Nanyang Technological University
HE2001 Microeconomics II

Solution 1

Question 1

(a) (x_1, x_2) is the bundle of asparagus and tomatoes purchased at prices (p_1, p_2) when Freddy has income m.

Since (15, 15) is actually bought at the given budget, it must satisfy the budget constraint with equality

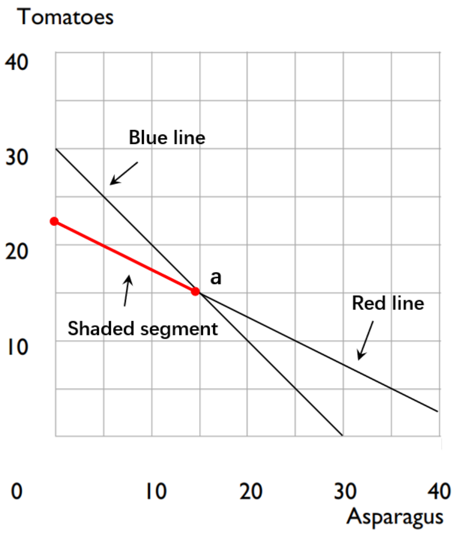
$$p_1x_1 + p_2x_2 = m.$$

Freddy's income $m = (\$1)(15) + (\$1)(15) = \$30$, i.e., the total expenditure he spent on consuming asparagus and tomatoes this week.

The budget line is the set of bundles (x_1, x_2) that just exhaust Freddy's income at price (\$1, \$1):

$$x_1 + x_2 = 30$$

drawn as the blue line in the graph below. His consumption bundle (15, 15) is labeled as point A.



(Blue line: $x_1 + x_2 = 30$*, the budget line for the first week*
Red line: $x_1 + 2x_2 = 45$*, the budget line for the second week.)*

(b) The new income is given by the cost of the old bundle at the new price (\$1, \$2):

$$m = (\$1)(15) + (\$2)(15) = \$45.$$

Same as part (a), the new budget line is the bundles of goods that cost exactly \$45, shown with the red line in the graph

$$x_1 + 2x_2 = 45.$$

Point A (15, 15) costs exactly \$45, so the new budget line goes through it. The slope of this line is -1/2.

(c) With an income of \$45 and a price of \$1 per pound, Freddy can afford 45 pounds of asparagus (the horizontal intercept of the red line).

(d) \$45, as explained in part (b).

(e) The bundles that he will never purchase are namely those "worse bundles" to which point A is directly revealed preferred, because they lie within the old budget line, as shown by the shaded line segment in red.

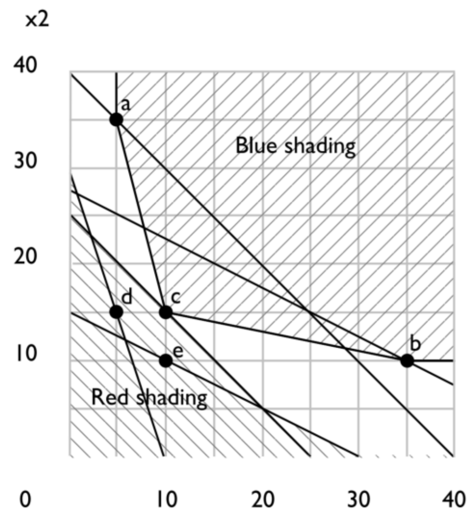
Therefore, Freddy's chosen bundle is restricted to the rest of the points on the new budget line. For any of these possible bundles, he consumes fewer tomatoes compared to bundle A.

Question 2

(a) At the given budget, the bundle that Ronald chooses must satisfy the budget constraint with equality. We can calculate his income under each situation and obtain the budget line based on the price and income.

Situation	Income	Budget line ($p_1x_1 + p_2x_2 = m$)	Chosen point
A	\$40	$x_1 + x_2 = 40$	(5, 35)
B	\$55	$x_1 + 2x_2 = 55$	(35, 10)
C	\$25	$x_1 + x_2 = 25$	(10, 15)
D	\$30	$3x_1 + x_2 = 30$	(5, 15)
E	\$30	$x_1 + 2x_2 = 30$	(10, 10)

The lines and points are shown in the graph:



(The equations for the fives lines are given in the table above. The red shading contains all points worse than bundle C. The blue shading contains all points at least as good as bundle C.)

(b) Checking violations of WARP.

A: At $(p_1, p_2) = (\$1, \$1)$ the choice of $(x_1, x_2) = (5, 35)$ cost \$40.
The costs of the alternative four choices are \$45, \$25, \$20, and \$20.
So (35, 10) was unaffordable while (5, 35) was chosen.
The remaining three were affordable while (5, 35) was chosen.
(5, 35) DRP (10, 15); (5, 35) DRP (5, 15); (5, 35) DRP (10, 10).

B: At $(p_1, p_2) = (\$1, \$2)$ the choice of $(x_1, x_2) = (35, 10)$ cost \$55.
The costs of the alternative four choices are \$75, \$40, \$35, and \$30.
So (5, 35) was unaffordable while (35, 10) was chosen.
The remaining three were affordable while (35, 10) was chosen.
(35, 10) DRP (10, 15); (35, 10) DRP (5, 15); (35, 10) DRP (10, 10).

C: At $(p_1, p_2) = (\$1, \$1)$ the choice of $(x_1, x_2) = (10, 15)$ cost \$25.
The costs of the alternative four choices are \$40, \$45, \$20, and \$20.
So (5, 35) and (35, 10) were unaffordable while (10, 15) was chosen.
The remaining two were affordable while (10, 15) was chosen.
(10, 15) DRP (5, 15); (10, 15) DRP (10, 10).

D: At $(p_1, p_2) = (\$3, \$1)$ the choice of $(x_1, x_2) = (5, 15)$ cost \$30.
The costs of the alternative four choices are \$50, \$115, \$45, and \$40.
So all other choices were unaffordable while (5, 15) was chosen.

E: At $(p_1, p_2) = (\$1, \$2)$ the choice of $(x_1, x_2) = (10, 10)$ cost \$30.
The costs of the alternative four choices are \$75, \$55, \$40, and \$35.
So all other choices were unaffordable while (10, 10) was chosen.

In summary:
(5, 35) DRP (10, 15); (5, 35) DRP (5, 15); (5, 35) DRP (10, 10);
(35, 10) DRP (10, 15); (35, 10) DRP (5, 15); (35, 10) DRP (10, 10);
(10, 15) DRP (5, 15); (10, 15) DRP (10, 10).

<u>No violation of WARP.</u>

(c) **Direct Preference Revelation:**
The chosen bundle C is directly revealed preferred to all other bundles on the corresponding budget line and all bundles below that budget line.

Indirect Preference Revelation:
From part (b), bundle C is directly revealed preferred to bundles D and E. Bundles D and

E are directly revealed preferred to all other bundles on their respective budget line and all bundles below their budget line, so those points are also worse than bundle C.

Their union set gives all points that are worse than bundle C, the red shading in the graph.

(d) From part (b), bundles A and B are both directly revealed preferred to C. Ronald has convex preferences, so all bundles on the line segments AC and BC are preferred to C as well.

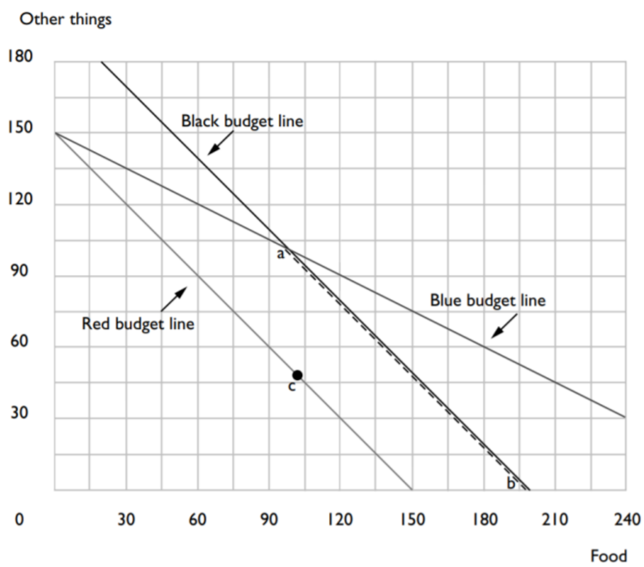
Ronald's preference is also monotonic, so any bundle provides a larger consumption than points on the two line segments can be further included. This gives us the boundary of the blue region.

Question 3

(a) The initial budget is \$150. Denote the amount of money they spent on food as x_1 and on other things as x_2. The initial budget line $\underline{x_1 + x_2 = 150}$ is drawn as the red line in the graph. Their current choice (100, 50) is labeled as point C.

If they received a grant of \$50, they would have a new budget of \$200. As the grant can be spent on anything, the budget line would shift upward parallelly: $\underline{x_1 + x_2 = 200}$ (black line).

If they chose the coupon option and spent all their money on it, <u>they could buy food worth \$300</u> (equivalent to a food price reduction by half). If they bought no food, <u>they could spend \$150 on other things</u>. The slope of the budget line would change. The new budget line becomes $\underline{0.5x_1 + x_2 = 150}$ (blue line).



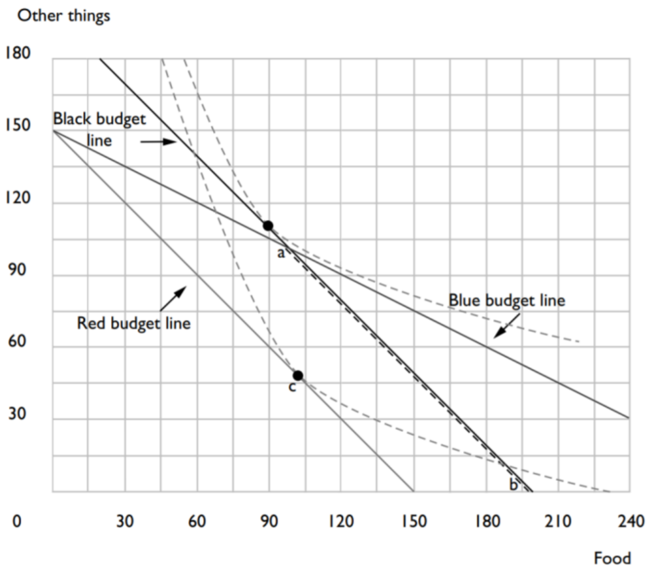
(The red line is the initial budget line without any welfare program. The black line is the budget line with the grant. The blue line is the budget line with the food coupon.)

(b) The lump-sum grant generates income effects. Given that food is a normal good, the increase in income will lead to an increase in demand. Hence only bundle points containing a larger amount of food than bundle C (i.e., buy more than $100 of food) should be included. Label the two ends with points A and B.

(c) <u>They should choose coupons.</u>
For any possible bundle on line segment AB under the lump-sum grant option, if you choose the coupon option instead, you can always get more food even when other expenditures are constant.

(d) If it is uncertain that food is a normal good, an increase in income could result in a reduction in food consumption. Thus, on the lump-sum grant budget line (black line), their chosen bundle could contain less food and more of other things compared to bundle C (to the upper left of point C), for example, at the given point in the graph below.

The possible bundles are not restricted to line segment AB anymore, and the newly chosen point under the lump-sum grant can make the McCawbers better off because it is on a higher indifference curve compared to what the food coupon can reach. One possible situation is provided below. Therefore, you cannot say for sure which option is better.



(The red line is the initial budget line without any welfare program. The black line is the budget line with the grant, and the black point on this line is the chosen bundle. The blue line is the budget line with the food coupon.)

Question 4

(a) Cost of 1850 and 1890 bundles at various year's prices:

Cost	1850 bundle	1890 bundle
Cost at 1830 Prices	44.1	61.6
Cost at 1850 Prices	<u>49.0</u>	<u>68.3</u>

Cost at 1890 Prices	63.1	91.1
Cost at 1913 Prices	78.5	113.7

(b) At 1890 prices, 1850 bundle was affordable while 1890 bundle was chosen, so the 1890 bundle is revealed preferred to the 1850 bundle.

(c) Laspeyres quantity index:

$$L_q = \frac{(0.14)(220) + (0.34)(42) + (0.08)(180) + (0.044)(200)}{(0.14)(165) + (0.34)(22) + (0.08)(120) + (0.044)(200)} = 1.39$$

(d) Paasche quantity index:

$$P_q = \frac{(0.16)(220) + (0.66)(42) + (0.10)(180) + (0.051)(200)}{(0.16)(165) + (0.66)(22) + (0.10)(120) + (0.051)(200)} = 1.44$$

(e) Laspeyres price index:

$$L_q = \frac{(0.16)(165) + (0.66)(22) + (0.10)(120) + (0.051)(200)}{(0.14)(165) + (0.34)(22) + (0.08)(120) + (0.044)(200)} = 1.29$$

(f) This is consistent with the Laspeyres quantity index, so to afford the 1890 bundle, he would have to spend 1.39 times as much as a typical 1850 bundle, at the 1850 prices.

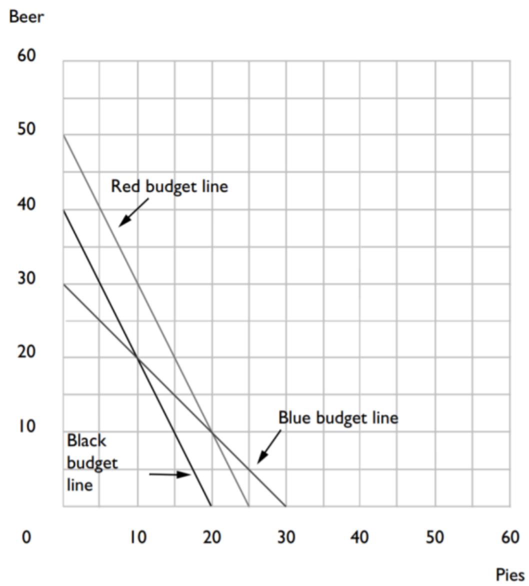
(g) This is consistent with the inverse of the Paasche quantity index, so the fraction of the amount is $1/1.44 = 0.69$.

Question 5

(a) At the prices ($2, $1) and the net income of $50, their old budget line

$$2x_1 + x_2 = 50$$

is given by the red line in the graph:



(Red line: $2x_1 + x_2 = 50$, the initial budget line with income tax.
Blue line: $x_1 + x_2 = 30$, the budget line with sales tax on beer only.
Black line: $2x_1 + x_2 = 40$, the budget line with sales tax on both beer and meat pies.)

Given the prices ($2, $1) and budget constraint of $50, their bundle of goods was (10, 30), i.e., <u>they bought 10 meat pies</u>.

(b) At the new prices ($2, $2) and the income of $60, their new budget line

$$\underline{2x_1 + 2x_2 = 60 \text{ or } x_1 + x_2 = 30}$$

is given by the blue line in the graph.

(c) When the tax scheme changes, the new prices became ($2, $2) while the income rose to $60. Their bundle of goods became (10, 20). <u>The consumption of meat pies stayed the same at 10.</u>
With $1 imposed on each can of beer, <u>this tax raised $20 from them</u>.

(d) Assume the tax rate on meat pies and beer are both t. The prices will become $(\$2(1 + t), \$1(1 + t))$. The new budget line will be given by

$$2(1 + t)x_1 + (1 + t)x_2 = 60 \text{ or } 2x_1 + x_2 = \frac{60}{1 + t}$$

There is no change in the slope, so the effect of this universal sales tax is the same as an income tax. Also, the amount collected by this new tax scheme should be exactly $20, so the government needs to make sure the effect of this sales tax is equivalent to a $20 reduction in net income:

$$60 - \frac{60}{1 + t} = 20 \rightarrow t = 50\%$$

<u>At this tax rate, the price of meat pies and beer will rise from ($2, $1) to ($3, $1.5). The government can collect $1 for every meat pie and $0.5 for every can of beer sold.</u>

If this revenue is raised from directly taxing the income, a tax amount as large as $20 needs to be imposed.

(e) At the same tax rate of 50% on both goods, the budget line

$$\underline{2x_1 + x_2 = 40}$$

is given by the black line in the graph.

When taxing beer only, the chosen bundle is (10, 20), which is also on the budget line for taxing both goods (black line). We know Norm and Sheila would have a most preferred bundle on the black budget line. This bundle will be chosen when (10, 20) is available, so it is preferred to bundle (10, 20).

But rather than allow them to simply face a reduction in income and choose the preferred point, we are distorting their behavior by shifting the slope (or the price ratio) when only taxing beer. If they choose any other point (including (10, 20)) rather than that preferred point on the black line, they would become worse off by Revealed Preference.

Hence, taxing beer alone generates distortion and makes consumers worse off by causing them to pick a non-preferred point, while a preferred bundle can be chosen under the tax scheme of taxing both goods (or equivalently a lump-sum income tax).