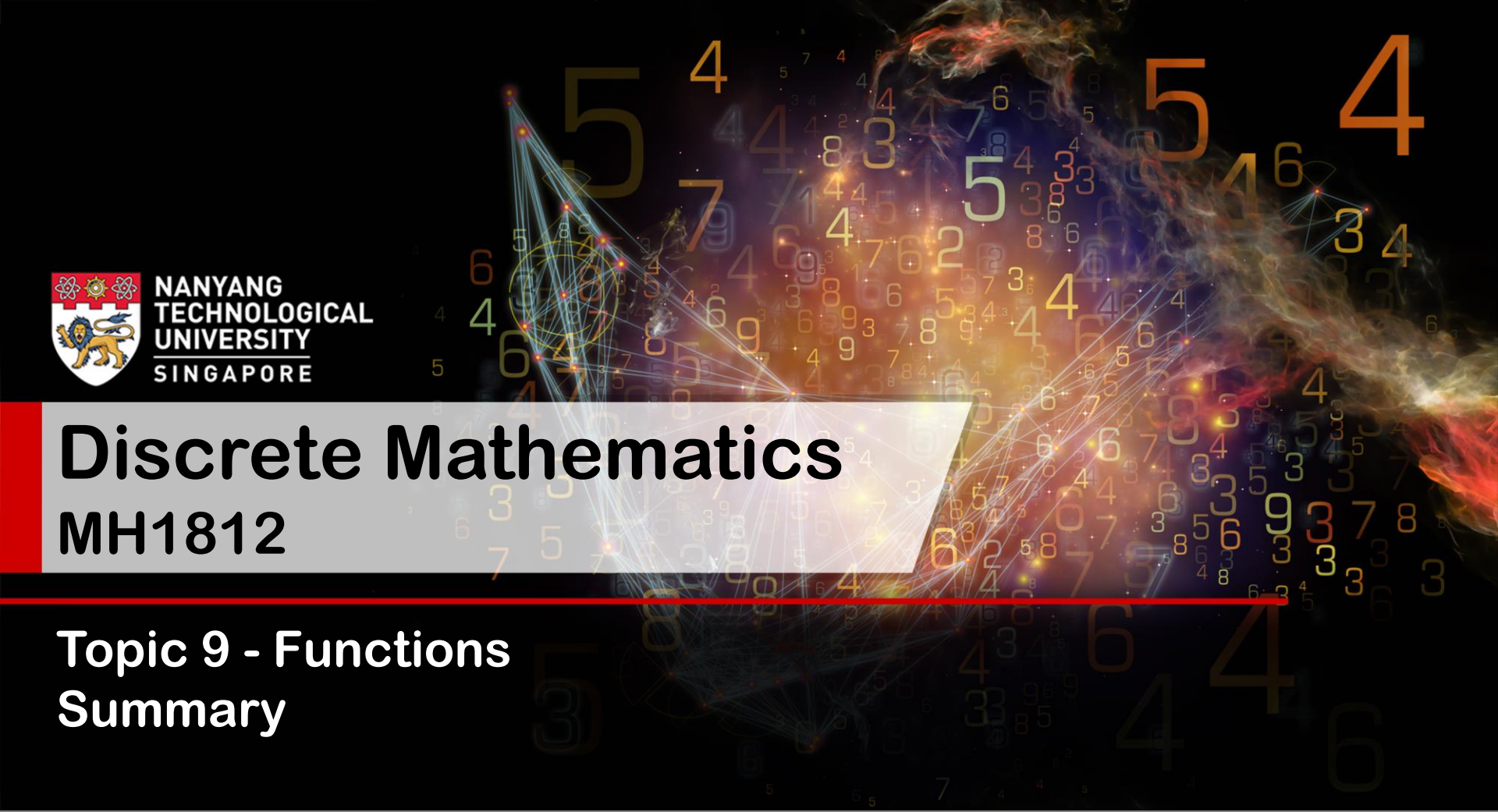


NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Discrete Mathematics

MH1812

Topic 9 - Functions Summary

Introduction to Functions: Definition

Let X and Y be sets. A **function** f from X to Y is a rule that assigns every element x of X to a unique y in Y. We write $f: X \to Y$ and $f(x) = y$.

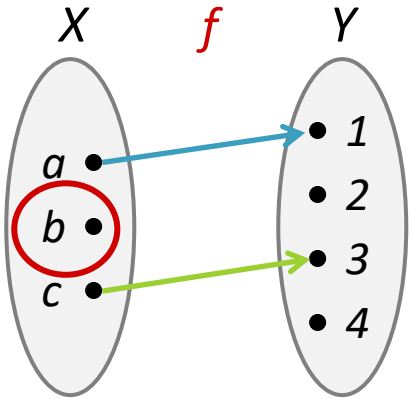
$$(\forall x \in X\ \exists y \in Y, y = f(x)) \wedge (\forall x_1, x_2 \in X, f(x_1) \neq f(x_2) \to x_1 \neq x_2)$$

X =	Domain
Y =	Codomain
y =	**Image** of x under f
x =	**Preimage** of y under f
Range =	Subset of Y with preimages

Introduction to Functions: Functions vs. Non-functions

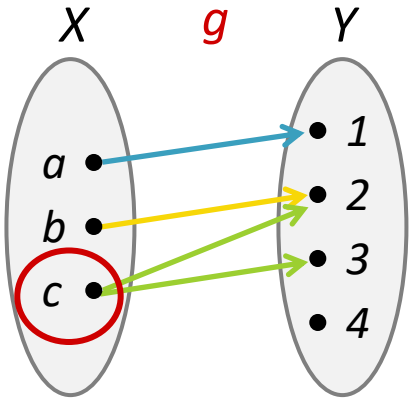
$$(\forall x \in X\ \exists y \in Y, y = f(x)) \wedge (\forall x_1, x_2 \in X, f(x_1) \neq f(x_2) \rightarrow x_1 \neq x_2)$$

$X = \{a,b,c\}$ to $Y = \{1,2,3,4\}$



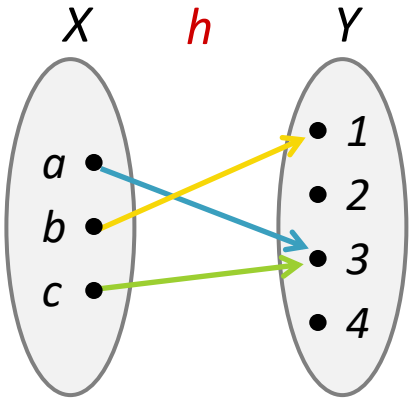
No!

(*b* has no image)



No!

(*c* has two images)



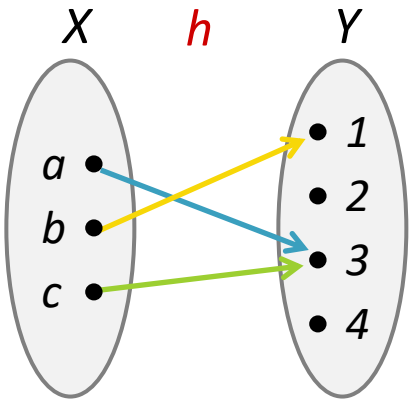
Yes!

(Each element of *X* has exactly one image)

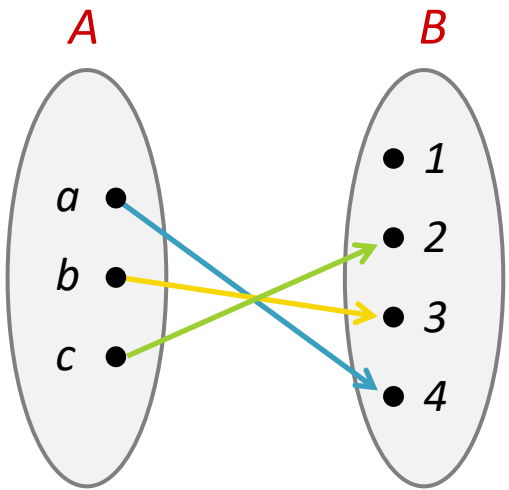
Introduction to Functions: Functions vs. Non-functions

$$(\forall x \in X\ \exists y \in Y, y = f(x)) \wedge (\forall x_1, x_2 \in X, f(x_1) \neq f(x_2) \rightarrow x_1 \neq x_2)$$

$X = \{a,b,c\}$ **to** $Y = \{1,2,3,4\}$

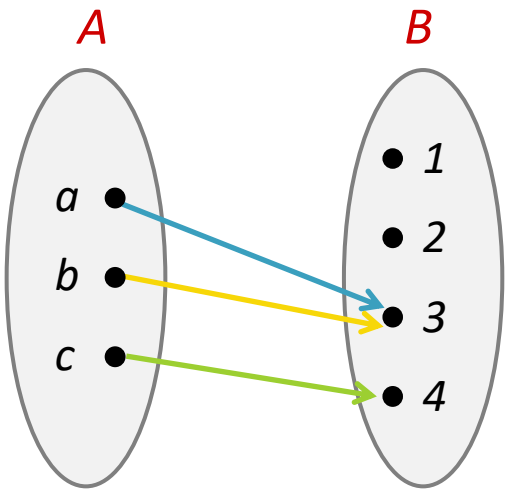


Injectivity: One-to-one Example



One-to-one

(All elements in *A* have a different image)



Not one-to-one

(*a* and *b* have the same image)

Example

$f: \mathbb{R} - \{0\} \to \mathbb{R},\ f(x) = \frac{x+1}{x}$

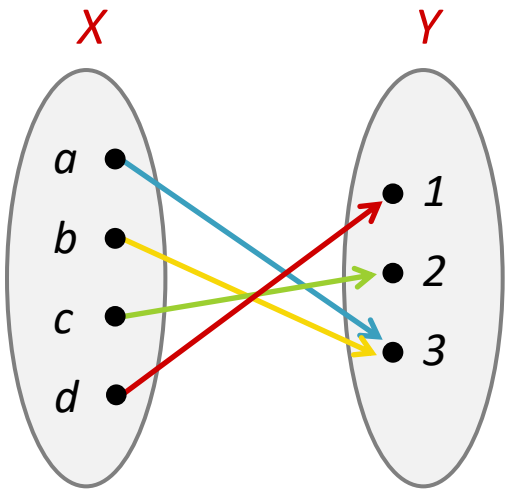
Is f injective?

Example

$f: \mathbb{R} \to \mathbb{R},\ f(x) = \frac{x}{x^2+1}$

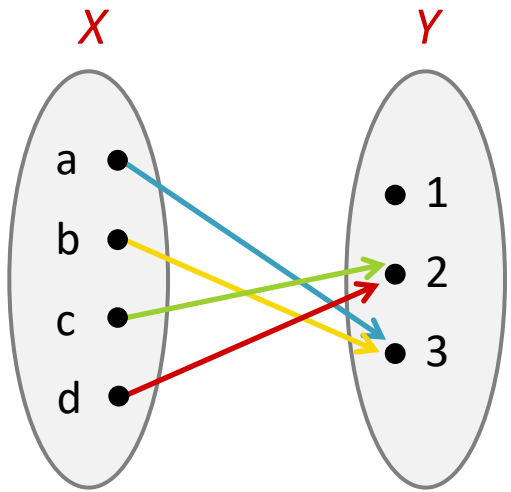
Is f injective?

Surjectivity: Onto Example



Onto

(All elements in *Y* have a preimage)



Not onto

(*1* has no preimage)

Example

$f: \mathbb{R} - \{0\} \to \mathbb{R} - \{1\},\ f(x) = \frac{x+1}{x}$

Is f surjective?

Example

$f: \mathbb{R} \to \mathbb{R},\ f(x) = \frac{x}{x^2+1}$

Is f surjective?

Example

$$f, g: \mathbb{R} \to \mathbb{R},\ f(x) = x + 3; \qquad g(x) = -x^3$$

Find $f^{-1}, g^{-1},\ g \circ f$, and $f \circ g$.