

Name:

Matric. no.:

Tutor group:

February 2023

CA1

TIME ALLOWED: 50 minutes

QUESTION 1. (15 marks)

- (a) [5 marks] For each element $x \in \{0, \dots, 4\}$ find which element of $\{0, \dots, 4\}$ is congruent to x^2 modulo 5. Fill in the following table accordingly.

$x \pmod{5}$	0	1	2	3	4
$x^2 \pmod{5}$					

- (b) [5 marks] For each $a \in \{0, 1, \dots, 4\}$ evaluate the truth value of the following statement

$$\exists x \in \{0, 1, \dots, 4\} \text{ such that } 2x - 1 \equiv a \pmod{5}.$$

Fill in the following table accordingly.

a	0	1	2	3	4
T/F					

- (c) [5 marks] Let S be the set of integers that are congruent to 2 modulo 4. Is S closed under multiplication? Justify your answer.

For graders only:	Question	1(a)	1(b)	1(c)	2(a)	2(b)	2(c)	3(a)	3(b)	3(c)	Total
	Marks										

QUESTION 2. (15 marks)

Let \mathbb{Q} denote the set of rational numbers and S denote the set of odd integers. Determine the truth value of the following statements. Justify your answers.

- (a) [5 marks] $\forall x \in \mathbb{Q}, \exists y \in \mathbb{Q}, \exists z \in S, xyz = 2023$;

- (b) [5 marks] $\exists x \in S, \exists y \in \mathbb{Q}, \forall z \in S, x + yz = 2023$;

- (c) [5 marks] $\neg(\forall x \in S, \forall y \in \mathbb{Q}, \exists z \in S, xy + z = 2023)$.

QUESTION 3. (20 marks)

- (a) [5 marks] Write out the truth table for the compound propositions $(\neg p \rightarrow q) \vee \neg q$ and $\neg(q \wedge p) \vee q$. Is $(\neg p \rightarrow q) \vee \neg q \equiv \neg(q \wedge p) \vee q$?

- (b) [5 marks] Identify all the critical rows for the argument below. Write out the truth values for p , q , r , and s for each critical row that you find.

$$\begin{aligned} &\neg p \rightarrow q; \\ &r \rightarrow q; \\ &s \rightarrow \neg p; \\ &\neg s; \\ &\therefore \neg r. \end{aligned}$$

- (c) For each of the following arguments, decide whether or not it is valid. If it is invalid give a counterexample, if it is valid then demonstrate how the conclusion follows from the premises, pointing out which inference rule you are using at each step. You may need the following inference rules: modus ponens, modus tollens, and disjunctive syllogism.

- (i) [5 marks]

$$\begin{aligned} &p \wedge q; \\ &\neg r \rightarrow s; \\ &q \vee r; \\ &p \vee s; \\ &\therefore r. \end{aligned}$$

- (ii) [5 marks]

$$\begin{aligned} &\neg p \rightarrow r; \\ &r \rightarrow s; \\ &\neg p \vee q; \\ &\neg q; \\ &\therefore s. \end{aligned}$$