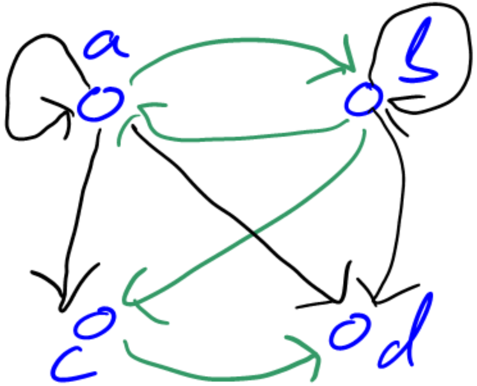
$R^{-1}=\{(b,a),(a,b),(c,b),(d,c)\}$

- **Is R^t an equivalence relation, a partial order?**

No

"add shortcuts"

$R^t = \{(a,b),(b,a),(b,c),(c,d),(a,a),(a,c),(a,d),(b,b),(b,d)\}$



Example

Consider the relation $R = \{(x,y) \mid x \equiv y \bmod 2\}$ **on the set** $A = \mathbb{N}$.

- **Show that** R **is an equivalence relation.**
- **Show that the equivalence classes of** R **partition the set** A.

compare to online video lessons.