Swarm Intelligence

Extensions of Ant System

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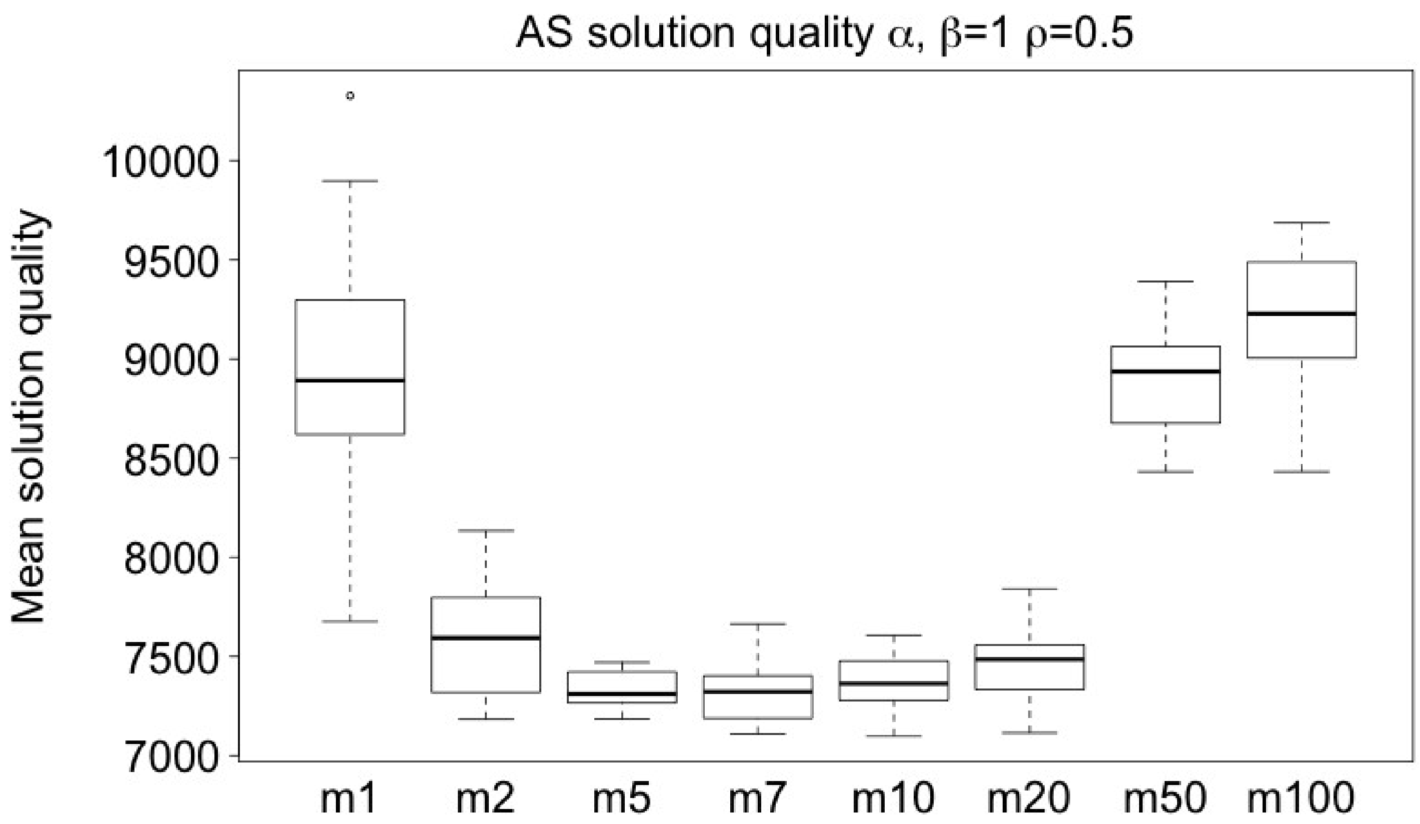
Outline

1. Implementation exercise
2. Review of AS
3. MAX-MIN Ant System (MMAS)
4. Ant Colony System
5. Elitist Ant System
6. Rank-based Ant System
7. Best-worst Ant System
8. ACOTSP

Implementation exercise 1

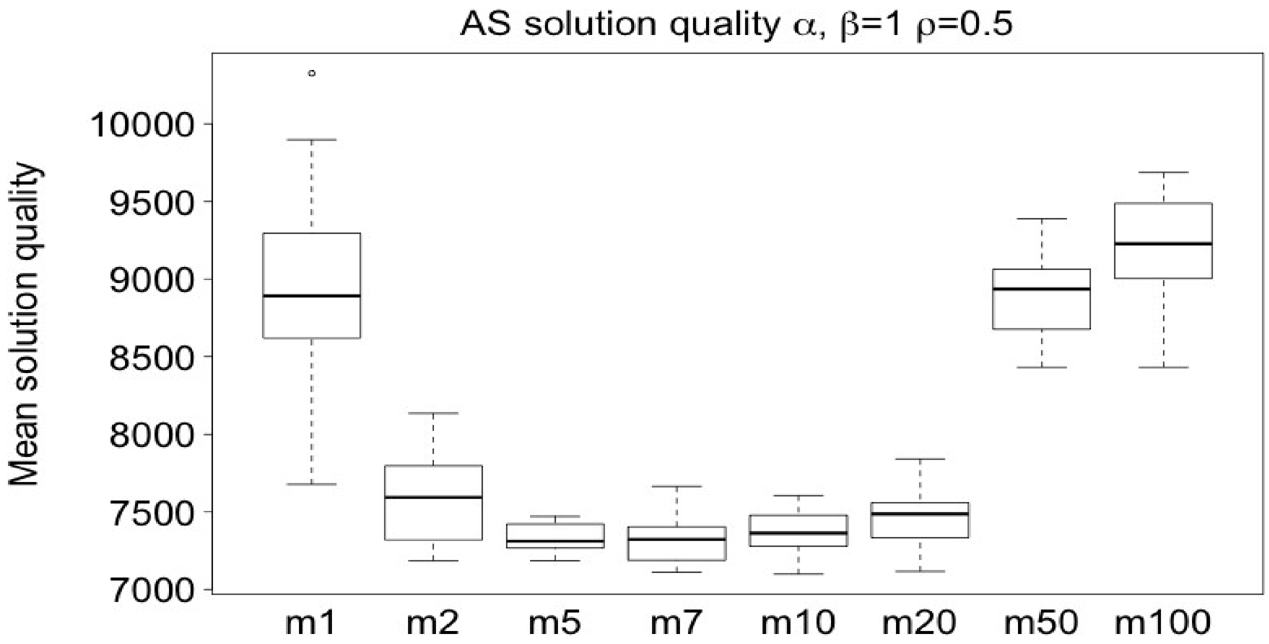


Compare results statistical tests

- Is there a **statistically significant** difference between the solution quality generated by the different algorithms?

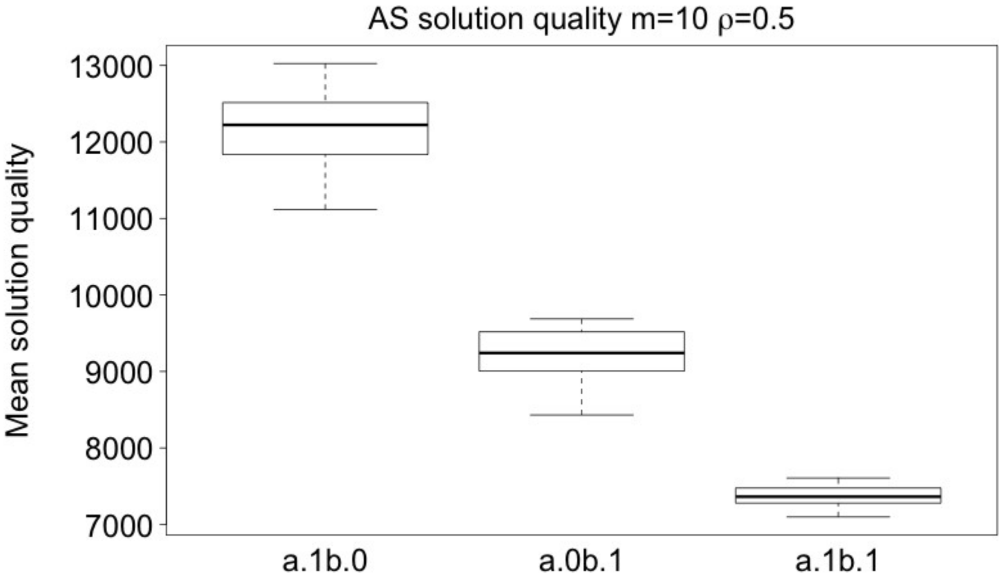
- **Null hypothesis**: The statement to be tested.
  - Example: For the Wilcoxon signed-rank test, the null hypothesis is that ‘the median of the differences is zero’
- The **significance level (α)** determines the maximum allowable probability of incorrectly rejecting the null hypothesis
- The null hypothesis is rejected if this p-value is smaller than the previously chosen significance level

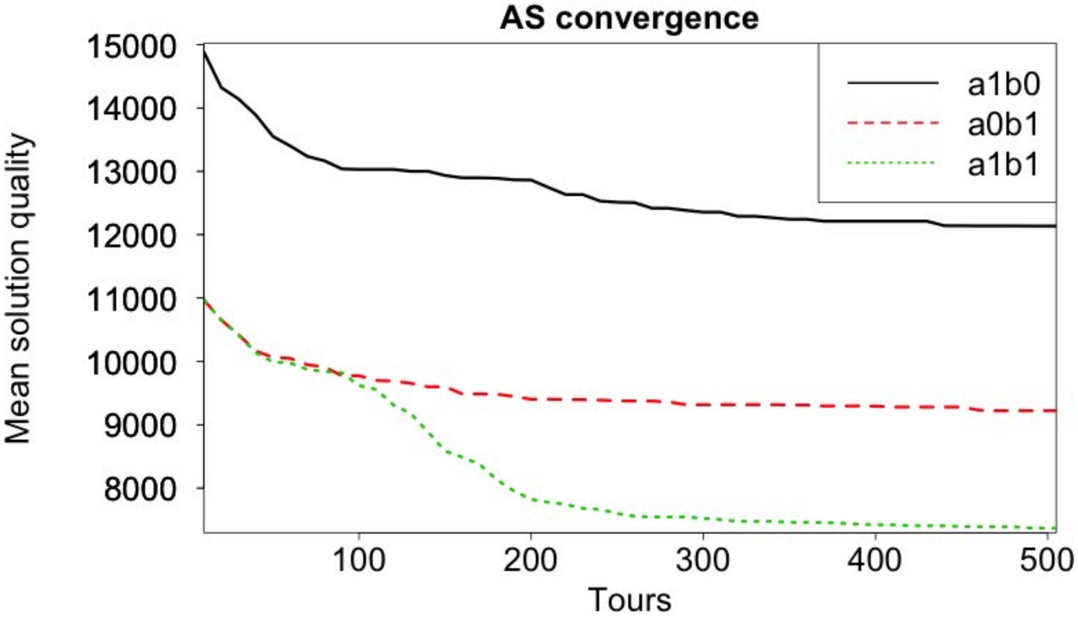
Implementation exercise 1



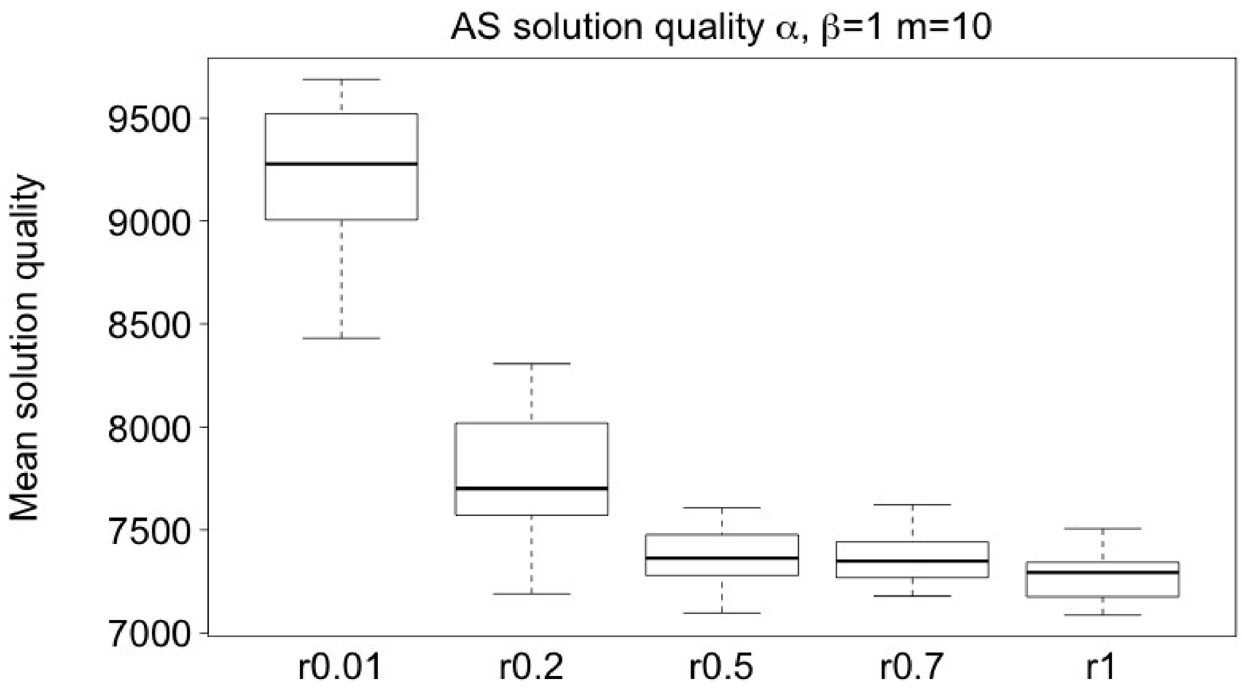
- Wilcoxon test p-values: — Corrected (Bonferroni)
  - m1 vs. m2: 1.907e-06 — 0.00001
  - m2 vs. m5: 0.003654 — 0.011
  - m5 vs. m7: 0.6676 — 1.000

Implementation exercise 1





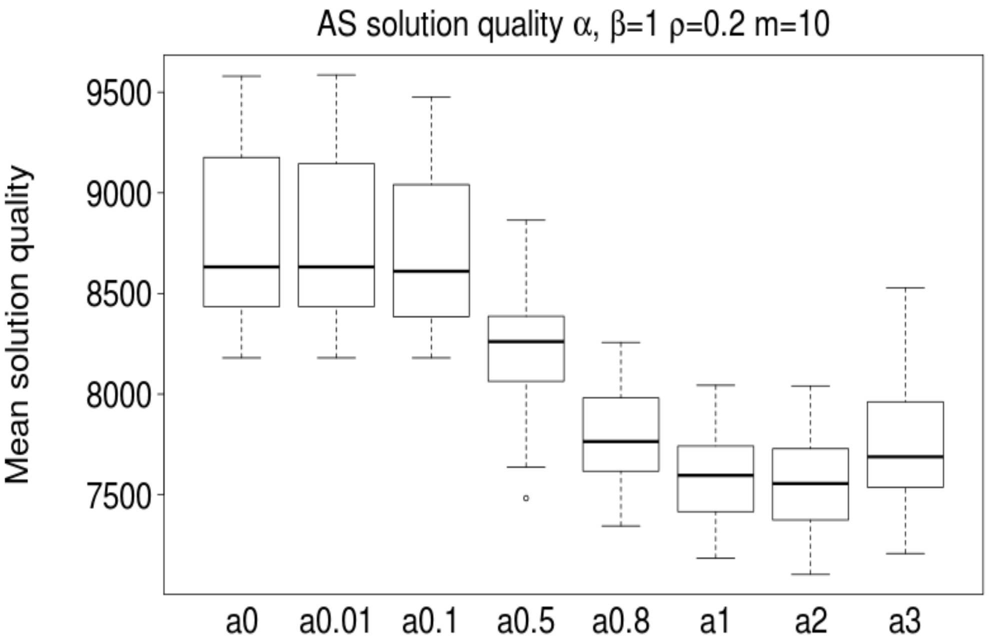
Implementation exercise 1

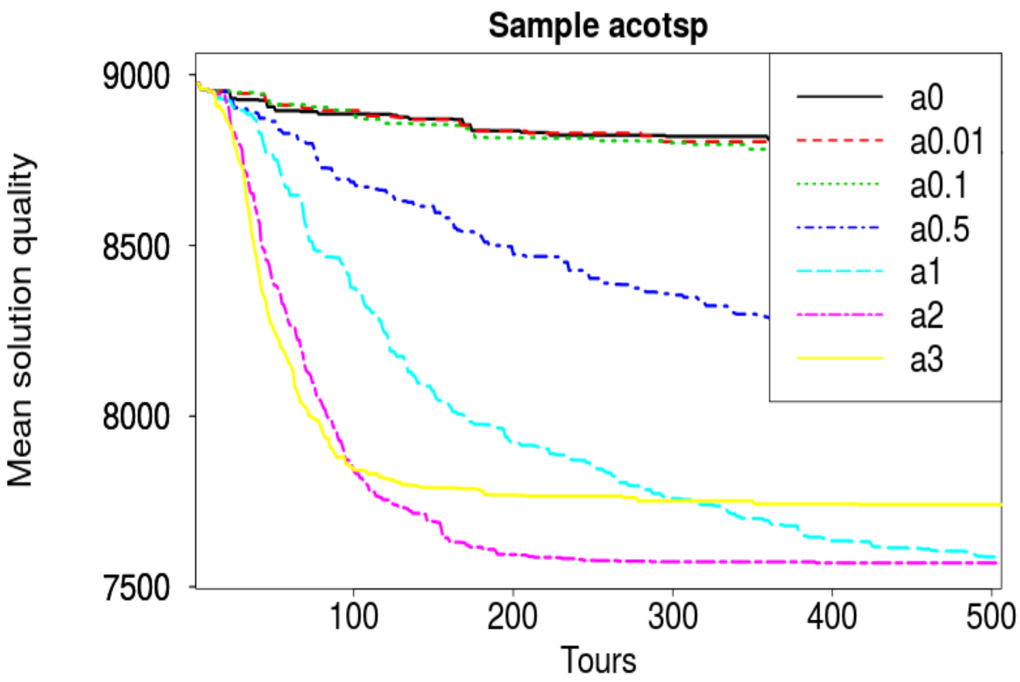


Wilcoxon test p-values				
	0.01	0.2	0.5	1
0.01	-	4.67E-009	4.97E-009	4.67E-009
0.2	-	-	1	1
0.5	-	-	-	1
1	-	-	-	-

Implementation exercise 1

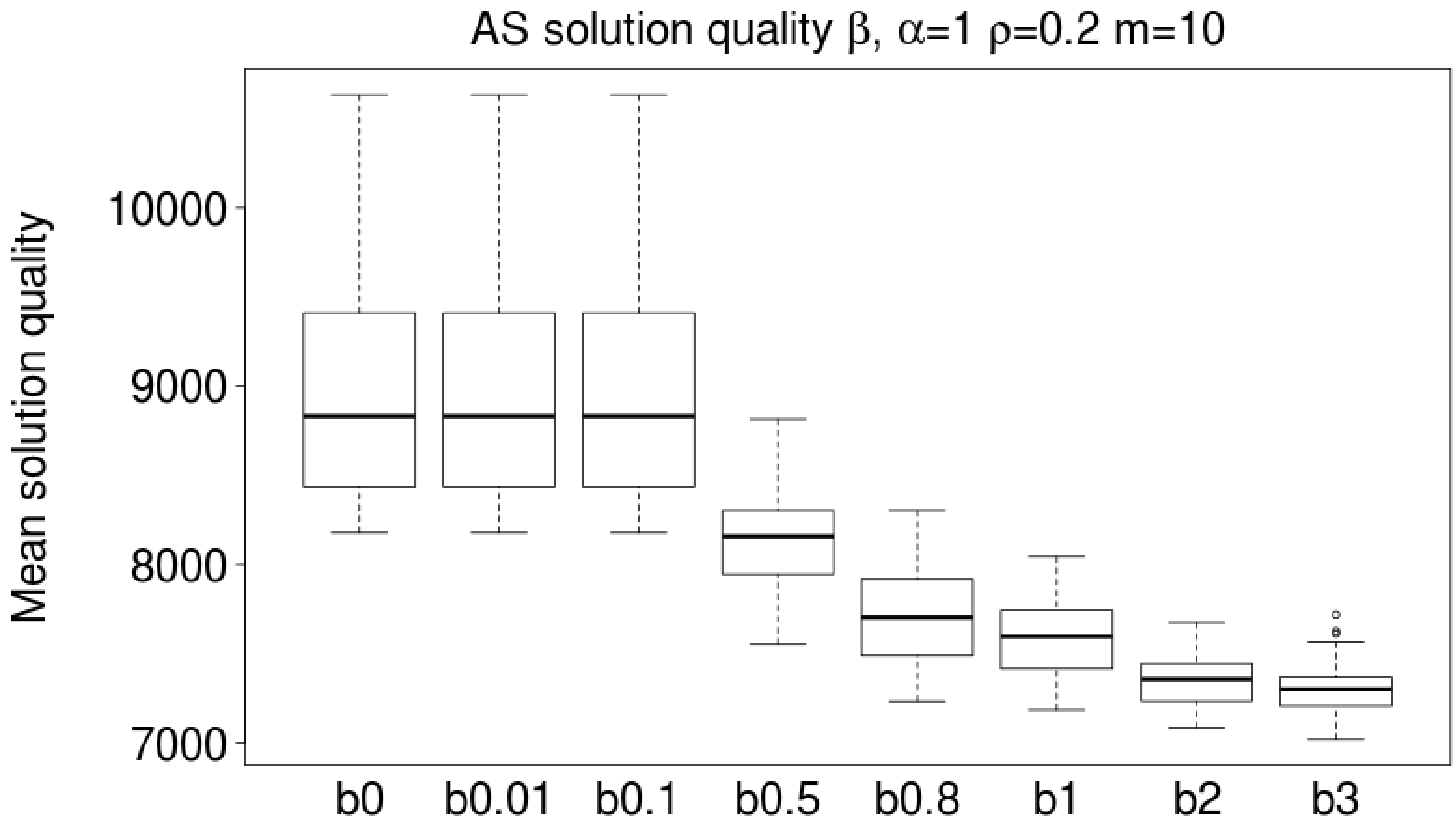
- More examples of parameter analysis:





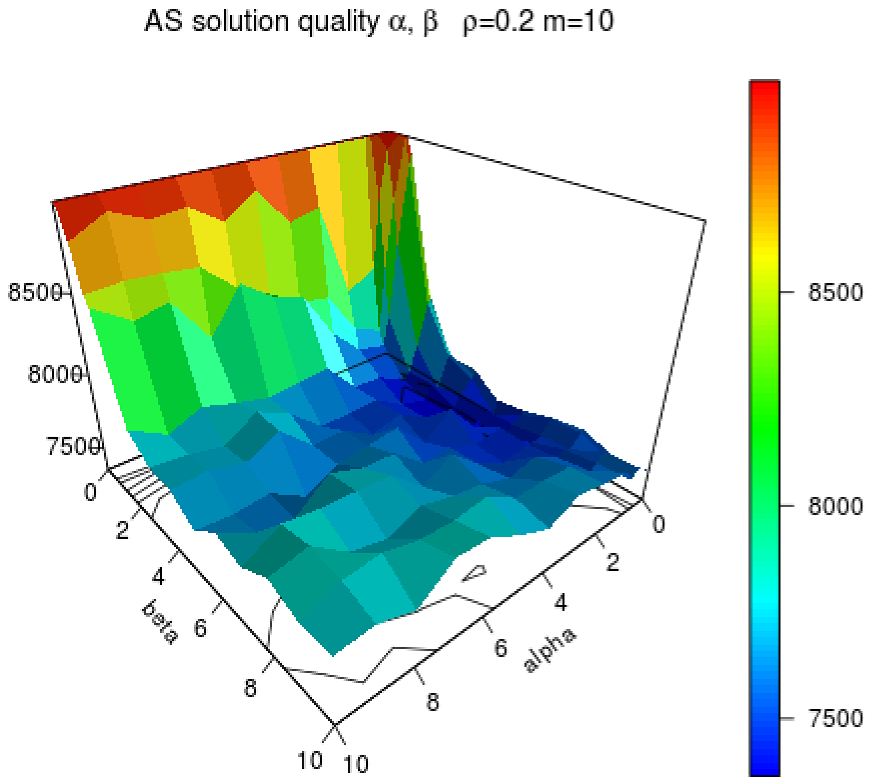
Implementation exercise 1

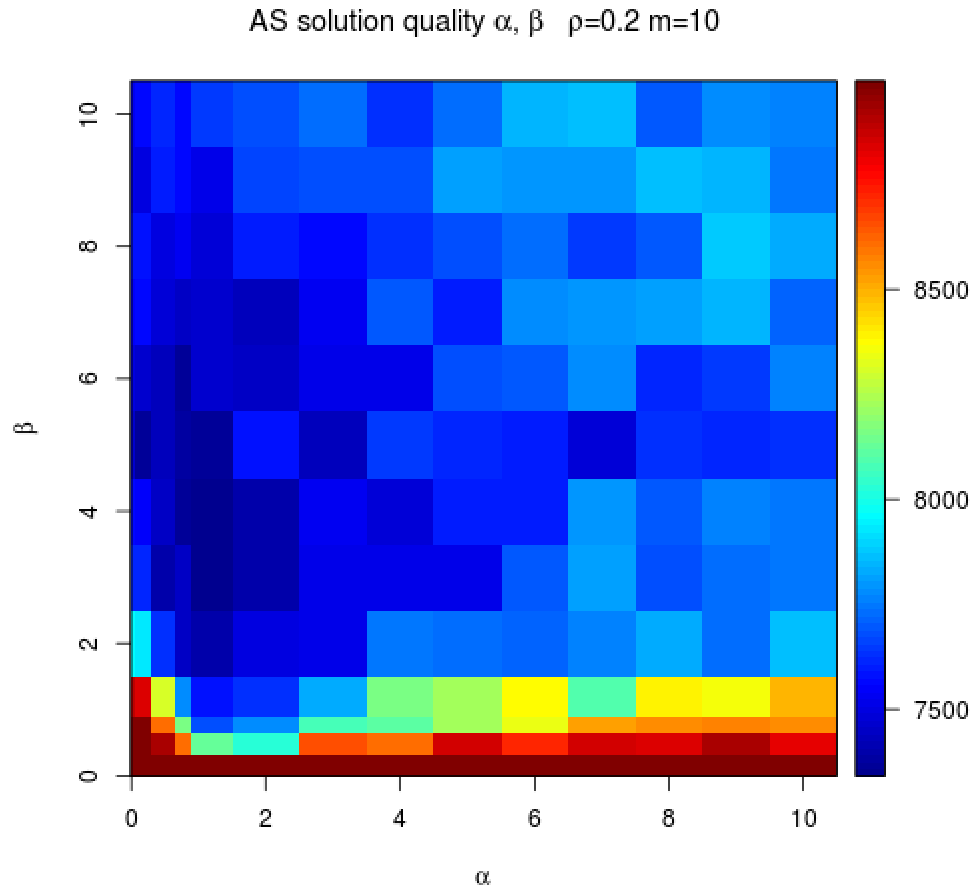
- More examples of parameter analysis:



Implementation exercise 1

- We can also analyze interactions:





Extensions of AS

- MAX-MIN Ant System (MMAS)
  - Only iteration best or best-so-far ants update pheromone
  - Pheromone trails have explicit upper and lower limits
  - Pheromone trails initialized to upper limit
  - Pheromone trails are re-initialized when stagnated
- Ant Colony System (ACS)
  - Pheromone is updated also while building the solution
  - Only iteration best or best-so-far ants update pheromone

MAX-MIN Ant System (MMAS)

- Only iteration best or best-so-far ants update pheromone

$$\tau_{ij}(t) = (1-\rho)\cdot\tau_{ij}(t-1) + \Delta\tau_{ij}^{best}$$

$$\Delta\tau_{ij}^{best} = \frac{1}{L_{best}}, \quad if \quad arc(i,j) \in best\,tour$$

  - L_{best}: length of the shortest tour found

MAX-MIN Ant System (MMAS)

- Pheromone trail values are subject to bounds

$$\tau_{min} \leqslant \tau_{ij} \leqslant \tau_{max}$$

$$\tau_{max} = \frac{1}{\rho \cdot L^{opt}}$$

$$\tau_{min} = \frac{\tau_{max}}{a}$$

$$\tau'_{max} = \frac{1}{\rho \cdot L^{bs}}$$

$$\tau'_{min} = \frac{\tau_{max}}{2 \cdot n}$$

$$\tau_0 = \infty$$

MAX-MIN Ant System (MMAS)

- Pheromone trails are re-initialized:
  - When the algorithm converges
  - When no improving solution has been generated for a certain number of consecutive iterations

Ant Colony System (ACS)

- Three main ideas:
  - Different state transition rule
  - Different global pheromone update rule
  - New local pheromone update rule

- Goal is: better control on exploration/exploitation

Ant Colony System (ACS)

- State transition (pseudo-random proportional) rule, which is biased towards:
  - exploitation with probability q_0

$$j = argmax_{j \in N_i^k} \left(\tau_{ij} \cdot \eta_{ij}^{\beta} \right) \quad if \; q \leqslant q_0$$

  - exploration with probability 1-q_0

    j is chosen according to the usual proportional transition rule

Ant Colony System (ACS)

- **Local update rule** (to enhance exploration): while building a solution, each ant updates pheromone on visited edges

$$\tau_{ij} = (1-\rho) \cdot \tau_{ij} + \rho \cdot \tau_0$$

Ant Colony System (ACS)

- **Global update rule**: pheromone updated **only on edges of the best tour** found so far

$$\tau_{ij} = (1-\rho) \cdot \tau_{ij} + \rho \cdot \Delta \tau_{ij}^{bs} \quad \forall (i, j) \in T^{bs}$$

$$\Delta \tau_{ij}^{bs} = \frac{1}{L^{bs}}$$

Ant Colony System for TSP
Simple pseudo code

```
While !termination()
  For each ant Do
    select random initial starting city
    While tour is not complete
      select next city using state transition rule
      apply local pheromone update rule
    EndWhile
  EndFor
  Apply global pheromone update rule
EndWhile
```

Elitist Ant System

- Elitism refers to favor best individuals to guide the search
- After each iteration the **global best ant** deposit pheromone along with the others
- Introduce a new parameter ***e*** that controls the contribution of the global best ant to the pheromone update

$$\tau_{ij}(t)=(1-\rho)\cdot\tau_{ij}(t-1)+\sum_{k=1}^{m}\Delta\tau_{ij}^{k}+e\,\Delta\tau_{ij}^{bs}$$

Rank-based Ant System

- A number of the best ants are allowed to update pheromone
- All the ants are **ranked** regarding their tour quality and the best ω-1 are selected
- They deposit pheromone according to their rank. So the best ones contribute more
- Parameter ω controls the **number of ants** allowed to deposit pheromone (usually 25%) and also controls the **amount of pheromone** contributed by each ant
- The global best ant deposit pheromone with the others

$$\tau_{ij}(t)=(1-\rho)\cdot\tau_{ij}(t-1)+\sum_{r=1}^{\omega-1}(\omega-r)\Delta\tau_{ij}^{r}+\omega\Delta\tau_{ij}^{bs}$$

Best-worst Ant System

- Transition rule and pheromone evaporation as in Ant System
- Pheromone update after each iteration:
  - The global best ant contributes positively to the pheromone update
  - The worst ant contributes negatively to the pheromone update (additional evaporation)
    - This is only applied in the edges present in the worst ant and absent in the global best ant.
- Pheromone trails **mutation** → exploration
- Restart of the search when stagnation (τ_0)

Ant System

Class exercise #2 – test your knowledge of ACO

- Open the file MMAS_ACS_SYSTEM-class_exercise.pdf and answer the eight question of the exercise.
  - The goal of this exercise is for you to determine how well you understood the differences among the most popular ACO algorithms
  - Once you finished the exercise, compare and discuss your answers with one of your classmates. *Note that you will need a classmate for the next implementation exercise*

ACOTSP

- ACOTSP developed by Thomas Stutzle, provides the implementation of a set of ACO algorithms to solve TSP.
- Which algorithms are implemented?
  - Ant System
  - Elitist Ant System
  - Max-min Ant System
  - Rank based Ant System
  - Best-worst Ant System
  - Ant Colony System

ACOTSP
Options: Algorithms

- How to specify the algorithm?
  - ***--as*** *: Ant System*
  - ***--eas*** *: Elitist Ant System*
  - ***--ras*** *: Rank-based version of Ant System*
  - ***--mmas*** *: MAX-MIN ant system*
  - ***--bwas*** *: Best-worst ant system*
  - ***--acs*** *: Ant colony system*
- Look for other parameters using ***./acotsp --help***
- Related parameters:

  --q0: prob. of best choice in tour construction (ACS)

  --elitistants: number of elitist ants (MMAS)

  --rasranks: number of ranks in rank-based Ant System (RAS)

ACOTSP
Options: Other

- Other general parameters

  --tries: number of independent trials (runs)

  --***tours***: number of steps in each trial (max tours evaluated per trial)

  --time: maximum time for each trial (seconds)

  --seed: seed for the random number generator

  --optimum: to stop if tour better or equal optimum is found

  --ants: number of ants

  --nnants: nearest neighbours in tour construction

  - To use of candidate list to construct solutions

  --alpha: alpha (influence of pheromone trails)

  --beta: beta (influence of heuristic information)

  --rho: rho (pheromone trail evaporation)

  --localsearch: 0: no local search   1: 2-opt   2: 2.5-opt   3: 3-opt

ACOTSP

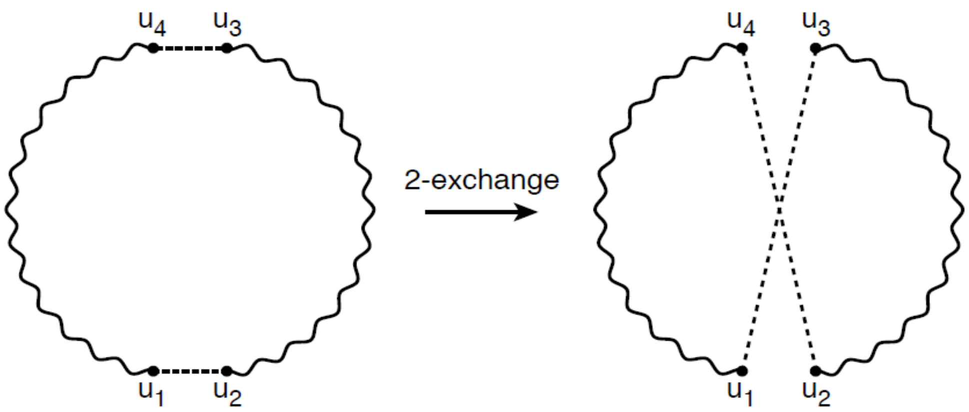
Options: Local search

- **Local search** starts from a solution already constructed and moves through the search space from one neighbor to other
- ACOTSP offers the possibility to apply a local search procedure to improve the tours found
- The options are:
  - 2-opt
  - 2.5-opt
  - 3-opt

ACOTSP

Options: Local search

- 2-opt
  - Heuristic: Select two edges and exchange them (2-exchange)
  - Repeat this process for all the edges combinations looking for improvement



- 3-opt follows the same idea using 3 edges, also 2-opt moves are evaluated
- 2.5-opt: Evaluates the insertion of a node coming from edge (A-B) between the nodes of other edge (C-D). Ex. A-C-B-D

Extensions of Ant System

Implementation exercise #1 – implementation of AS

- Open the file Implementation_Exercise1.pdf and solve point 1, which consists in implementing Ant System according to the provided template in C++