Nanyang Technological University
HE2001 Microeconomics II

Tutorial 6

1. Willy owns a small chocolate factory, located close to a river that occasionally floods in the spring, with disastrous consequences. Next summer, Willy plans to sell the factory and retire. The only income he will have is the proceeds of the sale of his factory. If there is no flood, the factory will be worth \$500,000. If there is a flood, then what is left of the factory will be worth only \$50,000. Willy can buy flood insurance at a cost of \$.10 for each \$1 worth of coverage. Willy thinks that the probability that there will be a flood this spring is 1/10. Let c_F denote the contingent commodity *dollars if there is a flood* and c_{NF} denote *dollars if there is no flood.* Willy's von Neumann-Morgenstern utility function is $U(c_F, C_{NF}) = 0.1\sqrt{c_F} + 0.9\sqrt{c_{NF}}$

   (a) If he buys no insurance, then in each contingency, Willy's consumption will equal the value of his factory. What is his contingent commodity bundle?

   (b) To buy insurance that pays him \$$x$ in case of a flood, Willy must pay an insurance premium of $0.1x$. (The insurance premium must be paid whether or not there is a flood.) If Willy insures for \$$x$, then if there is a flood, he gets \$$x$ in insurance benefits. Suppose that Willy has contracted for insurance that pays him \$$x$ in the event of a flood. Then after paying his insurance premium, how much will he consume? If Willy has this amount of insurance and there is no flood, how much will he be able to consume?

   (c) You can eliminate x from the two equations for c_F and c_{NF} that you found above. This gives you a budget equation for Willy. Of course there are many equivalent ways of writing the same budget equation, since multiplying both sides of a budget equation by a positive constant yields an equivalent budget equation. If the "price" of c_{NF} is 1, write down the budget equation.

   (d) Willy's marginal rate of substitution between the two contingent commodities, *dollars if there is no flood* and *dollars if there is a flood*, is $MRS(c_F, c_{NF}) = -\frac{0.1\sqrt{c_{NF}}}{0.9\sqrt{c_F}}$. To find his optimal bundle of contingent commodities, what value should the MRS be? Solving this equation, what is the ratio that you find Willy will choose to consume the two contingent commodities at?

   (e) Since you know the ratio in which he will consume c_F and c_{NF}, and you know his budget equation, you can solve for his optimal consumption bundle, what is it? If Willy buys, what amount will be paid to him in the case of a flood? What is the amount of insurance premium?

2. Hjalmer Ingqvist's son-in-law, Earl, has not worked out very well. It turns out that Earl likes to gamble. His preferences over contingent commodity bundles are represented by the expected utility function

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 c_1^2 + \pi_2 c_2^2$$

   (a) Just the other day, some of the boys were down at Skoog's tavern when Earl stopped in. They got to talking about just how bad a bet they could get him to take. At the time, Earl had \$100. Kenny Olson shuffled a deck of cards and offered to bet Earl \$20 that Earl would not cut a spade from the deck. Assuming that Earl believed that Kenny wouldn't cheat, the probability that Earl would win the bet was 1/4 and the probability that Earl would lose the bet was 3/4. How much will he have if he wins? How much if he loses? What is Earl's expected utility if he takes the bet? What is his expected utility if not taking the bet?

   (b) Just when they started to think Earl might have changed his ways, Kenny offered to make the same bet with Earl except that they would bet \$100 instead of \$20. What is Earl's expected utility if he takes this bet? Will he take the bet?

(c) Let Event 1 be the event that a card drawn from a fair deck of cards is a spade. Let Event 2 be the event that the card is not a spade. Write down the equation that represents Earl's preferences between income contingent on Event 1, c_1, and income contingent on Event 2, c_2. Sketch Earl's indifference curve passing through the point $(100, 100)$

(d) On the same graph, let us draw Hjalmer's son-in-law Earl's indifference curves between contingent commodities where the probabilities are different. Suppose that a card is drawn from a fair deck of cards. Let Event 1 be the event that the card is black. Let event 2 be the event that the card drawn is red. Suppose each event has probability $1/2$. Write down the formula that represents Earl's preferences between income contingent on Event 1 and income contingent on Event 2. On the graph, show two of Earl's indifference curves, including the one that passes through $(100, 100)$.

3. The *certainty equivalent* of a lottery is the amount of money you would have to be given with certainty to be just as well-off with that lottery. Suppose that your von Neumann-Morgenstern utility function over lotteries that give you an amount x if Event 1 happens and y if Event 1 does not happen is $U(x, y, \pi) = \pi\sqrt{x} + (1 - \pi)\sqrt{y}$ where π is the probability that Event 1 happens and $1 - \pi$ is the probability that Event 1 does not happen.

   (a) If $\pi = 0.5$, calculate the utility of a lottery that gives you \$10,000 if Event 1 happens and \$100 if Event 1 does not happen.

   (b) If you were sure to receive \$4,900, what would your utility be?

   (c) Given this utility function and $\pi = 0.5$, write a general formula for the certainty equivalent of a lottery that gives you \$$x$ if Event 1 happens and \$$y$ if Event 1 does not happen.

   (d) Calculate the certainty equivalent of receiving \$10,000 if Event 1 happens and \$100 if Event 1 does not happen.