Nanyang Technological University
HE2001 Microeconomics II

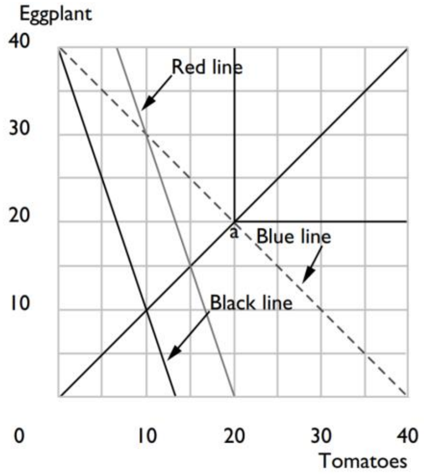
Solution 3

Question 1

(a) The monetary value of his endowment is $(\$5)(10) + (\$5)(30) = \$200$.

(b) The budget line is $5x + 5y = 200$, drawn as the blue line in the graph. The indifference curves for two perfect complements are L-shaped.

We need to solve the utility maximization for perfect complements. For perfect complements, goods will always be consumed in a fixed ratio: one to one in this case. Intuitively, any optimal consumption point must have equal amounts of eggplants and tomatoes. Otherwise, the budget is not used effectively (i.e., the chosen bundle is always on the 45-degree line, so it's the intersection of the budget line and the line $y = x$).
He ends up consuming 20 pounds of both. This chosen bundle (20, 20) is labeled with point A.



(Blue budget line: $5x + 5y = 200$*, the initial budget line for part (a);*
Red budget line: $15x + 5y = 300$ *for part (c);*
Black budgtline line: $15x + 5y = 200$ *for part (d))*

(c) With the new prices (\$15, \$5), the endowment value is $(\$15)(10) + (\$5)(30) = \$300$.

The new budget line is $15x + 5y = 300$, drawn as the red line. The new intersection of the budget line and $y = x$ is at (15, 15), so he now consumes 15 pounds of both.

(d) With an income of \$200, the budget line is $15x + 5y = 200$, drawn as the black line in the graph. Find the intersection again. Mario now consumes 10 pounds of both: (10, 10). Note that this situation describes what we are measuring with the standard income effect.

Price change which occurs after selling endowment at old prices. (Does not include the change in income from endowment change).

(e) For perfect complements, you cannot substitute between goods, so the pure substitution effect of a price change is zero.

The ordinary income effect is consistent with the case in part (d), where Mario still has an income of \$200 after the price change. The increase in tomato price is like a decrease in income. From the bundle of (20, 20) to (10, 10), the change in the demand for tomatoes due to the ordinary income effect is -10.

When considering the total change in the demand, we take the change in the endowment value into account. This is consistent with the case in part (c), where Mario has a higher income of \$300 after the price change. (The increase in tomato value raises his income.) From (20, 20) to (15, 15), the total change in the demand for tomatoes is -5.

With zero substitution effect, the change in the demand for tomatoes due to the endowment income effect is +5 (the difference between -10 and -5).

Alternatively,

$$\frac{\Delta x_{endowment\ effect}}{\Delta p_1} = \frac{\Delta x_1^m}{\Delta \mathrm{m}}\omega_1$$

$$\Delta x_{endowment\ effect} = \frac{\Delta x_1^m}{\Delta \mathrm{m}}\omega_1 \Delta p_1 = \frac{5}{100} \times 10 \times 10 = 5$$

(Note that $\frac{\Delta x_1^m}{\Delta \mathrm{m}}$ here is constant no matter what the initial reference optimal consumption point we choose, but this may not always be the case. It is hence better to use the diagram to illustrate effects- it will also better demonstrate your understanding of the material.)

Question 2

(a) With an endowment of (100, 200) and prices of (1, 2), Lucetta's budget is the total value of the endowment, 500. The budget line is given by $a + 2b = 500$. The initial endowment is labeled with point E.

(b) The utility maximization problem:

$$\max_{a,b} U(a,b) = ab\,, s.t.\, a + 2b = 500$$

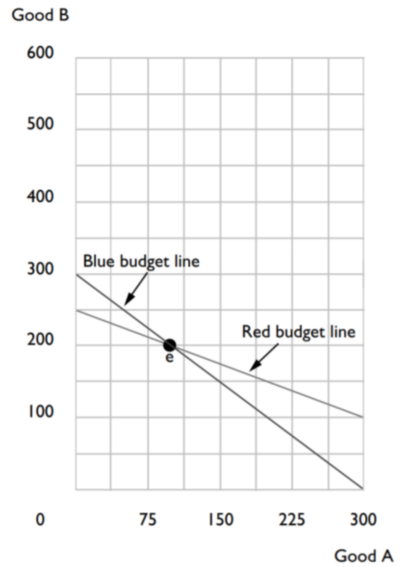
or equivalently $\max_{b} U(b) = (500 - 2b)b = -2b^2 + 500b.$

Alternatively, we set $MRS = price\ ratio$:

$$\frac{MU_a}{MU_b} = \frac{p_a}{p_b}$$

$$\frac{b}{a} = \frac{1}{2}$$

Lucetta's gross demand for A and B is given by (250, 125).



(Red budget line: $a + 2b = 500$*, the initial budget line for part (a);*
Blue budget line: a+b=300, the budget line for part (d))

(c) Compare the initial endowment of (100, 200) and the chosen bundle (250, 125), the net demand for A is 150, and the net demand for B is -75.

(d) The budget is reduced to 300. The new budget line is $a + b = 300$, drawn as the blue budget line in the graph.

(e) Solve the utility maximization problem again:

$$\max_{a,b} U(a,b) = ab\,, s.t.\, a + b = 300$$

or equivalently $\max_{b} U(b) = (300 - b)b = -b^2 + 300b.$

Alternatively, we set $MRS = price\ ratio$ again to obtain

$$\frac{b}{a} = 1$$

The maximum is obtained at $a = b = 150$, so the consumption of good B rises by 25, and the consumption of good A decreases by 100. She is still a net seller of B after the price change.

(f) By selling all goods before the price change, Lucetta's endowment value is still 500. But she faces lower prices when buying the consumption bundle later. Her utility maximization problem becomes:

$$\max_{a,b} U(a,b) = ab\,, s.t.\, a + b = 500$$

or equivalently $\max_{b} U(b) = (500 - b)b = -b^2 + 500b$.

The $MRS = price\ ratio$ formula gives the same ratio $\frac{b}{a} = 1$.

Lucetta's gross demand for A and B is (250, 250).

(g) Lucetta is a net seller of good B. If she is holding goods at the time of the price change, she will become poorer as her income, depending on selling good B, will be reduced by its price fall. (This is the change from part (a) to (d). If she was a net buyer instead, she would not be so hurt.)

If she has sold all goods before the price falls and is holding money, the income will remain unchanged for her. Furthermore, since she will face a lower price now, she will become better off because she can afford a larger consumption bundle. (This is the change from part (a) to (f). The situation is independent of whether she is a net seller or buyer. Selling goods before the price falls means that she will always have a higher income.)

The main cause that makes her better off in (f) than in (d) is that she can avoid the negative endowment income effect. However, the magnitude of the difference likely depends on whether Lucetta is a net seller or buyer of good B.

Question 3

(a) Mac's utility maximization problem is given by:

$$\max_{c,r} U(c,r) = cr\,, s.t.\, c + 5r = 400$$

or equivalently $\max_{r} U(r) = 5(80 - r)r = -5r^2 + 400r$.

Alternatively, we set $\frac{MU_c}{MU_r} = \frac{p_c}{p_r}$ to obtain $\frac{r}{c} = \frac{1}{5}$.

The maximum is obtained at $r = 40$, and the remaining hours will be allocated to work. So Mac will choose to work 4o hours each week.

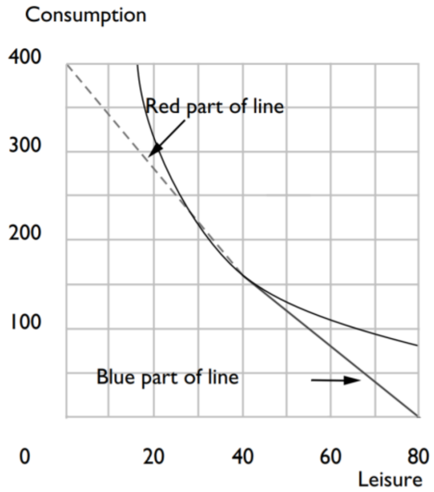
(b) If Wendy works for 40 hours or less each week, her budget is given by

$$c = 4(80 - r) = -4r + 320 \text{ } or \text{ } c + 4r = 320$$

If she works for more than 40 hours, her budget is given by

$$c = (4)(40) + 6(80 - r - 40) = -6r + 400 \text{ } or \text{ } c + 6r = 400$$

Note that the slopes of the budget lines are different depending on her working hours. The cutoff is 40 hours. Draw the two budget lines and you will see the kink is at the point of (40, 160).



(Solid/blue part: Wendy's budget line $c = -4r + 320$ *if she does not work overtime;*
Dashed/red part: Wendy's budget line $c = -6r + 400$ *if she works overtime)*

(c) As explained in part (b), the equation for the solid line is $c + 4r = 320$.
The equation for the dashed line is $c + 6r = 400$.

(d) When Wendy receives a constant wage rate, her utility maximization problem is given by:

$$\max_{c,r} U(c,r) = cr\,, s.t.\, c + 4r = 320$$

or equivalently $\max_{r} U(r) = -4r^2 + 320r$.

Alternatively, we set $\frac{MU_c}{MU_r} = \frac{p_c}{p_r}$ to obtain $\frac{r}{c} = \frac{1}{4}$.

The maximum is obtained at $r = 40$, and the rest hours will be allocated to work. So Wendy will choose to work 40 hours and earn a total wage of $160 each week. Draw the indifference curve through the point (40, 160) in the graph.

Notice that here, the overtime cutoff is set exactly at the point where she chose to work without overtime. An interesting point is that since (40, 160) passes through both budget lines, by revealed preference, she will always choose a higher than work amount in the overtime budget. This outcome is similar to one of the questions in your previous tutorials.

(e) If Wendy does not work overtime, the highest utility she can obtain is $U = (160)(40) = 6400$.

(f) If Wendy works overtime, her utility maximization problem is given by:

$$\max_{c,r} U(c,r) = cr\,, s.t.\, c + 6r = 400$$

or equivalently $\max_{r} U(r) = -6r^2 + 400r$.

Alternatively, we set $\frac{MU_c}{MU_r} = \frac{p_c}{p_r}$ to obtain $\frac{r}{c} = \frac{1}{6}$.

The maximum is obtained at $r = 33.3$. So she will choose to work 46.7 hours and earn a wage of $200 each week. This is the best choice for Wendy from the dashed line segment. Note that this solution is only valid if r is less than 40 and she is indeed working overtime, which is true given our argument above.
The obtained utility $U = 6660$ from this choice is higher than the optimal utility she can have if she does not work overtime, so she will choose to work overtime with 46.7 hours each week.

(g) When Wendy is making the best choice (33.3, 200), her utility is 6,660. If she had Mac's job, she would choose (40, 200) and obtain a utility of 8,000, as in part (a).
With the same wage income and more leisure time (thus a higher utility overall), Mac's job is the better job.

Question 4

(a) Without wage, the consumption is fixed at the nonlabor income of $20. The utility function becomes $U(R) = 20 - (12 - R)^2$.
The maximum is obtained at $12 - R = 0$, so he will choose a leisure time of 12 hours.

(b) At a wage rate of $10, Dudley can have an additional labor income of $10(16 - R)$ for consumption. Now the utility maximization problem is given by:

$$\max_{C,R} U(C, R) = C - (12 - R)^2, s.t. C + 10R = 180$$

or equivalently $\max_{R} U(R) = -R^2 + 14R + 36$.

Alternatively, we set $\frac{MU_C}{MU_R} = \frac{p_C}{p_R}$ to obtain $\frac{1}{-2R+24} = \frac{1}{10}$.

The maximum is obtained at $R = 7$, so he will choose leisure time of 7 hours and working time of 9 hours.

(c) Observe the utility maximization problem. The nonlabor income will not change the optimal leisure time or work time chosen. It will only affect the size of the constant term and thus the overall utility. Therefore, when the nonlabor income decreases, Dudley will still choose to work 9 hours per week.

(d) The income tax will reduce the actual wage rate, so the utility maximization problem will be changed. It is now given by

$$\max_{C,R} U(C, R) = C - (12 - R)^2, s.t. C + 8R = 144$$

or equivalently $\max_{R} U(R) = -R^2 + 16R$.

Alternatively, we set $\frac{MU_C}{MU_R} = \frac{p_C}{p_R}$ to obtain $\frac{1}{-2R+24} = \frac{1}{8}$.

The maximum is obtained at $R = 8$, so he will choose leisure time of 8 hours and working time of 8 hours. Dudley reduces labor supply in response to wage rate decreases.