MH1820 Introduction to Probability and Statistical Methods
Tutorial 8 (Week 9)

Problem 1 (Joint PDF, Marginal PDF) Let $f(x,y) = (3/16)xy^2$, $0 \le x \le 2$, $0 \le y \le 2$, be the joint PDF of X and Y.

(a) Find $f_X(x)$ and $f_Y(y)$, the marginal PDF of X and Y respectively.

(b) Are the two random variables independent? As in the discrete case, two continuous-type random variables X and Y are independent provided $f(x,y) = f_X(x)f_Y(y)$.

(c) Compute the mean μ_X and variance σ_X^2 of X.

(d) Find $\mathbb{P}(X \le Y)$.

Problem 2 (Joint PDF, Marginal PDF, Conditional PDF) Let $f(x,y) = 1/40$, $0 \le x \le 10$, $10 - x \le y \le 14 - x$ be the joint PDF of X and Y.

(a) Sketch the region of the points (x,y) satisfying the inequalities $0 \le x \le 10$, and $10 - x \le y \le 14 - x$.

(b) Find $f_X(x)$, the marginal PDF of X.

(c) Determine $h(y|x)$, the conditional PDF of Y, given that $X = x$.

(d) Calculate $\mathbb{E}[Y|X = x]$, the conditional mean of Y, given that $X = x$.

Problem 3 (Conditional PDF, Conditional Expectation)

Let X and Y be continuous random variables with joint PDF

$$f(x,y) = \begin{cases} x + \frac{3}{2}y^2, & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the conditional PDF $f(x|y)$ for all x, y.

(b) Compute the conditional expectation $E[X|Y = y]$ for all y.

(c) Find the conditional probabilities (i) $P\left(X \le \frac{1}{2}|Y = \frac{1}{2}\right)$ and (ii) $P\left(\frac{1}{4} \le X \le \frac{3}{4}|Y = \frac{1}{2}\right)$.

Problem 4 (Joint PDF, Marginal PDF, Conditional probability)
Let X and Y have the joint PDF $f(x,y) = cx(1-y)$, $0 < y < 1$, and $0 < x < 1-y$, where c is a constant.

(a) Determine c.

(b) Compute $\mathbb{P}(Y < X \mid X \leq 1/4)$.

Answer Keys.

1(a) $f_X(x) = x/2$ for $0 \leq x \leq 2$, $f_Y(y) = \frac{3y^2}{8}$ for $0 \leq y \leq 2$ **1(b)** Yes **1(c)** $\mu_X = 4/3$, $\sigma_X^2 = 2/9$ **1(d)** 3/5 **2(b)** $f_X(x) = \frac{1}{10}$ for $0 \leq x \leq 10$. **2(c)** $h(y|x) = \frac{1}{4}$ for $10 - x \leq y \leq 14 - x$ **2(d)** $\mu_{Y|x} = 12 - x$ for $0 \leq x \leq 10$. **3(a)** $\frac{2x+3y^2}{3y^2+1}$ **3(b)** $\frac{9y^2+4}{6(3y^2+1)}$ **3(c)** (i) 5/4 (ii) 1/2 **4(a)** $c = \frac{1}{8}$ **4(b)** 29/93