Decomposition of Price Change Effect

Jubo Yan

Division of Economics, Nanyang Technological University

yanjubo@ntu.edu.sg

January 22, 2024

Overview

1. Price Change
2. Substitution and Income Effects
3. Rate of Change
4. Hicks Substitution Effect

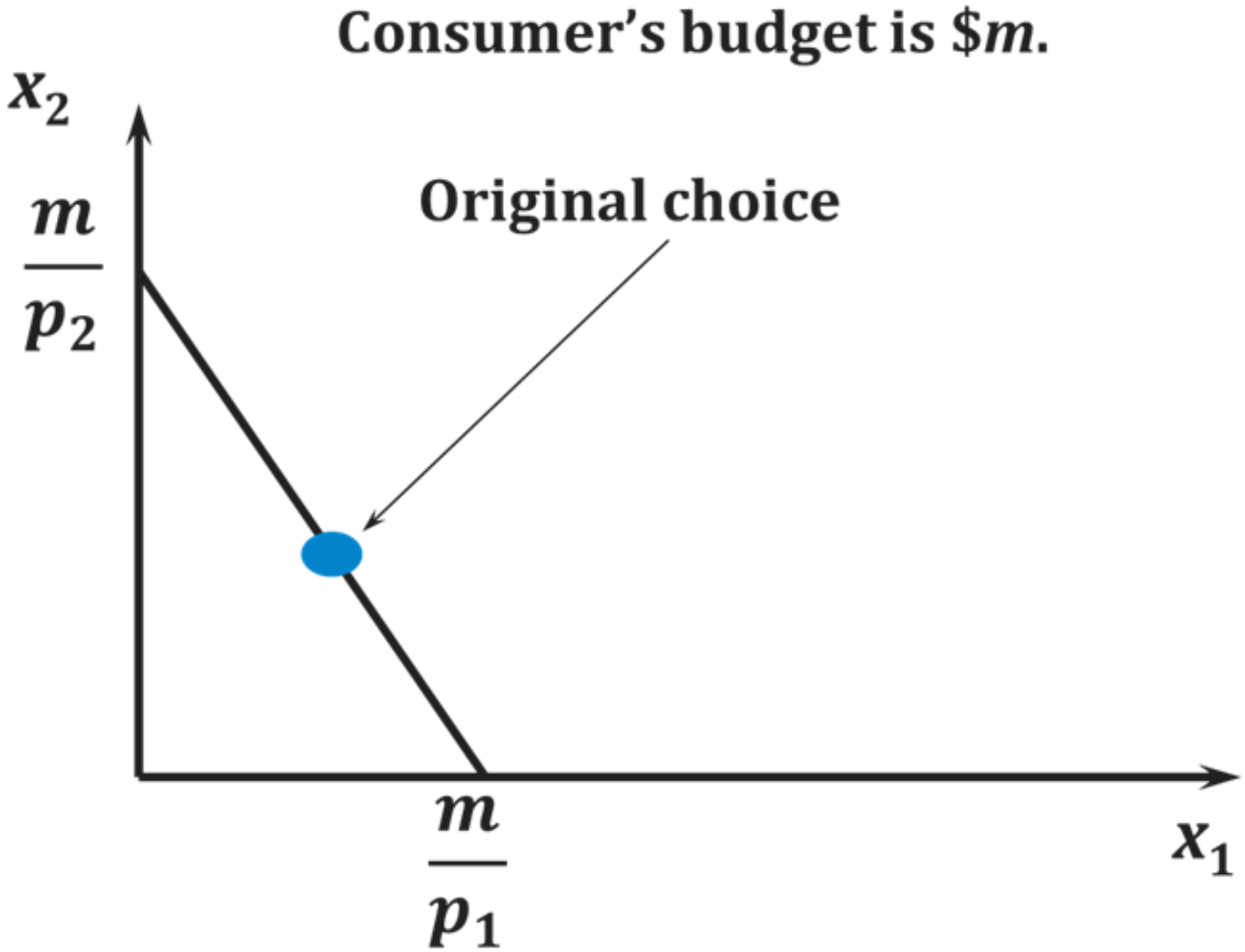
Motivation

- Economists often are concerned with how a consumer's behavior changes in response to changes in the economic environment
- We examine how a consumer's choice of a good responds to changes in its price
- Natural to think that when the price of a good rises the demand for it will fall
  - it is, however, possible to construct examples where the optimal demand for a good decreases when its price falls–Giffen good
  - another example is the number of working hours when wages change

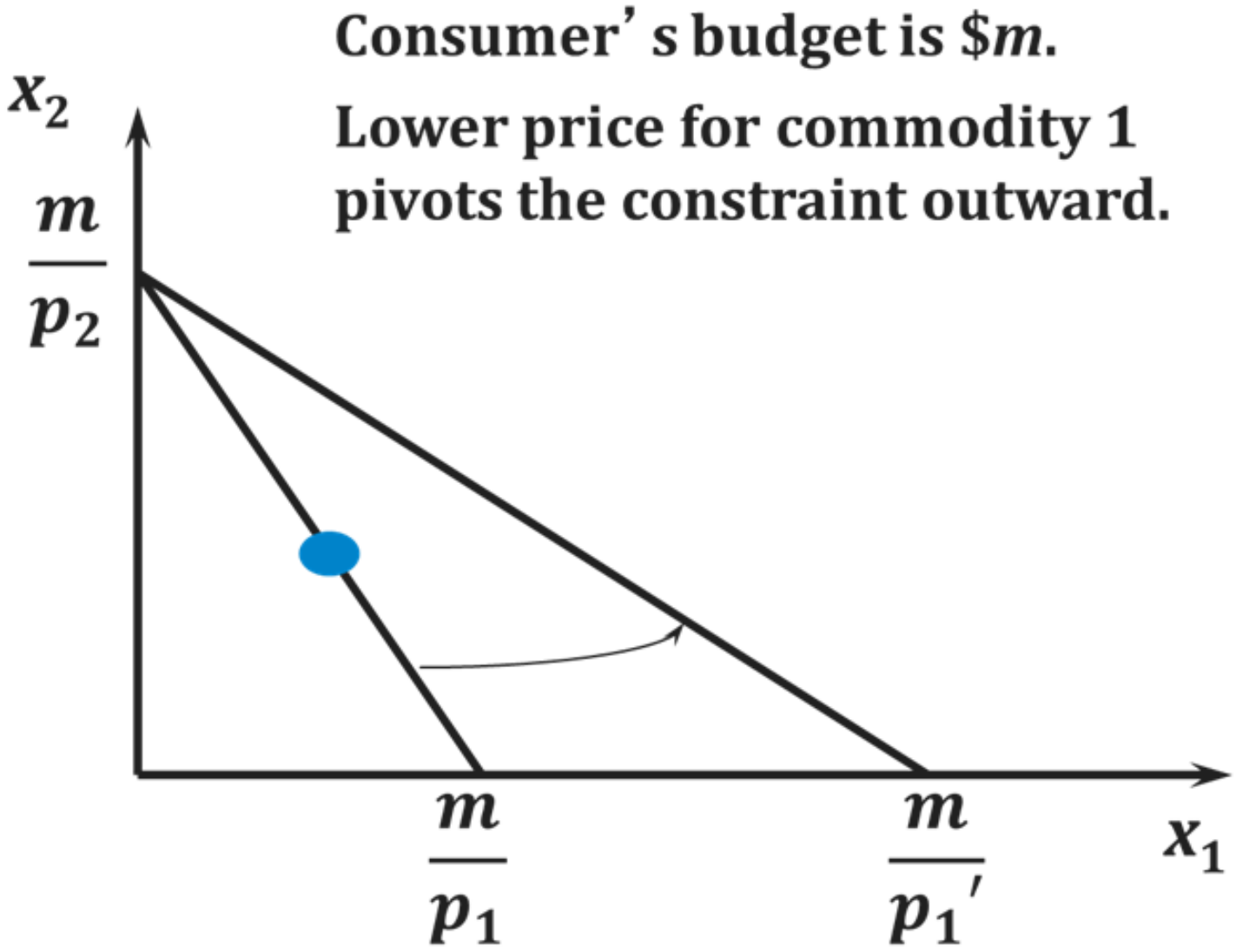
Effect of Price Change

- What happens when a commodity's price decreases?
- Substitution effect: the commodity is relatively cheaper, so consumers substitute it for now relatively more-expensive other commodities
- Income effect: the consumer's budget of $m can purchase more than before, as if the consumer's income rose, with consequent income effects on quantities demanded

Effect of Price Change



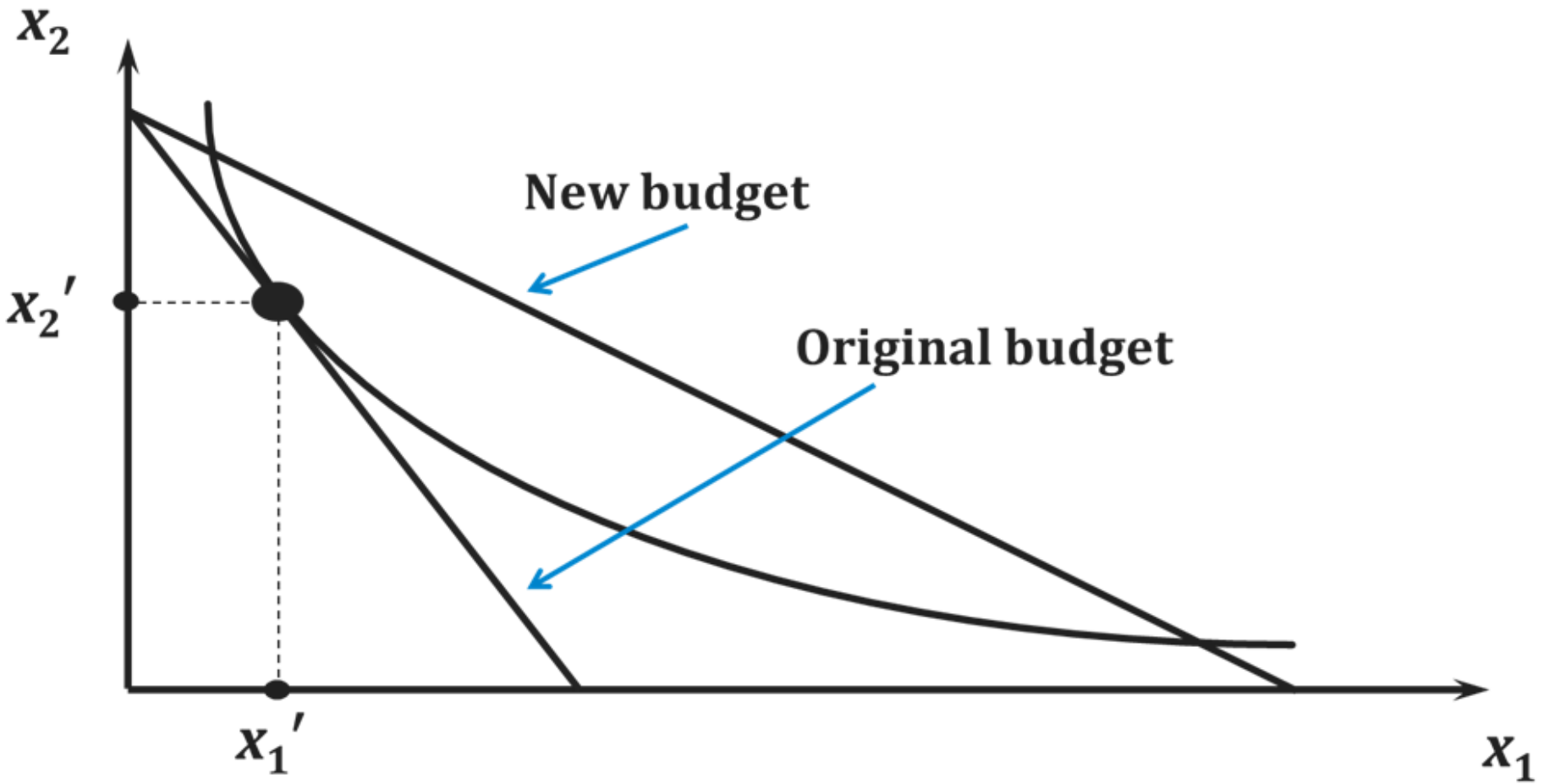
Effect of Price Change



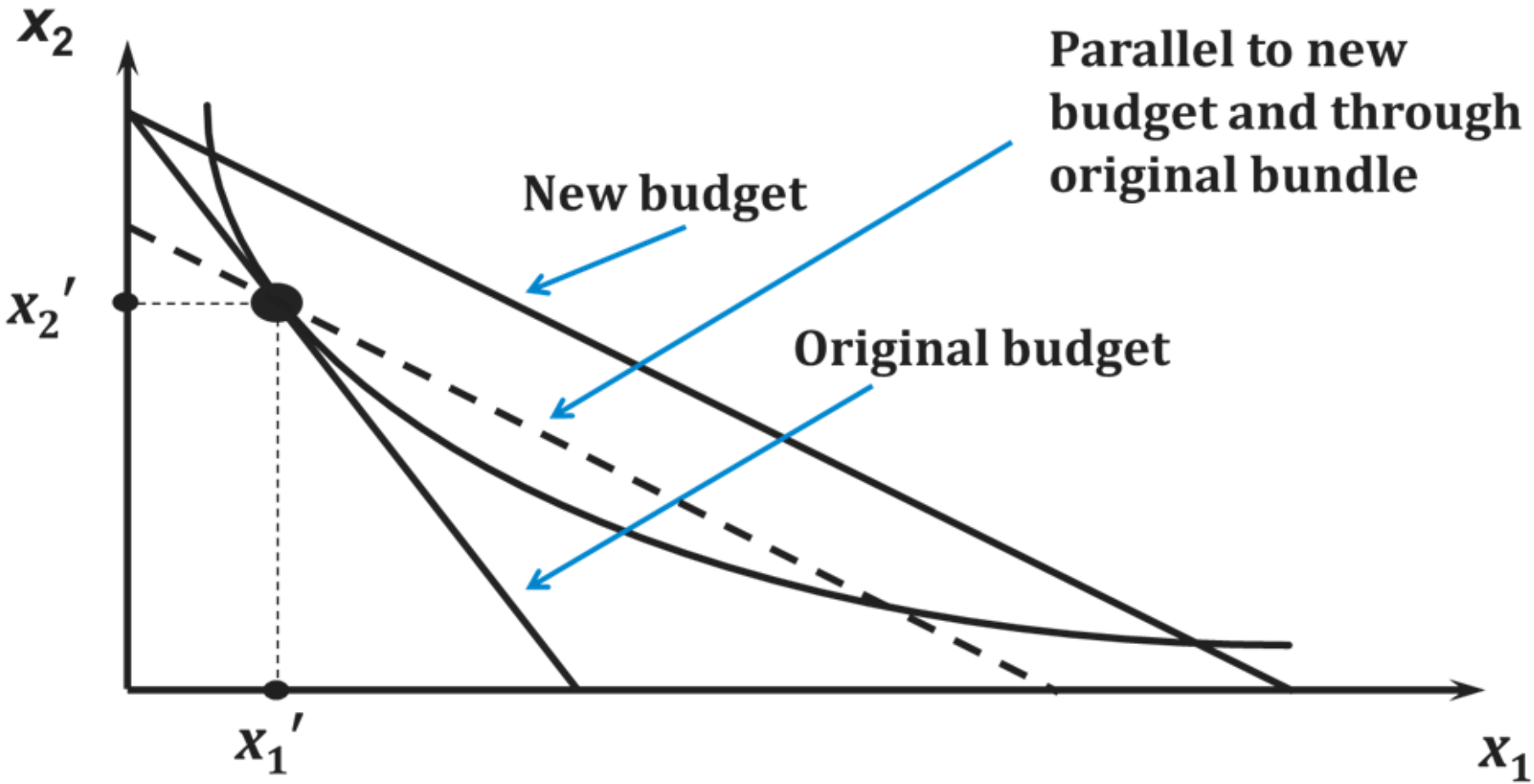
Substitution and Income Effects

- Slutsky discovered that changes to demand from a price change are always the sum of a pure substitution effect and an income effect
- We break the price movement into two steps:
  1. let relative prices change and keep purchasing power constant
  2. let purchasing power change while holding the prices constant
- Slutsky isolated the change in demand due only to the change in relative prices by asking:
  "*What is the change in demand when the consumer's income is adjusted so that, at the new prices, she can only just buy the original bundle?*"

Budget Change



Pure Substitution Effect



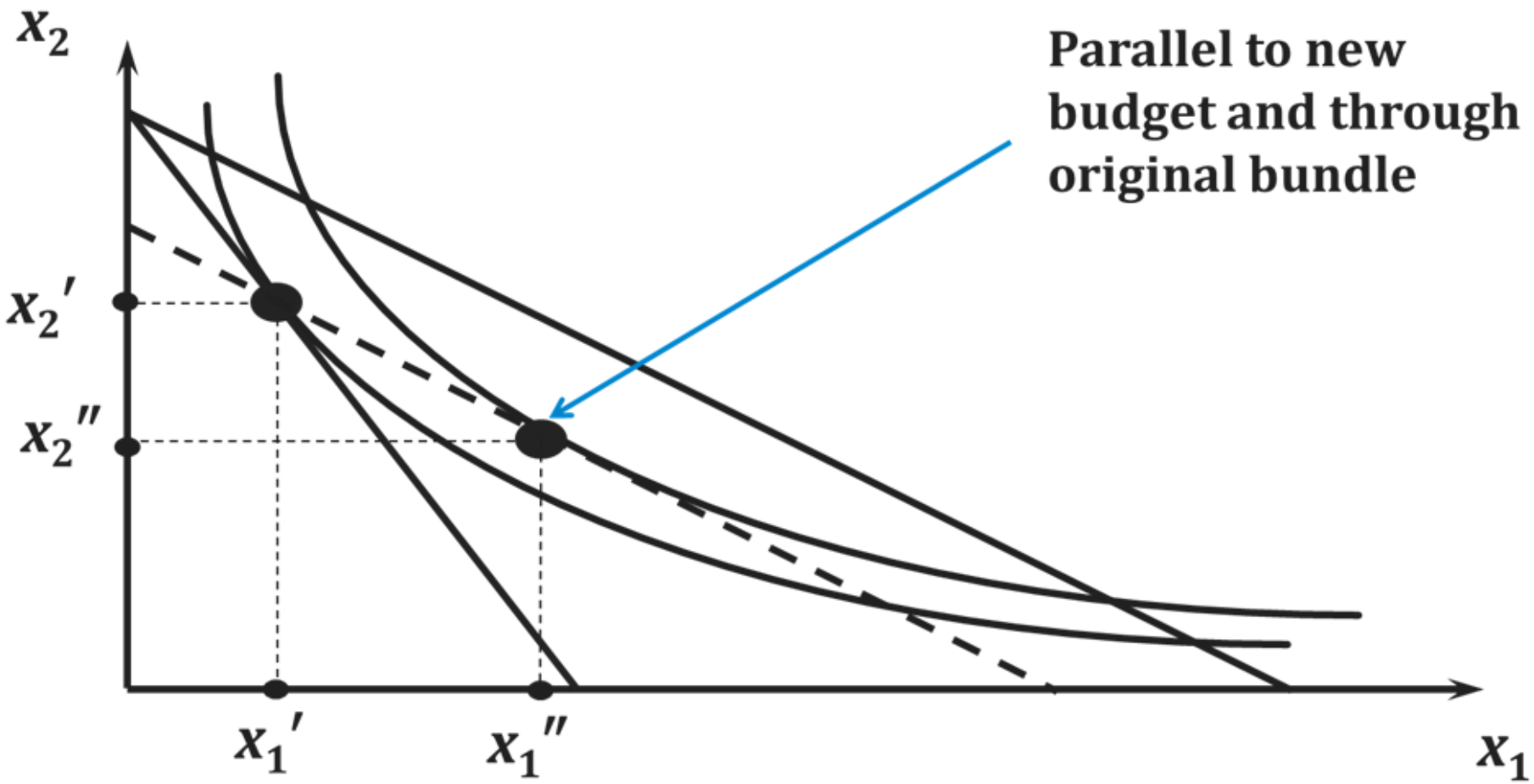
Pure Substitution Effect

- As shown in the graph, the original consumption bundle (x_1', x_2') is just affordable under the pivoted budget line
- How much do we need to adjust money income to achieve the pivoted budget line?
- Let m' be the amount of money income that will make (x_1', x_2') just affordable

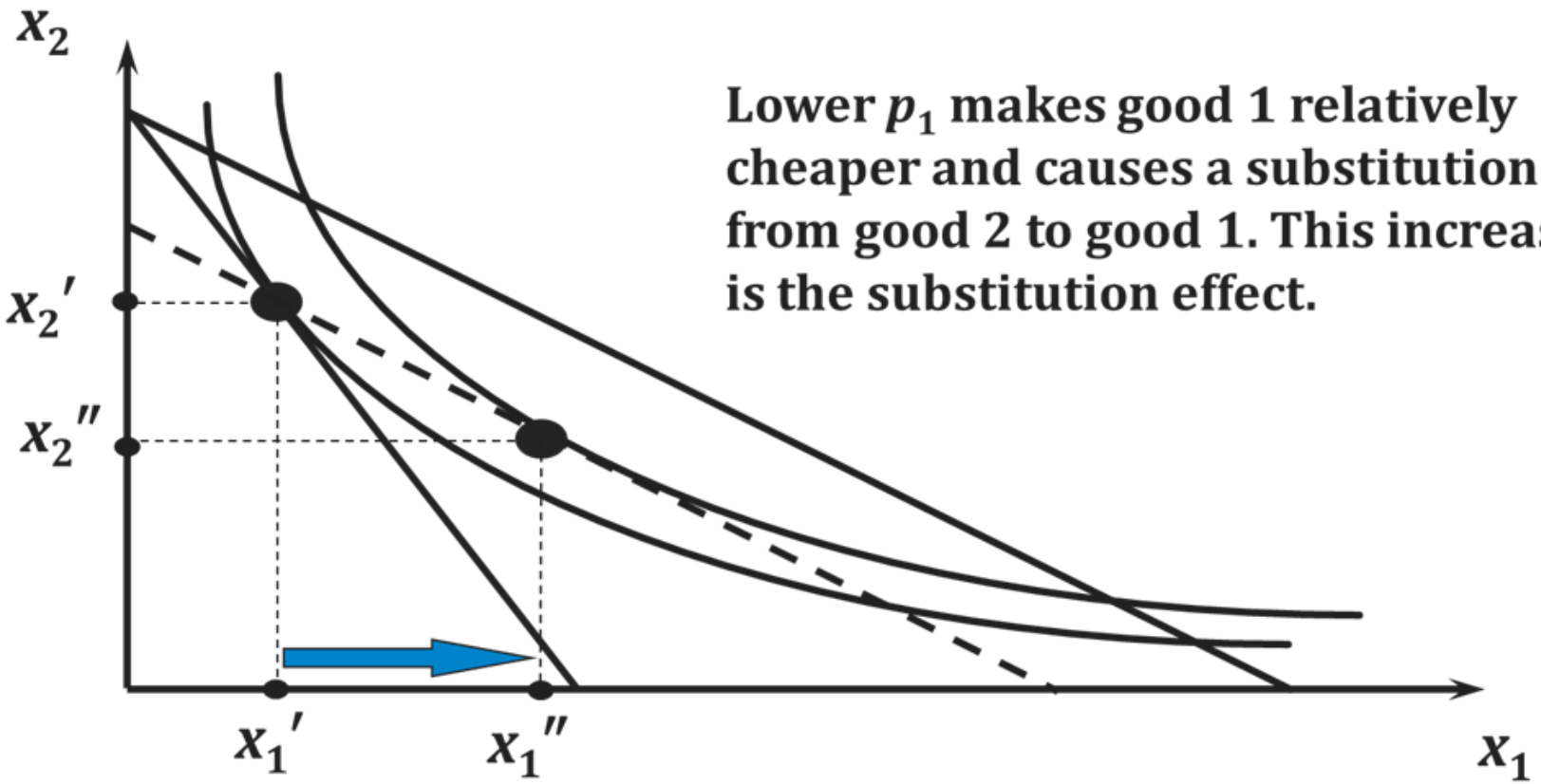
$$m' = p_1' x_1' + p_2 x_2'$$
$$m = p_1 x_1' + p_2 x_2'$$

- $\Delta m = m' - m = x_1' (p_1' - p_1) = x_1' \Delta p_1$ makes the old bundle (x_1', x_2') just affordable

Pure Substitution Effect



Pure Substitution Effect

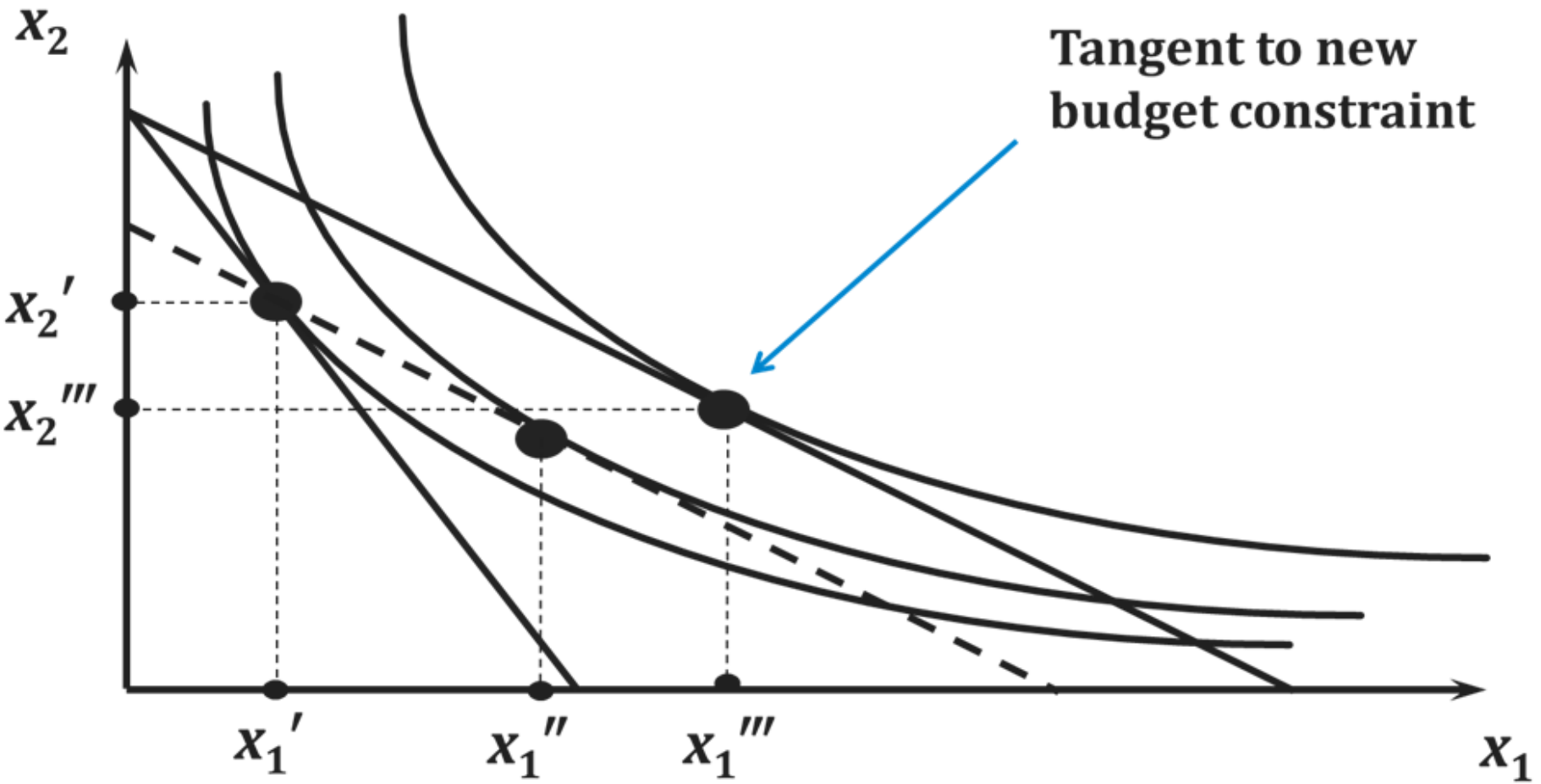


Pure Substitution Effect

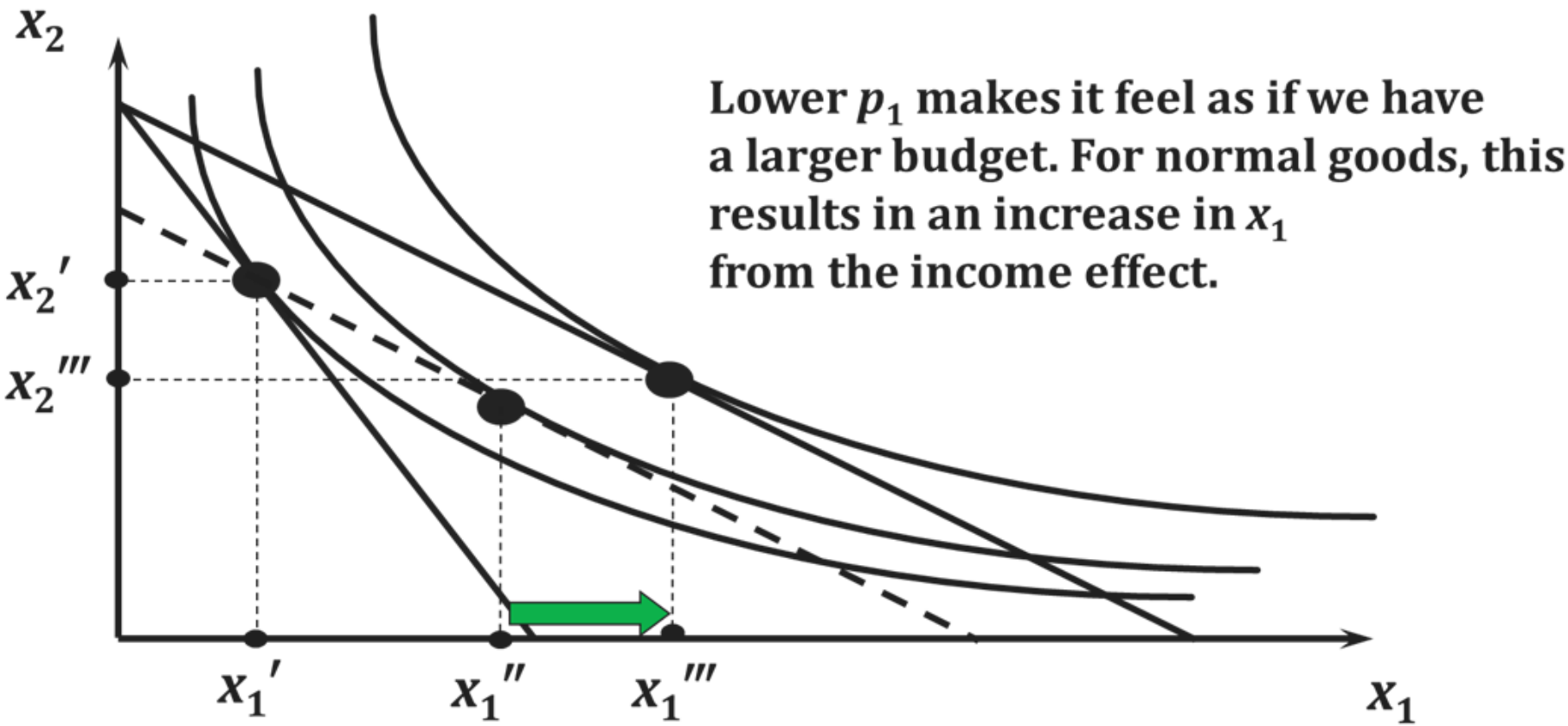
- Although (x_1', x_2') is still affordable, it is not generally the optimal purchase at the pivoted budget line
- The movement from (x_1', x_2') to (x_1'', x_2'') is known as substitution effect

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m)$$

Income Effect



Income Effect



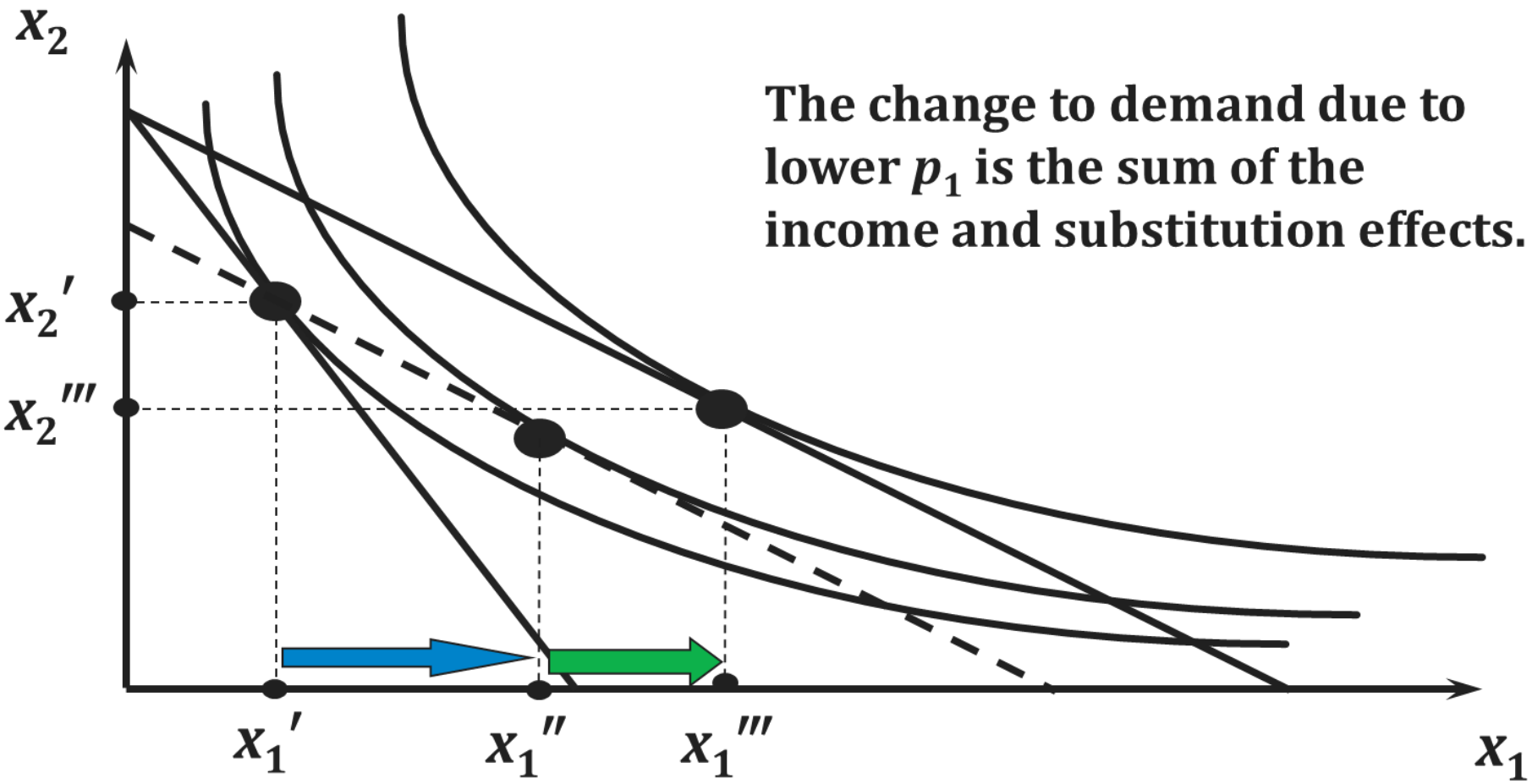
Income Effect

- For income effect, we just need to change income from m' to m keeping the prices constant at (p_1', p_2)

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m')$$

- When the price of a good decreases, we need to decrease income in order to keep purchasing power constant
  - normal good: income $\downarrow$ demand $\downarrow$
  - inferior good: income $\downarrow$ demand $\uparrow$

Overall Demand Change



Overall Demand Change

- The total change in demand Δx_1 can be decomposed into substitution and income effects

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

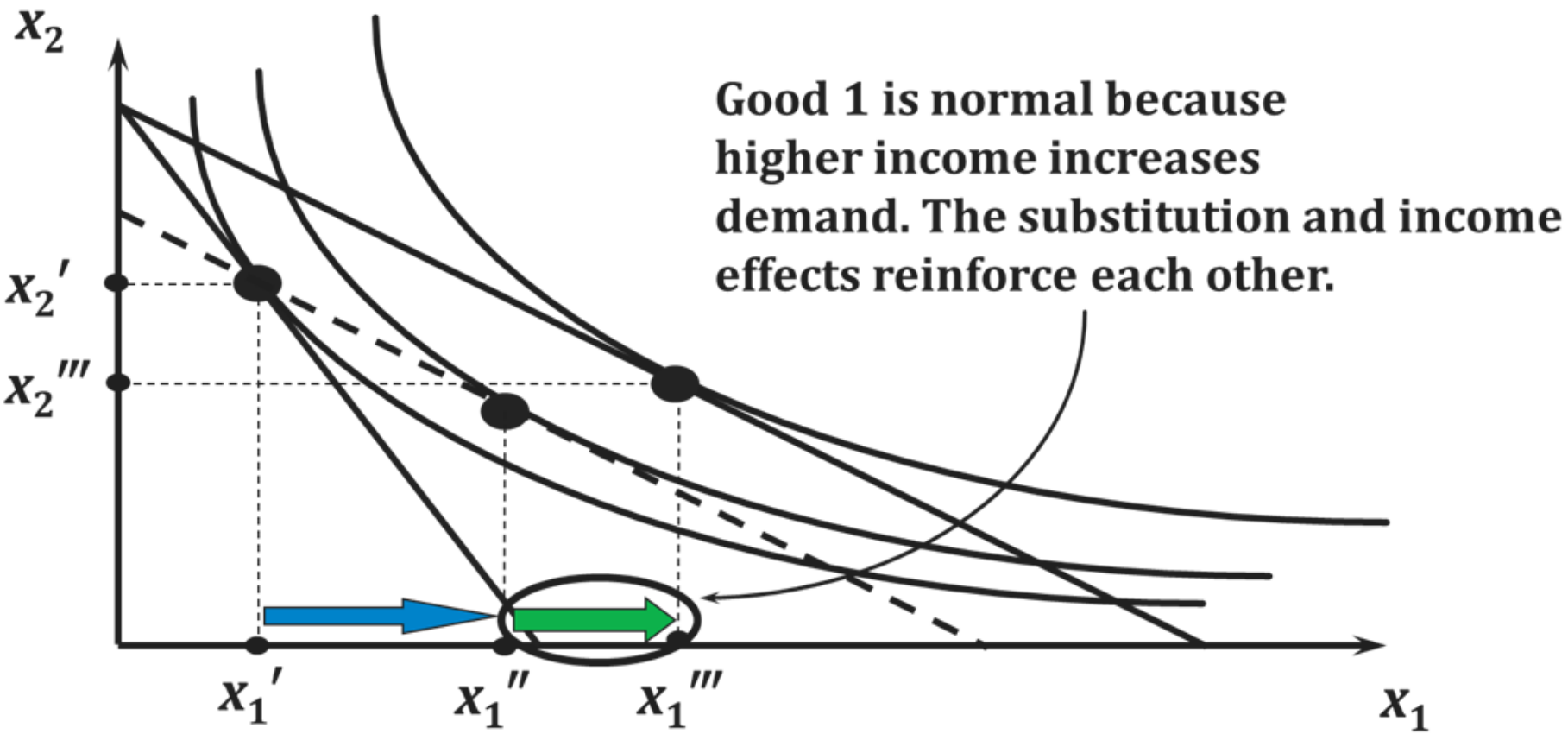
$$x_1(p_1', m) - x_1(p_1, m) = \underbrace{[x_1(p_1', m') - x_1(p_1, m)]}_{\textit{substitution effect}} + \underbrace{[x_1(p_1', m) - x_1(p_1', m')]}_{\textit{income effect}}$$

- This is called the Slutsky Identity which can be used to analyze the change of demand in response to price change
  - normal good: $p_1 \uparrow \Longrightarrow \Delta x_1^s \downarrow + \Delta x_1^n \downarrow \Longrightarrow \Delta x_1 \downarrow$
  - inferior good: $p_1 \uparrow \Longrightarrow \Delta x_1^s \downarrow + \Delta x_1^n \uparrow \Longrightarrow \Delta x_1 ?$
    - not too inferior: $|\Delta x_1^s| > |\Delta x_1^n| \Longrightarrow \Delta x_1 \downarrow$
    - Giffen good (very inferior): $|\Delta x_1^s| < |\Delta x_1^n| \Longrightarrow \Delta x_1 \uparrow$

Normal Goods

- Most goods are normal (i.e., demand increases with income)
  - the "estimated" indifference curve is meaningless if the consumer does not behave in this way
- The substitution and income effects reinforce each other when a normal good's own price changes
- Since both the substitution and income effects increase demand when own price falls, a normal good's ordinary demand curve slopes down
- The law of downward-sloping demand therefore always applies to normal goods

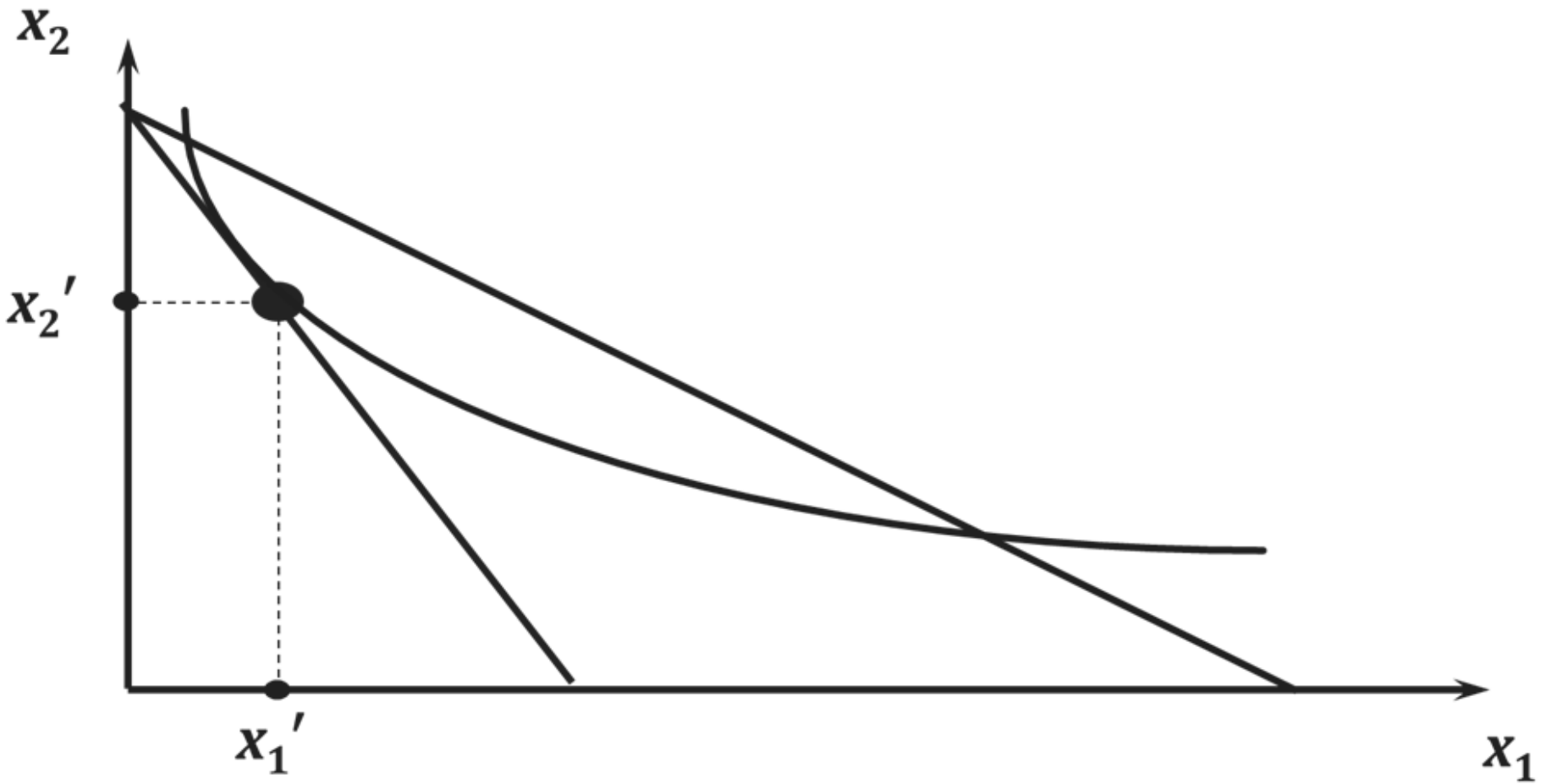
Normal Goods



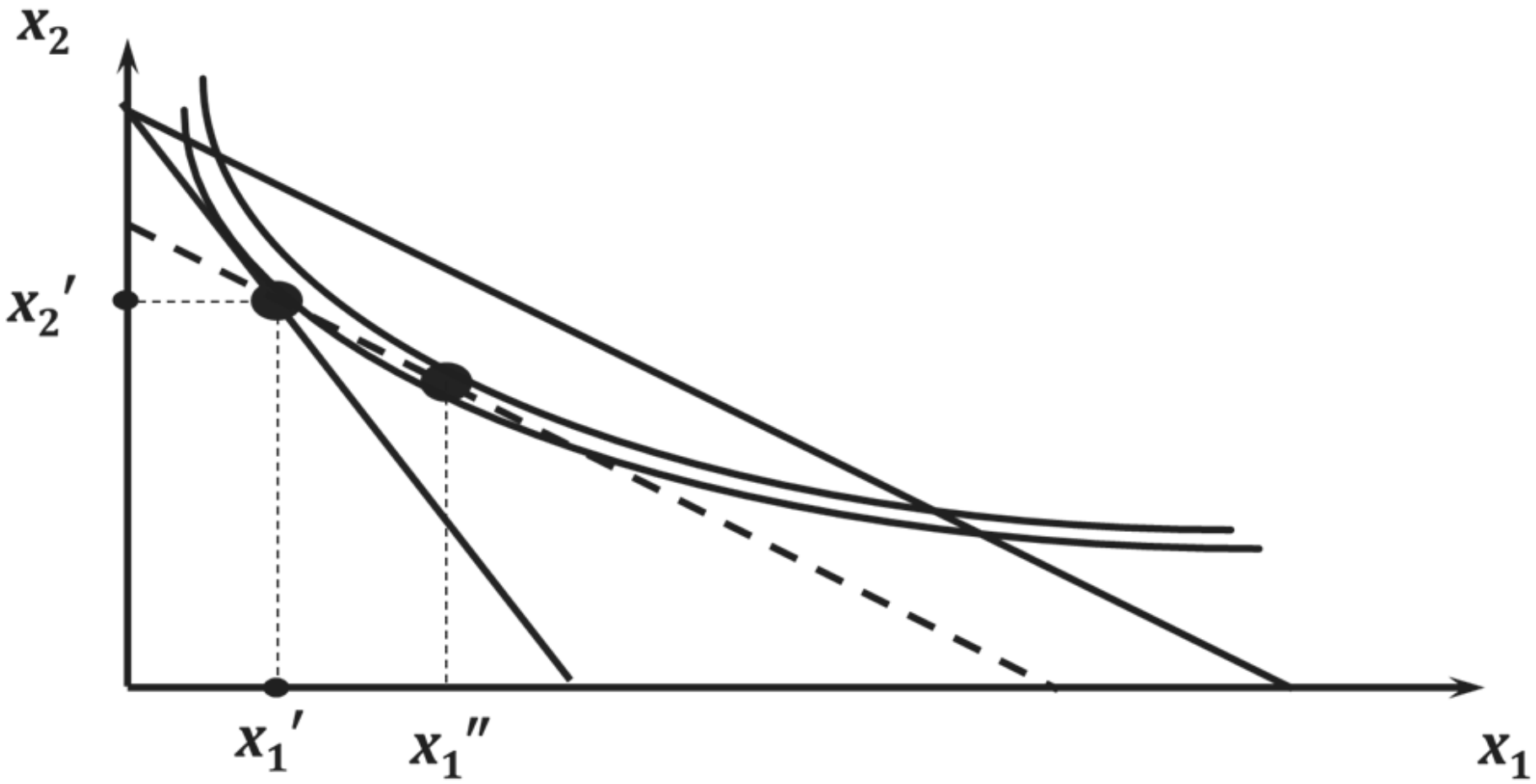
Inferior Goods

- Some goods are income-inferior
  - demand is reduced by higher income
- The substitution and income effects oppose each other when an income-inferior good's own price changes

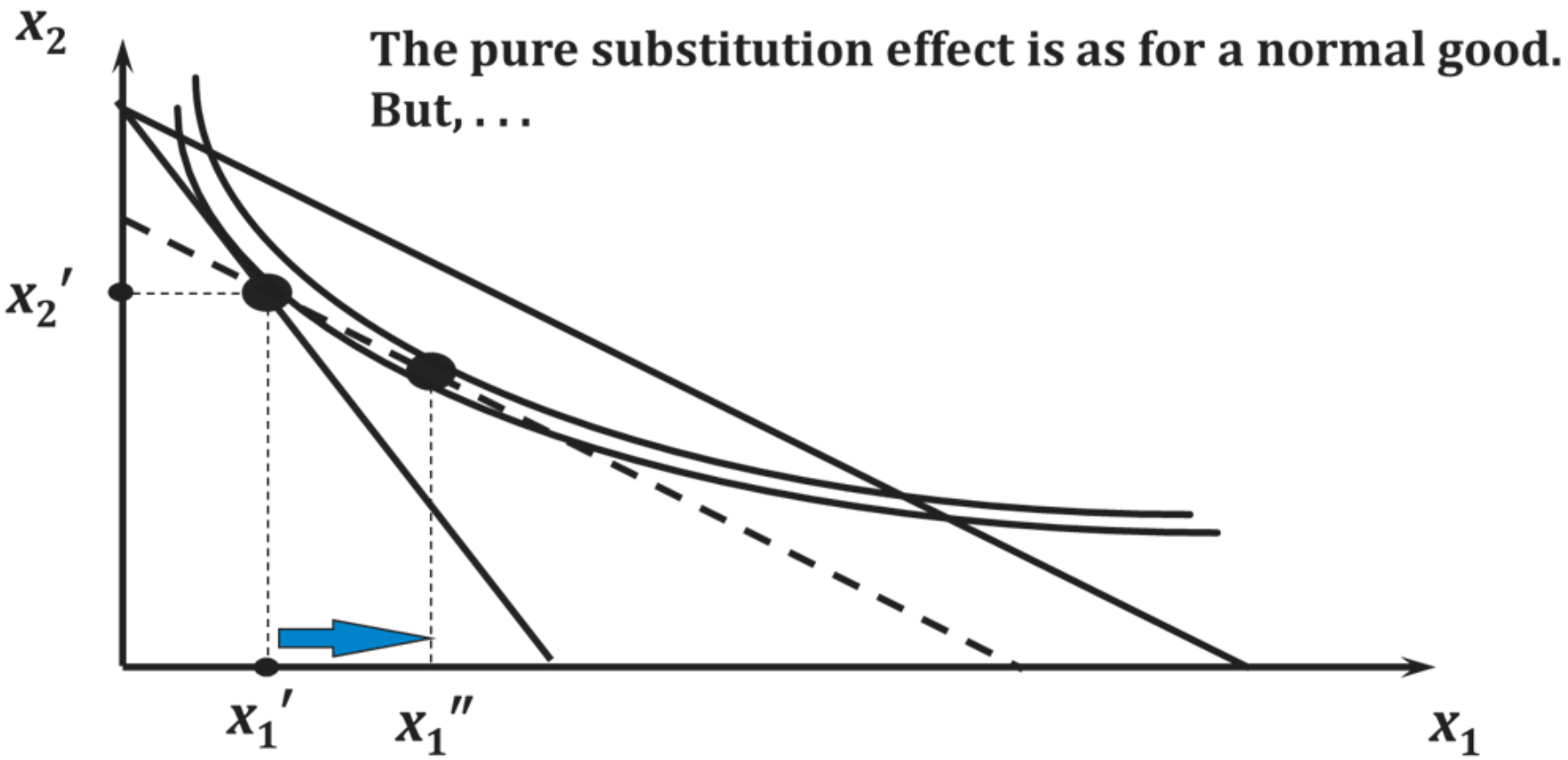
Inferior Goods



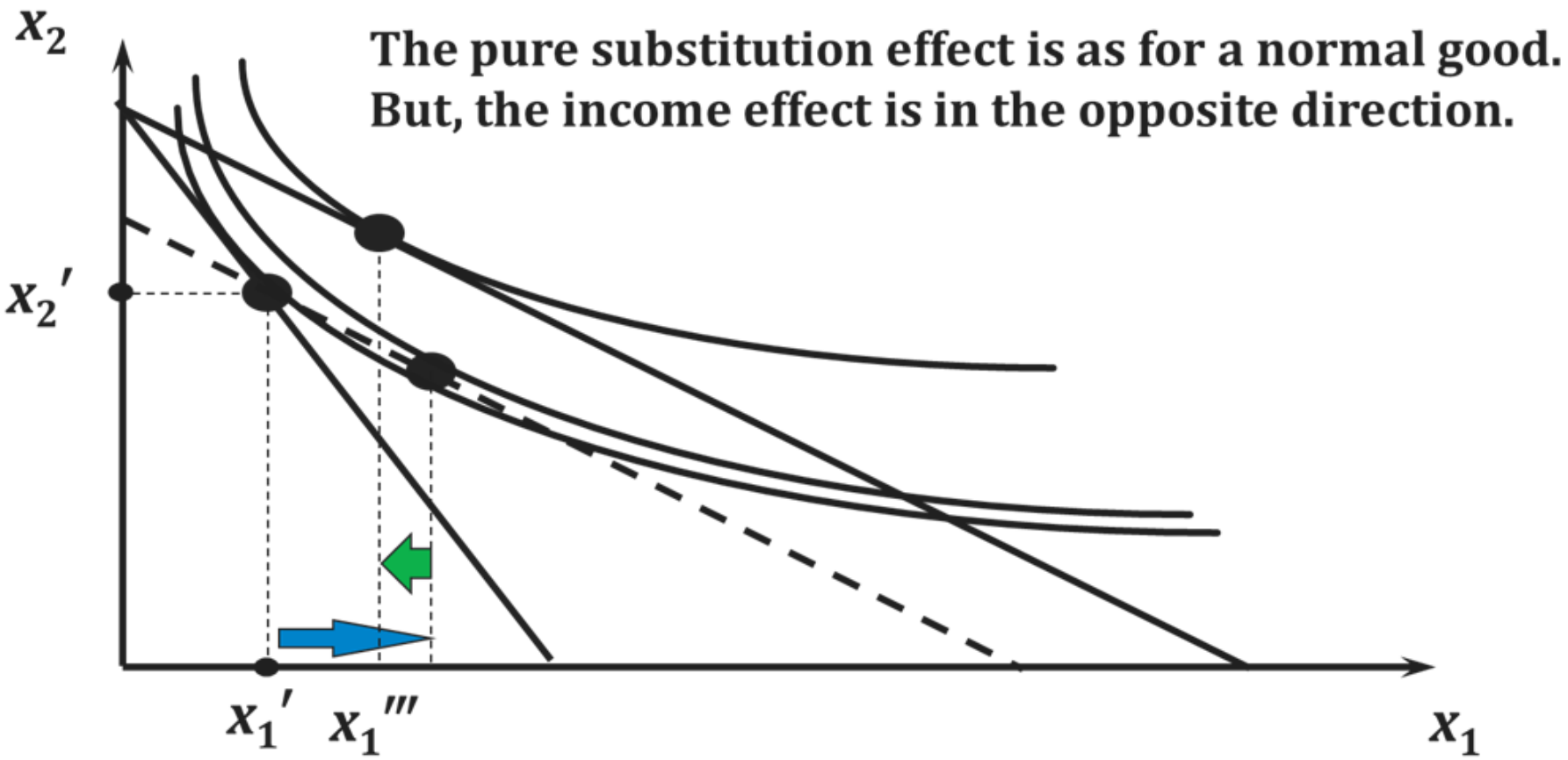
Inferior Goods



Inferior Goods



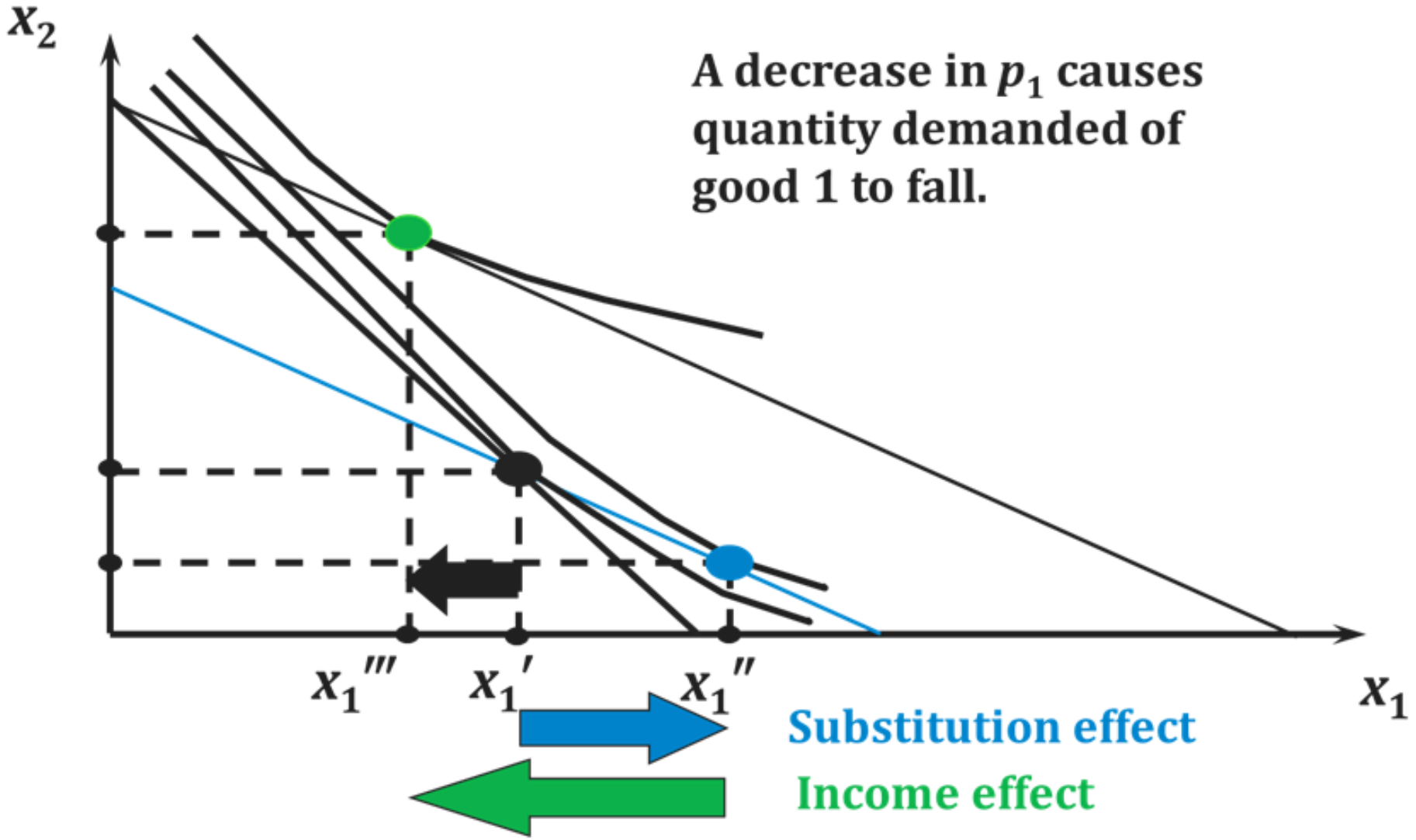
Inferior Goods



Giffen Goods

- In rare cases of extreme income-inferiority, the income effect may be larger in size than the substitution effect, causing quantity demanded to fall as own price falls
  - Giffen goods
- Slutsky's decomposition of the effect of a price change into a pure substitution effect and an income effect thus explains why the law of downward-sloping demand is violated for extremely income inferior goods

Giffen Goods



Rate of Change

- It is useful to express the Slutsky identity in terms of rates of change

$$\Delta x_1^m = x_1(p_1', m') - x_1(p_1', m) = -\Delta x_1^n$$

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m$$

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}$$

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1 \quad \text{because } \Delta m = x_1 \Delta p_1$$

- We can interpret this formulation as the following:
  - $\frac{\Delta x_1}{\Delta p_1}$: rate of change in demand as price changes, holding income fixed
  - $\frac{\Delta x_1^s}{\Delta p_1}$: rate of change in demand as the prices changes, adjusting income so as to keep the old bundle just affordable
  - $\frac{\Delta x_1^m}{\Delta m} x_1$: rate of change of demand holding prices fixed

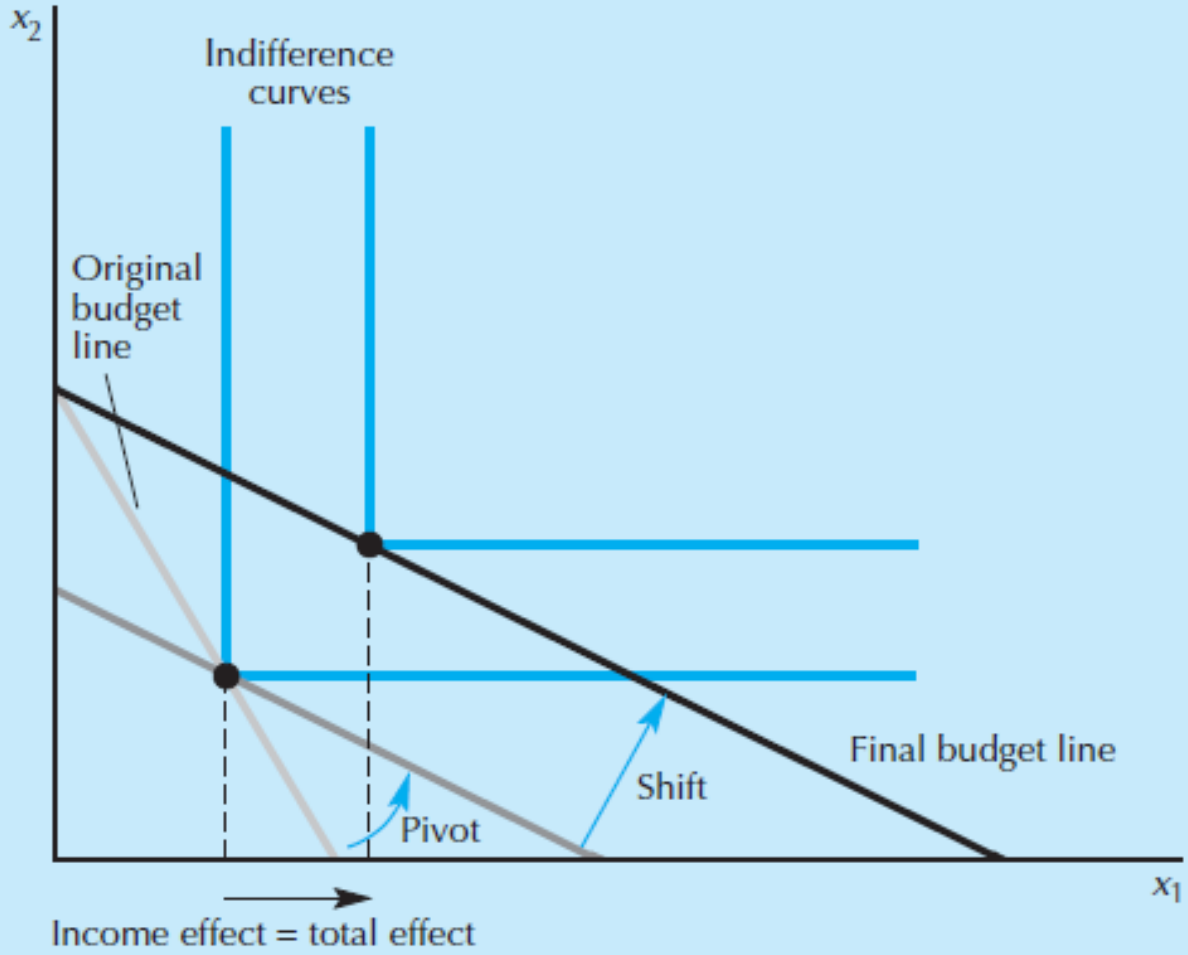
The Law of Demand

- Consumer theory seemed to have no particular content
  - demand could go up or down when a price increased
  - demand could go up or down when income increased
- Choices made by rational consumers must satisfy SARP
- Slutsky decomposition assures the negativity of substitution effect

Theorem (The Law of Demand)

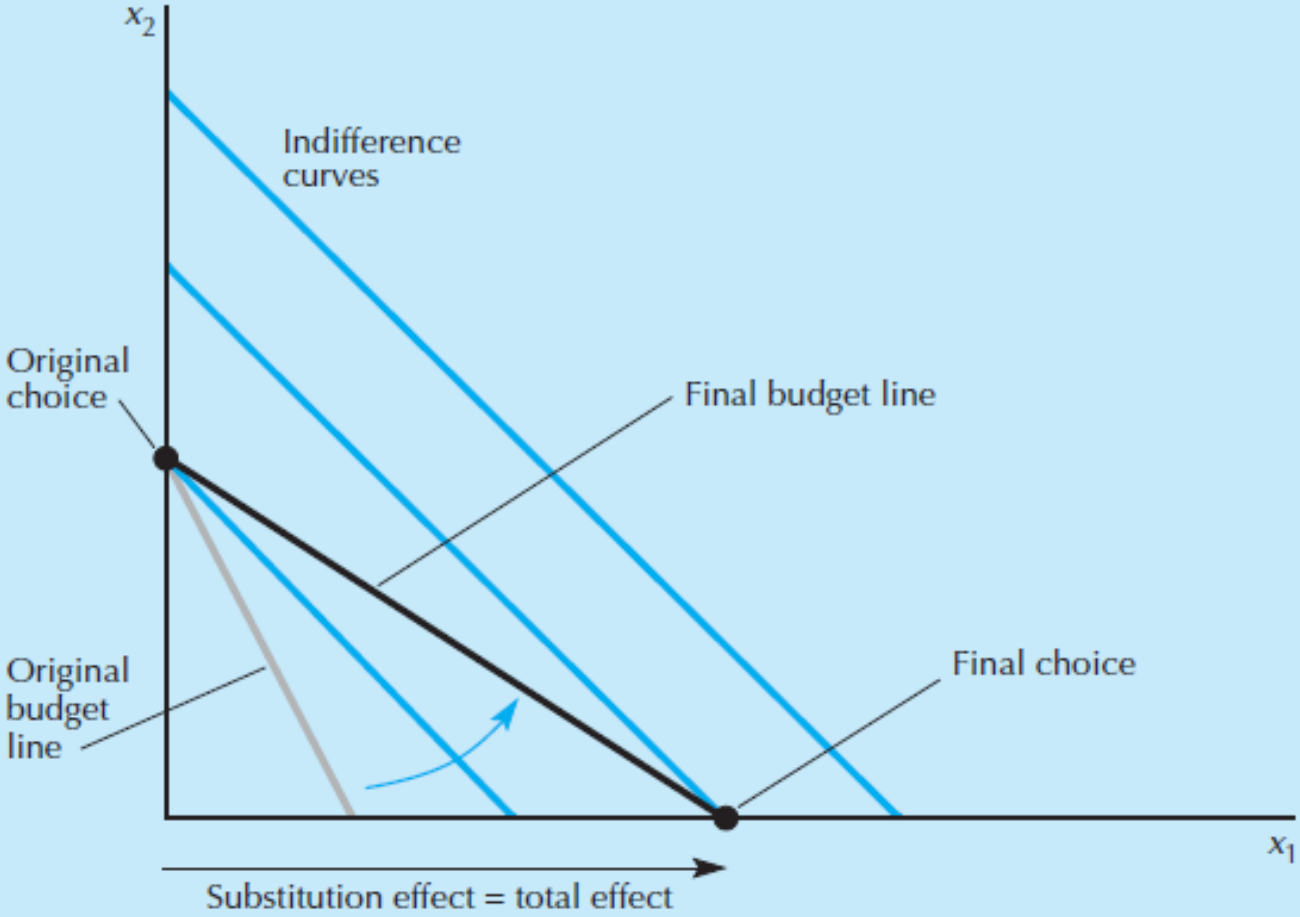
If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases

Examples of Substitution and Income Effects



Perfect complements. Slutsky decomposition with perfect complements.

Examples of Substitution and Income Effects

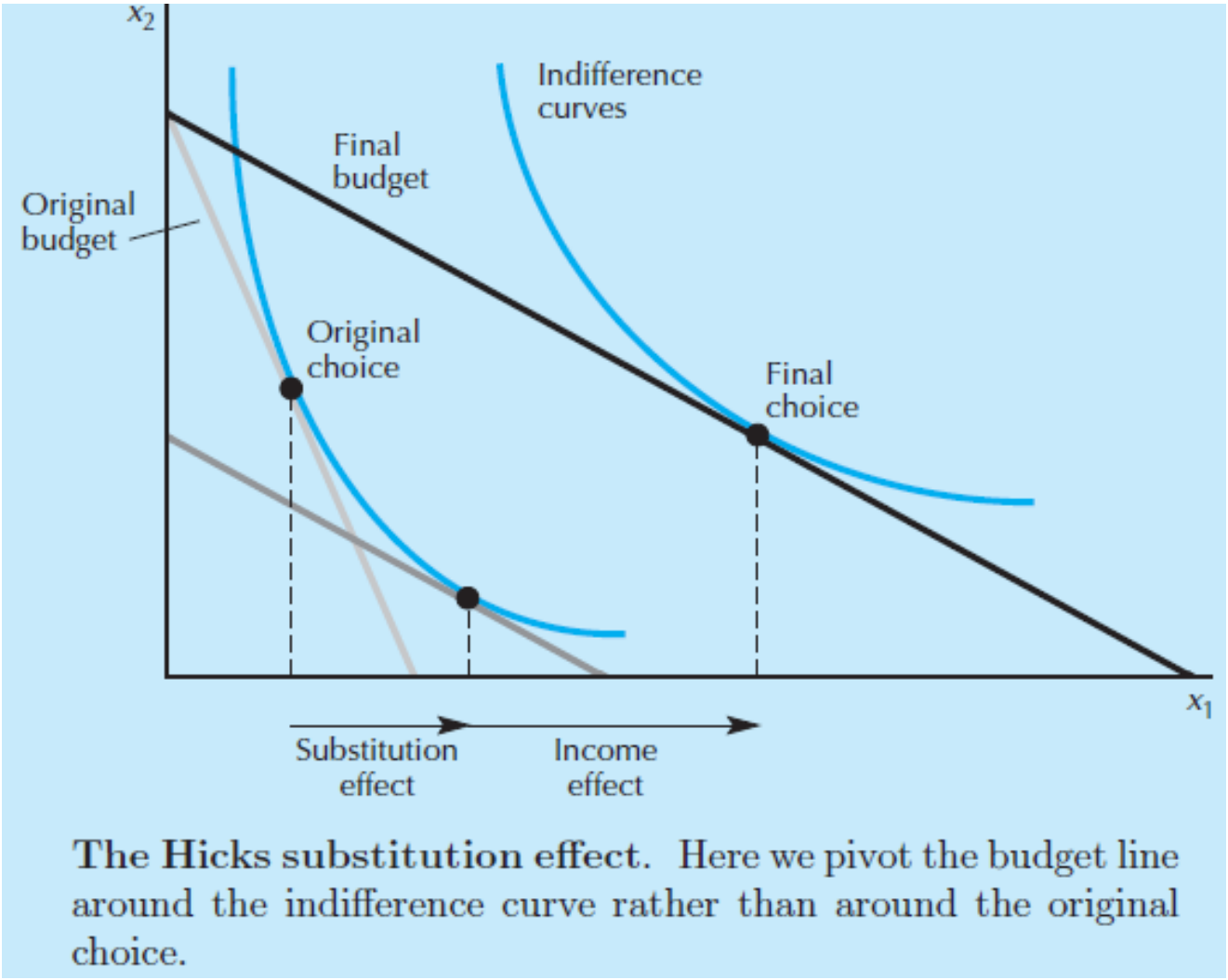


Perfect substitutes. Slutsky decomposition with perfect substitutes.

Slutsky and Hicks Substitution Effects

- The definition we have studied so far is called the Slutsky substitution effect
  - pivot the budget line around the original consumption bundle
  - same relative prices as the final budget line
  - the original bundle is just affordable
- We now describe the Hicks substitution effect
  - roll the budget line around the IC through the original bundle
  - same relative prices as the final budget line
  - original bundle no longer available, but sufficient purchasing power to be indifferent

Hicks Substitution Effect



The Hicks substitution effect. Here we pivot the budget line around the indifference curve rather than around the original choice.

Hicks Substitution Effect

- the Hicks substitution effect keeps utility constant rather than keeping purchasing power constant
  - must be negative

- Let (x_1, x_2) be a demanded bundle at some prices (p_1, p_2), and let (y_1, y_2) be a demanded bundle at some other prices (q_1, q_2). Suppose consumer is indifferent between (x_1, x_2) and (y_1, y_2)

$$p_1 x_1 + p_2 x_2 \leq p_1 y_1 + p_2 y_2$$

$$q_1 y_1 + q_2 y_2 \leq q_1 x_1 + q_2 x_2$$

$$(q_1 - p_1)(y_1 - x_1) + (q_2 - p_2)(y_2 - x_2) \leq 0$$

- Given $q_2 = p_2$

$$(q_1 - p_1)(y_1 - x_1) \leq 0$$

Compensated Demand Curves

- When prices change, we can choose to hold different factors fixed
  - income (standard case)
  - purchasing power (Slutsky substitution effect)
  - utility (Hicks substitution effect)
- We can draw the relationship between price and quantity demanded holding any of these three variables fixed
  - standard (or Marshallian) demand curve: can be upward sloping in theory (Giffen good)
  - Slutsky demand curve: always downward sloping
  - compensated (or Hicks) demand curve: always downward sloping