MH1820 Introduction to Probability and Statistical Methods
Tutorial 12 (Week 13)

Problem 1 Let $X_1, \ldots, X_{10} \sim N(\mu, \sigma^2)$ be i.i.d, where μ and σ are both unknown. Consider a test for $H_0 : \mu = 10$ against $H_1 : \mu \neq 10$ based on the test statistic $T = \frac{\overline{X}-\mu}{s/\sqrt{n}}$, where s^2 is the sample variance. Suppose we reject H_0 if and only if $|T| \geq t_0$.

(a) Find t_0 so that the size of the test is 0.05.

(b) Using the t_0 from part (a), is H_0 rejected for the following observations?

$$23.3,\ 3.5, -1.0,\ 40.3,\ 34.5,\ 9.6,\ 23.4,\ 18.5,\ 0.7,\ 9.0.$$

Problem 2 Let $X_1, \ldots, X_{10}$ be an i.i.d sample drawn from $\text{Exp}(\theta)$ where $\theta \in (0, \infty)$ is an unknown parameter. Consider a test for $H_0 : \theta = 1$ against $H_1 : \theta = \frac{1}{2}$ based on the test statistic $T = \sum_{i=1}^{n} X_i$.

(a) Find the observed value t_0 of T such that the p-value $\mathbb{P}(T \leq t_0 | H_0)$ is equal to 0.05.

(b) Using the t_0 from part (a), consider that test that rejects H_0 if and only if $T \leq t_0$. What is the size and the power of the test?

(c) Using the test from part (b), is H_0 rejected for the following observations?

$$0.1, 0.2, 0.1, 0.3, 0.5, 0.01, 1.2, 0.05, 0.001, 0.1$$

You may use the following online calculator for Gamma distribution (Notation: the shape parameter β on the website is our θ for Gamma distribution). `https://homepage.divms.uiowa.edu/~mbognar/applets/gamma.html`

Problem 3 Let $X_1, \ldots, X_5$ be an i.i.d sample drawn from Bernoulli(p) where $p \in [0, 1]$ is an unknown parameter. Consider the test for $H_0 : p = 0.2$ against $H_1 : p = 0.5$ which rejects H_0 if and only if $\sum_{i=1}^{5} X_i > 2$.

(a) Compute the probabilities for Type-I and Type-II Errors

(b) Find the size and the power of the test.

Answer Keys. **Q1(a)** $t_0 = 2.262$ **Q1(b)** Do not reject H_0 **Q2(a)** $t_0 \approx 5.425$ **Q2(b)** 0.643 **Q2(c)** Reject H_0 **Q3(a)** 0.05792, 0.5 **Q3(b)** 0.05792, 0.5