I.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & 0 & 2 \end{bmatrix}$$

$$\downarrow$$

$R_2 \leftrightarrow R_1$

~~R~~

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 4 \\ 1 & 1 & 1 & 1 & 6 \\ 1 & 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_3 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & 6 \end{bmatrix}$$

$R_2 \leftarrow R_2 - R_1$

$R_3 \leftarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 & 2 \end{bmatrix}$$

$R_3 \leftrightarrow R_2$

$$\begin{array}{c} \begin{array}{ccccc} w & x & y & z & \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix} \end{array}$$

$-z = -2 \Rightarrow z = 2$

$x = 2$

$w + y + z = 4$

$\Rightarrow w = 4 - y - z = 2 - y$

(a) Leading variables: w, x, z

(b) Free variables: y

(c) General solution:

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = y\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

(d) $w = 2 - y$

$= 2 - (5 + 2w)$

$= 2 - 5 - 2w$

$\Rightarrow 3w = -3$

$\Rightarrow w = -1 \quad \Rightarrow y = 3$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

(e) From last equation

$w + y = 2$

New equation is $-2w - 2y = -3$

$\Rightarrow w + y = \frac{3}{2}$

Inconsistent.

2. (a) $-m_1 x + y = b_1$

$-m_2 x + y = b_2$

$$\left[\begin{array}{ccc} -m_1 & 1 & b_1 \\ -m_2 & 1 & b_2 \end{array}\right] \quad R_2 \leftarrow R_2 - \frac{m_2}{m_1} R_1$$

$$\left[\begin{array}{ccc} -m_1 & 1 & b_1 \\ 0 & 1-\frac{m_2}{m_1} & b_2 - \frac{m_2}{m_1} b_1 \end{array}\right]$$

Pivots in each row $\Rightarrow$

$Ax = b$ has a soln. for each b.

$$\left(1 - \frac{m_2}{m_1}\right) y = b_2 - \frac{m_2}{m_1} b_1$$

Solve for y.

Then Solve for x.

(b) If $m_1 = m_2$, for second row to be all zero, $b_2 = b_1$

III.

A	B
F	T
T	F
T	F
T	F
F	F
F	F

IV. B

V. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} -2 \\ -4 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

$x = -2t + 2$
$y = -4t$
$z = t$

$\Rightarrow$ $x + 2z = \cancel{0}\ 2$
$y + 4z = 0$

$$\begin{bmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 4 & 0 \end{bmatrix}$$

C

$\underline{\text{VI}}$. C

$\underline{\text{VII}}$. (d)

$\underline{\text{VIII}}$. S_1 & S_2 not closed under scalar multiplication.

$\underline{\text{IX}}$. D

$\underline{\text{X}}$. $\begin{bmatrix} p \\ q \end{bmatrix}$ & $\begin{bmatrix} r \\ s \end{bmatrix}$ are l.d.

$$\Rightarrow \begin{bmatrix} p \\ q \end{bmatrix} = k \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} kr \\ ks \end{bmatrix}$$

$$\begin{aligned} p &= kr \\ q &= ks \end{aligned} \Rightarrow \frac{p}{q} = \frac{r}{s}$$

$$\Rightarrow ps = qr$$

(b)