

CE1107/CZ1107: DATA STRUCTURES AND ALGORITHMS

Binary Trees

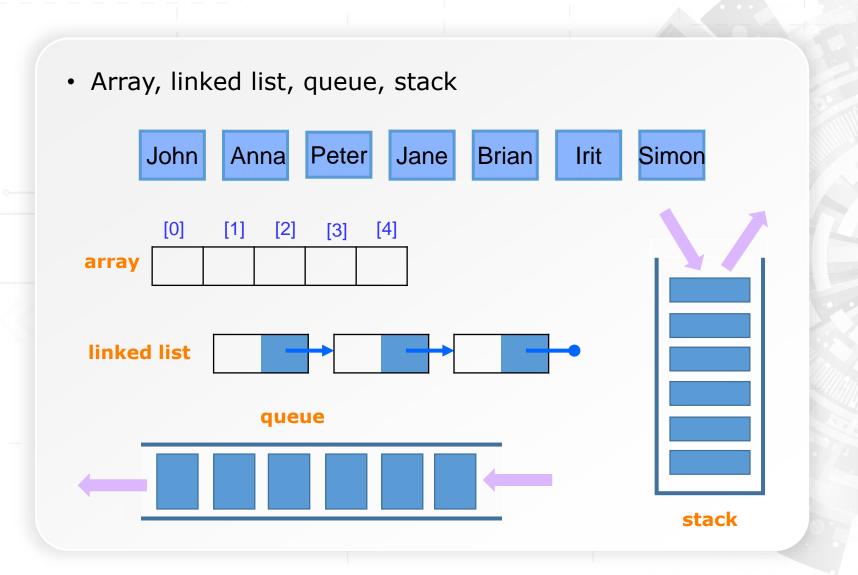
College of Engineering

School of Computer Science and Engineering

OUTLINE

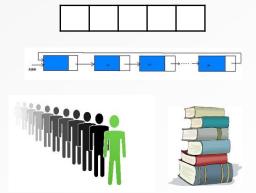
- Non-linear data structures
- Tree data structure
 - Binary trees
- Implement binary tree nodes in C
- Binary Tree Traversal
- Tree traversal order
 - Pre-order
 - In-order
 - Post-order
- Application examples
 - Count nodes in a binary tree
 - Find grandchild nodes
 - Calculate height of every node
- Level-by-level traversal
- Preorder traversal with a stack

LINEAR DATA STRUCTURE



DATA STRUCTURES SO FAR...

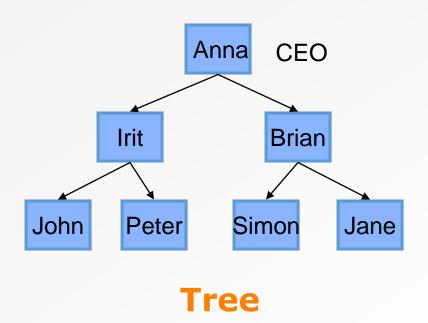
- Linear
 - Items all arranged one after another
 - Random access
 - Arrays
 - Sequential access
 - Linked list
 - Limited-access sequential
 - Stacks
 - Queues



- Used them to store lists of numbers, lists of people, lists of moves, etc
 - Linear data

NON-LINEAR DATA STRUCTURE

Suppose you have a set of names

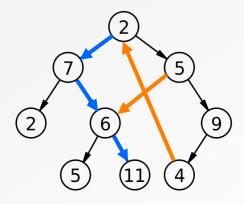


Company organization

Not good to use linear data structure to store <u>hierarchical relationships</u>

TREE DATA STRUCTURE

- Still using nodes + links representation
- New idea:
 - Each node can have links to more than one other node
 - No loop



Observe that:

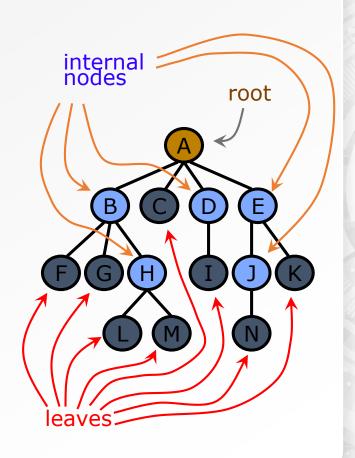
 If we follow one path of a tree, we get a linked list

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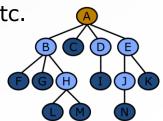
TREE DATA STRUCTURE

- A tree is composed of nodes
- · Each node contains a value
- Types of nodes
 - Root: only one in a tree, has no parent.
 - Internal (non-leaf):
 Nodes with children are called internal nodes
 - Leaf:nodes without children are calledleaves



WHY TREES?

- Model layouts with hierarchical relationships between items
 - Chain of command in the army
 - Personnel structure in a company
 - (Binary tree structure is limited because each node can have at most two children)
- Tree structures also allow us to
 - Some problems require a tree structure: some games, most optimization problems, etc.
 - Allow us to do the following very quickly: (we'll see that in the following lectures)
 - Search for a node with a given value
 - Add a given value to a list
 - Delete a given value from a list



TREE DATA STRUCTURE

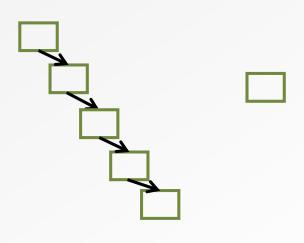
- Tree data structure looks like... a tree:
 - Only one root node (no nodes points to it)
 - Each node branches out to some number of nodes
 - Each node has only one "parent" node the node pointing to it (except the root node)

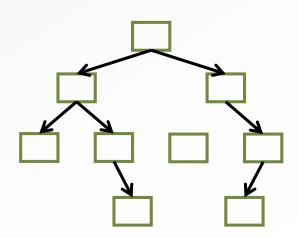


- General tree
 - Each node can have links to any number of other nodes
- Binary tree (we'll work with this in our course)
 - Each node can have links to at most two other nodes

POSSIBLE TREE CONFIGURATIONS

- We'll see later why not all trees configurations are desirable/useful
- Has to do with balance of a tree





OUTLINE

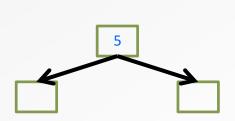
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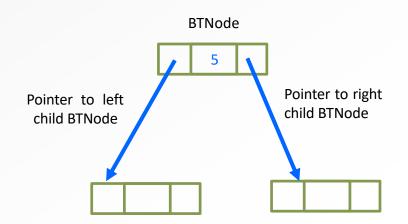
IMPLEMENTATION

- Recall implementation of LinkedList
 - Node has link to **at most one** other node
 - Defined a ListNode with one next pointer and a data item

```
typedef struct _listnode{
   int item;
   struct _listnode *next;
}ListNode;
```

- BinaryTree
 - Node has link to **at most <u>TWO</u>** other nodes
 - Define a BTNode with
 - Two pointers
 - A data item





BTNode

- Start with a simple BTNode that stores an integer
 - The type of item can be character, string, or structure, etc.

```
typedef struct _listnode{
    int item;
    struct _listnode *next;
}ListNode;

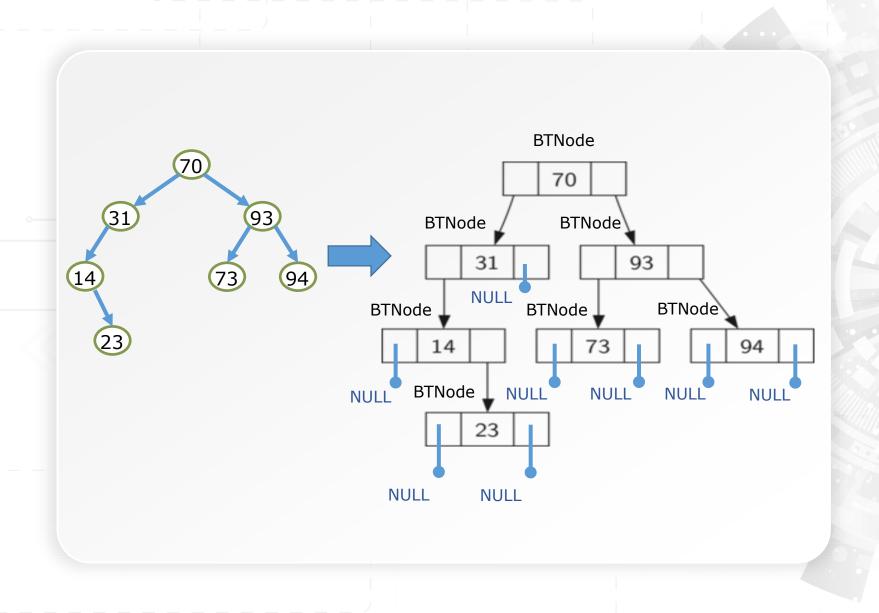
int item;
struct _btnode *left;
struct _btnode *right;

BTNode

Pointer to left
child BTNode

Pointer to right
child BTNode
```

EXAMPLE BINARY TREE

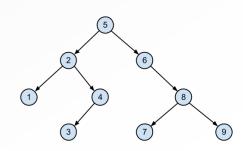


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TREE TRAVERSAL

- Given a linear data structure and a particular item, very obvious what the "next" item is
 - Each node has an obvious "previous" and "next" node
- Trees are non-linear structures
 - How to extract data from a binary tree?
 - What is the traversal sequence?left/left/left, then left/left/right, then...?
- Need a systematic way to visit every node in the tree
 - Clearly defined steps
 - No repeated visits to nodes

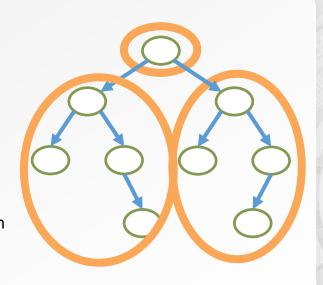


TREE TRAVERSAL

- Why is this important?
 - Tree traversal is foundation for many functions
- Very common function template:

Traverse tree

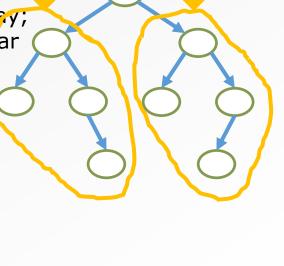
• At each node, perform some operation



- Example task: count # of nodes in a tree
 - At every node N, size of that subtree
 - size of N's left subtree
 - + size of N's right subtree
 - + N itself

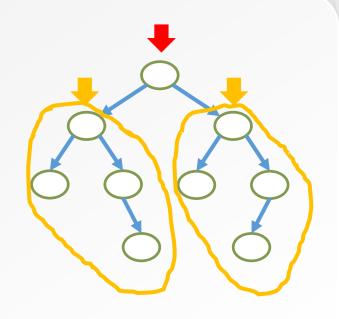
TREE TRAVERSAL

- Tree traversal is recursive
 - Recursion: is the process of repeating items in a <u>self-similar</u> way; divide a problem into several similar sub-problems.
 - At each node
 - Visit the node and both children
- Initial case + repeating case
 - (Visit root) + (visit children)
- When combined, guarantees that all nodes will be visited once and only once



TREE TRAVERSAL PROCESS

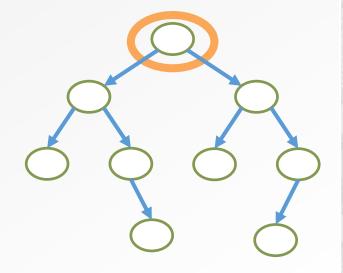
```
TreeTraversal(Node N):
    Visit N;
    If (N has left child)
        TreeTraversal(LeftChild);
    If (N has right child)
        TreeTraversal(RightChild);
    Return; // return to parent
```



TREE TRAVERSAL TEMPLATE #1

Pseudocode

```
TreeTraversal(Node N):
    Visit N;
    If (N has left child)
        TreeTraversal(LeftChild);
    If (N has right child)
        TreeTraversal(RightChild);
    Return; // return to parent
```



In main(), call TreeTraversal(root)

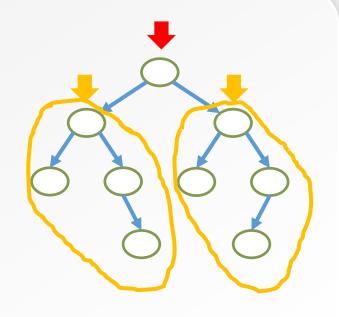
TREE TRAVERSAL TEMPLATE #2

Current function:

 Need to check for existence of left and right children before following them



- Always follow links to children
- Then check if the link is NULL
- In other words, not actually pointing at a BTNode



TREE TRAVERSAL TEMPLATE #2

<u>Pseudocode</u>

```
TreeTraversal2(Node N):

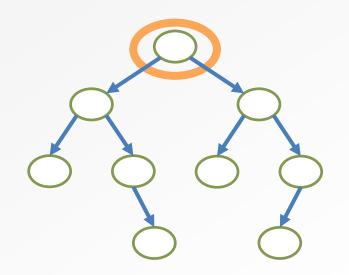
If N==NULL return;

Visit N;

TreeTraversal2(LeftChild);

TreeTraversal2(RightChild);

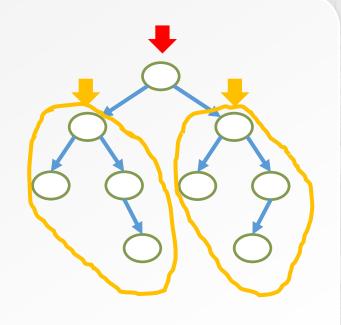
Return; // return to parent
```



In main(), call TreeTraversal2(root)

TreeTraversal2() IMPLEMENTATION

```
Void TreeTraversal2(BTNode *cur) {
    If (cur == NULL) return;
    PrintNode(cur); // visit cur
    TreeTraversal2(cur->left);
    TreeTraversal2(cur->right);
}
```



TREETRAVERSAL() FEATURES

Recursive

TreeTraversal() is called <u>from within</u>
 <u>its own body</u>

initial call TreeTraversal(root)

Depth-first

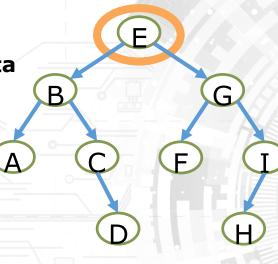
- The traversal goes as <u>deep</u> as possible before backtracking and going sideways
- Not level-by-level! (that is called breadth-first)

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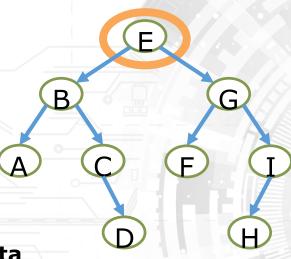
THREE "STANDARD" WAYS TO TRAVERSAL

- Pre-order
 - Process the current node's data
 - Visit the left child subtree
 - Visit the right child subtree
- In-order
- Post-order



THREE "STANDARD" WAYS TO TRAVERSAL

- Pre-order
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THREE "STANDARD" WAYS TO TRAVERSAL

Pre-order

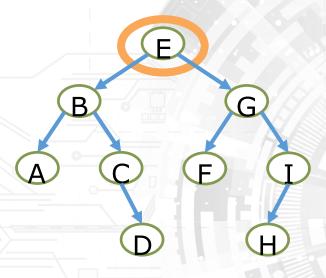
- Process the current node's data
- Visit the left child subtree
- Visit the right child subtree

In-order

- Visit the left child subtree
- Process the current node's data
- Visit the right child subtree

Post-order

- Visit the left child subtree
- Visit the right child subtree
- Process the current node's data



TREE TRAVERSAL - PRINT

- Recall the TreeTraversal() template (TT) Pre-order :
 - Simple task at each node: print out data in that node

```
void TreeTraversal(BTNode *cur) {
   if (cur == NULL)
      return;
```

// Do something with the current node's data

```
TreeTraversal(cur->left); //Visit the left child node
TreeTraversal(cur->right);//Visit the right child node
```

TREE TRAVERSAL - PRINT

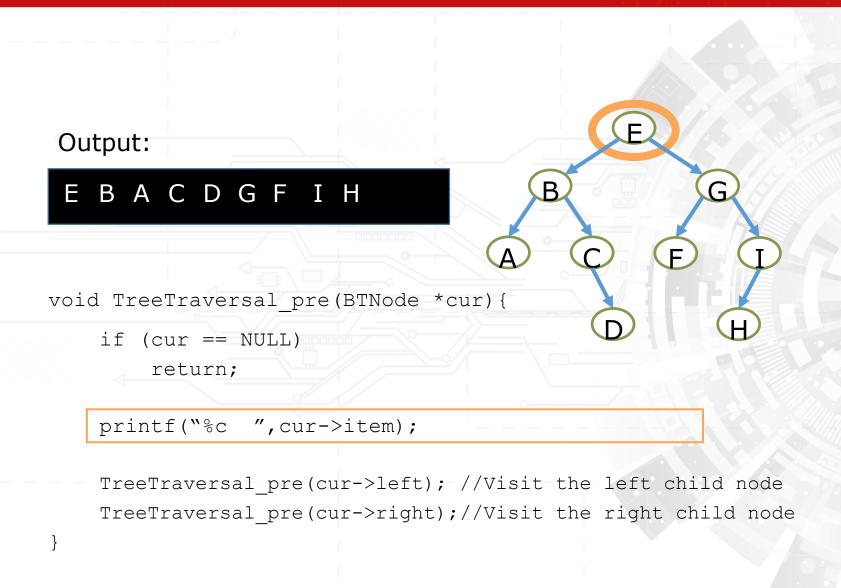
- Recall the TreeTraversal() template (TT) Pre-order :
 - Simple task at each node: print out data in that node

```
void TreeTraversal(BTNode *cur) {
   if (cur == NULL)
       return;

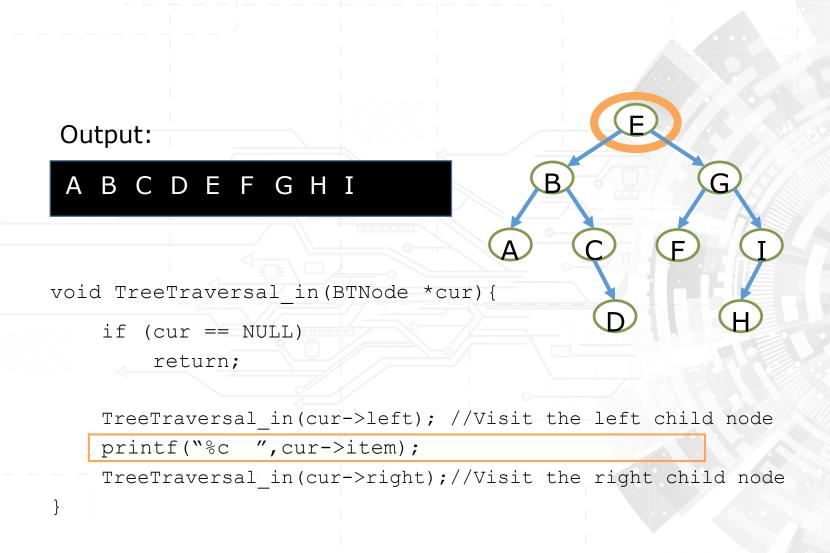
   printf("%c",cur->item);

   TreeTraversal(cur->left); //Visit the left child node
   TreeTraversal(cur->right);//Visit the right child node
}
```

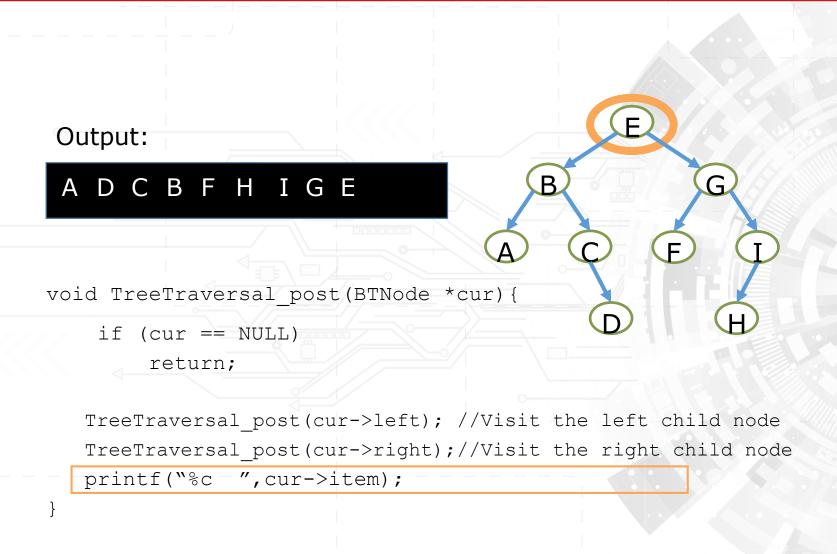
TREE TRAVERSAL PRE-ORDER: PRINT



TREE TRAVERSAL IN-ORDER: PRINT



TREE TRAVERSAL POST-ORDER: PRINT



PRE-ORDER, IN-ORDER AND POST-ORDER

Pre-Order Traversal

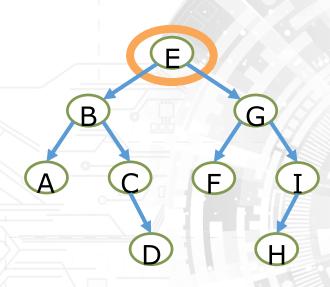
E B A C D G F I H

In-Order Traversal

ABCDEFGHI

Post-Order Traversal

ADCBFHIGE

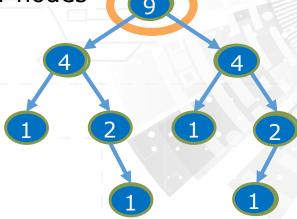


OUTLINE

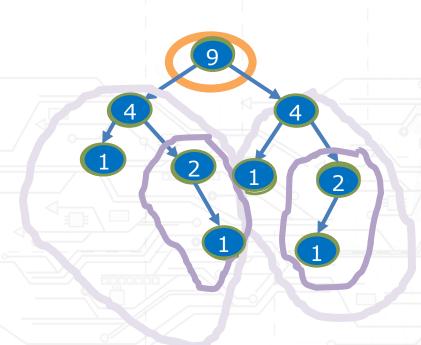
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COUNT NODES IN A BINARY TREE

- Recursive definition:
 - Number of nodes in a tree
 - = 1
 - + number of nodes in left subtree
 - + number of nodes in right subtree
- Each node returns the number of nodes in its subtree



COUNT NODES IN A BINARY TREE



- Each node returns the number of nodes in its own subtree
- Leaf nodes return 1
 Information propagates upwards as TreeTraversal returns from visiting leaf nodes
- Which is the first/last count to be returned?

- Return the size of your subtree to your parent node
- Leaf nodes must return 1 to parent node
- Root node returns size of entire tree

```
void TreeTraversal(BTNode *cur) {
    if (cur == NULL)
        return;

    //may do something with cur;

    TreeTraversal(cur->left);

    TreeTraversal(cur->right);

    //may do something with cur;
}
```

- Return the size of your subtree to your parent node
- Leaf nodes must return 1 to parent node
- Root node returns size of entire tree

```
int countNode(BTNode *cur) {
   if (cur == NULL)
      return ???;

   countNode(cur->left);
   countNode(cur->right);
   ??? //sum and get total;
}
```

- Leaf nodes must return 1
 - "Null" nodes should return 0
- Leaf node returns 1 + 0 + 0

```
int countNode(BTNode *cur) {
    if (cur == NULL)
        return 0;

l = countNode(cur->left);
    r = countNode(cur->right);
    return l+r+1;
}
```

- Leaf nodes must return 1
 - "Null" nodes should return 0
- Leaf node returns 1 + 0 + 0

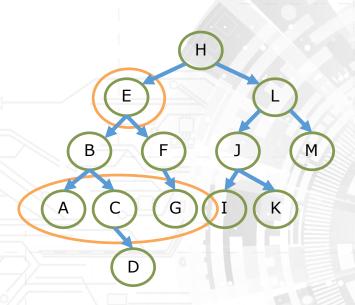
```
int countNode(BTNode *cur) {
   if (cur == NULL)
      return 0;

   return (countNode(cur->left)
      + countNode(cur->right)
      + 1);
}
```

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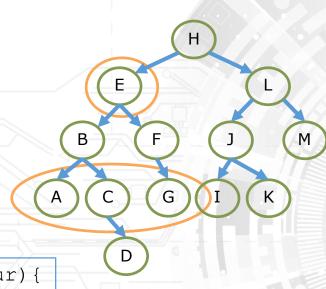
- Given a node X, find all the nodes that are X's grandchildren
- Given node E, we should return grandchild nodes A, C, and G
- What if we want to find klevel grandchildren?
 - Need a way to keep track of how many levels down we've gone



X->left->left X->left->right X->right->left X->right->right

2-level grandchildren

- We want to go down k "levels"
- Use a counter to track how far down we've gone
- At each TreeTraversal(child), increment counter



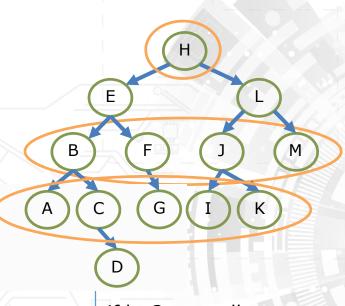
```
void TreeTraversal(BTNode *cur) {
   if (cur == NULL)
      return;
      Do something with the current node's data

// check counter
      Visit the left child node

TreeTraversal(cur->left);
      TreeTraversal(cur->right);
}
Visit the right child node
```

```
H
void main() { ...
    if (X = null) return;
    findgrandchildren(X,0);
                                        C=
1. void findgrandchildren (
                 BTNode *cur, int c)
       if (cur == NULL) return;
2.
       if (c == k) {
3.
          printf("%d ", cur->item);
4.
5.
           return;
6.
       if (c < k) {
7.
           findgrandchildren(cur->left, c+1);
8.
           findgrandchildren(cur->right, c+1);
9.
10.}
```

```
void main() { ...
    if (X = null) return;
    findgrandchildren(X,0);
}
```

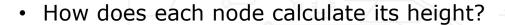


if k=2, we call
findgrandchildren(H,0),
what is the output?
How about k=3?
How about
findgrandchildren(H,1)?

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 Height of a node = number of links from that node to the deepest leaf node



- What is the height of node D, C, H?
- We found:
 - leaf.height= 0
 - Non-leaf node X

X.height=max(X.left.height, X.right.height)+1

2 B 1 F J 0 M

O A C G I K

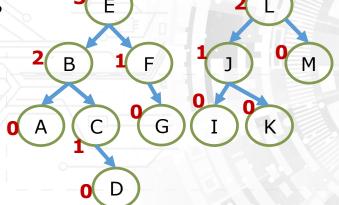
nt)+1 O D

Does information propagate upwards or downwards?

 Height of a node = number of links from that node to the deepest leaf node

How does each node calculate its height?

- What is the height of node D, C, H?
- Go through entire tree:
 calculate and store height of
 each node in the item field



Е

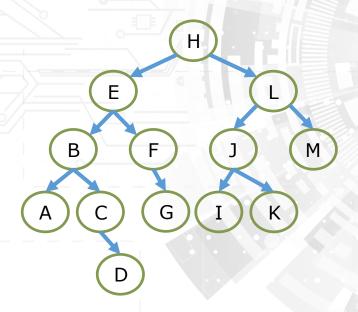
- We want each node to report its height
 - Leaf node must report 0

```
int TreeTraversal(BTNode *cur) {
   if(cur == NULL)
      return ;
   int l = TreeTraversal(cur->left);
   int r = TreeTraversal(cur->right);
   // do something here. Max( left, right)?
   return ;
}
```

We want each node to report its height - Leaf node must report 0 - At "null" condition, must report -1 int TreeTraversal(BTNode *cur) { if(cur == NULL) return -1; int l = TreeTraversal(cur->left); int r = TreeTraversal(cur->right); int $c = \max(1, r) + 1;$ return c;

QUESTIONS

- Does the tree traversal order matter?
- Depth of a node = number of links from that node to the root node. How does each node calculate its depth?



 Height of a node = number of links from that node to the deepest leaf node

```
We want each node to report its height
   - Leaf node must report 0
   - At "null" condition, must report -1
                                            В
   int TreeTraversal(BTNode *cur) {
       if(cur == NULL)
          return -1;
       int l = TreeTraversal(cur->left);
                                             n( D
       int r = TreeTraversal(cur->right);
       int c = \max(1, r) + 1;
       return c;
```

QUESTIONS

- Does the tree traversal order matter?
- Height of a node = number of links from that node to the deepest leaf node
- Depth of a node = number of links from that node to the root node. How does each node calculate its depth?

```
void TreeTraversal(BTNode *cur, int d) {
    if(cur == NULL)
        return;

    //print cur->item and d;
    TreeTraversal(cur->left, d+1);
    TreeTraversal(cur->right, d+1);
    return;
}
```

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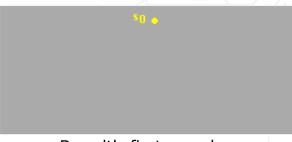
LEVEL-BY-LEVEL: BREADTH-FIRST SEARCH

Double Contractor

begins at the root and explores as far as possible along each branch before backtracking

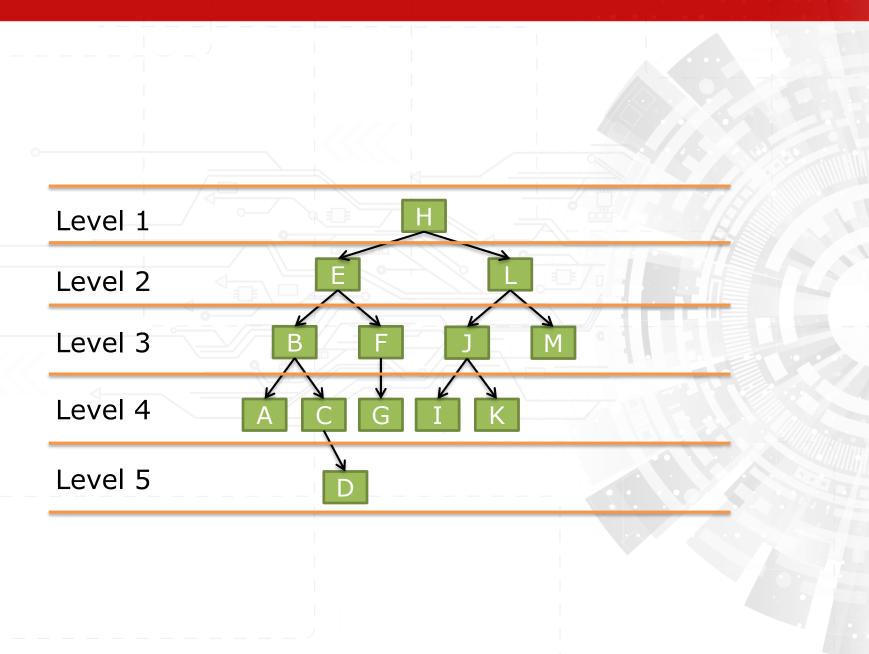
E.g. the post-order traversal

Depth-first search

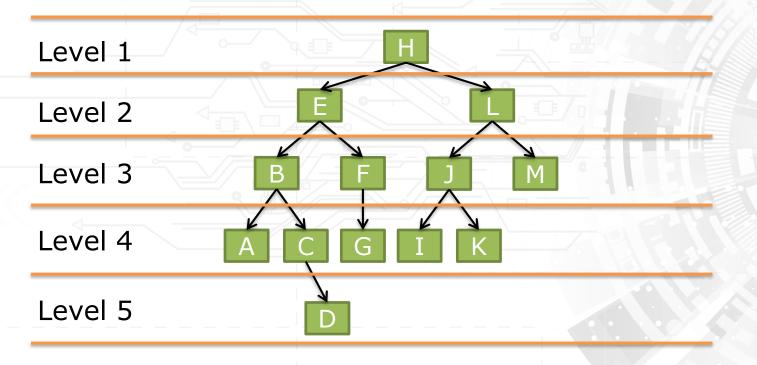


Breadth-first search

begins at a root node and inspects all its children nodes. Then for each of those children nodes in turn, it inspects their children nodes, and so on.

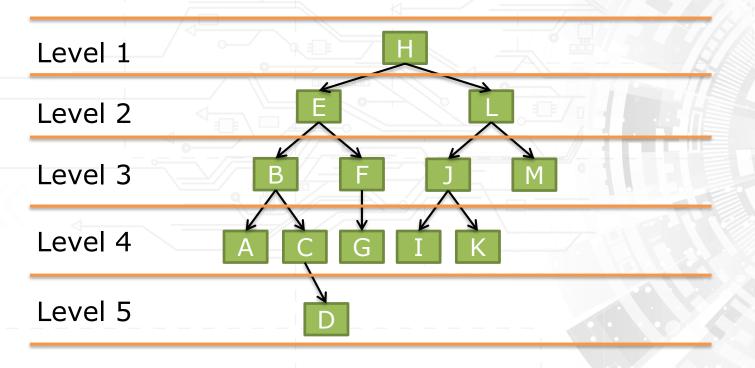


Hint: Make use of another data structure

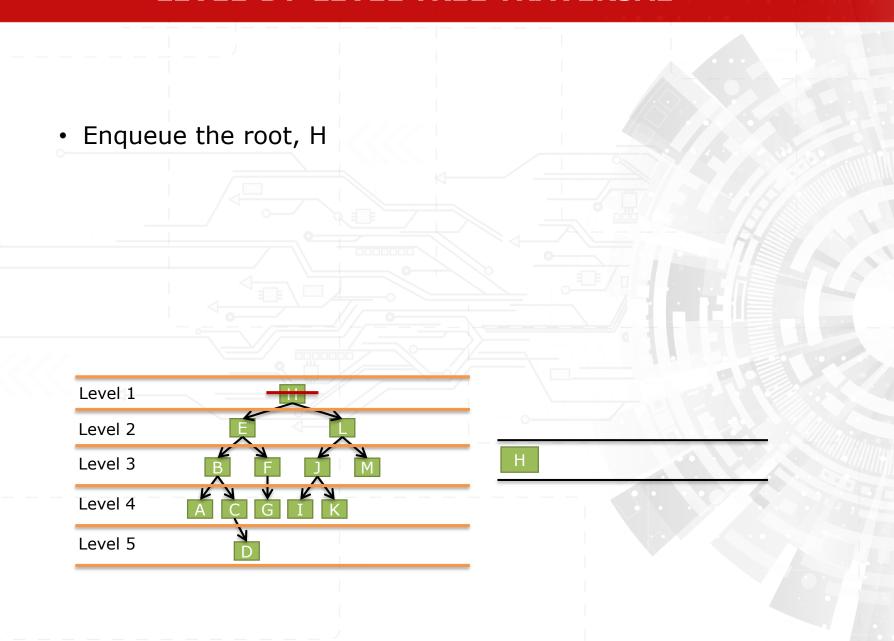


Nodes stored in order accessed in tree...

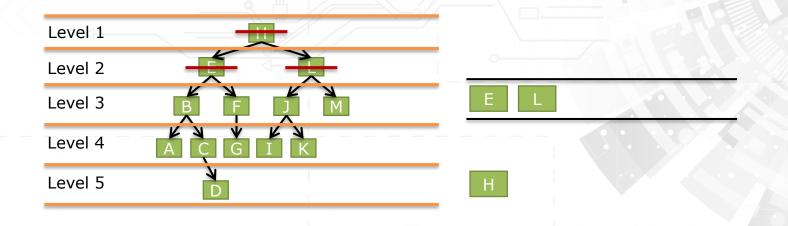
Use a queue! Root node should be first



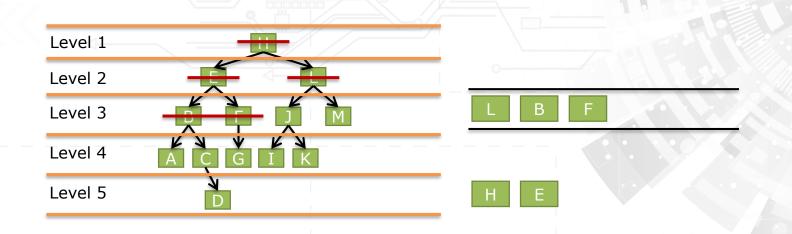
Nodes stored in order accessed in tree



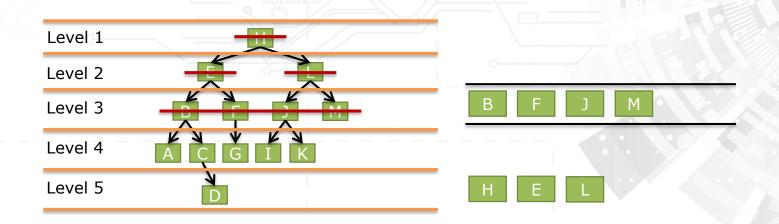
- Enqueue the root, H
- Dequeue H, and enqueue H's children



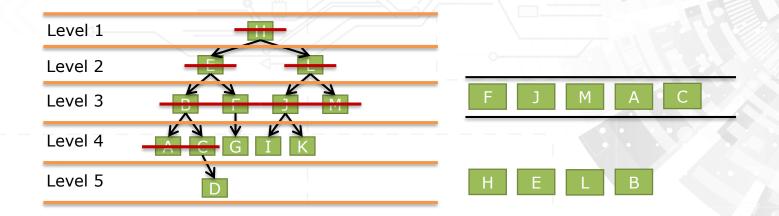
- Enqueue the root, H
- Dequeue H, and enqueue H's children
- Dequeue E, and enqueue E's children



- Enqueue the root, H
- Dequeue H, and enqueue H's children
- Dequeue E, and enqueue E's children
- Dequeue L, and enqueue L's children



- Enqueue the root, H
- Dequeue H, and enqueue H's children
- Dequeue E, and enqueue E's children
- Dequeue L, and enqueue L's children
- Dequeue B, and enqueue B's children

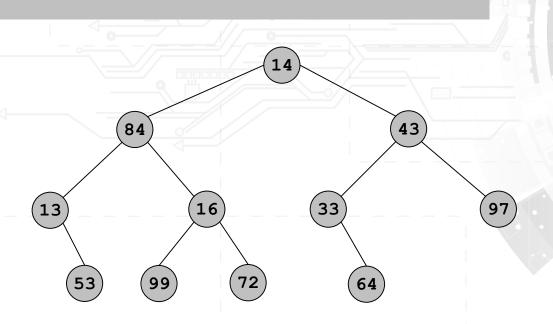


OUTLINE

- Non-linear data structures
- Tree data structure
 - Binary trees
- Implement binary tree nodes in C
- Binary Tree Traversal
- Tree traversal order
 - Pre-order
 - In-order
 - Post-order
- Application examples
 - Count nodes in a binary tree
 - Find grandchild nodes
 - Calculate height of every node
- Level-by-level traversal
- Preorder traversal with a stack

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

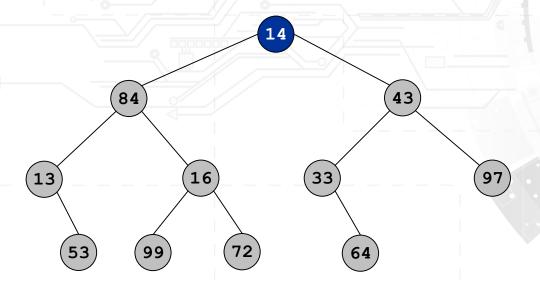


14

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14



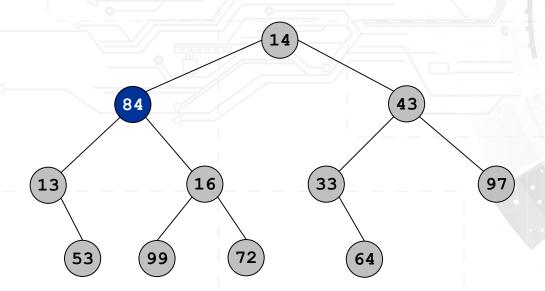
84

43

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84



13 16

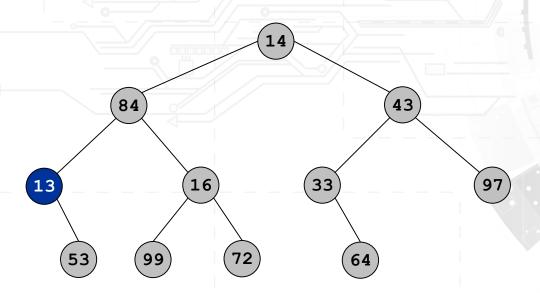
16

43

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13



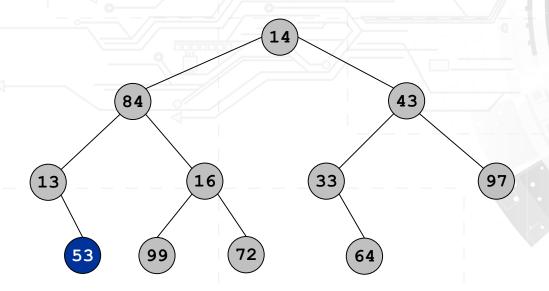
53 16

43

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53

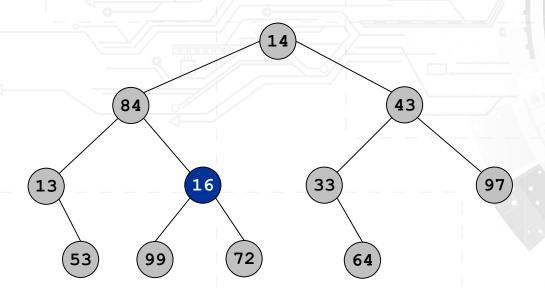


16 43

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16



99

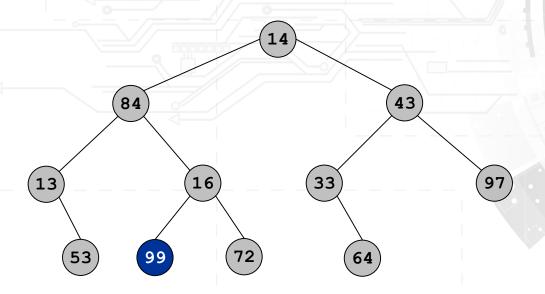
72

43

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99



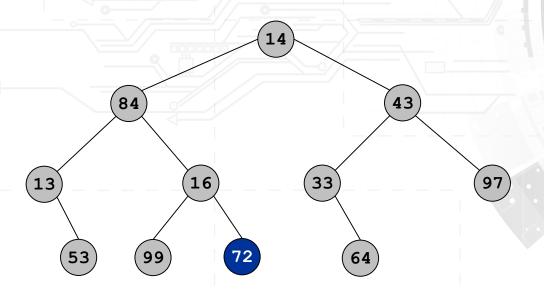
72

43

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72

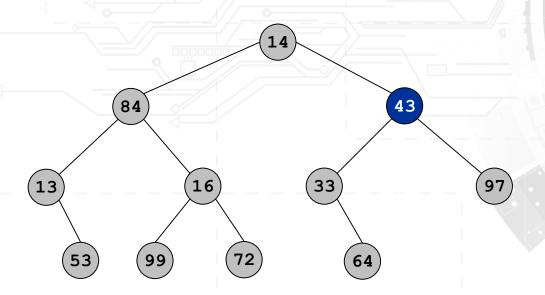


43

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43

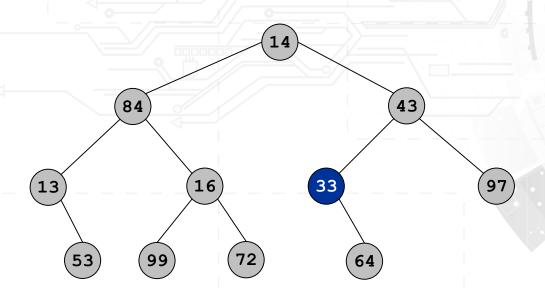


33 97

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33

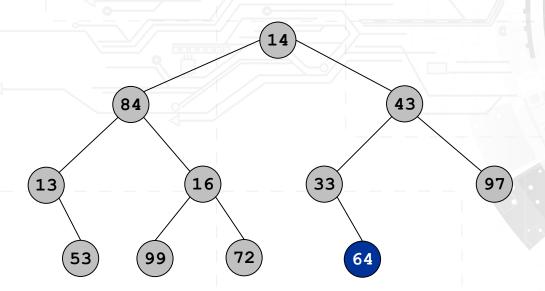


64 97

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33 64

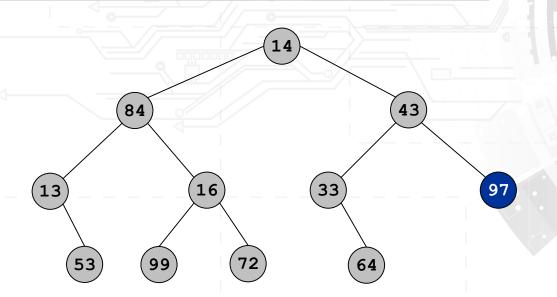


97

Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

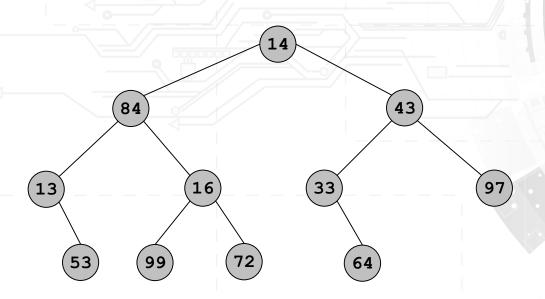
14 84 13 53 16 99 72 43 33 64 97



Push the root onto the stack.
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33 64 97



YOU SHOULD BE ABLE TO

- Binary tree Traverse:
 - Pre-order
 - In-order
 - Post-order
- Write recursive binary tree functions using the TreeTraversal template as a starting point
- Based on the traversal of the binary tree, do a lot of things: print, count numbers, count height/depth, find grandchildren,..., etc.