AY 22/23 MH1820 Midterm Test

Name: ..... **Matriculation Number:** .....

Tutorial Group: .....

Instructions

- This test consists of 5 multiple choice questions and 3 computational questions.
- For each of the multiple choice questions, there is only one correct answer. Tick the correct answer.
- Answer all questions. The marks for each question are indicated.
- For the computational questions, write down your answers in the space provided after the question. Express numerical values up to 4 decimal places.

1. [2 marks] How many 5-digit numbers are there that do not contain the digit 7? Note that the first digit of an n-digit number must be nonzero.

◯ 8^5 ◯ 9^5 ◯ $8 \cdot 9^4$ ◯ $9 \cdot 10^4$

2. [2 marks] In how many ways can 8 people P_1, P_2, ..., P_8 be seated in a row if P_1, P_2, P_3, P_4 and P_5 must sit next to each other?

◯ $5! \cdot 3!$ ◯ $5! \cdot 4!$ ◯ $5! \cdot 5!$ ◯ $6! \cdot 2!$

3. [2 marks] Five cards are randomly chosen from a standard poker deck of 52 cards. Which of the following events has the highest probability?

◯ Exactly three of the cards are kings.
◯ At least four cards are of spades.
◯ Three cards are of spades and two cards are of hearts.

4. [2 marks] A fair dice is rolled 3 times independently. What is the probability that at least one of the rolls is a 3 under the condition that the total rolled is 15?

◯ $\frac{1}{2}$ ◯ $\frac{1}{10}$ ◯ $\frac{3}{7}$ ◯ $\frac{3}{10}$

5. [2 marks] Cars arrive at a tollbooth at a mean rate of 4 cars every 6 minutes according to a Poisson distribution. What is the probability that 10 cars arrive at the tollboth in the first hour?

◯ $\frac{e^{-10}10^4}{4!}$ ◯ $\frac{e^{-4}4^{10}}{10!}$ ◯ $\frac{e^{-40}40^{10}}{10!}$ ◯ None of the above

6. [6 marks] A worker has asked her supervisor for a recommendation letter for a new job. She estimates that there is an 80% chance that she will get the job if she receives a strong recommendation, a 50% chance if she receives a moderate recommendation, and a 5% chance if she receives a weak recommendation. She further estimates that the probabilities that the recommendation will be strong, moderate and weak are 0.6, 0.3 and 0.1 respectively.

(a) What is the probability that she will receive the new job offer?

(b) Given that she does not receive the job offer, what is the probability that she received a strong recommendation?

(a)

(b)

7. [8 marks] Let X be a continuous random variable with PDF given by

$$f(x) = \frac{10}{x^2} \text{ for } x > 10 \text{ and } f(x) = 0 \text{ for } x \leq 10.$$

(a) Find $\mathbb{P}(X > 30)$.

(b) Compute the CDF $F(x)$ of X.

(c) Compute the PDF of the random variable $Y = X^3$.

(a)

(b)

(c)

8. [6 marks] The lifetime (in hours) X of a radio tube is a random variable with PDF given by $f(x) = \frac{1}{500}e^{-x/500}$ for $x > 0$, and $f(x) = 0$ otherwise. Five of these radio tubes operate independently in a radio set. The radio set fails if at least three of these radio tubes fail.

(a) Compute $\mathbb{P}(X > 300)$.

(b) Compute the probability that the radio set will operate for more than 300 hours.

(a)

(b)