

Name:

Matric. no.:
Tutor group:

April 2021
CA2
TIME ALLOWED: 50 minutes

QUESTION 1.
(15 marks)

- (a) Solve the following linear recurrence, that is, write a_n in terms of n : $a_n = 10a_{n-1} - 25a_{n-2}$ for $n \geq 2$, with initial conditions $a_0 = 7$, $a_1 = 10$.
- (b) Using induction, prove that

$$\sum_{i=0}^{n-1} (2i + 1) = n^2, \quad \forall n \in \mathbb{N}.$$

For graders only:	Question	1(a)	1(b)	2(a)	2(b)	2(c)	3(a)	3(b)	Total
	Marks								

QUESTION 2.
(15 marks)

Let A , B , and C be sets.

- (a) Prove that $A \cap \left(\overline{(C \cup B)} \cup (\overline{B} \cap C) \right) = A \cap \overline{B}$.
- (b) Show that $(A - B) - C \subseteq A - (B - C)$.
- (c) Is $(A - B) - C = A - (B - C)$? If yes, prove it, if no, give a counterexample.

QUESTION 3.**(20 marks)**

[Blank page for extra working]

(a) Consider the sets $A = \{0, 2, 4, 6\}$ and $B = \{1, 2, 3, 4\}$.

- (i) Write out each subset of A that has cardinality 2.
- (ii) Write out the cardinalities of $A \cup B$, $A - B$, and $A \times B$.
- (iii) Find the number of subsets of $A \times B$ that have at most 2 elements.

No justification is necessary for the answers of part (a).

(b) Consider the *distinguishable* permutations of the number 436314. (Note that each permutation has six digits.)

- (i) How many are there in total?
- (ii) How many are odd numbers?

Briefly justify your answers.