One-year real interest rate $r_t$ satisfies:

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}^e}$$

By definition, the real interest rate $r_t$ means 1 unit basket of a goods in current year can be saved and exchange for $1 + r_t$ units basket of goods in the next year.

To derive the real interest rate, suppose you have 1 unit basket of goods at time $t$. At time t, the price level is $P_t$. You sell the 1 unit basket of goods and receive $P_t$ dollars. You save the money and the current nominal interest rate is $i_t$. You receive $(1 + i_t)P_t$ dollars at $t + 1$. We then want to use the money to buy goods at $t + 1$. From the perspective of current period $t$, the true price level $P_{t+1}$ is unknown, so we form an expectation with respect to $P_{t+1}$. $P_{t+1}^e$ denotes the expected inflation rate at $t + 1$. The expected units of basket we can exchange at $t + 1$ will be $(1 + i_t)\frac{P_t}{P_{t+1}^e}$. This value should equal the $(1 + \text{real interest rate})$ since they mean the same thing.

Thus, we get

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}^e}.$$

The inflation rate is the percentage change of the aggregate price. So the realized inflation rate is

$$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t}.$$

The expected inflation rate is

$$\pi_{t+1}^e = \frac{P_{t+1}^e - P_t}{P_t}.$$

$$\begin{aligned} \pi_{t+1}^e &= \tfrac{P_{t+1}^e - P_t}{P_t} \\ &= \tfrac{P_{t+1}^e}{P_t} - 1 \end{aligned}$$

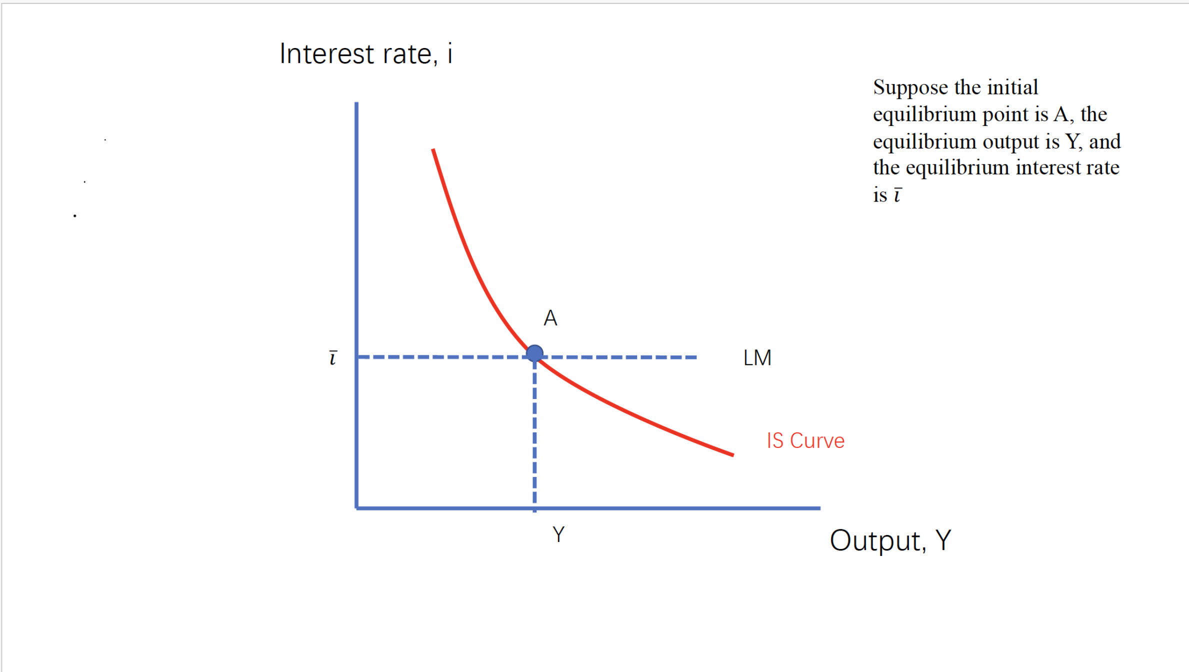
$$\Rightarrow 1 + \pi_{t+1}^e = \tfrac{P_{t+1}^e}{P_t}$$

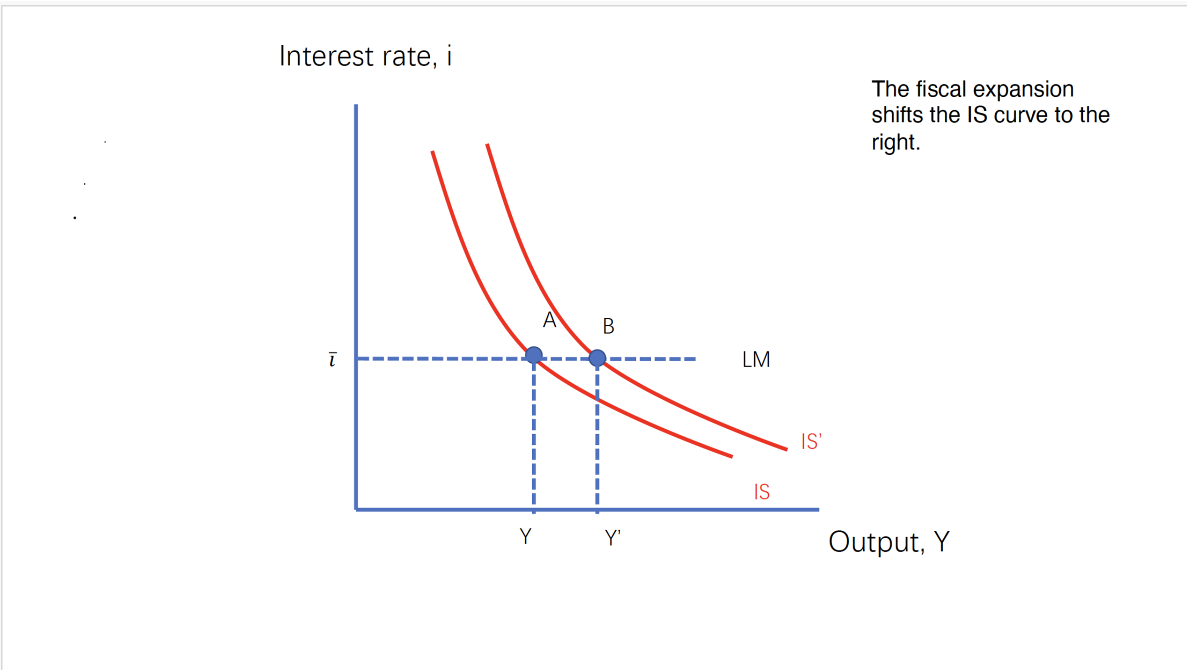
By referring to equation (4.1), we have $\frac{P_t}{P_{t+1}^e}$ in the equation.

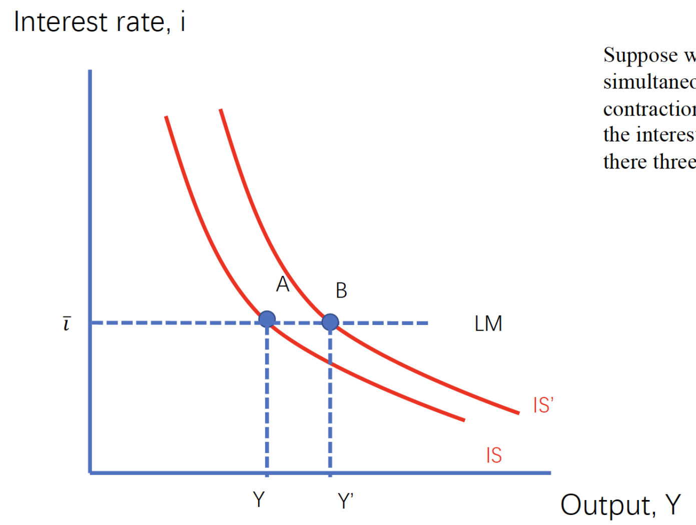
$$\Rightarrow 1 + r_t = (1 + i_t)\tfrac{1}{1+\pi_{t+1}^e} = \tfrac{1+i_t}{1+\pi_{t+1}^e}$$

Suppose there is a simultaneous fiscal expansion and monetary contraction. We know with certainty that the interest rate will increase.

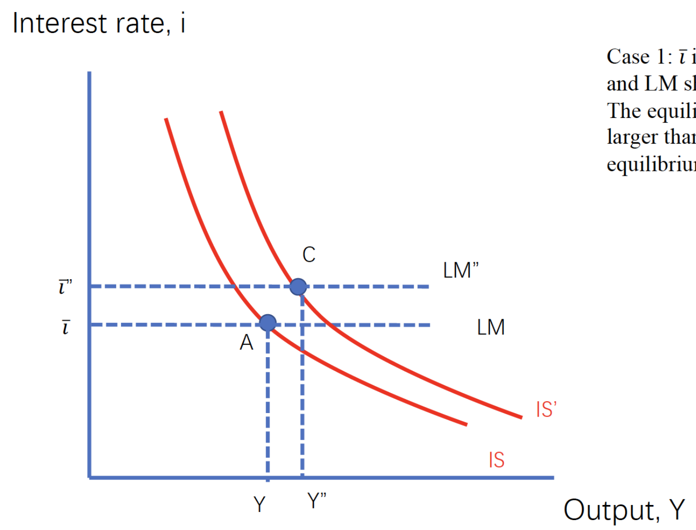
The equilibrium output can increase, decrease, or stay at the same level. The change depends on the size of the monetary contraction.



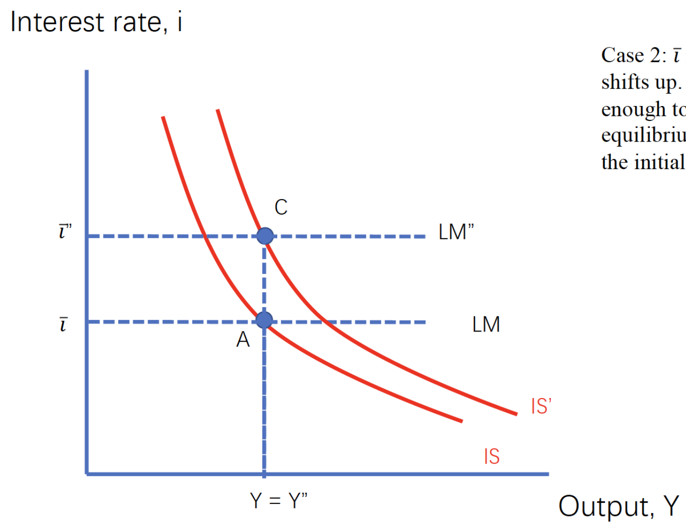





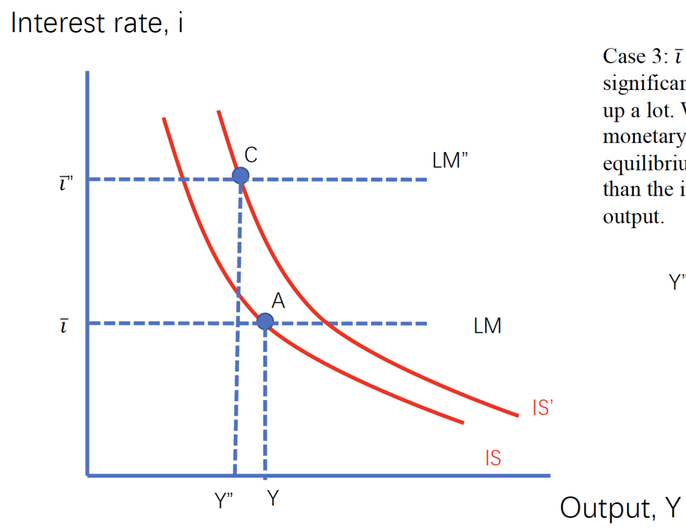

Suppose we have a simultaneous monetary contraction, which means the interest rate increases, there three possible cases.




Case 1: $\bar{\imath}$ increases a little and LM shifts up a little. The equilibrium output Y" is larger than the initial equilibrium output Y.




Case 2: $\bar{\imath}$ increases and LM shifts up. The size is just enough to make the equilibrium output equals the initial equilibrium output.




Case 3: $\bar{\imath}$ increases significantly and LM shifts up a lot. We have a large monetary contraction. The equilibrium output is smaller than the initial equilibrium output.

Y” < Y

When $x$ is small and close to zero, $\log(1+x) \approx x$. This is an approximation widely used in economics.

$$1 + r_t = \frac{1+i_t}{1+\pi^e_{t+1}}$$

Take logs on both side, we have

$$\log(1+r_t) = \log(1+i_t) - \log(1+\pi^e_{t+1})$$

If we have $\log(1+x) \approx x$ for x small and close to zero, we can get:

$$r_t \approx i_t - \pi^e_{t+1}$$

Side Note: If you learned calculus before, the following analysis shows you why $\log(1+x) \approx x$.
In mathematics, the Taylor series or Taylor expansion of a function is an infinite sum of terms that are expressed in terms of the function's derivatives at a single point. For most common functions, the function and the sum of its Taylor series are equal near this point.
In calculus, Taylor's theorem gives an approximation of a k-times differentiable function around a given point by a polynomial of degree k, called the k-th-order Taylor polynomial.
Here, we are taking the first-order Taylor polynomial as a linear approximation of the function $\log(1+x)$ around the point $x = 0$.
By Taylor's theorem, at a point $a$,

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ... + \frac{f^{(k)}(a)}{k!}(x-a)^k + \text{some remainder}$$

If $f(x) = \log(1+x)$, around point 0, we have

$$\log(1+x) = \log(1+0) + \frac{1}{1}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + ... \frac{f^{(k)}(0)}{k!}(x-0)^k + \text{some remainder.}$$

When x is close to zero, $(x-0)^2, (x-0)^3, ...., (x-0)^k$ will be very small, so

$$\log(1+x) \approx \log(1+0) + \frac{1}{1}(x-0) = x$$

$$r_t = i_t \text{ if } \pi^e_{t+1} = 0$$

$$r_t \approx i_t - \pi^e_{t+1}, \text{ if } \pi^e_{t+1} > 0, \; i_t > r_t$$

Given $i_t = \bar{i}$, if $\pi^e_{t+1}$ is higher, $r_t \approx i_t - \pi^e_{t+1}$ is lower.

Let $i$ be the nominal interest rate on a riskless bond, $x$ be the **risk premium**, and $p$ is the **probability of defaulting**, then to get the same expected return on the risky bonds as on the riskless bond:

$$(1+i) = (1-p) \times (1+i+x) + (p) \times (0)$$

so that the risk premium is

$$x = \frac{(1+i)p}{1-p}$$

Divide $(1-p)$ on both sides:

$$\frac{1+i}{1-p} - 1 - i = x$$

$$x = \frac{(1+i) - (1+i)(1-p)}{1-p} = \frac{(1+i)(1-1+p)}{1-p} = \frac{(1+i)p}{1-p}$$

## Explanations about Leverage Ratio Decision

Leverage and Lending:
When the asset value decreases, the liabilities of the bank are fixed, so the value of capital decreases with the same amount, leading to a higher value of leverage ratio. The bank is still solvent as long as the capital value is positive. This risk is higher because an additional decrease in the asset value will more likely draw the value of capital down to zero. When the capital value is negative, the bank is insolvent. If the owners of the bank do not want to suffer more loss, they will choose to default.

Option 1: the investors, here the owners of the bank, in particular, can provide more funds from their pockets. By doing this, the assets increase, and the value of capital increases. Initially, this is just the additional money from the owners. The leverage ratio will decrease. (e.g., asset is 100, capital is 20, leverage ratio is 5. Increase the capital by 20, asset increased by 20 up to 120, and the leverage ratio decreases as $120/40 = 3$). The larger the capital value is, the more room for the value of assets to decrease before getting insolvent.

Option 2: Call back their loans $\Rightarrow$ ask the borrowers to pay back the money lent by the bank $\Rightarrow$ use the money to pay some of the liabilities, then both values of assets and liabilities decrease, with the capital value fixed $\Rightarrow$ leverage ratio decreases.

Why does a higher leverage ratio imply a higher expected profit rate?

It is because leverage magnifies the returns on a firm's assets. The economic idea behind leverage refers to the use of debt or other financial instruments to finance a firm's or banks' investments, and a higher leverage ratio means that the firm/bank has a greater amount of debts/liabilities relative to its equity/capital. There are different terms with similar meanings. So here when a bank uses leverage, it can earn a return on its liabilities as well as on its own capital. With a same amount of assets and expected total profits to be earned from the assets, the expected profit rate is higher when capital is smaller. Note that the expected profits divided by the capital is the expected profit rate.