

Revealed Preference

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January 12, 2024

Overview

- 1 Course Overview
- 2 Revealed Preference
- 3 WARP and SARP
- 4 Index Numbers and Price Indices

Course Instructors

- Jubo YAN
 - assistant prof. in economics
 - office: SHHK-04-68
 - office hour: by appointment
- Jonathan Yeo
 - assistant prof. in economics
 - office: SHHK-04-63
 - office hour: TBD
- Xiaojie ZHANG
 - PhD student in economics
 - office hour: TBD

Course Aims

- In the first half of the course, we will continue our discussion on individual decision making in the market
 - further examine market transactions by including more details (i.e., **revealed preference**, different **price effects**, and **endowment**)
 - introduce **time**, **risk**, and **uncertainty** into market exchange and investigate their impacts on market transactions
- In the second half of the course, the goal is to have a solid understanding of the functioning of the market and when it can provide socially desirable outcomes
 - we will study topics in welfare economics, learning how socially desirable outcomes can be defined
 - we will look at the conditions under which markets can attain social efficiency

Topics (Week 1 to Week 7)

- ① Revealed Preference (week 1)
- ② Decomposition of Price Change (week 2)
- ③ Exchange Economy (week 3)
- ④ Intertemporal Choice (week 4)
- ⑤ Asset Markets (week 4)
- ⑥ Uncertainty and Risky Assets (week 5 & 6)

Learning Outcomes

- Use microeconomics concepts and theories to describe and explain welfare measurement and sources of market failure

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- Apply fundamental microeconomic models to analyse decision-making by individuals/government and the impacts on social welfare under different contexts

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- Apply fundamental microeconomic models to analyse decision-making by individuals/government and the impacts on social welfare under different contexts
- Communicate economic reasoning, in speech and writing to explain and evaluate how decision processes and/or market frictions influence economic outcomes and welfare

Class Organization (1st half Assessment)

- Class participation (5%) – lecture and tutorial

Class Organization (1st half Assessment)

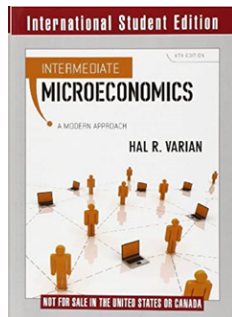
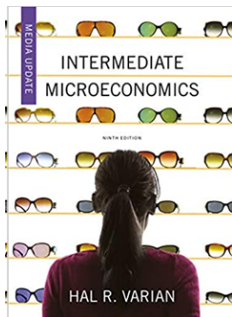
- Class participation (5%) – lecture and tutorial
- Online quizzes (5%×2)
 - quizzes are optional but students are encouraged to complete them as the top two (2) scores in the first half will count towards your grades
 - total 3 MCQ quizzes will be given after topics 2, 4, and 6
 - each quiz will be open for **two (2) weeks** and you can complete it whenever you want
 - has to be completed in **one-sitting** within time limit – only **first attempt** counts

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- Bisemester test (35%)
 - in-class tests consisting of composite questions
 - in Week 7 (Feb 26th)

Class Organization

- Textbook “*Microeconomics: A Modern Approach*” (9th edition) by Hal R. Varian. *W.W.Norton & Company*
- Most course materials are derived from the textbook (read it!)
- Any later editions should suffice



Class Organization

- Lectures will be in person in LT 20 throughout the semester
 - slides are to be uploaded to NTULearn before each lecture
 - we follow SSS UG policy in releasing lecture recordings
 - any exceptions should be addressed to UG office and be approved by them

Class Organization

- Lectures will be in person in LT 20 throughout the semester
 - slides are to be uploaded to NTULearn before each lecture
 - we follow SSS UG policy in releasing lecture recordings
 - any exceptions should be addressed to UG office and be approved by them
- Tutorials will be in person
 - attempt the tutorial questions BEFORE coming to sessions to maximize your learning outcomes
 - sample answers will be provided after each tutorial session
 - participation scores will be given based on your presentation of the tutorial questions (everyone should present)

Class Organization

- Try only ask clarification questions over emails and make an appointment if you need more thorough discussions
- It is also a good time to ask any questions before/after lectures/tutorials
- The discussion board on NTULearn has been enabled. You can post questions on the discussion board to foster more discussions among students

Questions?

- 1 Course Overview
- 2 Revealed Preference**
- 3 WARP and SARP
- 4 Index Numbers and Price Indices

Preference and Behavior

- We have been thinking what preferences could tell us about people's behavior
 - preferences are not directly observed in real life
 - need to discover people's preferences from observing behaviors

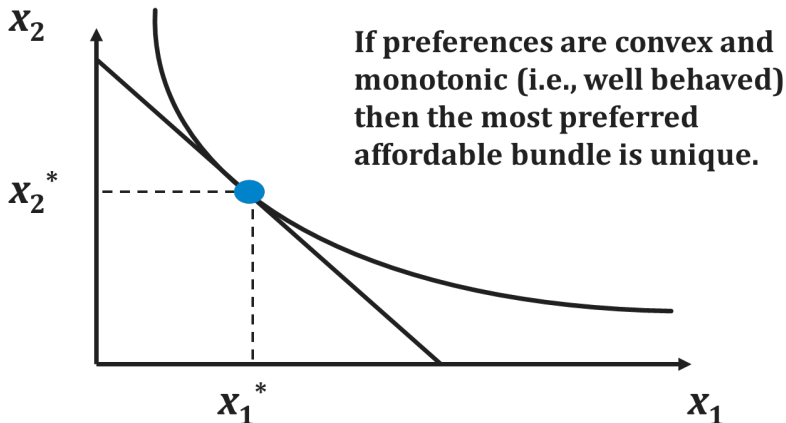
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- We have been thinking **what preferences could tell us about people's behavior**
 - preferences are **not** directly observed in real life
 - need to discover people's preferences from observing behaviors
- Assume that preference is **stable** over the period of observation
 - acceptable for short term
- Preferences are also assumed to be **strictly convex** and **monotonic**
 - most preferred affordable bundle is **unique**

Preference and Behavior



Preference and Behavior

- Suppose we observe the demands (consumption choices) that a consumer makes for **different** budgets
 - test the behavioral hypothesis that a consumer chooses the most preferred bundle from those available
 - discover the consumer's preference relation

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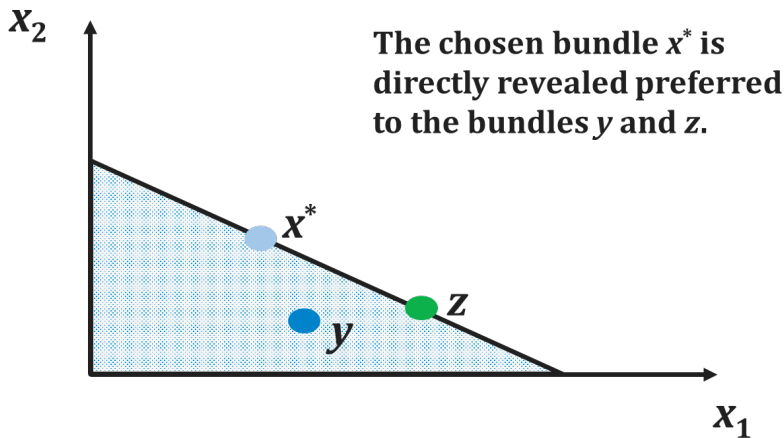
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$$\begin{aligned}
 p_1 y_1 + p_2 y_2 &\leq m & p_1 x_1^* + p_2 x_2^* &= m \\
 \Rightarrow p_1 x_1^* + p_2 x_2^* &\geq p_1 y_1 + p_2 y_2
 \end{aligned}$$

Direct Preference Revelation



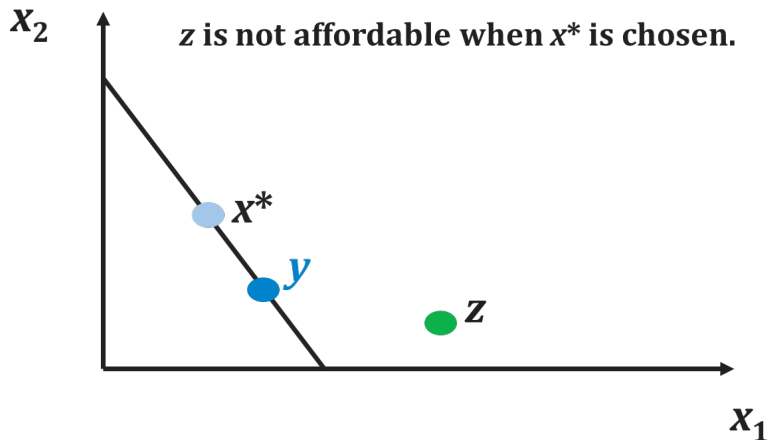
Indirect Preference Revelation

Definition (IRP)

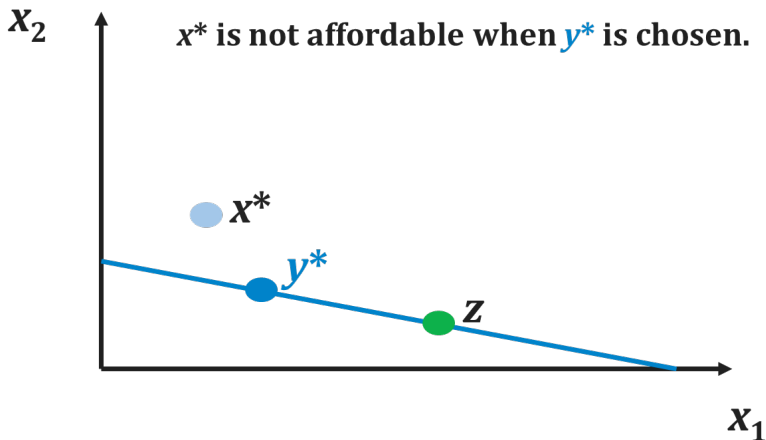
Suppose x is directly revealed preferred to y , and y directly revealed preferred to z . Then, by transitivity, x is **indirectly revealed preferred (IRP)** to z

- In more succinct notation, $x \succ y$ and $y \succ z$, then $x \succ z$

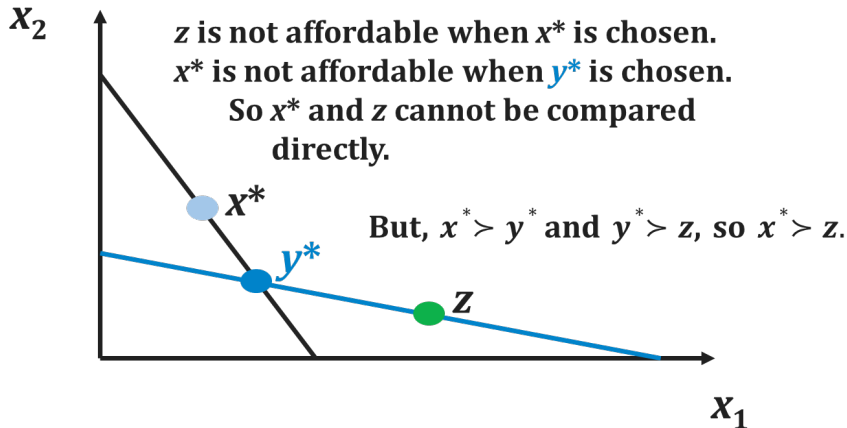
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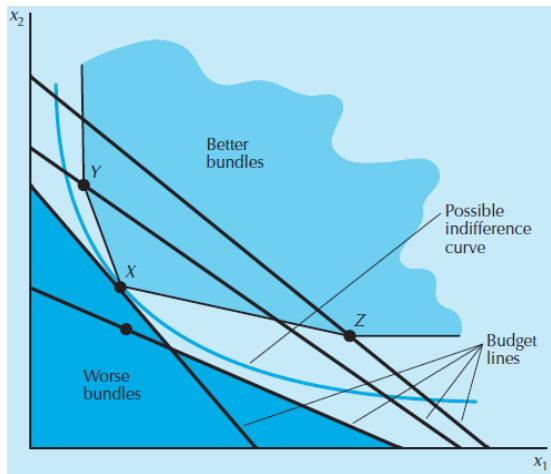


Recovering Preferences

- We can learn about consumer's preference by observing her choices
- Need to make further assumptions of preference

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- The discussion so far relies on the assumption that the consumer **has** preferences **and** she always chooses the best bundle she can afford
 - the “estimated” indifference curve is meaningless if the consumer does not behave in this way

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 - what kind of observation would lead us to conclude that the consumer was **not** maximizing?

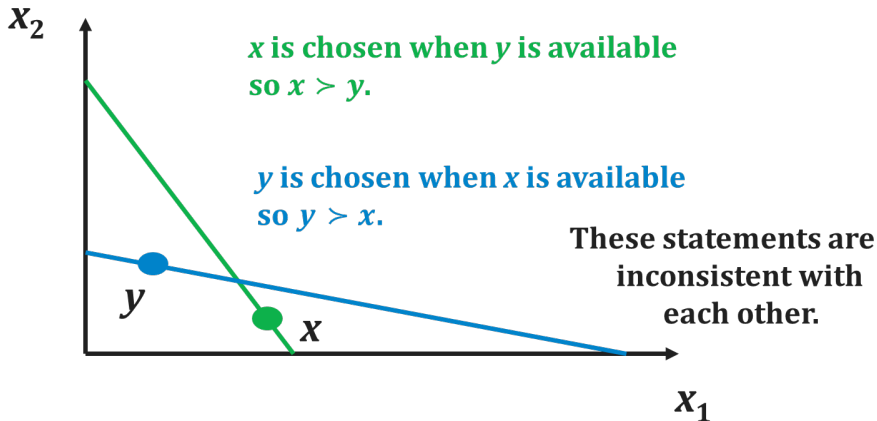
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Theorem (WARP)

Weak Axiom of Revealed Preference (WARP): *If the bundle x is revealed directly as preferred to the bundle y then it is never the case that y is revealed directly as preferred to x*

WARP



WARP

- If a bundle x is purchased at prices (p_1, p_2) and a different bundle y is purchased at prices (q_1, q_2) , then **if**

$$p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$$

- It must **not** be the case that

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- A person that makes choices in violation of the **WARP** is inconsistent with economic rationality. The WARP is a **necessary** condition for applying economic rationality to explain observed choices

Checking Violations of WARP

- A consumer makes the following choices:
 - at prices $(p_1, p_2) = (\$2, \$2)$ the choice was $(x_1, x_2) = (10, 1)$
 - at prices $(p_1, p_2) = (\$2, \$1)$ the choice was $(x_1, x_2) = (5, 5)$
 - at prices $(p_1, p_2) = (\$1, \$2)$ the choice was $(x_1, x_2) = (5, 4)$
- Is the WARP violated by these data?

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- Is the WARP violated by these data?
- At $(p_1, p_2) = (\$2, \$2)$ the choice of $(x_1, x_2) = (10, 1)$ cost $(\$2)(10) + (\$2)(1) = \$22$
 - could $(x_1, x_2) = (5, 5)$ be affordable? $(\$2)(5) + (\$2)(5) = \$20 \leq \22 .
So $(5, 5)$ was affordable while $(10, 1)$ was chosen. **(10, 1) DRP (5, 5)**
 - could $(x_1, x_2) = (5, 4)$ be affordable? $(\$2)(5) + (\$2)(4) = \$18 \leq \22 .
So $(5, 4)$ was affordable while $(10, 1)$ was chosen. **(10, 1) DRP (5, 4)**

Checking Violations of WARP

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So $(10, 1)$ was **unaffordable** while $(5, 5)$ was chosen
 - could $(x_1, x_2) = (5, 4)$ be affordable? $(\$2)(5) + (\$1)(4) = \$14 \leq \15 .
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So $(5, 5)$ was **unaffordable** while $(5, 4)$ was chosen
 - could $(x_1, x_2) = (10, 1)$ be affordable? $(\$1)(10) + (\$2)(1) = \$12 \leq \13 .
So $(10, 1)$ was affordable while $(5, 4)$ was chosen. **(5, 4) DRP (10, 1)**

Checking Violations of WARP

- In summary:
 - $(10, 1)$ $\text{DRP } (5, 5)$
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 - $(10, 1)$ DRP $(5, 5)$
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 - $(5, 5)$ DRP $(5, 4)$
 - $(5, 4)$ DRP $(10, 1)$
- The WARP states that it **cannot** be that both $(10, 1)$ DRP $(5, 4)$ and $(5, 4)$ DRP $(10, 1)$

SARP

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Theorem (SARP)

Strong Axiom of Revealed Preference: *If the bundle x is revealed preferred (directly or indirectly) to the bundle y and $x \neq y$, then it is never the case that the y is revealed preferred (directly or indirectly) to x*

SARP

- The observed choice data satisfy the SARP is both a **necessary and sufficient** condition for there to be a well-behaved preference relation that “rationalizes” the data

SARP

- The observed choice data satisfy the SARP is both a **necessary and sufficient** condition for there to be a well-behaved preference relation that “rationalizes” the data
- If the SARP is **not** violated, the behavior is considered rational. We can discover approximately where the consumer’s indifference curves are

Checking Violations of SARP

- Checking violations of SARP is more complicated as it requires to find any **arbitrary** chain that would violate SARP

		Bundles		
		1	2	3
Prices	1	20	10*	22 ^(*)
	2	21	20	15*
	3	12	15	10

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- Simple computer programs can be used to calculate IRP

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Index Numbers

- Over time, many prices change. Are consumers better or worse off “overall” as a consequence?
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 - index numbers give approximate answers to such questions
- There are two basic types of indices
 - price indices
 - quantity indices
- Each index compares expenditures in a **base period** and in a **current period** by taking the ratio of expenditures

Quantity Index Numbers

- A quantity index is a price-weighted average of quantities demanded; i.e., (p_1, p_2) can be base period prices (p_1^b, p_2^b) or current period prices (p_1^t, p_2^t)

$$I_q = \frac{p_1 x_1^t + p_2 x_2^t}{p_1 x_1^b + p_2 x_2^b}$$

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- If $(p_1, p_2) = (p_1^b, p_2^b)$, then we have the **Laspeyres quantity index**:

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Quantity Index Numbers

- If $L_q < 1$, then $p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b$
 - consumers overall were **worse off** in the current period than they were in the base period (how about $L_q > 1$?)

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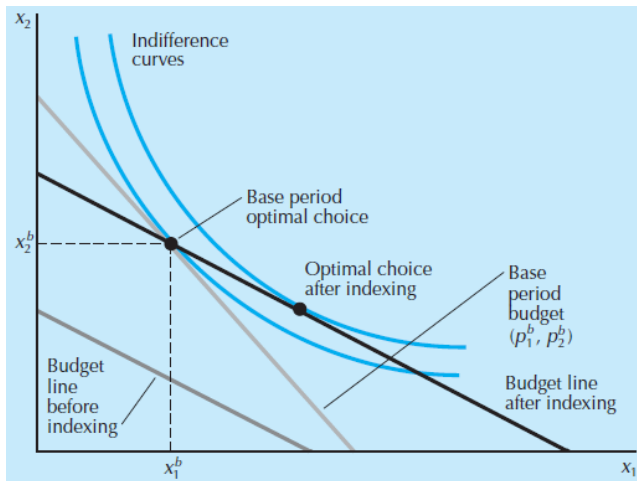
Indexing Social Security Payments

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- There have been attempts to adjust Social Security payments in a way that will keep purchasing power constant even when prices change—referred to as **indexing**
- One proposal of indexing is the following:
 - in some base year b , economists measure the average consumption bundle of senior citizens
 - in each subsequent year the Social Security system adjusts payments so that the “purchasing power” of the average senior citizen remains constant in the sense that the average Social Security recipient is just able to afford the consumption bundle available in year b

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- This indexing scheme is that the average senior citizen will almost always be better off than he or she was in the base year b
- Intuitively, it is because that consumers are able to adjust their consumption bundles (to achieve higher utility) even the consumption bundle in the base period b is affordable
- The analysis will become clearer when we learn the [Slutsky Equation](#) to decompose the [price effect](#)