MH1812 Discrete Mathematics: Quiz (CA) 1

Name: Tutorial Group:

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There are 3 (THREE) questions, please try all of them, and justify all your answers! Best of luck!

Question 1 (30 points)

a) Compute 40^{1234} modulo 2 (10 points).

b) Consider the set S of odd natural numbers, with respective operator Δ.

- Let Δ be the multiplication. Is S closed under Δ? Justify your answer (10 points).
- Let Δ be the addition. Is S closed under Δ? Justify your answer (10 points).

Solution.

a) We have that $40^{1234} \equiv 0$ modulo 2 because $40 = 2 \cdot 20 \equiv 0$ modulo 2.

b)
- The S of odd integer numbers is closed under multiplication. To see that, notice that an odd integer number is of the form $2a + 1$ for a some integer number. Then $(2a + 1)(2b + 1) = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1$ which is again an odd integer number.
- The S of odd integer numbers is not closed under addition. To see that, notice that an odd integer number is of the form $2a+1$ for a some integer number. Then $(2a+1)+(2b+1) = 2a+2b+2 = 2(a + b + 1)$ which is even number. Alternatively, one example can do. For example, take 3 and 5, they are both odd, 3+5 is 8 which is even, thus S is not closed under addition.

Question 2 (40 points)

a) Prove or disprove the following statement (20 points):

$$p \wedge (\neg(q \rightarrow r)) \equiv (p \rightarrow r).$$

b) Decide whether the following argument is valid (20 points):

$(p \vee q) \rightarrow \neg r;$
$\neg r \rightarrow s;$
$p;$
$\therefore s$

Solution.

a) One should disprove the statement. There are several ways to do so. For example, note that

$$p \wedge (\neg(q \rightarrow r)) \equiv p \wedge (\neg(\neg q \vee r)) \equiv p \wedge (q \wedge \neg r) \equiv p \wedge q \wedge \neg r$$

using the conversion theorem and De Morgan law. Now we see, for example, that if p is true and r is false, then $p \wedge q \wedge \neg r$ can take the value true when q is true, but $p \rightarrow r$ is then false, no matter the value of q. It is also possible to find the same conclusion by doing a truth table.

p	q	r	$q \rightarrow r$	$p \wedge \neg(q \rightarrow r)$	$p \rightarrow r$
T	T	T	T	F	T
T	T	F	F	T	F
T	F	T	T	F	T
T	F	F	T	F	F
F	T	T	T	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

b) Decide whether the following argument is valid (20 points):

$(p \vee q) \rightarrow \neg r;$
$\neg r \rightarrow s;$
$p;$
$\therefore s$

Solution. We start by noticing that
$p;$
$\therefore (p \vee q)$
Then
$(p \vee q) \rightarrow \neg r;$
$p \vee q;$
$\therefore \neg r$
Finally
$\neg r \rightarrow s;$
$\neg r;$
$\therefore s$
and we conclude that the argument is valid.

We can come to the same conclusion using a truth table. Note that we care only about the critical rows, those for which the premises are true. Thus in the table below, we assume that p is always true.

s	q	r	$p \vee q$	$p \vee q \to \neg r$	$\neg r \to s$	
T	T	T	T	F		
T	T	F	T	T	T	critical
T	F	T	T	F		
T	F	F	T	T	T	critical
F	T	T	T	F		
F	T	F	T	T	F	
F	F	T	T	F		
F	F	F	T	T	F	

We see that there are only 2 critical rows, for which s is true, therefore the argument is valid.

Question 3 (30 points)

Consider the domains $X = \{2, 4, 6\}$ and $Y = \{2, 3\}$, and the predicate $P(x, y) =$ "x is a multiple of y". What are the truth values of these statements:

a) $\forall x \in X,\ \exists y \in Y,\ P(x, y)$ (15 points).

b) $\neg(\forall x \in X,\ \forall y \in Y,\ P(x, y))$ (15 points).

Solution.

a) The first one is true. We check all values in X. For $x = 2$, there exists $y = 2$ such that $x = 2$ is a multiple of $y = 2$. For $x = 4$, there exists $y = 2$ such that $x = 4$ is a multiple of $y = 2$. For $x = 6$, there exists $y = 2$ such that $x = 6$ is a multiple of 2.

b) $\neg(\forall x \in X,\ \forall y \in Y,\ P(x, y))$ can be rewritten as

$$\exists x \in X,\ \exists y \in Y, \neg P(x, y).$$

So it is true. There exists an x, take $x = 2$, and there exists a y, take $y = 3$, such that $x = 2$ is not a multiple of $y = 3$.