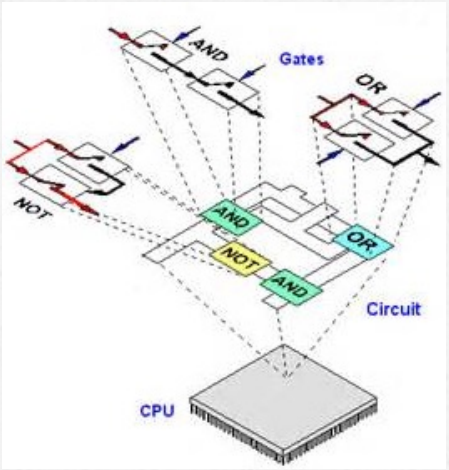
COMP1003 Computer Organization

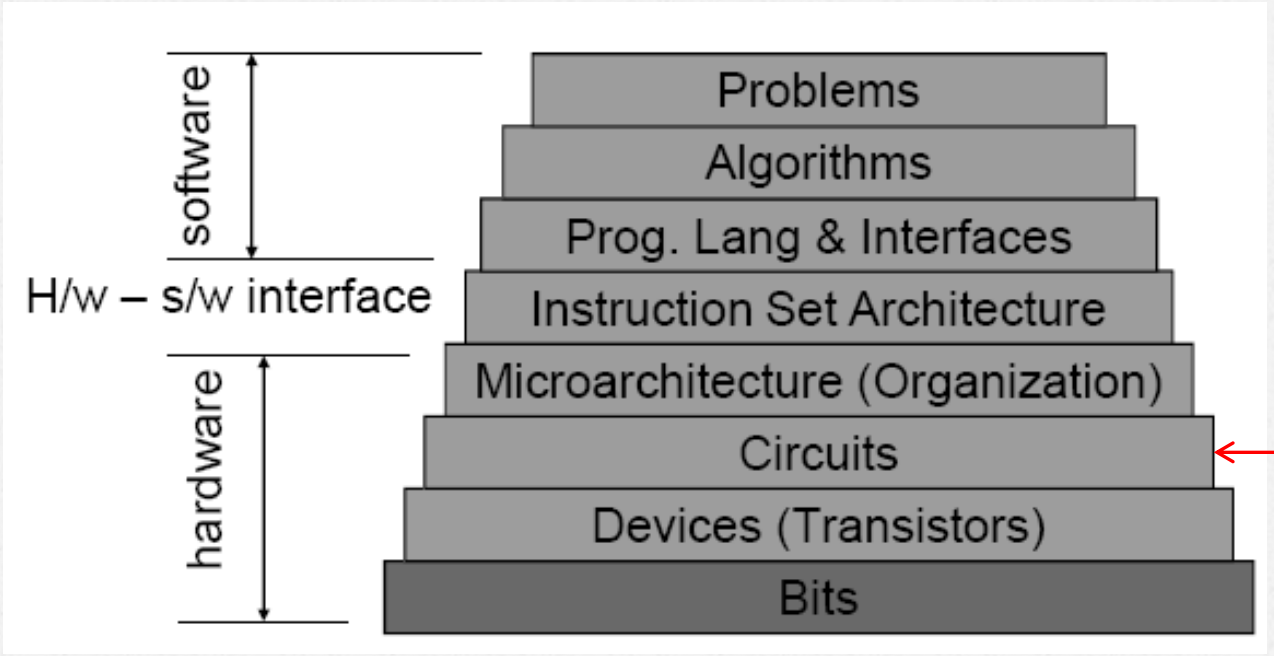
Lecture 5 From Gates to Circuits I: Combinational Circuits



United International College

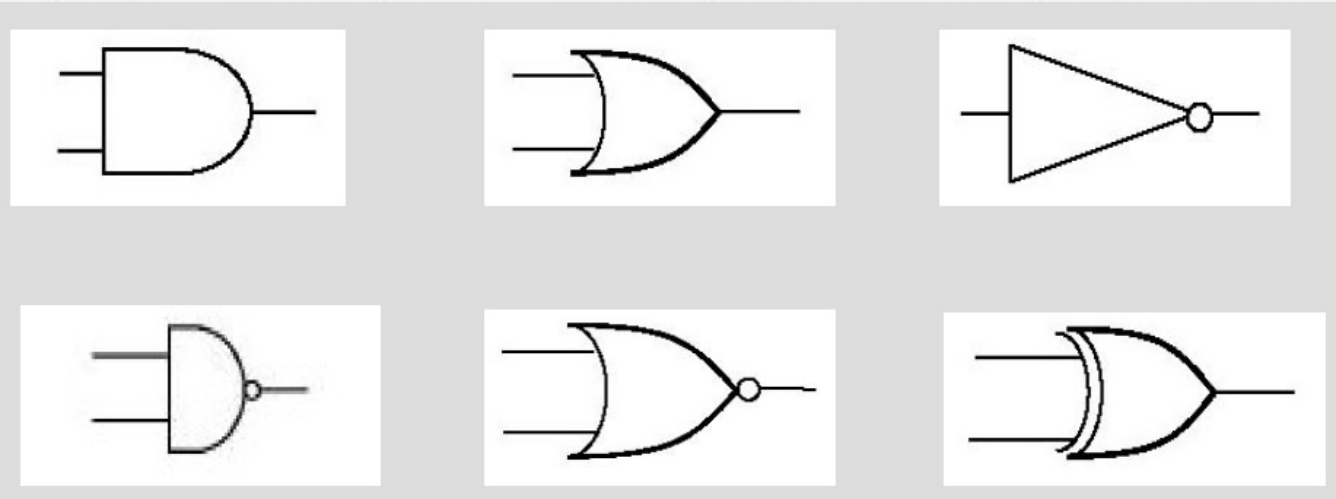
From Gates to Circuits

- How to use logic gates to build more complex and more powerful circuits



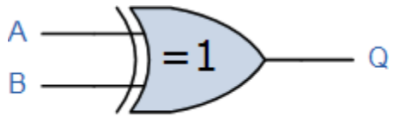
Basic Building Blocks

- A gate is a small, electronic device that computes various functions of two-valued signals



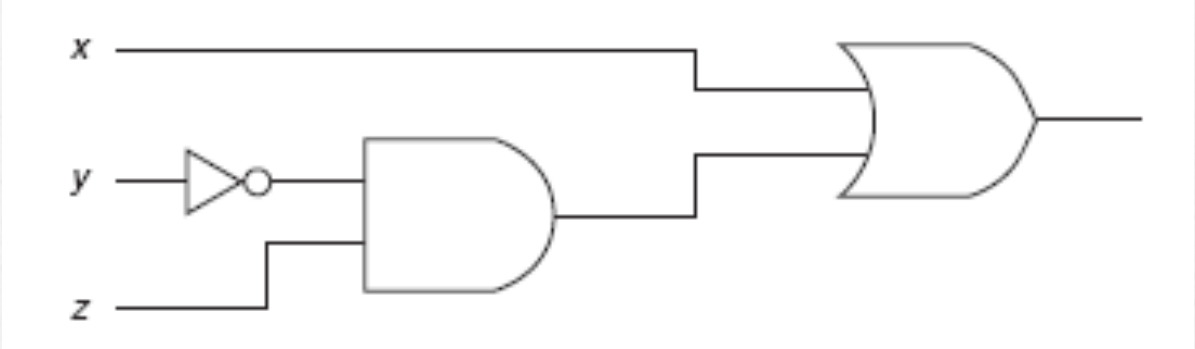
Representations of Boolean Functions

- 3 representations logically equivalent to one another
  1. Truth table
  2. Logic expression
  3. Logic circuit

Symbol	Truth Table		
 2-input Ex-OR Gate	A	B	Q
	0	0	0
	0	1	1
	1	0	1
	1	1	0
Boolean Expression $Q = A \oplus B$			

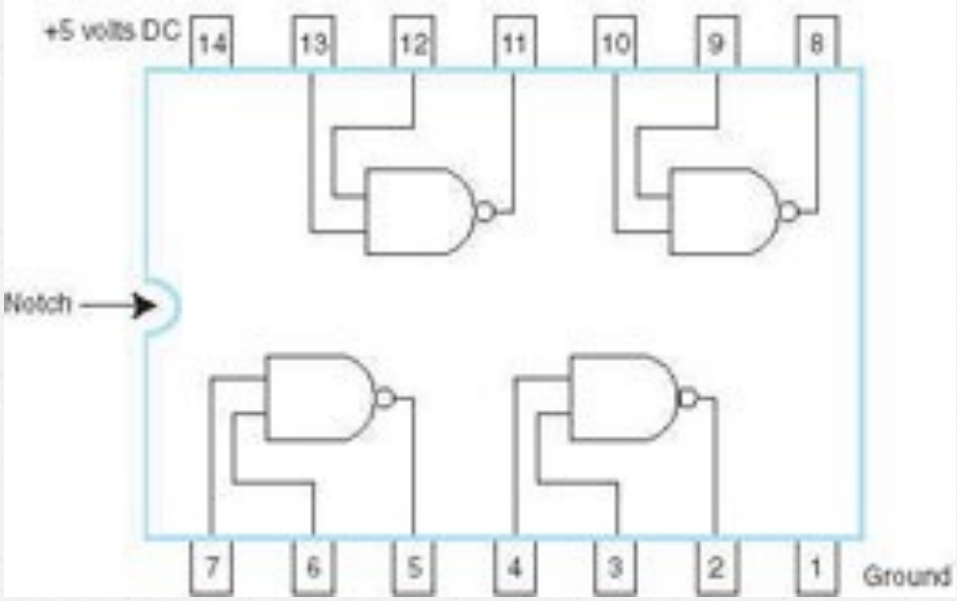
Logic Circuits

- A collection of gates connected together to implement Boolean functions
- Which Boolean function does the following circuit implement?



Integrated Circuits (IC)

- The integration of large numbers of tiny transistors into a small chip



Two Types of Circuits

- Combinational logic circuits
  - Its output depends solely on its current input.
  - No storage, memoryless
- Sequential logic circuits
  - Its output depends not only on its current input, but also on its current state (previous input)
  - Can remember previous input, storage devices

Combinational Circuit

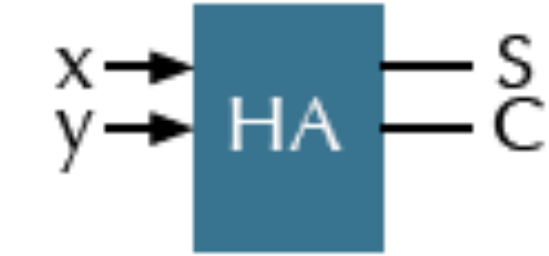
- Circuit with no memory
- Multiple inputs, multiple outputs, one Boolean function for each output
- Abstracted as a package or a black box
- Implementation described by one of
  - n-input-, m-output-column truth table
  - boolean function for each output variable
  - logic diagram (possibly using other packages)

Binary Addition

- How to build a circuit to perform binary addition?
- We start with the simplest case: adding two single bits
- The circuit is called half adder

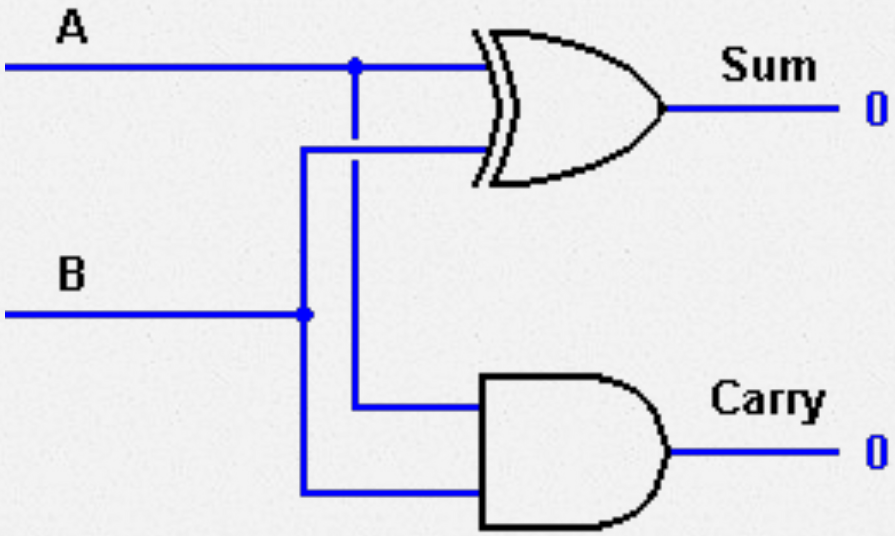
Half Adder

- Only add two single bits together (a, b), s is the sum and c is the carry bit
- Logic Expression?
- Logic Circuit?



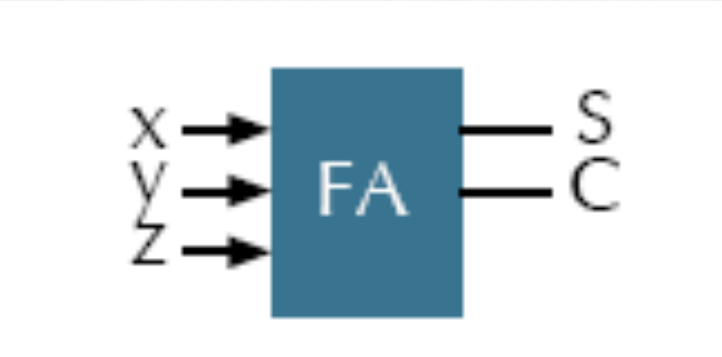
a_i	b_i	c_{i+1}	s_i
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Half Adder Circuit



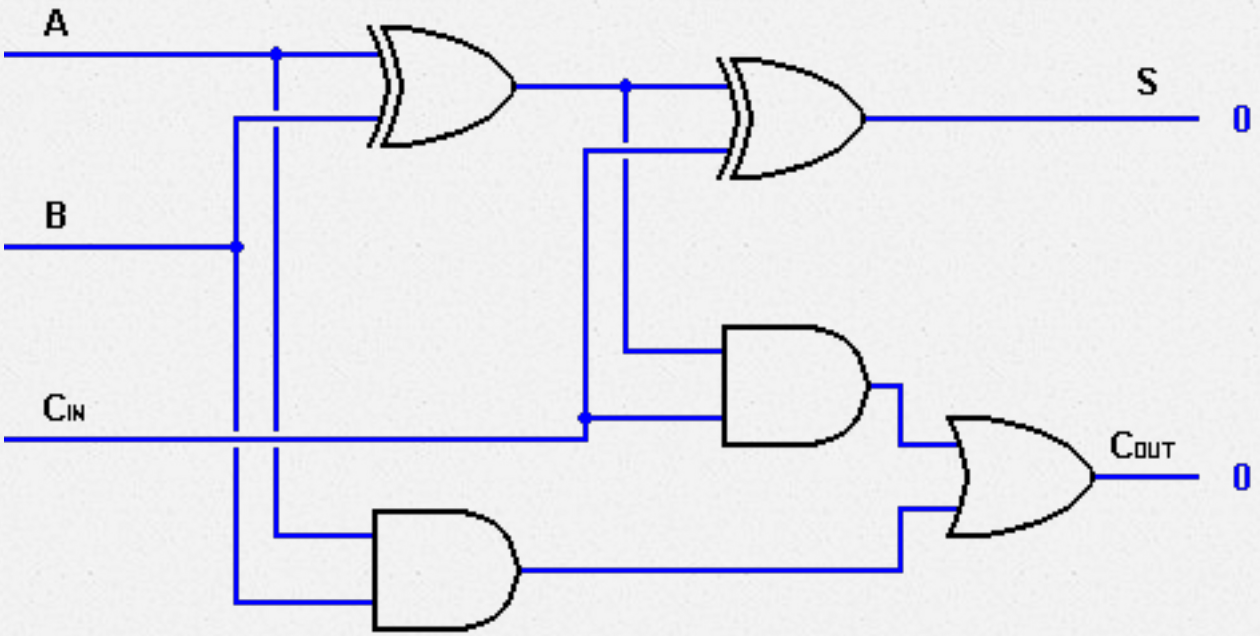
Full Adder

- Add 3 bits together (A, B, Carry-in)
- Logic expression?
- Logic circuits?



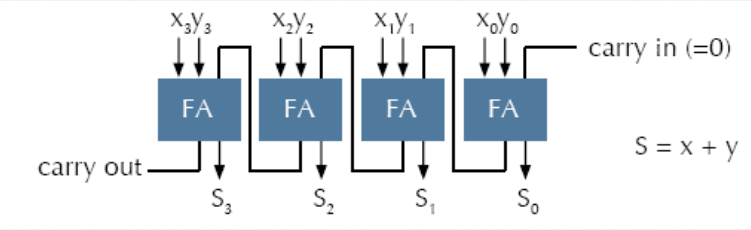
A_i	B_i	C_i	C_{i+1}	S_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

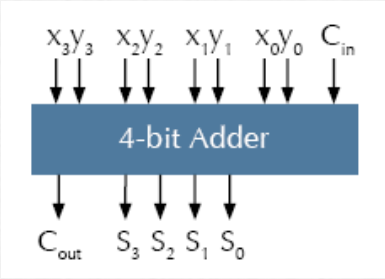
Full Adder Diagram



4-bit Adder

- Full adder only adds three bits together
- But we can build an adder capable of adding two 4-bit words by replicating the above circuit 4 times
  - feeding the Carry Out of one circuit into the Carry In of the circuit immediately to its left
- This is called a “ripple-carry adder” (very slow!)



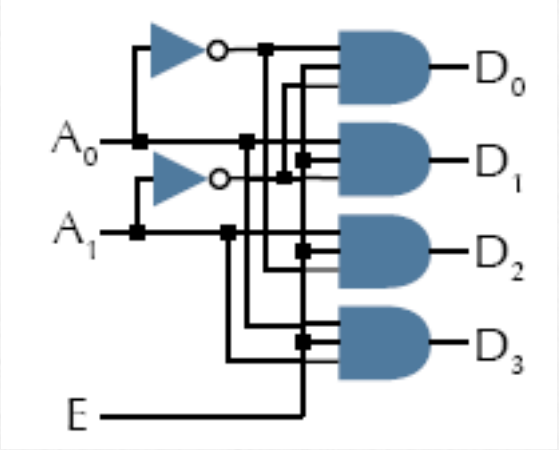


Decoder

- A decoder uses the inputs and their respective values to select one specific output line.
  - – from a set of n inputs to a maximum of 2^n outputs
  - – for a given input, one unique output line is asserted, or set to 1, while the other output lines are set to zero
- All memory addresses in a computer are specified as binary numbers.
  - When a memory address is referenced (whether for reading or for writing), the computer first has to determine the actual address.
  - This is done using a decoder

A 2-4 Decoder

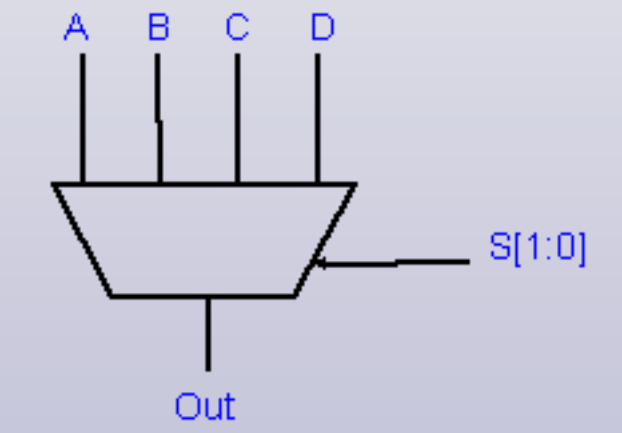
E	A_1	A_0	D_0	D_1	D_2	D_3
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
0	x	x	0	0	0	0



What are the logic expressions?

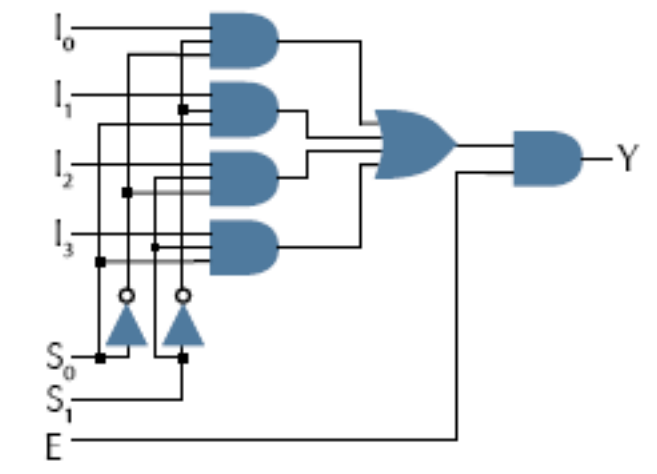
Multiplexer

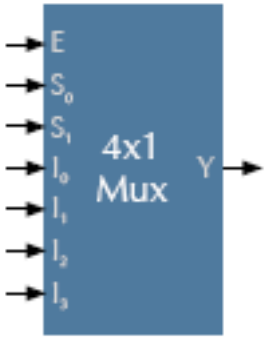
- A multiplexer behaves like a channel selector
- It selects a single output from several inputs.
- The particular input chosen for output is determined by the value of the multiplexer's control lines.
- To be able to select among n inputs, $\log_2 n$ control lines are needed.



4-to-1 Multiplexer

- What are the logic expressions?

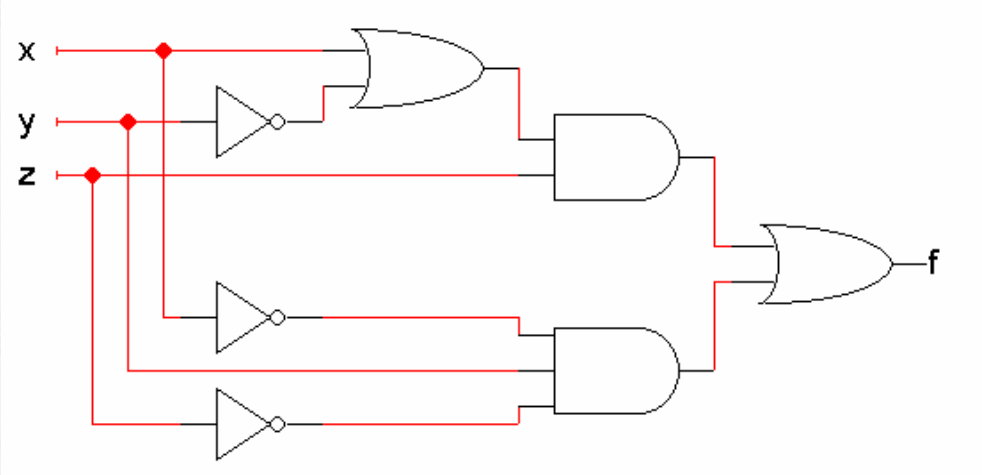




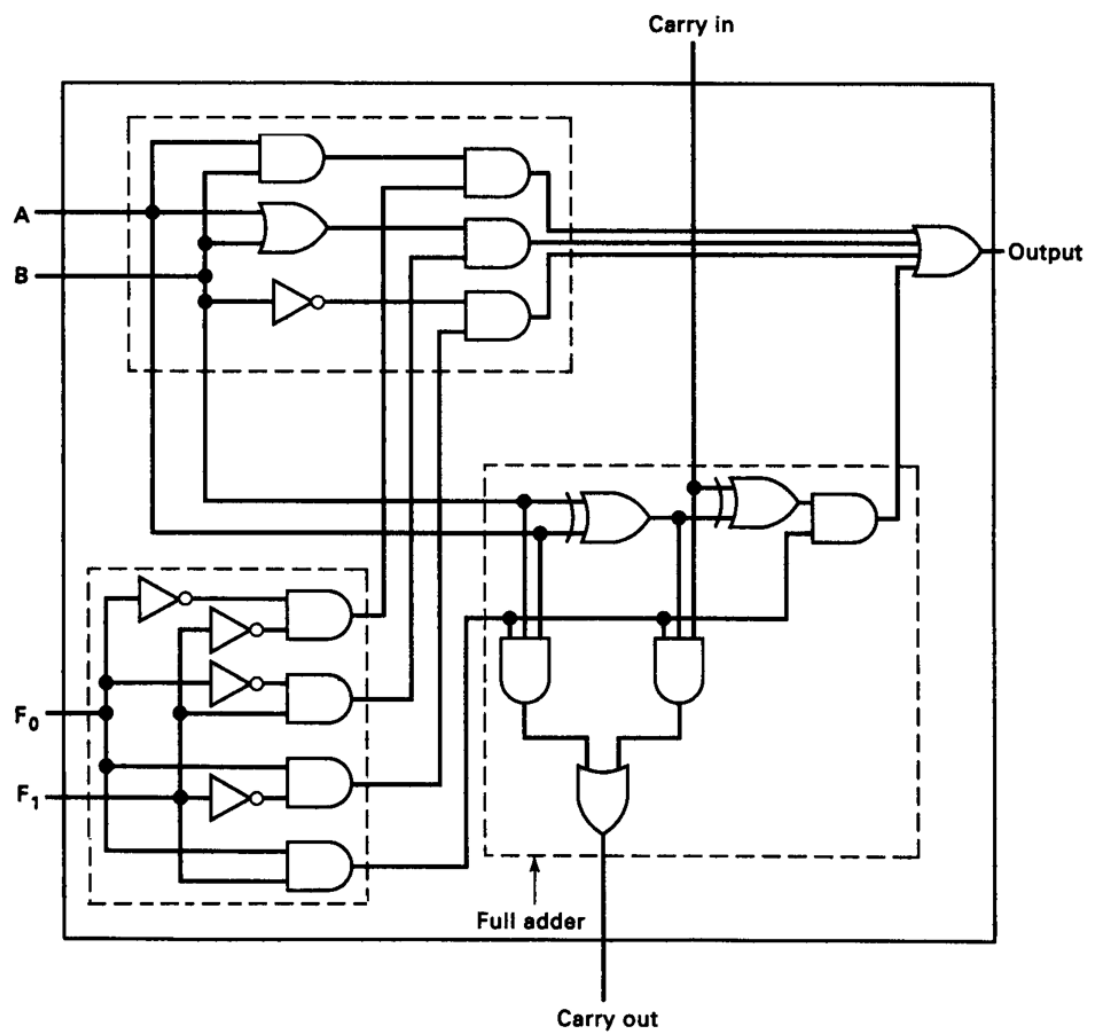
E	S_1	S_0	Y
1	0	0	I_0
1	0	1	I_1
1	1	0	I_2
1	1	1	I_3
0	x	x	0

Combinational Circuit Analysis

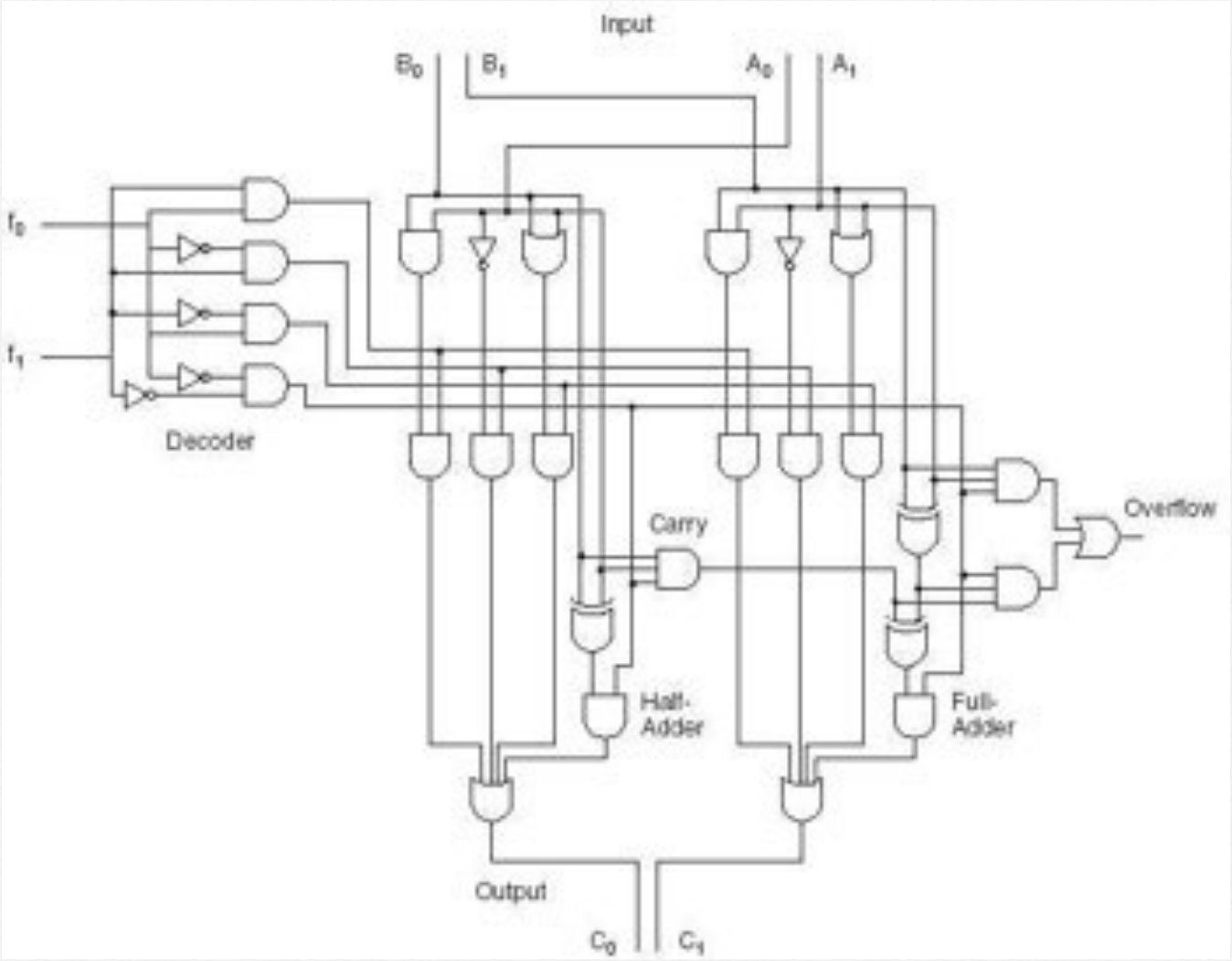
- find the logic function that the following circuit implement



1-bit ALU



2-bit ALU


Input
B_0
B_1
A_0
A_1
f_0
f_1
Decoder
Carry
Overflow
Half-
Adder
Full-
Adder
Output
C_0
C_1

Combination Circuit Design

1. From the design specification, obtain the truth table
2. From the truth table, derive the Sum of Products Boolean Expression.
3. Use Karnaugh Map to minimize the Boolean expression.
4. Use logic gates to implement the simplified Boolean Expression.
5. Verify the result.

Combination Circuit Design

- design a circuit that compares 1-bit number, A and B. The circuit should have
- three outputs:
  - – G (“Greater”) should be 1 only when A > B
  - – E (“Equal”) should be 1 only when A = B
  - – L (“Lesser”) should be 1 only when A < B

Truth Table

- How many inputs?
- How many outputs?
- Fill in the truth table

Inputs		Outputs		
B	A	A > B	A = B	A < B
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

Derive the Logical Expression

- Write the logical expression for each output in canonical forms (SOP or POS)
- Simplify the logical expression

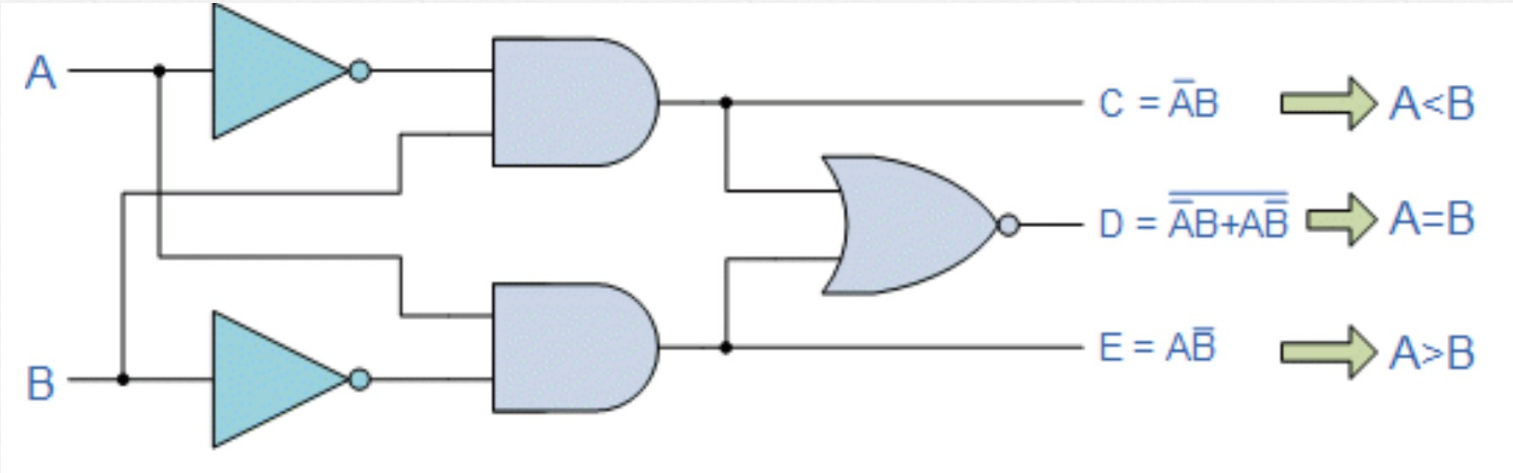
$C = \bar{A}B \Rightarrow A<B$

$D = \overline{\bar{A}B + A\bar{B}} \Rightarrow A=B$

$E = A\bar{B} \Rightarrow A>B$

Inputs		Outputs		
B	A	A > B	A = B	A < B
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

Draw the Logical Diagram



Summary

- Combinational Circuit
  - No memory
  - Basic building blocks
  - Abstracted as package
- Examples
  - Half adder
  - Full adder
  - Decoder
  - Multiplexer
  - ALU
- Combinational Circuit Analysis & Design