Nanyang Technological University
HE2001 Microeconomics II

Solution 5

Question 1

(a) The rate of growth of the market value in year *t* is

$$\frac{W'(t)}{W(t)} = \frac{(0.2 - 0.002t)e^{0.2t-0.001t^2}}{e^{0.2t-0.001t^2}} = 0.2 - 0.002t$$

To make sure the growth rate is greater than 5%, we need

$$0.2 - 0.002t > 0.05 \rightarrow t < 75$$

(b) At the bank interest rate of 5%, he should let the trees get 75 years of age when their rate of return will fall to 5%. After that, it is better to sell the trees and put the proceeds in the bank to earn the interest return.

(c) The value is maximized when

$$W'(t) = (0.2 - 0.002t)e^{0.2t-0.001t^2} = 0 \rightarrow t = 100$$

Question 2

(a) The after-tax interest rate of ordinary bonds is $(10\%)(1 - 40\%) = 6\%$, worse than the municipal bonds, so he should buy municipal bonds.

(b) For Hunter, the after-tax interest rate of ordinary bonds is $(10\%)(1 - 25\%) = 7.5\%$, better than the municipal bonds, so he should buy ordinary bonds.
(Note that the fact that different interest rates arise in equilibrium is perhaps due to different tax schemes for different kinds of people, so there are different levels of demand.)

(c) Fisher buys municipal bonds, so he pays zero tax.

Hunter buys ordinary bonds, so he pays a tax of $(\$10{,}000)(10\%)(25\%) = \$250$.

(d) Without any interest tax, both will choose ordinary bonds which give a higher return.
(Note that the rate of interest is actually determined endogenously in the market. Here we are assuming that it remains the same.)

For Fisher, the increase in income comes from the higher interest rate when adjusting his portfolio from municipal bonds to ordinary bonds. So his income will increase by $(\$1{,}000{,}000)(10\% - 7\%) = \$30{,}000$.

For Hunter, the increase in income comes from the reduced tax rate. So his income will increase by $(\$10{,}000)(10\%)(25\%) = \$250$ (exactly the tax he pays in part (c)).

(e) The demand for municipal bonds would be reduced to zero.
(Note that now, with no taxes on normal bonds, there are no frictions that prevent different interest rates in the market. Arbitrage means that they should have exactly the same yields/effective interest rates.)

(f) The new municipal bonds need to pay at least a 10% interest rate.

(g) With declining demand, the price of the old bonds will fall, so their actual interest rate is increasing until their yield equals 10%.

Question 3

(a) The bond is worth its present value for the investor:

$$PV = \frac{\$200}{1.1} + \frac{\$200}{1.1^2} + \frac{\$200 + \$2{,}000}{1.1^3} = \$2{,}000$$

(b) Given a lower interest rate, the value becomes:

$$PV = \frac{\$200}{1.05} + \frac{\$200}{1.05^2} + \frac{\$200 + \$2{,}000}{1.05^3} = \$2{,}272.32$$

(c) Dr. No should say yes only if the money offered ($2,200) is more than the present value of the bond. So he should say YES if the interest rate is 10% and NO if the interest rate is 5%.

(d) The required money flow is exactly the same as that generated by the bond, so Dr. No will need $2,272.32 for this program, equal to the present value of the bond calculated in part (b).

If the interest rate were 10%, he would only need $2,000. It becomes more dangerous since the plan now becomes cheaper.