Intertemporal Choice

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Overview

Motivation

- People often receive income in "lumps" (e.g., monthly salary) but spending usually happens over a period of time
- How is a lump of income spread over a period of time?
    - saving now for consumption later (e.g., the following month)
    - borrowing now against income to be received later (e.g., at the end of the month)

Present and Future Values

- We will assume there are two periods, 1 and 2. Let r denote the interest rate per period
  - if $r = 0.1$, then \$100 saved at the start of period 1 becomes \$110 at the start of period 2
- The value next period of \$1 saved now is the future value of that dollar
  - given an interest rate r the future value one period from now of $\$m$ is $FV = (1 + r)m$

Present and Future Values

- Would you pay \$1 to obtain \$1 at the start of next period?
  - you can save your \$1 now so you will have $\$(1+r) > \$1$ at the start of next period
- The amount of money that needs to be saved now to obtain \$1 at the start of the next period is the present value of \$1
  - $m(1+r) = 1 \Longrightarrow m = \frac{1}{1+r}$
  - the present value of \$m available at the start of next period is $PV = \frac{m}{1+r}$
- If $r = 0.1$, the most you should pay now for \$1 available next period is $PV = \frac{1}{1+0.1} = \$0.91$

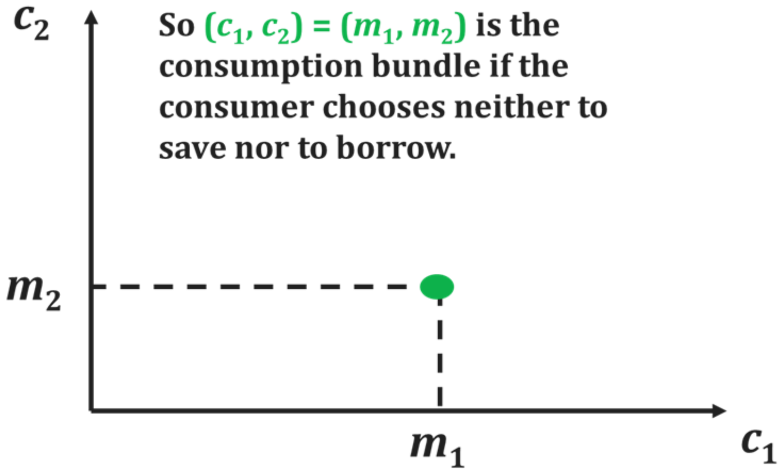
Intertemporal Budget Constraints

- We use subscripts to denote different periods (i.e., 1 for period 1 and 2 for period 2)
    - m_1 and m_2 are incomes received; c_1 and c_2 are consumptions made; p_1 and p_2 are prices
- Given (m_1, m_2) and (p_1, p_2), what is the most preferred intertemporal consumption bundel (c_1, c_2)?
    - intertemporal budget constraint?
    - intertemporal preference?
- For simplicity, we first assume $p_1 = p_2 = 1$

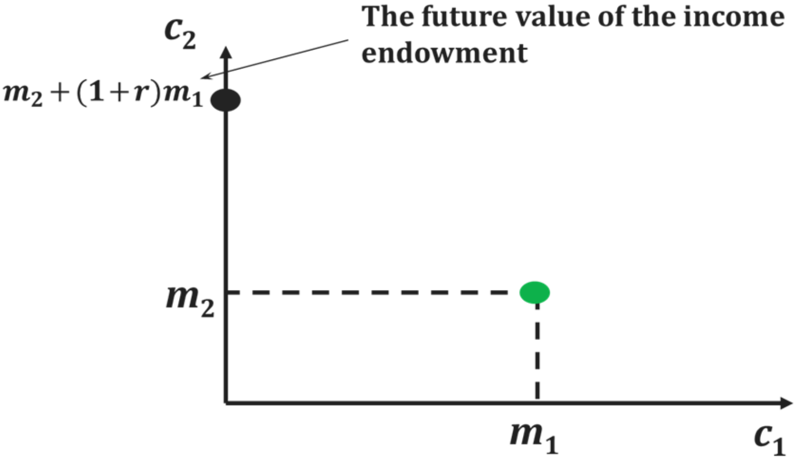
Intertemporal Budget Constraints

- Suppose consumer does not save or borrow
  - $c_1 = m_1$ and $c_2 = m_2$
- Now suppose consumer saves all income in period 1 and only consumes in period 2
  - $c_1 = 0$ and $c_2 = m_2 + (1 + r)m_1$
- Now suppose consumer spends everything in period 1 by borrowing against m_2
  - $c_1 = m_1 + \frac{m_2}{1+r}$ and $c_2 = 0$

Intertemporal Budget Constraint



Intertemporal Budget Constraint



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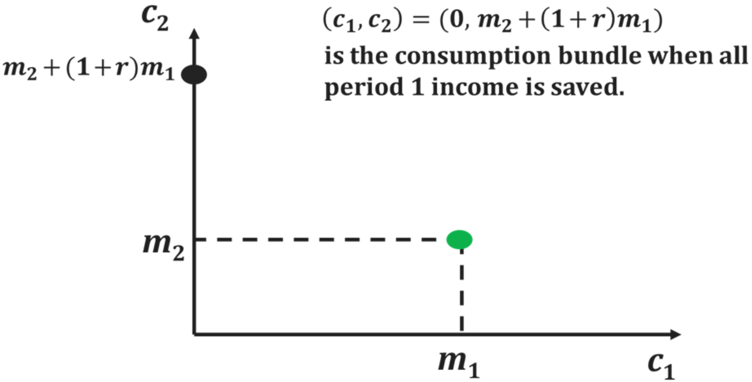

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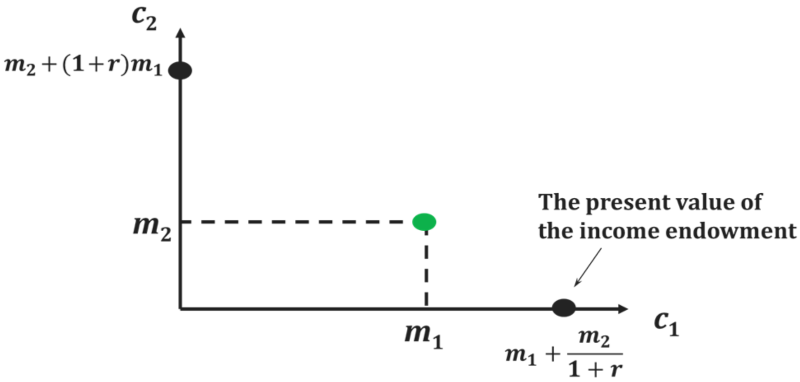

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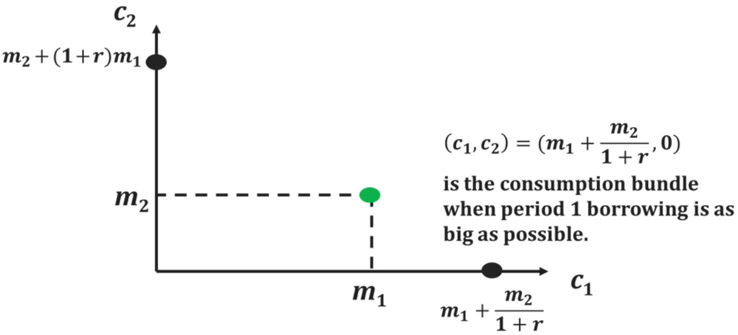


Intertemporal Budget Constraint

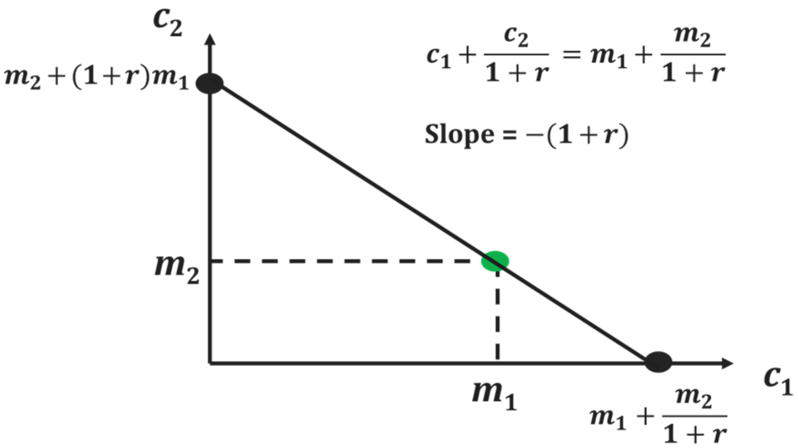
- Suppose c_1 units are consumed in period 1. This costs \$$c_1$ and leaves $m_1 - c_1$ saved. Period 2 consumption will then be

$$c_2 = m_2 + (1 + r)(m_1 - c_1)$$
$$c_1 + \frac{c_2}{1 + r} = m_1 + \frac{m_2}{1 + r}$$

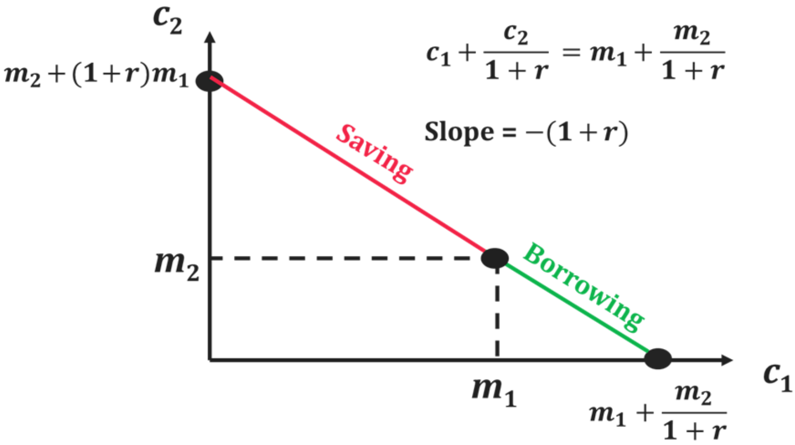
- Now we add prices p_1 and p_2 back

$$p_1 c_1 + \frac{p_2}{1 + r} c_2 = m_1 + \frac{m_2}{1 + r}$$

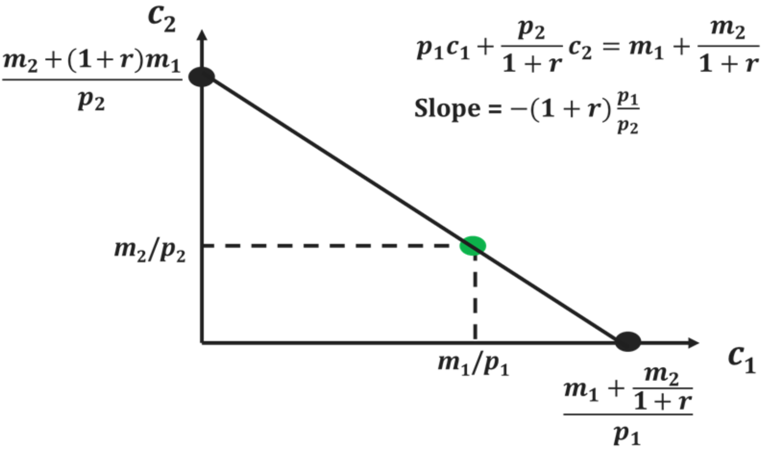
Intertemporal Budget Constraint



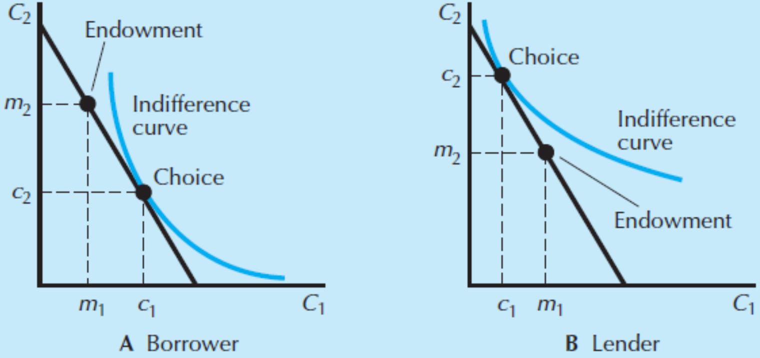
Intertemporal Budget Constraint



Intertemporal Budget Constraint ▷



Borrower and Lender

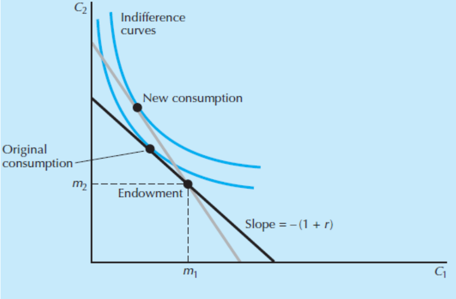


Borrower and lender. Panel A depicts a borrower, since $c_1 > m_1$, and panel B depicts a lender, since $c_1 < m_1$.

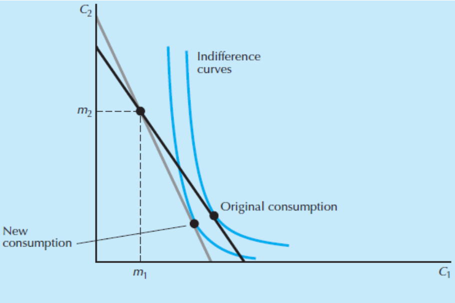
Interest Rate Change

- Similar to the case of consumption with endowment, we can also learn something about how the choice of being a borrower or a lender changes as the interest rate changes
- When interest rate decreases
  - lender $\rightarrow$ lender: welfare decreasing
  - lender $\rightarrow$ borrower: uncertain
  - borrower $\rightarrow$ borrower: welfare increasing
- When interest rate increases
  - borrower $\rightarrow$ borrower: welfare decreasing
  - borrower $\rightarrow$ lender: uncertain
  - lender $\rightarrow$ lender: welfare increasing

Interest Rate Change



If a person is a lender and the interest rate rises, he or she will remain a lender. Increasing the interest rate pivots the budget line around the endowment to a steeper position; revealed preference implies that the new consumption bundle must lie to the left of the endowment.



A borrower is made worse off by an increase in the interest rate. When the interest rate facing a borrower increases and the consumer chooses to remain a borrower, he or she is certainly worse off.

Intertemporal Slutsky Equation

- The use of Slutsky equation is also similar to the case of consumption with endowment
- Write the intetemporal budget constraint in terms of future value

$$p_1 c_1 + p_2 c_2 = m_1(1+r) + m_2 \quad \text{with } p_1 = 1 + r, p_2 = 1$$

- Increasing the interest rate r is equivalent to an increase in period 1 price p_1

$$\frac{\Delta c_1^t}{\Delta p_1} = \underbrace{\frac{\Delta c_1^s}{\Delta p_1}}_{-} + \underbrace{(m_1 - c_1)}_{\substack{\text{lender: } + \\ \text{borrower: } -}} \underbrace{\frac{\Delta c_1^m}{\Delta m}}_{\substack{\text{normal: } + \\ \text{inferior: } -}}$$

Price Inflation

- Define the inflation rate by π where $p_1(1+\pi) = p_2$
  - $\pi = 0.2$ means 20% inflation while $\pi = 1$ means 100% inflation

- For simplicity, assume that $p_1 = 1$ so that $p_2 = 1 + \pi$

$$
\begin{aligned}
c_1 + \frac{1+\pi}{1+r} c_2 &= m_1 + \frac{m_2}{1+r} \\
c_2 &= -\frac{1+r}{1+\pi} c_1 + \frac{1}{1+\pi}\left((1+r)m_1 + m_2\right)
\end{aligned}
$$

Real Interest Rate

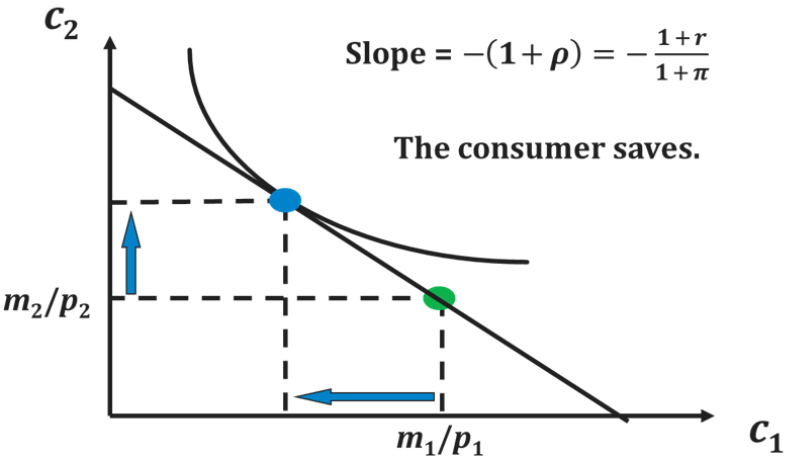
- With inflation, the slope of the intertemporal budget line is

$$-\frac{1+r}{1+\pi} = -(1+\underbrace{\rho}_{\text{real interest rate}})$$

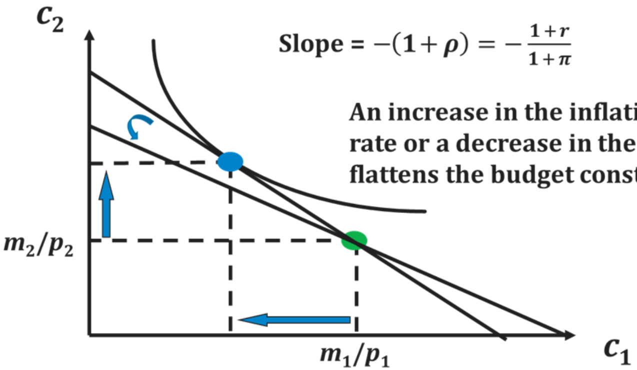
$$\rho = \frac{r-\pi}{1+\pi}$$

- For small π ($\pi \to 0$), $\rho \to r - \pi$

Inflation Rate Change



Inflation Rate Change



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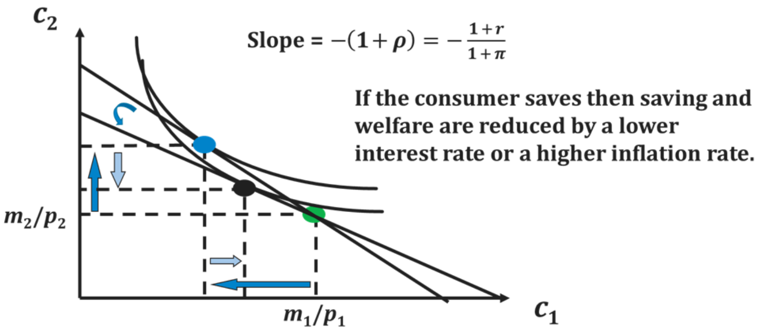

Securities

- A financial security is a financial instrument that promises to deliver an income stream
  - a security pays m_1 at end of year 1, m_2 at end of year 2, and m_3 at end of year 3
- How should the security be priced?
- Calculate the PV of m_1, m_2, and m_3
  - $\frac{m_1}{1+r}$, $\frac{m_2}{(1+r)^2}$, and $\frac{m_3}{(1+r)^3}$
  - the PV of the security is the sum of the three PVs

Bonds

- A bond is a special type of security that pays a fixed amount $\$x$ for T years (its maturity date) and then pays its face value $\$F$
- How should the bond be priced?
- Applying the calculation of security payment

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots + \frac{x}{(1+r)^T} + \frac{F}{(1+r)^T}$$

Consols

- A consol (or perpetuity)is a bond which never terminates, paying $\$x$ per period forever
- How should the consol be priced?
- The present value can be calculated by applying the calculation of security payment

$$\begin{aligned} PV &= \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots + \frac{x}{(1+r)^t} + \ldots \\ &= \frac{1}{1+r}\left[x + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \ldots\right] \\ &= \frac{1}{1+r}\left[x + PV\right] \end{aligned}$$

- Hence $PV = \frac{x}{r}$