Uncertainty

Jubo Yan

Division of Economics, Nanyang Technological University

yanjubo@ntu.edu.sg

February 9, 2024

Overview

Prevalent Uncertainty

- Uncertainty is prevalent. What are the uncertainties in economy?
  - tomorrow's prices; future wealth; future availability of commodities; present and future actions of other people
- What are rational responses to uncertainty?
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods

Prevalent Uncertainty

- Imagine you now have \$ 100 as your wealth and you need to decide whether to buy lottery ticket number 13 that costs \$5 and pays \$200 if 13 is drawn
- If you choose NOT to buy:
  - \$100 if 13 is not drawn
  - \$100 if 13 is drawn
- If you choose to buy:
  - \$95 if 13 is not drawn
  - \$295 if 13 is drawn

Prevalent Uncertainty

- For a different example, suppose an individual has \$35,000, but there is possibility that he may lose \$10,000 with $p = 0.01$ probability
- An insurance contract that will pay the person \$100 if the loss occurs in exchange for \$1 premium (paid before knowing the actual outcome)
- If he pays \$100 premium
  - \$34,900 with $p = 0.01$; \$34,900 with $p = 0.99$
- In reality, the person may choose to purchase any amount of insurance (between 0 and \$10,000)
  - depends on the person's preference
- We treat money under different circumstances as different goods and confine our discussions to money

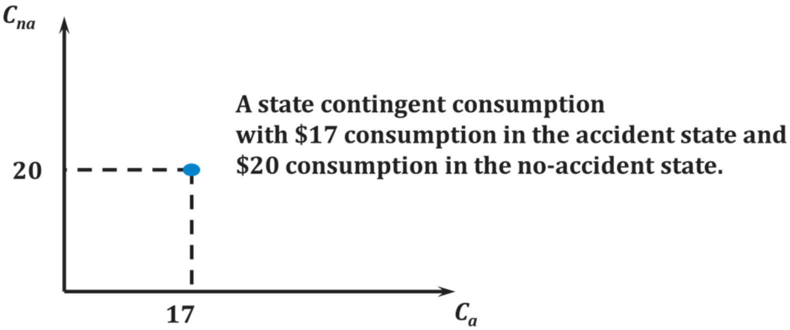
Contingencies

- Possible states of nature can be defined as whether an event happens
  - "car accident" (a) vs. "no car accident" (na)
  - an accident occurs with probability π_a and does not occur with probability π_{na} with $\pi_a + \pi_{na} = 1$
- A contract implemented only when a particular state of nature occurs is state contingent
  - the insurer pays only if there is an accident
- A state contingent consumption plan is implemented only when a particular state of nature occurs
  - take a vacation only if there is no accident

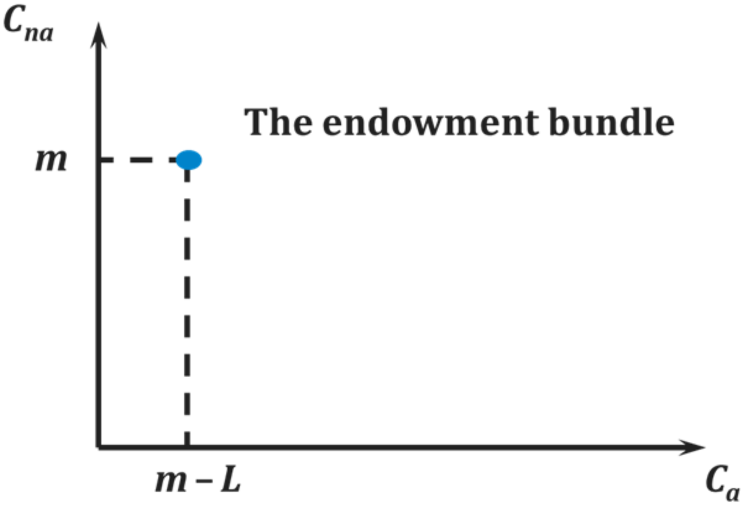
State Contingent Budget Constraints

- Each \$1 of accident insurance costs γ and consumer has m of wealth. C_{na} is consumption in the no-accident state and C_a is consumption in the accident state. L is loss should the accident happen
- Without insurance
  - $C_a = m - L$ and $C_{na} = m$

State Contingent Budget Constraints



State Contingent Budget Constraints



State Contingent Budget Constraints

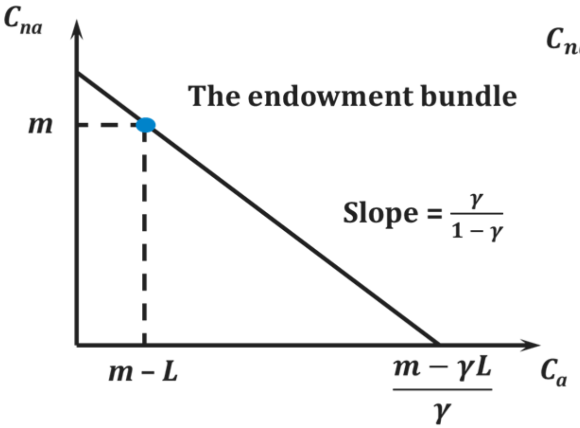
- Buy $\$K$ of accident insurance

$$C_{na} = m - \gamma K$$
$$C_a = m - L - \gamma K + K = m - L + (1-\gamma)K$$
$$K = \frac{C_a - m + L}{1-\gamma}$$

- Therefore

$$C_{na} = m - \frac{\gamma(C_a - m + L)}{1-\gamma}$$
$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma}C_a$$

State Contingent Budget Constraints



$$C_{na} = \frac{m-\gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$

Utility Function and Probabilities ▷

- In general, how a person values consumption in one state as compared to another will depend on the **probability** that the state in question will actually occur
- To describe preferences under uncertainty, we write the utility function as depending on the probabilities as well as on the consumption levels
  - $u(c_1, c_2, \pi_1, \pi_2)$
  - $\pi_1 + \pi_2 = 1$ as state 1 and state 2 are mutually exclusive
  - c_1 and c_2 are consumption in state 1 and 2

Expected utility

- One particularly convenient form that the utility function might take is the following

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

- Can be interpreted as the weighted sum of utilities with probabilities being the weights
- It is named expected utility function or von Neumann-Morgenstern utility function

Expected utility

- Any monotonic transformation of an expected utility function is a utility function that describes the same preferences
  - the additive form representation might be lost
- Expected utility function is unique up to an affine transformation
  - $v(u) = au + b$ with $a > 0$
  - the transformation will generate another utility function that preserves expected utility function properties

Expected Utility

- The expected utility assumes independence across different states
  - suppose you face a possibility that your house being burnt down
  - how much money you would be willing to sacrifice now to get a little more money if the house burns down is irrelevant to
  - how much consumption you will have in the other state of nature–how much wealth you will have if the house is not destroyed

- The independence assumption implies that the utility function for contingent consumption will take an additive form

$$U(c_1, c_2, c_3) = \pi_1 u(c_1) + \pi_2 u(c_2) + \pi_3 u(c_3)$$

$$MRS_{12} = -\frac{\Delta U(c_1, c_2, c_3)/\Delta c_1}{\Delta U(c_1, c_2, c_3)/\Delta c_2} = -\frac{\pi_1 \Delta u(c_1)/\Delta c_1}{\pi_2 \Delta u(c_2)/\Delta c_2}$$

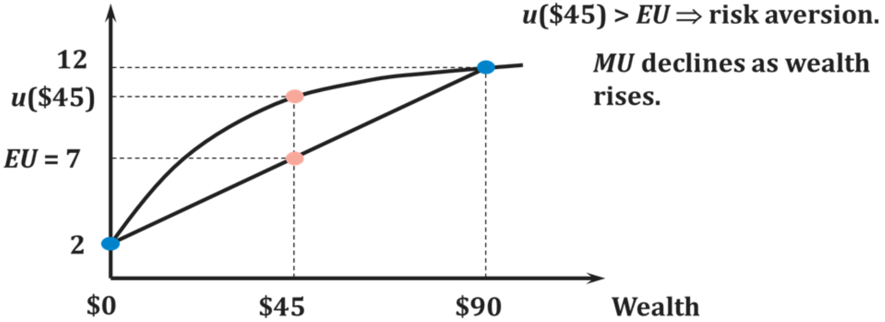
Risk Aversion

- A lottery: win \$90 with probability 1/2 and win \$0 with probability 1/2. $u(\$90) = 12$ and $u(\$0) = 2$

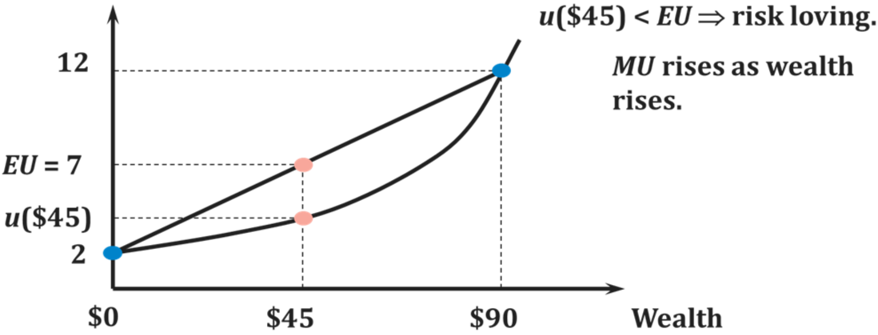
$$EM = \frac{1}{2}(\$90) + \frac{1}{2}(\$0) = 45$$
$$EU = \frac{1}{2}u(\$90) + \frac{1}{2}u(\$0) = \frac{1}{2}(12) + \frac{1}{2}(2) = 7$$

- Comparing EU and $u(EM)$
  - $u(\$45) > 7 \Rightarrow$ risk aversion
  - $u(\$45) < 7 \Rightarrow$ risk loving
  - $u(\$45) = 7 \Rightarrow$ risk neutrality

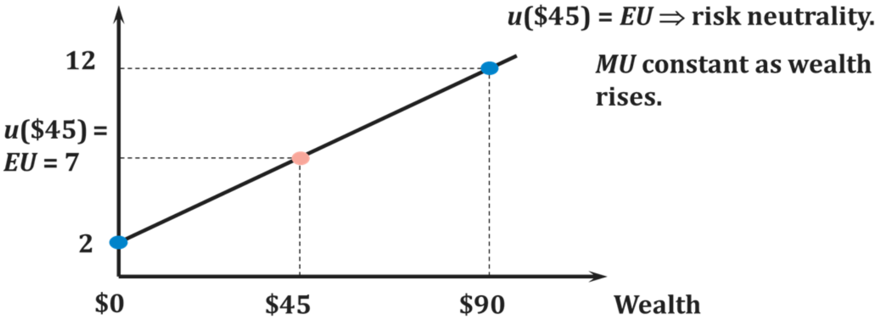
Risk Aversion



Risk Aversion



Risk Aversion

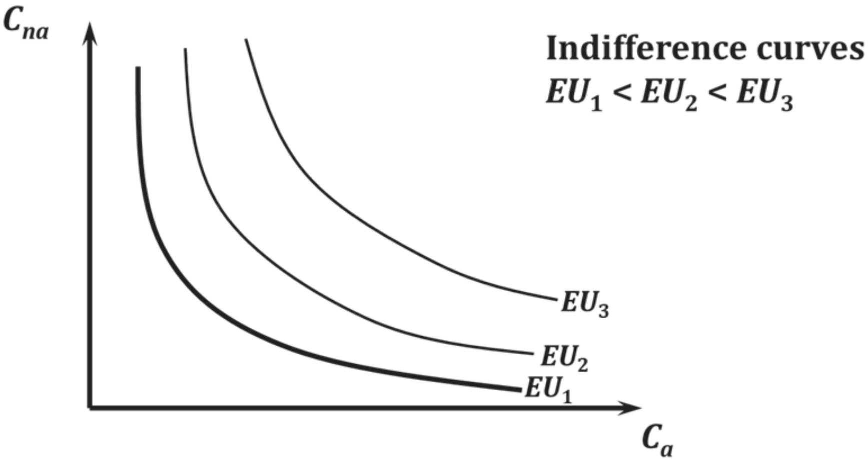


Preference under Uncertainty

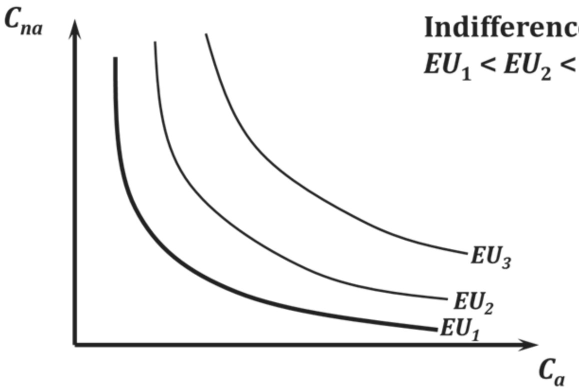
- State contingent consumption plans that give equal expected utility are equally preferred

$$EU = \pi_1 u(c_1) + \pi_2 u(c_2)$$
$$dEU = \pi_1 MU(c_1) dc_1 + \pi_2 MU(c_2) dc_2 = 0$$
$$\frac{dc_2}{dc_1} = -\frac{\pi_1 MU(c_1)}{\pi_2 MU(c_2)}$$

Preference under Uncertainty



Preference under Uncertainty

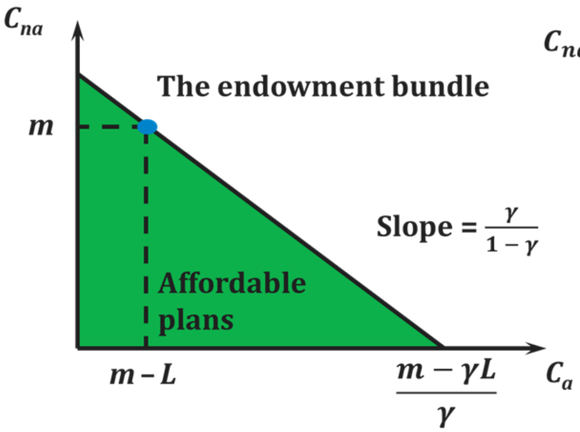


Indifference curves

$EU_1 < EU_2 < EU_3$

$$\frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$

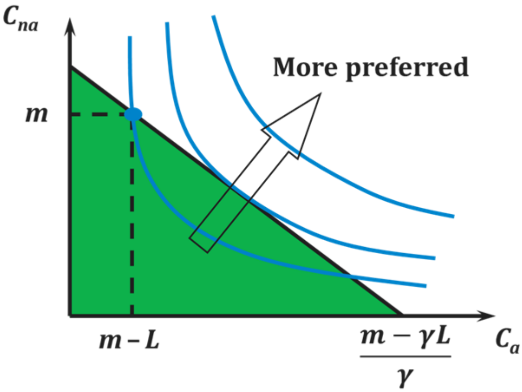
Choice under Uncertainty



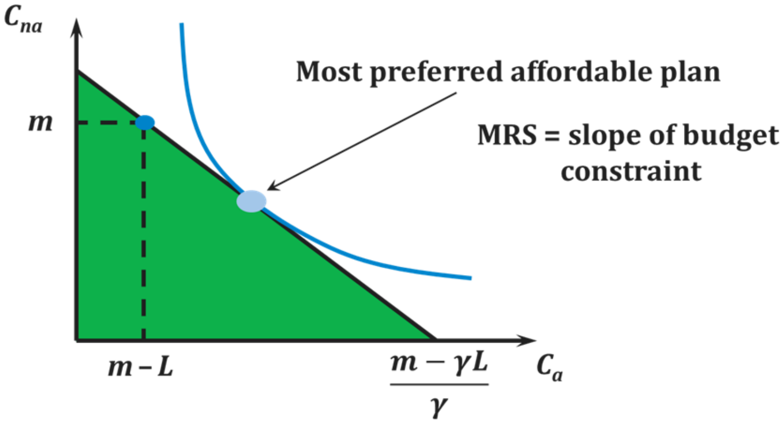
$$C_{na} = \frac{m-\gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$

Choice under Uncertainty

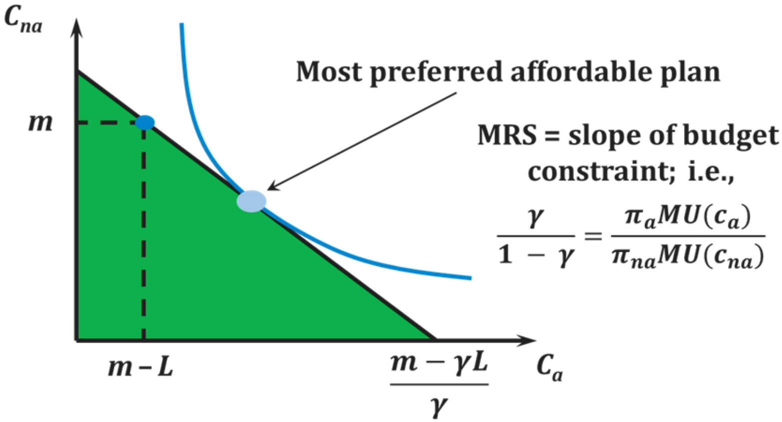
- Choose the most preferred affordable state contingent consumption plan



Choice under Uncertainty



Choice under Uncertainty



Insurance

- If insurers earn zero economic profit

$$\gamma K - \pi_a K - (1-\pi_a)0 = (\gamma - \pi_a)K = 0 \quad \Rightarrow \quad \gamma = \pi_a$$

- If price of \$1 insurance = accident probability, then insurance is fair

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$
$$MU(c_a) = MU(c_{na})$$

- How much fair insurance does a risk-avers consumer buy?
  - risk aversion $\Rightarrow MU(c) \downarrow$ as $c \uparrow$
  - full insurance $\Rightarrow c_a = c_{na}$

Insurance

- If insurers earn positive economic profit

$$\gamma K - \pi_a K - (1-\pi_a)0 = (\gamma - \pi_a)K > 0 \quad \Rightarrow \quad \gamma > \pi_a$$

- Rational choice requires

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})} > \frac{\pi_a}{1-\pi_a}$$
$$MU(c_a) > MU(c_{na})$$

- $c_a < c_{na}$ for a risk averter so he buys less than full "unfair" insurance

Diversification

- Two firms A and B. Shares cost \$10
  - with $p = 0.5$, A's profit is \$100 and B's profit is \$20
  - with $p = 0.5$, A's profit is \$20 and B's profit is \$100
- How should you invest with \$100?
  - buy only A's (B's) shares, you earn \$1,000 with $p = 0.5$ and \$200 with $p = 0.5$; expected earning is \$600
  - buy 5 shares from each firm, you earn \$600 with certainty
- Typically, diversification lowers expected earnings in exchange for lowered risk

Diversification

- Imagine 1,000 individuals each has \$35,000 and faces an *independent* 0.01 probability of a \$10,000 loss

- Each individual bears a large amount of risk–losing \$10,000 and will have an expected wealth of

$$0.99 \times \$35,000 + 0.01 \times \$25,000 = \$34,900$$

- If each individual sells some of his risk to other individuals, he could diversify the risk
  - each individual pays \$100 for certain to build up a cash reserve to compensate those who suffer the loss