Swarm Intelligence — Class Exercises **3** (SOLVED)

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1. What is the inertia coefficient, and how is it related to exploration and exploitation?
   Answer: The value of the inertia coefficient (ω) allows to control the magnitude of the velocity vector. If the value of ω is large, particles move through the search space with larger step sizes, allowing a better exploration of the search space. On the other hand, if ω is small, particles can focus on specific regions of the search space, which results in better exploitation.

2. What happens if we set $\psi_1 = 0$ and $\psi_2 = 1$? What happens if we instead set $\psi_1 = 1$ and $\psi_2 = 0$?
   Answer: Since ψ_1 controls the cognitive influence (i.e., the influence of the particle's personal best solution) and ψ_2 the social influence (i.e., the influence of the best solution found by a neighbor of the particle), setting $\psi_1 = 0$ and $\psi_2 = 1$ will result in a velocity update rule that ignores the personal influence, whereas setting $\psi_1 = 1$ and $\psi_2 = 0$ will result in one that ignores the social influence.

3. What is the purpose of the U_1 and U_2 matrices?
   Answer: The purpose of the random diagonal matrices U_1 and U_2 is to perform random changes to the magnitude and direction of the personal and social components in the velocity update rule, thus providing diversity to particles' movement.

4. What is the role of topologies? What happens if a particle can communicate with the entire swarm? What if instead it can only communicate with a few other particles?
   Answer: The topology determines the particles in the swarm from which each particle can received and transmit information (that is, the neighbors of the particle). The topology plays a key role in determining how rapidly information will be spread among particles, which may result in a more exploratory or exploitative behavior. In the fully-connected topology, the information of the best-so-far solution will always be communicated to the entire swarm, which may be useful to enhance the exploitation of the search space, while in a ring topology this communication will be slower, and therefore, the behavior of the swarm will be more exploratory.

5. Assume you want to minimize the Rosenbrock function in $N = 4$ dimen-

sions

$$\sum_{i=1}^{N-1} 100 \times (x_{i+1} - x_i^2)^2 + (1 - x_i)^2 \tag{1}$$

when $\omega = 1$, $\psi_1 = 1$ and $\psi_2 = 2$. At the current iteration, particle 1 has position $[0.43, 1.25, -3.2, 2.4]$ with value 8546.17, and velocity $[-0.8, 0.2, -1.2, 0.1]$; the personal best is $[1.2, 2.5, -2.1, 2.2]$ with value 7584.92 and the global best known by the particle in its neighbourhood is $[1.4, 1.1, -0.4, 3.1]$ with value 1199.66. The diagonals of the matrices U_1 and U_2 are respectively $[0.21, 0.43, 0.12, 0.84]$ and $[0.63, 0.12, 0.92, 0.43]$. Update the velocity and position of particle 1. What is the value of the solution now?

Answer: Using the standard velocity

$$v_i(t+1) = \omega \vec{v}_i(t) + \psi_1 \big(\vec{p}_i(t) - \vec{x}_i(t)\big) + \psi_2 \big(\vec{g}(t) - \vec{x}_i(t)\big)$$

and position

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}(t+1)$$

update rules of PSO, we get:

$$\omega \vec{v}_{i=1}(t) = [-0.8, 0.2, -1.2, 0.1]$$

$$\begin{aligned}
\psi_1\big(\vec{p}_{i=1}(t) - \vec{x}_{i=1}(t)\big) &= [0.21 * (1.2 - (0.43)),\\
&\quad 0.43 * (2.5 - (1.25)),\\
&\quad 0.12 * (-2.1 - (-3.2)),\\
&\quad 0.84 * (2.2 - (2.4))]\\
&= [0.1617, 0.5375, 0.132, -0.168]
\end{aligned}$$

$$\begin{aligned}
\psi_2\big(\vec{g}(t) - \vec{x}_{i=1}(t)\big) &= [2 * 0.63 * (1.4 - (0.43)),\\
&\quad 2 * 0.12 * (1.1 - (1.25)),\\
&\quad 2 * 0.92 * (-0.4 - (-3.2)),\\
&\quad 2 * 0.43 * (3.1 - (2.4))]\\
&= [1.2222, -0.036, 5.152, 0.602]
\end{aligned}$$

$$\begin{aligned}
v_{i=1}(t+1) &= [-0.8 + 0.1617 + 1.2222,\\
&\quad 0.2 + 0.5375 + (-0.036),\\
&\quad -1.2 + 0.132 + 5.152,\\
&\quad 0.1 + (-0.168) + 0.602]\\
&= [0.5839, 0.7015, 4.084, 0.534]
\end{aligned}$$

$$\begin{aligned}
x_{i=1}(t+1) &= [0.43 + 0.5839,\\
&\quad 1.25 + 0.7015,\\
&\quad -3.2 + 4.084,\\
&\quad 2.4 + 0.534]\\
&= [1.0139, 1.9515, 0.884, 2.934]
\end{aligned}$$

$$\begin{aligned}
f_{\text{Rosenbrock}}\big(\vec{x}(t+1)\big) &= \big((1.9515 - (1.0139)^2)^2 + (1 - 1.0139)^2\big) +\\
&\quad \big((0.884 - (1.9515)^2)^2 + (1 - 1.9515)^2\big) +\\
&\quad \big((2.934 - (0.884)^2)^2 + (1 - 0.884)^2\big)\\
&= 0.853058001 + 9.457188332 + 4.646901672\\
&= 14.957148005
\end{aligned}$$