Portfolio Optimization Report

Group PO 1

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Executive Summary

In financial mathematics, an investment portfolio refers to a combination of assets, typically stocks, which is assembled for the purpose of generating long term returns. Typically, an investor would like to choose the composition of the portfolio in such a way to simultaneously maximise returns and minimise risks. While the return can usually be measured by the increase in value of the portfolio, it is less clear how one should quantify the risk of the portfolio. Some common measures of risks include variance or standard deviation, value-at-risk (VaR) and expected shortfall. Each of these can be measured at different time intervals (daily, weekly, etc.). By carefully selecting the composition of the portfolio, the investor would aim to optimise some objective function which balances the return with the risk, by maximising the Sharpe ratio. How we make these careful compositions is known as the study of Stock Portfolio Optimisation.

In this project, many in-sample optimisation strategies were tested to meet the project aim of measuring the effectiveness of modern trading strategies. More specifically, key concepts of Modern Portfolio Theory were put to the test to see if their complicated implementations are worth the rewards they reap. This investigation was conducted with the initial hypothesis being that although some improvements would be made to portfolio performance, no improvement would be worth the complicated implementation process of the strategy. This hypothesis which would later be proven false, was made on the idea that markets are inherently both random and complicated and thus a wildly successful strategy would not be so simple to come by.

Throughout the project several distinct types of portfolio weighting and stock choosing strategies were considered. In experimenting with the stocks and weights, 7 strategies were shortlisted, each with a different combination of choice of stocks and choice of weights. The first of these strategies was to purely test the effectiveness of weight optimisation and consisted of choosing stocks at random and then adjusting their weights each year to maximise Sharpe Ratio. The next three strategies chose stocks by maximising expected return while using a variety of methods to adjust the weights of stocks. Similarly, the third strategy involved choosing stocks to maximise Sharpe ratio of the portfolio and each of the same variety of weight adjustment methods were tested. These weight adjustment methods were weighting the stocks equally and letting it run, adjusting weights every 5 years to maximise the portfolio's Sharpe ratio and adjusting weights every 5 years to minimise the portfolio's variance. Each of these strategies were then compared to a baseline strategy in which stocks were chosen at random and given equal weights. Ideally, the tested strategies will outperform this baseline in KPI's such as return and Sharpe Ratio by a significant amount thus implying its effectiveness and worth.

Here is a key question to this project: What is a "better" stock portfolio, or, more precisely, what is counted as a "good" stock portfolio? This question was explored through the different strategies and the comparisons between those strategies, however, a clear goal needed to be specified. The chosen goal was minimising risk and maximising return. A straightforward way to analyse this metric of risk and return is by using a formula called the Sharpe ratio. This is the ratio of return over risk and produces a metric that can easily be analysed to compare strategies. In addition, returns were also taken into consideration, due to the end goal of investing in stocks being to earn money. In summary it was defined that a good portfolio was one with a high return value with a high Sharpe ratio.

Upon creating the baseline strategy, it was found that the strategy produced unstable results. Not only was the overall strategy producing inconsistent a large variety of results, meaning that it can return an extremely low amount, but also an extremely high amount, each individual portfolio produced was also unstable. This was then visible through the strategies relatively low average Sharpe Ratio of 0.81.

The next strategy (strategy 1 in the report) then introduced dynamic trading principles such as adjusting weight yearly to maximise the portfolio's Sharpe ratio. Using this strategy led to a more table set of portfolios by firstly, generating a smaller range of portfolios and improving average Sharpe Ratio to be 0.93. This meant that the strategy behaved more predictably in a holistic sense, but also individual portfolios behaved more predictably thus incurring less risk. Furthermore, as the strategy had a similar return it was an effective method of optimising portfolios and confirmed that weight adjustment was a worthwhile practice for trading.

After this the rest of the trading strategies were all compared to not only find the best way to choose stocks to include in the portfolio but to also find the best way to weight the stocks in the portfolio. For these, there were 3 weight optimization methods that were tested and two stock choosing methods as previously mentioned. All portfolio's performed far better than the previously discussed strategies thus implying the conclusion that optimal stock choice was more important than weight adjustment when its portfolio optimisation. The most effective of these strategies and the ones that would later be recommended to the audience were adjusting weights to maximize Sharpe Ratio while choosing stocks by maximizing the portfolio's Sharpe Ratio and adjusting weights to minimize volatility while choosing stocks by maximizing the portfolio Sharpe Ratio. The former of these strategies produced an impressive Sharpe Ratio of 1.992 and the latter demonstrated remarkably stable growth.

Ultimately, these realisations led us to the conclusion that our hypothesis was false and that these strategies were capable of effectively producing optimised portfolios that performed well in portfolio KPI's. Namely, the strategy of adjusting weights to maximize Sharpe Ratio while choosing stocks by maximizing the portfolio's Sharpe Ratio had such strong performance that despite its difficult implementation, it would be a worthwhile strategy to use in the real world.

Like most things the project was not without its flaws. Comparisons with ETF's and questionable figures brought about the realisation that although the analysis was done correctly, some biases in the data that was used meant that investigation was not adjacent to the real world. Examples of this included data being comprised of already high performing stocks, strategies unintentionally drawing on future events and ignoring costs such as taxes and trading fees. Nonetheless, all comparisons within the project, barring comparison to an ETF, still had merit as the strategies were still tested on a level playing field. Furthermore, these issues could act as points to discuss in further projects in which more data could be used, and more factors could be incorporated. Moreover, findings of this project could act as a framework to further test the altering the parameters of portfolios such as testing readjustment intervals or portfolio size.

Introduction

Purpose

Aim

This experiment has the aim of testing if modern trading strategies can supply significant improvements to portfolio performance and aid in achieving stock portfolio optimization. More specifically, assessing the effectiveness of various stock choosing and weight adjustment strategies.

Hypothesis

It was hypothesized that the strategies would supply detectable improvements to a portfolio's optimization, however they would not supply exceptional and ground-breaking benefits. This hypothesis is due to the inherent randomness involved in the stock market and the fact that if an incredibly profitable strategy were found it would either be everywhere, or it would not ever been shared.

Necessary Context

As this project is founded heavily within the financial domain, this report and the project itself lean heavily on several concepts and jargonistic terms. As these terms are vital to understanding the report they will be defined as follows.

Stock Portfolio

First, by definition, a financial portfolio is a collection of financial investments like stocks, bonds, commodities, cash, and cash equivalents" (Tardi, 2023). For this report, however, it can just be thought of as a basket of all the investment assets that you own such as stocks, houses, or bonds. More specifically, a stock portfolio is a kind of financial portfolio where all its assets can be traded are "market-traded securities" (Velazquez, 2022) which for this report can be assumed to just be things traded on the stock market.

In Lamens terms: A stock portfolio is just the basket of stocks that you own.

Weights

When dealing with stock portfolios not every stock in the portfolio may contribute equally to the value of the entire portfolio. A stock's "weight" within a portfolio is how much it contributes to the portfolio's total value (Chen, 2023). For example, if you own $100 of Tesla stock and your whole portfolio is worth $1000, the weight of Tesla stock in your portfolio is 0.1 meaning that Tesla stock makes up 10% of your portfolio.

In Lamens terms: How much of the portfolio each stock takes up.

Publicly Traded

A company is considered a publicly traded company if its stock can be bought and sold in a public stock market. Consequentially, this also means that at some point the company sold some of its stock to the public (Banton, 2022).

In Lamens terms: A publicly traded company is on the stock market.

Index

Much like a stock portfolio, an index is a basket of stocks, however, its main purpose is to track the performance of different segments of the market. Due to the complexities of trading, many investors will invest in securities that mimic these indexes as they are often simple and supply reliable growth (Chen, 2023).

S&P 500

The S&P 500, short for "Standard and Poor's 500 index" is an index that tracks the stocks of the top five hundred performing publicly traded companies in the US economy (Kenton, 2023). This ranking is made mostly on the company's market cap, how much a company is worth on the stock market (Fernando, 2023), however, several other factors influence these rankings to a lesser extent. As shown in Figure 1, these stocks are weighted in the index by how much they contribute to the total market cap of all the stocks in the index.

$$\text{Company Weighting in S \& P} = \frac{\text{Company market cap}}{\text{Total of all market caps}}$$

Figure 1: Formula for S&P 500 stock weights.
(https://www.investopedia.com/terms/s/sp500.asp)

Sharpe Ratio

The Sharpe ratio of a security or portfolio is a metric that considers most investors' goals of making the most money while taking on as negligible risk as possible while doing so. As shown by its mathematical definition in Figure 2, the metric does this by dividing volatility by return to obtain a ratio of the two goals. A good Sharpe ratio is one or above.

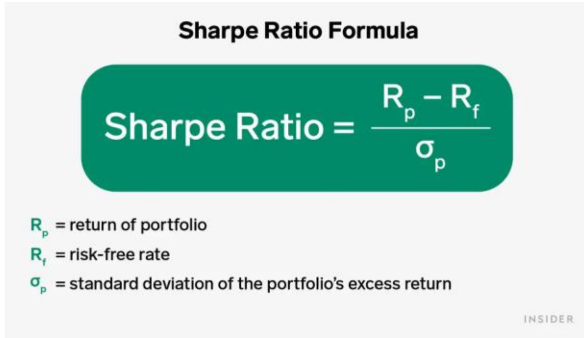

Figure 2: Formula for Sharpe Ratio (https://www.businessinsider.com/personal-finance/sharpe-ratio)

Portfolio Optimization

Defined as a 'formal mathematical approach to making investment decisions across a collection of financial instruments or assets' (MathWorks, n.d), portfolio optimization is the process of creating a portfolio that meets the goals of the traded. These goals are often to make as much money as possible (maximize return) while taking on the least risk (minimize volatility). As we have a metric to track both goals we can, therefore, consider stock portfolio optimization to be the practice of maximizing a stock portfolio's Sharpe ratio. There are multiple methods to do this, however, the most prominent is called modern portfolio theory.

Modern Portfolio Theory

Modern Portfolio Theory, A.K.A mean-variance analysis, is a method that uses mathematical analysis to help investors achieve the goal of minimizing the Sharpe ratio of the portfolio thus in turn reaping low-risk rewards. The main property of modern portfolio theory that makes it so effective is the idea that although choosing assets with high Sharpe ratios may positively influence a portfolio's optimization, it is much more beneficial to see things from the perspective of the entire portfolio when making investment decisions. What this means is that although on their own neither a high-risk high-return stock nor a low-risk low-return stock performs well, however, when paired together they can create an effective portfolio (The Investopedia Team, 2023).

The way that modern portfolio theory achieves these goals is by using an "efficient frontier." An efficient frontier is a group portfolio of portfolios that maximize investor outcomes by maximizing the Shape Ratio of portfolios (Baldridge, 2023). Such a group is useful as it can be represented graphically as shown by the blue line in Figure 3. In this graphic, each green point stands for a portfolio and each of these points are plotted according to their risk and return. Points will differ by factors such as stock weight or included stocks. Portfolios that are closer to the curve are better at achieving our goals of stock portfolio optimization and thus we want to choose those portfolios that are close to or on the efficient frontier. From where on the line, we choose these portfolios are dictated by the investor's risk tolerance.

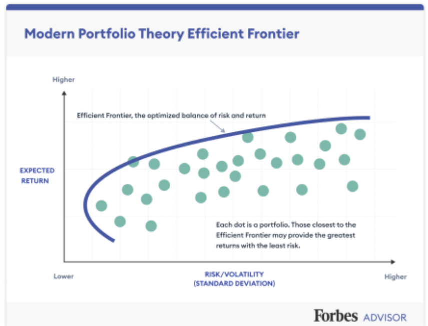

Figure 3: Efficient Frontier diagram. (https://www.forbes.com/advisor/investing/modern-portfolio-theory/)

The Data

To complete this project, we were given a single CSV file that held some price data for us to work with. This price data took the form of the daily closing price (the price at the end of the day) for all stocks in the S&P 500 for the period starting on 7/9/1993 and ending on 30/7/2019. The data was bare bones as shown by the diagram in Figure 4 as we were not given any other variables for each stock other than price.

Ticker Symbols

	Date	0111145D US Equity	0202445Q US Equity	0203524D US Equity	0226226D US Equity	0376152D US Equity	0440296D US Equity	0544749D US Equity	0574018D US Equity	0598884D US Equity	...
0	19930907	13.2719	13.6829	8.4429	8.1042	11.000	57.3245	17.8887	6.8315	28.1246	...
1	19930908	13.3263	13.5315	8.2147	7.9590	11.000	57.2096	17.8064	6.8315	27.5051	...
2	19930909	13.7070	13.3800	8.7852	8.0627	11.125	59.1625	17.6831	6.8315	27.7529	...
3	19930910	13.3807	13.4810	9.4127	8.0368	11.125	59.6220	17.6420	6.8773	27.5051	...
4	19930911	13.3807	13.4810	9.4127	8.0368	11.125	59.6220	17.6420	6.8773	27.5051	...

Dates

Stock price at marker close

Figure 4: The CSV file provided for analysis.

A crucial point of note is that the data was interlaced with many NaNs as shown in Figure 5. These NaNs reflect the fact that the S&P 500 is an index that is based on ranking. Resultantly, stocks will enter and exit the index as the rankings change throughout the years. These dispersed NaNs represented when a stock was not in the S&P 500 either because it had gone bankrupt or because it was no longer in the top 500 performing US companies.

...	YNR US Equity	YRCW US Equity	YUM US Equity	YUMC US Equity	ZBH US Equity	ZETHQ US Equity
...	NaN	3.35	114.02	45.31	134.50	NaN
...	NaN	3.35	114.02	45.31	134.50	NaN
...	NaN	3.18	114.10	45.43	134.53	NaN
...	NaN	3.18	113.24	44.00	136.67	NaN
...	NaN	3.18	113.24	44.00	136.67	NaN

Figure 5: Examples of NaNs in the data set

Some other remarks about the data are that the period covered a total of 9459 price entries and there was data on a total of 1199 stocks (1 column had the dates in it). This is shown by the screenshot in Figure 6 and what it means is that we had more than enough data to work with for our experiments. Additionally, if a stock price stays the same for multiple days in a row it is because the price data included non-trading days such as weekends.

```
stock_data.shape
```

```
(9459, 1200)
```

Figure 6: Shape of the Data

Data Cleaning

Although not performed at the start of the project due to changing algorithmic requirements, data cleaning did still make up a small part of the project. One such example is that although it is important to acknowledge when stocks leave and renter the S&P 500 when trading in the real world, this would grow complex if we factored it into our analysis as we do not have any data for the stocks that are not there. As a result, for much of our analysis, we would only look at the stocks that stayed in the index for the whole period we were looking at. As most high-performing companies stay high performing this would only reduce our pool of stocks to work with to around four hundred. Another bit of data cleaning that was performed throughout the project was changing the data such that the dates column replaced the indexes. This was especially vital for using the Python financial package pyportfolioopt as its functions required data to be input with datetime indexes.

Data Quality - Exploratory Data Analysis

As this was the first time working with stock data for much of the group, the main purpose of our EDA was to supply some statistical context to the data and supply insight as to how variable values ranged.

General Analysis

As mentioned, there are a lot of NaNs in the data, which shows that there will inevitably be some companies which may not be on the S&P 500 market at times. As a result, we agreed to investigate the performance of the stocks in smaller periods. To do this we first removed all rows in the data that were not in the range of 07/09/2000 to 06/10/2000. We then removed any columns with a NaN in them thus leaving us with all the data on stocks that were in the S&P 500 for the 30 days.

	Date	0111145D US Equity	0202445Q US Equity	0203524D US Equity	0226226D US Equity	0376152D US Equity	0544749D US Equity	0574018D US Equity	0772031D US Equity	0848680D US Equity	...	XL US Equity	XLNX US Equity	XOM US Equity	XRAY US Equity	XRX US Equity
2557	2000-09-07	21.7458	52.3221	21.5922	13.0700	6.3125	26.6109	16.7186	38.5419	23.6473	...	42.1552	63.7788	25.9731	10.3589	31.7959
2558	2000-09-08	22.7548	51.3936	20.8791	12.8673	6.3750	26.0178	16.8440	38.5419	23.0121	...	43.4136	62.0944	25.8956	10.9993	31.0478
2559	2000-09-09	22.7548	51.3936	20.8791	12.8673	6.3750	26.0178	16.8440	38.5419	23.0121	...	43.4136	62.0944	25.8956	10.9993	31.0478
2560	2000-09-10	22.7548	51.3936	20.8791	12.8673	6.3750	26.0178	16.8440	38.5419	23.0121	...	43.4136	62.0944	25.8956	10.9993	31.0478
2561	2000-09-11	22.9368	51.2298	21.3355	13.7188	6.5000	26.6366	17.3038	38.1466	23.2075	...	43.4922	62.0034	26.3547	10.7545	30.6737
2562	2000-09-12	22.8515	52.2129	19.8523	13.5431	6.5625	26.3788	17.1784	37.7513	22.4258	...	43.6889	59.0443	26.5933	10.2836	30.5490
2563	2000-09-13	22.2249	52.4314	19.8523	13.6917	6.6250	25.9146	17.0112	37.6524	23.1587	...	43.8461	61.4116	26.6127	10.3683	30.7984
2564	2000-09-14	21.8195	52.6498	20.1661	14.1918	6.5313	26.3530	16.9694	38.2952	22.4747	...	44.7113	60.8653	26.4443	10.5473	31.9206
2565	2000-09-15	22.3470	50.5744	19.5100	13.1916	6.7500	27.0750	16.7186	37.5046	21.6929	...	44.5933	58.8622	27.2911	10.4343	35.4120
2566	2000-09-16	22.3470	50.5744	19.5100	13.1916	6.7500	27.0750	16.7186	37.5046	21.6929	...	44.5933	58.8622	27.2911	10.4343	35.4120
2567	2000-09-17	22.3470	50.5744	19.5100	13.1916	6.7500	27.0750	16.7186	37.5046	21.6929	...	44.5933	58.8622	27.2911	10.4343	35.4120
2568	2000-09-18	22.2618	48.1713	19.0251	12.8132	6.2500	27.1008	16.3842	35.6269	21.5464	...	46.4022	55.9032	27.8011	10.3825	34.4144
2569	2000-09-19	21.5246	47.0244	20.3087	12.9213	6.6250	26.4304	16.7186	37.1583	22.0838	...	46.3236	61.5481	27.2911	10.6791	32.7935
2570	2000-09-20	21.6352	43.1466	20.1946	12.9483	6.9375	26.0951	16.6350	36.9117	20.9601	...	45.4191	63.2781	26.8259	10.7168	33.9157
2571	2000-09-21	21.1561	41.7812	17.5704	12.9754	7.6875	26.2756	16.5514	36.4665	20.1783	...	45.9303	61.8213	26.7678	10.9055	30.9231
2572	2000-09-22	21.1561	41.6720	19.1677	12.8402	7.6563	25.7599	16.5932	37.9987	19.4455	...	45.9303	61.5937	26.8259	10.7923	33.0428
2573	2000-09-23	21.1561	41.6720	19.1677	12.8402	7.6563	25.7599	16.5932	37.9987	19.4455	...	45.9303	61.5937	26.8259	10.7923	33.0428

Figure 7: A smaller subset of data for EDA

After cleaning the data into its smaller subset shown in Figure 7, we produced some graphs to explore the metrics involved in stock portfolio optimization. These histograms, shown in Figure 8, illustrate the distributions of volatility, return and Sharpe ratio of stocks. Although the return and volatility graphs supplied little information, the Sharpe ratio histogram supplies valuable insight that Sharpe ratios can realistically range anywhere from -4 to 4. Additionally, the distribution was positively skewed which is likely to be a result of inflation raising the price of stocks with time. Another important observation was that all volatilities were positive. This is valuable as volatility should always be positive and thus the results imply that the calculations were correct.

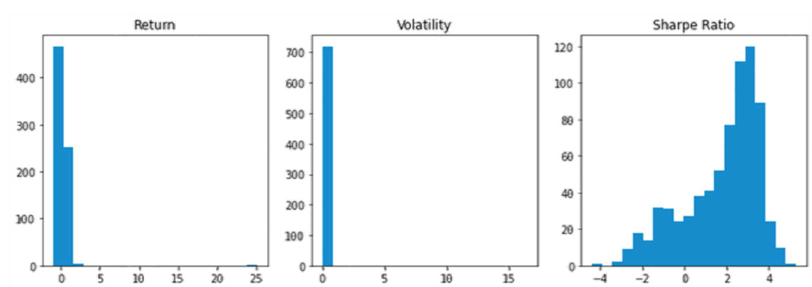

Figure 8: Histograms exploring variable distributions.

Another area of exploration ship was the relationships between stock return and volatility. We investigated this by creating the scatter plot in Figure 9 in which the graph shows a general which illustrates that return is often directly proportional to volatility. Furthermore, it shows the principle that to achieve high returns you need high volatility as although several stocks were seen to have low returns with high volatility, there are almost no stocks that have high returns and low volatility. This point is accentuated by all points living below the dotted line thus implying an upper bound on return for a given volatility.

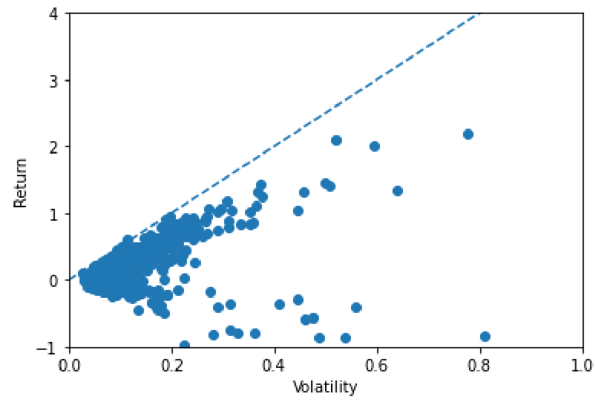

Figure 9: Scatter plot illustrating the relationship between stock return and volatility.

The next part that we explored was the correlation between stocks' Sharpe ratio and more specifically the correlation of the top twenty stocks' Sharpe ratio. What this shows is how diverse a portfolio consisting of the top twenty stocks by Sharpe ratio would be and thus illustrate how much the stock values rise and fall together. Ideally, the stocks in the portfolio should have low correlation to each other, so that if a stock falls in value, there will be some that rise or stay. As Figure 10 shows, this is not achieved by the sample of the top twenty stocks, as we see a high correlation between the stocks, with their maximum at 0.73. Therefore, we can conclude that just picking the top twenty stocks in terms of individual Sharpe ratio does not make an effective strategy further implying the importance of holistic approaches such as MPT.

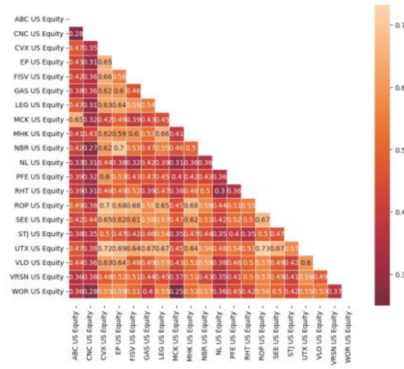

Figure 10: Heatmap showing the correlation between the top twenty stocks ranked by individual Sharpe ratio.

After that, we investigated the correlation of twenty random stocks' Sharpe ratios as shown by the heatmap in Figure 11. This yielded low correlations, especially compared to that of the top twenty stocks, with the highest correlation score being 0.55. This shows the potential of a random selection of stocks presenting a better result as low correlations between stocks in a portfolio is desirable.

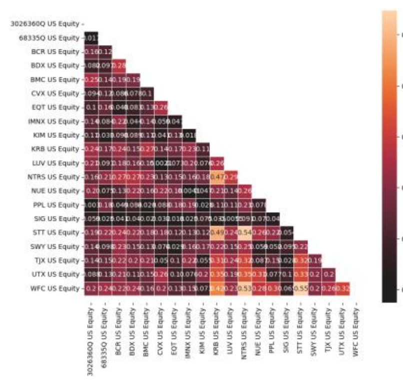

Figure 11: Heatmap showing the correlation between random stocks.

Efficient Frontier

We wanted to explore the relationship between volatility and returns and how it related to MPT given our data set using what is called a Monte Carlo Simulation. To do this we took ten random stocks from the S&P 500 data set and assigned max Sharpe ratio weights to the portfolio (Weights of stocks were adjusted to maximize portfolio. This process was then repeated one thousand times with the stocks in each portfolio being returned. The resultant graph is shown in Figure 12 where we can see how using an efficient frontier allows us to choose the most optimized portfolio.

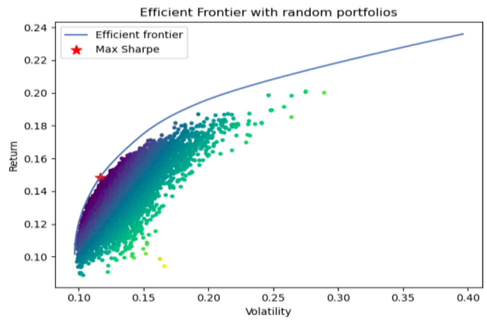

Figure 12: Graph showing an efficient frontier using the data provided.

Figure 12 illustrates that most of the portfolios derived from the S&P 500 perform well optimally as they tend to exist close to the frontier. Furthermore, there were a few outliers which drifted off to the right of the curve thus showing most portfolios were made up of stocks in the data set would perform well. This is likely to be a consequence of the S&P 500 being a very high-performing index due to the stocks included.

Model Development

Throughout the project, the package PyPortfolioOpt was of significant use and resulted in little modelling having to be completed. All weight optimization and portfolio choosing could be done through this package using Monte Carlo Simulations and thus the only work needed was to implement it into the trading strategies that would be tested. The implementation of each strategy varied individually, however, most involved creating the portfolio and then simulating its performance over the years, adjusting at regular intervals. The strategies chosen were designed to grow in complexity throughout the project to see if each addition was worth using. These strategies were.

Baseline Trading Strategy: Random Stock Choice with Equal Weights
This strategy was intended to simulate how an average person would trade and supply a point of comparison for other strategies. Stocks were chosen at random, and money was distributed equally across the stocks. At the end of the year, the portfolio value was then redistributed across the stocks.

(Code for this strategy in the jupyter notebook titled "Basic Strategy 4")

Strategy 1: Random Stock Choice with Weight Optimization
This strategy again involved stocks being chosen at random at the start of the simulation, however, at the end of each year the weights of the stocks in the portfolio were adjusted such that the portfolio's Sharpe Ratio was maximized.

(Code for this strategy in the jupyter notebook titled "Random Selection + Weight Adjustment Version 2")

Strategy 2: Expected Return Stocks Choices with Weight Optimization
This strategy involved choosing stocks to include in the portfolio by maximizing the highest expected return. Then weights were adjusted every 5 years using three different strategies. These strategies were: keeping weights equal, maximizing the portfolio's Sharpe ratio, and minimizing portfolio volatility.

(Code for this strategy in jupyter notebook title "AdvancedOptimisationStrategies")

Strategy 3: Sharpe Ratio Stocks Choices with Weight Optimization
This strategy was identical to strategy two, however, stocks were chosen to maximize their portfolio's Sharpe ratio rather than maximize its expected return.

(Code for this strategy in jupyter notebook title "AdvancedOptimisationStrategies")

Results

Baseline Strategy

The baseline strategy was assessed by simulating creating one hundred portfolios that applied the strategy and tracking them over a period of 01/01/1998 to 31/12/2018. As shown in Figure 13, the strategy had the potential to generate massive returns as shown by the few portfolios with final values above $20000, however, most portfolios had around 700% growth as shown by the high concentration of final values around this point. Furthermore, Figure 13 also shows a lack of stability in the strategy as the large spread of values shows that the strategy does not perform predictably, and the inconsistent and jagged portfolios imply that the portfolios the strategy creates are not very stable.

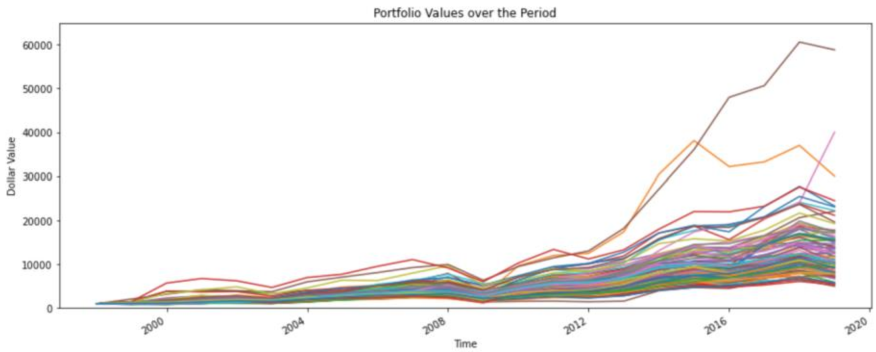

Figure 13: Performance of portfolios generated using the baseline strategy (Each line is a portfolio's value over time)

Portfolios generated with this strategy had an average return of 1177.11% of the original value and an average Sharpe Ratio of 0.81.

Strategy 1: Random Stock Choice with Weight Optimization

As strategy one involved the use of random processes, it was also tested by creating 100 portfolios and simulating their progress over the aforementioned period. As shown by Figure 14, this strategy had remarkably comparable results to the baseline strategy, however, some subtle differences will be discussed in the next section. Portfolios generated with this strategy had an average return of 1178.11% and an average Sharpe ratio of 0.93.

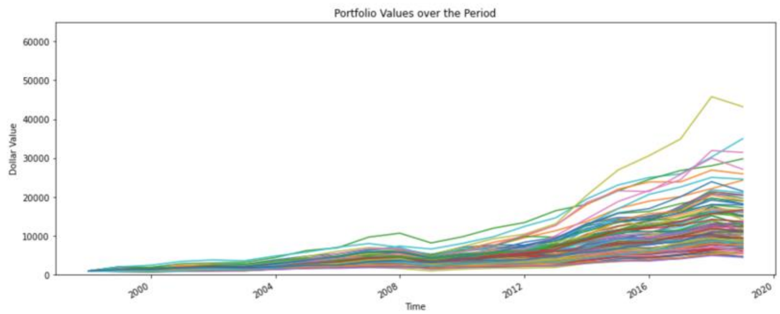

Figure 14: Performance of portfolios generated using strategy 1 (Each line stands for a portfolio's value over time)

Comparison of Strategy 1 to the Baseline Strategy

(Code for this comparison in jupyter notebook title “Comparison of Trading Strategy 1 to Baseline Version 1”)

With Strategy 1 and the Baseline strategy having similar average returns of 1177.11% and 1178.11% respectively, neither strategy outperforms the other when it comes to its ability to generate returns. The main difference, however, is in the stability of the two strategies. As shown not only in the differences between Figures 13 and 14 but also in Figure 15, the values of portfolios in Strategy 1 have a much tighter distribution at each point in time. This shows that the strategy performs more predictably.

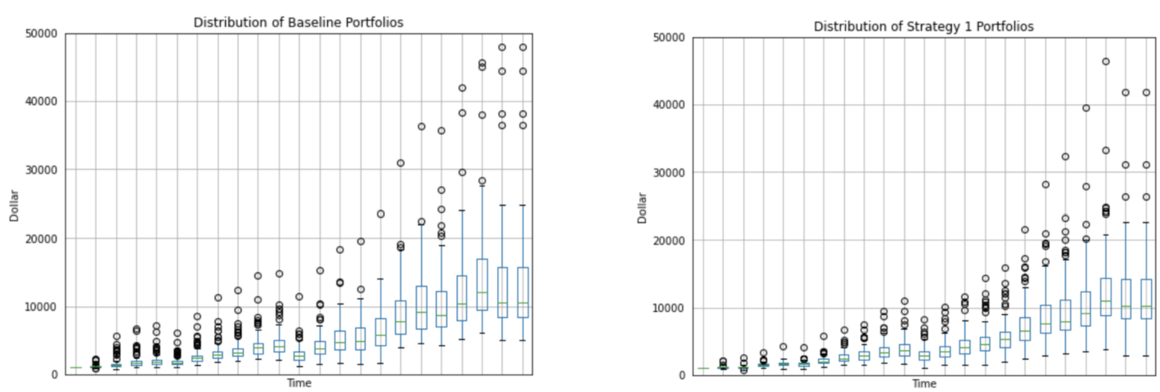

Figure 15: Side by Side comparison of each strategies distribution of values at each point in time

A probable cause of this higher predictability in strategy one is shown in Figure 16 which illustrates that portfolios generated by strategy one has significantly higher Sharpe Ratios. This fact in conjunction with the average returns of the strategies being similar shows that portfolios generated by strategy one is less volatile and less risky.

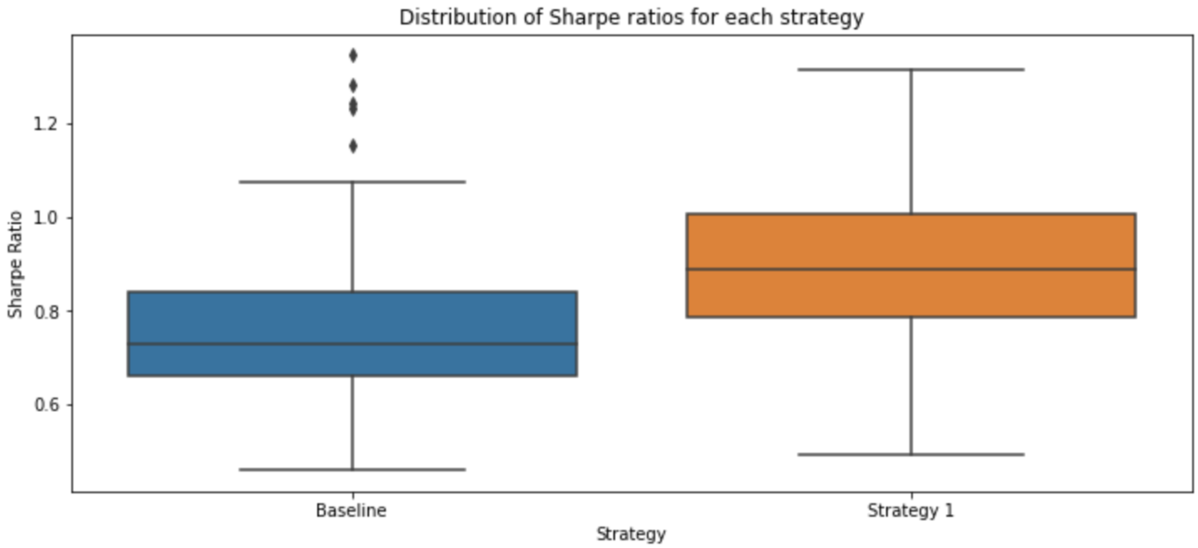

Figure 16: Comparison of distributions of portfolio Sharpe ratios across strategies 1 and 2

Main Findings:
Strategy one's main benefits over the baseline strategy were that it performed more predictably, and its portfolios had less risk involved all while keeping similar returns. Thus, it was successful in moving towards the goal of portfolio optimization. These findings are because if a portfolio is constantly tuned to maximize its Sharpe Ratio, the portfolio is going to in turn have a higher average Sharpe Ratio over its life span.

Side Note:
Returns did initially seem unrealistically high, however, not only was inflation not accounted for in the portfolios, but the data was already a bit biased. As stocks are chosen from a list of stocks that stay in the S&P 500 for the whole period, as this is the only data we had on them, they are already the best of the best when it comes to stocks. As the experiments were taken under the same conditions the comparison did not heavily on the comparison's merit.

Strategy 2: Expected Return Stocks Choices with Weight Optimization

As previously said, this strategy involved assessing a multitude of different weight optimization strategies while the stock choosing criteria remained constant. Each of these weighting strategies was applied every 5 years and the results for each weighting strategy are shown below:

- Equal Weights: The first weight optimization strategy was to have equal weights across the whole testing period. This strategy resulted in a **Sharpe ratio of 1.621** and a **final return of $5691.**
- Max Sharpe Weights: The second weight optimization strategy maximized the portfolio's Sharpe ratio. This strategy resulted in a high **Sharpe ratio of 1.842** and a **final return of $6140**.
- Minimum Volatility: The final weight optimization strategy was aimed to minimize the portfolio's volatility. This strategy resulted in a high **Sharpe ratio of 1.739** and a **final return of $4656**.

Figure 17: Performance of each weight optimization strategy

As illustrated by Figure 17 these results showed that optimizing weights for the largest Sharpe ratio resulted in the highest cumulative returns ($6137) and the highest Sharpe ratio (1.842). The equally weighted portfolio performed second best in terms of cumulative returns ($5691) and had a Sharpe ratio (1.621). Interestingly however, although the minimum volatility portfolio had the worst cumulative returns ($4656), it still had a higher Sharpe ratio compared to the equally weighted portfolio (1.739).

Trading Strategy 3: Sharpe Ratio Stocks Choices with Weight Optimization

This strategy was a continuation of the last, but instead of stocks being chosen for the portfolio by highest expected return, stocks were instead sampled based on Sharpe Ratio from 1998 to 2003 (The period just before the portfolio was tracked). Once again weights were adjusted every 5 years. The results for each optimization strategy were as follows:

- Equal Weights: This strategy resulted in a **Sharpe ratio of 1.495** and a **final return of $7034.**
- Max Sharpe Weights: This strategy resulted in a high **Sharpe ratio of 1.992** and a **final return of $5437.**
- Minimum Volatility: This strategy resulted in a high **Sharpe ratio of 1.659** and a **final return of $7041.**

(Optimization strategies were explained in the results of trading strategy 2)

Figure 18: Comparison of weight optimization strategies

Based on the results the comparison between each of the weight optimization strategies was not as clear cut as it was in strategy two. In terms of plain statistics, the equal weight ($7034) and minimum volatility ($7041) strategies performed similarly when it came to returning with Maximum Sharpe ratio weight adjustment ($5437) underperforming. Conversely, when it came to Sharpe Ratio, adjusting weights periodically to maximize the Sharpe ratio was the best strategy with a very respectable Sharpe ratio of 1.992. This in conjunction with Figure 18's illustration that Maximum Sharpe ratio weight adjustment was competitive when it came to returns for most of the period means that maximum Sharpe ratio weight adjustment performed the best when it comes to optimization stock portfolios.

Comparing Strategies 2 and 3 and their various Weight Optimization Methods

(Code for this comparison in jupyter notebook title "AdvancedOptimisationStrategies")

Figure 19: Comparing all permutations of stock choice and weight adjustment strategy.

Overall, all the strategies performed well over the period, however, Figure 19 illustrates that some of these strategies did perform significantly higher than others. The most prominent example of this is the strategy that chose stocks to achieve the highest portfolio Sharpe ratio and then adjusted weight to minimize total portfolio variance. As shown by the cyan-colored line in Figure 19, this portfolio grew very steadily and predictably, seeming unfazed by events that caused major drops in the value of other portfolios. This predictability while keeping high rewards means it was especially effective at meeting our goals of maximizing return and minimizing risk. Moreover, it was also worth noting that the strategy that picked weights and stocks of maximum Sharpe ratio performed the highest when it came to our metric for successful optimization (Sharpe Ratio) with a remarkably high score of 1.992.

Main Finding:
The main finding of this comparison was that the most effective strategies for creating optimal portfolios were adjusting weights to maximize Sharpe Ratio while choosing stocks by maximizing the portfolio's Sharpe Ratio and adjusting weights to minimize volatility while choosing stocks by maximizing the portfolio Sharpe Ratio. Furthermore, risk-averse investors should use the latter strategy as it was incredibly predictable while those who are comfortable with more risk should use the former as it was the most optimal by our metrics. The success of these strategies also implies that overall choosing stocks by maximizing the Sharpe ratio is more effective than choosing them to maximize expected return, however, nothing concrete was obtained to show this claim.

Comparing our strategies to an ETF

As investing can be quite a complicated endeavor, many investors opt to put their money into ETFs as they can supply reliable returns. As a result, comparing the best strategy generated to a high-performing ETF would answer the question of: "Is all this work worth it?" The ETF that we made this comparison to is an S&P 500 trust ETF and after putting $1000 in for the same period that had been used for earlier testing the final value of the portfolio was $3722.74 as shown in Figure 20.

Figure 20: Performance of a similar ETF fund over the same testing period

When compared with the investment strategies assessed in this report, investment strategies the ETF significantly underperformed all strategies. More importantly, the most optimized strategy that was assessed, Maximum Sharpe ratio weights and stock choices, had a final value of almost 1.5x the ETF while keeping a respectable Sharpe ratio of 1.992. Furthermore, similar reactions to exogenous events such as the global financial crisis implied that the tested trading strategy was not significantly more volatile than the ETF.

This finding seems a bit suspicious as it is unrealistic for some second-year data science students to outperform an ETF, however, there are several reasons this may occur. Firstly, there are certain costs such as taxes and management costs involved in the real world that we have not included in our experiment. These costs, however, are reflected in the value of the ETF as rewards are diluted and losses are worsened. Secondly, all the stocks used in the earlier experiments were not only chosen from an index of high-performing stocks, but they were also chosen from the even higher-performing subset of these stocks that stayed for the entire period. As ETF managers do not have this fortune-telling ability, they were at a disadvantage in this comparison. As a result of these two factors, the comparison between the two investment strategies is not a fair one, however, these factors do not affect the validity of other comparisons throughout this report.

Main Finding:
The comparison between our trading strategy and an ETF is inconclusive due to the lack of fairness in the experiment. The strategy wildly outperformed the ETF and thus it may be worth using these strategies rather than investing in a similar ETF.

Conclusion:

In conclusion, this report was able to achieve its aim of testing if modern trading strategies can supply significant improvements to portfolio performance and aid in achieving stock portfolio optimization through several experiments and strategies. In doing this the hypothesis that strategies assessed would not supply exceptional and ground-breaking benefits was disproved by the report. To elaborate on this, several strategies significantly outperformed the baseline point of comparison with the most effective strategy for portfolio optimization, choosing stocks and regularly adjusting weights to maximize the portfolio's Sharpe ratio. This strategy's Sharpe ratio of 1.992 meant that it performed incredibly, and the effort taken in implementing this strategy was worth the hassle and this report recommends that this strategy is used. Furthermore, although no trading strategy was able to outperform the baseline in terms of average return every strategy outperformed it in terms of Sharpe ratio thus highlighting that when implementing these advanced trading strategies, the main benefit is low volatility and predictable performance rather than high returns.

As discussed in the comparison between the best strategy and an ETF this experiment inherently had some flaws due to working around the data provided. In future experiments, we may want to look at a larger data set to broaden our analysis of these trading strategies' effectiveness. Furthermore, strategies such as altering investments in reaction to market events and varying parameters such as portfolio size and investment frequency should be included by investigator in further experimentation.

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