

# Machine Learning (DS4023) Assignment 3

Deadline: Nov. 11, 2024.

## Problem 1: Hard-margin SVM. (18 pts)

You are given the following two sets of data points, each belonging to one of the two classes (class 1 and class -1):

- Class 1 (labeled as +1):

$$(1, 2), (2, 3)$$

- Class -1 (labeled as -1):

$$(2, 1), (3, 2)$$

Please find the optimal separating hyperplane using a linear SVM and derive the equation of the hyperplane. Assume the hard-margin SVM.

1. Write down the formulation of SVM, including the separation hyperplane, the constraints and the final optimization problem with parameters. (4 pts)
2. Write down the Lagrangian form for this problem using the parameters and Lagrange multipliers. Please also write out its dual form. (10 pts)
3. Assume that the Lagrangian multipliers  $\alpha_i$ 's are all 0.5 and that the point (1, 2) is a support vector for ease of calculation. Please calculate the values of weight vector  $w$  and bias  $b$ . Write out the explicit form of the hyperplane. (4 pts)

## Problem 2: Soft-margin SVM. (20 pts)

Suppose we have the data points  $x \in \mathbb{R}^{n \times d}$  with corresponding labels  $y \in \mathbb{R}^n$ . We want to use a soft-margin SVM to classify these data points with a regularization parameter  $C = 1$ .

1. Write down the formulation of soft-margin SVM for this problem using  $w, x, y, b$  and  $\xi$ . Write out explicitly their dimensions. (3 pts)
2. Write down the Lagrangian form and derive the dual for the problem. Write down the detailed derivation steps. (12 pts)
3. Obtain the decision boundary. (3 pts)
4. Explain why  $\xi$  disappears in the dual. (2 pts)

## Problem 3: Kernel SVM. (17 pts)

Consider the following 2D dataset with four training points:

$$\mathbf{x}_1 = (1, 2), \quad y_1 = 1$$

$$\mathbf{x}_2 = (2, 3), \quad y_2 = 1$$

$$\mathbf{x}_3 = (3, 1), \quad y_3 = -1$$

$$\mathbf{x}_4 = (4, 3), \quad y_4 = -1$$

We want to use the **polynomial kernel**  $k(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i^\top \mathbf{x}_j + 1)^2$  to classify these points with a soft-margin SVM. The regularization parameter  $C = 1$ .

1. Compute the kernel matrix  $K$ . **(6 pts)**
2. Set up the dual optimization problem. You can use the results from Problem 2. **(4 pts)**
3. Suppose the Lagrange multipliers  $\alpha$ 's are

$$\alpha_1 = 0.0182, \quad \alpha_2 = 0.0068, \quad \alpha_3 = 0.0250, \quad \alpha_4 = 0,$$

and  $\mathbf{x}_3$  is a support vector. Please compute the bias term  $b$ . **(2 pts)**

4. Classify a new point  $\mathbf{x}_5 = (2, 1)$  using the learned kernel SVM model. **(5 pts)**

## Problem 4: Programming (45 pts)

Complete the jupyter notebook attached on programming for ensemble learning and SVM. Submit the completed file.

### To submit:

1. A file containing written answers to the Problems 1 – 3.
2. The Jupyter notebook with solutions.