

NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I (CA1)

MH1812 – Discrete Mathematics

February 2017

TIME ALLOWED: 40 minutes

Name:

Matric. no.:

Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1.**(40 marks)**

- (a) Which integer $a \in \{0, 1, 2\}$ is congruent to $2017 + 2020 + 2023$ modulo 3? (10 marks)
- (b) Write down each integer $a \in \{0, 1, 2, 3\}$ for which there exists an integer n such that $a \equiv n^2 \pmod{4}$? (10 marks)
- (c) Decide whether the set S is closed under the operation Δ when
- $S = \{\text{even integers}\}$ and Δ is subtraction. (10 marks)
 - $S = \{\text{irrational numbers}\}$ and Δ is multiplication. (10 marks)

Briefly justify your answers.

QUESTION 2.**(40 marks)**

- (a) Prove or disprove the following statement (20 marks):

$$(p \wedge \neg q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow q.$$

- (b) Decide whether or not the following argument is valid (20 marks):

$$\begin{aligned} p &\rightarrow q; \\ \neg p &\rightarrow r; \\ \neg(r \wedge q); \\ \therefore \neg p \end{aligned}$$

Briefly justify your answer.

QUESTION 3.**(20 marks)**

Consider the domains $X = \mathbb{Z} = \{\text{integers}\}$ and $Y = \{0, 1, 2\}$, and the predicate $P(x, y) = \text{"3 divides } x - y\text{"}$. Determine the truth values of the following statements:

- (a) $\forall x \in X, \exists y \in Y, P(x, y)$; (10 marks)
- (b) $\neg(\forall y \in Y, \exists x \in X, \neg P(x, y))$. (10 marks)

Briefly justify your answers.