Tutorial 3

Determinants

1. If a 3×3 matrix A has $|A| = -1$, find $|\frac{1}{2}A|, |-A|, |A^2|$ and $|A^{-1}|$.

2. Reduce $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ to U to find $|A|$ as the product of pivots.

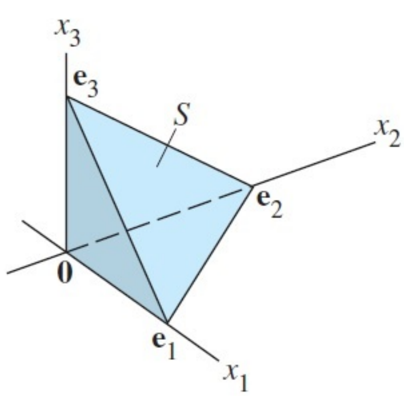
3. Using variables a, b, c, construct a 3×3 skew-symmetric matrix ($A = -A^T$). Show that the determinant of such a matrix is equal to 0.

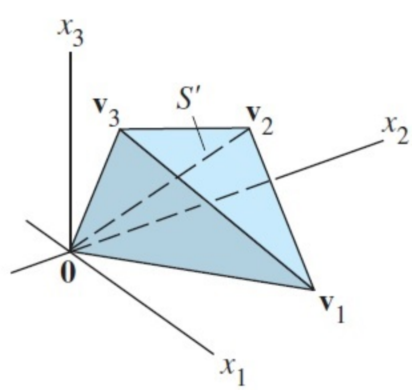
4. Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1, 3, 0), (-2, 0, 2)$ and $(-1, 3, -1)$.

5. In the figure below, let S be the tetrahedron in $\mathbb{R}^3$ with vertices at the vectors $\mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ ($\mathbf{e}_i$'s are standard unit vectors) and let S' be the tetrahedron with vertices at vectors $\mathbf{0}, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

   a. Find the standard matrix for the linear transformation that maps S to S'.

   b. Find a formula for the volume of the tetrahedron S'. (Volume of a tetrahedron = (1/3) × (area of base) × height).





Answers

1. -1/8, 1, 1, -1

2. 1

3.

4. 18

5. a. $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$ b. $(1/6) \times \mathrm{abs}(|A|)$

End