Nanyang Technological University
School of Social Sciences

HE2002 Macroeconomics II

Solution to Tutorial 10

Chapter 11, Q7. Answer

1. $F(xK, xN) = (xK)^{1/3}(xN)^{2/3} = xF(K, N)$
   Yes. The Cobb-Douglas production function satisfies the property of constant returns to scale.

2. Yes. The Cobb-Douglas production function satisfies the the property of decreasing returns to capital.

3. Yes. The Cobb-Douglas production function satisfies the the property of decreasing returns to labor.

4. $Y/N = (K/N)^{1/3}$

5. In steady state, $s(Y/N) = \delta(K/N)$, which, given the production function in part (d), implies $K/N = (s/\delta)^{3/2}$

6. $Y/N = (s/\delta)^{1/2}$

7. $Y/N = (0.32/0.08)^{1/2} = 2$

8. $Y/N = (0.16/0.08)^{1/2} = 2^{1/2}$

Chapter 11, Q8. Answer

1. Substituting from problem 7 part (e) implies $K/N = (0.1/0.1)^{3/2} = 1$.

2. Substituting from problem 7 part (f), $Y/N = (0.1/0.1)^{1/2} = 1$.

3. $K/N = (0.1/0.2)^{3/2} = 0.35; Y/N = (0.1/0.2)^{1/2} = 0.71$

4. In the initial period, $K_t/N = 1$ and $Y_t/N = 1$. In this economy the depreciation rate is higher than the saving rate now, so capital per worker will be decreasing.

   Note that when capital depreciation rate may potentially differ over, a more general expression of capital accumulation equation is expressed as:

   $$K_{t+1} = K_t(1 - \delta_t) + sY_t.$$

   Therefore, when the value of δ changes in t, it will affect the value of $\frac{K_{t+1}}{N}$ onwards. In the following periods:
   $K_{t+1}/N = 0.8K_t/N + 0.1Y_t/N = 0.9$; $Y_{t+1}/N = (K_{t+1}/N)^{1/3} = 0.97$.
   $K_{t+2}/N = 0.8K_{t+1}/N + 0.1Y_{t+1}/N = 0.82$; $Y_{t+2}/N = (K_{t+2}/N)^{1/3} = 0.93$.
   $K_{t+3}/N = 0.8K_{t+2}/N + 0.1Y_{t+2}/N = 0.75$; $Y_{t+3}/N = (K_{t+3}/N)^{1/3} = 0.91$.

	K/N	Y/N
t	1.00	1.00
t+1	0.90	0.97
t+2	0.82	0.93
t+3	0.75	0.91

Chapter 11, Q9. Answer

1. Assume the production function is $Y = \sqrt{K}\sqrt{N}$.
   Suppose the economy is in its initial steady state in period $j \leq t$. The initial steady-state capital per work is $K/N = (s/\delta)^2 = (0.12/0.1)^2 = 1.44$. The initial steady-state output per worker is $Y/N = s/\delta = 0.12/0.1 = 1.2$.
   Note that when saving rate may potentially change over time, a more general expression of capital accumulation equation is:

   $$K_{t+1} = K_t(1-\delta) + s_t Y_t.$$

   Therefore, when saving rate starts to change in period $t+1$, $\frac{K_j}{N}$ will start to deviate from their initial steady state levels from $j \geq t+2$ onwards. In the following periods, the declining saving rate becomes smaller than the steady-state level, so both capital per worker K/N and output per worker Y/N will decline over time.
   $K_{t+1}/N = 0.9K_t/N + 0.12Y_t/N = 1.44$; $Y_{t+1}/N = \sqrt{K_{t+1}/N} = 1.2$.
   $K_{t+2}/N = 0.9K_{t+1}/N + 0.11Y_{t+1}/N = 1.428$; $Y_{t+2}/N = \sqrt{K_{t+2}/N} = 1.195$.
   $K_{t+3}/N = 0.9K_{t+2}/N + 0.1Y_{t+2}/N = 1.405$; $Y_{t+3}/N = \sqrt{K_{t+3}/N} = 1.185$.

	K/N	Y/N
t	1.440	1.200
t+1	1.440	1.200
t+2	1.428	1.195
t+3	1.405	1.185

2. Given the change in the depreciation rate, the new steady-state capital per work is $K/N = (s/\delta)^2 = (0.1/0.12)^2 = 0.6944$. The new steady-state output per worker is $Y/N = (K/N)^{1/2} = 0.8333$.
   During the transition, when both saving rate and capital depreciation rate may vary over time, the general expression for capital accumulation equation is:

   $$K_{t+1} = K_t(1-\delta_t) + s_t Y_t.$$

   This leads to $\frac{K_j}{N}$ to deviate from initial steady-state levels from $j \geq t+1$ onwards.
   $K_{t+1}/N = 0.88K_t/N + 0.12Y_t/N = 1.4112$; $Y_{t+1}/N = \sqrt{K_{t+1}/N} = 1.1879$.
   $K_{t+2}/N = 0.88K_{t+1}/N + 0.11Y_{t+1}/N = 1.3725$; $Y_{t+2}/N = \sqrt{K_{t+2}/N} = 1.1715$.
   $K_{t+3}/N = 0.88K_{t+2}/N + 0.1Y_{t+2}/N = 1.3250$; $Y_{t+3}/N = \sqrt{K_{t+3}/N} = 1.1511$.

	K/N	Y/N
t	1.44	1.2
t+*1*	1.4112	1.1879
t+*2*	1.3725	1.1715
t+*3*	1.3250	1.1511