

# Route Assignment for Autonomous Vehicles

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**Abstract.** We demonstrate a self-organizing, multi-agent system to generate approximate solutions to the route assignment problem for a large number of vehicles across many origins and destinations. Our algorithm produces a set of mixed strategies over the set of paths through the network, which are suitable for use by autonomous vehicles in the absence of centralized control or coordination. Our approach combines ideas from co-evolutionary dynamics in which many species coordinate and compete for efficient navigation, and ideas from swarm intelligence in which many simple agents self-organize into successful behavior using limited inter-agent communication. Experiments demonstrate a marked improvement of both individual and total travel times as compared to greedy uncoordinated strategies, and we analyze the differences in outcomes for various routes as the simulation progresses.

**Keywords:** Swarm intelligence · Vehicle routing · Autonomous vehicles · Multi-agent systems · Co-evolution · Coordination games

## 1 Introduction

As autonomous vehicles become a significant portion of road traffic, the routing decisions made by those vehicles will have a strong impact on the congestion and efficiency of the road network. At present, it is acceptable for any autonomous or autonomously-routed vehicle to simply greedily select the most efficient route from its origin to its destination. However, once these vehicles represent the majority (or even a large minority) of traffic, a problem with this greedy approach arises: if all vehicles take the apparent shortest route, that route will quickly become overloaded and travel will slow to a crawl. Such a situation could be avoided by more intelligent coordination between the vehicles - if they were to spread themselves out across a variety of routes, congestion could be avoided and travel times improved. This problem falls into the class of coordination games, in which players must simultaneously select a strategy, and the utility of a strategy decreases as more players select it. The solution to such problems is for each player to adopt an equilibrium mixed strategy over the set of possible choices, such that no one player can improve their expected utility by unilaterally deviating from that mixed strategy.

The problem of vehicle route selection is further complicated by the fact that travel time depends not only on the route selection of other vehicles traveling from the same origin to the same destination, but is also affected by the

choices made by vehicles traveling between other origin/destination pairs, even those which share neither the origin nor destination. Additionally, the domain of autonomous vehicles gives particular importance to the requirement that strategies be in equilibrium. If a particular traveler believed that a better route choice was available, he or she would simply direct the vehicle to use that route instead. This prevents arrangements in which a small subset of vehicles are assigned poor routes with the goal of alleviating congestion for the majority of vehicles. This concept of equilibrium is based on early work by Wardrop in which he defines “user equilibrium” on a travel network as a state in which every individual agent is acting in the way which minimizes its own travel time [16].

In this paper, we present a multi-agent simulation from which a set of equilibrium mixed strategies for route selection may be gleaned. Our algorithm begins from the greedy assignment of vehicles to the shortest route, and uses a combination of exploration of alternative routes, and reinforcement of successful, faster routes, to arrive at a set of routing strategies which are more efficient, and from which no individual agent can profitably deviate.

The study of route selection for vehicles and its effect on traffic congestion is not new. It has been studied since at least the 1950s to assist in the planning of extensions to the road network and the prediction of their effects on traffic flow. The problem has traditionally been modeled as optimization under constraints, in which a variety of simplifying assumptions are made about traffic dynamics to reduce the problem to an instance of convex non-linear programming which can be approximated through gradient descent methods such as the Frank-Wolfe Algorithm [6]. Further work has found an assortment of improvements [7] and alternative algorithms [4, 10] within this paradigm.

More recently, there have been a variety of attempts to apply both evolutionary algorithms and swarm intelligence [2] algorithms to various traffic problems, including signal control [11, 15], network layout [12], and scheduling [14]. There have also been efforts to adopt ant colony optimization [5] approaches to both the signal control [3] and vehicle routing problems.

## 2 Algorithm

Our algorithm works by modeling the road network as a graph of edges (also called links) and nodes, a set of vehicles, and a set of origin-destination pairs of nodes in the graph between which vehicles travel. Each vehicle is assigned an origin node and a destination node, and has a path of edges which leads from its origin node to its destination node. The number of vehicles travelling between a particular origin-destination pair remains constant throughout the simulation. For simplicity, the set of all vehicles traveling between a particular origin-destination pair is referred to as a *species*.

The algorithm proceeds by simulating the flow of these vehicles across the network. When a vehicle reaches its destination, it returns to its origin and begins another journey. It also informs another randomly selected vehicle of the same species of the route it took. The second vehicle will then adopt that strategy

for its next trip, with a small chance of alteration through the addition of a detour. In this way, successful strategies spread throughout the population, and new strategies are explored through the occasional addition of detours.

The time it takes for a particular vehicle to pass through an edge is determined by a congestion equation, which calculates the delay due to other vehicles on the same edge. Each edge has a length which defines the base travel time  $t$ , and a capacity  $c$ , which defines the number of vehicles an edge can carry before congestion effects take over. The number of vehicles currently travelling on a link is denoted by  $v$ . In this work, we use a slight modification of the standard Bureau of Public Roads congestion equation [1]. The travel time on a link,  $S$ , is determined by the equation:

$$S = t * (1 + 0.15 * (v/c)^4)$$

A known difficulty with this equation is that the exponential form predicts unrealistic travel times in the case of severe congestion [13]. To avoid this, we model congestion with queuing once the traffic load passes a certain threshold (in our experiments, 2.5 times the capacity of the link). If a vehicle attempts to enter a link which is at the specified threshold, it instead enters the back of the queue for that link. Whenever a vehicle exits a link, if its queue is not empty, the vehicle at the front of the queue is removed and enters the link.

## 2.1 Spread of Strategies

When a vehicle reaches its designated destination, it has the potential to either spread its current path to another vehicle of its species (that is, a vehicle with the same origin and destination), or to replace its own path with one it has received from another vehicle. Upon reaching its destination, a vehicle returns to its starting node, and checks whether it has received a replacement path. If so, it adopts that path, and follows it through the network. If that vehicle has not received a replacement path, then it instead communicates its current path to another vehicle of the same species.

The exact procedure for communicating the path of a successful vehicle is as follows. Let  $v$  be the vehicle that has just successfully completed its route. Randomly select another vehicle  $w$  of the same species as  $v$ . Mark  $w$  as having received a replacement path, and set its replacement path to the path of  $v$ . With a small random chance (in our experiments, a chance of 5% was used), modify the  $w$ 's new replacement path with a detour, as explained in the next section. If  $w$  has previously received a replacement route, that route is overwritten with the new route from  $v$ . The vehicle  $w$  will continue on its current route, and will not adopt the new replacement route until it reaches its destination and begins a new journey. Note that  $w$  is selected at random, without consideration for the efficiency of its current path - the frequency of a path is modulated by its rate of reproduction, not by its rate of removal.

The result of this system of spreading successful strategies is that the average rate at which a particular path increases its share of the population is dependent on the speed at which vehicles following that path complete their route as

compared to the average speed across all vehicles of that species. A path which allows vehicles to complete their journeys faster than this average speed will tend to increase its share of the population. As its share of the population increases, congestion effects will slow it, until eventually vehicles traveling along it are no longer faster than the average, and its share of the population levels off. Eventually, all paths used by a particular species will take roughly the same amount of time. In this case, each will spread at the same rate, and population will be stable (the random nature of path replacement and detours will of course allow some amount of variation to remain). Thus, the population will have found a set of equilibrium strategies for navigating the network - no vehicle could improve its time by switching, because the feasible paths all take approximately equal time.

## 2.2 Detours

It is not sufficient to only increase the frequency of efficient paths within the population. We also must explore new routes that are not currently in use. It may be that one of those is more efficient than any route being currently taken. To accomplish this exploration, we apply a detour (with a small random chance) to paths that are passed from one vehicle to another.

A detour is formed by randomly selecting two nodes from the current path to serve as the start and end of the detour,  $D_s$  and  $D_e$  such that  $D_s$  occurs before  $D_e$  in the path. Next, a random node is selected from the set of all nodes in the graph to serve as the midpoint of the detour,  $D_m$ . We then replace the section of the original path from  $D_s$  to  $D_e$  with the concatenation of the shortest (in distance, not apparent travel time) route from  $D_s$  to  $D_m$  followed by the shortest route from  $D_m$  to  $D_e$ . If this would result in any loops in the new route, those loops are removed.

Despite the name, the detour operator does not create only more circuitous routes. Because the paths between the start, midpoint and end of the detour are the shortest paths available, a detour operator can also make a path more direct. If  $D_m$  lies on the shortest path from  $D_s$  to  $D_e$  (or even a shorter path than is currently being taken), then the effect of this detour will actually be to simplify the route.

## 2.3 Initialization and Termination

It is necessary to select a starting set of routes for the vehicles in the simulation. In our experiments, we have each vehicle begin by taking the shortest path from its origin to its destination. Once this is done, simulation begins with a winding-up period in which no mutation and reproduction occurs - vehicles simply travel along their paths, and return to the start once complete. This allows any initial congestion due to the simultaneous start of many vehicles to dissipate before evolution begins. Once travel times along routes have roughly stabilized (or a pre-determined number of iterations have been performed), evolution can begin as described above.

Once sufficient evolution has occurred (as determined by a lack of further improvement across the simulation), a short period of simulation without further evolution is useful to allow the traffic patterns to stabilize before observing the final results. From these results, the routes taken for each origin-destination pair can be read off to create a mixed strategy for travel between that pair. Each route is assigned a probability equal to its relative frequency in the population.

### 3 Experiments

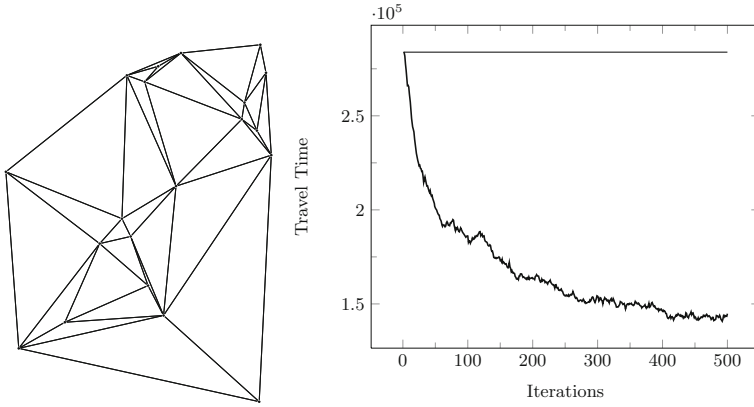
This algorithm was tested on a randomly-generated network consisting of 20 nodes and 98 edges, which was embedded in 2D space. The layout of the experiment network can be seen in Fig. 1. The base free-flow travel times for links were determined by the distance between the start and end of that link. The capacity of a link is proportional to its length. The origin-destination pairs used were all pairs of non-adjacent nodes on the convex hull of the network, giving 40 different origin-destination pairs. Adjacent nodes were omitted because they tend to lack feasible alternative routes, and therefore are not subject to meaningful evolution. Each origin-destination pair was loaded with a number of vehicles equal to the total capacity of the shortest path between those nodes. This would allow relatively uncongested travel for a single pair, but creates significant congestion when applied across all pairs. In total, over 1,100 vehicles were simulated.

Each experiment ran for five million iterations (that is, five million simulated edge traversals), with an additional five hundred thousand iterations each of initialization and termination periods to ensure stable starting and ending conditions. Data points about the current travel times in the network were collected every ten thousand iterations. Travel times were calculated based on the length of each vehicle's last completed trip. We conducted ten experiments in which we measured the total average travel time experienced across the network, and the average travel times experienced for each individual origin-destination pair. Additionally, we examined the average time of each individual path between a single origin-destination pair to determine whether travel times were approximately equal, which would be the expected outcome if the simulation had reached equilibrium.

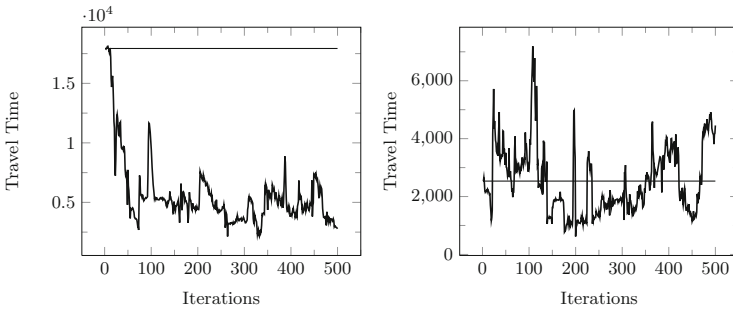
### 4 Results

Over the course of the simulation, the average total travel time fell from roughly 310,000 simulation ticks to roughly 158,000 ticks, a reduction of nearly half, as seen in Fig. 1. A steep initial improvement occurs as the most severely congested routes clear out, followed by a period of slowing, somewhat unsteady improvement as vehicles settle into efficient equilibria.

Figure 2 shows two representative graphs which demonstrate the different types of behavior an origin-destination pair can display. On the left we have the data for vehicles traveling from Node 12, in the upper left corner of the network, to Node 4, near the lower left. Here we see the same steep initial improvement



**Fig. 1.** The experiment network, and total travel time across all routes as the simulation progresses.



**Fig. 2.** Average travel times for two different origin-destination pairs. The horizontal lines represent the baseline travel time.

as in the overall average, followed by slower improvement marked by significant spikes in travel time. This occurs when the vehicles on other origin-destination pairs make an adjustment to their route choices such that more vehicles occupy the links being used to travel from 12 to 4. Once the congestion becomes severe, alternative routes are found to once again lower travel times.

On the right side of Fig. 2, we have the data for vehicles traveling from Node 4 to Node 3, both in the lower left corner of the network, separated by only two links. Here, instead of a sharp initial improvement, we see a brief, small improvement followed by a severe spike. Fluctuations occur throughout the rest of the simulation, but the travel time ultimately ends up worse than how it started. This is because the links along the initial path between these nodes were not used by the initial paths for other species. As evolution progressed, those other species discovered these little-used links, and congestion increased. We thus see that an overall improvement in travel time does not indicate uniform improvement across all origin-destination pairs.

$3 \Rightarrow 14 \Rightarrow 8 \Rightarrow 17 \Rightarrow 1 \Rightarrow 11 \Rightarrow 15$	39.4%	6025 ticks
$3 \Rightarrow 14 \Rightarrow 8 \Rightarrow 5 \Rightarrow 1 \Rightarrow 10 \Rightarrow 15$	36.4%	6266 ticks
$3 \Rightarrow 13 \Rightarrow 4 \Rightarrow 11 \Rightarrow 15$	18.2%	6205 ticks
$3 \Rightarrow 0 \Rightarrow 8 \Rightarrow 1 \Rightarrow 10 \Rightarrow 6 \Rightarrow 12 \Rightarrow 15$	6.1%	7320 ticks

**Fig. 3.** Routes from Node 3 to Node 15 at the end of an experiment run, showing the relative frequency of each route and the time taken in simulation ticks.

As an example of the routes generated for a particular origin-destination pair, consider the set of routes from Node 3 to Node 15 seen in Fig. 3. The first two routes are similar, passing through several common nodes. The third route uses an almost entirely different set of nodes, and passes around the outer edge of the graph, but still requires roughly the same amount of travel time. Only the fourth strategy, which has a very low share of the population, takes significantly longer due to its use of congested links (the direct  $8 \Rightarrow 1$  link in particular, which is avoided by the other routes) and a circuitous detour to Node 12 near the end. This strategy likely represents a relatively recent mutation which has not yet been competed out of the population.

The fact that the travel times are roughly equal (aside from the small outlier) is to be expected - this represents an equilibrium state in which all three strategies will reproduce at approximately the same rate. The equality is only approximate, however, because of the discrete nature of the simulation and ongoing changes in the co-evolutionary fitness landscape up until the end of the simulation.

## 5 Future Work and Conclusion

Our algorithm offers several directions for potentially fruitful future work. In this paper we use a standard model for congestion, the BPR equation. This model is ultimately a simplification of real traffic behavior, made in order to allow efficient computation and provide mathematically convenient properties (in particular, a nicely concave shape for gradient descent approaches as well as full independence of delays along different links). Our algorithm does not rely on these properties of congestion modeling, and any desired model could be used instead, including those which account for interdependence between the congestion on links (such as models which account for the delay at intersections caused by cross-traffic [8], or cascading congestion across multiple links [9]). Further work is needed, however, to determine the performance of our algorithm under these more detailed congestion models.

Our experiments in this paper demonstrate the effectiveness on a moderately-sized, randomly-generated network, but more work is needed to study the behavior of this approach on a real-life road network. It may be necessary to alter the method of generating detours when working on a very large-scale network - perhaps by favoring local detours over distant ones.

We have demonstrated an algorithm by which a large number of autonomous agents can arrive at a set of equilibrium strategies for navigating a network. Our algorithm significantly outperforms a baseline greedy approach in which each agent independently selects its apparent fastest route. Further work is needed to adapt this approach to real-world road networks and vehicle behavior.

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