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Industrialization and the Evolution of Enforcement Institutions

Toshihiko Mukoyama · Latchezar Popov

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Abstract We construct a dynamic general equilibrium model with incomplete contracts to examine the interaction between factor accumulation, institutions of contract enforcement, and political-economy frictions. In the cross-country data, an economy's exposure to enforcement frictions is correlated with the degree of industrialization. Theoretically, we find the incompleteness of contracts leads to underinvestment in relation-specific capital and causes production inefficiency due to misallocation. Misallocation occurs through two channels: unbalanced inputs for each product and unbalanced production across products. In addition to production inefficiency, the imperfect contract enforcement leads to distortions in factor supplies. We analyze the dynamic patterns of enforcement institutions by allowing the government to invest in the improvement of the contractual environment ("institutional capital"). We analyze how different types of governments choose different patterns of institutional investment over time. A higher level of institutional capital can enhance industrialization through directly improving production efficiency and indirectly encouraging physical-capital accumulation. As institutional capital is accumulated, the economy shifts production toward industries that are more vulnerable to contractual enforcement. We highlight the role of government commitment in the equilibrium accumulation of institutional capital.

Keywords: Industrialization, Institution, Incomplete contract, Misallocation
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https://sites.google.com/site/toshimukoyama/MP_ET_Online_Appendix.pdf

T. Mukoyama
Department of Economics, Georgetown University, 37th and O Streets, N.W., Washington D.C., 20057-1036, USA
Tel.: +1-202-687-5601
Fax: +1-202-687-6102
E-mail: tm1309@georgetown.edu

L. Popov
Department of Economics, Texas Tech University, P.O. Box 41014 Lubbock, Texas, 79409-1014, USA
Tel.: +1-806-834-5178
Fax: +1-806-742-1137
E-mail: latchezar.popov@ttu.edu

1 Introduction

Many researchers have emphasized the importance of well-functioning economic institutions in the process of long-run economic growth. In their survey of the recent literature, Acemoglu, Johnson, and Robinson (2005a) summarize the empirical evidence and conclude institutions are the fundamental cause of income differences and long-run growth. Helpman (2004) echoes this view by stating institutions promoting the rule of law, enforce contracts, and limit the power of rulers are important for economic development.[1]

As a general concept, economic institutions can be considered a broad set of rules. For example, in his influential work, North (1990) defines institutions as the "rules of the game" in a society. Since the pioneering work of De Soto (1989) and (2000), the fact that such "rules of the game" vary substantially across countries has been widely documented.[2] In this paper, we focus on one particular element of such economic institutions: *enforcement of contracts*. This aspect of institutions is often viewed as essential in promoting economic development. North forcefully argues the inability of societies to develop effective, low-cost enforcement of contracts is the most important source of both historical stagnation and contemporary underdevelopment in the Third World. In our analysis, we consider one particular effect of imperfect contract enforcement: underinvestment in relation-specific capital.

The goal of this paper is to provide a dynamic framework that allows us to analyze both the effects of such institutions on the economy and their evolution under different governments. Our model extends Acemoglu, Antràs, and Helpman's (2007) framework into a dynamic environment and allows the government to engage in investment that improves the contractual environment, such as better protection of property rights and better-functioning courts. The importance of the role of the government has been emphasized by various authors, such as North and Thomas (1973), North (1990), and Acemoglu, Johnson, and Robinson (2005a).[3] Treating the evolution of institutions as capital accumulation follows the view that institutional building is a slow-moving process (see, e.g.,

[1] See also North and Weingast (1989) and Acemoglu, Johnson, and Robinson (2005b).

[2] See, for example, Djankov et al. (2002, 2003).

[3] Alternatively, for example, private agents who benefit from better contract enforcement might spend their own resources in order to improve the contracting environment. Popov (2014) considers a related principal-agent model in which the principal chooses the level of enforcement.

North (1990) and Hough and Grier (2014)). We use the term "institutional capital" as an analogy to physical capital stock in production.[4]

To motivate the model analysis, we first empirically investigate the relationship between economic development and the importance of contract enforcement. In particular, we look at the economy's industry composition as an indicator of the importance of the contracting environment. The main idea is that industries requiring a larger amount of specific investments in their production process are affected to a larger degree by the extent of enforcement frictions. We call the degree of exposure to enforcement frictions of an industry its *contract sensitivity.* For the measurement of contract sensitivity, we utilize an index constructed by Nunn (2007).[5] Nunn's measure is constructed as the fraction of intermediate inputs that are nonstandardized, that is, not traded on an exchange and not having a reference price. These intermediate goods correspond to the notion of specific investments. This measure conforms closely to the way we model contractual incompleteness. The details of data and further formal analyses are described in Appendix A and Online Appendix F and G.

Figure 1 plots average contract sensitivity in manufacturing, weighted by value added in different industries for four countries: the United Kingdom (UK), Korea, China, and Poland. We find the following patterns: (1) Average contract sensitivity for the already developed countries, such as the UK, does not exhibit a strong trend; and (2) countries that industrialize successfully, such as South Korea and China, experience a marked increase in average contract sensitivity over time. Poland has exhibited moderate growth in recent years, but not as rapid as the industrialization of South Korea and China; its contract sensitivity exhibits an intermediate pattern between the UK and the rapidly growing economies. In sum, we observe that countries that grow rapidly shift toward more contract-sensitive industries.

[4] North (1981, Chapter 15) is explicit about the analogy to capital stock in describing the incremental nature of institutional change. Some other researchers use a similar term, "social capital," with somewhat different connotation—for example, Knack and Keefer (1997) consider "trust" and "civic norms" as the contents of social capital. Hall and Jones (1999) consider a concept similar to our social capital, and they call it "social infrastructure." They use an index of government antidiversion policy, originally used by Knack and Keefer (1995), and openness to trade in constructing their social infrastructure measure. They argue that social infrastructure is essential in explaining cross-country income differences. Acemoglu, Johnson, and Robinson (2001) and Rodrik, Subramanian, and Trebbi (2004) also find that measures of property rights and rule of law are important in explaining cross-country income differences using different empirical strategies. La Porta et al. (1999) analyze the determinants of government quality, which include the index of property rights, corruption, and bureaucratic delays, in a cross section of countries. In a more specific context of default enforcement, Arellano and Kocherlakota (2014) and Drozd and Serrano-Padial (2018) consider limited capacity of legal system and courts in enforcing financial contract. Drozd and Serrano-Padial (2018) call this *enforcement capacity*, which is a concept very similar to our institutional capital.

[5] Nunn (2007) calls this measure "contract intensity."

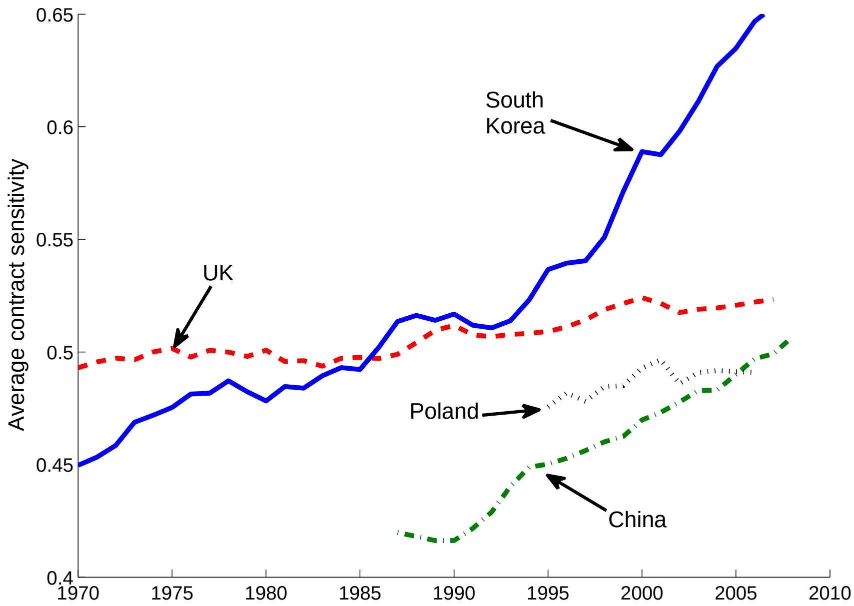

Fig. 1 Average contract sensitivity, weighted by value added

The main contribution of this paper is theoretical. Our model results are related to several strands of literature. We show that, in the model equilibrium, the inefficiencies due to incompleteness of contracts are attributed to two different elements. The first is the loss of productivity due to misallocation. When the investment level is not contractible for some production activities, the investment level of these activities is smaller than the contractible investment, which leads to inefficiently unbalanced input. When the degree of incompleteness is different across products, the quantity produced becomes unbalanced across products as well. This productivity loss shows up as a loss of total factor productivity (TFP) in the aggregate production function in our model. One can view our results as providing a novel microfoundation for the causes of misallocation emphasized in the recent literature, such as Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). The second inefficiency comes from the gap between private and social returns to factor inputs. Contractual incompleteness increases the wedge between the private returns of investment (accruing to the supplier firms that make investment decisions) and its social returns. As a result, the demand of production factors is distorted, which eventually affects the equilibrium factor supply. Our model can be viewed as providing a microfoundation for distortions affecting factor supply, emphasized by Chari, Kehoe, and McGrattan (1997). Many other authors also emphasized the role of investment-good prices on international income differences; see, for example, Restuccia and Urrutia (2001) and Hsieh and Klenow (2007).

Boehm (2018) also analyzes contract enforcement and aggregate productivity. He uses a different index of contract sensitivity (in our terminology) based on litigation data in his cross-country regressions, and his main focus is on the input-output relationship across industries. His empirical study relates the input-output relationship across industries to the enforcement environment and contract sensitivity, whereas our empirical focus in Figure 1 is the time-series pattern of the average contract sensitivity weighted by value added. Boehm's empirical result suggests relation-specific inputs are used less when the contracting environment is worse; this pattern is consistent with what we find for the country averages of contract sensitivity, presented in Appendix A. Boehm's model is static and he mainly analyzes cross-country patterns. By contrast, our focus is the dynamic evolution of contract enforcement institutions over time.

This paper is also related to recent literature that emphasizes the importance of input-output linkages in the context of economic development.[6] Our paper focuses on a particular type of friction in the production linkages: contract enforcement frictions in specific investment.[7]

A large recent literature studies the role of enforcement frictions in financial markets on aggregate outcome. Examples include Erosa and Hidalgo-Cabrillana (2008), Amaral and Quintin (2010), Buera, Kaboski, and Shin (2011), Midrigan and Xu (2014), and Moll (2014). In contrast to this literature, we focus on the enforcement friction in the production process.

Our framework allows us to analyze the interactions between factor accumulation, institutions of contract enforcement, and political-economy frictions. Among the possible political-economy frictions, we focus on the role of government commitment. We analyze a setting where a benevolent government maximizes the representative consumer's utility. We focus on the *positive* implications of the equilibrium outcome: Our setting can, for example, be viewed as the result of a political outcome where the policy is determined by probabilistic voting.[8] We consider three different types of government: (i) the

[6] See, for example, Jones (2011, 2013), Bartelme and Gorodnichenko (2015), and Osotimehin and Popov (2018).

[7] Boehm and Oberfield (2018) analyze firms' production choices in a network using a static model and Indian data on court quality.

[8] In the standard model of probabilistic voting, political competition leads to a policy outcome that maximizes the weighted sum of voters' welfare. See, for example, Persson and Tabellini (2000) for a textbook treatment. With a

government that can commit and dictate the accumulation of both physical capital and institutional capital, (ii) the government that can commit and dictate only the accumulation of institutional capital, and (iii) the government that accumulates institutional capital but cannot commit to its future policy. We show the possibility of commitment can have a large impact.

Our model of institutional evolution features a benevolent social planner who tries to maximize the representative consumer's utility. When a representative agent does not exist, how to transfer the efficiency gains from better institutions across different agents is an additional consideration.[9] Another important implication of heterogeneity is the possibility of political conflict. An influential strand of literature (see, e.g., Acemoglu (2006) for an overview) models the role of political conflict in the persistence of inefficient institutions. Our model abstracts from these considerations. It features different political-economy frictions in the sense that it focuses on the role of government commitment. Thus, our paper complements this literature.

The paper is organized as follows. The next section develops a model of production under incomplete contracts. Section 3 analyzes a model in which the evolution of the institutions is endogenous. Section 4 concludes.

2 Model

Motivated by the empirical regularities above, this section constructs a dynamic general equilibrium model in which contracts between firms can be incomplete. We show that incompleteness of contracts affects aggregate productivity through factor misallocation. It also influences the incentives for factor supply through factor prices.

Our model extends Acemoglu, Antràs, and Helpman (2007). We not only embed their model in a dynamic environment, but also provide characterizations of distortions in the economy with incomplete contract enforcement. To focus on the distortions from the contracting frictions and study the effects of misallocation in more depth, we abstract from the issue of technology adoption, which is the main focus of Acemoglu, Antràs, and Helpman (2007).

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different political setting, distortions can exist in policy choices. See, for example, Mukoyama and Popov (2014) for a recent example with lobbying.

[9] See, for example, Koeppl, Monnet, and Quintin (2014).

Production has three layers: final goods, intermediate goods, and raw materials. Final goods are introduced mainly for the ease of accounting, and our main focus is on the differentiated intermediate-good sector. Each intermediate good is produced by a monopolist who has to engage in the production activity with many suppliers. The contract between an intermediate-good producer and his suppliers is incomplete, and the degree of incompleteness differs across goods. Therefore, each intermediate good in this model corresponds to an industry in the earlier empirical analysis. In section 3, we allow this incompleteness to change over time through the government's activities.

2.1 Environment

The economy consists of a representative consumer and three sectors of production. The final good can be used for consumption and investment and is the numeraire.

2.1.1 Consumers

The representative consumer's utility is

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t),$$

where $\beta \in (0,1)$ is the discount factor, $u(\cdot,\cdot)$ is the period utility function, which is strictly increasing and strictly concave, C_t is consumption at time t, and L_t is the labor supply at time t. The total time endowment is normalized to one.

The household supplies labor, owns a portfolio of shares of all firms in the economy, and owns and accumulates physical capital.[10] Hence, the budget constraint for the consumer is

$$K_{t+1} + C_t + T_t = (1 + r_t - \delta_K)K_t + w_t L_t + \Pi_t,$$

where K_t is the physical capital stock, T_t is the lump-sum tax from the government, and δ_K is depreciation rate of physical capital, r_t is the rental rate of physical capital, and w_t is the wage rate. Π_t is the profit from all firm sectors.

[10] Because a representative consumer exists, explicitly considering a trading of shares is not necessary.

2.1.2 Raw-material producers

Raw materials, which are necessary for producing intermediate-goods inputs, are produced from physical capital and labor in a competitive sector. The representative producer has the production function $F(K, L)$ that has constant returns to scale, is strictly concave, and is differentiable. We also assume standard properties for the marginal product of physical capital: $\lim_{K\to 0} F_1(K, L) = \infty$ and $\lim_{K\to\infty} F_1(K, L) < \delta_K$.

Raw materials are used for investment in creating specialized inputs, which are then used for intermediate-good production. One unit of raw material is priced at q_t. Thus, in equilibrium, the equations

$$r_t = q_t F_1(K_t, L_t) \tag{1}$$

and

$$w_t = q_t F_2(K_t, L_t) \tag{2}$$

determine the factor prices, where we use the notational convention of $F_1(\cdot,\cdot)$ and $F_2(\cdot,\cdot)$ to denote the partial derivatives with respect to the first and the second term.

2.1.3 Intermediate-good producers

A unit measure of *intermediate goods* is indexed by $z \in [0, 1]$. Each intermediate good is produced by a monopolist, and thus we use the index z to also represent each intermediate-good producer. Intermediate goods are made from specialized input $X_t(j)$, where $j \in [0, 1]$, each of which is provided by supplier j. The production function for the intermediate good z is:

$$y_t(z) = \left[\int_0^1 X_t(j)^\alpha dj\right]^{1/\alpha}, \tag{3}$$

where $\alpha \in (0, 1)$. Each supplier provides the specialized input $X_t(j)$, which is made of $x_t(i, j)$ specific investments, where $i \in [0, 1]$ is a type of investment. The input $X(j)$ is produced from

$$X_t(j) = \exp\left[\int_0^1 \ln x_t(i, j) di\right].$$

The specific investment is done by purchasing the corresponding amount of raw materials, whose unit price is q_t. Thus, to invest x_t units, the supplier has to pay $q_t x_t$.

2.1.4 Final-good producers

Final goods are assembled from the intermediate goods under perfect competition. The final good is used as the numéraire, and thus its price is one. The production function is

$$Y_t = \left[\int_0^1 y_t(z)^\phi dz \right]^{1/\phi}, \tag{4}$$

where $y_t(z)$ is the quantity of intermediate good z. We assume $\phi \in (0, 1)$, which implies elasticity of substitution between varieties equal to $1/(1-\phi) > 1$. In equilibrium, the output is used for consumption, investment in physical capital, and government spending:

$$K_{t+1} - (1-\delta)K_t + C_t + T_t = Y_t.$$

The price of each intermediate good z at time t is $p_t(z)$. Because of the constant-returns-to-scale production function and perfect competition, the production of final goods yields no profit, and we can also assume a representative producer.

2.2 An economy with complete contracts

For the purpose of comparison, we first consider a situation in which the contract between the intermediate-good producers and the suppliers can be fully specified. In this case, all investments $\{x_t(i, j)\}$ and the payments to each suppliers $\tau_t(j)$ are determined in advance. We further assume the intermediate-good producers can offer a take-it-or-leave-it contract to the suppliers. Because the suppliers' outside option is zero, the intermediate-good producer can extract all surplus.

For the intermediate-good producer, setting all investments equal to $x_t(i, j) = X_t(j) = y_t(z)$ for all (i, j) is optimal; therefore, the cost to produce $y_t(z)$ units of output is simply $q_t y_t(z)$. From the profit maximization in the final-good sector, the demand for intermediate good z is [11]

$$y_t(z) = p_t(z)^{\frac{1}{\phi-1}} Y_t.$$

[11] Note that because the price of the final good is normalized to one, the demand function can be written in the familiar form for the constant elasticity of substitution demand structure $y_t(z) = (p_t(z)/1)^{\frac{1}{\phi-1}} Y_t$.

Thus, the profit-maximization problem for the intermediate-good producer z is

$$\max_{p_t(z)} \quad \pi_t(z) \equiv (p_t(z) - q_t) p_t(z)^{\frac{1}{\phi-1}} Y_t,$$

and the solution is

$$p_t(z) = \frac{q_t}{\phi}. \tag{5}$$

In terms of quantity, the solution can also be expressed as

$$x_t(i,j) = Y_t q_t^{-\frac{1}{1-\phi}} \phi^{\frac{1}{1-\phi}}. \tag{6}$$

From the production functions, $x_t(i,j) = y_t(z) = Y_t$, which implies $q_t = \phi$ from (6). Note $p(z) = 1$ holds from (5).

2.3 An economy with incomplete contracts

Now we turn to the economy with frictions; that is, contracts are incomplete, so not all investments $\{x(i,j)\}$ can be specified in advance. We first solve the within-period problem. As will become clear, the decisions of intermediate- and final-good producers are static in nature. We show that, within a period, all quantities and prices can be determined as a function of the physical capital, labor, and by the current contractual environment.

2.3.1 The intermediate-good-producer problem

First, we tackle the problem of an intermediate-good producer. The contract between the intermediate-good producers and the suppliers is incomplete. For product z, we assume (for every supplier j) the contract is complete for the investments $0 \leq i \leq \mu(z)$ and incomplete for $\mu(z) < i \leq 1$. That is, for the first $\mu(z)$ investments, the level of investments $x_t(i,j)$, where t denotes time t, is contractible, but for the rest of the investments, $x_t(i,j)$ is determined by suppliers. The departure from Acemoglu, Antràs, and Helpman's (2007) framework is that we allow $\mu(z)$ to be heterogeneous across z. Note that $(1 - \mu(z))$ corresponds to the *contract sensitivity* that we measured for Figure 1. In this section, we take $\mu(z)$ as given and constant. In section 3, we allow $\mu(z)$ to change over time. We rearrange the index z so that $\mu(z)$ is nondecreasing.

Following Acemoglu, Antràs, and Helpman (2007), we adopt the following timing in the game between the intermediate-good producer and the specialized input suppliers.

1. The intermediate-good producer z offers a contract $[\{x_{c,t}(i,j)\}_{i=0}^{\mu(z)}, \tau_t(j)]$ for every $j \in [0,1]$. Here, $x_{c,t}(i,j)$ is the contractible investment level and $\tau_t(j)$ is the upfront payment to supplier j.
2. Specialized input suppliers decide whether to accept the contract.
3. For $0 \le i \le \mu(z)$, the suppliers invest $x_t(i,j) = x_{c,t}(i,j)$ that is specified in the contract. For $\mu(z) < i \le 1$, they *decide* $x_t(i,j)$.
4. The suppliers and the intermediate-good producer bargain over the division of the revenue.
5. Output is produced, sold, and the revenue is distributed following the bargaining agreement.

Note the intermediate-goods producer can anticipate any rent gained by suppliers in the bargaining step and offset the flat payment $\tau_t(j)$. For the noncontractible investment, the payment to the suppliers cannot be contingent on the investment level. The only incentive the suppliers have to put nonzero noncontractible investments is that the anticipated bargaining outcome depends on the noncontractible investment.

The symmetric subgame perfect equilibrium (SSPE) of this game can be defined and characterized in a similar manner as Acemoglu, Antràs, and Helpman (2007). Below, for simplicity, we suppress the dependence on variables in t and z in this subsection.

To solve for the SSPE, we move backwards, starting from the bargaining stage. As in Acemoglu, Antràs, and Helpman (2007), we use the Shapley value as the bargaining solution among the suppliers and the intermediate-good producer. Online Appendix H contains the details of the derivation of the Shapley value. Intuitively, the intermediate goods producer and the suppliers each receive their marginal contribution to the total revenue. The order in which the marginal contribution is calculated affects the result, and the Shapley value is computed as the average of this marginal contribution over all possible orderings of the suppliers and the intermediate-good firm. Following the same steps as Acemoglu, Antràs, and Helpman, we obtain that supplier j receives

$$s_j = (1-\gamma)Y^{1-\phi}\left(\frac{x_n(j)}{x_n}\right)^{(1-\mu)\alpha} x_c^{\phi\mu} x_n^{\phi(1-\mu)}, \tag{7}$$

where $\gamma \equiv \alpha/(\alpha+\phi)$, when it makes $x_n(j)$ units of noncontractible investment, the other suppliers make x_n units of noncontractible investment, and x_c is the amount of contractible investment. Because contractible and noncontractible investments are complements, x_c increases the marginal contribution of every supplier and encourages the investment in $x_n(j)$.

The intermediate-good producer i receives

$$s_i = \gamma Y^{1-\phi} x_c^{\phi\mu} x_n^{\phi(1-\mu)}. \tag{8}$$

Foreseeing the bargaining outcome, each supplier decides the noncontractible investment to maximize its profit. Thus, in a symmetric equilibrium,

$$x_n = \arg\max_{x_n(j)} \quad (1-\gamma)Y^{1-\phi}\left(\frac{x_n(j)}{x_n}\right)^{(1-\mu)\alpha} x_c^{\phi\mu} x_n^{\phi(1-\mu)} - q(1-\mu)x_n(j).$$

Solving this problem, the optimal noncontractible investment by the supplier for a given x_c is[12]

$$x_n(x_c) = \left[\frac{\alpha(1-\gamma)}{q} Y^{1-\phi}\right]^{\frac{1}{1-\phi(1-\mu)}} x_c^{\frac{\mu\phi}{1-\phi(1-\mu)}}. \tag{9}$$

The elasticity of x_n with respect to x_c is $\mu\phi/(1-\phi(1-\mu)) \in (0,1)$. We find, in equilibrium, x_n/x_c is always less than one. When x_n is inefficiently low (it should be equal to x_c for efficiency), the intermediate-good producer can encourage the supplier to increase x_n by increasing x_c. But it faces diminishing returns for this ability, given that $\mu\phi/(1-\phi(1-\mu))$ is less than one. Because $\mu\phi/(1-\phi(1-\mu))$ is increasing in μ, this diminishing-returns problem is more severe when the contract incompleteness is more severe.

The intermediate-good producer i solves the problem

$$\max_{x_c,\tau} \quad s_i - \tau$$

subject to (9) and

$$s_j + \tau \geq \mu q x_c + (1-\mu) q x_n,$$

[12] Another possible equilibrium is $x_n = 0$. We rule out this equilibrium.

where s_i is the surplus received by the intermediate-good producer i, solved in (8), and s_j is the surplus for a supplier, solved in (7). The solution to this problem is

$$x_c = Yq^{-\frac{1}{1-\phi}}\Phi^{\phi(1-\mu)}B(\mu)^{1-\phi(1-\mu)},$$

where

$$\Phi \equiv [\alpha(1-\gamma)]^{\frac{1}{1-\phi}}$$

and

$$B(\mu) \equiv \left\{\frac{\phi}{1-\phi(1-\mu)}[1-(1-\mu)\alpha(1-\gamma)]\right\}^{\frac{1}{1-\phi}}. \tag{10}$$

Comparing this expression with the investment level without enforcement problem, shown in (6), we can see that this level is always lower and coincides with (6) when $\mu = 1$.

By substituting the above x_c in (9), x_n can be solved as

$$x_n = Yq^{-\frac{1}{1-\phi}}\Phi^{1-\mu\phi}B(\mu)^{\mu\phi}.$$

It is straightforward to show x_n is always strictly smaller than the outcome in (6). Because the supplier receives only $(1-\gamma)$ fraction of the reward from investment, it invests too little—this result is a classic hold-up problem. With Lemma 2 (in Appendix B), we show x_c and x_n are increasing in μ, for given q and Y. Because the change in demand for the raw material with respect to μ is

$$\frac{\partial}{\partial\mu}[\mu x_c + (1-\mu)x_n] = x_c - x_n + \mu\frac{\partial x_c}{\partial\mu} + (1-\mu)\frac{\partial x_n}{\partial\mu}$$

and $x_c > x_n$ (which is straightforward to show), this expression is always positive, which is why the equilibrium factor price q (analyzed in the following section) is higher when the enforceability is better. The factor demand is more distorted by a lower μ for three reasons: (i) x_c is lower, (ii) x_n is lower, and (iii) the weight of non-contractible investment becomes larger. When μ is small, the intermediate-good producer has more difficulty inducing a higher (closer to the efficient) level of x_n by choosing a higher level of x_c, which leads to lower values of x_c and x_n in (i) and (ii).

Substituting the investments into the production function, output is

$$y = x_c^{\mu}x_n^{1-\mu} = Yq^{-\frac{1}{1-\phi}}D(\mu), \tag{11}$$

where

$$D(\mu) \equiv \Phi^{1-\mu} B(\mu)^{\mu}. \tag{12}$$

Before we turn to the general equilibrium, we consider the inefficiency at the individual intermediate-good producer level. The TFP at the firm level is defined as output divided by the total amount of raw materials embodied in the various kinds of investments:

$$TFP(\mu) = \frac{y(\mu)}{\mu x_c(\mu) + (1-\mu) x_n(\mu)} = \frac{r(\mu)^{1-\mu}}{\mu + (1-\mu) r(\mu)},$$

where $r(\mu) = x_n(\mu)/x_c(\mu)$, which is less than one and increasing in μ. First, it is straightforward to see that the fact that $r(\mu) < 1$ for $\mu < 1$ is the source of inefficiency. When $\mu = 1$, $r(\mu)$ is irrelevant and the TFP is equal to one. Thus, the *unbalanced input*, which is a form of misallocation, is the source of inefficient production, because the cost-minimizing outcome is to make all investment equal. Second, as μ increases, the ratio $r(\mu)$ increases. This increase in $r(\mu)$ improves efficiency for each investment (intensive margin), because the balanced input is better for efficiency. At the same time, when μ increases from a value close to zero, the balance across investments can become worse, because "some investments are at a higher level" is less balanced than "almost all investments are at a low level" (extensive margin). Taken together, we find that, somewhat surprisingly, $TFP(\mu)$ is U-shaped in μ and attains a minimum at an intermediate level of μ. We return to this property when we discuss the aggregate productivity. This insight is important, because it implies that at the very low level of μ, the social marginal returns to a better enforcement institution from the perspective of the TFP may be low (or even negative).

2.3.2 General equilibrium

Finally, we turn to characterize the aggregate outcome. Acemoglu, Antràs, and Helpman (2007) also analyze the general equilibrium properties of their framework, in which they explain how changes in contractibility affect firm-specific technology choices. The framework here allows a sharper characterization of TFP losses, misallocation, and barriers to capital accumulation. Using (4) and (11), the price of raw materials can be solved as:

$$q_t = \left[\int_0^1 D(\mu(z))^{\phi} dz \right]^{\frac{1-\phi}{\phi}}. \tag{13}$$

Notice that the price of raw materials depends only on the distribution of $\mu(z)$. If this distribution is unchanged, the value of q_t stays constant.

Next, we turn to aggregate output. The total demand for the raw materials is

$$\int_0^1 [\mu(z)x_{c,t}(\mu(z)) + (1-\mu(z))x_{n,t}(\mu(z))]dz = Y_t q^{-\frac{1}{1-\phi}} \int_0^1 H(\mu(z))dz,$$

where

$$H(\mu(z)) \equiv \mu(z)\Phi^{\phi(1-\mu(z))}B(\mu(z))^{1-\phi(1-\mu(z))} + (1-\mu(z))\Phi^{1-\mu(z)\phi}B(\mu(z))^{\mu(z)\phi}. \tag{14}$$

Here, $H(\mu)$ is the normalized demand for inputs by a firm with enforceability μ. Because the supply of the raw materials is $F(K_t, L_t)$, this equation implies (using (13))

$$Y_t = \Theta F(K_t, L_t),$$

where

$$\Theta \equiv \frac{\left[\int_0^1 D(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}}}{\int_0^1 H(\mu(z))dz}. \tag{15}$$

Note Θ depends only on the distribution of $\mu(z)$, and thus the existence of $\mu(z) < 1$ acts as the change in Hicks-neutral technology in this framework. In addition to the unbalanced input that we highlighted above, another source of misallocation is present here: *unbalanced production* across different intermediate goods. If the values of $\mu(z)$ are different across different products, the amounts of production are different. This outcome is inefficient, because the final goods are produced most effectively with equal amounts of intermediate goods.

2.4 Characterizing the economy with incomplete contracts

In this subsection, we characterize the properties of the incomplete-contracts model discussed above. We first develop several general equilibrium properties of the model. Appendix B shows some properties at the level of individual intermediate-good producers. First, the relative output between different producers are independent of K_t and L_t, and only depends on their respective μ and parameters. This result implies the model is inconsistent with the observation in Figure 1. As we show later, once we allow μ to change over time, the industry composition in the model also changes over time. Second, x_c and x_n are increasing in μ. The output for each intermediate-good producer is also increasing in

μ. That is, a firm that faces fewer problems with imperfect enforcablility of contracts produce more. Third, Θ is strictly less than one when μ is strictly less than one for a strictly positive measure of producers. As we discussed earlier, Θ can be less than one for two reasons. The first is an unbalanced input for a given intermediate good z whose $\mu(z) \in (0,1)$. When $\mu(z) \in (0,1)$, the contractible investment x_c and noncontractible investment x_n are different, because they are governed by different incentives. The second is unbalanced production across intermediate goods. If $\mu(z)$ for some z is smaller than others, these intermediate goods are produced at different amounts.

We have seen that, at each point in time, the production side of the economy has an equilibrium such that all prices and all quantities (up to a multiplicative factor $F(K,L)$) depend only on the distribution of enforceability $\mu(z)$. The coefficient Θ describes the effect of these distortions in production. However, enforceability problems also affect the demand for physical capital and labor and therefore their prices. As a result, the supply of labor and the accumulation of physical capital can be distorted due to the incompleteness of the contracts.

Several special cases are notable. First, when $\Theta = 1$ and $q = 1$, the economy is Pareto efficient, as is the case, for example, when the contract is complete and $\phi = 1$. Second, when $\phi < 1$ and contracts are complete, $\Theta = 1$ and $q = \phi < 1$. In this case, even with complete contracts, distortions exist that are reflected to factor income, due to imperfect competition in the intermediate-good sector. Third, if the contractual enforcement is imperfect and $\phi < 1$, then $\Theta < 1$, and $q < \phi < 1$ hold, whereas their exact values depend on the whole distribution of enforcabilities $\mu(z)$. For the second and third cases, the allocation is not Pareto efficient.

In our model, both Θ and q are influenced by the distribution of $\mu(z)$ and are therefore affected by the contractual incompleteness. A small value of Θ means that a large production inefficiency exists because of the two misallocations we discussed earlier. A value of q less than Θ means that the factor suppliers are not able to receive their marginal product. The effect of μ on Θ and q can be distinctive. For example, take an extreme case where $\mu(z) = 0$ for all z. In this case, $\Theta = 1$ holds, because for given K and L, all factors are used in a balanced manner across tasks and products. Thus, no misallocations that reduce Θ are present. However, the value of $q = \alpha\phi/(\alpha + \phi)$ is lower than ϕ

(which is the value of q in the complete-contract case) in this case, because the incompleteness of the contract affects the factor demand. In general, the following proposition holds for q.

Proposition 1 *Consider two otherwise identical economies with enforceability $\mu_1(z)$ and $\mu_2(z)$ such that the probability measure induced by μ_2 strictly first-order stochastically dominates the probability measure induced by μ_1. Then $q_2 > q_1$, where q_1 is the equilibrium value of q associated with μ_1, and q_2 is the equilibrium value of q associated with μ_2.*

Proof See Appendix E. ■

Stochastic dominance here means that, for a given $\bar{\mu}$, the fraction of intermediate-good producers with $\mu_2(z) \geq \bar{\mu}$ is larger than the fraction of producers with $\mu_1(z) \geq \bar{\mu}$. In other words, the values of $\mu_2(z)$ are on average larger than the values of $\mu_1(z)$. Recall that we have rearranged the index z so that $\mu(z)$ is nondecreasing in z, which implies that for a given z, $\mu_2(z) \geq \mu_1(z)$ holds. In other words, the producers in the economy with μ_2 face fewer contractual issues. The proposition implies the factor rewards are higher in an economy with fewer contractual problems. The intuition behind this proposition is the effect of μ on the factor demand, analyzed in section 2.3.1. Smaller values of μ lead to lower demand for the raw materials, which leads to a smaller value of q.

Note the effect of the friction on factor supply is distinct from its effect on production efficiency. To see this difference, consider a social planner that takes the economy's TFP (i.e., Θ) as fixed, but can directly choose labor supply and investment in capital. Then, the marginal products of capital and labor that the social planner faces involve Θ, instead of q in (1) and (2). Thus, the ratio Θ/q measures the divergence between social and private returns to factors. The following proposition implies that in the economy with contractual incompleteness, this discrepancy is larger than in the complete economy ($1/\phi$). Moreover, the discrepancy increases in the degree of contractual incompleteness. Therefore, the enforcement friction reduces factor supply even below the level that is directly implied by the lower productivity.

The intuition for a low factor demand under a low value of μ was discussed in section 2.3.1. Under incomplete contracts, the non-contractible investment is always too little, due to the hold-up problem. Because production factors are complementary, this effect is stronger when more investment suffers from this issue. Moreover, the contractible investment is also too little, and this problem is more severe when μ is smaller, because the influence of a high contractible investment in mitigating the underinvestment in non-contractible capital is smaller. In addition, a smaller μ implies a higher share of investment is non-contractible, and non-contractible investments face a more severe underinvestment problem than contractible investments.

Proposition 2 *If $\mu(z) \in [0, 1)$ for a strictly positive measure of z, then $\Theta/q > 1/\phi$. Let $\Theta(\mu)$ be the value of Θ in an economy where all firms have enforceability μ. Define $q(\mu)$ similarly. Then $\Theta(\mu)/q(\mu)$ is a strictly decreasing function of μ.*

Proof See Appendix E. ■

The first part of the proposition implies that when contract enforcement is imperfect, the gap between social and private returns is larger than in the perfect-enforcement case. Here the imperfection in contract enforcement worsens the factor-supply distortions. The second part of the proposition deals with a special case in which all industries have the same μ. In this special case, we can obtain a sharper characterization: The gap between the social and private returns widens monotonically as μ becomes smaller. Thus, the factor supply is more distorted in the economy where contract enforcement is more difficult.[13] Intuitively, as we discussed earlier, q is hurt by the incompleteness of contracts not only because of the misallocation (which reduces the productivity of inputs and also is the main driver of Θ), but also because of the hold-up problem. Here, the direct consequence is that Θ/q decreases with μ; that is, the impact of μ for q is stronger than the impact for Θ.

The inefficiency due to the misallocation is closely related to the recently evolving literature on misallocation, such as Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Chari, Kehoe, and McGrattan (1997) also emphasizes a similar distortion to q. Our model can be viewed as providing a microfoundation for these distortions.

Restuccia and Rogerson (2008) emphasize that the correlation between the productivity of individual production units and the distortion that induces misallocation is an important determinant of

[13] Appendix C derives explicit expressions for the steady-state L and K/L for specific functional forms of the production function and the utility function. It shows that both Θ/q and q matter for the equilibrium values of L and K/L.

the aggregate consequences of misallocation. In our context, the corresponding correlation is the one between productivity of industries (or intermediate-good firms) and contract sensitivity. In Appendix D, we calculate this correlation using the U.S. data, and find the correlation between the productivity of industries and their contract sensitivity is positive. This finding implies the aggregate productivity loss from misallocation is more pronounced than in the case of no correlation. Although our baseline model does not feature heterogeneous productivity among intermediate-good firms, Appendix D extends our model to the case of heterogeneous productivity and finds the correlation between the productivity of industries and their contract sensitivity is indeed positive. For example, Θ is 17% lower in the imperfect-enforcement case compared to the perfect-enforcement case when we assume no correlation exists between productivity and distortion, whereas the corresponding productivity loss is 24% when we calibrate the model to the positive correlation we observe in the data.

3 Evolution of institutions

In our baseline model of section 2, the distribution of intermediate outputs is constant over time if the distribution of μ is constant, which is at odds with the empirical pattern shown in Figure 1. As we discussed in the introduction, historical studies indicate that the evolution of institutions played an important role in the process of industrialization. Therefore, analyzing the evolution of the distribution of μ is the natural next step. Here we focus on the institutional evolution led by government actions. In this section and the next, we use the baseline model of section 2 and abstract from endogenous labor supply.

Through an analogy with physical capital, we model institutions by *institutional capital* G_t, which allows contracts to be enforced. The institutional capital influences μ in the following manner:

$$\mu(z, G_t) = \underline{\mu}(z) + h(G_t)[1 - \underline{\mu}(z)], \tag{16}$$

where we assume the function $h(\cdot)$ is differentiable, strictly increasing, strictly concave, and $0 = h(0) < h(G) < 1$ for any $G > 0$. We assume $\underline{\mu}(z)$ is a function of z. The function $\underline{\mu}(z)$ is fixed over time, and in the context of the model in this section and the next section, $1 - \underline{\mu}(z)$ corresponds to Nunn's (2007) contract sensitivity measure for industry z, which we use to construct Figure 1 and to conduct additional empirical analysis in Appendix A. One interpretation of (16) is $1 - \underline{\mu}(z)$ activities require certain contract-enforcement action by the government, and $h(G_t)$ is the probability that this enforcement action is successful. The production of different goods requires different combinations of specialized inputs, which gives rise to heterogeneity in $1 - \underline{\mu}(z)$.

We assume that improving the institutions increases the probability of successful enforcement. A natural interpretation of this assumption, with respect to reality, is that the government's investment in the court system increases the number of disputes that can be resolved. Intermediate-goods producers try to enforce their contracts with their suppliers, but they may not have the opportunity to use the court because the number of disputes that can be tried is limited. Arellano and Kocherlakota (2014) and Drozd and Serrano-Padial (2018) use similar setups to study default externalities, caused by crowding out of enforcement capacity when more agents default.

The institutional capital can be accumulated by government investment I_t^G:

$$G_{t+1} = (1 - \delta_G)G_t + I_t^G - \tau(G_t, G_{t+1}),$$

where δ_G is the depreciation rate of the institutional capital. We assume the investment I_t^G is made in final goods and that institutional investment involves adjustment cost $\tau(G_t, G_{t+1}) \geq 0$.

In our model, institutions, represented by G_t, affect economic outcomes by changing the enforceability characteristics of firms $\mu(z, G_t)$. As a result, institutions determine the distribution of μ, and hence the economy's productivity Θ. In the following, we denote this dependence as $\Theta(G_t)$. Similarly, the price of raw materials is also a function of G_t and expressed as $q(G_t)$.

In this section, we extend the previous section's model by endogenizing the process of institutional building over time.[14] Throughout, we assume the government is benevolent and maximizes the representative consumer's utility. The purpose of this section is to theoretically examine how different *types* of governments make choices regarding institutional dynamics. In this sense, our analysis is *positive* rather than normative.[15]

[14] Our model bears some resemblance to a model of investment in public capital. See, for example, Glomm and Ravikumar (1997) and Azzimonti, Sarte, and Soares (2009). Similar to our paper, Herrera and Martinelli (2013) model *state capacity* (the ability of the state to levy taxes and deter crime) as a type of capital. In contrast to this study, their focus is on political economy considerations. They do not consider the effect of state capacity on the productive efficiency of the economy.

[15] As is argued in the introduction, casting our model in a political-economy context using a probabilistic voting model is also possible, because the policy choice maximizes social welfare in the standard formulation.

Our model of institutional determination is based on resource cost. If perfect enforcement can be achieved without cost, it will be chosen under any of the different scenarios we consider. An alternative view focuses on the political process that leads to choices of public policies or institutions. When the gains of better institutions are heterogeneous, the majority of agents may prefer inefficient institutions, even though total output will be increased by better institutions. For example, Koeppel, Monnet, and Quintin (2014) show the optimal level of loan-enforcement capacity can be obtained only if an endowment redistribution scheme is implemented.

An influential line of research is based on the fact that economic advantage leads to political advantage, as forcefully expounded by Acemoglu and Robinson (2012). In a formal model, Acemoglu (2006) shows a ruling elite may be hurt by better institutions in a variety of ways: (i) Better institutions bid up factor prices, and hence cut into the elite's income; (ii) if a competing group benefits disproportionately from better institutions, the elite loses its economic advantage and hence may lose its political supremacy; and (iii) a variety of inefficient institutions lead to a direct transfer of resources to the elite from the rest of society. Sonin (2003) explores the operation of these mechanisms in the context of the protection of property rights. Acemoglu, Johnson, and Robinson (2005b) model how exogenous events that increase the de facto power of the group desiring better institutions (e.g., opening up transatlantic trade) will break the persistent equilibrium and lead to institutional change. Finally, weak economic and political institutions may interact: Institutional change may require concentration of power in one of the member of the elite, who will no longer have an incentive to implement changes.[16]

Our approach focuses on a different tradeoff and is largely complementary to the political-economy view in that we consider the process of institutional building once a such a decision has been made. We also show that even with a benevolent government, commitment is key to achieving positive institutional change.

We consider three different types of governments. First, we consider a government that can control both investment in physical capital I_t^K and investment in institutional capital I_t^G. The government can coerce private agents to save and invest in physical capital, in addition to deciding the institutional

[16] See, for example, Guriev and Sonin (2009). Acemoglu, Egorov, and Sonin (2012) explore the problem of stability of institutions more generally.

investment. Thus, we call it a *coercive government*. Many governments have conducted "forced saving" policies—recent examples are found in China and Singapore. The coercive government here tries to emulate this type of government.

The second and third governments let the market decide its physical capital. Thus we call these cases *market economies*. For the second, we assume the government can commit to future I_t^G. For the third, we assume the government cannot commit, and non-trivial interactions occur between the government and the private agents. We analyze the Markov perfect equilibrium of the game between the government and the private agents.

Below, the representative consumer's utility is specified as

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

where $u(\cdot)$ is a strictly increasing and strictly concave function. The consumer's budget constraint is

$$K_{t+1} + C_t + T_t = (1 + r_t - \delta_K)K_t + w_t + \Pi_t.$$

Note we normalize labor supply to 1. The production structure is the same as in section 2.3. The tax T_t is used for financing the government investment, that is, the government budget constraint is $T_t = I_t^G$.[17]

3.1 Coercive government

The coercive government's optimization problem is time-consistent and can be expressed with a standard Bellman equation:

$$v_c(K, G) = \max_{C, K', G'} u(C) + \beta v_c(K', G')$$

subject to the resource constraint

$$C + K' + G' = \Theta(G)f(K) + (1 - \delta_K)K + (1 - \delta_G)G - \tau(G, G'),$$

where $v_c(K, G)$ is the value function. We follow the conventional notation that a prime ($'$) is used for the next-period values. This problem can be solved by the standard value function iteration method.

[17] Because we assume lump-sum (nondistortionary) taxation, we can assume a per-period balanced government budget without loss of generality.

3.2 Market economy: Commitment

In the cases with a market economy, the government has to respect the private agents' decision about I_t^K. As in the classic problem of finding optimal distortionary taxation, future government policies on I_t^G matter for the current decision of private agents, and therefore the ability of government to commit to future policies has important consequences. First, we set up the problem for the case in which the government can commit to the future policies. In the next section, we consider the case without commitment.

The sequential problem for the benevolent government is

$$v^*(K_0, G_0) = \max_{\{C_t, K_{t+1}, G_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{SP1}$$

subject to

$$C_t + G_{t+1} + K_{t+1} \leq \Theta(G_t) f(K_t) + (1-\delta_K)K_t + (1-\delta_G)G_t, \tag{17}$$

$$C_t > 0, \tag{18}$$

$$K_{t+1} > 0, \tag{19}$$

and

$$u'(C_t) = \beta(1 + q(G_{t+1}) f'(K_{t+1}) - \delta_K) u'(C_{t+1}). \tag{20}$$

Constraint (17) is the resource constraint, and C_{t+1} and K_{t+1} are assumed to be positive in (18) and (19). Constraint (20) is the Euler equation for consumers.

We transform this problem into a recursive formulation by using the technique pioneered by Kydland and Prescott (1980). In particular, we introduce an additional state variable λ, which is equal to the right-hand side of equation (20). The problem becomes:

$$v(K, G, \lambda) = \sup_{C>0, K'>0, G'\geq 0, \lambda'} u(C) + \beta v(K', G', \lambda'), \tag{RP1}$$

subject to

$$C + K' + G' \leq \Theta(G) f(K) + (1-\delta_K)K + (1-\delta_G)G, \tag{21}$$

$$u'(C) = \lambda', \tag{22}$$

$$\beta[1-\delta_K+q(G)f'(K)]u'(C)=\lambda, \tag{23}$$

and

$$\lambda' \in \Omega(K',G'). \tag{24}$$

We call this problem (RP1). The term $\beta[1-\delta_K+q(G)f'(K)]u'(C)$ governs the individual's incentive to save in the current period. By choosing λ', the government commits to future policies that provides the appropriate incentives for current saving; condition (23) is the constraint that the government keeps its past promises. The initial period has no promises to be kept, and therefore the government can choose λ_0 with $\lambda_0 = \arg\max v(K_0,G_0,\lambda_0)$.

Here, $\Omega(K',G')$ is called the "admissible set" and it provides the feasible values for λ'. In particular, if we choose λ' that is too low, the future consumption that is dictated by (23) may not be feasible. In choosing λ', we have to guarantee the sequence of (K_t,G_t) that satisfies all future constraints is nonempty. We state the formal definition of $\Omega(K',G')$ in Online Appendix I. Online Appendix I also describes further technical aspects of this problem.

To solve this problem, we first solve for $\Omega(K',G')$. Proposition I.1 in Online Appendix I describes the iterative method that we can use to find $\Omega(K',G')$. Constraint (23) pins down C uniquely by

$$C = u'^{-1}\left[\frac{\lambda}{\beta(1-\delta_K+q(G)f'(K))}\right].$$

Similarly, (22) and (23) imply

$$\lambda' = \frac{\lambda}{\beta(1-\delta_K+q(G)f'(K))}.$$

Then the planner's problem is reduced to the choice of (K',G') subject to the resource and admissibility constraints.

3.3 Market economy: Non-commitment

The solution in the previous section is not time-consistent. A social planner invests in institutions to improve economy-wide productivity and to reduce the gap between private and social returns to physical capital. It has an incentive to announce a high I_t^G in the future in order to facilitate the private investment in physical capital. However, after private agents accumulate physical capital, the planner may have an incentive to cut institutional investment compared to the announcement and save resources.

In this section, we analyze the relationship between the government and the private agents as a game. In each period, the government chooses institutional investment, taking into account the response of the private agents; similarly, consumers making investment decisions choose optimally given their beliefs about the government's reaction function.

We use the concept of MPE, subgame perfect equilibria in which strategies are conditioned only on payoff-relevant variables.[18] We assume that after output is produced, the government chooses its institutional investment (and hence future G). After observing the government action, households choose their savings (and collectively determine the future capital stock). This timing convention implies the government can commit to the next period's institutional capital; however, it cannot commit to institutions beyond the next period, which affects investment in physical capital in the current period.[19]

Let $C(K, G, K', G')$ denote consumption if the current state of the economy is (K, G) and the government and the private sector choose (K', G'). It is given by:

$$C(K, G, K', G') = \Theta(G) f(K) + (1 - \delta_K) K + (1 - \delta_G) G - G' - \tau(G, G') - K'.$$

Now, to find an MPE, we consider the government's one-period deviation for G' from its equilibrium policy. In doing so, the government needs to take into account the private agents' responses. Let $\nu(K, G)$ be the continuation government policy function for G', and let $M(K, G)$ be the private agents' decision of physical capital K'. Given the government's choice of G', the private agents individually make a consumption/saving choice that that satisfies their Euler equation. Using the fact that they all make the same choice and that $r' = q(G') f'(K')$, we find that:

$$u'(C(K, G, K', G')) = \beta(1 - \delta_K + q(G') f'(K')) u'(C(K', G', M(K', G'), \nu(K', G'))). \tag{25}$$

We denote the solution of the equation as $\tilde{M}(K, G, G')$.

[18] See Klein, Krusell, and Ríos-Rull (2008) for the details and properties of the MPE in this type of dynamic game. Our computational algorithm largely follows Azzimonti, Sarte, and Soares (2009).

[19] In the case of coercive government, K and G are chosen by the same decision maker, so timing is irrelevant. A government with commitment power chooses the entire path of G at date zero before private agents make any decisions.

Let the value from following the policy functions ν and M be denoted by v. Then v solves the Bellman equation:

$$v(K,G) = u(C(K,G,M(K,G),\nu(K,G))) + \beta v(M(K,G),\nu(K,G)). \tag{26}$$

Finally, given the value function v and the private agents' response function, the government solves:

$$\max_{G'} \quad u(C(K,G,\tilde{M}(K,G,G'),G'))) + \beta v(\tilde{M}(K,G,G'),G').$$

In choosing G', the government takes into account the direct effect of increasing future productivity and the effect on the choices of consumers and future governments.

Definition 1 An MPE is the collection of functions $(\nu, M, \tilde{M}, v)$, such that:

1. The perceived government policy function is optimal

$$\nu(K,G) \in \arg\max_{G'} \quad u(C(K,G,\tilde{M}(K,G,G'),G')) + \beta v(\tilde{M}(K,G,G'),G').$$

2. The private Euler equation is satisfied: $\tilde{M}(K,G,G')$ solves equation (25).
3. The value function v satisfies equation (26).
4. The law of motion for physical capital is consistent: $M(K,G) = \tilde{M}(K,G,\nu(K,G))$.

The generalized Euler equation (GEE) has been commonly used to develop intuitions for the working of an MPE in this type of environment.[20] We derive the GEE in our environment in the following proposition. First, we introduce some notation: Let u_1 be the first derivative of the utility function, let C_i be the partial derivative of $C(K,G,K',G')$ function with respect to the ith term, and let $\tilde{M}_i$ be the partial derivative of $\tilde{M}(K,G,G')$ function with respect to the ith term. Also, let $'$ denote the value in the next period, and let $''$ denote the value two periods ahead.

Proposition 3 *Define* $\Delta_K \equiv u_1 C_3 + \beta u_1' C_1'$ *and similarly* $\Delta_G \equiv u_1 C_4 + \beta u_1' C_2'$*. If the equilibrium policy functions* $\tilde{M}$ *and* ν *are differentiable, the following GEE holds:*

$$\Delta_G + \tilde{M}_3 \Delta_K - \frac{\beta(\tilde{M}_2' + \tilde{M}_3 \tilde{M}_1')\tilde{M}_1''}{\tilde{M}_2'' + \tilde{M}_3' \tilde{M}_1''}(\Delta_G' + \tilde{M}_3' \Delta_K') + \beta(\tilde{M}_2' + \tilde{M}_3 \tilde{M}_1')\Delta_K' = 0.$$

Proof See Appendix E. ■

[20] See, for example, Klein, Krusell, and Ríos-Rull (2008).

Variable Δ_K represents the marginal effects of having additional physical capital on the consumer's welfare while holding all other variables fixed. A similar interpretation holds for Δ_G with respect to the institutional capital. The envelope theorem implies that in the coercive planner's problem, we can ignore the induced changes in future optimal choices. Therefore, in that case, it is optimal to set $\Delta_K = 0$ and $\Delta_G = 0$. In the MPE, however, capital investment is not optimal, because the private sector makes the saving decision based on $q(G')$ (instead of $\Theta(G')$) in (25). Therefore, the government tries to balance the costs and benefits from an additional investment in the institutional capital.

To see how the government makes this choice, it is useful to consider a heuristic derivation of the GEE. Consider the equilibrium sequence of the physical capital and the institutional capital. Now, perturb the institutional capital in period $t+1$ by $\epsilon_{g,t+1}$ and in period $t+2$ by $\epsilon_{g,t+2}$, so that both the institutional capital and the physical capital in period $t+3$ are back to the original equilibrium value. This perturbation induces changes in private investment behavior. At period t, K_{t+1} changes by $\epsilon_{k,t+1} \equiv \tilde{M}_{3,t}\epsilon_{g,t+1}$. At period $t+1$, K_{t+2} changes by $\tilde{M}_{1,t+1}\epsilon_{k,t+1} + \tilde{M}_{2,t+1}\epsilon_{g,t+1} + \tilde{M}_{3,t+1}\epsilon_{g,t+2}$ units due to the changes in three variables that affect the decision for K_{t+2}. If we substitute in the expressions for period $t+3$, we can see that $\epsilon_{k,t+3} = 0$ if we set $\epsilon_{g,t+2} = -(\tilde{M}_{2,t+1} + \tilde{M}_{3,t}\tilde{M}_{1,t+1})\tilde{M}_{1,t+2}\epsilon_{g,t+1}/(\tilde{M}_{2,t+2} + \tilde{M}_{3,t+1}\tilde{M}_{1,t+2})$. Each of these changes (G_{t+1}, K_{t+1}, G_{t+2}, and K_{t+2}) have effects on the consumer's welfare. The government sets the total marginal effect to zero at the optimum. This procedure derives the GEE.

In light of this intuition, we can highlight the differences between the non-commitment case and the commitment case. In the non-commitment case, when G_{t+1} is perturbed as above, the equilibrium value of K_{t+1} is determined by taking into account that K_{t+1} (and also the perturbation of G_{t+1}) influences the government's decision on G_{t+2}. This influence is represented as the $\nu(K', G')$ in (25). Because of this structure, the government without commitment may have difficulty influencing the decision of K_{t+1} by changing G_{t+1} (and G_{t+2}). For example, this would be the case when the effect of the perturbation of G_{t+1} and the resulting change in G_{t+2} (dictated by $\nu(K', G')$) are offsetting. In the commitment case, a similar perturbation would also affect K_{t+1} (and K_{t+2}). In the commitment case, however, the private sector does not link the change in G_{t+1} to the change in G_{t+2}, because the path of the institutional capital is pre-committed. Another important difference is that, whereas

β	δ_K	δ_G	ζ	ϕ	α	κ
0.95	0.1	0.1	0.3	0.8	0.5	0.3

Table 1 Parameter values

the deviation in G_{t+1} would only affect the value of K after time $t+1$ in the non-commitment case, it would cause the entire path of K to change in the commitment case, because the deviation has to be pre-committed at time 0. This logic suggests the government's incentive for investing in the institutional capital in the commitment case is stronger than in the non-commitment case, because it can exert a stronger influence on the private investment in K. Moreover, because K_{t+1} and G_{t+1} competes for the same pool of resources, encouraging a higher K_{t+1} by raising G_{t+1} is difficult (i.e., $\tilde{M}_3$ tends to be negative).

3.4 Results

In the following, we characterize the equilibrium of this economy numerically. The outcome should be viewed as a numerical example rather than a carefully calibrated quantitative exercise, because many of parameters in this model are difficult to pin down from the data. We specify the utility function as $u(C_t) = \log(C_t)$, the production function as $f(K_t) \equiv F(K_t, 1) = K_t^{\zeta}$, and the adjustment cost function as $\tau(G_t, G_{t+1}) = \kappa[(G_{t+1} - G_t)/G_t]^2 G_t$.

Note the relationship between institutions and the contractual incompleteness frictions are captured by the functions $\Theta(G)$ and $q(G)$ that we derived in section 2. We use these micro-founded functions in our quantitative simulation.

The parameter values are set as in Table 1. We consider one period to be one year, and set $\beta = 0.95$, $\delta_K = 0.1$, and $\delta_G = 0.1$. For production technologies, we set $\zeta = 0.3$, $\phi = 0.8$, and $\alpha = 0.5$. The adjustment cost parameter $\kappa = 0.3$. The function $h(G)$ is assumed to be $G/(\xi + G)$, where ξ is set to be 1.0. The contract sensitivity of each industry, $1 - \underline{\mu}(z)$, is assumed to be distributed uniformly on $[0, 1]$. The computational details of the model is described in Online Appendix K.

Figure 2 describes the time path of various economic variables with three different governmental arrangements. A coercive government faces the social return to physical capital accumulation $\Theta(G)f'(K)$, instead of the private return $q(G)f'(K)$. As we have seen in Proposition 2, when contract

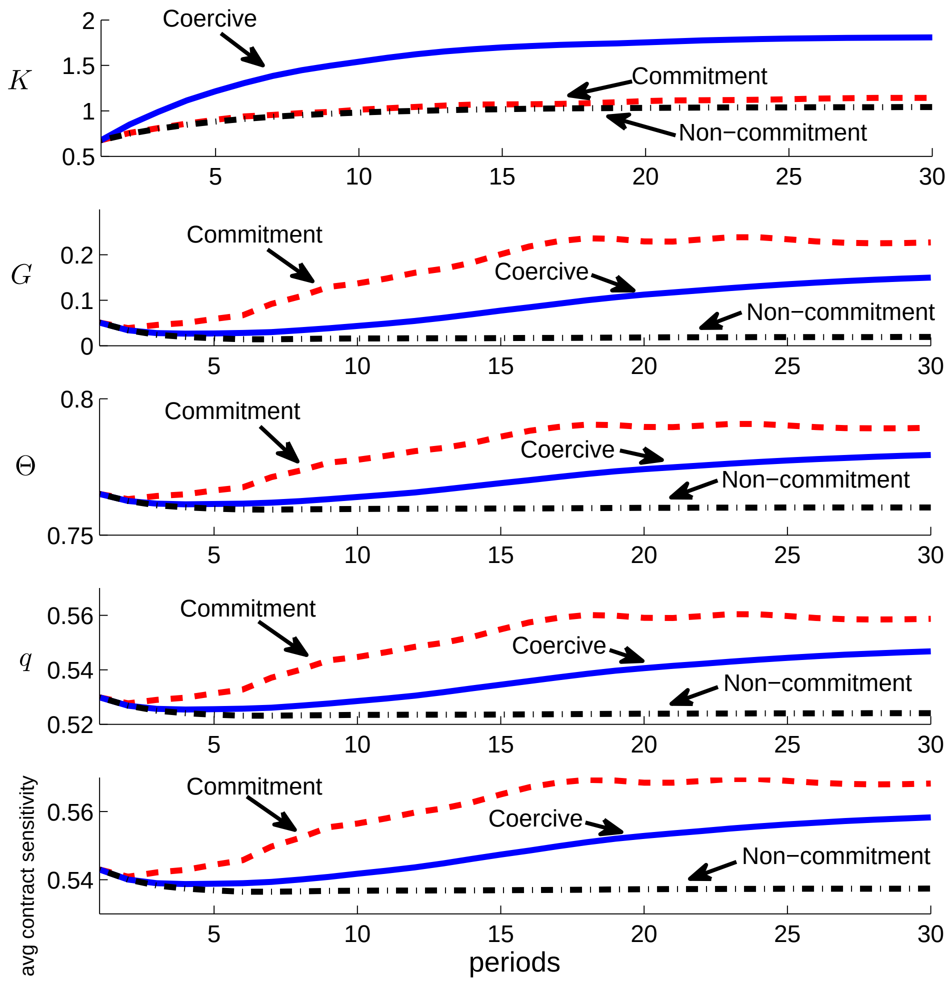

Fig. 2 Results with endogenously evolving institutions

enforcement is imperfect, $\Theta(G) > q(G)$ holds and a discrepancy exists between social return and private return. The physical capital stock K of the coercive-government case is larger than in both market-economy cases for that reason.

Comparing the institutional capital accumulation G, we see the market-economy case with commitment has a higher rate of investment in G than the coercive government, because to facilitate the investment in K, knowing the social return is high, the government in the market economy has to induce private agents to invest in K. For that purpose, increasing $q(G)$ and therefore G is necessary.

The path of G is reflected in $\Theta(G)$, $q(G)$, and the average contract sensitivity, which are all functions of G. An interesting implication is that the government with many policy means does not necessarily have the largest incentive to build institutions. If the government has direct means of influencing the economic outcome, rather than through the institutions, it may resort to these direct methods rather than conducting costly institution building. In our framework, the coercive government faces looser constraints than the government in the market-economy case with commitment, and thus the former outcome provides a higher welfare to consumers. The government with commitment over-invests in G, trying to correct the distortion that is created by the fact that $q(G) < \Theta(G)$. Even after the over-investment, however, it cannot achieve the high level of K that can be reached by the coercive government.

Comparing the commitment case and non-commitment case, the effect of the time-consistency problem shows up strongly in the results. In the case of non-commitment, the government cannot credibly promise the future $q(G)$ will be high, because once K is accumulated, it has an incentive not to follow through on the promise of building a costly institution. In our case, the institutional capital G in the MPE of the non-commitment case collapses to almost zero. This result underlines the importance of commitment in institutional building.[21]

As we discussed earlier when interpreting the GEE, the effect of commitment can be quantitatively large for two potential reasons. First, in the non-commitment case, the future reaction of the institutional investment, which is perceived in the private sector's optimization, can offset the change in the current incentive, making the change in G_{t+1} ineffective in influencing the private sector's behavior. Second, the change in G_{t+1} in the non-commitment case can only affect the incentives for the private sector's investment at time $t+1$ and beyond, whereas in the commitment case, it can also affect the investment patterns up to time t. Because of the forward-looking nature of investment, the difference in incentives because of this difference in the time horizon can be sizable. In general, encouraging a higher K_{t+1} by raising G_{t+1} is difficult because they compete for the same resources. The commitment case can use $G_{t+2}, G_{t+3}, ...$ to influence K_{t+1}, whereas the the government with no commitment cannot resort to anything other than G_{t+1}.

[21] Acemoglu (2003) emphasizes the importance of commitment in the context of inefficient institutions. Our results are also consistent with North and Weingast's (1989) emphasis on commitment in the Glorious Revolution, discussed in the introduction.

For both channels, the crucial part of the insight from our model is that when the government decides to increase G_{t+1}, an important part of the reason is to induce the future K to be larger, in addition to the direct benefit of G on productivity. If the impact of G_{t+1} on future K is small, the government has less incentive to invest in G_{t+1}. A large K in the future, in turn, makes it more likely that the government would want to invest in G further in future, because G and K are complementary in production. This reinforcement mechanism also amplify the impact of commitment. In addition, the effect of G on productivity does not have an Inada-like property: As shown in section 2.3.1, the marginal improvement of productivity by improving μ can be small or even negative even when μ is small. This finding implies a "big push" may be necessary to capture substantial gain from investment in G, which may be difficult under non-commitment.

4 Conclusion

This paper examines the relationship between industrialization and contract enforcement. In the data, we find that throughout the process of industrialization, a country tends to shift its production to more contract-sensitive sectors.

Motivated by the empirical correlation between industrialization and sectoral composition, we build a dynamic general equilibrium model in which production features contract incompleteness by extending Acemoglu, Antràs, and Helpman (2007). We show that the contract incompleteness affects aggregate productivity through misallocation in production. In addition, it affects the factor-supply behavior through factor prices. We offer simple characterizations of these distortions in the model economy.

The dynamic model is used to analyze the government's endogenous institutional investment. In the model, the institutional arrangement evolves together with economic development. This association is broadly consistent with the relationship between industrialization and contract sensitivity that we find in the data. We compare how different government settings can give rise to different dynamic patterns of institutional building.

We find that when the government can commit to its investment in institutional capital but cannot control the private agents' investment in physical capital, it has an incentive to over-invest in the institutional capital, when compared to the case in which the government can control the investment in both types of capitals. The reason is that the private returns from investment are lower in the market economy compared to social returns, and the government tries to correct this distortion by improving institutions.

We also find the possibility of commitment has an important impact on the dynamic accumulation of institutional capital. Inducing private agents' investment in physical capital tends to be more difficult for the government without commitment, because it cannot credibly promise the future institutional environment will remain good. In the numerical example, the government invests little to improving institutions when it cannot commit, because it cannot induce a high investment in private capital and the social return from institution building therefore remains low.

Appendix

A Empirical analysis

Here, we investigate the interaction of relationship specificity and contract enforcement using each country's industrial composition. Industries that require a larger amount of specific investments in their production process are affected to a larger degree by the extent of enforcement frictions. We call the degree of an industry's exposure to enforcement frictions its *contract sensitivity*. Our hypotheses are twofold: First, more severe enforcement frictions tilt industrial composition away from contract-sensitive industries; second, a country's industrial composition changes over time with industrialization, reflecting the change in enforcement frictions.

To classify industries according to their contract sensitivity, we build on the influential work of Nunn (2007), who constructs a measure of exposure to enforcement frictions for a variety of industries.[22] Nunn's measure is constructed as the fraction of intermediate inputs that are nonstandardized, that is, not traded on an exchange and not having a reference price. These intermediate goods correspond to the notion of specific investments. This measure conforms closely to the way we model contractual incompleteness in the next section.

Focusing on international trade, Nunn (2007) empirically shows better enforcement quality confers a comparative advantage in more contract-sensitive industries. Note our application differs from his analysis in two important dimensions. First, Nunn considers exports, whereas we are interested in production.[23] Second, Nunn focuses on the cross-sectional patterns of comparative advantage, whereas we investigate the dynamics of industrial composition.

In addition to Nunn's (2007) measure, we use data on output and value added of industries from the World KLEMS initiative. The initiative coordinates efforts by national and regional statistical offices to construct comparable data on output and productivity at the industry level. We combine Nunn's score of contract sensitivity with output and value added from the KLEMS database to construct weighted scores of contract sensitivity over time and across countries.[24] We also use measures of Purchasing Power Parity GDP per capita from the Penn World Tables version 7. Finally, we use World Governance Indicators from the World Bank as measure of institutions quality. We focus on the manufacturing sector, because the structural transformation of an economy (the shift from agriculture to manufacturing to services) is mainly driven by entirely different mechanisms.

Examples of several countries are presented in Figure 1 of the main text. Here, we proceed to analyze the data more formally. We explore the relationship between the measure of contract sensitivity, various measures of institutional quality, and output per capita. First, we demonstrate the empirical link between average contract sensitivity in a country and the quality of its economic institutions. We use measures of institutional quality from the World Bank's World Governance Indicators. We focus on the variables such as rule of law and government effectiveness, which are most closely related to the notion of contract enforcement. We estimate the following equations:

$$\mathcal{S}_c = \beta_0 + \beta_1 X_c + \beta_2 \ln(y_c) + \epsilon_c \tag{27}$$

$$\mathcal{S}_c = \beta_0 + \beta_1 X_c + \beta_2 y_c + \epsilon_c, \tag{28}$$

where c is the country index, $\mathcal{S}_c$ is the average contract sensitivity in country c, X_c are measures of institutional quality in country c, y_c is the GDP per capita of country c from the Penn World Tables, and ϵ_c is the error term. For the cross-sectional analysis, all variables are for year 2007.

The results for regressions (27) and (28) are displayed in Table 2. Even with a modest sample size, enforcement institutions (interpreted via the rule of law and government effectiveness variables) have a statistically and economically significant effect on the contract sensitivity of industrial composition. By contrast, corruption appears to have little effect. In terms of the magnitude, an improvement in the rule of law variable from the 10th to the 90th percentile implies an increase of average contract sensitivity by 0.076.

Next, we consider the link between economic growth and contract sensitivity. We run a panel regression of the change in the contract sensitivity on the change in log real GDP per capita and fixed effects:

$$\Delta\mathcal{S}_{c,t} = \beta_0 + \beta_1 \Delta \ln(y_{c,t}) + \beta_2 Y_{c,t} + \epsilon_{c,t}, \tag{29}$$

where $Y_{c,t}$ denotes the fixed-effect variables.

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[22] Nunn (2007) calls this measure "contract intensity."

[23] As a robustness check, we also analyze exports in Online Appendix G. Our conclusions remain the same.

[24] KLEMS data are in ISIC 3.1 classification at the two-digit level, whereas Nunn's measure is in the BEA IO classification. We use a crosswalk between the two classifications using NAICS (North American Industry Classification System) with concordances from the BEA and the Census. For more details on the data construction, see Online Appendix F.

	[1] $\mathcal{S}$	[2] $\mathcal{S}$	[3] $\mathcal{S}$	[4] $\mathcal{S}$	[5] $\mathcal{S}$	[6] $\mathcal{S}$
Rule of law	0.049*	0.047**				
	(.025)	(.02)				
Government effectiveness			0.049*	0.051**		
			(.027)	(.023)		
Control of corruption					0.028	0.033*
					(.021)	(.018)
$\ln$ GDP per capita	-0.036		-0.028		-0.20	
	(.035)		(.033)		(.036)	
GDP per capita ($\times 10^{-6}$)		-1.4		-1.35		-1.28
		(1.08)		(1.06)		(1.15)
N	32	32	32	32	32	32

The dependent variable is average contract sensitivity weighted by value added.
Standard errors in parenthesis.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.
Constant omitted. Institutions measures are on a -2.5: 2.5 scale.

Table 2 Cross-sectional results; value added weights

	[1] $\Delta\mathcal{S}$	[2] $\Delta\mathcal{S}$	[3] $\Delta\mathcal{S}$	[4] $\Delta\mathcal{S}$
$\Delta \ln$ GDP per capita	0.0339***	0.0317***	0.0278***	0.0277***
	(.007)	(.007)	(.007)	(.008)
Year fixed effect	No	No	Yes	Yes
Country fixed effect	No	Yes	No	Yes
N	945	945	945	945

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3 Panel regression results; value added weights

The results for regression (29) are summarized in Table 3. The regression results confirm our conclusions from Figure 1: Countries that successfully industrialize switch to more contract-sensitive industries. These results suggest institutional development plays an important role in the process of industrialization.[25] [26]

B Characterization of the individual intermediate-good production

First, we look at the distribution of output across intermediate good producers. The following property is directly related to our empirical observations in the introduction.

Lemma 1 *The relative output of intermediate goods, $y(\mu(z'))/y(\mu(z))$, is independent of K_t and L_t.*

[25] In Online Appendix G, we repeat the same exercises using the gross output as a weight, instead of value added, and obtain similar results.

[26] We are running an unbalanced panel regression. The available years are as follows: USA: 1950 - 2009; Canada: 1961 - 2009; Australia, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Korea, Luxemburg, Netherlands, Portugal, Spain, Sweden and UK: 1970 – 2007; Japan: 1973 – 2007; Taiwan: 1981 - 2007; China: 1987 – 2007; Hungary: 1991 – 2007; Czech Republic, Cyprus, Estonia, Latvia, Lithuania, Malta, Poland, Russia, Slovak Republic and Slovenia: 1995 – 2007.

Proof This lemma is straightforward from the fact that

$$\frac{y(\mu(z'))}{y(\mu(z))} = \frac{D(\mu(z'))}{D(\mu(z))}$$

and that $D(\mu)$ only depends on μ and parameters. ■

Therefore, as long as the distribution of μ is invariant over time, the normalized distribution of y is also invariant over time. In a dynamic setting of our model, the aggregate output changes over time as K_t changes, as in the standard neoclassical growth model. However, if the institutions, and hence the firms' enforceability μ, do not change over time, the relative output will be constant. This finding is inconsistent with the empirical regularities presented in Figure 1. Although this outcome is a consequence of the particular formulation we adopted here, it shows that replicating the pattern in Figure 1 along the growth path is not a trivial task.

We can further characterize the cross section of output as the following.

Lemma 2 *If $\mu(z) > \mu(z')$, then $x_{ct}(\mu(z)) > x_{ct}(\mu(z'))$ and $x_{nt}(\mu(z)) > x_{nt}(\mu(z'))$, and $y_t(\mu(z)) > y_t(\mu(z'))$.*

Proof See Appendix E. ■

An intermediate firm z with a larger value of $\mu(z)$ faces less problems with imperfect contractability with the supplier. Lemma 2 shows that their output is larger.

Next, we turn to the aggregate consequences of the contract incompleteness.

Lemma 3 *If $\mu(z) \in (0,1)$ for a strictly positive measure of z, then $\Theta < 1$.*

Proof See Appendix E. ■

C An analytical example from section 2.4

Here consider a special case in which we can obtain an analytical solution. Let $F(K,L) = K^{\zeta}L^{1-\zeta}$, where $\zeta \in (0,1)$, and $u(C, 1-L) = (C^{\eta}(1-L)^{1-\eta})^{\sigma}/(1-\sigma)$, where $\eta \in (0,1)$, $\sigma > 0$, and $\sigma \neq 1$.

The first-order conditions at any date are:

$$\frac{w_t}{c_t} = \frac{(1-\eta)/\eta}{1-L_t}$$

$$c_t^{\eta(1-\sigma)-1}(1-L_t)^{(1-\eta)(1-\sigma)} = \beta(1-\delta_K + r_{t+1})c_{t+1}^{\eta(1-\sigma)-1}(1-L_{t+1})^{(1-\eta)(1-\sigma)}$$

Consider the steady-state values and omit the time subscripts. Then,

$$\frac{w}{Y - \delta_K K} = \frac{(1-\eta)/\eta}{1-L} \tag{30}$$

and

$$r = \frac{1}{\beta} - 1 + \delta_K \tag{31}$$

hold.

Let $X = K^{\zeta}L^{1-\zeta}$ be the steady-state production of raw materials. Final output is $Y = \Theta X$. Then the steady-state factor prices are:

$$w = \frac{(1-\zeta)X}{L}q$$

and

$$r = \frac{\zeta X}{K}q. \tag{32}$$

Plugging r in (31), we obtain

$$\frac{X}{K} = \frac{m}{q}, \tag{33}$$

where $m = (1/\beta - 1 + \delta_K)/\zeta$ and m is independent of the distribution of μ.

Using the expression for w, we rewrite (30) as

$$\frac{\frac{(1-\zeta)X}{L}q}{\Theta X - \delta_k K} = \frac{(1-\eta)/\eta}{1-n}$$

or

$$\frac{\frac{(1-\zeta)X}{K}q\frac{1}{L}}{\Theta\frac{X}{K}-\delta_K}=\frac{(1-\eta)/\eta}{1-L}.$$

Using (33),

$$\frac{\frac{(1-\zeta)m}{q}q\frac{1}{L}}{\Theta\frac{m}{q}-\delta_K}=\frac{(1-\eta)/\eta}{1-L}.$$

Then solving for L, and substituting m back, we obtain

$$L=\left[\frac{1-\eta}{\eta(1-\zeta)}\left(\frac{\Theta}{q}-\frac{\delta_K\zeta}{1/\beta-1+\delta_k}\right)+1\right]^{-1}.$$

From (31) and (32), the expression of K/L can be derived as:

$$K=\left[\frac{\zeta q}{1/\beta-1+\delta_K}\right]^{\frac{1}{1-\zeta}}\left[\frac{1-\eta}{\eta(1-\zeta)}\left(\frac{\Theta}{q}-\frac{\zeta\delta_K}{1/\beta-1+\delta_K}\right)+1\right]^{-1}.$$

The level of L is a function of Θ/q, which we characterized in Proposition 2. In particular, a large value of Θ/q, which can result from a small μ in Proposition 2, leads to a small value of L. For K, the level of q has an effect in addition to Θ/q. Note, once again, that these distortions on K and L are in addition to the production inefficiency, that is, $\Theta<1$, that we highlighted in section 2.3.

D Extension: Heterogeneous productivity

In this section, we extend our baseline model in section 2 to quantify the effects of contract incompleteness in the presence of specific investments. In our baseline model, industries (and firms) differ only by their contract-sensitivity measure, $(1-\mu(z))$; however, in reality, large differences exist in productivity across industries, and this heterogeneity is important in, for example, analyzing the misallocation of inputs across industries. To accommodate this heterogeneity, we extend the model to allow heterogeneity in productivity as well as in contract sensitivity.

In the extended model, we find heterogeneity in productivity does not interact with contracting frictions if the enforcement frictions $(1-\mu(z))$ and firm productivity are uncorrelated. However, because the distortions in our model affect firms proportionally to their undistorted optimal size, the same degree of enforcement friction will lead to larger output loss if it is applied to a more productive firm. Quantitatively, we find that the correlation between productivity and potential enforcement frictions is large and has sizable implications, a finding consistent with the rapidly expanding literature on misallocation.

D.1 A simple model with productivity dispersion

First, for the purpose of illustration, we introduce heterogeneity in productivity in a simple manner: in addition to a different enforceability parameter $\mu(z)$ every firm has a different productivity parameter $A(z)$. The demand for the differentiated product and the relationship between the firm and its suppliers is assumed to be the same as in the baseline model. The intermediate output is now given by:

$$y(z)=A(z)\left[\int_0^1 X_j(z)^\alpha dj\right]^{1/\alpha} \tag{34}$$

instead of (3).

It is straightforward to show that the Shapley value for the firm and each supplier is equal to the values in the baseline model multiplied by $A(z)^\phi$. It can also be shown that the game and the firm's problem are isomorphic to a firm with productivity one and demand parameter $YA(z)^{\frac{\phi}{1-\phi}}$ instead of Y. Therefore, the equilibrium $x_{c,t}(z)$ and $x_{n,t}(z)$ can be derived as:

$$x_{c,t}(z)=Y_tA(z)^{\frac{\phi}{1-\phi}}q_t^{-\frac{1}{1-\phi}}\left[\alpha(1-\gamma)\right]^{\frac{\phi(1-\mu(z))}{1-\phi}}B(\mu(z))^{1-\phi(1-\mu(z))}$$

and

$$x_{n,t}(z) = Y_t A(z)^{\frac{\phi}{1-\phi}} q_t^{-\frac{1}{1-\phi}} [\alpha(1-\gamma)]^{\frac{1-\phi\mu(z)}{1-\phi}} B(\mu(z))^{\mu(z)\phi},$$

where $B(\cdot)$ is given as (10).

Plugging the expression above into the production function, we obtain

$$y_t(z) = Y_t A(z)^{\frac{1}{1-\phi}} q_t^{-\frac{1}{1-\phi}} D(\mu(z)),$$

where $D(\cdot)$ is given as (12).

We can find the general equilibrium following the same steps as in section 2.3.2. As a result, we obtain

$$q_t = \left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^{\phi} dz\right]^{\frac{1-\phi}{\phi}}$$

and

$$\Theta = \frac{\left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}}}{\int_0^1 A(z)^{\frac{\phi}{1-\phi}} H(\mu(z)) dz},$$

where $H(\cdot)$ is given as (14).

It is straightforward to see that if $\mu(z) = 1$ for all z,

$$\Theta = \left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} dz\right]^{\frac{1-\phi}{\phi}} \tag{35}$$

holds. This is the TFP of the frictionless economy.

By interpreting $Z = [0, 1]$ as a sample space with the usual measure, we can think of $A(z)$ and $\mu(z)$ as random variables. If they are independent,

$$\Theta = \left[\int_0^1 A(z)^{\frac{\phi}{1-\phi}} dz\right]^{\frac{1-\phi}{\phi}} \frac{\left[\int_0^1 D(\mu(z))^{\phi} dz\right]^{\frac{1}{\phi}}}{\int_0^1 H(\mu(z)) dz}.$$

This expression is simply a multiplication of the TFP in the frictionless economy in (35) and Θ in the homogeneous-productivity case in (15). Therefore, the productivity dispersion does not interact with the misallocation problem if productivity and contract sensitivity are independent. Similarly, the factor-supply distortion Θ/q is unaffected by the distribution of productivity and depends only on the distribution of enforcement frictions $\mu(z)$. Heterogeneity in productivity and the misallocation problem from contracting frictions interact only if these two are correlated. In the following, we investigate whether we observe correlations across these two factors in the U.S. industries.

D.2 Measurement

We use the NBER-CES Manufacturing Industry database to map the model to the data. The database contains 473 industries in the 1997 NAICS classification. After data cleaning, we are left with 464 industries. Nunn's (2007) contract-sensitivity measure is in the 1997 IO classification, so we use a concordance (provided by the BEA) between the IO classification and the NAICS.

We assume no enforcement frictions are present in the U.S. economy.[27] To map the model to the data, we have to modify the intermediate-good production function slightly. The production function (34) can be viewed as $y(z) = A(z)W(z)$, where $W(z)$ is the quantity of input: $W(z) = [\int_0^1 X_j(z)^{\alpha} dj]^{1/\alpha}$. Now, assume that there are m types of inputs, with each quantity $W_i(z)$, where $i = 1, ..., m$:

$$y(z) = A(z) \prod_{i=1}^{m} W_i(z)^{\eta_i},$$

where η_i, $i = 1, ..., m$ are parameters. Denote p_i as the price of input i.

[27] Recall that $1 - \mu$ is a measure of potential distortions and not actual distortions in an industry. The underlying assumption is that effective enforcement in the U.S. economy solves the potential contractibility issues.

Then the problem for firm z in the frictionless economy is:

$$\max_{\{W_1(z),\ldots,W_m(z)\}} Y^{1-\phi}[A(z)\prod_{i=1}^{m} W_i(z)^{\eta_i}]^{\phi} - \sum_{i=1}^{m} p_i W_i(z).$$

The first-order conditions imply

$$\eta_i\phi = \frac{p_i W_i(z)}{R(z)},$$

where $R(z)$ is the revenue of firm z: $R(z) \equiv Y^{1-\phi}[A(z)\prod_{i=1}^{m} W_i(z)^{\eta_i}]^{\phi}$. Denote the *observed cost share of input i* by $\tilde{\eta}_i \equiv \eta_i\phi$. This cost share can be calculated for each industry in our dataset.

Taking logs of the definition of the revenue, we obtain

$$\log R(z) = (1-\phi)\log Y + \phi \log A(z) + \sum_i \tilde{\eta}_i \log W_i(z).$$

Therefore, $A(z)$ can be measured using

$$A(z) \propto \exp\left(\frac{\log R(z) - \sum_i \tilde{\eta}_i \log W_i(z)}{\phi}\right). \tag{36}$$

We conduct this measurement assuming an industry z in the data corresponds to a continuum of intermediate-good firms with measure $\chi(z)$. We assume the elasticity of substitution between any two intermediate goods is the same, regardless of whether they belong to the same industry.

Then the formulas for q and Θ can be modified as

$$q = \left[\sum_z \chi(z)A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^{\phi}\right]^{\frac{1-\phi}{\phi}} \tag{37}$$

and

$$\Theta = \frac{\left[\sum_z \chi(z)A(z)^{\frac{\phi}{1-\phi}} D(\mu(z))^{\phi}\right]^{\frac{1}{\phi}}}{\sum_z \chi(z)A(z)^{\frac{\phi}{1-\phi}} H(\mu(z))}. \tag{38}$$

Total sales of an industry in an economy without frictions are proportional to $\chi(z)A(z)^{\frac{\phi}{1-\phi}}$. Therefore, we set $\chi(z)$ so that the ratio of sales between two industries in the model economy is equal to the corresponding ratio we observe in the data:

$$\frac{\chi(z)}{\chi(z')} = \frac{R(z)/R(z')}{(A(z)/A(z'))^{\frac{\phi}{1-\phi}}}.$$

The ratio $A(z)/A(z')$ can be calculated from equation (36). This condition and the normalization $\sum_z \chi(z) = 1$ pin down the values of $\chi(z)$.

Because q and Θ in (37) and (38) are homogeneous of degree one in $\{A(z)\}$, we can normalize the measured $A(z)$. Therefore we calculate the productivity $A(z)$ from

$$A(z) = C\exp\left(\frac{\log R(j) - \sum_i \tilde{\eta}_i \log W_i(j)}{\phi}\right),$$

where we can observe $R(z)$, $\tilde{\eta}_i$, and $W_i(z)$ in the data. The value of ϕ is set at 0.8, and the constant C is chosen so that the aggregate Θ is one when $\mu(z) = 1$ for all z.

Figure 3 presents a scatterplot of this measured productivity against contract sensitivity for the final year in the sample, 2009. The data reveals that more productive industries tend to be more contract sensitive, with a correlation coefficient of 0.38 when industries are not weighted and 0.45 when we use the computed weights χ.[28]

D.3 Model results

Next, we turn to a model-based evaluation of the importance of the correlation between productivity and contract sensitivity. Restuccia and Rogerson (2008) emphasize this correlation. For every industry, we set $\mu(z)$ to be one minus

[28] This relationship is valid for all years in the sample, with a minimum for the correlation of 0.27.

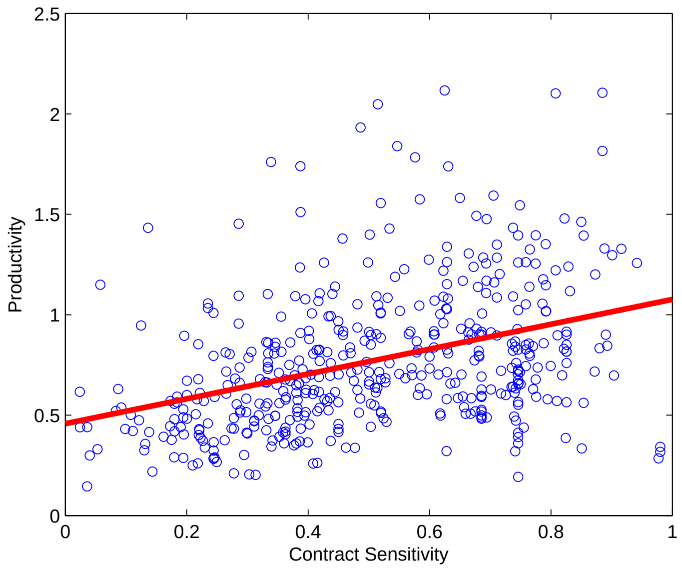

Fig. 3 Contract sensitivity and productivity. The red line is a linear fit.

	Θ	q	Θ/q
(i) Perfect enforcement	1.00	0.80	1.25
(ii) Imperfect enforcement, no correlation	0.83	0.62	1.34
(iii) Imperfect enforcement, correlation	0.76	0.52	1.46

Table 4 The values of Θ and q, for different correlations between contract sensitivity and productivity

	Θ/Θ^*	K/K^*	$Y/Y^* = \frac{\Theta}{\Theta^*}(K/K^*)^\zeta$
(i) Perfect enforcement	1.00	1.00	1.00
(ii) Imperfect enforcement, no correlation	0.83	0.69	0.74
(iii) Imperfect enforcement, correlation	0.76	0.54	0.63

Table 5 Steady-state value of K and Y for different correlations between contract sensitivity and productivity

the measure of contract sensitivity. Then given the parameter values (we use $\phi = 0.8$ and $\alpha = 0.5$ as in Table 1), we compute Θ (TFP) and q (price of raw materials) for three cases: (i) an economy with perfect enforcement ($\mu(z) = 1$ for all z); (ii) the economy with imperfect-enforcement ($\mu(z)$ values set from the observed contract sensitivity) but no correlation between productivity and contract sensitivity; and (iii) the economy with imperfect enforcement and the observed correlation between productivity and contract sensitivity (0.45).

Table 4 describes the results. The mechanism in the paper generates quantitatively significant misallocation and hence TFP loss. Moreover, the correlation between productivity and contract sensitivity is an important amplification mechanism for our model. As a result of the positive correlation, TFP drops by an additional 7% and misalignment between private and social marginal returns worsens by 9%.

In addition to the inefficient allocation of inputs, the enforcement friction leads to suboptimal labor-supply and capital-accumulation decisions, driven by the ratio of private to social factor returns, Θ/q, which we display in the last column of the table. The enforcement friction worsens this disparity (which is present even in the full-enforcement economy due to imperfect competition). We assume labor is supplied inelastically and that $F(K, 1) = K^\zeta$ with $\zeta = 0.3$. Given this structure, we can compare the steady states of the three scenarios without specifying other parameters (δ_G, δ_K, κ, and preference parameters). The results are described in Table 5 (* denotes variables in the complete-contracts case). In total, the barriers to capital accumulation leads to an additional 13% drop (0.76 vs. 0.63) in output at the steady state.

E Proofs

Proof of Lemma 2:

Define $\tilde{x}_c(\mu) = \varphi^{\frac{\phi(1-\mu)}{1-\phi}} B(\mu)^{1-\phi+\phi\mu}$; then $x_c(\mu) = Yq^{-\frac{1}{1-\phi}}\tilde{x}_c(\mu)$. Substituting the expression for $B(\mu)$ and collecting terms shows

$$\tilde{x}_c(\mu) = \left[\frac{\varphi}{\phi}\frac{1-\phi+\phi\mu}{1-\varphi+\varphi\mu}\right]^{-\frac{1-\phi+\phi\mu}{1-\phi}} \equiv f(\mu)^{-\frac{1-\phi+\phi\mu}{1-\phi}}.$$

Differentiating:

$$\begin{aligned}\frac{d}{d\mu}\tilde{x}_c(\mu) &= -\frac{1-\phi+\phi\mu}{1-\phi}h(\mu)^{-\frac{1-\phi+\phi\mu}{1-\phi}-1}\frac{\varphi}{\phi}\frac{\phi-\varphi}{(1-\varphi+\varphi\mu)^2} - \frac{\phi}{1-\phi}f(\mu)^{-\frac{1-\phi+\phi\mu}{1-\phi}}\ln f(\mu)\\ &= \frac{1}{1-\phi}f(\mu)^{-\frac{1-\phi+\phi\mu}{1-\phi}}\left[-\frac{\phi-\varphi}{1-\varphi+\varphi\mu} - \phi\ln\left(\frac{\varphi}{\phi}\frac{1-\phi+\phi\mu}{1-\varphi+\varphi\mu}\right)\right].\end{aligned}$$

Consider φ and μ as parameters and define $g(x) = -\frac{x-\varphi}{1-\varphi+\varphi\mu} - x\ln\left(\frac{\varphi}{x}\frac{1-x+x\mu}{1-\varphi+\varphi\mu}\right)$. Then $\frac{d}{d\mu}\tilde{x}_c(\mu) > 0$ if and only if $g(\phi) > 0$. Note that $g(\varphi) = 0$. Differentiating:

$$g'(x) = -\frac{1}{1-\varphi+\varphi\mu} + \frac{1}{1-x+x\mu} - \ln\left(\frac{\varphi}{x}\frac{1-x+x\mu}{1-\varphi+\varphi\mu}\right).$$

Again, $g'(\varphi) = 0$. Finally,

$$g''(x) = \frac{1}{x}\frac{1}{1-x+x\mu} + \frac{1-\mu}{(1-x+x\mu)^2} > 0.$$

Then, because $\phi > \varphi$, the second-order Taylor expansion of $g(x)$ around $x = \varphi$ implies $g(\phi) > 0$ and hence $\frac{d}{d\mu}x_c(\mu) > 0$.

Define $\tilde{x}_n(\mu) = \varphi^{\frac{1-\phi\mu}{1-\phi}} B(\mu)^{\mu}$; then $x_n(\mu) = Yq^{-\frac{1}{1-\phi}}\tilde{x}_n(\mu)$. Next , we show that $\tilde{x}_n(\mu)$ is increasing in μ. We can express $\tilde{x}_n(\mu) = \tilde{x}_c(\mu)\frac{\varphi}{\phi}\frac{1-\phi(1-\mu)}{1-\varphi(1-\mu)}$. Since $\frac{1-\phi(1-\mu)}{1-\varphi(1-\mu)}$ and $\tilde{x}_c(\mu)$ are both increasing in μ, $\tilde{x}_n(\mu)$ is increasing too.

Using these two facts, we can also show $D(\mu)$ is increasing. Note that in (11), y_t moves together with $D(\mu)$. $\ln(D(\mu)) = \mu\ln(\tilde{x}_c(\mu)) + (1-\mu)\ln(\tilde{x}_n(\mu))$. Then

$$\frac{D'(\mu)}{D(\mu)} = \mu\frac{\tilde{x}_n'(\mu)}{\tilde{x}_n(\mu)} + (1-\mu)\frac{\tilde{x}_n'(\mu)}{\tilde{x}_n(\mu)} + \ln[\tilde{x}_c(\mu)/\tilde{x}_n(\mu)] > 0,$$

where we used that $\tilde{x}_c'(\mu) > 0$, $\tilde{x}_n'(\mu) > 0$ (established above) and that $\tilde{x}_c(\mu) > \tilde{x}_n(\mu)$. ■

Proof of Lemma 3:

Define

$$\eta(\mu) \equiv \frac{\mu x_c(\mu) + (1-\mu)x_n(\mu)}{y(\mu)}.$$

(Here, we are explicit about the dependence of x_c, x_n, and y on μ.) From the production function (11),

$$\eta(\mu) = \frac{\mu + (1-\mu)x_n(\mu)/x_c(\mu)}{(x_n(\mu)/x_c(\mu))^{1-\mu}}.$$

It is straightforward to show that $\eta(\mu) > 1$ when $x_n(\mu)/x_c(\mu) \neq 1$.

From the expressions on $x_n(\mu)$ and $x_c(\mu)$ in the main text,

$$x_n(\mu)/x_c(\mu) = \alpha(1-\gamma)B(\mu)^{-(1-\phi)}.$$

Substituting the expression for $B(\mu)$ in the main text, we can see that

$$\frac{d}{d\mu}\frac{x_n(\mu)}{x_c(\mu)} = \frac{\alpha(1-\gamma)}{\phi}\frac{\phi-\alpha(1-\gamma)}{(1-(1-\mu)\alpha(1-\gamma))^2} > 0.$$

Then for all $\mu < 1$, $x_n(\mu)/x_c(\mu) < x_n(1)/x_c(1) = \alpha/(\alpha+\phi) < 1$. Therefore $\eta(\mu) > 1$ for any $\mu < 1$.

Since the market for raw materials clears in equilibrium,

$$\begin{aligned}
F(K,L) &= \int_0^1 [\mu(z)x_c(\mu(z)) + (1-\mu(z))x_n(\mu(z))]dz \\
&= \int_0^1 \eta(\mu(z))y(\mu(z))dz \\
&> \int_0^1 y(\mu(z))dz \\
&= \left[\int_0^1 y(\mu(z))dz\right]^{\frac{\phi}{\phi}} \\
&\geq \left[\int_0^1 y(\mu(z))^{\phi}dz\right]^{\frac{1}{\phi}} = Y,
\end{aligned}$$

where the third line follows from the fact that $y(\mu(z)) > 0$ for all z and $\eta(\mu(z)) > 1$ for a positive measure of z. The fifth line follows from Jensen's inequality. Thus, the chain of inequalities implies $Y < F(K,L)$, and therefore $\Theta < 1$. ■

Proof of Proposition 1:

In the proof of Lemma 2, we show $D(\mu)$ is strictly increasing, and hence $D(\mu)^{\phi}$ is also strictly increasing. This fact, equation (13), the fact that $1/\phi - 1 > 0$, and the definition of first-order stochastic dominance imply the result. ■

Proof of Proposition 2:

We show the second statement first. From the derivation in the main text, we know

$$\frac{\Theta}{q} = \frac{\frac{\left[\int_0^1 D(\mu(z))^{\phi}dz\right]^{\frac{1}{\phi}}}{\int_0^1 H(\mu(z))dz}}{\left[\int_0^1 D(\mu(z))^{\phi}dz\right]^{\frac{1-\phi}{\phi}}} = \frac{\int_0^1 D(\mu(z))^{\phi}dz}{\int_0^1 H(\mu(z))dz}.$$

Then, in the case in which all intermediate-good producers have the same μ, we obtain

$$\frac{\Theta(\mu)}{q(\mu)} = \frac{D(\mu)^{\phi}}{H(\mu)} = D(\mu)^{\phi-1}\frac{D(\mu)}{H(\mu)}.$$

From the derivation in the main text,

$$D(\mu)^{\phi-1} = [\alpha(1-\gamma)]^{-(1-\mu)}B(\mu)^{-\mu(1-\phi)} = \varphi^{-(1-\mu)}B(\mu)^{-\mu(1-\phi)},$$

where we defined $\varphi \equiv \alpha(1-\gamma) = \alpha\phi/(\alpha+\phi)$. Note that $\varphi \in (0,1)$.

From the definitions of H and D, it follows that

$$\frac{D(\mu)}{H(\mu)} = \frac{\varphi^{1-\mu}B(\mu)^{-(1-\mu)(1-\phi)}}{\mu + (1-\mu)\varphi B(\mu)^{-(1-\phi)}}$$

Combining the two expressions above,

$$\frac{\Theta(\mu)}{q(\mu)} = \frac{\varphi^{-(1-\mu)}B(\mu)^{-\mu(1-\phi)}\varphi^{1-\mu}B(\mu)^{-(1-\mu)(1-\phi)}}{\mu + (1-\mu)\varphi B(\mu)^{-(1-\phi)}} = \frac{B(\mu)^{-(1-\phi)}}{\mu + (1-\mu)\varphi B(\mu)^{-(1-\phi)}} \equiv \frac{f(\mu)}{g(\mu)},$$

where f and g represent the numerator and denominator, respectively. Because

$$\frac{d}{d\mu}\frac{\Theta(\mu)}{q(\mu)} = \frac{1}{f(\mu)g(\mu)}\left[\frac{f'(\mu)}{f(\mu)} - \frac{g'(\mu)}{g(\mu)}\right],$$

it is clearly sufficient to show that $\frac{f'(\mu)}{f(\mu)} - \frac{g'(\mu)}{g(\mu)} < 0$ for all μ.

First, we derive f'/f. Substituting the expression for $B(\mu)$, we obtain

$$f(\mu) = B(\mu)^{-(1-\phi)} = \frac{1-\phi+\phi\mu}{\phi[1-\varphi+\varphi\mu]}.$$

Taking derivatives, we obtain

$$\frac{f'(\mu)}{f(\mu)} = \frac{\phi - \varphi}{(1 - \varphi + \varphi\mu)(1 - \phi + \phi\mu)}.$$

Next, we turn to g'/g. Again, we substitute the expression for $B(\mu)$ to show

$$g(\mu) = \mu + (1 - \mu)\frac{\varphi}{\phi}\frac{1 - \phi + \phi\mu}{1 - \varphi + \varphi\mu}.$$

Taking derivatives yields,

$$\frac{g'(\mu)}{g(\mu)} = \frac{1 - \frac{\varphi}{\phi}\frac{1-\phi+\phi\mu}{1-\varphi+\varphi\mu} + (1-\mu)\frac{\varphi}{\phi}\frac{\phi-\varphi}{(1-\varphi+\varphi\mu)^2}}{\mu + (1-\mu)\frac{\varphi}{\phi}\frac{1-\phi+\phi\mu}{1-\varphi+\varphi\mu}}.$$

Simplifying,

$$\frac{g'(\mu)}{g(\mu)} = \frac{1}{1 - \varphi + \varphi\mu}\frac{\phi - \varphi}{\mu(\phi - \varphi + \varphi\phi) + \varphi - \varphi\phi}.$$

This implies

$$\begin{aligned}\frac{f'(\mu)}{f(\mu)} - \frac{g'(\mu)}{g(\mu)} &= \frac{\phi - \varphi}{1 - \varphi + \varphi\mu}\left[\frac{1}{1 - \phi + \phi\mu} - \frac{1}{\mu(\phi - \varphi + \varphi\phi) + \varphi - \varphi\phi}\right] \\ &= -\frac{(\phi - \varphi)(\mu\varphi(1 - \phi) + (1 - \varphi)(1 - \phi))}{(1 - \varphi + \varphi\mu)(1 - \phi + \phi\mu)[\mu(\phi - \varphi + \varphi\phi) + \varphi - \varphi\phi]} < 0\end{aligned}$$

(note that $\phi - \varphi > 0$ from the definition of φ), which concludes the proof that $\Theta(\mu)/q(\mu)$ is decreasing.

Next, we prove the first statement of the proposition. It will be convenient to restate it as $q/\Theta < \phi$ if $\mu(z) \in [0, 1)$ for a strictly positive measure of z, or

$$\frac{\int_0^1 H(\mu(z))dz}{\int_0^1 D(\mu(z))^\phi dz} < \frac{H(1)}{D(1)^\phi}.$$

This statement is equivalent to

$$\frac{\int H(\mu)d\nu(\mu)}{\int D(\mu)^\phi d\nu(\mu)} < \frac{H(1)}{D(1)^\phi},$$

where ν is a probability measure on the Borel sigma-algebra such that $\nu(\{1\}) < 1$. Let $h(\mu) \equiv H(\mu)/D(\mu)^\phi$. Then the statement can be rewritten as:

$$\frac{\int h(\mu)D(\mu)^\phi d\nu(\mu)}{\int D(\mu)^\phi d\nu(\mu)} < h(1).$$

Define the Borel probability measure ν_1 by $\nu_1(A) = \int_A D(\mu)^\phi d\nu(\mu)/\int D(\mu)^\phi d\nu(\mu)$. By the change-of-variables theorem, the statement is equivalent to:

$$\int h(\mu)d\nu_1(\mu) < \int h(\mu)d\nu_2(\mu),$$

where ν_2 is the probability measure such that $\nu_2(\{1\}) = 1$. Because h is strictly increasing (from the first part of the proof), it is sufficient to show ν_2 first-order stochastically dominates ν_1 strictly. By construction $\nu_1([0, a]) \geq 0 = \nu_2([0, a])$ for all $a < 1$ and $\nu_1([0, 1]) = \nu_2([0, 1]) = 1$. Because $D(\mu) > 0$ and $\nu([0, a]) > 0$ for some $a \in [0, 1)$, $\nu_1([0, a]) > 0 = \nu_2([0, a])$ for some $a \in [0, 1)$, which shows ν_2 first-order stochastically dominates ν_1 strictly, which concludes the proof. ∎

Proof of Proposition 3:

If the policy functions are differentiable, the value function v is also differentiable. Let $R_1 \equiv u_1(C_1 - \tilde{M}_1)$, $R_2 \equiv u_1(C_2 - \tilde{M}_2)$, and $R_3 \equiv u_1(C_4 - \tilde{M}_3)$. The government's first-order condition is:

$$R_3 + \beta[v_1'\tilde{M}_3 + v_2'] = 0, \tag{E.39}$$

where v_i is the partial derivative with respect to the ith term, and $'$ represents the next period. By definition,

$$v(K, G) = u(C(K, G, \tilde{M}(K, G, \nu(K, G)), \nu(K, G))) + \beta v(\tilde{M}(K, G, \nu(K, G)), \nu(K, G)).$$

Differentiating the value function we get:

$$v_1 = R_1 + \beta v_1'\tilde{M}_1 \tag{E.40}$$

$$v_2 = R_2 + \beta v_1'\tilde{M}_2, \tag{E.41}$$

where we used the FOC to eliminate all terms involving derivatives of ν.

Updating equations (E.39), (E.40), and (E.41) one period, we obtain a system of six linear equations and six unknowns: v_1, v_2, v_1', v_2', v_1'', v_2'', which we can express in matrix notation as:

$$\begin{bmatrix} 1 & 0 & -\beta\tilde{M}_1 & 0 & 0 & 0 \\ 0 & 1 & -\beta\tilde{M}_2 & 0 & 0 & 0 \\ 0 & 0 & \beta\tilde{M}_3 & \beta & 0 & 0 \\ 0 & 0 & 1 & 0 & -\beta\tilde{M}_1' & 0 \\ 0 & 0 & 0 & 1 & -\beta\tilde{M}_2' & 0 \\ 0 & 0 & 0 & 0 & \beta\tilde{M}_3' & \beta \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_1' \\ v_2' \\ v_1'' \\ v_2'' \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ -R_3 \\ R_1' \\ R_2' \\ -R_3' \end{bmatrix}.$$

The system can be solved analytically with standard tools of linear algebra. Using Gaussian elimination, we obtain:

$$v_1 = R_1 + \beta\tilde{M}_1 R_1' - \frac{\beta\tilde{M}_1\tilde{M}_1'}{\tilde{M}_2' + \tilde{M}_1'\tilde{M}_3}[R_2' + R_3/\beta + R_1'\tilde{M}_3]$$

$$v_2 = R_2 + \beta\tilde{M}_2 R_1' - \frac{\beta\tilde{M}_2\tilde{M}_1'}{\tilde{M}_2' + \tilde{M}_1'\tilde{M}_3}[R_2' + R_3/\beta + R_1'\tilde{M}_3].$$

Then updating the expressions for v_1 and v_2 one period and plugging into the first-order condition, we obtain the following expression:

$$\begin{aligned} R_3 \ &+\beta\tilde{M}_3 R_1' + \beta^2\tilde{M}_3\tilde{M}_1' R_1'' - \frac{\beta^2\tilde{M}_3\tilde{M}_1'\tilde{M}_1''}{\tilde{M}_2'' + \tilde{M}_1''\tilde{M}_3'}[R_2'' + R_3'/\beta + R_1''\tilde{M}_3'] \\ &+\beta R_2' + \beta^2\tilde{M}_2' R_1'' - \frac{\beta^2\tilde{M}_2'\tilde{M}_1''}{\tilde{M}_2'' + \tilde{M}_1''\tilde{M}_3'}[R_2'' + R_3'/\beta + R_1''\tilde{M}_3'] = 0. \end{aligned}$$

Finally, substituting the expression for R_i and grouping, we obtain the expression in the text. ■

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