March 2022   **CA2**   TIME ALLOWED: 50 minutes

QUESTION 1.   **(16 marks)**

(a) [8 marks] Solve the following linear recurrence, that is, write a_n in terms of n:

$a_n = 9a_{n-2}$ for each $n \geqslant 2$, with initial conditions $a_0 = 0$, $a_1 = 2$.

(b) [8 marks] A sequence $b_0, b_1, b_2, \ldots$ is defined by letting $b_0 = 3$ and $b_k = (b_{k-1})^2$ for every integer $k \geqslant 1$. Using induction, show that $b_n = 3^{2^n}$ for every integer $n \geqslant 0$.

QUESTION 2.   **(17 marks)**

In this question **no justification is required**.
A computer programming team has 13 members.

(a) [2 marks] How many ways can a group of seven be chosen to work on a project?

(b) Suppose seven team members are women and six are men.

  (i) [3 marks] How many groups of seven can be chosen that contain four women and three men?

  (ii) [3 marks] How many groups of seven can be chosen that contain at least one man?

  (iii) [3 marks] How many groups of seven can be chosen that contain at most three women?

(c) [3 marks] Suppose two team members refuse to work together on projects. How many groups of seven can be chosen to work on a project?

(d) [3 marks] Suppose two team members insist on either working together or not at all on projects. How many groups of seven can be chosen to work on a project?

QUESTION 3.   **(17 marks)**

(a) Let $A = \{a, b\}$, $B = \{1, 2\}$, and $C = \{2, 3\}$. Find each of the following sets.

    (i) [3 marks] $A \times (B \cup C)$

    (ii) [3 marks] $(A \times B) \cap (A \times C)$

    (iii) [3 marks] The power set $P(B - C)$

    (iv) [3 marks] The power set $P(P(\emptyset))$

(b) [5 marks] For all sets A and B, is the power set $P(A \times B)$ equal to $P(A) \times P(B)$? If so then prove it, if not then give a counterexample.