Exercises for Chapter 1

Exercise 1. Show that 2 is the only prime number which is even.

Exercise 2. Show that if n^2 is even, then n is even, for n an integer.

Exercise 3. The goal of this exercise is to show that $\sqrt{2}$ is irrational. We provide a step by step way of doing so.

1. Suppose by contradiction that $\sqrt{2}$ is rational, that is $\sqrt{2} = \frac{m}{n}$, for m and n integers with no common factor. Show that m has to be even, that is $m = 2k$.

2. Compute m^2, and deduce that n has to be even too, a contradiction.

Exercise 4. Let n be an integer greater than 1. Suppose that $a \equiv a' \mod n$ and $b \equiv b' \mod n$. Show that

1. $(a + b) \mod n \equiv (a' + b') \mod n$,

2. $(a \cdot b) \mod n \equiv (a' \cdot b') \mod n$.

Exercise 5. Compute the addition table and the multiplication tables for integers modulo 4.

Exercise 6. Show that $\frac{m(m+1)}{2} \equiv 0 \pmod{m}$ for m an odd number.

Exercise 7.

1. Compute $7 \cdot 8 \cdot 9 \cdot 10$ modulo 3.

2. Show that $n^3 - n$ is always divisible by 3, for n any positive integer.

Exercise 8. Compute 40^{1234} modulo 2.

Exercise 9. Consider the set S of odd natural numbers, with respective operator Δ.

- Let Δ be the multiplication. Is S closed under Δ? Justify your answer.
- Let Δ be the addition. Is S closed under Δ? Justify your answer.

Exercise 10. Consider the following sets S, with respective operator Δ.

- Let S be the set of rational numbers, and Δ be the multiplication. Is S closed under Δ? Justify your answer.

- Let S be the set of natural numbers, and Δ be the subtraction. Is S closed under Δ? Justify your answer.
- Let S be the set of irrational numbers, and Δ be the addition. Is S closed under Δ? Justify your answer.