Macroeconomics II

mini lecture on government debt dynamics

(reference: Blanchard Chapter 22.1-22.3*)*

1. Introduction

- Government debt, deficits, and budget are fascinating topics.

- We are going to look at:
  - Deficit; Debt repayment.
  - Dangers of high debt & debt spirals.

Decreasing Debt-to-GDP after WW II

Table 1 Changes in Debt Ratios Following World War II

	1	2	3	4	5	6
Country	Start/End Year	Start/End Debt Ratio	Primary Balance	Growth Rate	Real Interest Rate	Inflation Rate
Australia	1946–1963	92–29	1.1	4.6	–2.3	5.7
Canada	1945–1957	115–59	3.6	4.3	–1.4	4.0
New Zealand	1946–1974	148–41	2.3	3.9	–2.9	4.9
United Kingdom	1946–1975	270–47	2.1	2.6	–1.5	5.5

Columns 2 and 3: Percent of GDP. Columns 4 to 6: Percent.

Source: S.M.A. Abbas et al., "Historical Patterns and Dynamics of Public Debt: Evidence from a New Database," IMF Economic Review 2011 59 (November): pp. 717–742

Singapore's case

- As of 2020, the IMF measured Singapore's national debt-to-GDP ratio as 131.19%, the 6th highest in the world when expressed as a percentage of GDP.
- However, no one seems to be worried about the country's national debt.
- That's because the headline figure reported was gross national debt.
- When we examined Singapore's net national debt, the country owes nothing at all.
- Feb 16 speech -- 2023: 0.3% deficit-to-GDP; 2024: 0.1 surplus-to-GDP (expected)

2. Debts and deficits

- The official measure of the deficit is given by:

$$official \quad deficit_t = iB_{t-1} + G_t - T_t$$

- However, the correct measure of the deficit (inflation-adjusted) is:

$$deficit_t = rB_{t-1} + G_t - T_t$$

- This measure can be found by subtracting an amount πB_{t-1} from the official measure.

Government budget constraint

- Government budget constraint:

$$B_t - B_{t-1} = deficit_t$$

- Combining the last two equations:

$$B_t - B_{t-1} = rB_{t-1} + G_t - T_t$$

Debts, deficits, government budget - summary

- Change in the debt: $B_t - B_{t-1} = rB_{t-1} + G_t - T_t$.
- Interest payments: rB_{t-1}.
- Primary deficit: $G_t - T_t$.
- Alternatively: $B_t = (1 + r)B_{t-1} + G_t - T_t$.

Repayment of the debt – example

- Assume: initial debt $B_0 = 0$ and $G_0 - T_0 = 0$.

- Assume that government lowers taxes T_1 by an amount X in year 1. The resulting deficit is financed by borrowing, and the resulting debt is repaid in year 2.

Repayment of the debt – example

- Debt B_1 at the end of year 1:

$$B_1 = (1 + r)B_0 + G_1 - T_1 = X.$$

- The debt equation in year 2 equals:

$$B_2 = (1 + r)B_1 + G_2 - T_2$$

- Because of repayment in year 2, $B_2 = 0$. Then the necessary primary surplus equals:

$$T_2 - G_2 = (1 + r)B_1 = \ (1 + r)X$$

Repayment of the debt – in period t

- Now assume that the government waits to repay the debt in year t.

- Assume that from year 2 to year $t-1$ the primary deficit is equal to zero: $G_t - T_t = 0$.

- Then the debt at the end of year 3 will be equal to:

$$B_3 = (1+r)B_2 + 0 = (1+r)^2 X$$

Repayment of the debt – in period t

- Solving for debt at the end of year 4, and so on, the debt grows at a rate equal to the interest rate (assuming primary deficit is zero).

- Hence the debt at the end of year $t-1$ is given by:

$$B_{t-1} = (1+r)^{t-2}X$$

Repayment of the debt – in period t

- Despite the fact that the government runs primary deficits equal to zero, the debt continues to increase over time.

- The reason is that debt is positive, interest payments continue to add to the stock of debt (assuming zero primary deficit).

- Note, however, that the ***net present value*** is equal to X, ***irrespective*** of year of repayment!

Repayment of the debt – in period t

- In year t , the year in which the government decides to repay the debt, the budget constraint is given by:

$$B_t = (1 + r)B_{t-1} + G_t - T_t = 0$$

- If the debt is repaid at the end of year t, we know that $B_t = 0$. Rearranging gives us the required primary surplus:

$$T_t - G_t = (1 + r)B_{t-1} = (1 + r)^{t-1}X$$

Repayment of the debt - conclusions

- If government spending is unchanged, a decrease in taxes in year 1 must eventually be offset by an increase in taxes in the future.
- The longer the government waits to increase taxes, or the higher the real interest rate, the higher the eventual increase in taxes must be.
- However, the present value of the future tax increase ***remains the same***:

$$(1+r)^{t-1}X/(1+r)^{t-1} = X.$$

3. Debt stabilization

- Suppose the government decides to stabilize the debt from year 2 on.

- Remember the budget constraint for year 2:

$$B_2 = (1 + r)B_1 + G_2 - T_2$$

- Debt stabilization implies that $B_2 = B_1$:

$$B_1 = (1 + r)B_1 + G_2 - T_2$$

Debt stabilization's implication

- This can be rewritten in the following way:

$$T_2 - G_2 = (1 + r)B_1 - B_1 = rB_1$$

- This implies that the primary surplus must be equal to the real interest payments on the debt.

- Intuition: to prevent the interest payments adding to the debt, they must be paid from tax revenues.

General government debt dynamics

- To consider the debt-to-GDP ratio, we divide the government budget constraint in year t by Y_t:

$$\frac{B_t}{Y_t} = \frac{(1+r)B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

- Dividing and multiplying by Y_{t-1} gives:

$$\frac{B_t}{Y_t} = (1+r)\left(\frac{Y_{t-1}}{Y_t}\right)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Evolution of debt-GDP ratio

- We write $\frac{Y_{t-1}}{Y_t} = \frac{1}{1+g}$ with g the growth rate of output.
- We use the approximation $\frac{(1+r)}{(1+g)} \approx 1 + r - g$ to rewrite the debt accumulation equation:

$$\frac{B_t}{Y_t} = (1 + r - g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Evolution of debt-GDP ratio

- This can be rearranged as:

$$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r - g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

- Hence the change in the debt-GDP ratio is equal to the sum of two terms:
  - The difference between the real interest rate and the growth rate multiplied by the initial debt-GDP ratio (term 1).
  - Primary deficit as a percentage of GDP (term 2).

Evolution of debt-GDP ratio

- Change in debt-GDP ratio:

$$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r - g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\boldsymbol{\Delta\tilde{b} = (r - g)\tilde{b} + d}$$

- Increase in debt-GDP ratio will be larger:
  - The higher the real interest rate.
  - The lower the growth rate of output.
  - The higher the initial debt-GDP ratio.
  - The higher the ratio of the primary deficit to GDP.

Debt dynamics: $\boldsymbol{\Delta\tilde{b} = (r - g)\tilde{b} + d}$

Example 1:

$$d = 0, g = 0, r > 0, \tilde{b} > 0$$

- Therefore

$$\Delta\tilde{b} = r\tilde{b}$$

- The debt-to-GDP ratio grows over time

$$\tilde{b}_{t+1} = (1 + r)\tilde{b}_t$$

Example 2:

$$d = 0, g > 0, r = 0, \tilde{b} > 0$$

- Therefore

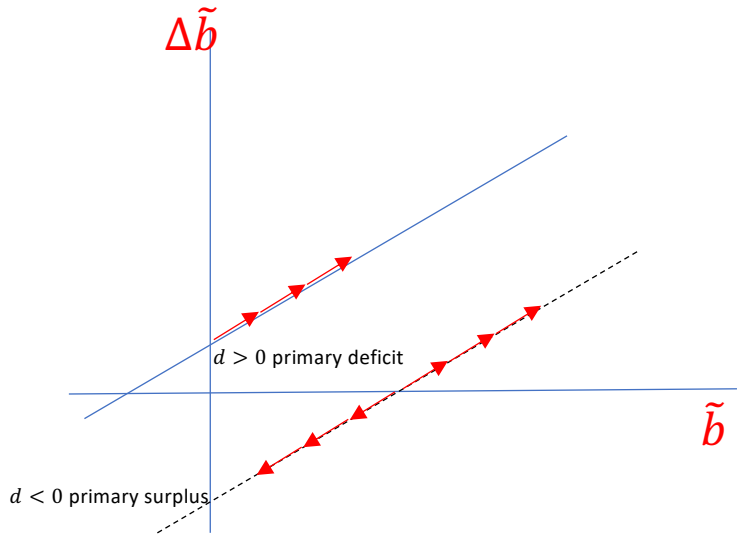
$$\Delta\tilde{b} = -g\tilde{b}$$

- The debt-to-GDP ratio falls overtime

$$\tilde{b}_{t+1} = (1 - g)\tilde{b}_t$$

Case 1: $r - g > 0$

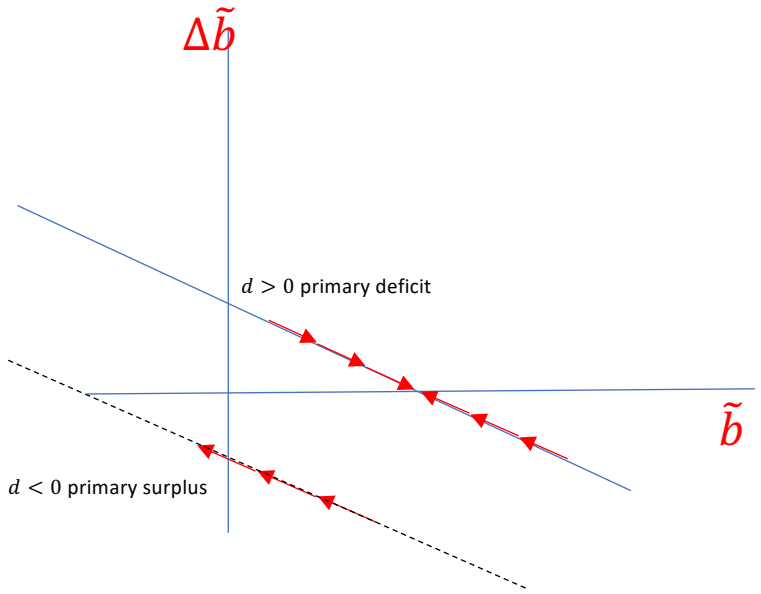
- On the graph



- The interest payments grow faster than GDP (and therefore tax revenues)
- Debt-to-GDP ratio grows/falls without limit
- Even with primary surplus, debt-to-GDP might not be stationary

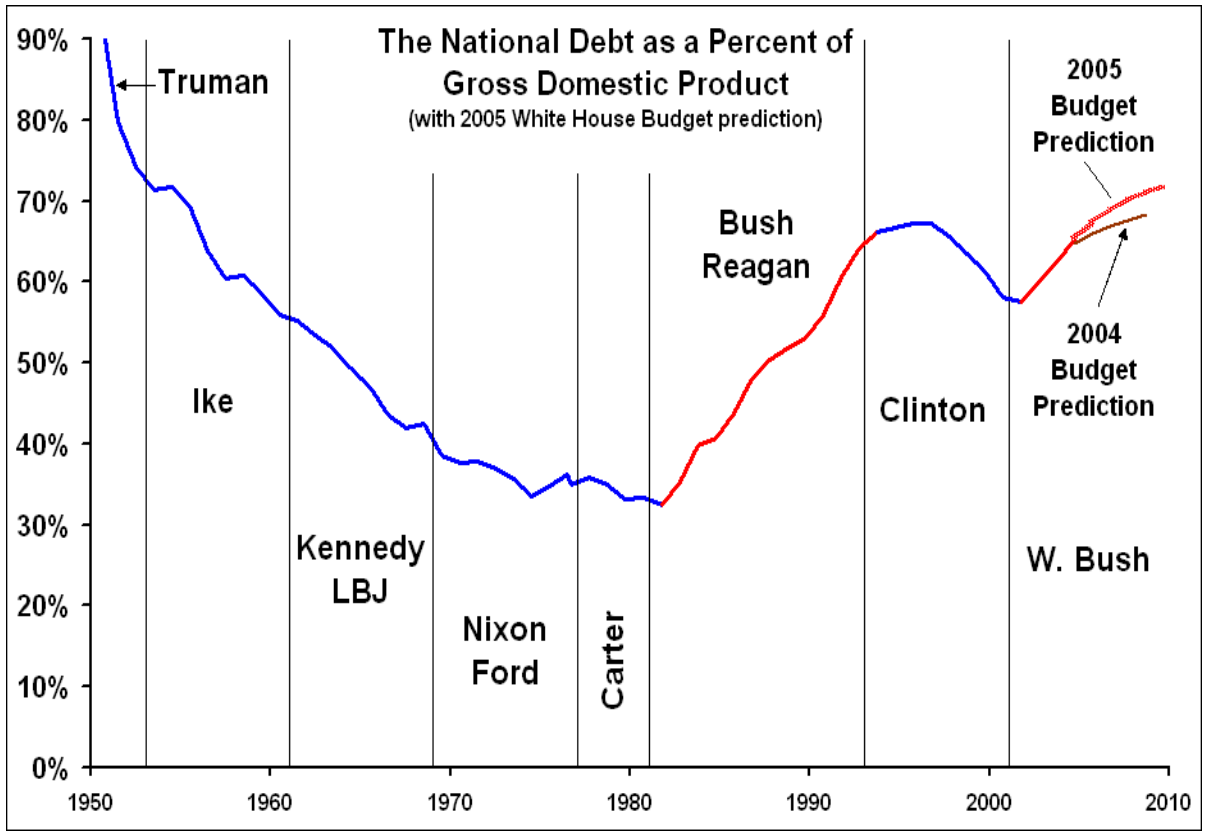
Case 2: $r - g < 0$

- On the graph



- The interest payments grow slower than GDP (and therefore tax revenues)
- Debt-to-GDP ratio shrinks unless there is primary deficit (the stationary point does not represent debt)
- Recal that: debt ratios of many countries declined following world war II.
- Declines were not so much the result of primary fiscal surplus (term 2), but the result of high growth and negative real interest rates (term 1).

In US history…



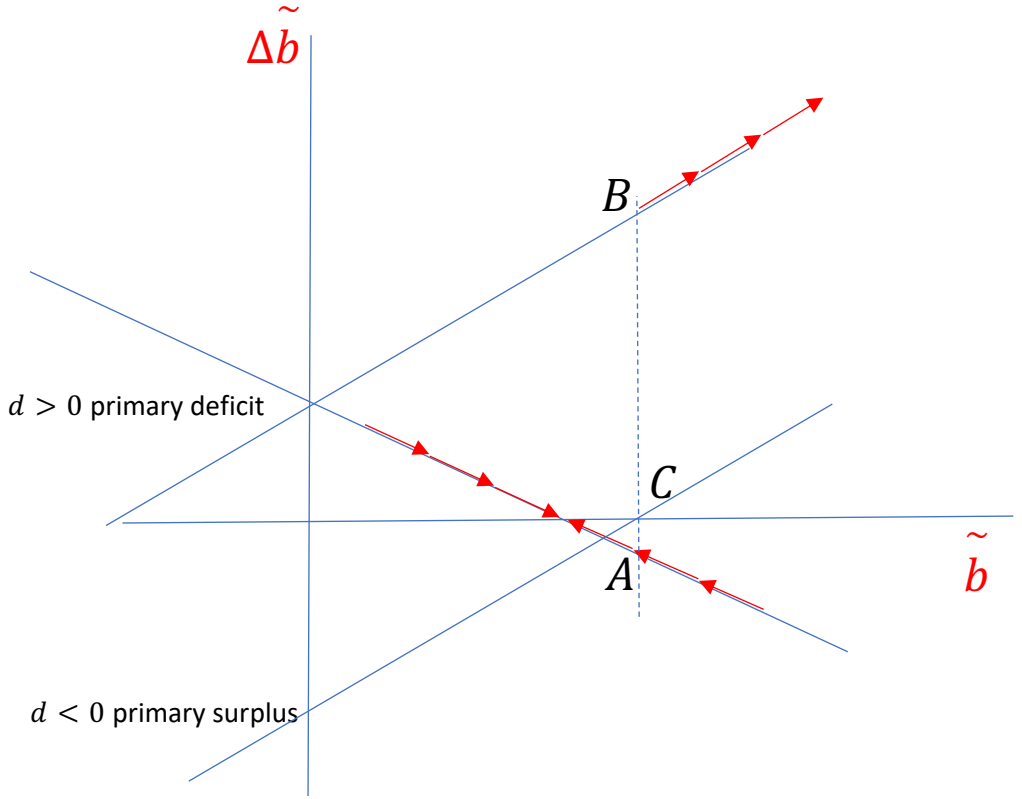
- Deficit most of the time in the US
- 50s / 60s: high growth and low rate (Keynesian policies)
- 70s: low growth and low rate
- 80s: low growth and high rate (disinflation)
- 90s: high growth and high rate

Other effects

- in the above analysis, we've assumed r and g are exogenous
- but government actions might affect them
- increased borrowing may increase the interest rate as default is seen to be more likely
- measures to address the deficit (eg tax rises) might have supply side effects and reduce the growth rate

Case 3: switching from $r < g$ to $r > g$

- On the graph



- A sudden increase of interest rate push the economy from point A to point B
- Instead of having a shrinking debt-to-GDP ratio, we have a rising one
- Need enough primary surplus to bring to point C (why point C?)