Firm Dynamics and the Macroeconomy: Basics

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From today, we focus on L

$\gamma \in (0, 1)$

- If the capital is fixed,

$$A_i(K_i^\alpha L_i^{1-\alpha})^\gamma = \boxed{B_i} L_i^\beta$$

where

$$B_i = \underbrace{A_i K_i^{\alpha\gamma}}$$

and

$$\beta = \underbrace{\gamma(1-\alpha)} \in (0, 1).$$

From today, we focus on L

- If the rental market for capital is perfectly competitive,

$$\max_{K_i} \; A_i (K_i^{\alpha} L_i^{1-\alpha})^{\gamma} - r K_i$$

imply

$$K_i = \left(\frac{\alpha\gamma}{r}\right)^{\frac{1}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}} L_i^{\frac{\gamma-\alpha\gamma}{1-\alpha\gamma}}$$

Plugging this solution into the production function

$$A_i (K_i^{\alpha} L_i^{1-\alpha})^{\gamma} = \left(\frac{\alpha\gamma}{r}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}} L_i^{\frac{\gamma-\alpha\gamma}{1-\alpha\gamma}}$$

Thus we can write a new production function

$$\boxed{B_i L_i^{\beta}}$$

where

$$B_i = \left(\frac{\alpha\gamma}{r}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}}$$

$\gamma \in (0,1)$

and

$$\beta = \frac{\gamma - \alpha\gamma}{1-\alpha\gamma} \in (0,1)$$

Misallocation

types of

First, continuing with the discussion in the last class, let us talk about the misallocation. An example:

- There are two firms, firm 1: $Y_1 = A_1 L_1^{\alpha}$ and firm 2: $Y_2 = A_2 L_2^{\alpha}$ where $\alpha \in (0, 1)$.
- Let $A_1 = 1$ and $A_2 = 2$. $\alpha = 1/2$.
- Assume that there are firm-specific distortions τ_i $(i = 1, 2)$. We can think of τ_i as a tax.
- Firm i maximizes the profit

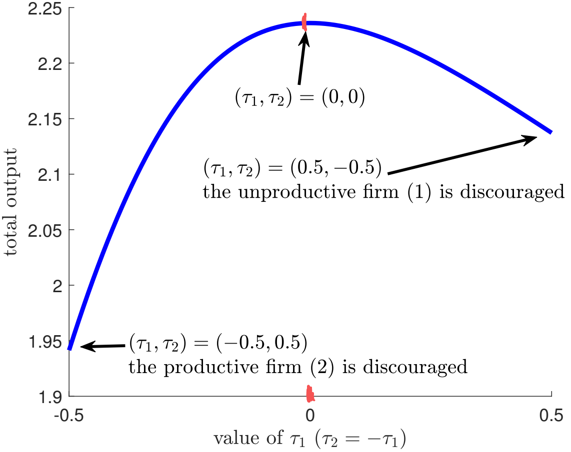
$$(1 - \tau_i) A_i L_i^{\alpha} - w L_i.$$

  Because the actual output is $A_i L_i^{\alpha}$, the firm's decision problem is distorted.
- Assume that the total labor is fixed at 1. Thus w is deterimined by $L_1 + L_2 = 1$.
- The total output is computed as

$$Y = Y_1 + Y_2 = A_1 L_1^{\alpha} + A_2 L_2^{\alpha}$$

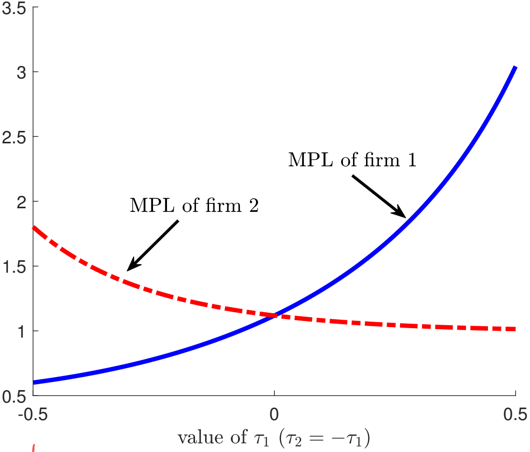
Misallocation

$\frac{Y}{L} = Y$



- The total output is reduced the most with positive correlation between the distortion (discouragement) and productivity (Restuccia and Rogerson, 2008)

Misallocation



- MPL ($\alpha A_i L_i^{\alpha-1}$) dispersion is the source of the productivity loss.

Situations where misallocations can occur: Examples

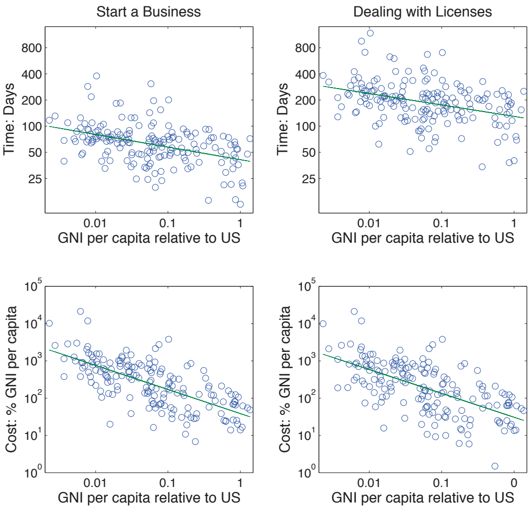
- Tax rates are different depending on firm identity/characteristics
- Regulations that depend on firm size
- Firing/hiring taxes/subsidies
- Entry/exit taxes/subsidies, some other frictions
- Financial frictions
- Contract enforcement

Notes:

- The cost of reduced entry depends on the post-entry importance of entrants.
- The importance of financial frictions depends on the persistence of shocks (whether the firm can overcome the friction by self-financing); see Moll (2014). The shock does seem to be persistent; see Lee and Mukoyama (2015). There still can be effects for young firms and potential entrants.
- The importance of contract enforcement at the industry level positively correlates with industry productivity (Mukoyama and Popov, 2020)

Entry barriers

From Moscoso Boedo and Mukoyama (2012)



- Why? Political economy considerations (Mukoyama and Popov, 2014)

Misallocation as a theory of TFP

- Misallocation can change the measured TFP (measured by Y/L^{α}, for example) without changing A_1 and A_2.
- The effect can be sizable but not as much as 10-folds differences between rich and poor countries.
- For the development questions, the determination of A_i (growth of productivity at the firm level) is still important.

Firm growth

On firm growth

$T = \int_0^\infty A_\tau e^{\gamma a} da$

$e^{-\delta a}$

$A_0 e^{g(t-a)}$

Two small points first:

- First, note that individual firm growth is not necessary or sufficient for aggregate growth.
- Second, the loss from missing entry can be large if we take firm growth into account.

An example:

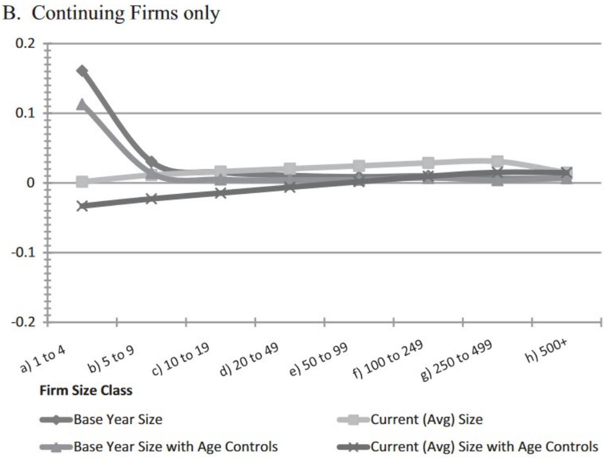
- Labor supply is elastic (employment is demand-determined). One firm hires one worker.
- cohort, age
  The production of a firm who enters at time τ and age a (today is $t = \boxed{a + \tau}$) is $\boxed{A_\tau e^{\gamma a}}$. $\gamma \geq 0$ is the firm growth rate. Assume that $A_\tau = A_0 e^{g\tau}$.
  time.
- The surviving firms at age a is $e^{-\delta a}$. Assume $\delta > \gamma$.
- The mass of entrants is 1.
- Outcome: The total employment is $\int_0^\infty e^{-\delta a} da = 1/\delta$. The aggregate production is $A_0 e^{gt}/(\delta + g - \gamma)$.
- If Δ units of entrants are lost, the immediate loss is $\Delta A_\tau dt$ but the present value of loss is $\Delta A_\tau/(\rho + \delta - \gamma)$, where ρ is the discount rate.

T

- $g = 0, \gamma > 0$: T const, firms grow
- $g > 0, \gamma = 0$: T grow, firms stay the same size

Some facts on firm growth

Figures from Haltiwanger et al. (2013)

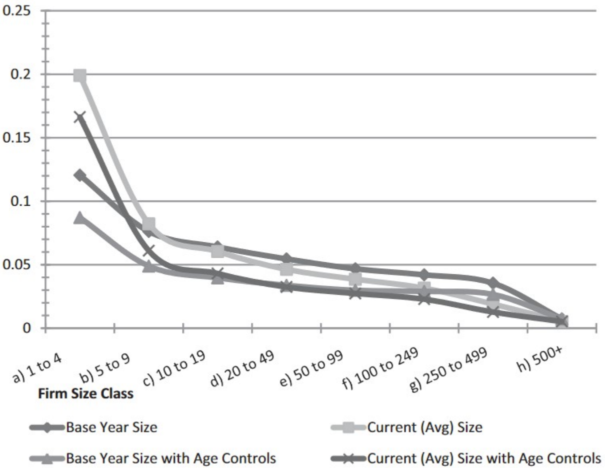


- The growth rate of a firm is independent of size: "Gibrat's Law" (mixed supports in the data)

Some facts on firm growth

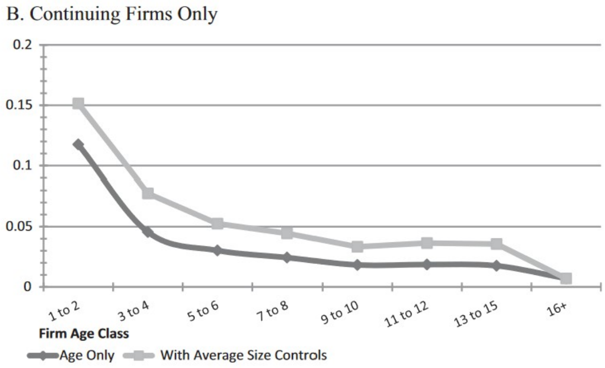
Figures from Haltiwanger et al. (2013)

FIGURE 3.—FIRM EXIT BY FIRM SIZE



Some facts on firm growth

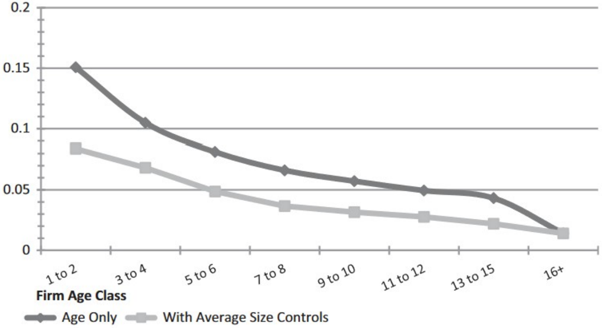
Figures from Haltiwanger et al. (2013)



Some facts on firm growth

Figures from Haltiwanger et al. (2013)

Figure 5.—Firm Exit by Firm Age



Gibrat's law implies a Pareto tail

Let us go back to the previous example, but with growth in terms of size.

- A firm is born with size $A > 0$.
- It grows at the rate γ.
- It exits at the rate δ.
- At the stationary distribution, for any size $x > A$, the density $s(x)$ has to satisfy

  $$s(xe^{\gamma dt})\Delta e^{\gamma dt} = e^{-\delta dt} s(x)\Delta$$

  for small dt and Δ.
- Guess that the distribution is Pareto: $s(x) = Fx^{-(\kappa+1)}$ (for $x \geq A$), where $F > 0$ and κ is the shape parameter. Then

  $$F(xe^{\gamma dt})^{-(\kappa+1)} e^{\gamma dt} = e^{-\delta dt} F x^{-(\kappa+1)}$$

  and therefore

  $$\kappa = \frac{\delta}{\gamma}.$$

  A large γ or a small δ implies a small κ (thick tail).

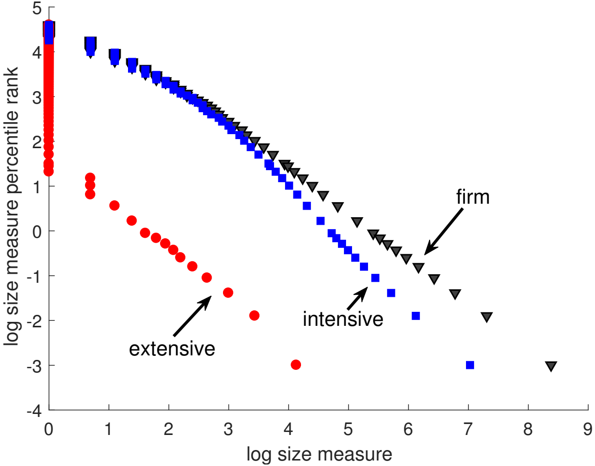
Gibrat's law implies a Pareto tail

$$s(xe^{\gamma dt})\Delta e^{\gamma dt} = e^{-\delta dt}s(x)\Delta$$

The distributions in the US, from QCEW

From Cao et al. (2020):

- Intensive margin: average employment per establishment in each firm
- Extensive margin: number of establishments in each firm



US versus Japan

Tables from Mukoyama (2009)

Table 2 Entry and Exit Rates

Annual, percent

	United States	Japan
Entry rate	11.6	4.4
Exit rate	10.2	4.4

Table 4 Establishments in the United States and Japan: Average Sizes

	United States	Japan
Average size of all establishments	17.6	9.4
Average size of opening establishments	8.3	9.6
Average size of closing establishments	9.0	7.9

- Low average size in Japan (despite a large entrant size) with low exit rates: lack of growth in establishments.

A simplified version of Mukoyama and Osotimehin (2019)

One way of looking at this paper is Hopenhayn and Rogerson (1993) with endogenous productivity shocks (and growth).

- Representative consumer:

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t [\log(C_t) - \xi L_t],$$

  where $\beta \in (0,1)$ and $\xi > 0$.

- Final good (used for consumption and R&D):

$$Y_t = \left(\int_{\mathcal{N}_t} {q_{jt}}^{\psi} {y_{jt}}^{1-\psi} dj \right)^{\frac{1}{1-\psi}}.$$

- Quality q_{jt} can be improved by incumbent intermediate producer's innovation or entrants' creative destruction ("quality ladders").

- Aggregate quality (productivity) index: $Q_t \equiv \bar{q}_t^{\frac{\psi}{1-\psi}}$.

Intermediate-good firms (monopolistic competition)

- Good j only produced by the cutting-edge producer (monopoly).
- Produced only by labor.

$$y_{jt} = \ell_{jt}.$$

- Exit if entrant innovates on product j or if hit by an exogenous shock δ.
- Firing costs
  - Tax for each worker fired τw.
  - The tax is transferred lump-sum to the consumer.
- Incumbents can innovate on their own products.
- Entrants innovate randomly across different products.
- Innovation is stochastic.

$$q_{jt} = \begin{cases} (1+\lambda_i) q_{j,t-1} & \text{if innovates} \\ q_{j,t-1} & \text{if does not innovate} \end{cases}$$

where $i = I, E$.

Quality ladders

Model: innovation

- Incumbents
  - Improves the quality of its own product by R&D (in final goods).
  - Probability of successful innovation: x_{Ijt}
  - Innovation cost:

$$\mathbf{r}_{Ijt} = \theta_I Q_t \frac{q_{jt}}{\bar{q}_t} {x_{Ijt}}^{\gamma}.$$

- Entrants
  - First pay the entry cost ϕQ_t (become a potential entrant) and then conduct R&D
  - Probability of successful innovation: x_{Et}
  - Innovation cost:

$$\mathbf{r}_{Et} = \theta_E Q_t {x_{Et}}^{\gamma}.$$

  - Cost is increasing in the aggregate productivity Q_t.
  - Creative destruction rate: $\mu = m x_E$, where m is the number of potential entrants.

Model: firm's problem

$$
\begin{aligned}
& V_t(q_t, \ell_{t-1}) \\
& \quad = \max_{\ell_t, x_{It}} \quad \Pi_t(q_t, \ell_{t-1}, \ell_t, x_{It}) \\
& \qquad + \frac{1}{1+r} \Big\{ (1-\mu_t) \left[(1 - x_{It}) Z_{t+1}(q_t, \ell_t) \right. \\
& \qquad \qquad \left. + x_{It} Z_{t+1}((1+\lambda_I) q_t, \ell_t) \right] \\
& \qquad \qquad - \mu_t \tau w_{t+1} \ell_t \Big\}
\end{aligned}
$$

where

$$
Z_t(q_t, \ell_{t-1}) = (1-\delta) V_t(q_t, \ell_{t-1}) - \delta \tau w_t \ell_{t-1}
$$

and

$$
\begin{aligned}
& \Pi_t(q_t, \ell_{t-1}, \ell_t, x_{It}) \\
& \quad = (p_t - w_t)\ell_t - \theta_I Q_t \frac{q_t}{\bar{q}_t} {x_{It}}^{\gamma} - \tau w_t \max\langle 0, \ell_{t-1} - \ell_t \rangle,
\end{aligned}
$$

with $p_t = q_t^{\psi} {y_{jt}}^{-\psi} {Y_t}^{\psi}$.

Model: entry

- Free-entry condition

$$\max_{x_{E_t}} \left\{ -\theta_E Q_t {x_{Et}}^{\gamma} - \phi Q_t + \frac{1}{1+r} x_{Et} \bar{V}_{E,t+1} \right\} = 0,$$

- $\bar{V}_E$ is the expected value of entry
- Creative destruction rate $\mu = m x_E$

Model: solving for the stationary equilibrium

1. Normalized model: $\hat{Y} \equiv Y_t/Q_t$, $\hat{w} \equiv w_t/Q_t$, $\hat{q} \equiv q_t/\bar{q}_t$,....
2. Given g_q, μ, $\hat{Y}$, $\hat{w}$
   - Compute value functions and decision functions
   - Stationary distribution of firms over $\hat{q}$, α and ℓ_{-1}
3. Stationary GE conditions: find g_q, μ, $\hat{Y}$, $\hat{w}$ such that
   (i) $\hat{Y}$ consistent with firms' output decision
   (ii) $\hat{\hat{V}}_E$ satisfies the free entry condition
   (iii) $\hat{Y} = \hat{C} + \hat{R}$
   (iv) $\frac{1}{N} \int \int \hat{q} f(\hat{q}, \ell) d\ell dq = 1$

The steps are very similar to the standard firm dynamics model (Hopenhayn and Rogerson, 1993) and the standard heterogeneous-agent models (Bewley-Huggett-Aiyagari)

Model: stationarized problem

$$\hat{V}(\hat{q},\ell) = \max_{\ell' \geq 0, x_I} \hat{\Pi}(\hat{q},\ell,\ell',x_I) \\ + \beta\left((1-\mu)\hat{S}(x_I,\hat{q}/(1+g_q),\ell') - \mu\tau\hat{w}\ell'\right),$$

where

$$\hat{S}(x_I,\hat{q}/(1+g_q),\ell') = (1 - x_I\hat{Z}(\hat{q}/(1+g_q),\ell') \\ + x_I\hat{Z}((1+\lambda)\hat{q}/(1+g_q),\ell').$$

$$\hat{\Pi}(q,\alpha,\ell,\ell',x_I) = (\hat{q}^{\psi}\ell'^{-\psi}\hat{Y}^{\psi} - \hat{w})\ell' - \theta_I\hat{q}{x_I}^{\gamma} - \tau\hat{w}\max\langle 0,\ell-\ell'\rangle.$$

- The frictionless case can be solved analytically. (Next slide)
- For the case with $\tau > 0$, there is one more step to computation: rewrite ℓ as the deviation from the frictionless level of ℓ (which can be computed from static optimization). This step is important because ℓ can have a very long tail.

Model: frictionless benchmark

- The value function $\hat{Z}$ is linear in productivity q.
- The innovation decision is independent of q ($\rightarrow$ Gibrat's law)
- Note: the normalized productivity next period is $(1+\lambda_I)\hat{q}/(1+g_q)$ with successful innovation, but without innovation $\hat{q}/(1+g_q) < \hat{q}$. $\rightarrow$ the firm has to contract if it does not innovate.
- Right tail of the productivity distribution is Pareto

$$F(\hat{q} > u) \propto u^{-\kappa},$$

where κ is the solution to:

$$1 = (1-\delta)\left[(\mu + (1-\mu)x_I)\gamma_i^{\kappa} + (1-\mu-(1-\mu)x_I)\gamma_n^{\kappa}\right].$$

- Growth rate of $\bar{q}$:

$$1 + g_q = (1-\delta)[(1+\lambda_I x_I)(1-\mu) + (1+\lambda_E)\mu] + \delta(1+\lambda_E)\bar{q}^h$$

Results: the effect of a higher firing tax

Innovation rate of entrants is lower.

- period profit (tax payment/distortion/wages) $(-)$

Innovation rate of incumbents can be higher or lower.

- period profit (tax payment/distortion/wages) $(-)$
- creative destruction effect (lower μ) $(+)$
- tax-escaping effect (escape tax payment by innovating) $(+)$

References

Cao, D., H. R. Hyatt, T. Mukoyama, and E. Sager (2020). Firm Growth through New Establishments. `https://toshimukoyama.github.io/MyWebsite/CHMS.pdf`.

Haltiwanger, J., R. S. Jarmin, and J. Miranda (2013). Who Creates Jobs? Small versus Large versus Young. *Review of Economics and Statistics*.

Hopenhayn, H. and R. Rogerson (1993). Job Turnover and Policy Evaluation: A General Equilibrium Analysis. *Journal of Political Economy 101*, 915–938.

Lee, Y. and T. Mukoyama (2015). Productivity and Employment Dynamics of US Manufacturing Plants. *Economics Letters 136*, 190–193.

Moll, B. (2014). Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation? *American Economic Review 104*, 3186–3221.

Moscoso Boedo, H. and T. Mukoyama (2012). Evaluating the Effects of Entry Regulations and Firing Costson International Income Differences. *Journal of Economic Growth 17*, 143–170.

Mukoyama, T. (2009). On the Establishment Dynamics in the United States and Japan. *Monetary and Economic Studies 27*, 53–74.

Mukoyama, T. and S. Osotimehin (2019). Barriers to Reallocation and Economic Growth: The Effects of Firing Costs. *American Economic Journal: Macroeconomics 11*, 235–270.

Mukoyama, T. and L. Popov (2014). The Political Economy of Entry Barriers. *Review of Economic Dynamics 17*, 383–416.

Mukoyama, T. and L. Popov (2020). Industrialization and the Evolution of Enforcement Institutions. *Economic Theory 69*, 745–788.

Restuccia, D. and R. Rogerson (2008). Policy Distortions and Aggregate Productivity with Heterogeneous Establishments. *Review of Economic Dynamics 11*, 707–720.