Example Class 2

Propositional Logic

Outline

- Knights & Knaves
- Find the murderer’s knife

The Island of Knights & Knaves

"Alice in Wonderland" White Knight Concept Art by Michael Kutsche © Disney Enterprises Inc.

Knights never lie

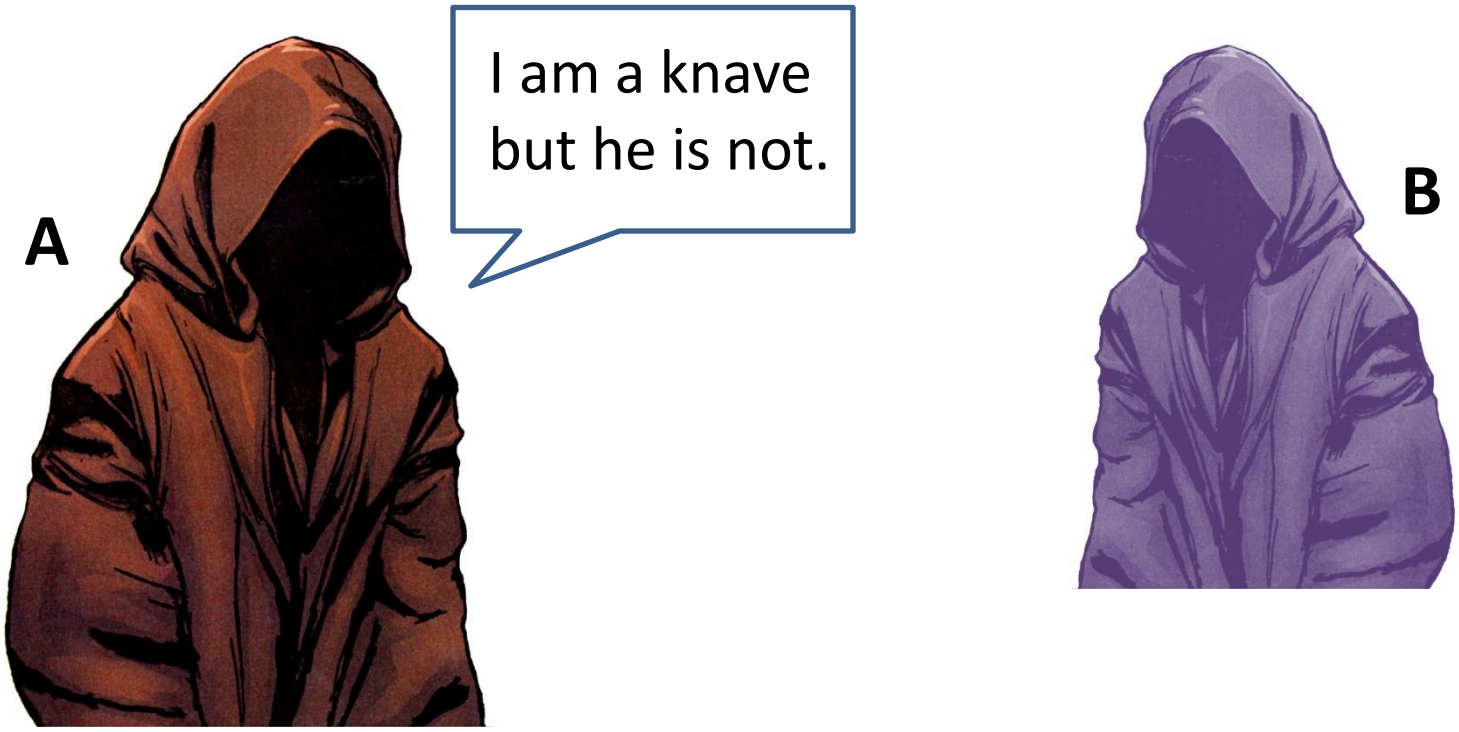
"Alice in Wonderland" Knave of Hearts Concept Art by Michael Kutsche © Disney Enterprises Inc.

Knaves always lie

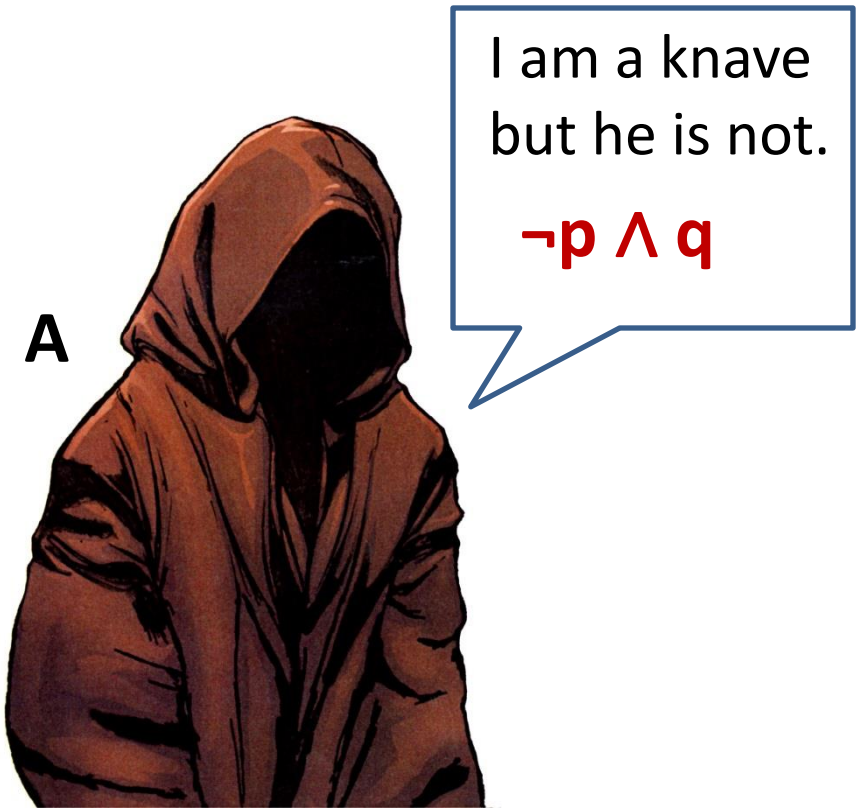
Knave = a dishonest or unscrupulous man,
in cards a jack.

Art work belongs to Michael Kutsche

Knights & Knaves I



Knights & Knaves I



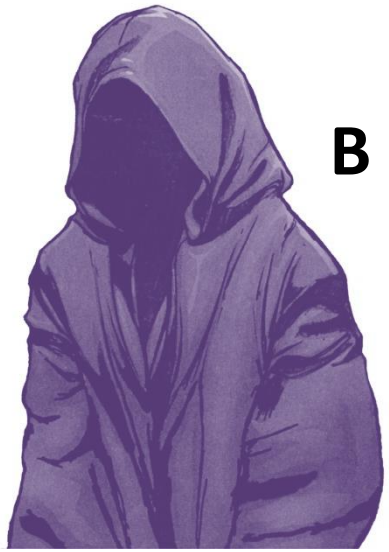
- p="A is a knight"
- q="B is a knight"

p	q	¬p ∧ q
T	T	F
T	F	F
F	T	T
F	F	F

- If A is a knight, then p = true, and ¬p ∧ q must be true.
- If A is a knave, then p=false, and ¬p ∧ q must false.

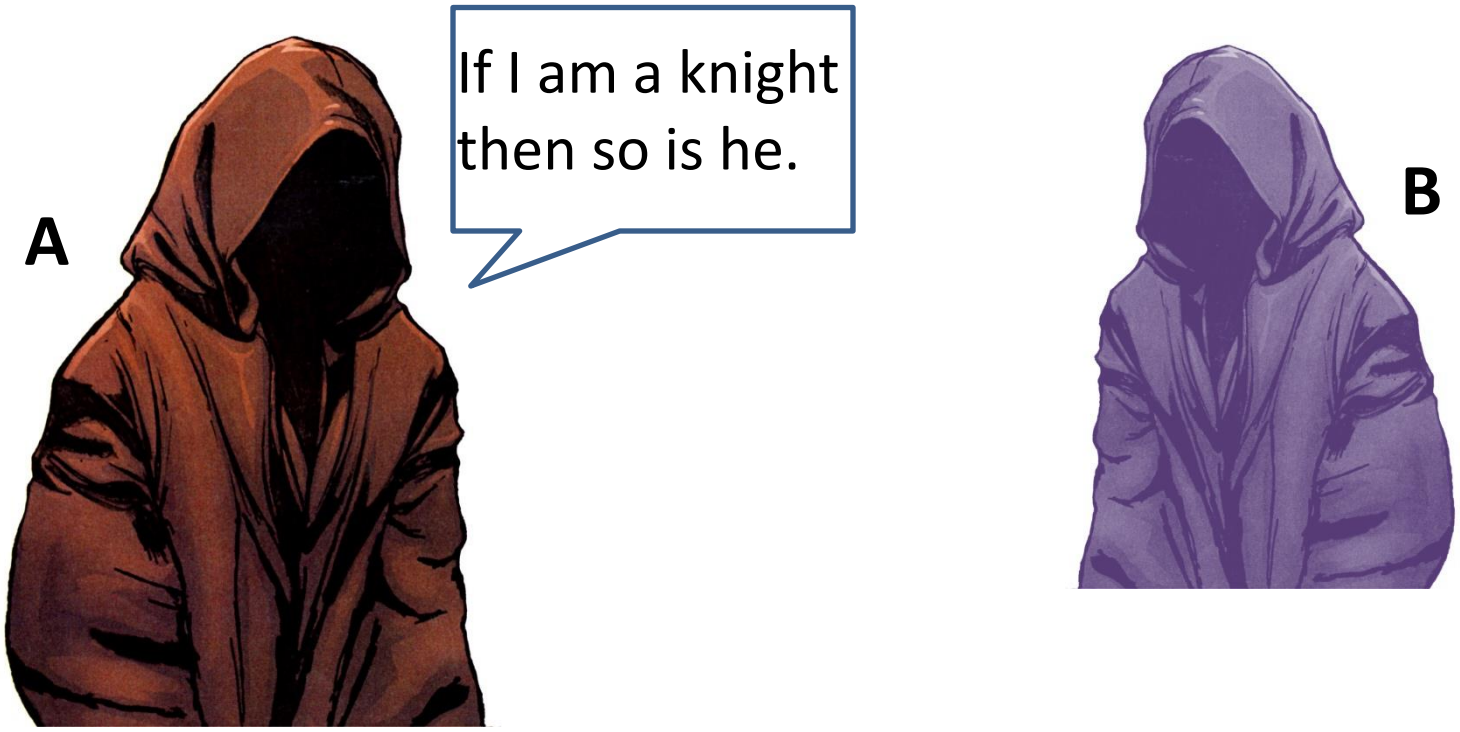
Knights & Knaves I

A is a Knave!

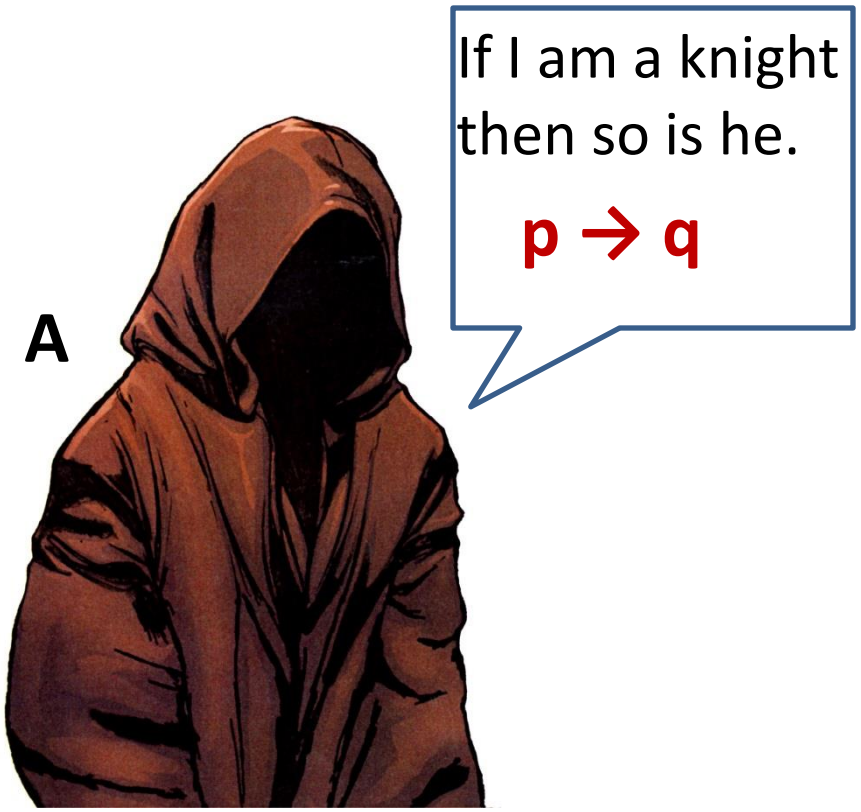


B is a Knave!

Knights & Knaves II



Knights & Knaves II



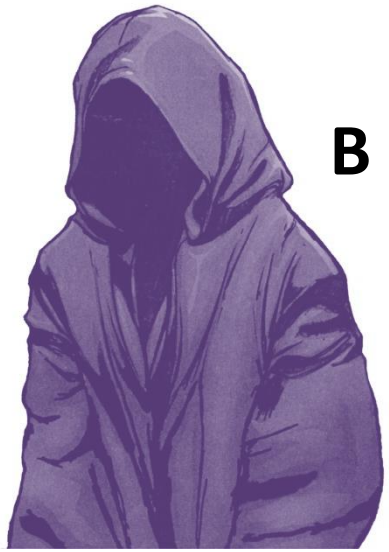
- p="A is a knight"
- q="B is a knight"

p	q	p → q
T	T	T
T	F	F
F	T	T
F	T	T

- If A is a knight, then p = true, and p → q must be true.
- If A is a knave, then p=false, and p → q must false.

Knights & Knaves II

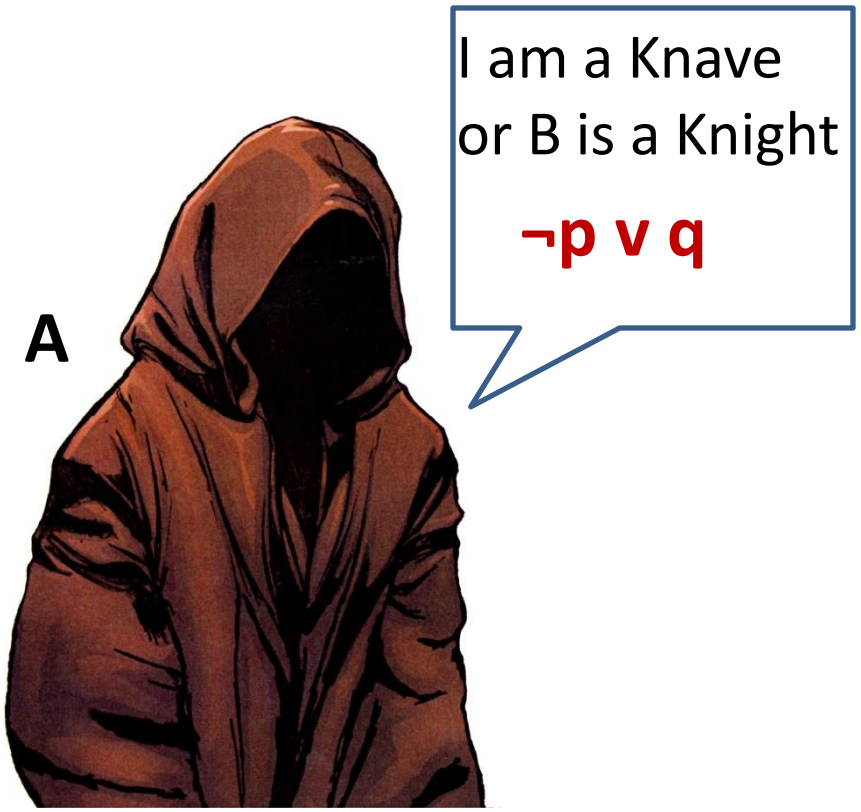
A is a Knight!



B is a Knight!

Knights & Knaves III

Knights & Knaves III



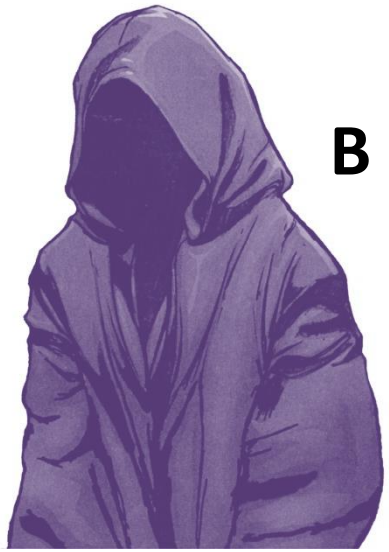
- p="A is a knight"
- q="B is a knight"

p	q	¬p v q
T	T	T
T	F	F
F	T	T
F	F	T

- If A is a knight, then p = true, and ¬p v q must be true.
- If A is a knave, then p=false, and ¬p v q must false.

Knights & Knaves III

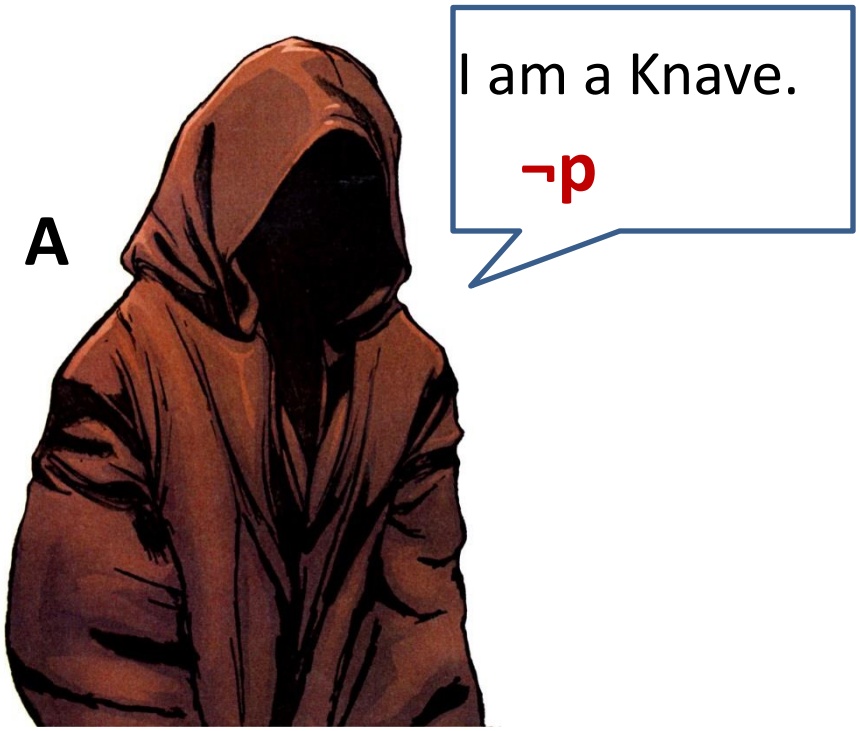
A is a Knight!



B is a Knight!

Knights & Knaves IV

- p="A is a knight"

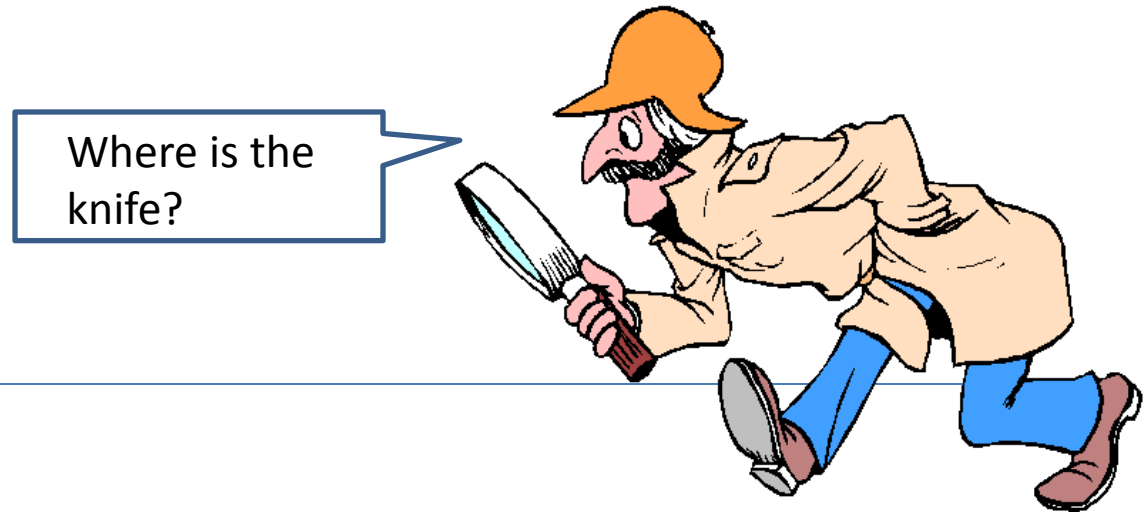


p	¬p
T	F
F	T

It's a paradox!

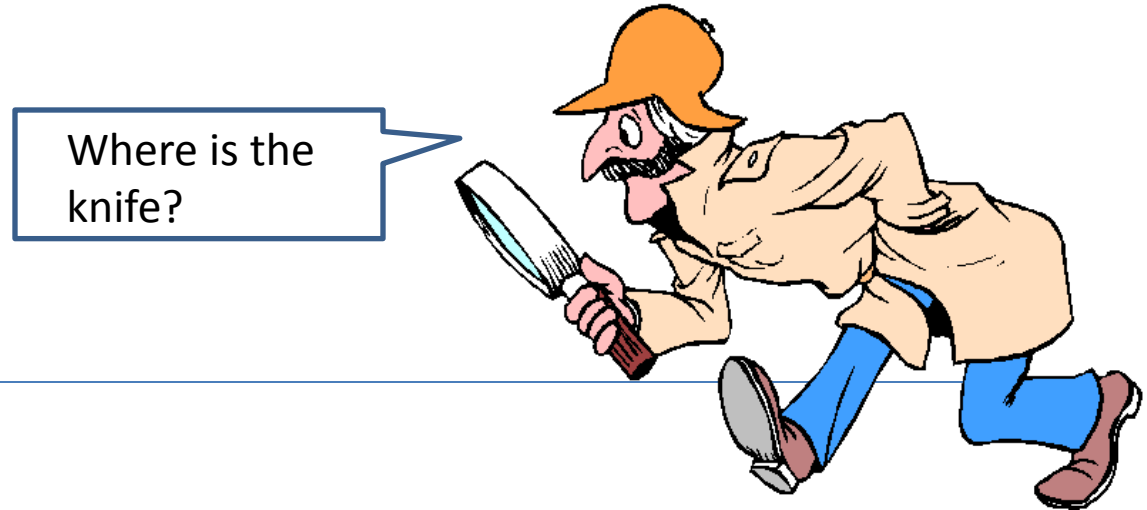
The Murder Clues

1. if the knife is in the **s**tore room, then we saw it when we **c**leared the store room;
2. the murder was committed at the **b**asement or inside the **a**partment;
3. if the murder was committed at the **b**asement, then the knife is in the **y**ellow dust bin;
4. we did not see a knife when we **c**leared the store room;
5. if the murder was committed **o**utside the building, then we are **u**nable to find the knife;
6. if the murder was committed inside the **a**partment, then the knife is in the **s**tore room.



Statements

1. if the knife is in the **s**tore room, then we saw it when we **c**leared the store room;
2. the murder was committed at the **b**asement or inside the **a**partment;
3. if the murder was committed at the **b**asement, then the knife is in the **y**ellow dust bin;
4. we did not see a knife when we **c**leared the store room;
5. if the murder was committed **o**utside the building, then we are **u**nable to find the knife;
6. if the murder was committed inside the **a**partment, then the knife is in the **s**tore room.



Applying Inference Rules

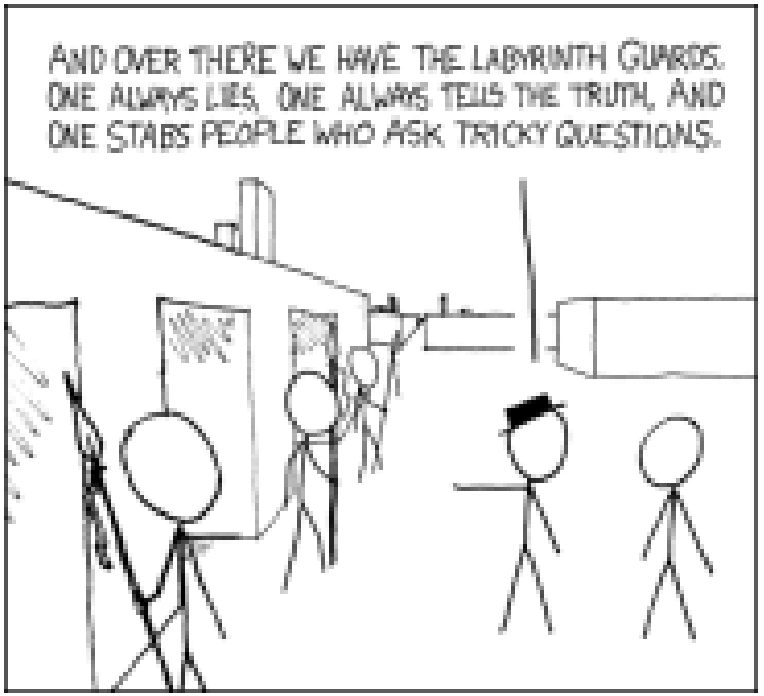
1. if **s**, then **c**;
2. **b** or **a**;
3. if **b**, then **y**;
4. not **c**;
5. if **o**, then **u**;
6. if **a**, then **s**

1. $s \rightarrow c$
2. $b \vee a$
3. $b \rightarrow y$
4. $\neg c$
5. $o \rightarrow u$
6. $a \rightarrow s$
7. $\neg s$ 1, 4; modus tollens
8. $\neg a$ 6, 7; modus tollens
9. b 2, 8; case elimination

$\therefore y$ 3, 9; modus ponens

Summary

- You will need logic for serious matters, such as programming or relational databases.
- Logic can learnt through puzzles: Knights & Knaves, or Find the murderer's knife



Cartoon belongs to xkcd