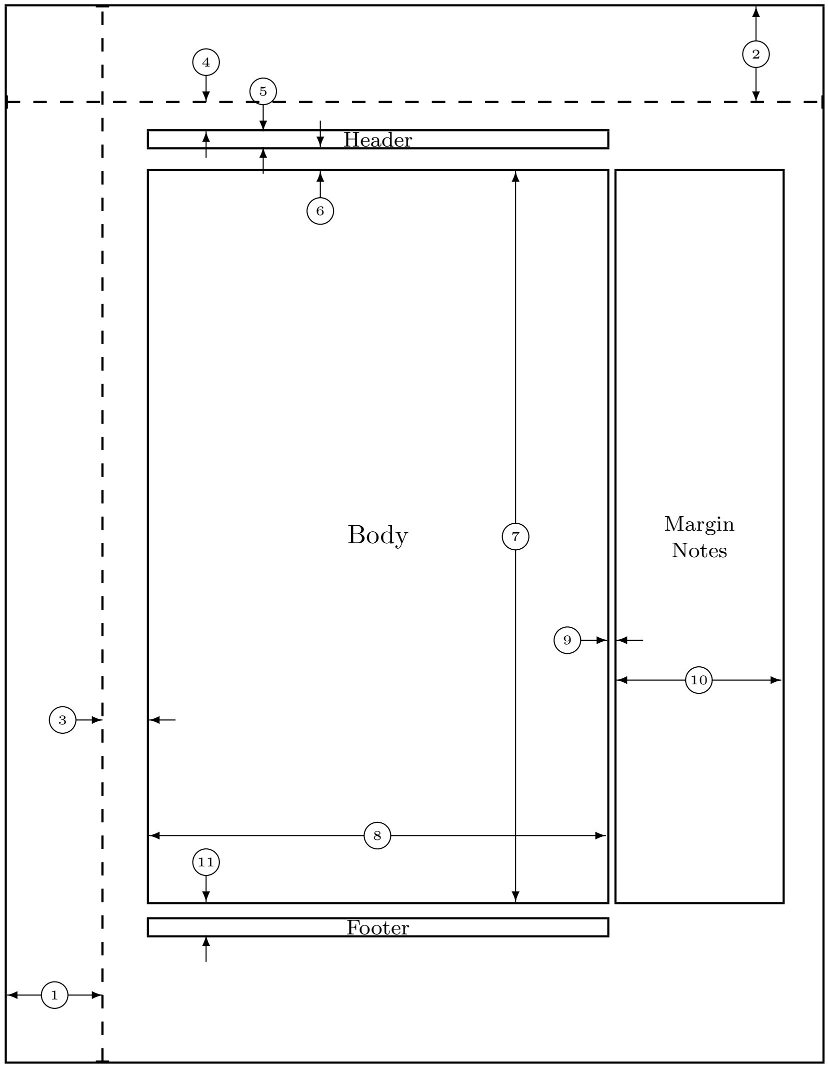
Residues and duality

June 22, 2023

Contents



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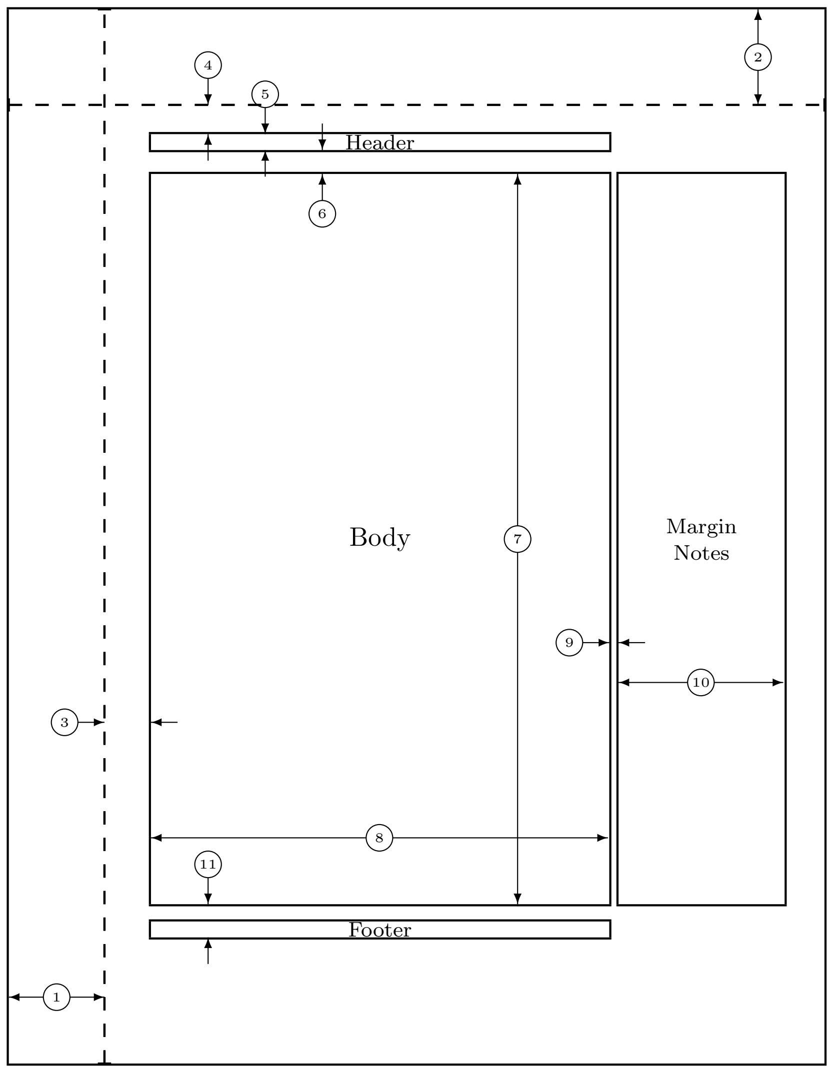

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Chapter 1

Introduction

The main purpose of these notes is to prove a duality theorem for cohomology of quasi-coherent sheaves, with respect to a proper morphism of locally noetherian preschemes. Various such theorems are already known. Typical is the duality theorem for a non-singular complete curve X over an algebraically closed field k, which says that

$$h^0(D) = h^1(K - D),$$

where D is a divisor, K is the canonical divisor, and

$$h^i(D) = \dim_K H^i(X, L(D))$$

for any i, and any divisor D. (See e.g. [16, Ch. II] for a proof.)

Various attempts were made to generalize this theorem to varieties of higher dimension, and as Zariski points out in his report [20], his generalization of a lemma of Enriques-Severi [19] is equivalent to the statement that

Bibliography

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[16] Jeqn-Pierre Serre, *Groupes algébriques et corps de classes*, Paris, Herman (1959).