AY 21/22 MH1820 Midterm Test 1

Name: ..... **Matriculation Number:** .....

Tutorial Group: .....

Instructions

- This test consists of 5 multiple choice questions and 3 computational questions.
- For each of the multiple choice questions, there is only one correct answer. Tick the correct answer.
- Answer all questions. The marks for each question are indicated.
- For the computational questions, write down your answers in the space provided after the question.

1. [2 marks] How many 4-digit numbers are there all of whose digits are odd?

◯ 10^4 ◯ 5^4 ◯ $4 \cdot 5^3$ ◯ $9 \cdot 10^3$

2. [2 marks] Five persons $P_1, \ldots, P_5$ are randomly assigned to five car seats $S_1, \ldots, S_5$. What is the probability that P_1 is seated at S_1 and P_2 is seated at S_2?

◯ $\frac{1}{5}$ ◯ $\frac{1}{10}$ ◯ $\frac{1}{20}$ ◯ $\frac{1}{120}$

3. [2 marks] Three cards are randomly chosen from a standard poker deck of 52 cards. Which of the following events has the highest probability?

◯ Exactly two of the cards are kings.
◯ Exactly one of the cards is a king and exactly one is a queen.
◯ All three cards are of spades.

4. [2 marks] A fair dice is rolled 3 times. What is the probability that the total rolled is at most 4 under the condition that the first roll is a 1?

◯ $\frac{1}{3}$ ◯ $\frac{1}{4}$ ◯ $\frac{1}{6}$ ◯ $\frac{1}{12}$

5. [2 marks] A fair coin is tossed 5 times. Let X be the total number of heads that occur and let $F(x)$ be the CDF of X. Which of the following is equal to $F(1)$?

◯ $\frac{1}{16}$ ◯ $\frac{3}{16}$ ◯ $\frac{5}{16}$ ◯ $\frac{7}{16}$

6. [5 marks] A ball is drawn from one of 2 boxes. The boxes contain the following number of balls of colors blue (B) and red (R).

	B	R
Box 1	1	4
Box 2	6	2

The following procedure is used to draw the ball.

- One of the boxes is chosen at random: Box 1 is chosen with probability 0.2 and Box 2 with probability 0.8.
- A ball is drawn from the chosen box (each ball in the box is chosen with the same probability).

(a) What is the probability that a blue ball is drawn?

(b) If a blue ball is drawn, what is the probability that it was drawn from Box 1?

(a)

(b)

7. [5 marks] Let X be a continuous random variable with PDF given by

$$f(x) = 2x \text{ for } 0 \leq x \leq 1 \text{ and } f(x) = 0 \text{ otherwise.}$$

(a) Draw a graph of f.

(b) Compute the CDF F of X and draw of graph of F.

(c) Compute $E[X]$.

(a)

(b)

(c)

8. [5 marks] Let X and Y be a independent discrete random variables, both with PMF $f(x)$ given by

x	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

and $f(x) = 0$ otherwise.

(a) Compute $E[X]$ and $Var[X]$.

(b) Compute $E[XY]$ and $Var[2X - Y]$.

(a)

(b)