The Political Economy of Entry Barriers*

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Abstract

We study a political economy model of entry barriers. Each period the policymaker determines whether to impose a high barrier to entry, and the special interest groups try to influence the policymaker's decision. Entry is accompanied by creative destruction—when many new firms enter, old firms are more likely to be driven out of the market. Therefore the current incumbents (industry leaders) tend to lobby for a higher entry barrier and potential entrants (industry followers) are likely to lobby for a freer environment for entry. We analyze both static and dynamic versions of the model to examine what kind of environment supports a policy that blocks entry. In the dynamic model, the economy can exhibit various different dynamics. In particular, multiple steady states may arise in equilibrium.

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1 Introduction

This paper studies a political economy model of entry barriers. We investigate the evolution of industries when incumbents lobby for high entry barriers in order to reduce firm turnover. The analysis of entry barriers has a long history in the industrial organization and the law and economics literature.[1] We focus on a particular subset of entry barriers—barriers that are imposed by government policies. Examples include legal procedures and monetary costs required when starting a new firm, documented by de Soto (1989, 2000) and Djankov et al (2002).[2] Related to this, Rajan and Zingales (2003) argue that financial development in many countries has stagnated because large industry incumbents (and incumbent financiers) with political power opposed it in order to restrict competition. In their view, the industry incumbents want to keep financial costs high, so that the high costs can act as entry barriers. Another example is patent policy, which is often described to be influenced by lobbying.[3] These policies change over time as the economy evolves, and analyzing these policy changes calls for a dynamic framework. In this paper, we take a particular policy determination rule (a lobbying game by special interest groups) as given, and analyze how policies regarding entry barriers change over time as the economy evolves.

Many recent studies in the industrial organization literature suggest that the entry process contributes significantly to an industry's productivity growth. As an extreme example, Foster et al (2006) find that almost all the labor productivity growth in the U.S. retail trade sector in the 1990s is accounted for by more productive entering establishments replacing less productive exiting establishments.[4] In macroeconomics, the process of creative destruction has been at the center of the endogenous growth literature.[5] Recent model-based

[1]See, for example, McAfee et al (2004) for the history of concepts. Tirole (1988, Chapter 8) is the textbook treatment of the entry and exit behavior in industrial organization literature.

[2]The World Bank provides more recent updates (see `http://www.doingbusiness.org/`). It documents entry barriers such as entry fees, procedures to obtain permits, requirements for government-provided licenses, and minimum capital requirements.

[3]See Boldrin and Levine (2008) for various episodes.

[4]Using the Census of Manufactures data, Foster et al (2001, Table 8.4) attribute about half of the multifactor productivity growth in the U.S. manufacturing sector during 1977-1987 to the reallocation of production resources across plants. In particular, 26% of productivity growth is due to the entry and exit of plants during this 10 year period.

[5]See, for example, Grossman and Helpman (1991) and Aghion and Howitt (1992). Parente and Prescott

macroeconomic studies also deliver similar messages: Barseghyan and DiCecio (2011) and Moscoso Boedo and Mukoyama (2012) suggest that high entry costs contribute significantly to lowering productivity in poor countries.

This paper builds a simple political economy model of policy choice in an economy that is characterized by creative destruction. In building our model, we draw our inspiration from Olson (1982). Analyzing the behavior of special interest groups, he argues that "It would be in the interest of these groups that are organized to increase their own gains... This would include choosing policies that, though inefficient for the society as a whole, were advantageous for the organized groups because the costs of the policies fell disproportionately on the unorganized" (p. 37). As a result, a socially undesirable policy can be chosen due to the influence of the special interest groups. We model the determination of the policy on firm entry regulations as a special case of his analysis. In our model, there are two special interest groups, or "coalitions," who typically have opposite policy preferences. One is the group of incumbent firms who do not want to be driven out of the market by new entrants, and thus favor a policy that makes entry difficult. The other is the group of the potential entrant firms who want a policy that allows them to enter easily. We do not model consumers—they are "the unorganized" who suffer from the cost of low productivity or lack of new products when a policy that is detrimental to new entry is selected.[6]

A recent work by Acemoglu and Robinson (2012) expresses a related view. They argue that a society which is ruled by a narrow elite who pursue their own benefit at the expense of the mass of the people tends to stagnate. They call such an organization (rules) of society "extractive institutions." It is contrasted with "inclusive institutions," which allow

(1999) also share our view that the entry of new firms with new technology is essential in growth and development and thus barriers to entry can cause stagnation. Although our model in the main text does not have endogenous growth, Appendix A presents a version of our model with general equilibrium and endogenous growth. There we show that inhibiting creative destruction results in some welfare costs. Note that in the context of the innovation-based growth models, it is not always the case that promoting entry (by weakening the patent policy or providing entry subsidies) enhances growth, because monopoly rights can provide incentives for innovation. See, for example, Mukoyama (2003) and Aghion et al (2005) for detailed discussions. The implications for welfare is also complex, since these endogenous growth models typically involve both static and dynamic distortions.

[6] The growth model in Appendix A explicitly consider consumers and their welfare.

the participation of the mass of the people in economic activities. Inclusive institutions tend to promote entry of new businesses and creative destruction, while under extractive institutions, innovative new technologies are often blocked due to political opposition. They write "The fact that they [the elites] have much to lose from creative destruction means not only that they will not be the ones introducing new innovations but also that they will often resist and try to stop such innovations" (pp. 183–184). Furthermore, similarly to our own view, they emphasize the role of organized groups: "It is also necessary to consider more broadly the factors that determine how political power is distributed in society, particularly the ability of different groups to act collectively to pursue their objectives or to stop other people from pursuing theirs" (pp. 42–43).

The contribution of our paper is to propose a tractable model that treats both the distribution of political power and the policy regime as fully endogenous and dynamic (while treating the nature of the political process as given). We highlight the utility of this framework in two cases. First and most importantly, under some parameter values there are two possible long-run steady states: one with low incumbent power and lots of entry and exit and another with high degree of incumbent power and little turnover. This is a potential explanation for the observed variety of entry regulations around the world. Second, we show that for some parameter values the political power of incumbents increases over time, which leads to economic policies more favorable to existing firms, reinforcing the process. Under this situation, even an economy with a large amount of entry can eventually switch to a regime with high barriers to entry and little turnover, similarly to the postwar experience of the economy of the United Kingdom.

We build on the framework of menu auctions by Bernheim and Whinston (1986) to model the policy determination in an environment with special interest groups. In characterizing our model, we utilize Bergemann and Välimäki's (2003) dynamic extension of Bernheim and Whinston (1986). The menu auction framework is widely used in the context of international trade policy—for example, in Grossman and Helpman (1994), special interest groups lobby for trade policies. In this paper, we start from a static model that is a direct application of the Bernheim and Whinston (1986) framework, and then contrast the results with the dynamic model. In applying Bergemann and Välimäki's (2003) results, we have to make modifications due to the difference in settings. In particular, in our model, the state space is continuous. In Appendix C, we show the existence of an equilibrium in a discrete-state setting and then show that it converges to the equilibrium with continuous state space by making the state space finer and finer. While this extension may be of independent interest, we emphasize that our main contribution is the *application* of the theory of menu auctions to our specific context rather than the development of the tool.

There are some other recent studies that are influenced by Olson's work. Krusell and Ríos-Rull (1996) consider a model where agents vote on the adoption of new technology. While young agents like the new technology, older agents who have a vested interest in the old technology may want to block the adoption of new technology. There are two differences between their model and ours. First, they employ an overlapping-generations model and the generational conflict plays a large role in their mechanism, while in our model the conflict is among firms who live for an infinite horizon. Second and more importantly, they employ majority voting as the political process, while we consider lobbying by special interest groups.

More recent papers by Bridgman et al (2004) and Bellettini and Ottaviano (2005) are closely related to ours. Both papers consider a model where the special interest groups lobby to influence the adoption of new technology. They both employ an overlapping-generation model, where the old agents do not want a new technology and the young agents do. An important difference compared to our study is that in our model the *relative size* of each coalition changes over time and does so *endogenously*. That is, in our model, whether there are many incumbent firms in the following period depends on what policy is currently adopted. This type of feedback from the policy to the coalition size is absent in each of the above studies. The fact that the coalition size is endogenous is crucial in formally analyzing Olson's (1982) idea. The endogeneity of entrenched interests cannot be tackled by a static model, which is our motivation to extend our analysis to a dynamic setting.

Our dynamic model allows for richer possibilities for the time-series policy outcomes than

Krusell and Ríos-Rull (1996), Bridgman et al (2004), and Bellettini and Ottaviano (2005). These papers emphasize the possibility of cyclical policy dynamics, which our model also allows. Unlike these models, our model also permits multiple steady states: two economies with the same set of parameter values and different initial number of incumbents may converge to different steady states with different policy choices. This means that two similar economies may end up having very different levels of creative destruction in the long run.

The possibility of multiple steady states provides a key insight into the question of why policies persistently favor one group over another. In the situations where multiple steady states can arise, *history matters* for the political and economic outcome in the long run.[7] The initial distribution of political power affects the economy through the policy choice, and the economic outcome feeds into the future distributions of political power. The political process and the economic process can have a positive feedback on each other, and the initial "bias" of the political power can be strengthened over time. In such a situation, a one-time (temporary) intervention to the policy can have a permanent effect on the political and economic outcome. In an influential book, North (1990) emphasizes the interaction between the political process and the economic process, and writes "political rules in place lead to economic rules, but the causality runs both ways. That is, property rights and hence individual contracts are specified and enforced by political decision-making, but the structure of economic interests will also influence the political structure" (p. 48). North emphasizes the "path-dependent" nature of institutional and economic change, and our multiple steady state result can be viewed as one example of such a path-dependence. This result highlights that underlying fundamentals may not totally determine even long-run economic outcomes.

Our model also permits dynamics where an economy starts out from a situation with free entry policy and gradually converges to a steady-state with entry blocking policy. There, the fact that the special interest group "builds up" during the period of free entry is important in leading to the future policy switch towards the blocking of new entry. This parallels Olson's

[7]In the development economics literature, there are papers which construct models with multiple steady states (often referred to as "poverty traps")—such as Azariadis and Drazen (1990), Banerjee and Newman (1993), and Galor and Zeira (1993). The important difference of our model to this literature is that our mechanism emphasizes special interest politics and entry and exit of firms.

(1982) analysis of the post-WWII relative decline of the British economy. The literature in economic history reviewed by Crafts (2011) also suggest that this channel is relevant in explaining British relative decline.

The paper is organized as follows. In Section 2, we characterize a simple static model, which can be solved in closed-form, and it helps building intuitions for the main dynamic model. Section 3 presents the dynamic model. We first analyze the general characteristics of the equilibrium outcome, and then illustrate special cases to obtain a sharper characterization. Section 4 explores some extensions. Section 5 concludes.

2 Static model

The economy consists of a continuum of firms with a total population of one. There are two types of firms, *leaders* and *followers*. Leaders, with mass L, are the incumbent firms who are already in the market. The rest of the firms, with mass $(1-L)$, are followers who are potential entrants currently out of the market.

There are two possible policy regimes, $a \in \{e, b\}$, where e represents *free entry* and b represents *entry blocking*. After a is determined, each firm receives entry/exit shocks (a leader receives an exit shock and a follower receives an entry shock). We denote $\delta_a \in (0,1)$ as the exit probability of leaders and $\lambda_a \in (0,1)$ as the entry probability of followers. We assume that $\delta_e > \delta_b$ and $\lambda_e > \lambda_b$. After the entry and exit, a leader who did not exit earns profit $\pi_\ell > 0$ and a follower who entered earns profit π_f. (We use the notation ℓ for leaders and the subscript f for followers.) Leaders who exited and followers who did not enter earn zero profit.

The political process is modeled as a common agency game. We assume that the leaders form one coalition (special interest group) and the followers form another coalition.[8] The payoff from policy a is $V_\ell(a) \equiv L(1-\delta_a)\pi_\ell$ for the leader coalition and $V_f(a) \equiv (1-L)\lambda_a\pi_f$ for the follower coalition. Following Bernheim and Whinston (1986), we assume that coalitions

[8]Olson (1965) emphasizes the free-riding incentives in forming a coalition. In particular, in our context, it is perhaps more realistic to imagine that the followers suffer from the free-rider problem more than the leaders do. In order to analyze this issue, one can potentially extend the model so that only a subset of firms participates in the coalition.

bid in a first-price menu auction. The policymaker, who decides the policy regime, does not have her own preferences over the two possible regimes. Rather, her decision is influenced by monetary payments of coalitions. One can interpret these payments as campaign contributions, lobbying, or bribes. Each coalition i ($i = \ell, f$) first specifies the conditional payments $R_i(a)$ for each policy option a and the policymaker chooses the policy that maximizes the sum of these payments $R_\ell(a) + R_f(a)$. The resulting net gain for coalition i when the policy a is chosen is $N_i(a) \equiv V_i(a) - R_i(a)$. The policymaker has the option to reject all offers, which implies the restriction $R_i(a) \geq 0$.

In the current setting, this menu auction is equivalent to an auction to bid on the right to be a dictator for the policy outcome—this can also be considered as Bertrand pricing with different costs. In a Nash Equilibrium, the coalition i with higher $V_i(a_i) - V_i(a_{-i})$, where a_i is its desired policy and a_{-i} is the less preferred policy, wins the auction and a_i is implemented. We focus on a particular Nash Equilibrium where the winner bets zero for its less preferred policy and just enough to win for its preferred policy. There are many additional Nash Equilibria in this game—the losing coalition can bid more on losing policy, since it knows that it will end up not pay anything, but the winning coalition has to bid more in order to win. Bernheim and Whinston (1986) introduce a refinement called *Truthful Nash Equilibrium*, which is formally defined in Appendix B.[9] In the current context, this refinement is equivalent to eliminating the weakly dominated strategies for the losing coalition.

In this equilibrium, the policy is determined as the following:

$$a = \begin{cases} e & \text{if } L < L^* \\ \{e, b\} & \text{if } L = L^* \\ b & \text{if } L > L^*, \end{cases}$$

where the threshold L^* is

$$L^* = \frac{(\lambda_e - \lambda_b)\pi_f}{(\lambda_e - \lambda_b)\pi_f + (\delta_e - \delta_b)\pi_\ell}. \tag{1}$$

Hence, if the mass of leaders L is sufficiently large, the entry blocking policy is selected. Here L is given exogenously, but in reality L changes over time. In particular, the current value

[9]See Bernheim and Whinston (1986) for further details on the characteristics of the Truthful Nash Equilibrium.

of L is determined as a result of past entry and exit. This means that the past policies affect the current L, which in turn affects the current policy. In the following section, this dynamic interaction is explicitly analyzed.

3 Dynamic model

In this section, we extend the model in the previous section to a dynamic one. It turns out that our model permits a variety of equilibrium dynamics, depending on parameter values. In particular, we illustrate that there are situations that resemble the post-WWII decline of the British economy and an equilibrium with multiple steady states.

3.1 Setup

Time is discrete with an infinite horizon. Each firm lives infinitely and maximizes the present value of the sum of the expected profit minus the political payment, discounted by the discount factor $\beta \in (0, 1)$. The policymaker also lives infinitely, and discounts the future at the same rate as the firms.

The events during each period follow the same order as in the static model. At the start of period t, each coalition (the leader coalition or the follower coalition, details are discussed later) announces the schedule of conditional payments for the current period. Then the policymaker decides on the policy a_t. After entry and exit occur, each firm that is in the market earns profit.

We call a firm a *leader* if it operated in the market at period $t-1$ and a *follower* if it did not operate at period $t-1$. The mass of the leaders at the start of time t (before the time t entry and exit occur) is denoted by L_t. Again, the probability that a follower enters is λ_a and the probability that a leader exits is δ_a when the current policy is a. If a leader firm stays in the market, it earns π_ℓ amount of profit, and if a follower firms enters, it earns π_f. Firms outside the market earn zero. As we explain in detail later, we assume that both the firms and the policymaker cannot commit to their future behavior.

There are three important aspects that make the dynamic model different from the static model. First, the population of the leaders, L_t, changes over time. Moreover, its future values are influenced by the current policy. Since the value of L_t influences the policy decision at time t, there is an influence of current policy on the future policy through the movement of L_t. One particular point of interest here is how the policy is shaped in the long run, as L_t evolves endogenously over time. Second, since a leader can become a follower (and a follower can become a leader) in the future with some probability, and the transition probabilities are affected by the political decision, the policy has an effect on the "planning horizon" of the forward-looking firms. Third, since the policymaker is also forward-looking, it may become easier or harder for the firms to influence the policymaker compared to the static setting.

Since there is a continuum of firms, the transition of L_t follows a deterministic law of motion

$$L_{t+1} = \mathcal{L}(a, L_t),$$

when $a_t = a$. Here, $\mathcal{L}(a, L_t)$ denotes the value of L at period $t+1$ when the period-t value of L is L_t and the period-t policy is $a \in \{e, b\}$. Under our assumption of entry and exit (and abusing the law of large numbers as is common in models with continuum of agents), $\mathcal{L}(a, L_t)$ is a linear function of L_t for a given choice of a:

$$L_{t+1} = \lambda_a(1 - L_t) + (1 - \delta_a)L_t.$$

This can be rewritten as

$$L_{t+1} = \bar{L}_a + (L_t - \bar{L}_a)(1 - \lambda_a - \delta_a) = \bar{L}_a + (L_t - \bar{L}_a)\gamma_a,$$

where $\bar{L}_a \equiv \lambda_a/(\lambda_a + \delta_a)$ is the steady state value of L, associated with policy a (that is, the unique value of L such that $\mathcal{L}(a, L) = L$) and $\gamma_a \equiv 1 - \lambda_a - \delta_a$ is the persistence of the deviation from the steady state. When $|\gamma_a|$ is small, L_t approaches $\bar{L}_a$ quickly during the period in which a is chosen.

In the following, we assume that $\gamma_e = 1 - \lambda_e - \delta_e \geq 0$. This automatically implies that $\gamma_b = 1 - \lambda_b - \delta_b > 0$ since $\lambda_e > \lambda_b$ and $\delta_e > \delta_b$. This restriction rules out "mechanical" cyclical dynamics of L_t with the same policy. For example, suppose that $1 - \lambda_e - \delta_e < 0$ holds and

$L_t < \bar{L}_e$. If the equilibrium policy is $a_t = a_{t+1} = ... = e$, this implies that $L_{t+1} > \bar{L}_e$, $L_{t+2} < \bar{L}_e$, and so on—even when the policy choice is constant, the value of L_t oscillates. We do not believe that this cyclical behavior (as opposed to the "genuine" cyclical behavior we show in Appendix D) is interesting, since this is mechanically driven by the size of λ_a and δ_a (this dynamics appears when λ_a and δ_a are sufficiently large). Clearly, λ_a and δ_a are functions of the period length—when one period is long, an incumbent is less likely to survive towards the end of a period, and an outsider is more likely to enter before the end of a period. Thus, these dynamics, which are driven by large values λ_a and δ_a, are essentially an artifact of the assumption that entry and exit occur only between periods. Given that entry and exit occur continuously in reality, this type of dynamics is not realistically relevant.

With $\gamma_a \geq 0$, the convergence to the steady state is monotone and stable when the same policy is chosen over a period of time—when the same policy a is chosen forever, the long-run value of L_t monotonically converges to $\bar{L}_a$.

3.2 Equilibrium

An important feature of the dynamic environment is that the status of each firm as a leader or a follower changes over time. Moreover, since we assume that transition events are independent across firms (conditional on regime), the set of firms which share the common status at a given date will be different and each firm may have different preferences over policy regimes at a later period. Therefore, the interests of a set of firms are guaranteed to be aligned for one period only.

We assume that, at each period, firms with the same status (and hence identical interests over policy choices) form a coalition (special interest group) in order to advance their interests. We do not investigate the coalition formation process explicitly and we assume symmetric outcomes within a coalition. At the end of the period, the coalition is dissolved. Each coalition acts to maximize the present discounted value of the profit for its representative member.

We concentrate on *symmetric Markov equilibria.* This means (i) we assume that strategies do not depend on the identity of the firms, but only on their status as a leader or a follower;

and (ii) strategies can be conditioned only on payoff-relevant variables, which in our case include only the mass of leaders in the economy, L_t, and the firm's own state of being a leader or a follower.

Let us denote (consistent with the notation in the static model) the equilibrium present discounted value of the net payoff (profit minus the payment to the policymaker) for a firm of type i ($i = \ell, f$) as $n_i(L)$. Similarly, we denote the equilibrium present discounted value of the gross payoff to firm of type i ($i = \ell, f$) from the political action a ($a = e, b$) as $v_i(a; L)$. For the leaders, it satisfies the following equation:

$$v_\ell(a; L) = (1 - \delta_a)(\pi_\ell + \beta n_\ell(\mathcal{L}(a, L))) + \delta_a \beta n_f(\mathcal{L}(a, L)),$$

and for the followers the following equation is satisfied:

$$v_f(a; L) = \lambda_a(\pi_f + \beta n_\ell(\mathcal{L}(a, L))) + (1 - \lambda_a)\beta n_f(\mathcal{L}(a, L)).$$

Then the total gross value of the coalition is simply $V_\ell(a; L) = L v_\ell(a; L)$ and $V_f(a; L) = (1 - L) v_f(a; L)$.

As in the static model, the coalition chooses the menu of payments $R_i(a; L)$ in order to maximize $V_i(a; L) - R_i(a; L)$. We assume that the coalition can commit to the payment menu only for the current period.[10]

The individual equilibrium net value $n_i(L)$ is then given by:

$$n_\ell(L) = \frac{V_\ell(a^*(L); L) - R_\ell(a^*(L); L)}{L}$$

and

$$n_f(L) = \frac{V_f(a^*(L); L) - R_f(a^*(L); L)}{1 - L},$$

where $a^*(L)$ is the equilibrium policy action. Thus, defining $r_\ell(a; L) \equiv R_\ell(a; L)/L$ and $r_f(a; L) \equiv R_f(a; L)/(1 - L)$, the equilibrium payoffs satisfy the functional equations

$$n_\ell(L) = (1 - \delta_{a^*(L)})(\pi_\ell + \beta n_\ell(\mathcal{L}(a^*(L), L))) + \delta_{a^*(L)} \beta n_f(\mathcal{L}(a^*(L), L)) - r_\ell(a^*(L); L)$$

[10]Note that maximizing the total net value of the coalition is equivalent to maximizing the net value of a representative member of the coalition.

and

$$n_f(L) = \lambda_{a^*(L)}(\pi_f + \beta n_\ell(\mathcal{L}(a^*(L), L))) + (1 - \lambda_{a^*(L)})\beta n_f(\mathcal{L}(a^*(L), L)) - r_f(a^*(L); L).$$

The policymaker's present discounted value of the receipt of payment is denoted by $G(L_t)$. The Bellman equation for the policymaker is

$$G(L) = \max_{a \in \{e,b\}} \left\{ R_\ell(a; L) + R_f(a; L) + \beta G(\mathcal{L}(a, L)) \right\}.$$

Below, we follow Bergemann and Välimäki (2003) in extending Bernheim and Whinston's (1986) equilibrium concepts to a dynamic setting. In particular, we consider the Markov Perfect Equilibrium as the baseline equilibrium concept and define the *Truthful Markov Perfect Equilibrium* as the refinement we utilize.

3.2.1 Truthful Markov Perfect Equilibrium

This section defines and describes the Truthful Markov Perfect Equilibrium in our setting. As we noted earlier, we focus on *Markov strategies* in the sense that the actions of firms and the policymaker depend only on the payoff-relevant state variables. Here, the only payoff-relevant state for a firm is L, aside from whether one is a leader or a follower. For the policymaker, the only payoff-relevant state is L. The definition of the *truthful Markov strategy* is the following.[11]

Definition 1 *$R_i(a; L)$ (for $i = \ell, f$) is said to be a* ***truthful Markov strategy relative to*** *$(\hat{a}, L)$ if, for $a = e, b$, either*

$$N_i(a; L) = N_i(\hat{a}; L)$$

or

$$N_i(a; L) < N_i(\hat{a}; L) \text{ and } R_i(\hat{a}; L) = 0$$

holds.

The Truthful Markov Perfect Equilibrium is defined as:[12]

[11] This is a natural extension of the truthful strategy in the static model, defined in Appendix B.

[12] This is also a natural extension of the Truthful Nash Equilibrium in Appendix B.

Definition 2 *Let* $\mathcal{R}(L) \equiv \{R_i(a;L)\}_{i=\ell,f;a=e,b}$. $(\mathcal{R}(L), \hat{a}(\mathcal{R}(L);L), n_i(L), V_i(a,L), G(L))$ *is a **Markov Perfect Equilibrium** if*

(i) *For all* L *and* $\mathcal{R}'(L) \equiv \{R'_i(a;L)\}_{i=\ell,f;a=e,b}$, $\hat{a}(\mathcal{R}'(L);L)$ *is a solution to*

$$\max_{a\in\{e,b\}} R'_\ell(a;L) + R'_f(a;L) + \beta G(\mathcal{L}(a,L)),$$

and

(ii) *for each* i *and all* L, *there is no other* $R'_i(a;L)$ *such that*

$$V_i(\tilde{a};L) - R'_i(\tilde{a};L) > V_i(a;L) - R_i(a;L),$$

where $\tilde{a} = \hat{a}(\tilde{\mathcal{R}}(L);L)$, $\tilde{\mathcal{R}}(L) \equiv \{R'_i(a;L), R_{-i}(a;L)\}_{a=e,b}$, *and* $a = \hat{a}(\mathcal{R}(L);L)$.

(iii) *Given* $\mathcal{R}(L)$, $n_i(L), V_i(a,L)$ *and* $G(L)$ *satisfy the functional equations above.*

*It is called a **Truthful Markov Perfect Equilibrium** if it is a Markov Perfect Equilibrium and each coalition plays a truthful Markov strategy relative to* $(\hat{a}(\mathcal{R}(L);L), L)$.

We define

$$a^*(L) \equiv \hat{a}(\mathcal{R}(L);L),$$

where $\mathcal{R}(L)$ is the equilibrium reward (payment) function as above, and call $a^*(L)$ as the *equilibrium political outcome* when the state is L.

Proposition 1 below describes a Truthful Markov Perfect Equilibrium (if it exists—we discuss the issue of existence in Appendix C). It can easily be checked with the definition, and thus the proof is omitted. There are two cases, called the "Regular Case" and the "Irregular Case." The Regular Case corresponds to the one analogous to the static model—the leader coalition prefers the blocking policy and the follower coalition prefers the free-entry policy. In the Irregular Case, the dynamic forces (through future values) overturn one (or both) coalition's policy preferences. The leader coalition might like the free entry policy (Irregular Case I), the follower coalition might like the entry blocking policy (Irregular Case II), or both (Irregular Case III).

Proposition 1 *The following is a Truthful Markov Perfect Equilibrium:*

Regular Case: $V_\ell(b;L) \geq V_\ell(e;L)$ *and* $V_f(e;L) \geq V_f(b;L)$

- *If* $V_\ell(b;L)+V_f(b;L)+\beta G(L'_b) \geq V_\ell(e;L)+V_f(e;L)+\beta G(L'_e)$, $a^*(L)=b$ *is selected.*
  - *Case I: If* $V_f(e;L) - V_f(b;L) + \beta[G(L'_e) - G(L'_b)] \geq 0$,

$$(R_\ell(e;L), R_\ell(b;L)) = (0, V_f(e;L) - V_f(b;L) + \beta[G(L'_e) - G(L'_b)]),$$

$$(R_f(e;L), R_f(b;L)) = (V_f(e;L) - V_f(b;L), 0).$$

  - *Case II: Otherwise (note that* $G(L'_b) > G(L'_e)$ *in this case since* $V_f(e;L) > V_f(b;L)$*),*

$$(R_\ell(e;L), R_\ell(b;L)) = (0,0),$$

$$(R_f(e;L), R_f(b;L)) = (V_f(e;L) - V_f(b;L), 0).$$

- *Otherwise,* $a^*(L) = e$ *is selected.*
  - *Case I: If* $V_\ell(b;L) - V_\ell(e;L) + \beta[G(L'_b) - G(L'_e)] \geq 0$,

$$(R_\ell(e;L), R_\ell(b;L)) = (0, V_\ell(b;L) - V_\ell(e;L)),$$

$$(R_f(e;L), R_f(b;L)) = (V_\ell(b;L) - V_\ell(e;L) + \beta[G(L'_b) - G(L'_e)], 0).$$

  - *Case II: Otherwise (note that* $G(L'_e) > G(L'_b)$ *in this case since* $V_\ell(b;L) > V_\ell(e;L)$*),*

$$(R_\ell(e;L), R_\ell(b;L)) = (0, V_\ell(b;L) - V_\ell(e;L)),$$

$$(R_f(e;L), R_f(b;L)) = (0,0).$$

Irregular Case I: $V_\ell(b;L) < V_\ell(e;L)$ *and* $V_f(e;L) \geq V_f(b;L)$

- *If* $V_\ell(b;L)+V_f(b;L)+\beta G(L'_b) \geq V_\ell(e;L)+V_f(e;L)+\beta G(L'_e)$, $a^*(L)=b$ *is selected.*

$$(R_\ell(e;L), R_\ell(b;L)) = (V_\ell(e;L) - V_\ell(b;L), 0),$$

$$(R_f(e;L), R_f(b;L)) = (V_f(e;L) - V_f(b;L), 0).$$

- *Otherwise,* $a^*(L) = e$ *is selected.*

$$(R_\ell(e; L), R_\ell(b; L)) = (R_1, 0),$$

$$(R_f(e; L), R_f(b; L)) = (R_2, 0),$$

where $V_\ell(e; L) - V_\ell(b; L) \geq R_1 \geq 0$, $V_f(e; L) - V_f(b; L) \geq R_2 \geq 0$, *and* $R_1 + R_2 = \beta[G(L'_b) - G(L'_e)]$.

Irregular Case II: $V_\ell(b; L) \geq V_\ell(e; L)$ *and* $V_f(e; L) < V_f(b; L)$

- *If* $V_\ell(b; L) + V_f(b; L) + \beta G(L'_b) \geq V_\ell(e; L) + V_f(e; L) + \beta G(L'_e)$, $a^*(L) = b$ *is selected.*

$$(R_\ell(e; L), R_\ell(b; L)) = (0, R_1),$$

$$(R_f(e; L), R_f(b; L)) = (0, R_2),$$

where $V_\ell(b; L) - V_\ell(e; L) \geq R_1 \geq 0$, $V_f(b; L) - V_f(e; L) \geq R_2 \geq 0$, *and* $R_1 + R_2 = \beta[G(L'_e) - G(L'_b)]$.

- *Otherwise,* $a^*(L) = e$ *is selected.*

$$(R_\ell(e; L), R_\ell(b; L)) = (0, V_\ell(b; L) - V_\ell(e; L)),$$

$$(R_f(e; L), R_f(b; L)) = (0, V_f(b; L) - V_f(e; L)).$$

Irregular Case III: $V_\ell(b; L) < V_\ell(e; L)$ *and* $V_f(e; L) < V_f(b; L)$

- *If* $V_\ell(b; L) + V_f(b; L) + \beta G(L'_b) \geq V_\ell(e; L) + V_f(e; L) + \beta G(L'_e)$, $a^*(L) = b$ *is selected.*
  - *Case I: If* $V_\ell(e; L) - V_\ell(b; L) + \beta[G(L'_e) - G(L'_b)] \geq 0$,

$$(R_\ell(e; L), R_\ell(b; L)) = (V_\ell(e; L) - V_\ell(b; L), 0),$$

$$(R_f(e; L), R_f(b; L)) = (0, V_\ell(e; L) - V_\ell(b; L) + \beta[G(L'_e) - G(L'_b)]).$$

– *Case II: Otherwise (note that* $G(L_b') > G(L_e')$ *in this case since* $V_\ell(e;L) > V_\ell(b;L)$*),*

$$(R_\ell(e;L), R_\ell(b;L)) = (0, V_\ell(e;L) - V_\ell(b;L)),$$

$$(R_f(e;L), R_f(b;L)) = (0, 0).$$

- *Otherwise,* $a^*(L) = e$ *is selected.*

  – *Case I: If* $V_f(b;L) - V_f(e;L) + \beta[G(L_b') - G(L_e')] \geq 0$*,*

$$(R_\ell(e;L), R_\ell(b;L)) = (V_f(b;L) - V_f(e;L) + \beta[G(L_b') - G(L_e')], 0),$$

$$(R_f(e;L), R_f(b;L)) = (0, V_f(b;L) - V_f(e;L)).$$

  – *Case II: Otherwise (note that* $G(L_e') > G(L_b')$ *in this case since* $V_f(b;L) > V_f(e;L)$*),*

$$(R_\ell(e;L), R_\ell(b;L)) = (0, 0),$$

$$(R_f(e;L), R_f(b;L)) = (0, V_f(b;L) - V_f(e;L)).$$

The Irregular Cases require that the "future forces" overturn the immediate payoffs. As concrete examples, the Irregular Case I can occur (for all values of L) when $\beta = 0.95$, $\pi_\ell = 0.001$, $\pi_f = 1$, $\lambda_e = 0.3$, $\lambda_b = 0.1$, $\delta_e = 0.5$, and $\delta_b = 0.01$; the Irregular Case II can occur (for large values of L) when $\beta = 0.95$, $\pi_\ell = 1$, $\pi_f = 0.001$, $\lambda_e = 0.101$, $\lambda_b = 0.1$, $\delta_e = 0.8$, and $\delta_b = 0.01$. We have not found a numerical example that Irregular Case III occurs in our extensive search over a wide range of parameter values. We conjecture that Irregular Case III does not occur, but unfortunately we have not been able to prove it formally.[13] While they are theoretically possible, the Irregular Cases occur only under limited (extreme) values of parameters—in fact, in all numerical examples that we analyze in Section 3.4, the equilibrium falls onto the Regular Case for all L.

[13] If we allow the period profits π_ℓ and π_f to depend on the policy choice a, then the following numerical example falls into the Irregular Case III: $\beta = 0.8$, $\pi_\ell^e = \pi_\ell^b = 1$, $\pi_f^b = 100$, $\pi_f^e = 90$, $\lambda_e = 0.41$, $\lambda_b = 0.4$, $\delta_e = 0.25$, and $\delta_b = 0.1$. Here, π_i^j is the profit of firm i under policy j.

3.3 Characterizing the political outcome

In this section, we characterize a general property of the equilibrium policies $a^*(L)$. We show that, similarly to the static model, $a^*(L)$ exhibits a threshold property: the free entry policy is chosen when L is small and the blocking policy is chosen when L is larger than a certain threshold.

3.3.1 Equivalence to surplus maximizing

It turns out that, in our setting, the equilibrium political outcome $a^*(L)$ maximizes present discounted value of total surplus.[14] This can be shown directly using the equilibrium outcome in Proposition 1.

Proposition 2 *The equilibrium policy choice $a^*(L)$ is the policy function for the following dynamic programming problem:*

$$S(L) = \max_{a \in \{b,e\}} \left\{ (1-\delta_a)L\pi_\ell + \lambda_a(1-L)\pi_f + \beta S(\mathcal{L}(a,L)) \right\}. \tag{2}$$

Proof. See Appendix E. ■

We use this property extensively below when we characterize the properties of the equilibrium. We emphasize that the fact that the policy maximizes the present-value of total surplus does not mean that it is "optimal." In particular, consumers do not participate in the political process, and the welfare for the consumers is not considered when calculating the "total surplus." In our model, the consumers are not explicitly considered since we focus on the positive analysis of equilibrium policymaking. However, as we discussed in the Introduction, it is likely that the consumer would benefit from the entry of new firms (creative destruction). Thus, if entry blocking is chosen as the equilibrium policy, it is likely that this policy choice is very costly when we consider the economy-wide welfare including the consumers.

The following proposition establishes some general properties of the total surplus function $S(L)$. $S(L)$ satisfies the Bellman equation (2), which implies that S can be analyzed with

[14] Appendix B shows that the same property holds in the static version of the model.

standard recursive methods. Let the mapping in functional space defined by the right-hand side of (2) be denoted by T. Since the per-period surplus falls in the compact set $[0, \max\{\pi_\ell, \pi_f\}]$, T is a contraction. Hence equation (2) has a unique solution. Let $p_a \equiv \pi_\ell - \delta_a \pi_\ell - \lambda_a \pi_f$ for $a = e, b$.

Proposition 3 *The operator T is a contraction. It has a unique continuous and convex fixed point. The unique fixed point S is differentiable everywhere, except on a set of points that is at most countable. Moreover,*

$$S'(L) \in \left[\min_{a \in \{e,b\}} \frac{p_a}{1 - \beta\gamma_a}, \max_{a \in \{e,b\}} \frac{p_a}{1 - \beta\gamma_a} \right],$$

for all L where the derivative is defined.

Proof. See Appendix E. ■

3.3.2 Results: Threshold rule

In this section, we show that the equilibrium outcome for $a^*(L)$ is a threshold rule, similar to the static case. It requires some extra preparations in order to show the result formally.

First, let us introduce some notation. Let $q_a \equiv \lambda_a \pi_f$. Let $S_a(L)$ be the discounted sum of surplus if the economy starts with L leaders and the policy a is followed forever. It is straightforward to see that

$$S_a(L) = \frac{p_a \bar{L}_a + q_a}{1 - \beta} + \frac{p_a (L - \bar{L}_a)}{1 - \beta\gamma_a} \tag{3}$$

and

$$S_a(L) = p_a L + q_a + \beta S_a(\mathcal{L}(a, L))$$

hold.

Let $S_{a,a'}(L)$ denote the discounted sum of surplus if the economy starts at L, chooses a for the first period and a' thereafter. Then

$$S_{a,a'}(L) = p_a L + q_a + \beta S_{a'}(\mathcal{L}(a, L))$$

holds. The following proposition characterizes the threshold rules.

Proposition 4 *The optimal policy is determined as follows:*

1. *If* $p_e/(1-\beta\gamma_e) \geq p_b/(1-\beta\gamma_b)$*, then the optimal choice correspondence satisfies* $e \in a^*(L)$ *for all* L*.*

2. *If* $p_e/(1-\beta\gamma_e) < p_b/(1-\beta\gamma_b)$ *and* $\bar{L}_e \leq \bar{L}_b$*, then there exists some* L^* *such that* $e \in a^*(L)$ *for all* $L < L^*$*,* $b \in a^*(L)$ *for all* $L > L^*$*, and* $\{e,b\} = a^*(L^*)$*.*

   (a) *If* $S_b(\bar{L}_e) \geq S_{e,b}(\bar{L}_e)$*, either* $L^* < 0$ *or* L^* *solves the equation* $S_b(L^*) = S_{e,b}(L^*)$ *and* $L^* \leq \bar{L}_e$*.*

   (b) *If* $S_e(\bar{L}_b) \geq S_{b,e}(\bar{L}_b)$*, either* $L^* > 1$ *or* L^* *solves the equation* $S_e(L^*) = S_{b,e}(L^*)$ *and* $L^* \geq \bar{L}_b$*.*

   (c) *In all other cases,* L^* *solves the equation* $S_b(L^*) = S_e(L^*)$ *and* $L^* \in (\bar{L}_e, \bar{L}_b)$*.*

3. *If* $p_e/(1-\beta\gamma_e) < p_b/(1-\beta\gamma_b)$*,* $\bar{L}_b < \bar{L}_e$*, and*

   (a) $S_b(\bar{L}_b) \geq S_{e,b}(\bar{L}_b)$*, then* $e \in a^*(L)$ *for all* $L < L^*$*,* $b \in a^*(L)$ *for all* $L > L^*$ *and* $\{e,b\} = a^*(L^*)$*, where either* $L^* < 0$ *or* L^* *solves the equation* $S_b(L^*) = S_{e,b}(L^*)$ *and* $L^* \leq \bar{L}_b$*.*

   (b) $S_e(\bar{L}_e) \geq S_{b,e}(\bar{L}_e)$*, then* $e \in a^*(L)$ *for all* $L < L^*$*,* $b \in a^*(L)$ *for all* $L > L^*$ *and* $\{e,b\} = a^*(L^*)$ *where either* $L^* > 1$ *or* L^* *solves the equation* $S_e(L^*) = S_{b,e}(L^*)$ *and* $L^* \geq \bar{L}_e$*.*

   (c) *If neither of the conditions in (a) or (b) are satisfied and* $1 - \beta\gamma_e - \beta\gamma_b > 0$*, then there exists some* $L^* \in (\bar{L}_b, \bar{L}_e)$ *such that* $e \in a^*(L)$ *for all* $L < L^*$*,* $b \in a^*(L)$ *for all* $L > L^*$ *and* $\{e,b\} = a^*(L^*)$*.*

Proof. See Appendix E. ■

In cases above, the political decision follows a threshold rule: $a^*(L) = e$ when L is small and $a^*(L) = b$ when L is large.

In the second statement, there are some cases (when both $S_b(\bar{L}_b) \geq S_{e,b}(\bar{L}_b)$ and $S_e(\bar{L}_e) \geq S_{b,e}(\bar{L}_e)$ do not hold) in which we have not been able to prove the threshold property in general. This corresponds to the situation where L exhibits cyclical dynamics. For this case, we have solved the model numerically and experimented with many parameter values, including ones outside the parametric restriction in 3(c). In all our experiments, $a^*(L)$ turned out to possess a threshold property as in all the other situations.

3.4 Dynamics of the economy and the policy in some special cases

In this section, we examine the dynamics of the economy in special cases and obtain sharper characterizations. In particular, we highlight two examples where the dynamics of the model economy provides insights on real-world phenomena. The first is the situation where the economy converges to one particular steady state. Our focus is on the dynamics where an economy starts out with the free entry policy and gradually converges to a steady-state with entry blocking policy. This dynamics formalizes Olson's (1982) analysis of the post-WWII relative decline of the British economy. The second example is where the economy exhibits multiple steady states. In this case, the economy converges to different steady-state depending on the initial value of L.

In general, the dynamic paths of L_t depend largely on the relative values of L^*, $\bar{L}_e$, and $\bar{L}_b$, and we divide the cases by the values of these variables. Note that $\bar{L}_a$ $(a = e, b)$ is a function of λ_a and δ_a, while L^* depends on all parameters of the model. The possibilities (discussed in detail below) are summarized by Figure 1. The reader may want to refer to the Figure as we go along with the discussion below.

3.4.1 Political convergence and the post-WWII decline of the British economy

When the threshold L^* lies on the same side of $\bar{L}_e$ and $\bar{L}_b$ (that is, $L^* \leq \min\{\bar{L}_e, \bar{L}_b\}$ or $L^* \geq \max\{\bar{L}_e, \bar{L}_b\}$), L converges to either $\bar{L}_e$ or $\bar{L}_b$ in the long run. This corresponds to 1, 2(a)(b), and 3(a)(b) of Proposition 4 and the first four panels of Figure 1. We call this case *political convergence*, since the initial value of L does not matter in the long run, and all economies with different starting values of L (under the same fundamental parameter values) converge to the unique policy choice. Below we highlight two special cases.

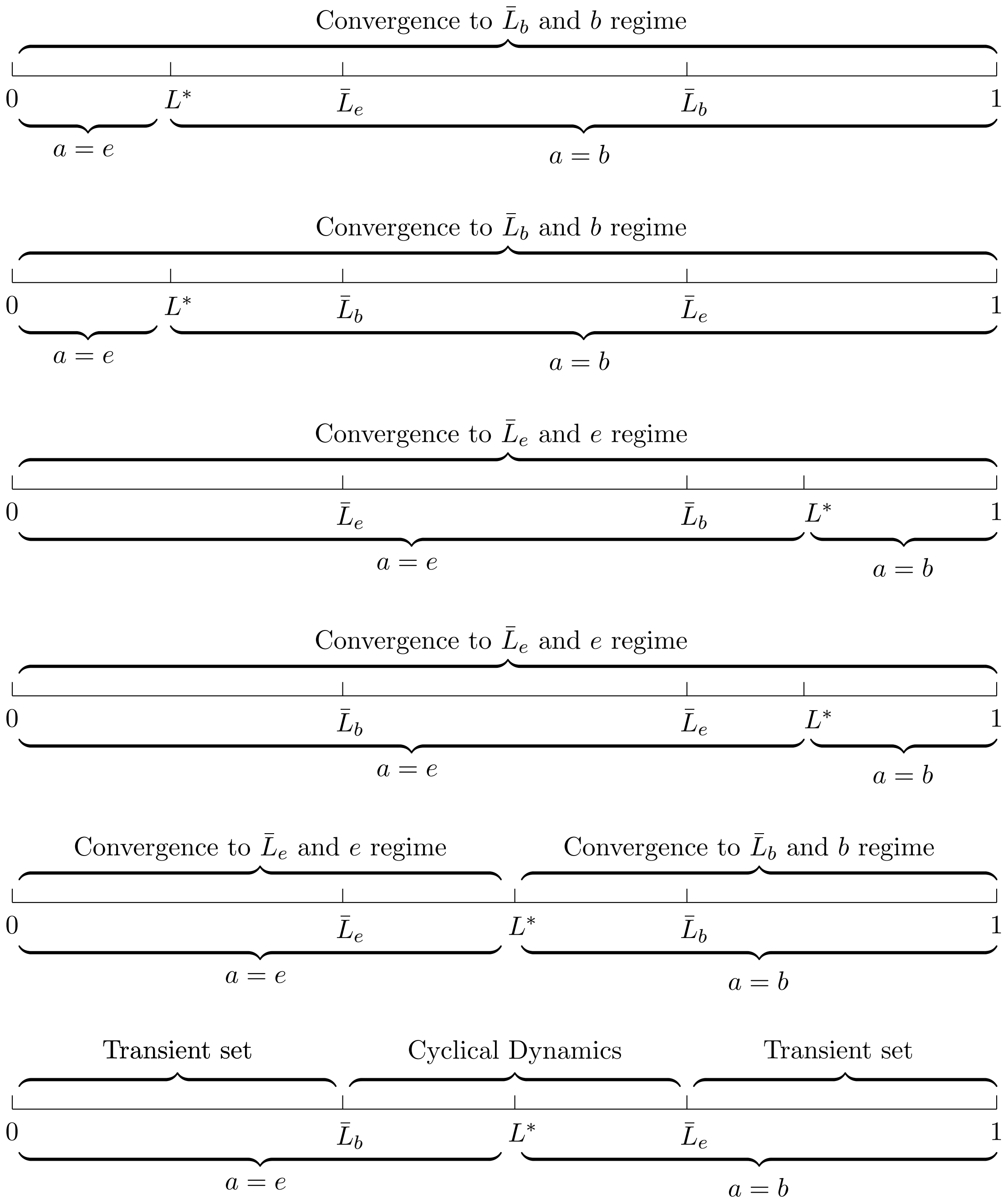

Figure 1: Possible dynamics. The mass of leaders L is on the horizontal axis. The first four panels exhibit convergence in the long run. The case of multiple steady states is in the fifth panel and the case with cyclical dynamics is in the last panel.

3.4.1.1 Political convergence I: Common steady state

One special case in which political convergence always occurs is when $\bar{L}_e = \bar{L}_b$. Clearly, in this case L^* is on the same side of $\bar{L}_e$ and $\bar{L}_b$. Call the common steady-state $\bar{L}$. Note that since $\bar{L}_a = \lambda_a/(\lambda_a + \delta_a)$, $\bar{L}_e = \bar{L}_b$ holds if and only if $\lambda_e/\delta_e = \lambda_b/\delta_b$.

One particular case that is relevant in light of Olson's (1982) argument is the case where $\pi_\ell > \pi_f$. In this case, an economy which starts out with low L and the free entry policy gradually builds up the special interest group of incumbents: L grows over time. At some point, the special interest group grows sufficiently large and surpasses L^*—after that, the economy stays in the blocking regime forever. Olson's analysis of post-WWII struggle in Great Britain, in contrast to the "miracles" in Germany and Japan, fits well with this type of dynamics.[15] Olson writes, "with age British society has acquired so many strong organizations and collusions that it suffers from an institutional sclerosis that slows its adaptation to changing circumstances and technologies" (p.78). The recent survey of history literature by Crafts (2011) echoes Olson's argument. Crafts writes, "A detailed review of the evidence suggests that the weakness of competition from the 1930s to the 1970s undermined productivity growth but since the 1970s stronger competition has been a key ingredient in ending relative economic decline" (from abstract). In fact, we can go further with our analysis—in the current case, the fundamental source of the convergence to the blocking regime is the fact that π_ℓ is larger than π_f. Therefore, one potential cure for this type of stagnation is to (permanently) raise the profitability of entrants in comparison to the incumbents.

Figure 2 is a numerical example of this case. The parameter values are: $\pi_\ell = 3$, $\pi_f = 1$, $\lambda_e = 0.2$, $\delta_e = 0.1$, $\lambda_b = 0.1$, $\delta_b = 0.05$, and $\beta = 0.8$. It draws the path of L_t when it starts from $L_1 = 0.4$. The economy starts from a free entry regime, but as L_t increases over time, the incumbent coalition gains more political power. At period 6, L_t exceeds L^* and the policy switches to entry blocking. L_t continues to increase, and the policy is locked into entry blocking forever.

[15] There are many other models, such as Jovanovic and Nyarko (1996), that can generate a gradual decline of technology adoption and stagnation. The difference here is the special role played by the special interest groups and competition from entry.

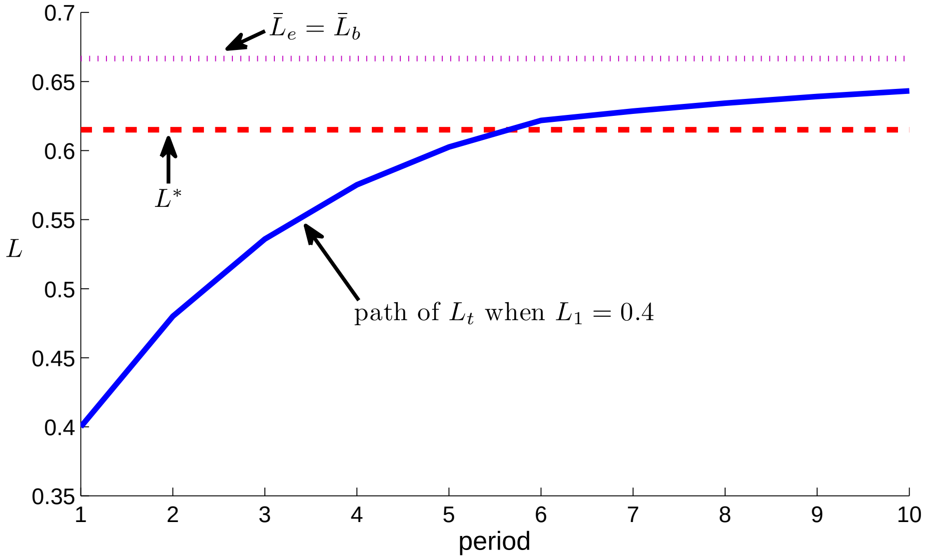

Figure 2: Example with "British decline" under common steady state

Figure 3 plots the path of the revenue flow for the policymaker that corresponds to the path of L_t in Figure 2. In Appendix B, it is analytically shown that the revenue of the policymaker is maximized when $L = L^*$ in the static model. Similarly, Figure 3 shows that the revenue flow is high when L_t is close to L^*.

The intuitions about the forces coming from different π's carry through even in the case with $\bar{L}_e \neq \bar{L}_b$. In fact, it is possible to show that the economy converges to $\bar{L}_b$ (with $a = b$) if π_ℓ/π_f is sufficiently high and the economy converges to $\bar{L}_e$ (with $a = e$) if π_ℓ/π_f is sufficiently low.

3.4.1.2 Political convergence II: Same profit

Now consider the case where $\pi_\ell = \pi_f$ (call the common profit as π). As we will see below, in this situation it is also the case that L^* is on the same side of $\bar{L}_e$ and $\bar{L}_b$ even when $\bar{L}_e \neq \bar{L}_b$. First, note that the Bellman equation (2) is reduced to

$$S(L) = \max_{a \in \{b,e\}} \{\mathcal{L}(a, L)\pi + \beta S(\mathcal{L}(a, L))\}.$$

With the standard argument (see Stokey, Lucas, and Prescott (1989)) it can be shown that

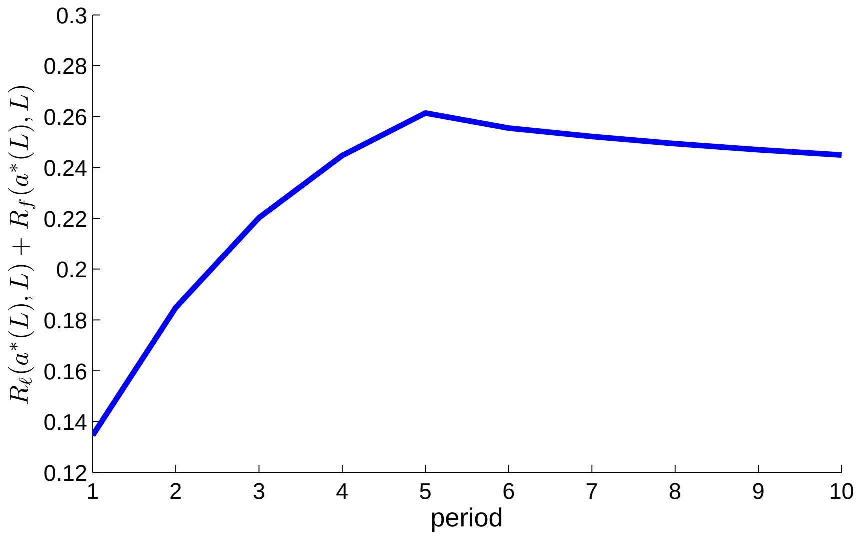

Figure 3: Example with "British decline" under common steady state: the revenue flow for the policymaker

$S(L)$ is strictly increasing. Therefore, at the right-hand side, a policy that maximizes $\mathcal{L}(a, L)$ is always chosen. The comparison between $\mathcal{L}(e, L)$ and $\mathcal{L}(b, L)$ leads us to the threshold rule for the policy:

$$a = \begin{cases} e & \text{if } L < L^* \\ \{e, b\} & \text{if } L = L^* \\ b & \text{if } L > L^*, \end{cases}$$

where the threshold L^* is

$$L^* = \frac{\lambda_e - \lambda_b}{(\lambda_e - \lambda_b) + (\delta_e - \delta_b)}.$$

Thus the threshold L^* here is identical to the threshold in the static model (1) with $\pi_\ell = \pi_f$. It is straightforward to show that $\bar{L}_b > \bar{L}_e > L^*$ if $\bar{L}_b > \bar{L}_e$ (as in the first panel of Figure 1) and $L^* > \bar{L}_e > \bar{L}_b$ if $\bar{L}_e > \bar{L}_b$ (as in the fourth panel of Figure 1). Thus L^* is always on the same side of $\bar{L}_e$ and $\bar{L}_b$ when $\bar{L}_e \neq \bar{L}_b$. (Clearly $L^* = \bar{L}_e = \bar{L}_b$ if $\bar{L}_e = \bar{L}_b$.)

In this case, the long-run behavior of the economy is determined by whether $\bar{L}_e$ is larger than $\bar{L}_b$. The long-run policy is e if $\bar{L}_e > \bar{L}_b$ and it is b if $\bar{L}_b > \bar{L}_e$. Therefore, the policy choice with free entry is likely to be sustained in an economy with a large value of $\bar{L}_e$, that is, either a large value of λ_e or a small value of δ_e.

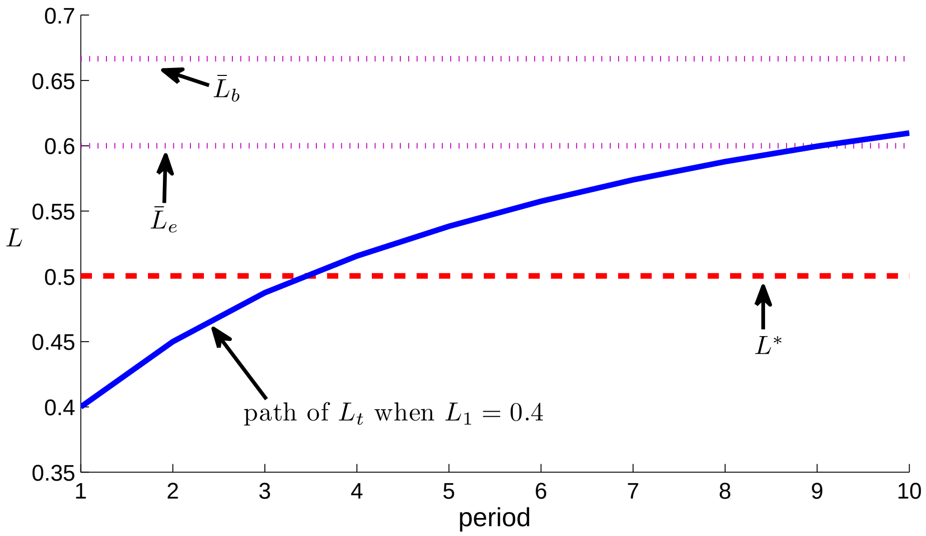

Figure 4: Example with "British decline" under common profit

As in the case of the common steady state, this case can also entail dynamics that resemble the post-WWII relative decline of the British economy—the economy starts out from free entry and moves into (and gets stuck in) the blocking regime—this happens when $\bar{L}_b > \bar{L}_e$. Figure 4 is a numerical example of this case. The parameter values are: $\pi_\ell = 1$, $\pi_f = 1$, $\lambda_e = 0.15$, $\delta_e = 0.1$, $\lambda_b = 0.1$, $\delta_b = 0.05$, and $\beta = 0.8$. It draws the path of L_t that starts from $L_1 = 0.4$. The economy starts from the free-entry regime. As time progresses, L_t increases, and the political power of the incumbent coalition is strengthened. At period 4, L_t surpasses L^*, and the policy switches to entry blocking. L_t converges to $\bar{L}_b$, and the policy remains entry blocking forever. Figure 5 plots the corresponding revenue for the policymaker. As in the previous example, the revenue is high when L_t is close to L^*.

In the current case, the fundamental cause for these dynamics is a relatively large value of $\bar{L}_b$. In this case, avoiding these dynamics requires raising δ_b or λ_e, or reducing δ_e or λ_b.

3.4.2 Multiple steady states

Under some parameter values, our economy can exhibit multiple steady states. This is the most important result of this paper, as it provides a potential explanations for why entry bar-

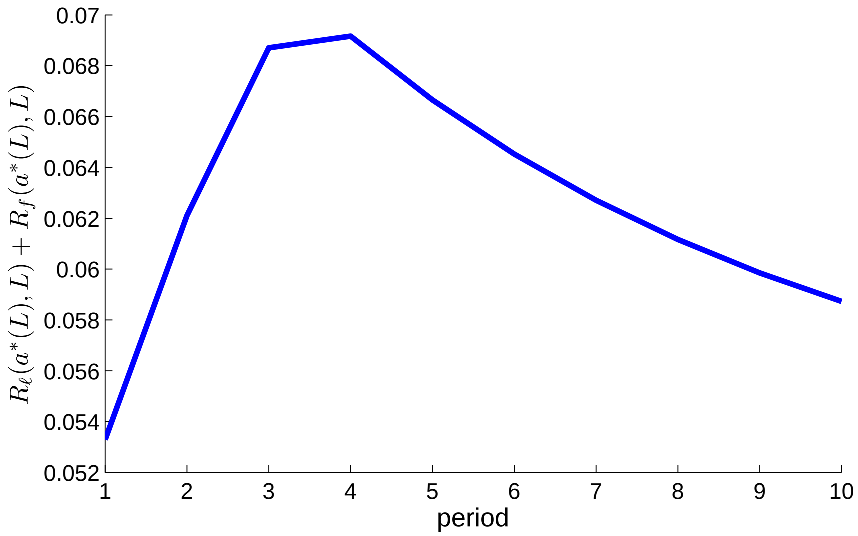

Figure 5: Example with "British decline" under common profit: the revenue flow for the policymaker

riers differ across countries (and why the difference persists). The mechanism highlights the importance of modeling both political and economic process endogenously—their interaction in a dynamic environment is the source of the multiplicity.

From the analysis of Section 3.4.1.2, we have seen that when $\pi_\ell = \pi_f$ and $\bar{L}_b > \bar{L}_e$, it is always the case that $\bar{L}_b > \bar{L}_e > L^*$, and the economy will eventually settle at the steady state with $L_t = \bar{L}_b$. As we increase π_f relative to π_ℓ, L^* tends to increase and when π_f is sufficiently larger than π_ℓ, L^* becomes larger than $\bar{L}_e$.

When $\bar{L}_b > L^* > \bar{L}_e$, the economy has multiple steady states. That is, depending on the initial value of L_t (call it L_0), the economy may end up in different steady states that have different political outcomes. Specifically, if $L_0 < L^*$, the policy outcome is $a^*(L_t) = e$ for all t and L_t converges to $\bar{L}_e$. If $L_0 > L^*$, the policy outcome is $a^*(L_t) = b$ for all t and L_t converges to $\bar{L}_b$. This case corresponds to the fifth panel of Figure 1.

Therefore, the initial condition matters in the long run, and some economies can get trapped in the policy that may not be desirable. This history dependence also implies that a *temporary* change in parameters, such as in π_ℓ, π_f, λ_e, λ_b, δ_e, or δ_b, can have a *permanent*

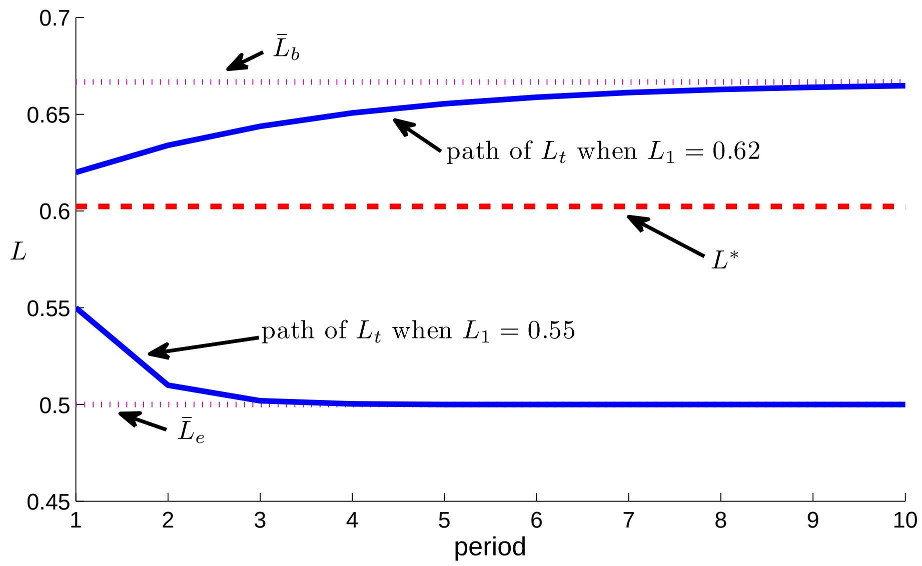

Figure 6: Example with multiple steady states

effect on the policy choice of the economy.

Figure 6 is a numerical example of the multiple steady states. The parameter values are: $\pi_\ell = 1$, $\pi_f = 2.2$, $\lambda_e = 0.4$, $\delta_e = 0.4$, $\lambda_b = 0.2$, $\delta_b = 0.1$, and $\beta = 0.8$. The two solid lines are two sample paths. In the path above, $L_1 = 0.62$ and, since L^* turns out to be 0.60 in this economy, the initial policy choice is $a^*(L_1) = b$. This raises L_t and over time L_t increases monotonically and approaches $\bar{L}_b$. The policy choice is $a^*(L_t) = b$ for all t. In the path below, $L_1 = 0.55$, which is below L^*, and $a^*(L_t) = e$ for all t. Figure 7 plots the corresponding paths of revenue for the policymaker. In both paths, revenue declines as L_t moves farther from L^*.

The possibility of multiple steady states suggests one answer to a question we posed in the Introduction: why do some countries institute heavy entry regulations while others do not? Even if the fundamental parameters are the same among countries, here *history matters*: if a country starts out with a large population (and therefore a large political power) of leaders who prefer entry blocking, a protective policy leads into more political power of the leaders in the future. If a country starts from relatively few leaders, a free environment for entry can

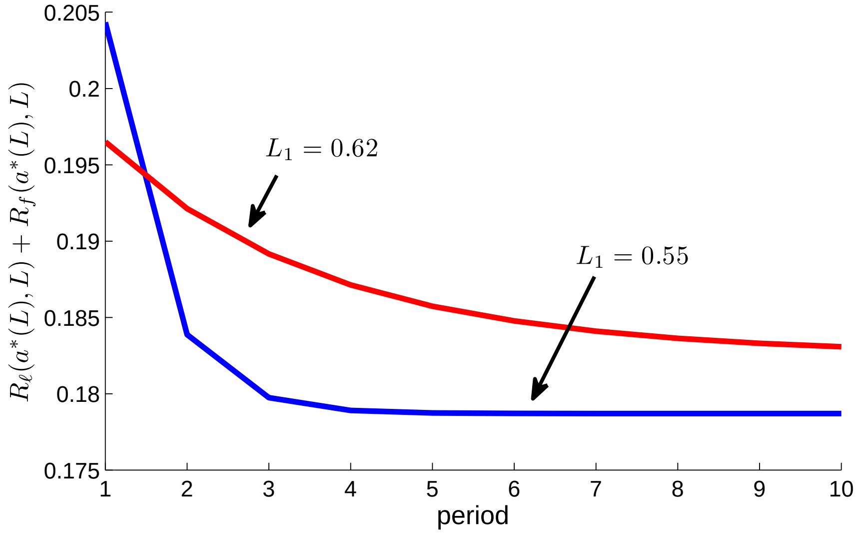

Figure 7: Example with multiple steady states: the revenue flow for the policymaker

persist.

When is it likely that multiple steady states exist in an economy? One key inequality is $\bar{L}_b > \bar{L}_e$, which is equivalent to $\lambda_b/\delta_b > \lambda_e/\delta_e$. This is the case when a blocking regime creates relatively more leaders (large λ_b) who tend to stay in the economy longer (small δ_b). As in the case in Section 3.4.1.2, this creates feedback between the policy and the economy—an entry blocking policy increases the size of the leader coalition and strengthens their political power, leading to further lobbying towards blocking. If $\pi_f = \pi_\ell$, this leads to a political convergence to $\bar{L}_b$, as we have seen in Section 3.4.1.2. However, when $\pi_f > \pi_\ell$ (a relatively strong vintage effect), this profit difference counteracts the previous force and tends to move L^* higher. When the difference between π_f and π_ℓ is small, $\bar{L}_e > L^*$ and the long run steady state is $\bar{L}_b$, again as in Section 3.4.1.2. If the difference is very large, $L^* \geq \bar{L}_b$ and the long run steady state is $\bar{L}_e$. At an intermediate value of the difference, multiple steady states may arise. This is the situation where two conflicting forces have relatively balanced strengths. On one hand, the number of incumbents can expand quickly in the blocking regime. On the other, entrants have a strong profit motive to enter. When the initial value of L_t is large, the first effect dominates, and when the initial value of L_t is small, the second effect dominates.

Once one effect dominates initially, that effect gains strength over time by the endogenous movement of L_t, and the economy gets stuck in that particular policy.

In this situation, the initial value of L_t matters. An important policy implication is that a *temporary* intervention that lowers L_t can have a *permanent* effect of moving the economy from the entry blocking regime to the free entry regime. One way to accomplish this goal in a manner that is consistent with the constraints imposed by the political process is by increasing the profitability of entrants π_f for certain amount of time. Proposition 5 formally shows this possibility.

Proposition 5 *Suppose that* $\bar{L}_e < L^* < \bar{L}_b$ *and* $L_t > L^*$. *There exists some* T *and* $\pi'_f > \pi_f$ *such that if* π_f *is increased to* π'_f *for* T *periods from period* t, *then* $a^*_{t'} = e$ *for all* $t' \geq t$.

Proof. See Appendix E. ■

As we have emphasized in Section 3.3.1, the "surplus-maximizing" nature of the equilibrium does not imply that the chosen policies are "optimal." In particular, since the consumers do not participate in the political process and the consumers are likely to benefit from the free entry regime, it is possible that the political outcome will end up being trapped in the entry-blocking regime even when the free entry policy is the desirable one from the entire economy's welfare standpoint.[16] Since neither the incumbent coalition nor the potential entrant coalition internalizes the consumer surplus, there is a role for some outside intervention in our model.

4 Extensions

In this section, we study the role of the assumptions we made for the dynamic model by providing two extensions of our baseline model.

4.1 Persistent policy

In our benchmark model, we assumed that the policymaker can change policy at every period. However, we often observe that policies are very persistent. Some policies are hard to change once they have been determined. Here, we consider an opposite extreme and assume that

[16] See Appendix A for such an example.

the policy is determined at time $t = 0$ and fixed over time. Thus, there is no feedback from economic evolution to policy determination in the future.

In this case, the lobbying occurs only once, and the structure of the game becomes a static one. Of course, this time the firms maximize the discounted sum of the net payoff, taking the future evolution of L (which depends on the policy choice) into account. It is straightforward to show that the Truthful Nash Equilibrium we used in Section 2 (and defined formally in Appendix B) maximizes total surplus. The total surplus is now defined as the discounted value of profits for all firms if the same policy a is followed forever. We have already denoted this object as $S_a(L)$ and have given an explicit formula in (3). Note that $S_a(L)$ is again linear in L. One can show that there are three possibilities:

(i) $S_e(L) > S_b(L)$ for all $L \in [0, 1]$. This implies that $a = e$ is chosen regardless of L. The economy eventually converges to $\bar{L}_e$.

(ii) $S_e(L) < S_b(L)$ for all $L \in [0, 1]$ and thus $a = b$ is chosen regardless of L. The economy eventually converges to $\bar{L}_b$.

(iii) Otherwise, there exists a threshold value of L, L^*, where $S_e(L^*) = S_b(L^*)$. When $L > L^*$, $a = b$ is chosen and the economy eventually converges to $\bar{L}_b$. When $L < L^*$, $a = e$ is chosen and the economy eventually converges to $\bar{L}_e$. (When $L = L^*$, either can be chosen.)

Note that the last case involves "multiple steady states" in the sense that the eventual steady state depends on the value of L at $t = 0$, but this result is not as deep as our baseline dynamic model, because in this case $t = 0$ is a special period when the policy is set. It is not so surprising that the economic situation of the period when the policy is set matters for the eventual outcome.

The "corner solution" (one particular policy is chosen regardless of L) cases (i) and (ii) do not occur in the static model. These can happen in the dynamic model because the future profit may overturn the preferences based on the current profit. With some algebra one can show, for example, that (i) can happen when π_ℓ/π_f is sufficiently small and $\lambda_e \approx \lambda_b$. (ii) can happen when π_f/π_ℓ is sufficiently small and $\lambda_e \approx \lambda_b$. The intuition is that, in case (i), the relative benefit from entry is sufficiently high so that even the incumbent firms (foreseeing that they will become entrant at some point) want the free entry policy. In case (ii), the relative benefit of being industry leaders is so large that even potential entrants (foreseeing that they will become incumbents at some point) want the blocking policy.

4.2 Change in the number of total firms

In our baseline model, the total number of firms is fixed at one. Thus when the number of leaders L_t increases, the number of followers $1 - L_t$ decreases. An alternative assumption is that the total number of firms can vary over time. In particular, it is possible at a stage of the industry life cycle that when the number of leaders increases, the number of followers also increases. Although the analysis of a general model is beyond the scope of this paper, here we present a modified dynamic model which incorporates this idea.

Suppose that at each period, the number of followers is always proportional to the number of leaders: the number of followers is ρL_t, where $\rho > 0$. We assume that the transition between leaders and followers follow the same probabilities as in the baseline model (λ_a and δ_a for $a = e, b$). The number of the followers is adjusted to ρL_t by the additional inflows from (and outflows to) the "outside."

With this assumption, the evolution of the number of leaders follow

$$L_{t+1} = (1 - \delta_a)L_t + \lambda_a \rho L_t = (1 - \delta_a + \rho\lambda_a)L_t. \tag{4}$$

Let $\theta_a \equiv 1 - \delta_a + \rho\lambda_a$ and assume that $\theta_a < 1/\beta$. If $\theta_a > 1$, L_t grows over time under the policy choice a. If $\theta_a < 1$, L_t shrinks over time.

Since the relative size of the leader and the follower is constant over time, it is straightforward to show that there exists a Truthful Markov Perfect Equilibrium where the same policy is chosen over time. Under this equilibrium, the total gross value of each coalition $V_i(a; L)$ and the payment $R_i(a; L)$ are both proportional to L. In this model, when the policymaker compares the values of

$$R_\ell(a; L) + R_f(a; L) + \beta G(\theta_a L)$$

in order to determine a (here, functions $R_\ell(a;L)$, $R_f(a;L)$, and $G(L)$ are defined similarly to the baseline model, using ρL as the follower's population instead of $1-L$ and using (4) for determining the next period L), she ends up comparing

$$r_\ell(a) + \rho r_f(a) + \beta\theta_a g$$

(because $G(L)$ is linear in L), where $r_\ell(a) \equiv R_\ell(a;L)/L$, $r_f(a) \equiv R_f(a;L)/(\rho L)$, and $g \equiv G(L)/L$ are determined in equilibrium. Clearly ρ has an important impact in policy choice because it affects the relative size of coalitions. Another notable parameter is θ_a, which affects the policy decision through the change in the *future* size of the policymaker's revenue.

In this version of the model, the policy outcome is completely determined by the parameter values and the same policy persists forever. Other than the size of L (and the total number of the firms $(1+\rho)L$), everything stays the same over time. In this sense, the policy choice is persistent (even without any commitment), but there is no "feedback effect" that is highlighted in the baseline model.

5 Conclusion

Why do policies that create entry barriers persist? In particular, why are there many governments that impose such high barriers for the entry of new firms? We consider this question by constructing a simple model of political economy. Our model can generate dynamics where the interaction of the economy and the endogenous policy can lead to a situation where the policymaker chooses a policy with high entry barriers for a long time. A particularly important case is where the economy can exhibit multiple steady states. In that case, economies with the same fundamentals (same parameter values) can end up choosing very different policies in the long run if they start from two different initial situations. We described it in detail using numerical simulations. When multiple steady states exist, a temporary intervention can have a permanent effect on the equilibrium policy outcome and on economic performance.

In addition to multiple steady states, our model permits very rich possibilities of the dynamics of the economy. Using numerical simulations, we described the dynamics that resemble the post-WWII relative decline of the British economy, where an economy that starts out with a free entry environment may develop special interest groups in the long run and end up blocking entry of new firms.

We have shown that the equilibrium policy maximizes the (present-value of) total surplus. This does not mean that the policy chosen is the "optimal" one. It is important to emphasize that the "total surplus" here includes only the surpluses of the participants in the political process. In our model, the firms and the policymaker participate in the policy decision, but the consumers do not participate. The consumer's welfare is ignored in the "total surplus." Hayek (1979), referring to Olson (1965), writes, "it is impossible in principle to organize all interests ... in consequence the organization of certain large groups assisted by government leads to a persistent exploitation of unorganized and unorganizable groups" (p. 97). The consumer here is one example of "unorganized and unorganizable groups" that can suffer from the political influences of the organized special interest groups (in particular, the firms that prefer "blocking"). As we discussed in the Introduction, it is likely that the consumers benefit from entry of new firms. The economic growth literature based on the idea of creative destruction has emphasized the importance of new firm entry in the context of economic growth and development. Incorporating consumers' welfare explicitly and analyzing the political economy of entry and exit in a model with an explicit growth process are important future research topics.[17] With these models, we will be able to further analyze the normative consequences of these political economy outcomes.

The result that an economy can get trapped into a blocking steady state, as in the case of "British decline" or in the case with multiple steady states, is a consequence of the political system that is influenced by lobbying. In this paper, we take the political institutions (the rule of the lobbying game that determines the political outcome) as exogenous. Future research should be directed towards analyzing *endogenous institutions* that promote efficient policies—policies that encourages entry when entry improves economic performance. Acemoglu and Robinson (2012) emphasize the role of "inclusive institutions" in economic development.

[17]The model in Appendix A is one such attempt.

They write, "the presence of markets is not by itself a guarantee of inclusive institutions. Markets, left to their own devices, can cease to be inclusive, becoming increasingly dominated by the economically and politically powerful. Inclusive economic institutions require not just markets, but inclusive markets that create a level playing field and economic opportunities for majority of the people" (p. 323). Such inclusive institutions would help avoiding development traps that are described in our paper.

Another important extension of our analysis is incorporating exogenous aggregate shocks into the model economy. In such an economy, it may turn out that a temporary aggregate shock can move the economy into a situation that leads to a different long-run outcome.[18] There, history would matter not only in the sense that the initial state matters, but also in the sense that the entire realizations of the aggregate shocks matter for the long-run outcome of the economy.

[18] Acemoglu and Robinson (2012) use the term "critical juncture" in referring to such events that "disrupt the existing political and balances in one or many societies" (p. 431). Their examples include the Black Death, the opening of Atlantic trade routes, and the Industrial Revolution.

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Appendix

A An endogenous growth model

This section embeds a "quality ladder" style endogenous growth model into our dynamic political economy model in the main text. The model is considerably more complex than the one in the main text.

We obtain a similar result to the original model. In particular, we present an example where multiple balanced growth paths exist. One benefit of the formulation here is that we can explicitly introduce the consumers who are not in organized coalitions and analyze the welfare properties of the model.

A.1 Model setup

There are three types of agents in the economy: entrepreneurs (e), workers (w), and a policymaker (p). The population of the entrepreneurs is 1 and the population of the workers is N. All agents maximize the present value of their expected consumption:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t c_t^j\right],$$

where $\beta \in (0,1)$ and $j \in \{e, w, p\}$. There is a continuum of industries on $[0,1]$ that produces differentiated intermediate goods, and each industry corresponds to an entrepreneur (that is, entrepreneur i produces in industry i). Each worker supplies one unit of labor inelastically for the production of the final (consumption) good. The final (consumption) good is produced by the production function

$$O_t = N^{1-\alpha}\int_0^1 A_{it}^{1-\alpha}x_{it}^{\alpha}di,$$

where $\alpha \in (0,1)$, $A_{it} > 0$ is the "quality" of the intermediate good from industry i, and x_{it} is the quantity of the intermediate good i that is used for the production. The final good industry is perfectly competitive, and pays the marginal product of labor to the workers and and the marginal product of the intermediate input to the intermediate good producer.

At a point in time, each intermediate good industry is either "monopolistic" or "competitive." When monopolistic, the intermediate good of that industry is produced solely by the industry leader who invented the cutting-edge quality of good i. (One interpretation is that when the cutting-edge quality is invented, the government provides the entrepreneur a monopoly right by, for example, granting a patent protection.) When the cutting-edge quality is imitated (patent expires), the industry becomes competitive, and good i is produced by the "competitive fringe" which acts as a price-taker.

A monopolistic industry at the start of time t becomes competitive with probability δ_a. (As in the main text, δ_a depends on the policy regime a.) If it becomes competitive, the competitive fringe produces, and the entrepreneur is forced out of the market (and becomes a "potential entrant") until it can innovate the next quality. If it remains monopolistic (with probability $(1-\delta_a)$), the entrepreneur produces the intermediate input. The production of one unit of intermediate good requires one unit of final good. Since the price of the intermediate good is its marginal product in final good production, the maximization problem for the monopolist is:

$$\max_{x_{it}} \ (\alpha N^{1-\alpha} A_{it}^{1-\alpha} x_{it}^{\alpha-1} - 1) x_{it}.$$

The solution is

$$x_{it} = \alpha^{\frac{2}{1-\alpha}} A_{it} N,$$

and the profit is

$$\pi_{it} = \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha) A_{it} N. \tag{A.1}$$

In the competitive industry, the price is equal to the marginal cost, therefore

$$x_{it} = \alpha^{\frac{1}{1-\alpha}} A_{it} N.$$

For simplicity, we set $N = 1$ in the following. Let the number of monopolistic industries at the beginning of period t be L_t. Then, at period t, $\delta_a L_t$ number of industries among them operate as competitive industries and $(1-\delta_a)L_t$ number of industries among them operate as monopolistic.

If an industry starts as a competitive industry, there is a probability λ_a that the potential entrant succeeds in innovating the next generation of quality and becomes a monopolist. Suppose that the new quality is $\gamma > 1$ times more productive than the previous quality (that is, $\gamma A_{i,t-1}$ is the "true" productivity if this intermediate good is used). Here, we allow the "actual" productivity at the period of the innovation to be larger or smaller than the true productivity: $\theta\gamma A_{i,t-1}$. If θ is less than one, the first period "actual" productivity is lower than the "true" productivity, possibly because of the necessity of learning for the new technology. If θ is larger than one, then the actual productivity is higher at the initial period than the true productivity, possibly because the new vintage of technology provides a temporary boost to the productivity.

Since $\lambda_a(1-L_t)$ industries innovate and $(1-\lambda_a)(1-L_t)$ industries remain competitive in this part of the economy, the average productivity for the whole economy at the end of period t is

$$\bar{A}_t \equiv \int_0^1 A_{it}di = \lambda_a(1-L_t)\gamma\bar{A}^c_{t-1} + (1-\lambda_a)(1-L_t)\bar{A}^c_{t-1} + L_t\bar{A}^m_{t-1},$$

where $\bar{A}^c_{t-1}$ is the average productivity of the competitive industries and $\bar{A}^m_{t-1}$ is the average productivity of the monopolistic industries. The averages evolve according to:

$$\bar{A}^m_t = \frac{\lambda_a(1-L_t)\gamma\bar{A}^c_{t-1} + (1-\delta_a)L_t\bar{A}^m_{t-1}}{\lambda_a(1-L_t) + (1-\delta_a)L_t},$$

and

$$\bar{A}^c_t = \frac{(1-\lambda_a)(1-L_t)\bar{A}^c_{t-1} + \delta_a L_t\bar{A}^m_{t-1}}{(1-\lambda_a)(1-L_t) + \delta_a L_t}.$$

In order to facilitate the mapping to the main text, we make a simplifying assumption.

Assumption 1 *We assume that at the end of the period, the productivity of all industries equalize to the average productivity. That is,*

$$A_{it} = \bar{A}_t = \bar{A}^c_t = \bar{A}^m_t$$

for all i and t, at the end of period t. The industry structure (monopolistic or competitive) remains the same.

Thus, at the end of period t,

$$\bar{A}_t = \bar{A}_{t-1} + \bar{A}_{t-1}\lambda_a(1-L_t)(\gamma-1)$$

and the gross growth rate of the technology is

$$\frac{\bar{A}_t}{\bar{A}_{t-1}} = g_a(L_t) \equiv \lambda_a(1-L_t)(\gamma-1)+1.$$

At the individual industry level, $A_{it} = \bar{A}_{t-1}$ if the industry does not innovate at time t and $A_{it} = \theta\gamma\bar{A}_{t-1}$ if it innovates. Clearly, Assumption 1 is extreme and unrealistic. Without this assumption, we can still define and compute the Truthful Markov Perfect Equilibrium, but the surplus-maximizing property (Proposition 2 in the main text) does not hold, and therefore the characterization becomes much more complex.

A.2 Constrained social optimum

In this section, we introduce a utilitarian social welfare problem. In a subsequent section, we will contrast it with the political-economy equilibrium.

The hypothetical social planner, who makes the political decision, is utilitarian. She makes the policy decision based on the total welfare of the economy, and is not swayed by the political process. We assume that she gives equal weight to the utility of all participants in the economy. Since the utility functions are all linear, the social welfare function is simply discounted net output and is given by:

$$\sum_{t=0}^{\infty} \beta^t \left[\int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di - \int_0^1 x_{it} di \right].$$

We assume that the fundamental parameter values (which reflect the patent system and the system for financing innovation) and the market structure (monopoly/competitive) cannot be altered by the social planner. (Thus, the resulting outcome is a "constrained social optimum.") The only choice variable in the social planning problem is the choice of the entry regimes (between free entry and entry blocking).

Assumption 1 ensures that the state variables in this problem are average productivity $\bar{A}_{t-1}$ and the mass of monopolistic industries L_t.

The contribution to net output of a monopolistic firm with productivity A_{it} is

$$A_{it}^{1-\alpha} x_{it}^{\alpha} - x_{it} = A_{it}[\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}].$$

Similarly, for a competitive industry,

$$A_{it}^{1-\alpha} x_{it}^{\alpha} - x_{it} = A_{it}[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}].$$

Then the current period, output for given L_t, $\bar{A}_{t-1}$ and a is:

$$(\delta_a L_t + (1-\lambda_a)(1-L_t))z_c \bar{A}_{t-1} + ((1-\delta_a)L_t + \lambda_a(1-L_t)\theta\gamma)z_m \bar{A}_{t-1},$$

where $z_c \equiv \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}$ and $z_m \equiv \alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}$. Note that $z_m = \alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha)z_c$.

Denote $L = L_t$ and $\bar{A} = \bar{A}_{t-1}$. The problem for the social planner is given by the following Bellman equation:

$$W(L,\bar{A}) = \max_{a\in\{e,b\}} (\delta_a L + (1-\lambda_a)(1-L))z_c \bar{A} + ((1-\delta_a)L + \lambda_a(1-L)\theta\gamma)z_m \bar{A} + \beta W(\mathcal{L}(a,L), g_a(L)\bar{A}).$$

It is easy to see that $W(L,\bar{A}) = \bar{A}W(L,1)$ for all $\bar{A}, L$. So if we denote $\tilde{W}(L) = W(L,1)$, the equation above can be rewritten more simply as:

$$\tilde{W}(L) = \max_{a\in\{e,b\}} (\delta_a L + (1-\lambda_a)(1-L))z_c + ((1-\delta_a)L + \lambda_a(1-L)\theta\gamma)z_m + \beta g_a(L)\tilde{W}(\mathcal{L}(a,L)). \tag{A.2}$$

We make the following assumption:

Assumption 2

$$\max_{a\in\{e,b\}, L\in[0,1]} g_a(L) < \frac{1}{\beta}.$$

This implies that the Bellman operator defined above is a contraction mapping with a bounded and continuous fixed point.

A.3 Political equilibrium

Here we introduce the political economy model in this environment. As before, we will concentrate on *symmetric Markov equilibria*, but now one of the relevant state variables is the average productivity $\bar{A}$. An old monopolist makes a profit of $\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)\bar{A}$, whereas a new monopolist makes a profit of $\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)\theta\gamma\bar{A}$. Let $\kappa \equiv \alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha)$.

$n_i(L,\bar{A})$ and $v_i(a,L,\bar{A})$ are defined similarly as in the main body of the paper. They satisfy the following equations:

$$v_\ell(a,L,\bar{A}) = (1-\delta_a)(\kappa\bar{A} + \beta n_\ell(\mathcal{L}(a,L), g_a(L)\bar{A})) + \delta_a\beta n_f(\mathcal{L}(a,L), g_a(L)\bar{A}),$$

and

$$v_f(a,L,\bar{A}) = \lambda_a(\kappa\theta\gamma\bar{A} + \beta n_\ell(\mathcal{L}(a,L), g_a(L)\bar{A})) + (1-\lambda_a)\beta n_f(\mathcal{L}(a,L), g_a(L)\bar{A}).$$

The total gross values of the coalitions are simply $V_\ell(a,L,\bar{A}) = Lv_\ell(a,L,\bar{A})$ and $V_f(a,L,\bar{A}) = (1-L)v_f(a,L,\bar{A})$.

The menu of payment R, the equilibrium net value of a coalition n and the government's value G are defined similarly as in the main text; a truthful strategy and the Truthful Markov Perfect Equilibrium are also similarly defined.

We conjecture that $v_i(a,L,\bar{A}) = \bar{A}v_i(a,L,1)$ and $n_i(L,\bar{A}) = \bar{A}n_i(L,1)$. Denoting $\tilde{v}_i(a,L) \equiv v_i(a,L,1)$ and $\tilde{n}_i(L) \equiv n_i(L,1)$, we have:

$$\tilde{v}_\ell(a,L) = (1-\delta_a)(\kappa + \beta g_a(L)\tilde{n}_\ell(\mathcal{L}(a,L))) + \delta_a\beta g_a(L)\tilde{n}_f(\mathcal{L}(a,L)),$$

$$\tilde{v}_f(a,L) = \lambda_a(\kappa\theta\gamma + \beta g_a(L)\tilde{n}_\ell(\mathcal{L}(a,L))) + (1-\lambda_a)\beta g_a(L)\tilde{n}_f(\mathcal{L}(a,L)).$$

From the definition of n and above, $\tilde{v}$ and $\tilde{n}$ do not depend on $\bar{A}$, and thus the conjecture is verified. Following a similar argument as in Appendix C we can establish the existence of discrete approximation of the equilibrium.

We define the total surplus as $S(L,\bar{A}) \equiv Ln_\ell(L,\bar{A}) + (1-L)n_f(L,\bar{A}) + G(L,\bar{A})$. In equilibrium, it satisfies the equation (the proof is similar to the one for Proposition 2):

$$S(L,\bar{A}) = \max_{a\in\{e,b\}} (1-\delta_a)L\kappa\bar{A} + \lambda_a(1-L)\kappa\theta\gamma\bar{A} + \beta S(\mathcal{L}(a,L), \bar{A}g_a(L)).$$

By a standard argument, we can establish that the Bellman equation above is a contraction mapping and $S(L,\bar{A}) = \bar{A}S(L,1)$. Thus again, by defining $\tilde{S}(L) \equiv S(L,1)$, we have

$$\tilde{S}(L) = \max_{a\in\{e,b\}} (1-\delta_a)L\kappa + \lambda_a(1-L)\kappa\theta\gamma + \beta g_a(L)\tilde{S}(\mathcal{L}(a,L)). \tag{A.3}$$

A.4 The growth rate and the individual welfare in the steady state

In the following, we focus on a steady state (balanced growth path). As in the main text, there are parametric configurations where the Truthful Markov Perfect Equilibrium does not involve a steady state—we rule out such parametric configurations in the analysis below.[19]

[19]The current framework can handle these situations, but here we ignore this issue for simplicity.

The steady state value of L_t, if the same policy regime continues forever, is $\bar{L} = \lambda_a/(\lambda_a + \delta_a)$. The steady-state growth rate of $\bar{A}_t$ is

$$\frac{\bar{A}_t}{\bar{A}_{t-1}} = \bar{g}_a \equiv (\gamma - 1)\frac{\lambda_a \delta_a}{\lambda_a + \delta_a} + 1.$$

It is straightforward to see that $\bar{g}_a$ is increasing in λ_a and δ_a. Given our assumption that $\lambda_e > \lambda_b$ and $\delta_e > \delta_b$, $\bar{g}_a$ is higher with the free entry regime.

As in the previous section, let the present discounted value of a "leader" entrepreneur (who entered the market at time t as a monopolist) as $v_{\ell t}$, and let $\tilde{v}_{\ell t} \equiv v_{\ell t}/\bar{A}_{t-1}$. In the steady state, $\tilde{v}_{\ell t}$ is constant, and thus we denote it as $\tilde{v}_\ell$. Similarly, we denote the present discounted value of a "follower" entrepreneur (who entered a market at time t as a potential entrant) be v_{ft} and define $\tilde{v}_{ft} \equiv v_{ft}/\bar{A}_{t-1}$. Again, $\tilde{v}_{ft}$ is constant in the steady state and we denote it $\tilde{v}_f$. Let $\pi_{\ell t}$ be the profit in (A.1) and $\tilde{\pi}_\ell \equiv \pi_{\ell t}/\bar{A}_{t-1}$ ($\tilde{\pi}_\ell$ is always constant). Similarly, let π_{ft} be the profit in (A.1) and $\tilde{\pi}_f \equiv \pi_{ft}/\bar{A}_t$. Note that $\tilde{\pi}_f = \theta\gamma\tilde{\pi}_\ell$, and thus $\tilde{\pi}_f$ can be larger or smaller than $\tilde{\pi}_\ell$.

Similarly to the main text, let $\tilde{n}_\ell$ and $\tilde{n}_f$ be the net payoffs of a leader and a follower respectively, normalized by $\bar{A}_{t-1}$ as above, after making the payment to the policymaker. That is, denoting $\tilde{r}_\ell$ and $\tilde{r}_f$ to be the normalized per-entrepreneur payment to the policymaker,

$$\tilde{n}_\ell = \tilde{v}_\ell - \tilde{r}_\ell$$

and

$$\tilde{n}_f = \tilde{v}_f - \tilde{r}_f$$

hold. Then,

$$\tilde{v}_\ell = (1 - \delta_a)(\pi_\ell + \beta\bar{g}_a\tilde{n}_\ell) + \delta_a\beta\bar{g}_a\tilde{n}_f$$

and

$$\tilde{v}_f = \lambda_a(\pi_f + \beta\bar{g}_a\tilde{n}_\ell) + (1 - \lambda_a)\beta\bar{g}_a\tilde{n}_f.$$

determine the values of $\tilde{v}_\ell$ and $\tilde{v}_f$. Of course, a, $\tilde{r}_\ell$, and $\tilde{r}_f$ are equilibrium objects here, and when the Truthful Markov Perfect Equilibrium is defined in this economy (as in the main text), we have to evaluate the off the equilibrium payoffs (which involves off the steady state behavior).

The utility for the policymaker is (again, for given a, $\tilde{r}_\ell$, and $\tilde{r}_f$ and after normalizing)

$$\tilde{G} = \tilde{r}_\ell \bar{L}_a + \tilde{r}_f(1 - \bar{L}_a) + \beta \bar{g}_a \tilde{G}.$$

The above Bellman equations are very similar to the ones in the main text. The only difference is that the "effective discount factor" $\beta \bar{g}_a$ is policy variant. (It also depends on L_t outside the steady state.)

In order to calculate the utility for the workers, one needs to compute the income (which is equal to consumption with the linear utility) for the workers. Workers are paid their marginal product, which is equal to $(1-\alpha)\int_0^1 {A_{it}}^{1-\alpha} x_{it}^{\alpha} di$ in equilibrium. Let $X_{it} = {A_{it}}^{1-\alpha} x_{it}^{\alpha}$ be the quality adjusted contribution of intermediate good i. With Assumption 1, this is $X_{it} = \alpha^{\frac{2\alpha}{1-\alpha}} \theta \gamma \bar{A}_{t-1}$ in an industry that innovates, and $\alpha^{\frac{2\alpha}{1-\alpha}} \bar{A}_{t-1}$ in an industry that is monopolistic from the previous period. In a competitive industry, it is $\alpha^{\frac{\alpha}{1-\alpha}} \bar{A}_{t-1}$. Therefore,

$$c_i^w = \frac{\theta\gamma\lambda_a\delta_a\alpha^{\frac{2\alpha}{1-\alpha}} + (1-\lambda_a)\delta_a\alpha^{\frac{\alpha}{1-\alpha}} + \lambda_a(1-\delta_a)\alpha^{\frac{2\alpha}{1-\alpha}} + \lambda_a\delta_a\alpha^{\frac{\alpha}{1-\alpha}}}{\lambda_a + \delta_a}(1-\alpha)\bar{A}_{t-1}$$

is the income per worker in the steady state. Thus the flow benefit for the worker is proportional to

$$\frac{\theta\gamma\lambda_a\delta_a\alpha^{\frac{\alpha}{1-\alpha}} + (1-\lambda_a)\delta_a + \lambda_a(1-\delta_a)\alpha^{\frac{\alpha}{1-\alpha}} + \lambda_a\delta_a}{\lambda_a + \delta_a}\bar{A}_{t-1}$$

while the total flow surplus of the entrepreneurs and the policymaker (combined) is proportional to (we can see this by summing up the values for the entrepreneurs and the policymaker above, or directly from equation (A.3))

$$\frac{\theta\gamma\lambda_a\delta_a + \lambda_a(1-\delta_a)}{\lambda_a + \delta_a}\bar{A}_{t-1}.$$

The social planner in Section A.2 looks at the entire social welfare (the combination of the above two) and it is proportional to (from equation (A.2))

$$\frac{\theta\gamma\lambda_a\delta_a\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha) + (1-\lambda_a)\delta_a + \lambda_a(1-\delta_a)\alpha^{\frac{\alpha}{1-\alpha}}(1+\alpha) + \lambda_a\delta_a}{\lambda_a + \delta_a}\bar{A}_{t-1}.$$

λ_e	δ_e	λ_b	δ_b	γ	β	θ	α
0.70	0.50	0.25	0.10	1.10	0.80	1.00	0.30

Table 1: Parameter values

	w	e and p	total
$\bar{L}_e$	1.85	0.26	2.11
$\bar{L}_b$	1.54	0.28	1.82

Table 2: Steady-state welfare, starting from $\bar{A}_0 = 1$

The discrepancy comes from the fact that entrepreneurs capture the profit only in monopolistic industries, while the workers can receive benefit from both monopolistic and competitive industries (even more from the competitive industries). This discrepancy can lead to the policy choice that is beneficial to the entrepreneurs and the policymaker but not for the workers (and the whole economy).

A.5 A numerical example of multiple steady states

In this section, we provide a numerical example where there are multiple steady states in the political equilibrium.

The parameter values are shown in Table 1. The steady state values of L are $\bar{L}_e = 0.58$ and $\bar{L}_b = 0.71$. The steady-state (net) growth rate is $\bar{g}_e - 1 = 2.9\%$ under the free entry regime and $\bar{g}_b - 1 = 0.7\%$ under the entry blocking regime.

The social planner solves the problem (A.2), and the optimal solution is $a = e$ for all L. The political equilibrium is the solution to the problem (A.3), and the solution has the threshold property as in the main text. The threshold is $L^* = 0.61$. Thus, as in Section 3.4.2 (also see Figure 4), the political equilibrium exhibits the multiple steady states. If $L_1 > L^*$, the economy converges to the blocking regime with low growth, and if $L_1 < L^*$, the economy converges to the free entry regime with high growth. Thus, here, an outcome that maximizes the utilitarian social welfare is not chosen if $L_1 > L^*$.

In the political equilibrium, the outcome balances the high growth from entry (the benefit of free entry regime) and the profit from monopoly (the benefit of entry blocking regime).

From the social perspective, the monopoly is costly, in particular for the workers. Therefore, the social planner's solution favors the free entry regime compared to the political equilibrium outcome. To illustrate the discrepancy of interests, Table 2 compares the steady-state welfare (discounted sum of utility) for workers (w), entrepreneurs and policymakers combined (e and p), and the social welfare (total) starting from $\bar{A}_0 = 1$. The workers are better off in the free entry steady state, while the active participants of the political process (entrepreneurs and policymakers) are better off in the entry blocking steady state.

B Formal analysis of the Truthful Nash Equilibrium in the static model

In this section, we formally define and solve for the Truthful Nash Equilibrium, originally developed by Bernheim and Whinston (1986). In order to define the Truthful Nash Equilibrium, we first define a *truthful strategy.*

Definition 3 *$R_i(a)$ (for $i = \ell, f$) is said to be a* ***truthful strategy relative to*** *a^* if, for $a = e, b$, either*

$$N_i(a) = N_i(a^*)$$

or

$$N_i(a) < N_i(a^*) \text{ and } R_i(a^*) = 0.$$

holds.

Now, the definition of the Truthful Nash Equilibrium is as follows.

Definition 4 *$(\{R_i(a)\}_{i=\ell,f;a=e,b}, a^*)$ is a* ***Nash Equilibrium*** *if (i) $R_\ell(a^*)+R_f(a^*) \geq R_\ell(a)+R_f(a)$ for all a and (ii) for each i, there is no other strategy that yields a net payoff greater than $N_i(a^*)$ given $R_j(a)$ ($j \neq i$; $a = e, b$). It is called a* ***Truthful Nash Equilibrium*** *if it is a Nash Equilibrium and each coalition plays a truthful strategy relative to a^*.*

Bernheim and Whinston (1986) show that, in certain settings that contain this section's model as a special case, the Truthful Nash Equilibrium is unique (see their Corollary 1).

It is straightforward to check the following result. (Proof is omitted.)

Proposition B.1 *The Truthful Nash Equilibrium in the static model is:*

1. *If* $V_\ell(b) - V_\ell(e) \geq V_f(e) - V_f(b)$,

$$(R_\ell(e), R_\ell(b)) = (0, V_f(e) - V_f(b))$$

$$(R_f(e), R_f(b)) = (V_f(e) - V_f(b), 0)$$

and $a^* = b$ *is selected.*

2. *Otherwise,*

$$(R_\ell(e), R_\ell(b)) = (0, V_\ell(b) - V_\ell(e))$$

$$(R_f(e), R_f(b)) = (V_\ell(b) - V_\ell(e), 0)$$

and $a^* = e$ *is selected.*

The payoff (before the political contributions are made) for the leader coalition is maximized by the regime of entry blocking b. The excess payoff for the leader from the policy b is

$$V_\ell(b) - V_\ell(e) = L(\delta_e - \delta_b)\pi_\ell, \tag{B.1}$$

and this is the upper bound of the political contribution that the leader coalition is willing to pay in order to obtain the policy b. Similarly, the excess payoff from the policy e for the follower coalition is given by

$$V_f(e) - V_f(b) = (1 - L)(\lambda_e - \lambda_b)\pi_f. \tag{B.2}$$

When $L(\delta_e - \delta_b)\pi_\ell \geq (1 - L)(\lambda_e - \lambda_b)\pi_f$, the leaders can "outbid" the followers. Thus it is easy to see that the outcome policy will be b in any Nash equilibrium in this case. It is an equilibrium strategy for the follower coalition to offer any amount up to $L(\delta_e - \delta_b)\pi_\ell$ as $R_f(e)$ (and the leader coalition has to "match" this amount as $R_\ell(b)$ in order to win), since the follower coalition "loses the bid" in equilibrium and does not have to pay $R_f(e)$. Thus there are multiple Nash equilibria, with different values of $R_f(e)$ and $R_\ell(b)$, with the equilibrium policy b. But the equilibria with $R_f(e) > V_f(e) - V_f(b)$ are not very reasonable since this bid is similar to an "empty threat"—they are willing to offer it because they know that they will not pay it. This is the reason we need the refinement of Truthful Nash Equilibrium (which is unique here).

Using (B.1) and (B.2), the condition that $a^* = b$ is selected can be rewritten as:

$$\frac{L}{(1-L)}\frac{(\delta_e - \delta_b)\pi_\ell}{(\lambda_e - \lambda_b)\pi_f} \geq 1. \tag{B.3}$$

The term $L/(1-L)$ represents the "extensive margin" (the size of the coalitions). The term $(\delta_e - \delta_b)\pi_\ell/(\lambda_e - \lambda_b)\pi_f$ represents the "intensive margin." Each leader firm's net gain (in expected value) when policy b is chosen is $(\delta_e - \delta_b)\pi_\ell$ and each follower firm's net loss (in expected value) when policy b is chosen is $(\lambda_e - \lambda_b)\pi_f$. This condition shows that both margins matter.

The condition (B.3) can also be rewritten as a threshold rule:

$$a = \begin{cases} e & \text{if } L < L^* \\ \{e, b\} & \text{if } L = L^* \\ b & \text{if } L > L^*, \end{cases}$$

where the threshold L^* is

$$L^* = \frac{(\lambda_e - \lambda_b)\pi_f}{(\lambda_e - \lambda_b)\pi_f + (\delta_e - \delta_b)\pi_\ell}.$$

This is the equation (1) in the main text.

Our equilibrium formulation allows us to characterize the final payoff for each agent. Define the net payoff for each firm in coalition i $(i = \ell, f)$ as

$$n_\ell \equiv \frac{N_\ell(a^*)}{L}$$

and

$$n_f \equiv \frac{N_f(a^*)}{1-L}.$$

It is straightforward to show that

$$n_\ell = \begin{cases} (1-\delta_e)\pi_\ell & \text{if } L < L^* \\ \dfrac{L^*}{L}(1-\delta_e)\pi_\ell + \left(1 - \dfrac{L^*}{L}\right)((1-\delta_b)\pi_\ell + (\lambda_e - \lambda_b)\pi_f) & \text{if } L \geq L^*, \end{cases}$$

and

$$n_f = \begin{cases} \dfrac{1-L^*}{1-L}\lambda_b\pi_f + \left(1 - \dfrac{1-L^*}{1-L}\right)(\lambda_e\pi_f + (\delta_b - \delta_e)\pi_\ell) & \text{if } L < L^* \\ \lambda_b\pi_f & \text{if } L \geq L^*. \end{cases}$$

From these, we can see that n_ℓ is constant in L when $L < L^*$ and increasing in L when $L \geq L^*$. We can also see that n_f is decreasing in L when $L < L^*$ and constant in L if $L \geq L^*$. The revenue of the policymaker is

$$R_\ell(a^*) + R_f(a^*) = \begin{cases} L(\delta_b - \delta_e)\pi_\ell & \text{if } L < L^* \\ (1-L)(\lambda_e - \lambda_b)\pi_f & \text{if } L \geq L^*. \end{cases}$$

Thus the revenue is increasing in L up to L^* and decreasing in L after L^*. That is, the government revenue is maximized when $L = L^*$. The policymaker can extract more revenue from private agents when they have neck-and-neck political power. An implication of our model is that when the policymaker is revenue-maximizing, she prefers a situation in which two private coalitions have close political power, rather than where one of them strongly dominates the other. In this sense, the policymaker has a preference for "conflict" among private coalitions.

Finally, we present an alternative method of characterizing the equilibrium. Let $S(L)$ be the sum of all payoffs (i.e., payoffs of firms and the policymaker) in the Truthful Nash Equilibrium. Here we make the dependence of the payoff on L explicit. Then, by definition,

$$S(L) = N_\ell(a^*) + N_f(a^*) + R_\ell(a^*) + R_f(a^*) = V_\ell(a^*) + V_f(a^*).$$

Since $a^* = b$ when $V_\ell(b) + V_f(b) \geq V_\ell(e) + V_f(e)$ and $a^* = e$ otherwise,

$$S(L) = \max_{a \in \{e,b\}} V_\ell(a) + V_f(a)$$

holds. In other words, the Truthful Nash Equilibrium maximizes the sum of payoffs (we call it "total surplus"). In fact, this is a special case of Bernheim and Whinston's (1986) Theorem 2. As we show in the main text, this property also holds in the dynamic model.

Let $S_a(L) \equiv V_\ell(a) + V_f(a)$. From (B.1) and (B.2), we can see that $S_a(L)$ is a linear function of L:

$$S_a(L) = [(1-\delta_a)\pi_\ell - \lambda_a \pi_f]L + \lambda_a \pi_f.$$

It is easy to show that $S_e(0) > S_b(0)$ and $S_e(1) < S_b(1)$. Thus, there exists a threshold value of L, L^*, such that if $L \geq L^*$, $S_e(L) \leq S_b(L)$ holds and $a = b$ is chosen. In fact, this L^* corresponds to the one defined in (1). Thus this is an alternative route for getting to the threshold characterization.

C Existence of the Truthful Markov Perfect Equilibrium

In this section, we show the existence of the Truthful Markov Perfect Equilibrium. Unfortunately, since contraction arguments cannot be used, the normal techniques to prove existence of equilibrium are unavailable when the state variable L is continuous. However, the continuous state variable has a benefit of allowing us simple and tractable characterizations of the equilibrium policy action $a^*(L)$.

Bergemann and Välimäki (2003, Theorem 4) show that the Truthful Markov Perfect Equilibrium exists in an economy with finite state. We cannot use the same argument (applying Brouwer's fixed-point theorem) as theirs, because our state is continuous. It is not straightforward to apply other fixed-point theorems—for example, the compactness requirement in Schauder's theorem is not easy to be ensured in our context.

Thus, we take the following strategy. We first introduce a discrete version of the economy, in which we prove (following Bergemann and Välimäki (2003)) that an equilibrium exists. Then we show that the equilibrium policy action in the discrete-state economy converges to the one in the continuous-state economy as the discrete states become "finer" and approach a continuum.

C.1 Existence of discrete-state equilibrium

Below we will prove the existence of the equilibrium by modifying the proof by Bergemann and Välimäki (2003). The main difference between our model and the model considered by Bergemann and Välimäki (2003) is that Bergemann and Välimäki's (2003) model is with finite states, while in our model one of the state variables (L) is continuous. This difference forces us first to start from considering a model with discrete state variables that approximates our original model. In this section, we establish the existence of the equilibrium in this discrete-state economy. Note that our existence proof allows for the discount factors of firms and the policymaker to be different.

Consider a discrete-state version of our economy as in the following. There are m firms in the economy. We normalize the population and assume that each firm has a mass $1/m$, so that the total mass is one.[20] Denote the mass of leaders when there are i leaders in the economy as $L^i \equiv i/m$. At the beginning of each period, the status of every firm as a leader or a follower is publicly known; therefore the state of the economy is an m-dimensional vector of firm states ω. We denote the space of the states as Ω. Clearly the cardinality of Ω is 2^m. As before, the probability that a leader in the current period will remain a leader is given by $1-\delta_a$, where a is the policy regime; similarly the probability that a follower will switch to a leader is λ_a. We assume that the transitions are independent across firms.

If we condition on the number of leaders in the current period and on the policy regime, the number of leaders in the following period is a sum of a discrete and finite number of independent random variables with nonzero variance. Thus (unlike the continuum case) the number of leaders in the next period is stochastic. It is a sum of two random variables (the number of leaders that remained leaders and the number of followers that switched) which follow binomial distributions.

In order to construct the transition probabilities for the aggregate state, first consider a subgroup of firms (that can include both leaders and followers) and let the number of firms in the subgroup be k. Define $Q^k_{i,j}(a)$ to be the probability that this subgroup, which has i leaders in the current period, will have j leaders in the following period when the current period policy is a:

$$Q^k_{i,j}(a) = \begin{cases} \displaystyle\sum_{h=\max\{j-(k-i),0\}}^{\min\{i,j\}} \binom{i}{h}(1-\delta_a)^h \delta_a^{i-h} \binom{k-i}{j-h} \lambda_a^{j-h}(1-\lambda_a)^{k-i-(j-h)} & \text{if } 0 \le j \le k, \\ 0 & \text{otherwise.} \end{cases} \tag{C.1}$$

The first line of (C.1) represents the transition probability for a case that is possible to happen (there cannot be more than k leaders in this subgroup at any period). Each term in the first line consists of two parts. The first part is the probability that h firms among the current i leaders remain leaders next period. The second part is the probability that $j-h$ firms among the current $k-j$ followers become leaders next period. Let $P_{i,j}(a)$ be the conditional probability that there are j leaders in the entire economy in period $t+1$ (that

[20]Later on, we take a limit of $m \to \infty$. The equilibrium strategies and payoffs clearly depend on m. To simplify the notation, here we suppress this dependence.

is, $L_{t+1} = L^j$) when there are i leaders at time t (that is, $L_t = L^i$) and the time-t policy is a. Then, $P_{i,j}(a)$ is given by $P_{i,j}(a) = Q^m_{i,j}(a)$.

We assume that, at the beginning of the period, all leaders form a "leader coalition" and all followers form a "follower coalition" in order to pursue their respective collective interests. We do not investigate the process of coalition formation and multilateral bargaining inside the coalition, and will only look at symmetric outcomes. Firms can commit to the collective action only for one period and condition their strategies only on payoff-relevant variables (Markovian assumption). Since all firms have the same transition probabilities and are treated symmetrically, the coalition maximizes the expected discounted payoff of the representative firm in each coalition. The symmetry assumptions imply that the only payoff-relevant aggregate variable for a firm is the number of leaders i or, equivalently, the mass of leaders $L^i (= i/m)$. As in our baseline model, let $n_s(L^i)$ be the expected payoff of firm type s ($s = \ell, f$) if there is L^i mass of leaders; let $v_s(a, L^i)$ be the corresponding gross (before the payment to the policymaker) payoff.

Then $v_s(a, L^i)$ can be expressed as

$$v_\ell(a, L^i) = (1-\delta_a)\pi_\ell + \beta(1-\delta_a)\sum_{j=1}^{m} Q^{m-1}_{i-1,j-1}(a) n_\ell(L^j) + \beta\delta_a \sum_{j=0}^{m-1} Q^{m-1}_{i-1,j}(a) n_f(L^j) \quad \text{(C.2)}$$

for a leader and

$$v_f(a, L^i) = \lambda_a \pi_f + \beta\lambda_a \sum_{j=1}^{m} Q^{m-1}_{i,j-1}(a) n_\ell(L^j) + \beta(1-\lambda_a)\sum_{j=0}^{m-1} Q^{m-1}_{i,j}(a) n_f(L^j) \quad \text{(C.3)}$$

for a follower. The values for the entire coalitions are $V_f(a, L^i) = v_f(a, L^i)(1 - L^i)$ and $V_\ell(a, L^i) = v_\ell(a, L^i)L^i$, similarly to the baseline (continuous-state) model. Denoting the payment to the policymaker from coalition s ($s = \ell, f$) as $R_s(a, L^i)$ and the individual payments as $r_\ell(a, L^i) \equiv R_\ell(a, L^i)/L^i$ and $r_f(a, L^i) \equiv R_f(a, L^i)/(1 - L^i)$ (with value zero when the corresponding population is zero), $n_s(L^i)$ can be defined in the same manner as in the main text.

Similarly, the Bellman equation for the policymaker is as follows:

$$G(L_i) = \max_{a \in \{e,b\}} \left\{ R_\ell(a; L_i) + R_f(a; L_i) + \beta_g \sum_{j=1}^{m} P_{i,j} G(L_j) \right\} \quad \text{(C.4)}$$

Note that the policymaker's discount factor is denoted as $\beta_g \in (0,1)$ and we allow it to be different from the firm's discount factor β.

Then the Truthful Markov Perfect Equilibrium in the discrete economy is defined in essentially the same manner as in Definition 1. Proposition C.1 below shows that the Truthful Markov Perfect Equilibrium exists. Moreover, the political outcome $a^{m*}(L^i)$ is equivalent to the solution that maximizes the discounted present value (denoted $S^m(L^i)$ in Proposition C.1 below) of total profit (surplus) in each period $L^i(1-\delta_a)\pi_\ell + (1-L^i)\lambda_a\pi_f$. In the next section, we show that this equivalence holds even when L is continuous, and use this fact extensively in characterizing the Truthful Markov Perfect Equilibrium in Section 3.3.2 and Section 3.4.

Proposition C.1 *A Truthful Markov Perfect Equilibrium exists. Moreover, if $\beta = \beta_g$ the equilibrium action profile $a^{m*}(L^i)$ is the maximizer in the following Bellman equation:*

$$S^m(L^i) = \max_{a\in\{b,e\}} \left\{ L^i(1-\delta_a)\pi_\ell + (1-L^i)\lambda_a\pi_f + \beta \sum_{j=0}^{m} P_{i,j}(a) S^m(L^j) \right\}. \tag{C.5}$$

Proof. See Appendix E. ■

Next, we show that the political outcome from this discrete economy converges to the one with the continuous state variable L as $m \to \infty$. This allows us to view the continuous-state model as approximating a discrete-state model with many possible values of L, in which the existence of an equilibrium is guaranteed.

C.2 Convergence from discrete-state model to continuous-state model

In the following, we assume that $\beta_g = \beta$. Below, we show the convergence in terms of total surplus and equilibrium policy functions. Proposition C.2 below establishes the convergence of the total surplus function.

Proposition C.2 *Let $S^m(L^i)$ and $S(L)$ be expected discounted total surplus functions for the discrete-state model and continuous-state model. $S^m(L^i)$ solves (C.5) and $S(L)$ solves*

(2). Then

$$\lim_{m\to\infty} \max_{L^i\in\{0,1/m,\ldots,1\}} \left|S(L^i) - S^m(L^i)\right| = 0$$

holds. Here, $S(L^i)$ means the value of the $S(L)$ function evaluated at $L = L^i$.

Proof. See Appendix E. ■

Next we show that when Proposition 4 can be applied and thus the policy outcome has the threshold property, the policy outcome for the discrete-state economy $a^{m*}(L^i)$ converges to the policy outcome for the continuous-state model $a^*(L)$.

The intuition for the proof is that the loss from "taking the wrong action" increases as L moves farther from L^* (which is established in the proof of Proposition 4). With this fact and the fact that $S^m(L^i)$ converges to $S(L)$, we are able to show that the policy in the continuous-state model approximates the policy in the discrete-state model when the number of states is sufficiently large.

Proposition C.3 *Suppose that the (continuous-state) economy's environment is such that Proposition 4 can be applied. For the case of statement 1 in Proposition 4, there exists an m^{***} such that if $m \geq m^{***}$, $a^{m*}(L^i) = e$ holds for all $L^i \in \{0, 1/m, \ldots, 1\}$. For the cases of statements 2 and 3, let L^* be the threshold. Then there exist bounds $\mathbf{L}_l^m \in [0,1]$ and $\mathbf{L}_u^m \in [0,1]$ such that $\mathbf{L}_l^m < L^* < \mathbf{L}_u^m$, where $a^{m*}(L_i) = e$ if $L_i \leq L_l^m$ and $a^{m*}(L^i) = b$ if $L_i \geq L_u^m$ for $L^i \in \{0, 1/m, \ldots, 1\}$. Moreover, $\lim_{m\to\infty} \mathbf{L}_l^m = \lim_{m\to\infty} \mathbf{L}_u^m = L^*$.*

Proof. See Appendix E. ■

D Cyclical dynamics

Our framework permits a policy-induced cyclical dynamics. Many previous studies, such as Krusell and Ríos-Rull (1996), Bridgman et al (2004), and Bellettini and Ottaviano (2005), highlighted the ability of their models to generate policy cycles.

Cyclical dynamics occur when $\bar{L}_b < L^* < \bar{L}_e$. This corresponds to the last panel of Figure 1. This occurs when $\lambda_b/\delta_b < \lambda_e/\delta_e$ and π_ℓ is sufficiently larger than π_f (but not too large).

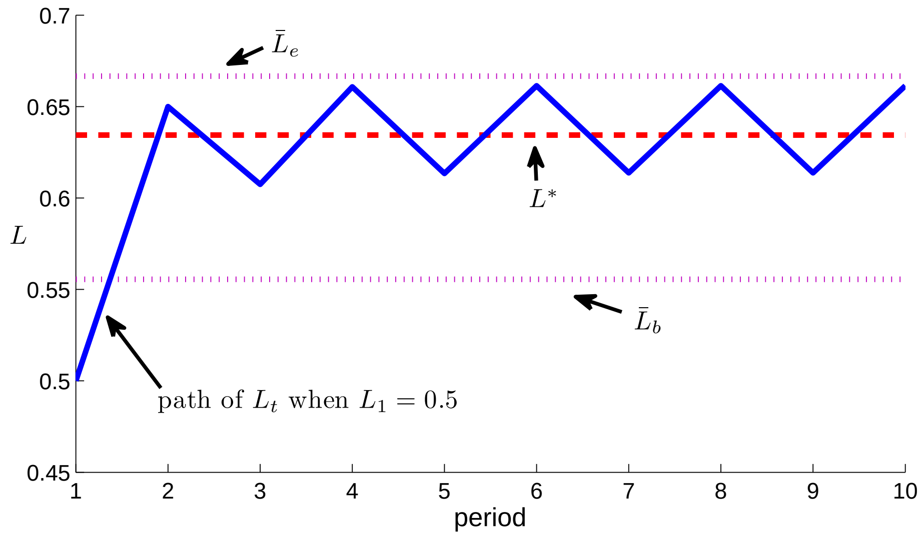

Figure 8: Example with cyclical dynamics

When the economy starts from the free entry regime, L_t increases, and this strengthens the incumbents' coalition and the entry blocking policy follows. The blocking regime leads to a small L_t, weakens the incumbents' political power, and it leads to the free entry regime. This interaction repeats, exhibiting cyclical dynamics in L_t and frequent policy switches.

Figure 8 is an example of cyclical dynamics. The parameter values are: $\pi_\ell = 7$, $\pi_f = 1$, $\lambda_e = 0.6$, $\delta_e = 0.3$, $\lambda_b = 0.25$, $\delta_b = 0.2$, and $\beta = 0.8$.[21] Figure 9 is the revenue path for the policymaker. As in the other examples, the revenue is high when L_t is close to L^*.

E Proofs

Proof of Proposition 2. Define $Z(L) \equiv Ln_\ell(L)+(1-L)n_f(L)+G(L)$. In equilibrium, the following holds. (Superscript * is added to clarify that all value functions are in equilibrium.)

$$\begin{aligned} Z^*(L) &= Ln^*_\ell(L) + (1-L)n^*_f(L) + G^*(L) \\ &= V^*_\ell(a^*(L), L) + V^*_f(a^*(L), L) + \beta G^*(\mathcal{L}(a^*(L), L)) \\ &= (1-\delta_{a^*(L)})L\pi_\ell + \lambda_{a^*(L)}(1-L)\pi_f + \beta Z^*(\mathcal{L}(a^*(L), L)), \end{aligned}$$

where the second equality uses the fact that the payment that the coalitions pay is equal to the payment that the policymaker receives in equilibrium, and the third equality uses the

[21] The equilibrium is the "Regular Case" at all t.

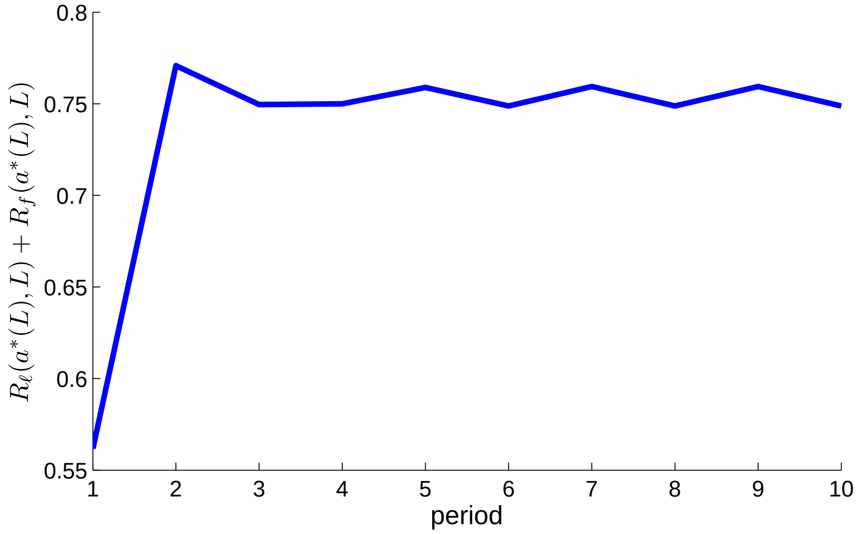

Figure 9: Example with cyclical dynamics: the revenue flow for the policymaker

definitions of $v_\ell(a, L)$, $v_f(a, L)$, $\mathcal{L}(a, L)$, and $Z(L)$.

Now, notice that in the Truthful Markov Perfect Equilibrium (Proposition 1), $V_\ell^*(a^*(L), L) + V_f^*(a^*(L), L) + \beta G^*(\mathcal{L}(a^*(L), L)) \geq V_\ell^*(a, L) + V_f^*(a, L) + \beta G^*(\mathcal{L}(a, L))$ for $a \neq a^*(L)$. From the above equation, this means that $(1 - \delta_{a^*(L)}) L\pi_\ell + \lambda_{a^*(L)}(1 - L)\pi_f + \beta Z^*(\mathcal{L}(a^*(L), L)) \geq (1 - \delta_a) L\pi_\ell + \lambda_a(1 - L)\pi_f + \beta Z^*(\mathcal{L}(a, L))$ for $a \neq a^*(L)$. This implies that $Z^*(L)$ satisfies

$$Z^*(L) = \max_a \left\{(1 - \delta_a) L\pi_\ell + \lambda_a(1 - L)\pi_f + \beta Z^*(\mathcal{L}(a, L))\right\},$$

which means that $Z^*(L)$ is a solution to the functional equation (2) and $a^*(L)$ is its policy function. ■

Proof of Proposition 3. The operator satisfies monotonicity and discounting, therefore by Blackwell's theorem it is a contraction on the space of bounded functions (see, for example, Stokey, Lucas, and Prescott (1989)). Let f be a convex function. Then $p_a L + q_a + \beta f(\mathcal{L}(a, L))$ is convex in L as composition of a convex and an affine function. Therefore Tf is convex (as a maximum of convex functions) if f is convex. Then since the space of bounded convex functions is closed, the unique fixed point is bounded and convex, and therefore continuous.

The statement that S is differentiable everywhere except on an (at most) countable set is equivalent to Theorem 25.3 in Rockafellar (1970).

Suppose that $p_b/(1-\beta\gamma_b) > p_e/(1-\beta\gamma_e)$. Let A be the set of bounded convex functions f such that $f'(L) \in [p_e/(1-\beta\gamma_e), p_b/(1-\beta\gamma_b)]$. If $f \in A$, Tf is differentiable almost everywhere, and where it is differentiable

$$(Tf)'(L) = p_a + \beta\gamma_a f'(\mathcal{L}(a, L))$$

holds, where a is the optimal action at L. The derivative can be bounded as

$$\begin{aligned}
(Tf)'(L) &= p_a + \beta\gamma_a f'(\mathcal{L}(a, L)) \\
&\leq p_a + \beta\gamma_a \frac{p_b}{1-\beta\gamma_b} \\
&= \frac{p_b}{1-\beta\gamma_b} + (1-\beta\gamma_a)\left(\frac{p_a}{1-\beta\gamma_a} - \frac{p_b}{1-\beta\gamma_b}\right) \\
&\leq \frac{p_b}{1-\beta\gamma_b}.
\end{aligned}$$

In a similar fashion, we establish that $(Tf)'(L) \geq p_e/(1-\beta\gamma_e)$. So the operator T maps A to itself. We will show that A is closed in the sup norm.

Let f_i be a sequence of functions, $f_i \in A$, that converges to f in the sup norm. Since the functions are convex and continuous, the condition $f \in A$ is equivalent to $(f(L_2) - f(L_1))/(L_2-L_1) \in [p_e/(1-\beta\gamma_e), p_b/(1-\beta\gamma_b)]$ for $L_1 \neq L_2$. Since $(f_i(L_2)-f_i(L_1))/(L_2-L_1) \in [p_e/(1-\beta\gamma_e), p_b/(1-\beta\gamma_b)]$ for all f_i, and $(f(L_2) - f(L_1))/(L_2 - L_1)$ is continuous in $f(L_1)$ and $f(L_2)$, $f(L)$ also satisfies this condition. Therefore $f \in A$, and thus A is closed in the sup norm. Then by the Contraction Mapping Theorem, $S \in A$.

The case when $p_b/(1-\beta\gamma_b) \leq p_e/(1-\beta\gamma_e)$ is analogous. ■

Proof of Statement 1 of Proposition 4. Let T be the mapping defined by the right-hand side of (2). Since T is a contraction, if we show that $TS_e = S_e$, we prove that $S_e = S$ and that $e \in a^*(L)$. From the definitions we see that $TS_e(L) = \max\{S_e(L), S_{b,e}(L)\}$. Then it will be sufficient to show that $g(L) \equiv S_e(L) - S_{b,e}(L) > 0$ for all L. We will show this by proving (i) $g(0) > 0$ and (ii) $g'(L) \geq 0$. First, (i) is shown as

$$\begin{aligned}
g(0) &= \lambda_e \pi_f - \lambda_b \pi_f + \beta \left(S_e(\lambda_e) - S_e(\lambda_b) \right) \\
&= (\lambda_e - \lambda_b) \left(\pi_f + \beta \frac{p_e}{1 - \beta\gamma_e} \right) \\
&= (\lambda_e - \lambda_b) \frac{\beta(1-\delta_e)\pi_\ell + (1 - \beta + \beta\delta_e)\pi_f}{1 - \beta\gamma_e} > 0,
\end{aligned}$$

where we used the fact that S_e is linear and the definitions. Second, (ii) is shown as

$$g'(L) = \frac{p_e}{1-\beta\gamma_e} - p_b - \beta\gamma_b \frac{p_e}{1-\beta\gamma_e} = (1-\beta\gamma_b)\left(\frac{p_e}{1-\beta\gamma_e} - \frac{p_b}{1-\beta\gamma_b} \right) \geq 0.$$

■

Proof of Statement 2 of Proposition 4.

(a) Direct calculation yields:

$$S_b'(L) - S_{e,b}'(L) = (1-\beta\gamma_e)\left(\frac{p_b}{1-\beta\gamma_b} - \frac{p_e}{1-\beta\gamma_e} \right) > 0. \tag{E.1}$$

We will divide this case into two subcases: (i) $S_b(0) \geq S_{e,b}(0)$ and (ii) $S_b(0) < S_{e,b}(0)$.

First, suppose that $S_b(0) \geq S_{e,b}(0)$. Then from the inequality above, it follows that $S_b(L) \geq S_{e,b}(L)$ for all L. Therefore $TS_b(L) = \max\{S_b(L), S_{e,b}(L)\} = S_b(L)$, so $S(L) = S_b(L)$ and $b \in a^*(L)$ for all $L \in [0, 1]$. We can consider this as a case where $L^* < 0$.

Second, suppose that $S_b(0) < S_{e,b}(0)$. Then (since both $S_b(0)$ and $S_{e,b}$ are linear) there exists some $L^* \in (0, \bar{L}_e]$ such that $S_b(L^*) = S_{e,b}(L^*)$. We will show that the threshold policy "$a(L) = e$ if $L < L^*$ and $a(L) = b$ if $L \geq L^*$" is optimal. Let $\hat{S}(L)$ be the value of following this policy.

Let $\mathcal{L}^{(i)}(a, L)$ be the value of L_{t+i} when the value of L_t is L and the policy a is conducted at time $t, t+1, ..., t+i-1$. Thus $\mathcal{L}^{(1)}(a, L) = \mathcal{L}(a, L)$, $\mathcal{L}^{(2)}(a, L) = \mathcal{L}(a, \mathcal{L}(a, L))$, and so on.

Let $n = \min\{i : \mathcal{L}^{(i)}(e, 0) \geq L^*\}$. That is, n is the minimum number of steps it takes to reach a value of L larger than L^* when L starts from zero and the policy

e is taken. Then define $L^{(i)}$, $i = 1, \ldots, n-1$ by $\mathcal{L}^{(i)}(e, L^{(i)}) = L^*$. That is, $L^{(i)}$ is the exact starting value of L which, after i periods of policy e, can lead to L^*. (Also let $L^{(0)} = L^*$ and $L^{(n)} = 0$ when necessary.) $L^{(i)}$ partitions the interval $[0, 1]$ into $[0, L^{(n-1)})$, $[L^{(n-1)}, L^{(n-2)})$, $[L^{(n-2)}, L^{(n-3)})$, ..., $[L^{(1)}, L^*)$, $[L^*, 1]$.

First consider L in the interval $[L^*, 1]$. Since $\bar{L}_b \geq L^*$, $\hat{S}(L) = S_b(L)$. Second, consider $L \in [L_1, L^*)$. Since L becomes larger than L^* after one period of policy e, $\hat{S}(L) = S_{e,b}(L)$. By definition $S_b(L^*) = S_{e,b}(L^*)$, so $\hat{S}$ is continuous at L^*. Since $\hat{S}(L) = p_e L + q_e + \beta \hat{S}(\mathcal{L}(e, L))$ for $L < L^*$, by induction, it follows that $\hat{S}$ is continuous everywhere. $\hat{S}(L)$ is differentiable everywhere, except at $L^{(i)}$ and L^*. Next consider the interval $L \in (L^{(i)}, L^{(i-1)})$,

$$\hat{S}'(L) = \sum_{j=1}^{i} p_e (\beta\gamma_e)^{j-1} + (\beta\gamma_e)^i \frac{p_b}{1 - \beta\gamma_b} = (1 - (\beta\gamma_e)^i) \frac{p_e}{1 - \beta\gamma_e} + (\beta\gamma)^i \frac{p_b}{1 - \beta\gamma_b}$$

Then $\hat{S}'(L)$ is increasing and $\hat{S}'(L) \leq p_b/(1 - \beta\gamma_b)$.

Let T be the mapping defined by the right-hand side of (2). Now we show that $T\hat{S} = \hat{S}$. Define

$$g(L) \equiv p_e L + q_e + \beta \hat{S}(\mathcal{L}(e, L)) - p_b L - q_b - \beta \hat{S}(\mathcal{L}(b, L)). \tag{E.2}$$

Suppose that $L \geq L^*$. Then $\mathcal{L}(a, L) \geq L^*$. Therefore:

$$\begin{aligned} T\hat{S}(L) &= \max\{p_e L + q_e + \beta \hat{S}(\mathcal{L}(e, L)), p_b L + q_b + \beta \hat{S}(\mathcal{L}(b, L))\} \\ &= \max\{S_{e,b}(L), S_b(L)\} \\ &= S_b(L) \\ &= \hat{S}(L). \end{aligned}$$

Note that here $g(L) = S_{e,b}(L) - S_b(L)$ is decreasing in L from (E.1). Suppose that $L < L^*$. Then $\hat{S}(L) = p_e L + q_e + \beta \hat{S}(\mathcal{L}(e, L))$. We will show that $g(L)$ defined in (E.2) is positive. Here, $g(L) = \hat{S}(L) - p_b L - q_b - \beta \hat{S}(\mathcal{L}(b, L))$. By the choice of L^*, $g(L^*) = 0$. Since g is continuous and differentiable always everywhere, it is sufficient to show that

$g'(L) < 0$ where it is defined.

$$\begin{aligned} g'(L) &= \hat{S}'(L) - p_b - \beta\gamma_b \hat{S}'(\mathcal{L}(b, L)) \\ &\leq \hat{S}'(L) - p_b - \beta\gamma_b \hat{S}'(L) \\ &= (1 - \beta\gamma_b)\left(\hat{S}'(L) - \frac{p_b}{1 - \beta\gamma_b}\right) < 0, \end{aligned}$$

where we used the fact that $\mathcal{L}(b, L) > L$ and $\hat{S}'(L)$ is increasing.

(b) The proof in this case is symmetric to case (a). Note that $g(L)$ defined as (E.2) is decreasing here as well.

(c) $S_b(\bar{L}_e) < S_{e,b}(\bar{L}_e) = p_e\bar{L}_e + q_e + \beta S_b(\bar{L}_e)$ implies that $S_b(\bar{L}_e) < (p_e\bar{L}_e + q_e)/(1 - \beta) = S_e(\bar{L}_e)$. Similarly, $S_b(\bar{L}_b) > S_e(\bar{L}_b)$. Since S_b and S_e are both linear, there exists L^* defined by

$$S_b(L^*) = S_e(L^*).$$

We will show that policy $a(L) = e$ if $L \leq L^*$, $a(L) = b$ if $L \geq L^*$ is optimal. The value of following this policy is:

$$\hat{S}(L) = \begin{cases} S_e(L) & \text{if } L < L^* \\ S_b(L) & \text{if } L \geq L^*. \end{cases}$$

It is sufficient to show that $T\hat{S} = \hat{S}$. Define $g(L)$ by (E.2). It will be sufficient to show that $g(L) \geq 0$ on $[0, L^*]$ and $g(L) \leq 0$ on $[L^*, 1]$. By the choice of L^*, $g(L^*) = 0$. It is straightforward to show that $\hat{S}(L)$ is continuous, therefore $g(L)$ is also continuous. Since $g(L)$ is continuous, it will be sufficient to show that $g'(L) < 0$ where it is defined; $g'(L)$ can be calculated as

$$g'(L) = p_e + \beta\gamma_e\hat{S}'(\mathcal{L}(e, L)) - p_b - \beta\gamma_b\hat{S}'(\mathcal{L}(b, L)).$$

If $L \leq L^*$, $\mathcal{L}(e, L) < L^*$, so

$$g'(L) \leq p_e + \beta\gamma_e\frac{p_e}{1 - \beta\gamma_e} - p_b - \beta\gamma_b\frac{p_e}{1 - \beta\gamma_e} = (1 - \beta\gamma_b)\left(\frac{p_e}{1 - \beta\gamma_e} - \frac{p_b}{1 - \beta\gamma_b}\right) < 0.$$

If $L > L^*$, $\mathcal{L}(b, L) > L^*$, so

$$g'(L) \leq p_e + \beta\gamma_e\frac{p_b}{1 - \beta\gamma_b} - p_b - \beta\gamma_b\frac{p_b}{1 - \beta\gamma_b} = (1 - \beta\gamma_e)\left(\frac{p_e}{1 - \beta\gamma_e} - \frac{p_b}{1 - \beta\gamma_b}\right) < 0.$$

■

Proof of Statement 3 of Proposition 4.

(a) Identical to the proof of case (a) of statement 2 of Proposition 4. Note that $g(L)$ defined as (E.2) is decreasing here as well.

(b) Identical to the proof of case (b) of statement 2 of Proposition 4. Note that $g(L)$ defined as (E.2) is decreasing here as well.

(c) Again, let T be the mapping defined by the right-hand side of (2). Now define $g(L)$ by

$$g(L) \equiv p_e L + q_e + \beta S(\mathcal{L}(e, L)) - p_b L - q_b - \beta S(\mathcal{L}(b, L)). \tag{E.3}$$

Note that the fixed point S is used in the definition. By Proposition 3, g is continuous and differentiable everywhere except on a countable set. We will show that there exists a $L^* \in (\bar{L}_b, \bar{L}_e)$ where $g(L^*) = 0$, $g(L) > 0$ for $L \in [0, L^*)$, and $g(L) < 0$ for $L \in (L^*, 1]$, which implies the proposition.

The bounds on $S'(L)$ (from Proposition 3) imply that

$$\begin{aligned} g'(L) &= p_e + \beta\gamma_e S'(\mathcal{L}(e, L)) - p_b L - \beta\gamma_b S'(\mathcal{L}(b, L)) \\ &\leq p_e + \beta\gamma_e \frac{p_b}{1 - \beta\gamma_b} - p_b - \beta\gamma_b \frac{p_e}{1 - \beta\gamma_e} \\ &= (1 - \beta\gamma_e - \beta\gamma_b) \left(\frac{p_e}{1 - \beta\gamma_e} - \frac{p_b}{1 - \beta\gamma_b} \right) \\ &< 0. \end{aligned}$$

Define the set B as $B \equiv \{L \in (\bar{L}_b, \bar{L}_e) : g(L) = 0\}$. Suppose $g(L) > 0$ for all $L \in (\bar{L}_b, \bar{L}_e)$. Then for all $L \in (\bar{L}_b, \bar{L}_e)$, $S(L) = p_e L + q_e + \beta S(\mathcal{L}(e, L))$. Since $\mathcal{L}(e, L) \in (\bar{L}_b, \bar{L}_e)$, it then follows that $S(L) = S_e(L)$ on $(\bar{L}_b, \bar{L}_e)$. Then by continuity, $S(\bar{L}_e) = S_e(\bar{L}_e)$. This leads to a contradiction, since by assumption $S_e(\bar{L}_e) < S_{b,e}(\bar{L}_e)$.

In a similar fashion, we show that $g(L) < 0$ for all $L \in (\bar{L}_b, \bar{L}_e)$ is impossible. Therefore $B \neq \emptyset$. Take an arbitrary $L^* \in B$. Since $g'(L) < 0$ almost everywhere, $g(L) > 0$ for $L \in [0, L^*)$ and $g(L) < 0$ for $L \in (L^*, 1]$. This also shows that B is a singleton.

$\blacksquare$

Before the proof of Proposition C.1, we need one technical lemma.

Lemma E.1 *For all integer i, j, k and action a:*

$$i(1-\delta_a)Q^{k-1}_{i-1,j-1}(a) + (k-i)\lambda_a Q^{k-1}_{i,j-1}(a) = jQ^k_{i,j}(a)$$

and

$$i\delta_a Q^{k-1}_{i-1,j}(a) + (k-i)(1-\lambda_a)Q^{k-1}_{i,j}(a) = (k-j)Q^k_{i,j}(a)$$

hold.

Proof. Suppose that there are i leaders and $k-i$ followers in the economy. Without loss of generality, index the leaders $h = 1,\ldots,i$ and the followers $h = i+1,\ldots,k$. Let W_h be a random variable that is 1 if in the next period (i) the firm h is a leader and (ii) there are exactly j leaders in the economy. Define $X \equiv \sum_{h=1}^{i} W_h$, $Y \equiv \sum_{h=i+1}^{k} W_h$, and $Z \equiv \sum_{h=1}^{k} W_h$. Then the expected values of these, conditional on there being i leaders and $k-i$ followers today, is

$$\begin{aligned}
E[Z] &= E\left[E\left[\sum_{h=1}^{k} W_h \,\middle|\, \#\text{ leaders next period} = j\right]\right] \\
&= E\left[\sum_{h=1}^{k} W_h \,\middle|\, \#\text{ leaders next period} = j\right] \times \Pr[\#\text{ leaders next period} = j] \\
&\quad + E\left[\sum_{h=1}^{k} W_h \,\middle|\, \#\text{ leaders next period} \neq j\right] \times \Pr[\#\text{ leaders next period} \neq j] \\
&= jQ^k_{i,j}(a),
\end{aligned}$$

$$E[X] = E\left[\sum_{h=1}^{i} W_h\right] = iE[W_1] = i(1-\delta_a)Q^{k-1}_{i-1,j-1}(a),$$

and

$$E[Y] = E\left[\sum_{h=i+1}^{k} W_h\right] = (k-i)E[W_{i+1}] = j\lambda_a Q^{k-1}_{i,j-1}(a).$$

Then the first statement follows from the fact that $X+Y = Z$, and thus $E[X]+E[Y] = E[Z]$.

The proof of the second statement is analogous. $\blacksquare$

Proof of Proposition C.1. The proof is an adaptation of the proof of Theorem 4 in Bergemann and Välimäki (2003). The proof first aims to establish the existence of a fixed point in the mapping from the list of net payoffs by firms and the policymaker, $\{\{n_\ell(L^i)\}_{i=0}^m$, $\{n_f(L^i)\}_{i=0}^m$, $\{G(L^i)\}_{i=0}^m\}$ to itself. That is, given $\{\{n_\ell(L^i)\}_{i=0}^m$, $\{n_f(L^i)\}_{i=0}^m$, $\{G(L^i)\}_{i=0}^m\}$ and using these as the continuation payoffs, one can assign (using a particular procedure) the current values (we will call them $\{\{n'_\ell(L^i)\}_{i=0}^m$, $\{n'_f(L^i)\}_{i=0}^m$, $\{G'(L^i)\}_{i=0}^m\}$) that correspond to a Truthful Markov Perfect Equilibrium to firms and policymakers. If we can find a current list of (net) payoffs that are identical to the list of payoffs we started with (i.e. the list of payoffs has a fixed point), the list constitutes the Truthful Markov Perfect Equilibrium (by construction), and we are done with the existence proof. In establishing the existence of the fixed point, we utilize Brouwer's fixed point theorem.

Let $\hat{\beta} = \max\{\beta, \beta_g\}$. Define $K = \max\{\pi_\ell, \pi_f\}/(1-\hat{\beta})$. Let $n_\ell = \{n_\ell(L^i)\}_{i=0}^m$, $n_f = \{n_f(L^i)\}_{i=0}^m$, $G = \{G(L^i)\}_{i=0}^m$ be arbitrary real $(m+1)-$dimensional vectors such that $n_\ell \in [0,K]^{m+1}$, $n_f \in [0,K]^{m+1}$, and $G \in [0, K/(1-\hat{\beta})]^{m+1}$.

Define $v_\ell(a, L^i)$ and $v_f(a, L^i)$ as in Appendix C and let $V_\ell(a, L^i) = L^i v_\ell(a, L^i)$, $V_f(a, L^i) = (1-L^i)v_f(a, L^i)$. Also, let

$$V_{f,\ell}(a, L^i) = V_\ell(a, L^i) + V_f(a, L^i)$$

and

$$V_\emptyset(a, L^i) = 0.$$

From (C.2), (C.3), and Lemma E.1,

$$V_{f,\ell}(a, L^i) = (1-\delta_a)L^i\pi_\ell + \lambda_a(1-L^i)\pi_f + \beta\sum_{j=0}^m P_{i,j}(a)[L^j n_\ell(L^j) + (1-L^j)n_f(L^j)].$$

Also let

$$W_S(L^i) = \max_{a\in\{e,b\}} V_S(a, L^i) + \beta_g \sum_{j=0}^m P_{i,j}(a)G(L^j),$$

where $S \subseteq \{f, \ell\}$. By the Maximum Theorem, W_S is continuous in the vectors n_ℓ, n_f, and G.

Set

$$N'_\ell(L^i) = W_{\ell,f}(L^i) - W_f(L^i), \tag{E.4}$$

$$N'_f(L^i) = \min\{W_{\ell,f}(L^i) - W_\emptyset(L^i) - N'_\ell(L^i), W_{\ell,f}(L^i) - W_\ell(L^i)\}, \tag{E.5}$$

and

$$G'(L^i) = W_{\ell,f}(L^i) - N'_\ell(L^i) - N'_f(L^i).$$

By Theorem 2 in Bernheim and Whinston (1986) (since these net payoffs are in the Pareto efficient frontier in their language), these payoff vectors can be supported by an action $a^*(L^i)$ that maximizes $V_{\ell,f}(a, L^i) + \beta_g \sum_{j=0}^m P_{i,j}(a)G(L^j)$ and a pair of bidding schedules that constitute a truthful Nash equilibrium. By the Maximum Theorem, the payoffs are continuous in the continuation vectors (that is, $\{\{n_\ell(L^i)\}_{i=0}^m, \{n_f(L^i)\}_{i=0}^m, \{G(L^i)\}_{i=0}^m\}$). Define $n'_\ell(L^i) = N'_\ell(L^i)/L^i$ if $L^i > 0$ and $n'_\ell(0) = 0$; and $n'_f(L^i) = N'_f(L^i)/(1 - L^i)$ if $L^i < 1$ and $n'_f(1) = 0$. Then $n'_j(L)$ is continuous in the continuation vectors. Next, we show that $n'_j \in [0, K]^{m+1}$ $(j \in \{\ell, f\})$ and $G \in [0, K/(1 - \hat{\beta})]^{m+1}$.

$n'_\ell(0) = 0$ by construction. When $L^i > 0$, $n'_\ell(L^i) = N'_\ell(L^i)/L^i = (V_\ell(a^*, L^i) - R_\ell(a^*))/L^i \leq V_\ell(a^*, L^i)/L^i = v_\ell(a^*, L^i)$. From (C.2) and the definition of K,

$$\begin{aligned}
v_\ell(a, L^i) &= (1 - \delta_a)\pi_\ell + \beta(1 - \delta_a)\sum_{j=1}^m Q^{m-1}_{i-1,j-1}(a)n_\ell(L^j) + \beta\delta_a \sum_{j=0}^{m-1} Q^{m-1}_{i-1,j}(a)n_f(L^j) \\
&\leq (1 - \hat{\beta})K + \beta(1 - \delta_a) \sum_{j=1}^m Q^{m-1}_{i-1,j-1}(a)K + \beta\delta_a \sum_{j=0}^{m-1} Q^{m-1}_{i-1,j}(a)K \\
&= (1 - \hat{\beta})K + \beta(1 - \delta_a)K + \beta\delta_a K \\
&\leq K.
\end{aligned}$$

Since a bidding schedule $R_\ell(e) = 0, R_\ell(b) = 0$ is feasible and $\{N'_\ell(L^i)\}$ is supported as a Nash equilibrium, we have that:

$$n'_\ell(L^i) = N'_\ell(L^i)/L^i \geq \min_a V_\ell(a, L^i)/L^i = \min_a v_\ell(a, L^i) \geq 0.$$

Then we have that $n'_\ell(L^i) \in [0, K]$ for all L^i. With similar steps, one can also prove that $n_f(L^i) \in [0, K]$. Finally, for $G'(L^i)$ we know that:

$$G'(L^i) = W_{\ell,f}(L^i) - N'_\ell(L^i) - N'_f(L^i) \leq W_{\ell,f}(L^i).$$

$W_{\ell,f}(L^i)$ satisfies the following inequality:

$$\begin{aligned}
W_{\ell,f} &= \max_{a\in\{e,b\}}\left\{(1-\delta_a)L^i\pi_\ell+\lambda_a(1-L^i)\pi_f+\beta\sum_{j=0}^{m}P_{i,j}(a)[L^in_\ell(L^i)+(1-L^i)n_f(L^i)]\right.\\
&\qquad\left.+\beta_g\sum_{j=0}^{m}P_{i,j}(a)G(L^i)\right\}\\
&\le \max_{a\in\{e,b\}}\left\{(1-\hat{\beta})K+\beta\sum_{j=0}^{m}P_{i,j}(a)K+\beta_g\sum_{j=0}^{m}P_{i,j}(a)K/(1-\hat{\beta})\right\}\\
&\le K/(1-\hat{\beta}).
\end{aligned}$$

Therefore, $G'(L^i)\le K/(1-\hat{\beta})$.

On the other hand, (E.4) and (E.5) imply that $N'_\ell(L^i)+N'_f(L^i)\le W_{\ell,f}(L^i)-W_\emptyset(L^i)$. So,

$$G'(L^i)=W_{\ell,f}(L^i)-N'_\ell(L^i)-N'_f(L^i)\ge W_\emptyset(L^i)=\beta_g\sum_{j=0}^{m}P_{i,j}(a)G(L^i)\ge 0.$$

Therefore, the mapping described above, from $\{\{n_\ell(L^i)\}_{i=0}^m,\ \{n_f(L^i)\}_{i=0}^m,\ \{G(L^i)\}_{i=0}^m\}$ to $\{\{n'_\ell(L^i)\}_{i=0}^m,\ \{n'_f(L^i)\}_{i=0}^m,\ \{G'(L^i)\}_{i=0}^m\}$, is continuous and maps the compact and convex set $[0,K]^{m+1}\times[0,K]^{m+1}\times[0,K/(1-\hat{\beta})]^{m+1}$ to itself. Then, by Brouwer's fixed point theorem, there exists a vector of payoffs that is a fixed point of the mapping described above. By Theorem 2 in Bernheim and Whinston (1986), these are equilibrium payoffs of the Truthful Nash Equilibrium of the one-shot game with specified continuation payoffs. Therefore, they are payoffs of a Truthful Markov Perfect Equilibrium.

Next, we show the second statement. Let $S^{m*}(L^i)\equiv L^in^*_\ell(L^i)+(1-L^i)n^*_f(L^i)+G^*(L^i)$.

(The * denotes that these functions constitute a Truthful Markov Perfect Equilibrium.) Then,

$$
\begin{aligned}
S^{m*}(L^i) &= L^i n_\ell^*(L^i) + (1-L^i)n_f^*(L^i) + G^*(L^i) \\
&= V_\ell^*(a^{m*}(L^i), L^i) - R_\ell^*(a^{m*}(L^i), L^i) + V_f^*(a^{m*}(L^i), L^i) - R_f^*(a^{m*}(L^i), L^i) \\
&\quad + \sum_{s\in\{\ell,f\}} R_s^*(a^{m*}(L^i), L^i) + \beta \sum_{j=0}^{m} P_{i,j}(a^{m*}(L^i))G^*(L^j) \\
&= V_\ell^*(a^{m*}(L^i), L^i) + V_f^*(a^{m*}(L^i), L^i) + \beta \sum_{j=0}^{m} P_{i,j}(a^{m*}(L^i))G^*(L^j) \\
&= (1-\delta_{a^{m*}(L^i)})L^i\pi_\ell + \lambda_{a^{m*}(L^i)}(1-L^i)\pi_f \\
&\quad + \beta \sum_{j=0}^{m} P_{i,j}(a^*(L^i))[L^j n_\ell^*(L^j) + (1-L^j)n_f^*(L^j) + G^*(L^j)] \\
&= \max_{a\in\{e,b\}} \Bigg\{ (1-\delta_a)L^i\pi_\ell + \lambda_a(1-L^i)\pi_f \\
&\qquad + \beta \sum_{j=0}^{m} P_{i,j}(a)[L^j n_\ell^*(L^j) + (1-L^j)n_f^*(L^j) + G^*(L^j)] \Bigg\} \\
&= \max_{a\in\{e,b\}} \left\{ (1-\delta_a)L^i\pi_\ell + \lambda_a(1-L^i)\pi_f + \beta \sum_{j=0}^{m} P_{i,j}(a) S^{m*}(L^j) \right\},
\end{aligned}
$$

where the fifth equation is from Theorem 2 in Bernheim and Whinston (1986) (there, it is shown that the equilibrium action $a^{m*}(L^i)$ attains $W_{\ell,f}(L^i)$). ■

Before proving Proposition C.2, we need some preparations.

Lemma E.2 *Let $\eta > 0$, $\epsilon > 0$ be arbitrary. For given L and a, there exists m^* such that for all $m \geq m^*$,*

$$\Pr[|L'_a - \mathcal{L}(a, L)| < \eta] > 1 - \epsilon, \quad \forall L \in \{0, 1/m, \ldots 1\}, a \in \{e, b\}$$

where L'_a is the mass of leaders in the next period if policy a is implemented.

Proof. This is an application of the Weak Law of Large Numbers. Let X_a^j be a random variable distributed according to a binomial distribution with parameter $(1-\delta_a)$ and j attempts. Let Y_a^j be a similar random variable with parameter λ_a. Then if the number of leaders is currently i, $L'_a = (X^i_a + Y^{m-i}_a)/m$. From the definitions, we see that $\text{var}(X^i_a) = i\delta_a(1-\delta_a)$ and $\text{var}(Y^{m-i}_a) = (m-i)\lambda_a(1-\lambda_a)$. Since the transitions of firms' states are independent events,

$$\text{var}(L'_a) = \frac{\text{var}(X^i_a) + \text{var}(Y^{m-i}_a)}{m^2} \leq \frac{\max\{\delta_a(1-\delta_a), \lambda_a(1-\lambda_a)\}}{m}.$$

The Chebyshev inequality implies that

$$\Pr[|L'_a - \mathcal{L}(a, L)| < \eta] \geq 1 - \frac{\text{var}(L'_a)}{\eta^2}.$$

Let $m^*_a = \lceil \max\{\delta_a(1-\delta_a), \lambda_a(1-\lambda_a)\}/\epsilon\eta^2 + 1\rceil$. Then if $m \geq m^*_a$, $\text{var}(L'_a)/\eta^2 < \epsilon$ for any L. Then if $m \geq m^*_a$,

$$\Pr[|L'_a - \mathcal{L}(a, L)| < \eta] > 1 - \epsilon, \quad \forall L \in \{0, 1/m, \ldots 1\}.$$

Finally, let $m^* = \max\{m^*_e, m^*_b\}$. This m^* satisfies all the requirements in the lemma. ■

Proof of Proposition C.2. Let T^m and T be the Bellman operators in the right-hand sides of (C.5) and (2) respectively. For a function $f : [0, 1] \to \Re$, define the function $H^m f : \{0, 1/m, \ldots 1\} \to \Re$ by $H^m f(i/m) = f(i/m)$. Defining the sup norm in the standard manner, the proposition is equivalent to $\lim_{m\to\infty} ||H^m S - S^m|| = 0$. Since T^m is a contraction mapping, $||T^m H^m S - S^m|| \leq \beta ||H^m S - S^m||$, where we used $T^m S^m = S^m$. From the triangle inequality, $||T^m H^m S - H^m S|| + ||T^m H^m S - S^m|| \geq ||H^m S - S^m||$. Combining these two, $||H^m S - S^m|| \leq ||T^m H^m S - H^m S||/(1-\beta)$ holds. Therefore, it is sufficient to show that $||T^m H^m S - H^m S|| \to 0$ as $m \to \infty$.

Set some arbitrary $\epsilon' > 0$, such that $\epsilon' < K/(1-\beta)$. Since S is uniformly continuous, there exists some $\eta > 0$ such that $|L' - L''| < \eta$ implies $|S(L') - S(L'')| < \epsilon'$ for any L' and L''. Set $\epsilon = (1-\beta)\epsilon'/(K\beta/(1-\beta) - \beta\epsilon')$. Let m^* be such that it satisfies the conclusion of Lemma E.2 for these η and ϵ. (Note that m^* only depends on ϵ' and not on L^i.) We will show that if $m \geq m^*$, then $|(T^m H^m S)(L^i) - S(L^i)| < \epsilon'$ for all $L^i \in \{0, 1/m, \ldots 1\}$, thus proving the proposition.

Let $f(a, L^i; S) \equiv (1-\delta_a)L^i\pi_\ell + \lambda_a(1-L^i)\pi_f + \beta S(\mathcal{L}(a, L^i))$ and $f^m(a, L^i; S) \equiv (1-\delta_a)L^i\pi_\ell + \lambda_a(1-L^i)\pi_f + \beta\sum_{j=0}^m p_{i,j}(a)S(L^j)$. Then $(T^m H^m S)(L^i) = \max_{a'} f^m(a', L^i; S)$ and $S(L^i) = \max_{a''} f(a'', L^i; S)$. Thus, if we establish that when $m \geq m^*$, $|f^m(a, L^i; S) - f(a, L^i; S)| < \epsilon'$ holds for all L^i and a, we are done. (If $f^m(a, L^i; S)$ is within ϵ' band of $f(a, L^i; S)$, then $\max_{a'} f^m(a', L^i; S)$ is within ϵ' band of $\max_{a''} f(a'', L^i; S)$.)

Now, suppose that $m \geq m^*$. Pick some L^i and a. Let $j_1 = \min\{j : j/m \geq \mathcal{L}(a, L_i) - \eta\}$ and $j_2 = \max\{j : j/m \leq \mathcal{L}(a, L_i) + \eta\}$. By the choice of m^*, it follows that $\sum_{j=j_1}^{j_2} p_{i,j}(a) \geq 1-\epsilon$ for $m \geq m^*$. By the choice of η, $|S(L_j) - S(\mathcal{L}(a, L_i))| < \epsilon'$ if $j \in \{j_1 \dots j_2\}$. Since $S(L) \in [0, K/(1-\beta)]$, $|S(L') - S(L'')| \leq K/(1-\beta)$ for any L' and L''. Therefore, for $m \geq m^*$,

$$\begin{aligned}
&|f(a, L_i; S) - f^m(a, L_i; S)| \\
&\quad = \beta\left|\sum_{j\neq[j_1,j_2]} p_{i,j}(a)(S(L_j) - S(\mathcal{L}(a, L_i))) + \sum_{j=j_1}^{j_2} p_{i,j}(a)(S(L_j) - S(\mathcal{L}(a, L_i)))\right| \\
&\quad \leq \beta\sum_{j\neq[j_1,j_2]} p_{i,j}(a)\frac{K}{1-\beta} + \beta\sum_{j=j_1}^{j_2} p_{i,j}(a)\epsilon' \\
&\quad \leq \beta\epsilon\frac{K}{1-\beta} + \beta(1-\epsilon)\epsilon' \\
&\quad = \epsilon'
\end{aligned}$$

holds. Since ϵ' was arbitrary, we are done. ■

Proof of Proposition C.3. For the first statement of Proposition 4, it is straightforward to show the result using the fact that $S^m(L)$ converges to $S(L)$ from Proposition C.2.

Consider the cases of statements 2 and 3 of Proposition 4. Set arbitrary $\mathbf{L}_l$ and $\mathbf{L}_u$ such that $\mathbf{L}_l < L^* < \mathbf{L}_u$. It will be sufficient to show that for all m sufficiently large, $a^{m*}(L) = e$ if $L \leq \mathbf{L}_l$ and $a^{m*}(L) = b$ if $L \geq \mathbf{L}_u$.

Define $g(L) \equiv p_e L + q_e + \beta S(\mathcal{L}(e, L)) - p_b L - q_b - \beta S(\mathcal{L}(b, L))$. This is the same $g(L)$ in the proof of Proposition 4 (i.e. (E.2) and (E.3)), because there we showed that $\hat{S}(L)$ is indeed $S(L)$ for (E.2). In the proof of Proposition 4, we have shown that it is a strictly decreasing function. Analogously, define $g^m(L^i) \equiv p_e L^i + q_e - p_b L^i - q_b + \beta\sum_{j=0}^m [p_{i,j}(e) - p_{i,j}(b)]S^m(L^j)$.

Then $a^{m*}(L) = e$ if $g^m(L) > 0$ and $a^{m*}(L) = b$ if $g^m(L) < 0$.

$$
\begin{aligned}
|g(L^i) - g^m(L^i)| &= \beta \left| \sum_{j=0}^{m} p_{i,j}(e)[S(\mathcal{L}(e,L)) - S^m(L^j)] - \sum_{j=0}^{m} p_{i,j}(b)[S(\mathcal{L}(b,L)) - S^m(L^j)] \right| \\
&\le \sum_{a\in\{e,b\}} \max_{i\in\{0,\ldots,m\}} \left| \sum_{j=0}^{m} p_{i,j}(a)[S(\mathcal{L}(a,L)) - S^m(L^j)] \right| \\
&\le \sum_{a\in\{e,b\}} \max_{i\in\{0,\ldots,m\}} \left| \sum_{j=0}^{m} p_{i,j}(a)[S(\mathcal{L}(a,L)) - S(L^j)] \right| + 2||S^m - H^m S||
\end{aligned}
$$

In the proof of Proposition C.2, we showed that both terms converge to 0 as $m \to \infty$. Therefore, there exists some M, such that for all $m \ge M$, $|g(L) - g^m(L)| < \min\{|g(\mathbf{L}_l)|, |g(\mathbf{L}_u)|\}$.

Then if $L \le \mathbf{L}_l$,

$$g_m(L) = g(L) + g_m(L) - g(L) \ge g(L) - |g_m(L) - g(L)| > g(L) - |g(\mathbf{L}_l)| \ge 0.$$

The last inequality follows since $L \le \mathbf{L}_l < L^*$, $g(L^*) = 0$, and $g(L)$ is strictly decreasing. Therefore, $a^*_m(L) = e$ if $L \le \mathbf{L}_l$. The proof for $L \ge \mathbf{L}_u$ is analogous. ■

Proof of Proposition 5. Choose some natural number T such that $(L_t - \bar{L}_e)\gamma_e^T + \bar{L}_e < L^*$. Such a T exists since $\gamma_e < 1$ and $\bar{L}_e < L^*$.

Let $\tilde{a}_{t+s}$, $s = 0, \ldots, T-1$ be an arbitrary sequence of policies, and $\tilde{L}_{t+s}$ be the corresponding sequence of leader masses. We will use $\hat{L}_{t+s}$ to denote the sequences with the policy choice $a_{t+s} = e$, $\forall s$. Note that since $L_t \ge L^* > \bar{L}_e$, $\hat{L}_{t+s} \le \tilde{L}_{t+s}$ holds, with strict inequality if the sequence contains at least one b before $t+s-1$. (This can be shown from $L_{t+1} = L_t\gamma_a + \bar{L}_a(1-\gamma_a)$, $\gamma_b > \gamma_e$, and $\bar{L}_b > \bar{L}_e$.)

The total surplus from the sequence of policy choices $\tilde{a}_s$ is then given by:

$$\mathbf{S}(\tilde{a}_t, \tilde{a}_{t+1}, \ldots, \tilde{a}_{t+T-1}) \equiv \sum_{s=0}^{T-1} \beta^s[(1-\delta_{\tilde{a}_s})\tilde{L}_{t+s}\pi_\ell + \lambda_{\tilde{a}_s}(1-\tilde{L}_{t+s})\pi_f'] + \beta^T S(\tilde{L}_{s+T}).$$

Therefore, $\mathbf{S}(\tilde{a}_t, \tilde{a}_{t+1}, \ldots, \tilde{a}_{t+T-1})$ is linear in π_f' and

$$\frac{\partial}{\partial \pi_f'}\mathbf{S}(\tilde{a}_t, \tilde{a}_{t+1}, \ldots, \tilde{a}_{t+T-1}) = \sum_{s=0}^{T-1} \beta^s \lambda_{\tilde{a}_s}(1-\tilde{L}_{t+s}).$$

.

Thus

$$\frac{\partial}{\partial \pi'_f}\mathbf{S}(e,e,\ldots,e) - \frac{\partial}{\partial \pi'_f}\mathbf{S}(\tilde{a}_t,\tilde{a}_{t+1},\ldots,\tilde{a}_{t+T-1}) = \sum_{s=0}^{T-1}\beta^s[\lambda_e(1-\hat{L}_{t+s}) - \lambda_{\tilde{a}_s}(1-\tilde{L}_{t+s})].$$

Let B be the set of all sequences of policy choices $\{\tilde{a}_{t+s}\}_{s=0}^{T-1}$ such that $\tilde{a}_{t+s} = b$ for some s. Let $(\tilde{a}_t,\ldots,\tilde{a}_{t+T-1}) \in B$ and let $(\tilde{L}_t,\ldots,\tilde{L}_{t+T-1})$ be the corresponding sequence of leader masses. Since $\lambda_e \geq \lambda_{\tilde{a}_s}$ and $1-\hat{L}_{t+s} \geq 1-\tilde{L}_{t+s}$ with at least one strict inequality (and $\hat{L}_{t+s}, \tilde{L}_{t+s} < 1$ for $s \geq 1$), it follows that

$$\frac{\partial}{\partial \pi'_f}\mathbf{S}(e,e,\ldots,e) > \frac{\partial}{\partial \pi'_f}\mathbf{S}(\tilde{a}_t,\tilde{a}_{t+1},\ldots,\tilde{a}_{t+T-1}).$$

Moreover, since the set B is finite,

$$\frac{\partial}{\partial \pi'_f}\mathbf{S}(e,e,\ldots,e) > \max_{(\tilde{a}_t,\ldots,\tilde{a}_{t+T-1})\in B}\frac{\partial}{\partial \pi'_f}\mathbf{S}(\tilde{a}_t,\tilde{a}_{t+1},\ldots,\tilde{a}_{t+T-1}).$$

Thus for π'_f sufficiently large, $\mathbf{S}(e,e,\ldots,e) > \mathbf{S}(\tilde{a}_t,\tilde{a}_{t+1},\ldots,\tilde{a}_{t+T-1})$ for all possible sequences $\tilde{a}_{t+s}$. Therefore $a_{t+s} = e$ is chosen for all $s < T$.

For $s \geq T$, by the choice of T, $L_{t+T} < L^*$ holds. Thus $a_{t+s} = e$ for all $s \geq T$ as well. ■

Additional References for Appendix

[1] Rockafellar, R. Tyrrell (1970). *Convex Analysis*, Princeton: Princeton University Press.