Nanyang Technological University
HE2001 Microeconomics II

Solution 6

Question 1

(Note that utility here is implicitly $u(x) = \sqrt{x}$, so we can tell that Willy is risk-averse.)

(a) If he buys no insurance, the contingent commodity bundle will be

$$(c_F, c_{NF}) = (50{,}000, 500{,}000).$$

(a) After paying the insurance premium $0.1x$, he will be able to get the compensation x in case of the flood, so he will consume

$$c_F = 50{,}000 + x - 0.1x = 50{,}000 + 0.9x$$

If there is no flood, he will consume

$$c_{NF} = 500{,}000 - 0.1x$$

(b) Eliminate x from the two equations above and we get the budget equation

$$0.1C_F + 0.9C_{NF} = 455{,}000.$$

(c) Look at the coefficients for C_F and C_{NF} in the budget equation above. The "effective price ratio" of the two contingent commodities is

$$\frac{P_F}{P_{NF}} = \frac{0.1}{0.9}$$

Willy's optimal contingent consumption bundle is obtained when MRS equals the price ratio

$$\left| -\frac{0.1\sqrt{C_{NF}}}{0.9\sqrt{C_F}} \right| = \frac{P_F}{P_{NF}} = \frac{0.1}{0.9}$$

$$\frac{C_{NF}}{C_F} = 1, i.e., there\ is\ full\ insurance$$

(Is insurance fair here? Yes --- net profit per unit of insurance sold is \$0 since price equals expected payout.)

(d) Given $0.1C_F + 0.9C_{NF} = 455{,}000$ and $C_{NF}/C_F = 1$, we can solve for

$$C_{NF} = C_F = 455{,}000$$
$$\text{and } x = 450{,}000$$

So Willy's optimal consumption bundle is $(c_F, c_{NF}) = (455{,}000, 455{,}000)$

He will buy an insurance policy that pays him $\$450{,}000$ if there is a flood. The amount of the insurance premium for this policy is $0.1x = \$45{,}000$.

Question 2

(Note that utility here is $u(x) = x^2$, which is convex, so we can tell that Earl is risk-seeking.)

(a) He will have \$120 if he wins and \$80 if he loses.
(The expected payoff is $(0.25)(120) + (0.75)(80) = 90$.)

The expected utility of taking the bet is

$$u(120, 80, 0.25, 0.75) = (0.25)(120^2) + (0.75)(80^2) = 8{,}400.$$

The expected utility of not taking the bet is

$$u(100) = 100^2 = 10{,}000$$

so he should refuse the bet.

(b) He will have \$200 if he wins and \$0 if he loses.
(The expected payoff is $(0.25)(200) + (0.75)(0) = 50$.)

The expected utility of taking the bet is

$$u(200, 0, 0.25, 0.75) = (0.25)(200^2) + (0.75)(0) = 10{,}000$$

so now he is just indifferent about taking it or not.

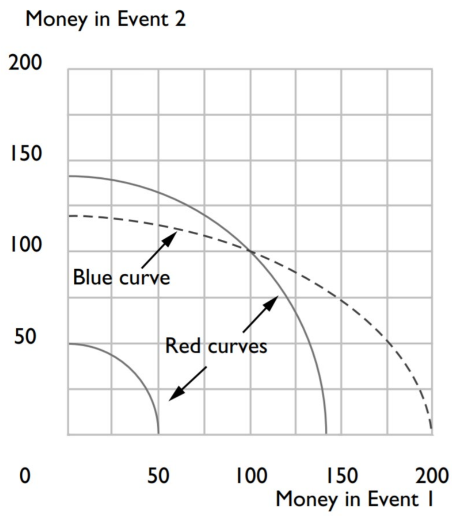
(Notice that the expected payoff is lower, but the variance is larger. The bet is thus more risky, and has less return. Earl prefers it more than the previous one due to this risk.)

(c) Earl's preference between the incomes can be represented as

$$u = \frac{1}{4}c_1^2 + \frac{3}{4}c_2^2.$$

Given this function, his indifference curves are in the shape of an ellipse in the first quadrant with center O and different widths/heights depending on the value of u. The ratio of the x width to the y height is $\sqrt{3}$.

For example, when $\frac{1}{4}c_1^2 + \frac{3}{4}c_2^2 = 100^2 \; or \; \frac{1}{40000}c_1^2 + \frac{3}{40000}c_2^2 = 1$, the height of the ellipse is $200/\sqrt{3}$ and the weight is 200, drawn as the blue curve in the graph.



(Blue curve: the indifference curve of drawing spades;
Red curves: two indifference curves of drawing a black card)

(d) The new events have different probabilities and thus the formula becomes:

$$u = \frac{1}{2}c_1^2 + \frac{1}{2}c_2^2.$$

Given this function, his indifference curves are in the shape of a circle in the first quadrant, with different radii depending on the value of u. Drawn as the red curves in the graph.

(Notice that for these kinds of indifference curves, given a straight budget line, he will always choose a risky option where he gets a lot of money in 1 state and none in the other.)

<u>Question 3</u>

(a) Here the individual is risk-averse. The expected value of the lottery is \$5,050.

The utility of the lottery is $U(10{,}000, 100, 0.5) = 0.5\sqrt{10{,}000} + 0.5\sqrt{100} = 55.$

(b) The utility is $U(4{,}900) = \sqrt{4{,}900} = 70.$

(c) The utility of this lottery is $U(x, y, 0.5) = 0.5\sqrt{x} + 0.5\sqrt{y}$, so the certainty equivalent should be $\left(0.5\sqrt{x} + 0.5\sqrt{y}\right)^2$.

(d) Replace $x = 10{,}000$ and $y = 100$ in the formula above, and we have the certainty equivalent $\left(0.5\sqrt{10{,}000} + 0.5\sqrt{100}\right)^2 = 3{,}025.$

Note that for risk-averse individuals the CE of a risky lottery is less than its expected value. For risk-seeking individuals, the CE of a risky lottery is more than its expected value.