AY 23/24 MH1820 Midterm Test

Name: ..... **Matriculation Number:** .....

Tutorial Group: .....

Instructions

- This test consists of 5 multiple choice questions and 3 computational questions.
- For each of the multiple choice questions, there is only one correct answer. Tick the correct answer.
- Answer all questions. The marks for each question are indicated.
- For the computational questions, write out the steps in your calculations clearly in the space provided after each question. Express numerical values up to 4 decimal places.

1. [2 marks] How many words can be formed from the letters in SWIFTIES? Here every permutation of these letters counts as a word.

◯ $8!$ ◯ $\frac{8!}{2!}$ ◯ $\frac{8!}{2!2!}$ ◯ $\frac{8!}{2!2!4!}$

2. [2 marks] Mr. Tan has 8 friends, of whom 5 will be chosen to attend a party. How many choices are there if 2 particular friends of Mr. Tan, say X and Y, will not attend the party together?

◯ 20 ◯ 30 ◯ 35 ◯ 36

3. [2 marks] Two fair dice (say A and B) are rolled independently. What is the conditional probability that **at least** one of the dice lands on an **even** number given that the two dice land on different numbers ?

◯ $\frac{1}{5}$ ◯ $\frac{2}{5}$ ◯ $\frac{3}{5}$ ◯ $\frac{4}{5}$

4. [2 marks] A website receives hits at a rate of 180 per hour according to a Poisson distribution. What is the probability that there are **at least** 10 hits in the first 2 minutes?

◯ $\sum_{x=10}^{\infty} e^{-180}\frac{180^x}{x!}$ ◯ $e^{-10}\frac{10^{-2}}{2!}$ ◯ $1-\sum_{x=0}^{9} e^{-6}\frac{6^x}{x!}$ ◯ $e^{-6}\frac{6^{10}}{10!}$

5. [2 marks] If $\mathbb{E}[X]=1$ and $\mathrm{Var}[X]=5$, find $\mathbb{E}[(2+X)^2]$.

◯ 12 ◯ 13 ◯ 14 ◯ None of the above

6. [6 marks] Consider the following game: Three fair dice are rolled independently. A player bets on one of the numbers 1 through 6. If the number bet by the player appears i times, where $i = 1, 2, 3$, then the player wins $\$(2 \times i)$; if the number bet by the player does not appear on any of the dice, then the player loses \$2 (i.e. win $-\$2$).

(a) What is the probability that the number bet by the player appears exactly two times?

(b) Let X denote the expected winning (in \$) of the player. Find $\mathbb{E}[X]$.

(a)

(b)

7. [6 marks] Let X be a continuous random variable with PDF given by

$$f(x) = 3x^2 \text{ for } 0 \leq x \leq 1 \text{ and } f(x) = 0 \text{ elsewhere.}$$

(a) Compute the CDF $F(x)$ of X.

(b) Compute the PDF of the random variable $Y = e^X$.

(a)

(b)

8. [8 marks] A student is waiting to receive notification mail telling her whether she has been accepted to a certain college. She estimates that her probability of being accepted is 0.6. She also estimates that the conditional probabilities of receiving notification mail on the following days of next week, given that she is accepted (or rejected) as follows:

Day	$\mathbb{P}$(mail|accepted)	$\mathbb{P}$(mail|rejected)
Monday	0.15	0.05
Tuesday	0.20	0.10
Wednesday	0.25	0.10

(a) What is the probability that she receives mail on Monday?

(b) What is the conditional probability that she receives mail on Tuesday given that she does not receive mail on Monday?

(c) What is the conditional probability that she will be accepted given that there is no mail received on all three days (i.e. Monday, Tuesday and Wednesday)?

(a)

(b)

(c)

End of Test