NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

CC0002 Navigating the Digital World

Module 2: Quantitative Reasoning Techniques

Testing Techniques

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Case Study

Does the drug you take for headache work as claimed?

- How do you **know for sure**?
- Can it be a **placebo** instead?
- How do you **test its efficacy**?

Quantitative Reasoning

Desired insights on the problem

A. Does the drug at all reduce your headache in reasonable time?

B. Does the drug manage to work better (faster) than a placebo?

Steps to obtain the desired insights

- How to frame concrete numerical questions?
- How to identify tools and data for analysis?
- How to build models to analyse the data?
- How to analyse the results you obtain?

Suppose you take the drug now, and your headache goes away within the next hour.

Does this mean the drug is effective?

(Write down what you think.)

Identify Your Data

What type of data is relevant?

- Binary: Did your headache subside?
- Continuous: How long did it take to subside?

```
Single trial of drug: YES/NO
Single trial of drug: 16 minutes
```

How much data do you need?

- Is it sufficient to have a single data point?
- Is it required to have a million data points?

```
Single trial: 16
Multiple trials: 16, 18, 24, 20, …
```

Do you want a comparison?

- Which base case would you compare with?
- Is it possible to get data for both the cases?

```
Drug trials: 16, 18, 24, 20, …
Placebo trials: 28, 23, 18, 21, …
```

Formulate Your Question

Which case seems to be better?

- Will better in any one of the trials suffice?
- Does it have to be better in all the trials?
- Is it fine if one is better on average?

Is average behaviour sufficient?

- What if the drug seems better on average?
- Do you know if the drug will always be better?
- How about being better most of the time?

Suddenly, things look not too obvious! ☺

Simulated Experiments

```
Drug trials: 16, 18, 24, 20, …
Placebo trials: 28, 23, 18, 21, …
Number of trials = 30 in each case
```

Compare the Means

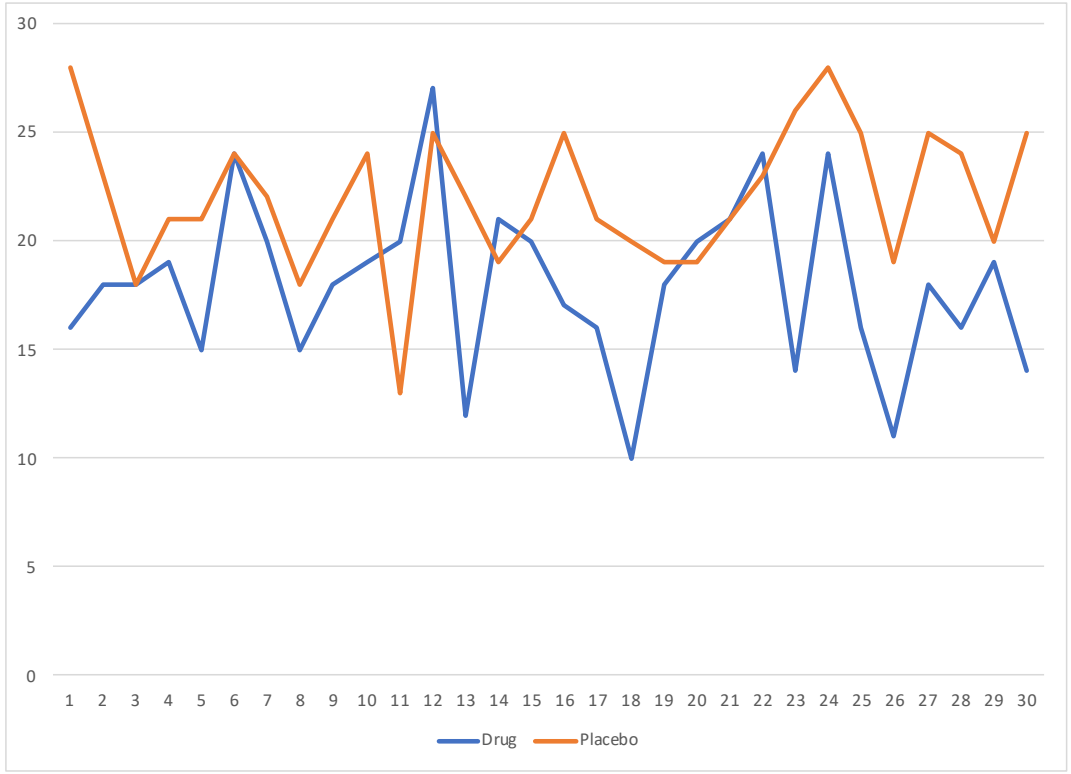
```
Mean(Drug)    : 18 minutes
Mean(Placebo) : 22 minutes
```

Mean is merely a single-point statistic from our entire data distribution.

What is Data Distribution?

Representation: Line plot

Plot data-points connected by lines



Simulated Experiments

Drug trials

16, 18, 24, 20, 18, 19, 15, 15, 18, 19,
20, 27, 12, 21, 20, 17, 16, 10, 18, 20,
21, 24, 14, 24, 16, 11, 18, 16, 19, 14.

Placebo trials

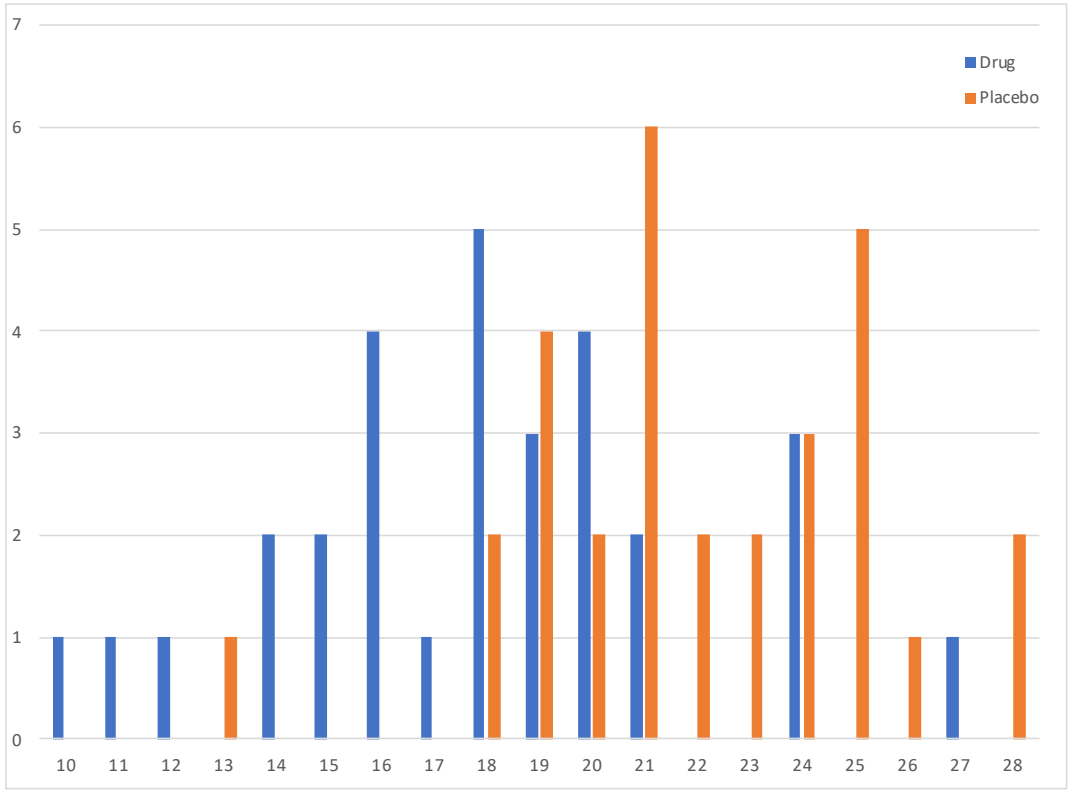
28, 23, 18, 21, 21, 24, 22, 18, 21, 24,
13, 25, 22, 19, 21, 25, 21, 20, 19, 19,
21, 23, 26, 28, 25, 19, 25, 24, 20, 25.

Number of trials = 30 in each case

What is Data Distribution?

Representation: Histogram

Count frequency across specific bins



Simulated Experiments

Drug trials

16, 18, 24, 20, 18, 19, 15, 15, 18, 19,
20, 27, 12, 21, 20, 17, 16, 10, 18, 20,
21, 24, 14, 24, 16, 11, 18, 16, 19, 14.

Placebo trials

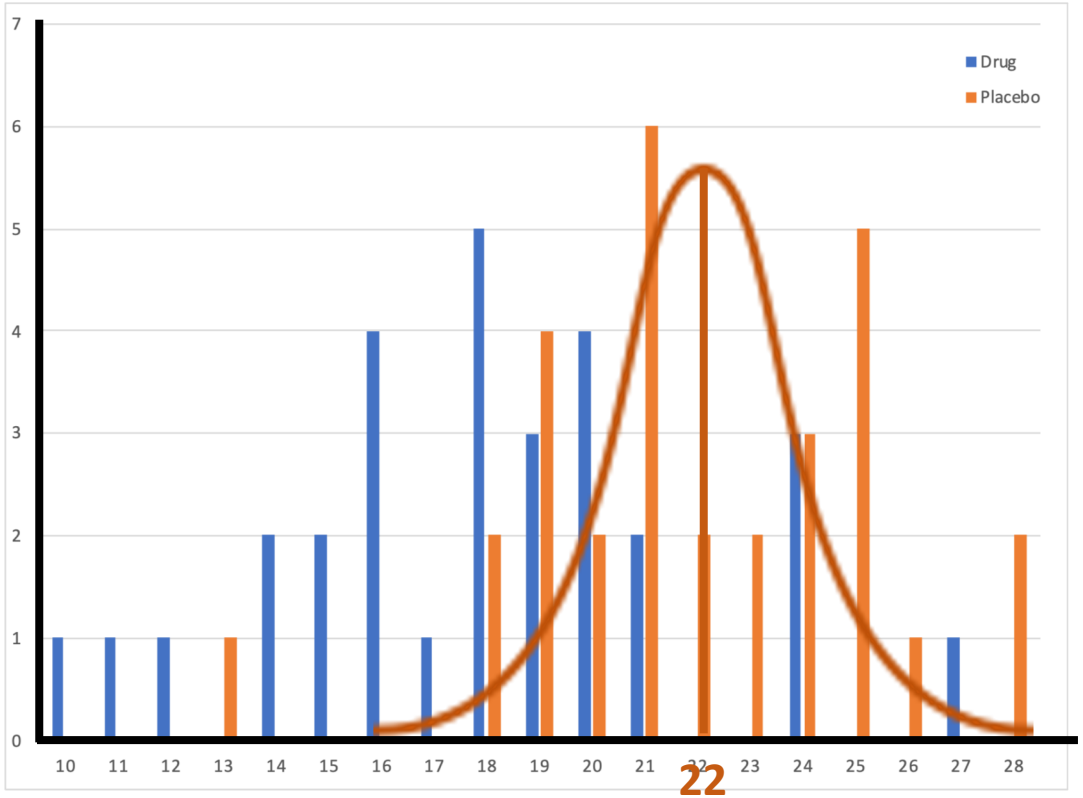
28, 23, 18, 21, 21, 24, 22, 18, 21, 24,
13, 25, 22, 19, 21, 25, 21, 20, 19, 19,
21, 23, 26, 28, 25, 19, 25, 24, 20, 25.

Number of trials = 30 in each case

Comparing Distributions

Core concept: Likelihood

What are the chances (likelihood) that the trials for the drug are identical to the placebo trials?



Idealise the Distribution

Placebo trials (idealised)

Assume a Normal/Gaussian distribution

Mean = 22 | Standard deviation = 3.21

Drug trials (actual ones)

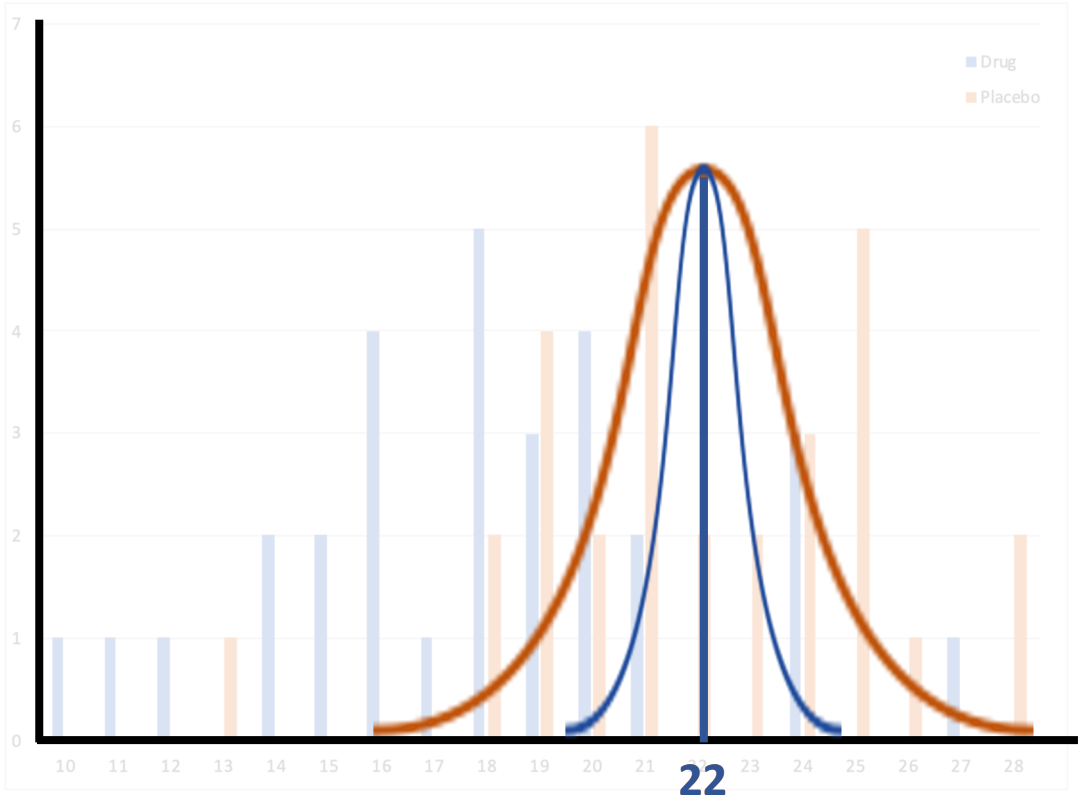
16, 18, 24, 20, 18, 19, 15, 15, 18, 19,
20, 27, 12, 21, 20, 17, 16, 10, 18, 20,
21, 24, 14, 24, 16, 11, 18, 16, 19, 14.

Number of trials = 30 in each case

Comparing Distributions

Core concept: Likelihood

What is the chance (likelihood) that the mean for drug trials is 18 if it is identical to placebo?



Recreate the Distribution

Placebo trials (idealised)

Drug trials (if identical)

Assume a Normal/Gaussian distribution

Mean = 22 | Standard deviation = 3.21

Mean of drug trials (recreated)

Mean = 22 (same as placebo mean)

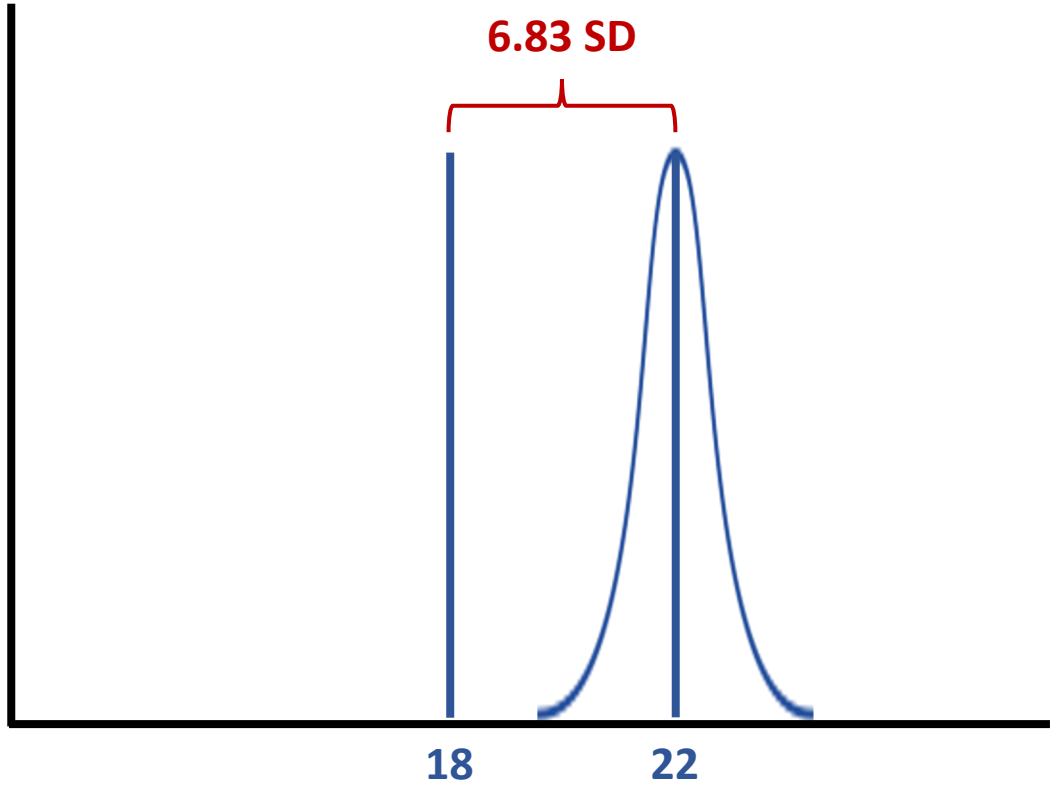
Std. Dev. = 3.21/sqrt(30) = 0.586

Number of trials = 30 in each case

Tool for Our Analysis

Likelihood using Normal Distribution

What is the chance (likelihood) that the actual mean for drug trials is 18 given our analysis?



Analyse the Distribution

Mean of drug trials (recreated)

Mean = 22 (same as placebo mean)

Std. Dev. = 3.21/sqrt(30) = 0.586

Mean of drug trials (actual)

Mean = 18 (from our dataset)

Distance between the means

= 22 - 18 = 4 units (mins.)

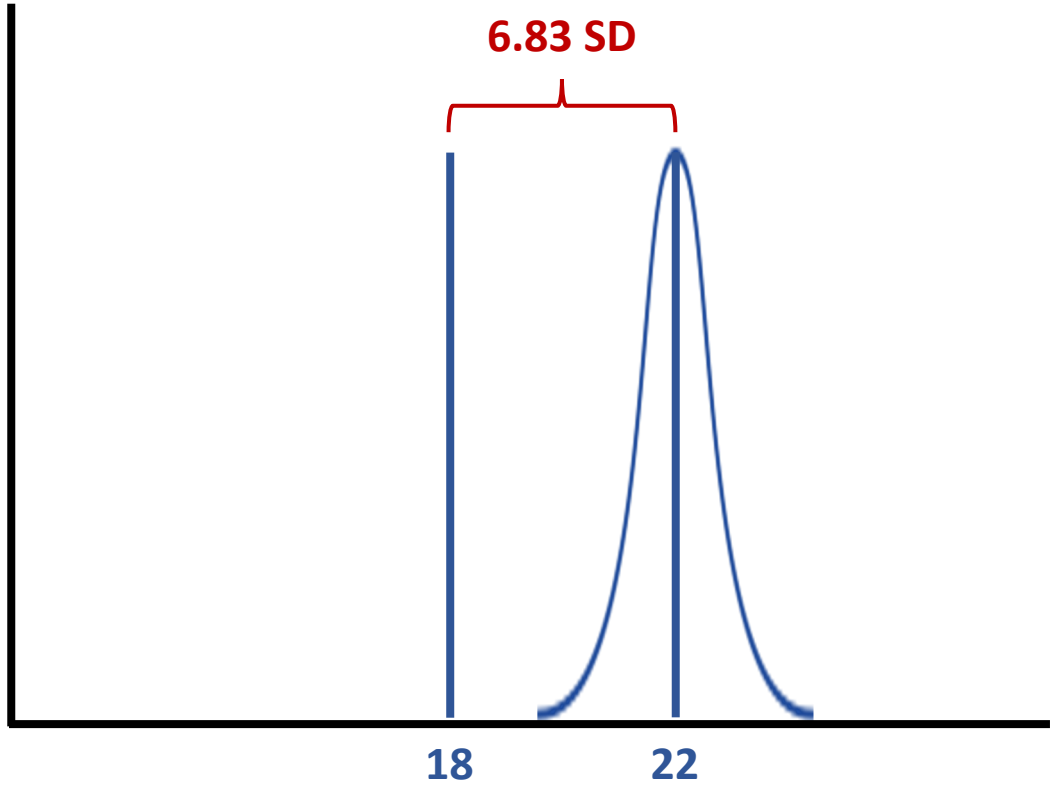
= 4/0.586 = 6.83 Std. Dev.

(quite far, in fact)

Result of Our Analysis

What we did is Hypothesis Testing

What is the chance (likelihood) that the actual mean for drug trials is 18 given our analysis?



Hypothesise a Distribution

Mean of drug trials (hypothesis)

Mean = 22 (same as placebo mean)

Std. Dev. = 3.21/sqrt(30) = 0.586

Mean of drug trials (actual)

Mean = 18 (from our dataset)

What is the probability?

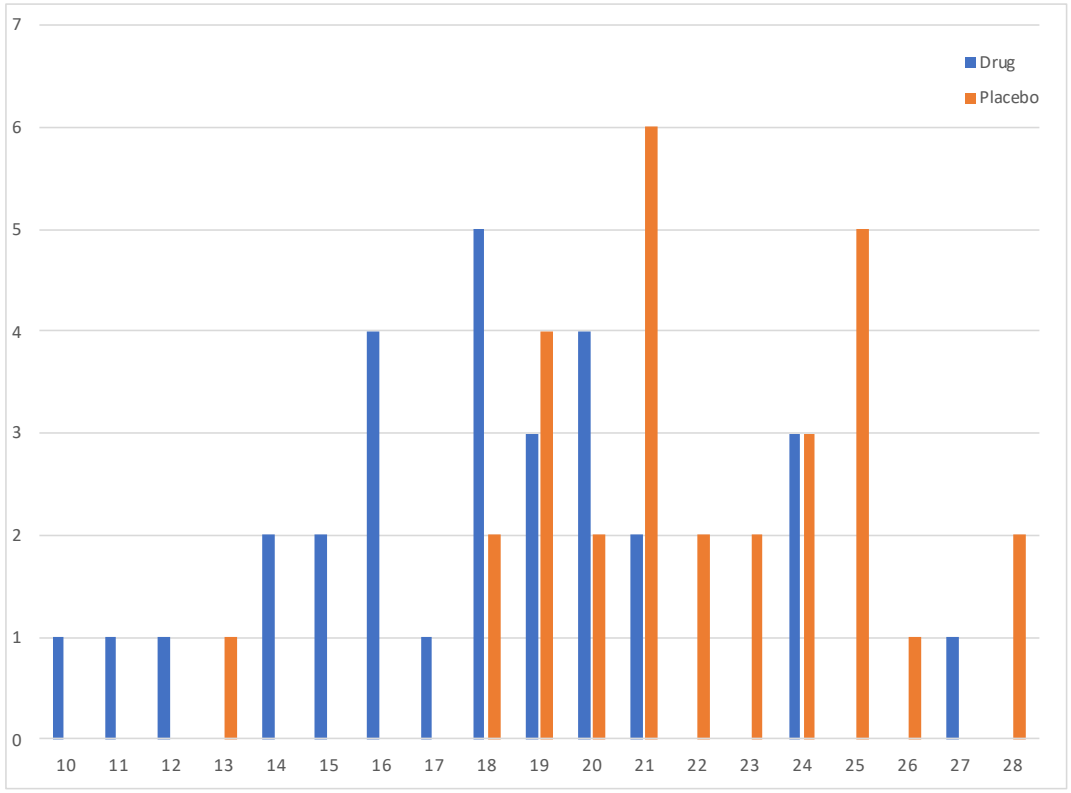
Probability of this happening in practice is **< 1 in 10 billions**

(calculated using the "z-score" for 6.83 SD)

Analysis of Our Result

What we did is Hypothesis Testing

What are the chances (likelihood) that the trials for the drug are identical to the placebo trials?



The Big Picture

Real-life hypothesis: The drug is identical to placebo in efficacy.

Data hypothesis: Drug trials are identical to the placebo trials.

Statistical hypothesis: Statistics of drug trials are identical as if they are drawn from placebo trials.

What is the probability?

Probability of this happening in practice is **< 1 in 10 billions**

Informed Decision

Decision on taking the drug still

A. depends on the difference between drug and placebo.

B. depends on your budget for medicine and pain tolerance.

C. depends on how the drug will act on you in case of headache!

Decision-making is still an individual's choice, and no analysis can decide for you.

We found that the drug is not identical to the placebo in case of our headache trials.

Will you now take the drug for headache?

(Write down what you think.)

Pause and Ponder

Which fertiliser is the most effective for plant growth?

- How do you **know for sure**?
- Can all of them **fail the test**?
- How do you **compare efficacy**?

Acknowledgements

Arranged in order of appearance

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