Online Appendix

D CPS data

D.1 Adjustment for the CPS redesign in January 1994

In computing the (un)employment rate, we use the multiplicative factor constructed in Polivka and Miller (1998) to correct the break attributable to the CPS redesign in January 1994. The adjusted (un)employment rate equals the adjusted number of (un)employment divided by the adjusted number of labor force, where the adjusted number of employment is given by the adjusted employment-to-population rate times the civilian noninstitutional population and the adjusted number of labor force is given by the adjusted labor participation rate times the civilian noninstitutional population.

Similarly, in calculating the adjusted full-time and part-time employment rates plotted in Figure 1, we use the multiplicative factor for the ratio of part-time employment to total employment.

D.2 Margin adjustment

In order to correct margin errors, we employ the method proposed by Elsby, Hobijn, and Şahin (2015). Below, let E_t^F, E_t^P, U_t^F, U_t^P, and O_t be the number of workers in the labor market states EF, EP, UP, UP, and O, respectively, at the beginning of period t. We define a vector $\mathbf{\Delta s}_t$ to be

$$\Delta \mathbf{s}_t = \mathbf{s}_t - \mathbf{s}_{t-1} = [E_t^F - E_{t-1}^F, E_t^P - E_{t-1}^P, U_t^F - U_{t-1}^F, U_t^P - U_{t-1}^P]'.$$

The identity that the change in the stock is the sum of the inflows out of the stock minus the outflows to the stock shows

$$\Delta \mathbf{s}_t = \mathbf{X}_{t-1}\mathbf{p},$$

where

$$\mathbf{X}_{t-1} =$$

$$\begin{bmatrix} -E_{t-1}^F & -E_{t-1}^F & -E_{t-1}^F & -E_{t-1}^F & E_{t-1}^P & 0 & 0 & 0 & U_{t-1}^F & 0 & 0 & 0 & U_{t-1}^P & 0 & 0 & 0 & O_{t-1} & 0 & 0 & 0 \\ E_{t-1}^F & 0 & 0 & 0 & -E_{t-1}^P & -E_{t-1}^P & -E_{t-1}^P & -E_{t-1}^P & 0 & U_{t-1}^F & 0 & 0 & 0 & U_{t-1}^P & 0 & 0 & 0 & O_{t-1} & 0 & 0 \\ 0 & E_{t-1}^F & 0 & 0 & 0 & E_{t-1}^P & 0 & 0 & -U_{t-1}^F & -U_{t-1}^F & -U_{t-1}^F & -U_{t-1}^F & 0 & 0 & U_{t-1}^P & 0 & 0 & 0 & O_{t-1} & 0 \\ 0 & 0 & E_{t-1}^F & 0 & 0 & 0 & E_{t-1}^P & 0 & 0 & 0 & U_{t-1}^F & 0 & -U_{t-1}^P & -U_{t-1}^P & -U_{t-1}^P & -U_{t-1}^P & 0 & 0 & 0 & O_{t-1} \end{bmatrix},$$

and

$\mathbf{p} =$

$$\left[\begin{array}{cccccccccccccccccccc} p_{E^FE^P} & p_{E^FU^F} & p_{E^FU^P} & p_{E^FO} & p_{E^PE^F} & p_{E^PU^F} & p_{E^PU^P} & p_{E^PO} & p_{U^FE^F} & p_{U^FE^P} & p_{U^FU^P} & p_{U^FO} & p_{U^PE^F} & p_{U^PE^P} & p_{U^PU^F} & p_{U^PO} & p_{OE^F} & p_{OE^P} & p_{OU^F} & p_{OU^P} \end{array}\right]',$$

with the element p_{ij} denoting the transition probability from state i to state j.

Given the vector of the transition probabilities in data $\hat{\mathbf{p}}$, the vector of the change of the stocks in data $\Delta\mathbf{s}_t$ and the matrix $\mathbf{X}_{t-1}$, the vector of corrected transition probabilities is chosen so as to minimize

$$\frac{1}{2}(\mathbf{p}-\hat{\mathbf{p}})'\mathbf{W}(\mathbf{p}-\hat{\mathbf{p}})$$

subject to

$$\Delta\mathbf{s}_t = \mathbf{X}_{t-1}\mathbf{p},$$

where the weight matrix satisfies

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{EF} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{EP} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W}_{UF} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{UP} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{N} \end{bmatrix}^{-1},$$

and

$$\mathbf{W}_{EF} = \begin{bmatrix} \frac{\hat{p}_{E^FE^P}(1-\hat{p}_{E^FE^P})}{E^F_{t-1}} & -\frac{\hat{p}_{E^FE^P}\hat{p}_{E^FU^F}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FE^P}\hat{p}_{E^FU^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FE^P}\hat{p}_{E^FN}}{E^F_{t-1}} \\ -\frac{\hat{p}_{E^FU^F}\hat{p}_{E^FE^P}}{E^F_{t-1}} & \frac{\hat{p}_{E^FU^F}(1-\hat{p}_{E^FU^F})}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^F}\hat{p}_{E^FU^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^F}\hat{p}_{E^FN}}{E^F_{t-1}} \\ -\frac{\hat{p}_{E^FU^P}\hat{p}_{E^FE^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^P}\hat{p}_{E^FU^F}}{E^F_{t-1}} & \frac{\hat{p}_{E^FU^P}(1-\hat{p}_{E^FU^P})}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^P}\hat{p}_{E^FN}}{E^F_{t-1}} \\ -\frac{\hat{p}_{E^FN}\hat{p}_{E^FE^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FN}\hat{p}_{E^FU^F}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FN}\hat{p}_{E^FU^P}}{E^F_{t-1}} & \frac{\hat{p}_{E^FN}(1-\hat{p}_{E^FN})}{E^F_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{EP} = \begin{bmatrix} \frac{\hat{p}_{E^PE^F}(1-\hat{p}_{E^PE^F})}{E^P_{t-1}} & -\frac{\hat{p}_{E^PE^F}\hat{p}_{E^PU^F}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PE^F}\hat{p}_{E^PU^P}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PE^F}\hat{p}_{E^PN}}{E^P_{t-1}} \\ -\frac{\hat{p}_{E^PU^F}\hat{p}_{E^PE^F}}{E^P_{t-1}} & \frac{\hat{p}_{E^PU^F}(1-\hat{p}_{E^PU^F})}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^F}\hat{p}_{E^PU^P}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^F}\hat{p}_{E^PN}}{E^P_{t-1}} \\ -\frac{\hat{p}_{E^PU^P}\hat{p}_{E^PE^F}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^P}\hat{p}_{E^PU^F}}{E^P_{t-1}} & \frac{\hat{p}_{E^PU^P}(1-\hat{p}_{E^PU^P})}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^P}\hat{p}_{E^PN}}{E^P_{t-1}} \\ -\frac{\hat{p}_{E^PN}\hat{p}_{E^PE^F}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PN}\hat{p}_{E^PU^F}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PN}\hat{p}_{E^PU^P}}{E^P_{t-1}} & \frac{\hat{p}_{E^PN}(1-\hat{p}_{E^PN})}{E^P_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{UF} = \begin{bmatrix} \frac{\hat{p}_{U^FE^F}(1-\hat{p}_{U^FE^F})}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^F}\hat{p}_{U^FE^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^F}\hat{p}_{U^FU^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^F}\hat{p}_{U^FN}}{U^F_{t-1}} \\ -\frac{\hat{p}_{U^FE^P}\hat{p}_{U^FE^F}}{U^F_{t-1}} & \frac{\hat{p}_{U^FE^P}(1-\hat{p}_{U^FE^P})}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^P}\hat{p}_{U^FU^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^P}\hat{p}_{U^FN}}{U^F_{t-1}} \\ -\frac{\hat{p}_{U^FU^P}\hat{p}_{U^FE^F}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FU^P}\hat{p}_{U^FE^P}}{U^F_{t-1}} & \frac{\hat{p}_{U^FU^P}(1-\hat{p}_{U^FU^P})}{U^F_{t-1}} & -\frac{\hat{p}_{U^FU^P}\hat{p}_{U^FN}}{U^F_{t-1}} \\ -\frac{\hat{p}_{U^FN}\hat{p}_{U^FE^F}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FN}\hat{p}_{U^FE^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FN}\hat{p}_{U^FU^P}}{U^F_{t-1}} & \frac{\hat{p}_{U^FN}(1-\hat{p}_{U^FN})}{U^F_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{UP} = \begin{bmatrix} \frac{\hat{p}_{U^PE^F}(1-\hat{p}_{U^PE^F})}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^F}\hat{p}_{U^PE^P}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^F}\hat{p}_{U^PU^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^F}\hat{p}_{U^PN}}{U^P_{t-1}} \\ -\frac{\hat{p}_{U^PE^P}\hat{p}_{U^PE^F}}{U^P_{t-1}} & \frac{\hat{p}_{U^PE^P}(1-\hat{p}_{U^PE^P})}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^P}\hat{p}_{U^PU^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^P}\hat{p}_{U^PN}}{U^P_{t-1}} \\ -\frac{\hat{p}_{U^PU^F}\hat{p}_{U^PE^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PU^F}\hat{p}_{U^PE^P}}{U^P_{t-1}} & \frac{\hat{p}_{U^PU^F}(1-\hat{p}_{U^PU^F})}{U^P_{t-1}} & -\frac{\hat{p}_{U^PU^F}\hat{p}_{U^PN}}{U^P_{t-1}} \\ -\frac{\hat{p}_{U^PN}\hat{p}_{U^PE^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PN}\hat{p}_{U^PE^P}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PN}\hat{p}_{U^PU^F}}{U^P_{t-1}} & \frac{\hat{p}_{U^PN}(1-\hat{p}_{U^PN})}{U^P_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{N} = \begin{bmatrix} \frac{\hat{p}_{NE^F}(1-\hat{p}_{NE^F})}{N_{t-1}} & -\frac{\hat{p}_{NE^F}\hat{p}_{NE^P}}{N_{t-1}} & -\frac{\hat{p}_{NE^F}\hat{p}_{NU^F}}{N_{t-1}} & -\frac{\hat{p}_{NE^F}\hat{p}_{NU^P}}{N_{t-1}} \\ -\frac{\hat{p}_{NE^P}\hat{p}_{NE^F}}{N_{t-1}} & \frac{\hat{p}_{NE^P}(1-\hat{p}_{NE^P})}{N_{t-1}} & -\frac{\hat{p}_{NE^P}\hat{p}_{NU^F}}{N_{t-1}} & -\frac{\hat{p}_{NE^P}\hat{p}_{NU^P}}{N_{t-1}} \\ -\frac{\hat{p}_{NU^F}\hat{p}_{NE^F}}{N_{t-1}} & -\frac{\hat{p}_{NU^F}\hat{p}_{NE^P}}{N_{t-1}} & \frac{\hat{p}_{NU^F}(1-\hat{p}_{NU^F})}{N_{t-1}} & -\frac{\hat{p}_{NU^F}\hat{p}_{NU^P}}{N_{t-1}} \\ -\frac{\hat{p}_{NU^P}\hat{p}_{NE^F}}{N_{t-1}} & -\frac{\hat{p}_{NU^P}\hat{p}_{NE^P}}{N_{t-1}} & -\frac{\hat{p}_{NU^P}\hat{p}_{NU^F}}{N_{t-1}} & \frac{\hat{p}_{NU^P}(1-\hat{p}_{NU^P})}{N_{t-1}} \end{bmatrix}.$$

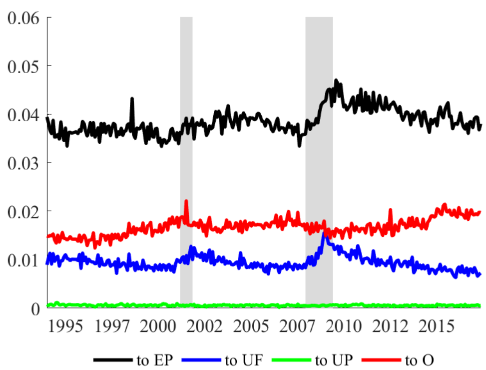
E Supplement on the CPS evidence

E.1 The monthly transition probabilities

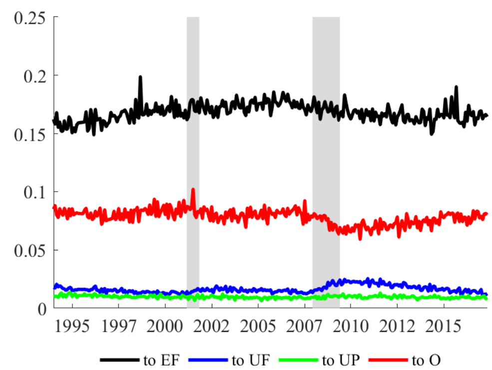
Figure E.1 displays the monthly transition probabilities from EF (Panel (A)), from EP (Panel (B)), from UF (Panel (C)), and from UP (Panel (D)) from 1994 to 2018.

Figure E.1: Transition probability

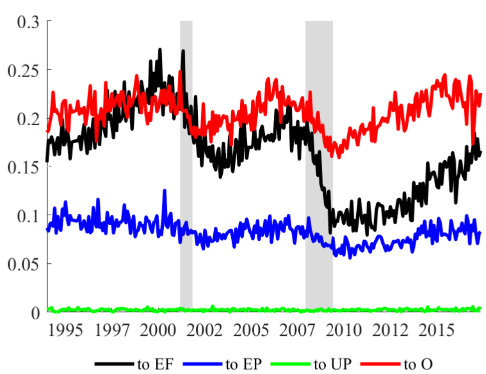
(A) Transition probability from EF



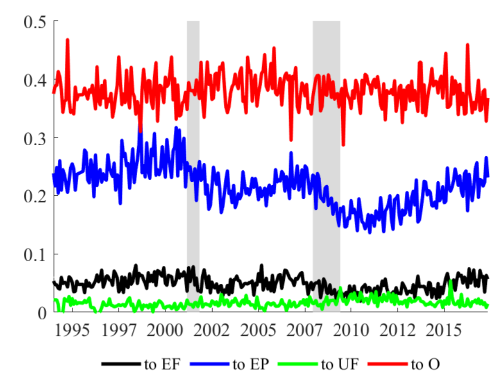
(B) Transition probability from EP



(C) Transition probability from UF



(D) Transition probability from UP



Note: All series are seasonally adjusted. Since there are missing observations in 1995 due to the failure of individual identifiers in the CPS, we use Tramo ("Time Series Regression with ARIMA Noise, Missing Observations, and Outliers")/Seats ("Signal Extraction in ARIMA Time Series") interface to interpolate the missing observations along with seasonal adjustment.

Table E.1 shows the average of the monthly transition probabilities. Due to the missing observations in 1995, we report the averages over 1996:M1–2018:M12. In this table, by construction, the sum of each row is 1. We find that about 24 percent of unemployed workers in full-time labor market find a job and 2/3 of them find a job in full-time position. Also, about 26 percent of unemployed workers in part-time labor market find a job and 4/5 of them find a job in part-time position.

Table E.1: Average transition probability (monthly)

		S^j_{t+1}				
		EF	EP	UF	UP	O
S^i_t	EF	0.934	0.039	0.009	0.001	0.017
	EP	0.169	0.727	0.016	0.010	0.078
	UF	0.163	0.081	0.548	0.002	0.206
	UP	0.049	0.215	0.016	0.340	0.380
	O	0.020	0.025	0.018	0.007	0.930

E.2 Net-flow decomposition analysis with alternative definition of the Great Recession period

Table E.2 reports the result of the net-flow decomposition using the alternative definition of of the Great Recession period, from December 2007 to November 2009.

Table E.2: Net-flow decomposition of employment stocks over 2007M12-2009M11

	$j = EF$	$j = EP$
Rate of change in stock of state j $(r^j_{GR} - \overline{r}^j)$	-0.48	0.33
Net flow rate from state i to state j $(\overline{f}^{ij}_{GR} - \overline{f}^{ij})$		
$i = EF$	–	0.74
$i = EP$	-0.16	–
$i = UF$	-0.22	-0.09
$i = UP$	-0.01	-0.13
$i = O$	-0.09	-0.19

Note: Average monthly flow (%) over the Great Recession period (December 2007 to November 2009), compared to the long-run average over the entire period (January 1996 to December 2018).

E.3 Involuntary part-time employment

In this section, we divide part-time employment further into voluntary part-time employment (EVP) and involuntary part-time employment (EIP) based on the CPS Questionnaire. Earlier work, such as Lariau (2017) and Warren (2017) emphasize this distinction. Table E.3 reproduces the transition matrix (Table E.1) with this distinction.

In the main text, we emphasize the gross flows between full time and part time. From Table E.3, one can see that a larger fraction of EIP moves to EF compared to EVP does. At the same time, in terms of the total number of people who flows into EF, EVP is also an important origin because the size of EVP stock is substantially larger than the size of EIP stock. (The average stock of EVP is about four times larger than the stock of EIP.) Therefore, both EVP and EIP are important sources of gross inflows

Table E.3: Average transition probability

		S^j_{t+1}					
		EF	EIP	EVP	UF	UP	O
	EF	0.945	0.007	0.021	0.009	0.001	0.017
	EIP	0.261	0.421	0.214	0.049	0.009	0.046
S^i_t	EVP	0.132	0.039	0.728	0.009	0.010	0.082
	UF	0.161	0.034	0.032	0.559	0.002	0.211
	UP	0.047	0.024	0.177	0.015	0.348	0.388
	O	0.018	0.002	0.018	0.017	0.007	0.938

Note: This table shows the average of the monthly transition probability between 1996 and 2018. EF is full-time employment, EIP is involuntary part-time employment, EVP is voluntary part-time employment, UF is the unemployed looking for a full-time job, UP is the unemployed looking for a part-time job, and O is out of labor force.

into EF. For the inflow from UP, the largest employment destination is EVP. Note, however, UP is also an important source of inflow for EIP, given that the size of EIP is much smaller than EVP.

In our baseline model, we do not make distinction between EIP and EVP. The first reason is that the important counterparts of the gross flows are similar (EF and UP) between these two. The second is the concern regarding the distinction: whereas there is a clear metric for distinction between full-time and part-time (usual hours of work), the difference between EIP and EVP is arguably more subjective. This subjective aspect, we suspect, is reflected in a large flow from EIP to EVP—in terms of economic intuitions, it is not clear why so many would change the status from "involuntary" to "voluntary" for each month.

Table E.4: Net-flow decomposition of employment stocks over the Great Recession period

	$j = EF$	$j = EIP$	$j = EVP$
The rate of change in stock of state j	−0.49	3.37	−0.31
Net flow rate from state i to state j			
$i = EF$	–	2.94	0.38
$i = EIP$	−0.09	–	−0.26
$i = EVP$	−0.07	1.30	–
$i = UF$	−0.22	−0.62	−0.06
$i = UP$	−0.01	−0.26	−0.11
$i = O$	−0.09	0.01	−0.27

Note: Average monthly flow (%) during the Great Recession period (December 2007 to November 2009), compared to the long-run average between January 1996 and December 2018.

Table E.4 repeats the analysis of Table 1 with the distinction between EVP and EIP. Two results emerge. First, as is pointed out in previous studies such as Lariau

(2017), Warren (2017), and Borowczyk-Martins and Lalé (2018) EIP stock is strongly countercyclical. Second, although EVP stock is procyclical, its flow components behave very similarly to the components of EIP (except for the net flow from O, which we ignore in the model). For example, the net flow from EF contributed positively to both EIP and EVP during the Great Recession. The decline in job finding from both UF and UP contribute strongly to both EIP and EVP stocks. This pattern is in contrast to EF stock, for which the UP contribution is almost zero. This similarity in the each component of the flows is another reason we put EIP and EVP together in the model specification.

As is emphasized in earlier studies, there are also some potentially important differences between EIP and EVP, especially from a quantitative viewpoint. For example, the cyclicality of EIP is an important feature that stands out, and the cyclical EF to EP flow, which we emphasize in the main text, is mainly due to the EF to EIP flow. As is explained in the main text, we decided to pool EIP and EVP in the baseline analysis due to the tractability reasons in the estimation and also over the measurement concerns. The pooling assumption is, from this viewpoint, a limitation of our analysis and it is an important future research to consider a model with explicit distinction between EIP and EVP as well as investigating the measurement issues further.

F Derivation of model equations

F.1 The optimization problem of the wholesale firms

Since all the subdivisions in the full-time and part-time division face the analogous problem except that the evolution equations of the employment stock are differ depending on the division to which the subdivision belongs, we describe the optimization problem of the subdivision in the full-time division.

Each subdivision's value function in period t is given by

$$\mathcal{F}_t^F(n_{j,t-1}^F, v_{j,t-1}^F; w_{j,t}^F) = \max_{\varrho_{j,t}^F, v_{j,t}^F, n_{j,t}^F, k_{j,t}^F} \left(\begin{array}{r} p_t^F y_{j,t}^F - \frac{w_{j,t}^{Fn}}{p_t} n_{j,t}^F - r_t^k k_{j,t}^F - \left(\mathcal{K}_t^F \left(\frac{q_{t-1}^F v_{j,t-1}^F}{n_{j,t-1}^F} \right) + \mathcal{A}_t^F(\varrho_{j,t}^F) \right) n_{j,t-1}^F \\ + \mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{F}_{t+1}^F(n_{j,t}^F, v_{j,t}^F; w_{j,t+1}^F) \right] \end{array} \right)$$

subject to

$$n_{j,t}^F \leq \varrho_{j,t}^F n_{j,t-1}^F + q_{t-1}^F v_{j,t-1}^F + n_t^{PF}, \tag{F.1}$$

and

$$v_{j,t}^F \geq 0. \tag{F.2}$$

Let $J_{j,t}^F$ and $\Theta_{j,t}^{v,F}$ be the Lagrange multipliers for the constraints (F.1) and (F.2), respectively. The optimality condition for capital input implies

$$r_t^k = p_t^F \alpha \left(\frac{y_{j,t}^F}{k_{j,t}^F} \right) = p_t^F \alpha \left(\frac{y_t^F}{k_t^F} \right).$$

At the optimum, (F.1) holds with equality. The first-order necessary conditions for $n_{j,t}^F$, $\varrho_{j,t}^F$, and $v_{j,t}^F$ are respectively given by

$$p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\partial \mathcal{F}_{t+1}^F(n_{j,t}^F, v_{j,t}^F; w_{j,t+1}^F)}{\partial n_{j,t}^F} \right] = J_{j,t}^F, \tag{F.3}$$

$$\frac{\partial \mathcal{A}_t^F(\varrho_{j,t}^F)}{\partial \varrho_{j,t}^F} = J_{j,t}^F, \tag{F.4}$$

and

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\partial \mathcal{F}_{t+1}^F(n_{j,t}^F, v_{j,t}^F; w_{j,t+1}^F)}{\partial v_{j,t}^F} \right] + \Theta_{j,t}^{v,F} = 0, \tag{F.5}$$

with $v_{j,t}^F \Theta_{j,t}^{v,F} = 0$.

The envelop theorem shows

$$\frac{\partial \mathcal{F}_t^F(n_{j,t-1}^F, v_{j,t-1}^F; w_{j,t}^F)}{\partial n_{j,t-1}^F} = \frac{\partial \mathcal{K}_t^F(x_{j,t-1}^F)}{\partial x_{j,t-1}^F} x_{j,t-1}^F - \left(\mathcal{K}_t^F(x_{j,t-1}^F) + \mathcal{A}_t^F(\varrho_{j,t}^F)\right) + J_{j,t}^F \varrho_{j,t}^F, \quad \text{(F.6)}$$

and

$$\frac{\partial \mathcal{F}_t^F(n_{j,t-1}^F, v_{j,t-1}^F; w_{j,t}^F)}{\partial v_{j,t-1}^F} = q_{t-1}^F \left(J_{j,t}^F - \frac{\partial \mathcal{K}_t^F(x_{j,t-1}^F)}{\partial x_{j,t-1}^F} \right), \quad \text{(F.7)}$$

where

$$x_{j,t}^F = \frac{q_t^F v_{j,t}^F}{n_{j,t}^F}.$$

Here, we focus on the interior optimum, at which $\Theta_{j,t}^{v,F} = 0$ for all $j \in \mathcal{J}^F$. Then (F.5) and (F.7) imply that

$$\mathbb{E}_t \left[\Lambda_{t,t+1} q_t^F \left(J_{j,t+1}^F - \frac{\partial \mathcal{K}_{t+1}^F(x_{j,t}^F)}{\partial x_{j,t}^F} \right) \right] = 0.$$

Because $q_t^F > 0$, for any states, it must hold that

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \left(J_{j,t+1}^F - \frac{\partial \mathcal{K}_{t+1}^F(x_{j,t}^F)}{\partial x_{j,t}^F} \right) \right] = 0.$$

Since $\mathcal{K}^F(\cdot)$ is a quadratic function, substitute (F.6) into (F.3) delivers[43]

$$J_{j,t}^F = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\mathcal{K}_{t+1}^F(x_{j,t}^F) - \mathcal{A}_{t+1}^F(\varrho_{j,t+1}^F) + J_{j,t+1}^F \varrho_{j,t+1}^F \right) \right]. \quad \text{(F.8)}$$

Then, the above equation shows $J_{j,t}^F = J_t^F(w_{j,t}^{Fn})$, (F.4) shows $\varrho_{j,t}^F = \rho_t^F(w_{j,t}^{Fn})$, and, by construction, $x_{j,t}^F = x_t^F(w_{j,t}^{Fn})$ respectively. We can rewrite (F.8) as

$$J_t^F(w_{j,t}^{Fn}) = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \begin{pmatrix} \mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) - \mathcal{A}_{t+1}^F(\rho_{t+1}^F(w_{j,t+1}^{Fn})) \\ +\rho_{t+1}^F(w_{j,t+1}^{Fn}) J_{t+1}^F(w_{j,t+1}^{Fn}) \end{pmatrix} \right]. \quad \text{(F.9)}$$

Therefore, using

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} x_t^F(w_{j,t}^{Fn}) J_{t+1}^F(w_{j,t+1}^{Fn}) \right] - \mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) \right],$$

---

[43]When $\mathcal{K}^F(\cdot)$ is a quadratic function, it must be held that $-\mathcal{K}_t^F(x_{j,t}^F) + \frac{\partial \mathcal{K}_t^F(x_{j,t}^F)}{\partial x_{j,t}^F} x_{j,t}^F = \mathcal{K}_t^F(x_{j,t}^F)$.

we rewrite (F.9) as

$$J_t^F(w_{j,t}^{Fn}) = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) + \mathcal{A}_{t+1}^F(\rho_{t+1}^F(w_{j,t+1}^{Fn}))\right)\right]$$
$$+ \mathbb{E}_t\left[\Lambda_{t,t+1}(x_t^F(w_{j,t}^{Fn}) + \rho_{t+1}^F(w_{j,t+1}^{Fn}))J_{t+1}^F(w_{j,t+1}^{Fn})\right].$$

F.2 Wage functions

- Worker's surplus

  We define the worker's surplus from being employed as $H_t^F(w_{j,t}^{Fn}) = V_t^F(w_{j,t}^{Fn}) - U_t$ and $H_t^P(w_{j,t}^{Pn}) = V_t^P(w_{j,t}^{Pn}) - U_t$ and denote their average conditional on being a newly employed worker by $H_{x,t}^F = V_{x,t}^F - U_t$ and $H_{x,t}^P = V_{x,t}^P - U_t$. By construction, $H_t^F(w_{j,t}^{Fn})$ and $H_t^P(w_{j,t}^{Pn})$ respectively are give by

$$H_t^F(w_{j,t}^{Fn}) = \frac{w_{j,t}^{Fn}}{p_t} - b_t + \mathbb{E}_t\left[\Lambda_{t,t+1}\begin{pmatrix}\rho_{t+1}^F(w_{j,t+1}^{Fn})H_{t+1}^F(w_{j,t+1}^{Fn}) \\ +\lambda^{FP}(1-\rho_{t+1}^F(w_{j,t+1}^{Fn}))H_{x,t+1}^P - s_{t+1}^F H_{x,t+1}^F\end{pmatrix}\right].$$

  and

$$H_t^P(w_{j,t}^{Pn}) = \frac{w_{j,t}^{Pn}}{p_t}\mu_b^P - \hat{\mu}_b^P b_t + \mathbb{E}_t\left[\Lambda_{t,t+1}\begin{pmatrix}\rho_{t+1}^P(w_{j,t+1}^{Pn})H_{t+1}^P(w_{j,t+1}^{Pn}) + \varphi H_{x,t+1}^F) \\ -s_{t+1}^P H_{x,t+1}^P\end{pmatrix}\right]$$

  where $\hat{\mu}_b^P = 1 - (1-\mu_b^P)b^P/b$.

- Firm's surplus

  The firm's surplus from hiring an additional employee is respectively given by

$$J_t^F(w_{j,t}^{Fn}) = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{\kappa_{t+1}^F}{2}(x_t^F(w_{j,t}^{Fn}))^2 + \mathcal{A}_t(\varrho_{t+1}^F(w_{j,t+1}^{Fn}))\right)\right]$$
$$+ \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\rho_{t+1}^F(w_{j,t+1}^{Fn}) + x_t^F(w_{j,t}^{Fn})\right)J_{t+1}^F(w_{j,t+1}^{Fn})\right],$$

  and

$$J_t^P(w_{j,t}^{Pn}) = p_t^P a_t^P - \frac{w_{j,t}^{Pn}}{p_t}\mu_b^P + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{\kappa_{t+1}^P}{2}(x_t^P(w_{j,t}^{Pn}))^2 + \varrho^P J_{t+1}^P(w_{j,t+1}^{Pn})\right)\right].$$

- Optimality conditions

$$\frac{\partial \mathcal{A}_t^F(\rho_t^F(w_{j,t}^{Fn}))}{\partial \rho_t^F(w_{j,t}^{Fn})} = J_t^F(w_{j,t}^{Fn}), \tag{F.1}$$

$$\frac{\partial \mathcal{A}^P_t(\rho^P_t(w^{Pn}_{j,t}))}{\partial \rho^P_t(w^{Pn}_{j,t})} = J^P_t(w^{Pn}_{j,t}),$$

$$\kappa^F_{t+1} x^F_t(w^{Fn}_{j,t})\mathbb{E}_t\left[\Lambda_{t,t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1} J^F_{t+1}(w^{Fn}_{j,t+1})\right], \tag{F.2}$$

and

$$\kappa^P_{t+1} x^P_t(w^{Pn}_{j,t})\mathbb{E}_t\left[\Lambda_{t,t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1} J^P_{t+1}(w^{Pn}_{j,t+1})\right].$$

- De-trending

  We convert the non-stationary variables into the stationary variables: $\bar{\Lambda}_{t,t+1} = (z_{t+1}/z_t)\Lambda_{t,t+1}$, $\bar{a}^F_t = a^F_t/z_t$, $\bar{a}^P_t = a^P_t/z_t$, $\bar{b}_t = b_t/z_t$ $\bar{J}^F_t = J^F_t/z_t$, $\bar{J}^P_t = J^P_t/z_t$, $\bar{H}^F_t = H^F_t/z_t$, $\bar{H}^P_t = H^P_t/z_t$, $\bar{w}^{Fn}_{j,t} = w^{Fn}_{j,t}/z_t$, $\bar{w}^{Pn}_{j,t} = w^{Pn}_{j,t}/z_t$, $\mathcal{A}^F(\varrho^F_t) = \mathcal{A}^F_t(\varrho^F_t)/z_t$, $\mathcal{A}^P(\varrho^P_t) = \mathcal{A}^P_t(\varrho^P_t)/z_t$, $\kappa^F = \kappa^F_t/z_t$, and $\kappa^P = \kappa^P_t/z_t$.

- Some definitions and lemmas

  We define

$$\mathcal{E}^F_t(w) \equiv \frac{\partial_\rho \mathcal{A}^F(\varrho^F_t(w))}{\varrho^F_t(w)\partial^2_\rho \mathcal{A}^F(\varrho^F_t(w))}.$$

  Recall that $\mathcal{A}^F(\cdot)$ takes a power function with an exponent ζ^F, so we have

$$\mathcal{E}^F_t(w) = \frac{1}{\zeta} \equiv \mathcal{E}^F.$$

  For later use, we present the following lemmas

  Lemma 1 *For any $\lambda > 0$,*

$$\frac{\partial \mathcal{A}^F(\rho^F_t(\lambda w))}{\partial w} = \lambda \mathcal{E}^F \varrho^F_t(\lambda w)\frac{\partial \bar{J}^F_t(\lambda w)}{\partial(\lambda w)},$$

  and

$$\frac{\partial \rho^F_t(\lambda w)\bar{J}^F_t(\lambda w)}{\partial w} = \lambda\left(\mathcal{E}^F + 1\right)\varrho^F_t(\lambda w)\frac{\partial \bar{J}^F_t(\lambda w)}{\partial(\lambda w)},$$

  We define the aggregate variables as follows:

$$w^\ell_t = \int_{\mathcal{J}^\ell} \frac{w^{\ell n}_{j,t}}{p_t}\left(\frac{n^F_{j,t}}{n^F_t}\right) dj, \qquad \text{for} \quad \ell = F, P \tag{F.3}$$

  In addition, we denote the average of the hiring rate and the retention rate weighted by the employment share by

$$x^\ell_t = \int_{\mathcal{J}^\ell} x^\ell_t(w^\ell_{j,t})\left(\frac{n^\ell_{j,t}}{n^\ell_t}\right) dj, \qquad \text{for} \quad \ell = F, P \tag{F.4}$$

and

$$\rho_t^\ell = \int_{\mathcal{J}^\ell} \varrho_t^F(w_{j,t}^\ell) \left(\frac{n_{j,t}^\ell}{n_t^\ell} \right) dj \qquad \text{for} \quad \ell = F, P. \tag{F.5}$$

The law of large numbers implies the dynamics of the average nominal wage for full-time workers follows

$$w_t^F = (1-\vartheta_w^F) w_t^{*F} \int_0^1 \frac{n_{j,t}^F(w_t^{*Fn})}{n_t^F} dj + \vartheta_w^F \int_0^1 \frac{\bar{\iota}_{w,t-1}}{\pi_t} w_{j,t-1}^F \frac{n_{j,t}^F(\bar{\iota}_{w,t-1} w_{j,t-1}^{Fn})}{n_t^F} dj.$$

The log-linearized equation is given by

$$\tilde{\bar{w}}_t^F = (1-\vartheta_w^F)\tilde{\bar{w}}_t^{*F} + \vartheta_w^F \left(\iota_w \tilde{\pi}_{t-1} + \tilde{\bar{w}}_{t-1}^F - \tilde{\varepsilon}_t^z - \tilde{\pi}_t \right). \tag{F.6}$$

where $\bar{w}_t^F = w_t^F / z_t$.

Analogously, the (log-linearized) evolution equation of the average real wage for part-time workers is given by

$$\tilde{\bar{w}}_t^P = (1-\vartheta_w^P)\tilde{\bar{w}}_t^{*P} + \vartheta_w^P \left(\iota_w \tilde{\pi}_{t-1} + \tilde{\bar{w}}_{t-1}^P - \tilde{\varepsilon}_t^z - \tilde{\pi}_t \right), \tag{F.7}$$

where $\bar{w}_t^P = w_t^P / z_t$.

F.2.1 Wages for full-time workers

We aim to find the expression for $\tilde{\bar{w}}_t^{*F}$.

- The surplus sharing rule

  The renegotiated nominal wage w_t^{*Fn} satisfies the following surplus sharing rule:

$$\chi_t^F(w_t^{*Fn}) \bar{J}_t^F(w_t^{*Fn}) = [1 - \chi_t^F(w_t^{*Fn})] \bar{H}_t^F(w_t^{*Fn}), \tag{F.8}$$

  where

$$\begin{aligned} \bar{J}_t^F(w_{j,t}^{Fn}) =& p_t^F \bar{a}_t^F - \frac{w_{j,t}^{Fn}}{p_t z_t} - \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\frac{\kappa^F}{2} (x_t^F(w_{j,t}^{Fn}))^2 + \mathcal{A}(\varrho_{t+1}^F(w_{t+1}^{*Fn})) \right) \right] \\ &+ \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\rho_{t+1}^F(w_{t+1}^{*Fn}) + x_t^F(w_{j,t}^{Fn}) \right) \bar{J}_{t+1}^F(w_{t+1}^{*Fn}) \right] \\ &- \vartheta_w^F \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left[\mathcal{A}(\varrho_{t+1}^F(\hat{\iota}_{w,t} w_{j,t}^{Fn})) - \mathcal{A}(\varrho_{t+1}^F(w_{t+1}^{*Fn})) \right] \right] \\ &+ \vartheta_w^F \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\begin{array}{l} \left(\rho_{t+1}^F(\hat{\iota}_{w,t} w_{j,t}^{Fn}) + x_t^F(\hat{\iota}_{w,t} w_{j,t}^{Fn}) \right) \bar{J}_{t+1}^F(\hat{\iota}_{w,t} w_{j,t}^{Fn}) \\ - \left(\rho_{t+1}^F(w_{t+1}^{*Fn}) + x_t^F(w_{t+1}^{*Fn}) \right) \bar{J}_{t+1}^F(w_{t+1}^{*Fn}) \end{array} \right) \right], \end{aligned} \tag{F.9}$$

and

$$\bar{H}_t^F(w_{j,t}^{Fn}) = \frac{w_{j,t}^{Fn}}{p_t z_t} - \bar{b}_t + \mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\begin{array}{l}\varrho_{t+1}^F(w_{t+1}^{*Fn})\bar{H}_{t+1}^F(w_{t+1}^{*Fn}) \\ +\lambda^{FP}(1-\rho_{t+1}^F(w_{t+1}^{*Fn}))\bar{H}_{x,t+1}^P - s_t^F\bar{H}_{x,t+1}^F\end{array}\right)\right]$$
$$+\vartheta_w^F\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\begin{array}{l}\left(\varrho_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})\bar{H}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \varrho_{t+1}^F(w_{t+1}^{*Fn})\bar{H}_{t+1}^F(w_{t+1}^{*Fn})\right) \\ -\lambda^{FP}\left(\varrho_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \varrho_{t+1}^F(w_{t+1}^{*Fn})\right)\bar{H}_{x,t+1}^P\end{array}\right)\right], \tag{F.10}$$

where $\hat{\iota}_{w,t} \equiv \gamma_z(\pi)^{1-\iota_w}(\pi_t)^{\iota_w}$.

- The effective bargaining power

  The effective workers' bargaining power satisfies

$$\chi_t^F(w_t^{*Fn}) = \frac{\eta_t^F}{\eta_t^F + (1-\eta_t^F)\mu_t^F(w_t^{*Fn})/\epsilon_t^F(w_t^{*Fn})} \tag{F.11}$$

where

$$\epsilon_t^F(w_{j,t}^{Fn}) \equiv p_t \partial H_t^F(w_{j,t}^{Fn})/\partial w_{j,t}^{Fn},$$
$$\epsilon_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) \equiv (z_t/z_{t+1})p_{t+1}\partial H_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})/\partial(\hat{\iota}_{w,t}w_{j,t}^{Fn}),$$
$$\mu_t^F(w_{j,t}^{Fn}) \equiv -p_t\partial J_t^F(w_{j,t}^{Fn})/\partial w_{j,t}^{Fn},$$

and

$$\mu_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) \equiv -(z_t/z_{t+1})p_{t+1}\partial J_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})/\partial(\hat{\iota}_{w,t}w_{j,t}^{Fn}).$$

- The recursive formulations for $\epsilon_t^F(w_{j,t}^{Fn})$ and $\mu_t^F(w_{j,t}^{Fn})$

  Using Lemma 1 and

$$\frac{\partial \bar{J}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})}{\partial(\hat{\iota}_{w,t}w_{j,t}^{Fn})} = \frac{1}{z_{t+1}}\frac{\partial J_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})}{\partial(\hat{\iota}_{w,t}w_{j,t}^{Fn})},$$

$\epsilon_t^F(w_{j,t}^{Fn})$ and $\mu_t^F(w_{j,t}^{Fn})$ can be formulated recursively as

$$\epsilon_t^F(w_{j,t}^{Fn}) =$$
$$1+\vartheta_w^F\mathbb{E}_t\left[\begin{array}{l}\bar{\Lambda}_{t,t+1}\varrho_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})\frac{p_t}{p_{t+1}}\hat{\iota}_{w,t} \\ \times\left(\begin{array}{l}\epsilon_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) \\ -\mathcal{E}^F\mu_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})\frac{(\bar{H}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\lambda^{FP}\bar{H}_{x,t+1}^P)}{\bar{J}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})}\end{array}\right)\end{array}\right] \tag{F.12}$$

and

$$\mu_t^F(w_{j,t}^{Fn}) = \\ 1 + \vartheta_w^F \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\varrho_{t+1}^F(\hat{\iota}_{w,t} w_{j,t}^{Fn}) + x_t^F(w_{j,t}^{Fn}) \right) \frac{p_t}{p_{t+1}} \hat{\iota}_{w,t} \mu_{t+1}^F(\hat{\iota}_{w,t} w_{j,t}^{Fn}) \right]. \tag{F.13}$$

Since $\vartheta_w^F \varrho^F \beta \in (0,1)$, we have in the balanced-growth steady state:

$$\epsilon^F = \frac{1 - \vartheta_w^F \varrho^F \beta \mathcal{E}^F \mu^F \frac{\bar{H}^F - \lambda^{FP} \bar{H}^P}{\bar{J}^F}}{1 - \vartheta_w^F \varrho^F \beta},$$

and

$$\mu^F = \frac{1}{1 - \vartheta_w^F(\varrho^F + x^F)\beta}.$$

Below, we denote the real wage by $\bar{w}_{j,t}^F = \bar{w}_{j,t}^{Fn}/p_t$ and $\bar{w}_{j,t}^P = \bar{w}_{j,t}^{Pn}/p_t$.

- Log-linearization

  First, log-linearizing the optimality conditions (F.1) and (F.2) deliver

$$\tilde{\varrho}_t^F(w_{j,t}^{Fn}) = \mathcal{E}^F \tilde{\bar{J}}_t^F(w_{j,t}^{Fn}), \tag{F.14}$$

  and

$$\tilde{x}_t^F(w_{j,t}^{Fn}) = \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn}) \right] + \vartheta_w^F \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t} w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn}) \right], \tag{F.15}$$

  respectively.

  Then, combined with the steady-state condition $\kappa^F x^F = \bar{J}^F$ and $\mathcal{A}^F(\varrho^F)/\partial_\rho \mathcal{A}^F(\varrho^F) = \varrho^F/(\zeta^F+1)$, the log-linearization of (F.8), (F.9) and (F.10) are respectively given by

$$\tilde{\bar{J}}_t^F(w_t^{*Fn}) + (1-\chi^F)^{-1} \tilde{\chi}_t^F(w_t^{*Fn}) = \tilde{\bar{H}}_t^F(w_t^{*Fn}), \tag{F.16}$$

  where

$$\begin{aligned} \tilde{\bar{J}}_t^F(w_{j,t}^{Fn}) =& \varkappa_a^F \left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F \right) - \varkappa_w^F \left(\tilde{w}_{j,t}^{Fn} - \tilde{p}_t - \tilde{z}_t \right) + \beta(\varrho^F + x^F) \mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn}) \\ &+ \left[(\varrho^F + x^F) - \left(x^F/2 + \varrho^F/(\zeta+1) \right) \right] \mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} \\ &+ \vartheta_w^F(\varrho^F + x^F)\beta \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t} w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn}) \right], \end{aligned} \tag{F.17}$$

with $\varkappa_a^F = p^F \bar{a}^F / \bar{J}^F$ and $\varkappa_w^F = \bar{w}^F / \bar{J}^F$, and

$$
\begin{aligned}
\tilde{\bar{H}}_t^F(w_{j,t}^{Fn}) =& \frac{\bar{w}^F}{\bar{H}^F}\left(\tilde{w}_{j,t}^{Fn} - \tilde{p}_t - \tilde{z}_t\right) - \frac{\bar{b}}{\bar{H}^F}\tilde{\bar{b}}_t - \beta s^F \mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F\right] \\
& + \beta\left(\varrho^F - s^F + \frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}(1-\varrho^F)\right)\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} \\
& + \beta\varrho^F\mathcal{E}^F\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn}) + \beta\varrho^F\mathbb{E}_t\tilde{\bar{H}}_{t+1}^F(w_{t+1}^{*Fn}) \\
& + \beta\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}(1-\varrho^F)\mathbb{E}_t\tilde{\bar{H}}_{x,t+1}^P \\
& - \beta\varrho^F\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}\mathcal{E}^F\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn}) \\
& + \vartheta_w^F\beta\varrho^F\mathcal{E}^F\mathbb{E}_t\left[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn})\right] \\
& + \vartheta_w^F\beta\varrho^F\mathbb{E}_t\left[\tilde{\bar{H}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{H}}_{t+1}^F(w_{t+1}^{*Fn})\right] \\
& - \vartheta_w^F\beta\varrho^F\mathcal{E}^F\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}\mathbb{E}_t\left[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn})\right].
\end{aligned}
\tag{F.18}
$$

- Find the expressions for $\mathbb{E}_t[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn})]$ and $\mathbb{E}_t[\tilde{\bar{H}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{H}}_{t+1}^F(w_{t+1}^{*Fn})]$

  With (F.17), it must be held that

$$
\begin{aligned}
&\mathbb{E}_t[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn})] \\
&\qquad = -\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{w}_{j,t}^{Fn} - \tilde{w}_{t+1}^{*Fn}\right] \\
&\qquad\quad + \vartheta_w^F(\varrho^F + x^F)\beta\mathbb{E}_t[\tilde{\bar{J}}_{t+2}^F(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+2}^F(\bar{\iota}_{w,t+1}w_{t+1}^{*Fn})] \\
&\qquad = -\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^F - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right] \\
&\qquad\quad - \vartheta_w^F(\varrho^F + x^F)\beta\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^F - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right] \\
&\qquad\quad - (\vartheta_w^F(\varrho^F + x^F)\beta)^2\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^F - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right] + \cdots.
\end{aligned}
$$

  Recall that $\mu^F = (1 - \vartheta_w^F(\varrho^F + x^F)\beta)^{-1}$. Iterating forward the above equation yields

$$
\begin{aligned}
&\mathbb{E}_t[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn}) - \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn})] \\
&\qquad = -(\varkappa_w^F\mu^F)\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^F - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right].
\end{aligned}
\tag{F.19}
$$

Similarly, with (F.18), it must be held that

$$
\begin{aligned}
&\mathbb{E}_t[\tilde{\bar{H}}^F_{t+1}(\hat{\iota}_{w,t}w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+1}(w^{*Fn}_{t+1})] \\
&\quad= \frac{\bar{w}^F}{\bar{H}^F}\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{w}^{Fn}_{j,t} - \tilde{w}^{*Fn}_{t+1}\right] \\
&\qquad + \vartheta^F_w\beta\varrho^F\mathbb{E}_t[\tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t}w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1}w^{*Fn}_{t+1})] \\
&\qquad + \frac{1}{\bar{H}^F}\vartheta^F_w\beta\varrho^F(\bar{H}^F - \lambda^{FP}\bar{H}^P)\mathcal{E}^F\mathbb{E}_t[\tilde{\bar{J}}^F_{t+2}(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t}w^{Fn}_{j,t}) - \tilde{\bar{J}}^F_{t+2}(\bar{\iota}_{w,t+1}w^{*Fn}_{t+1})] \\
&\quad= \frac{\bar{w}^F}{\bar{H}^F}\left(1 - \vartheta^F_w\beta\varrho^F\mu^F\mathcal{E}^F\frac{\bar{H}^F - \lambda^{FP}\bar{H}^P}{\bar{J}^F}\right)\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right] \\
&\qquad + \vartheta^F_w\beta\varrho^F\mathbb{E}_t[\tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t}w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1}w^{*Fn}_{t+1})].
\end{aligned}
$$

Iterating forward the above equation yields

$$
\begin{aligned}
&\mathbb{E}_t[\tilde{\bar{H}}^F_{t+1}(\hat{\iota}_{w,t}w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+1}(w^{*Fn}_{t+1})] \\
&\qquad\qquad = \frac{\bar{w}^F\epsilon^F}{\bar{H}^F}\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right].
\end{aligned}
\tag{F.20}
$$

- Find the expressions for $\tilde{\bar{J}}^F_t(w^F_{j,t})$ and $\tilde{\bar{H}}^F_t(w^F_{j,t})$

  Substituting (F.19) into (F.17) delivers

$$
\begin{aligned}
\tilde{\bar{J}}^F_t(w^F_{j,t}) &= \varkappa^F_a\left(\tilde{p}^F_t + \tilde{\bar{a}}^F_t\right) \\
&\quad - \varkappa^F_w\left(\tilde{\bar{w}}^F_{j,t} + \beta(\varrho^F + x^F)\vartheta^F_w\mu^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right]\right) \\
&\quad + \beta\mathbb{E}_t\left[x^F\tilde{\bar{J}}^F_{t+1}(w^{*F}_{t+1}) + X^F\tilde{\bar{\Lambda}}_{t,t+1}\right] + \beta\varrho^F\mathbb{E}_t\left[\tilde{\bar{J}}^F_{t+1}(w^{*F}_{t+1}) + \tilde{\bar{\Lambda}}_{t,t+1}\right].
\end{aligned}
\tag{F.21}
$$

with $X^F = (x^F + \varrho^F) - \varrho^F - \left(x^F/2 + \varrho^F/(\zeta^F + 1)\right) = x^F/2 - \varrho^F/(\zeta^F + 1)$.

Substituting (F.20) into (F.18) delivers

$$
\begin{aligned}
\tilde{\bar{H}}^F_t(w^F_{j,t}) =& \frac{\bar{w}^F}{\bar{H}^F}\left(\tilde{\bar{w}}^F_{j,t} + \vartheta^F_w\beta\varrho^F\hat{\epsilon}^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right]\right) \\
&- \frac{\bar{b}}{\bar{H}^F}\tilde{\bar{b}}_t - \beta s^F\mathbb{E}_t\left[\tilde{s}^F_t + \tilde{\bar{H}}^F_{x,t+1} + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\
&+ \beta\varrho^F\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} + \beta\varrho^F\mathbb{E}_t\left[\mathcal{E}^F\tilde{\bar{J}}^F_{t+1}(w^{*Fn}_{t+1}) + \tilde{\bar{H}}^F_{t+1}(w^{*F}_{t+1})\right] \\
&+ \beta\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}(1 - \varrho^F)\mathbb{E}_t\left[\tilde{\bar{H}}^P_{x,t+1} + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\
&- \beta\varrho^F\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}\mathcal{E}^F\mathbb{E}_t\tilde{\bar{J}}^F_{t+1}(w^{*F}_{t+1}),
\end{aligned}
\tag{F.22}
$$

where $\hat{\epsilon}^F = \epsilon^F - \mathcal{E}^F\mu^F(\bar{H}^F - \lambda^{FP}\bar{H}^P)/\bar{J}^F$.

- Find the expressions for $\tilde{\bar{w}}_t^{*F}$

  Substitute (F.21) and (F.22) into (F.16) and use the steady state condition $\chi^F \bar{J}^F = (1-\chi^F)\bar{H}^F$ to obtain

$$
\begin{aligned}
&\chi^F \frac{p^F \bar{a}^F}{\bar{w}^F}\left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F\right) - \chi^F\left(\tilde{\bar{w}}_t^{*F} + \beta\vartheta_w^F \mu^F \mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_t^{*F} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right]\right) \\
&+ \chi^F \beta x^F \frac{\bar{J}^F}{\bar{w}^F}\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) + \chi^F \beta X^F \frac{\bar{J}^F}{\bar{w}^F}\mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} + \chi^F \beta \varrho^F \frac{\bar{J}^F}{\bar{w}^F}\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) \\
&= (1-\chi^F)\left(\tilde{w}_t^{*F} + \vartheta_w^F \beta \varrho^F \hat{\epsilon}^F \mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_t^{*F} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right]\right) \\
&- (1-\chi^F)\frac{\bar{b}}{\bar{w}^F}\tilde{\bar{b}}_t - (1-\chi^F)\beta s^F \frac{\bar{H}^F}{\bar{w}^F}\mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\
&+ (1-\chi^F)\beta\varrho^F \frac{\bar{H}^F}{\bar{w}^F}\mathbb{E}_t \tilde{\bar{H}}_{t+1}^F(w_{t+1}^{*F}) + (1-\chi^F)\beta \frac{\lambda^{FP}\bar{H}^P}{\bar{w}^F}(1-\varrho^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\
&+ (1-\chi^F)\beta\varrho^F \frac{\bar{H}^F - \lambda^{FP}\bar{H}^P}{\bar{w}^F}\mathcal{E}^F \mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) - \chi^F(1-\chi^F)^{-1}\frac{\bar{J}^F}{\bar{w}^F}\tilde{\chi}_t^F(w_t^{*F}).
\end{aligned}
\tag{F.23}
$$

  Collect the terms to simplify (F.23) as

$$
\begin{aligned}
&\varphi_a^F\left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F\right) + (\varphi_x^F - \varphi_\rho^F)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) + \varphi_X^F \mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} + \varphi_b^F \tilde{\bar{b}}_t \\
&+ \varphi_s^F \mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_{\hat{\rho}}^F \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\
&+ \varphi_\chi^F\left[\tilde{\chi}_t^F(w_t^{*F}) - \beta\varrho^F \mathbb{E}_t \tilde{\chi}_{t+1}^F(w_{t+1}^{*F})\right] \\
&= \tilde{w}_t^{*F} + \psi^F \mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_t^{*F} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right]
\end{aligned}
\tag{F.24}
$$

  with

$$
\begin{aligned}
&\varphi_a^F = \chi^F \frac{p^F \bar{a}^F}{\bar{w}^F}, \quad \varphi_x^F = \chi^F \beta x^F \frac{\bar{J}^F}{\bar{w}^F}, \quad \varphi_X^F = \chi^F \beta X^F \frac{\bar{J}^F}{\bar{w}^F}, \quad \varphi_b^F = (1-\chi^F)\frac{\bar{b}}{\bar{w}^F}, \\
&\varphi_s^F = (1-\chi^F)\beta s^F \frac{\bar{H}^F}{\bar{w}^F}, \quad \varphi_\chi^F = \chi^F(1-\chi^F)^{-1}\frac{\bar{J}^F}{\bar{w}^F}, \quad \varphi_{\hat{\rho}}^F = (1-\chi^F)\beta\frac{\lambda^{FP}\bar{H}^P}{\bar{w}^F}(1-\varrho^F), \\
&\varphi_\rho^F = (1-\chi^F)\beta\frac{\bar{H}^F - \lambda^{FP}\bar{H}^P}{\bar{w}^F}\varrho^F \mathcal{E}^F, \quad \text{and} \quad \psi^F = (1-\chi^F)\vartheta_w^F \varrho^F \beta \hat{\epsilon}^F + \chi^F \beta \vartheta_w^F \mu^F.
\end{aligned}
$$

  Define $\tilde{\bar{w}}_t^o(w_t^{*F})$ as

$$
\begin{aligned}
\tilde{\bar{w}}_t^o(w_t^{*F}) = &\varphi_a^F\left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F\right) + (\varphi_x^F - \varphi_\rho^F)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) + \varphi_X^F \mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} + \varphi_b^F \tilde{\bar{b}}_t \\
&+ \varphi_s^F \mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_{\hat{\rho}}^F \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\
&+ \varphi_\chi^F\left[\tilde{\chi}_t^F(w_t^{*F}) - \beta\varrho^F \mathbb{E}_t \tilde{\chi}_{t+1}^F(w_{t+1}^{*F})\right].
\end{aligned}
\tag{F.25}
$$

Then, we simply (F.24) as

$$\tilde{\bar{w}}_t^{*F} = (1-\tau^F)\tilde{\bar{w}}_t^o(w_t^{*F}) - \tau^F \mathbb{E}_t \left[\iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right], \tag{F.26}$$

where

$$\tau^F = \psi^F/(1+\psi^F)$$

We note that if $\vartheta_w^F = 0$, then $\tilde{w}_t^{*F} = \tilde{\bar{w}}_t^o(w_t^{*F})$. Below, we will delve into the expression for $\tilde{w}_t^o(w_t^{*F})$ to express this in terms of difference between the newly-contracted wage w_t^{*F} and the aggregate wage w_t^F.

- Find the expression for $\tilde{\chi}_t^F(w_t^{*F})$

  We note that the expressions (F.19) and (F.20) hold in more general:

$$\tilde{\bar{J}}_t^F(w_t^F) - \tilde{\bar{J}}_t^F(w_t^{*F}) = -(\varkappa_w^F \mu^F)\left(\tilde{\bar{w}}_t^F - \tilde{\bar{w}}_t^{*F}\right), \tag{F.27}$$

and[44]

$$\tilde{\bar{H}}_t^F(w_t^F) - \tilde{\bar{H}}_t^F(w_t^{*F}) = (1-\eta^F)(\eta^F)^{-1}(\varkappa_w^F \mu^F)\left(\tilde{\bar{w}}_t^F - \tilde{\bar{w}}_t^{*F}\right). \tag{F.28}$$

Furthermore, (F.14), (F.15), (F.21), and (F.22) imply that[45]

$$\tilde{x}_t^F = \tilde{x}_t^F(w_t^F), \tag{F.29}$$

$$\tilde{\varrho}_t^F = \tilde{\varrho}_t^F(w_t^F) = \mathcal{E}^F \tilde{\bar{J}}_t^F(w_t^F),$$

and

$$\tilde{\bar{H}}_{x,t}^F = \tilde{\bar{H}}_t^F(w_t^F). \tag{F.30}$$

Thus, we have

$$\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) = \mathbb{E}_t \tilde{\bar{J}}_{t+1}^F - (\varkappa_w^F \mu^F)\mathbb{E}_t\left[\tilde{\bar{w}}_{t+1}^{*F} - \tilde{\bar{w}}_{t+1}^F\right]. \tag{F.31}$$

Now, we are ready to find the expression for $\tilde{\chi}_t^F(w_t^{*F})$. Log-linearizing (F.11), we express $\tilde{\chi}_t^F(w_t^{*F})$ in terms of $\tilde{\epsilon}_t^F(w_t^{*F})$ and $\tilde{\mu}_t^F(w_t^{*F})$ as follows:

$$\tilde{\chi}_t^F(w_t^{*F}) = (1-\chi^F)\left(\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\mu}_t^F(w_t^{*F})\right) + (1-\chi^F)(1-\eta^F)^{-1}\tilde{\varepsilon}_t^{\eta,F}.$$

[44]Use the steady-state condition $(\bar{w}^F/\bar{H}^F)\epsilon^F = (1-\chi^F)(\chi^F)^{-1}\epsilon^F\varkappa_w^F = (1-\eta^F)(\eta^F)^{-1}\varkappa_w^F\mu^F$.

[45]See (F.3), (F.4). and (F.5) for the definition of w_t^F, x_t^F, and ϱ_t^F.

Rearranging the terms in the above equation delivers

$$\begin{aligned}\tilde{\chi}_t^F(w_t^{*F}) =&(1-\chi^F)\left(\tilde{\epsilon}_t^F(w_t^F)-\tilde{\mu}_t^F(w_t^F)\right)+(1-\chi^F)(1-\eta^F)^{-1}\tilde{\varepsilon}_t^{\eta,F}\\ &+(1-\chi^F)\left[(\tilde{\epsilon}_t^F(w_t^{*F})-\tilde{\epsilon}_t^F(w_t^F))-(\tilde{\mu}_t^F(w_t^{*F})-\tilde{\mu}_t^F(w_t^F))\right].\end{aligned} \tag{F.32}$$

Let us define $\tilde{\epsilon}_t^F \equiv \tilde{\epsilon}_t^F(w_t^F)$, $\tilde{\epsilon}_{t+1}^F \equiv \tilde{\epsilon}_{t+1}^F(w_{t+1}^F)$, $\tilde{\mu}_t^F \equiv \tilde{\mu}_t^F(w_t^F)$, and $\tilde{\mu}_{t+1}^F \equiv \tilde{\mu}_{t+1}^F(w_{t+1}^F)$. We need to find the expressions for $\tilde{\epsilon}_t^F(w_t^{*F})-\tilde{\epsilon}_t^F$ and $\tilde{\mu}_t^F(w_t^{*F})-\tilde{\mu}_t^F$ and the dynamic equations for $\tilde{\epsilon}_t^F$ and $\tilde{\mu}_t^F$. For convince, define $\tilde{\Upsilon}_{t,t+1} \equiv \tilde{\bar{\Lambda}}_{t,t+1}-\tilde{\pi}_{t+1}+\iota_w\tilde{\pi}_t-\tilde{\varepsilon}_{t+1}^z$. Log-linearizing (F.12) yields

$$\begin{aligned}&\tilde{\epsilon}_t^F(w_t^{*F})\\ &=\vartheta_w^F\varrho^F\beta\mathbb{E}_t\left[\begin{array}{l}(1-e_o^F)\left(\tilde{\Upsilon}_{t,t+1}+\tilde{\epsilon}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})+\mathcal{E}^F\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})\right)\\ -e_o^F\left(\begin{array}{l}\tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})-\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})\\ +\frac{\bar{H}^F}{\bar{H}^F-\lambda^{FP}\bar{H}^P}\tilde{\bar{H}}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})-\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F-\lambda^{FP}\bar{H}^P}\tilde{\bar{H}}_{x,t+1}^P\end{array}\right)\end{array}\right]\end{aligned}$$

with $e_o^F \equiv \frac{1}{\epsilon^F}\mathcal{E}^F\mu^F\frac{\bar{H}^F-\lambda^{FP}\bar{H}^P}{\bar{J}^F}$. Log-linearizing (F.13) yields

$$\tilde{\mu}_t^F(w_t^{*F})=\vartheta_w^F(\varrho^F+x^F)\beta\mathbb{E}_t\left[\tilde{\Upsilon}_{t,t+1}+\tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})+m_o^F\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})\right] \tag{F.33}$$

with $m_o^F \equiv x^F+\varrho^F\mathcal{E}^F$. Then, substitute

$$\mathbb{E}_t\left[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})-\tilde{\bar{J}}_{t+1}^F(w_{t+1}^F)\right]=-(\varkappa_w^F\mu^F)\mathbb{E}_t\left[\iota_w\tilde{\pi}_t+\tilde{\bar{w}}_t^{*F}-\tilde{\varepsilon}_{t+1}^z-\tilde{\pi}_{t+1}-\tilde{\bar{w}}_{t+1}^F\right]$$

into (F.33) to obtain

$$\begin{aligned}\tilde{\mu}_t^F(w_t^{*F})-\tilde{\mu}_t^F(w_t^F)=&-\vartheta_w^F(\varrho^F+x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F}-\tilde{\bar{w}}_t^F\right)\\ &+\vartheta_w^F(\varrho^F+x^F)\beta\mathbb{E}_t\left[\tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t}w_t^{*F})-\tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t}w_t^F)\right]\\ =&-\vartheta_w^F(\varrho^F+x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F}-\tilde{\bar{w}}_t^F\right)\\ &+\vartheta_w^F(\varrho^F+x^F)\beta\left(-\vartheta_w^F(\varrho^F+x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F}-\tilde{\bar{w}}_t^F\right)\right)\\ &+(\vartheta_w^F(\varrho^F+x^F)\beta)^2\left(-\vartheta_w^F(\varrho^F+x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F}-\tilde{\bar{w}}_t^F\right)\right)\\ &+\cdots.\end{aligned}$$

Recall that $\vartheta_w^F(\varrho^F+x^F)\beta \in (0,1)$ and $\mu^F=(1-\vartheta_w^F(\varrho^F+x^F)\beta)^{-1}$. Iterating forward the above equation delivers

$$\tilde{\mu}_t^F(w_t^{*F})-\tilde{\mu}_t^F(w_t^F)=-(\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F\left(\tilde{\bar{w}}_t^{*F}-\tilde{\bar{w}}_t^F\right). \tag{F.34}$$

As for $\tilde{\epsilon}_t^F(w_t^{*F})$, we have

$$\begin{aligned}
&\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F(w_t^F) \\
&\quad= \vartheta_w^F \varrho^F \beta \left((1-e_o^F)\mathcal{E}^F + e_o^F\right) \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t} w_t^F)\right] \\
&\qquad - \vartheta_w^F \varrho^F \beta e_o^F \mathbb{E}_t \left[\tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t} w_t^F)\right] \\
&\qquad - \vartheta_w^F \varrho^F \beta e_o^F \frac{\bar{H}^F}{\bar{H}^F - \lambda^{FP}\bar{H}^P} \mathbb{E}_t \left[\tilde{\bar{H}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\bar{H}}_{t+1}^F(\hat{\iota}_{w,t} w_t^F)\right] \\
&\qquad + \vartheta_w^F \varrho^F \beta (1-e_o^F) \mathbb{E}_t \left[\tilde{\epsilon}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\epsilon}_{t+1}^F(\hat{\iota}_{w,t} w_t^F)\right].
\end{aligned}$$

Use (F.20) and (F.34) for $\mathbb{E}_t[\tilde{\bar{H}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\bar{H}}_{t+1}^F(\hat{\iota}_{w,t} w_t^F)]$ and $\mathbb{E}_t[\tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t} w_t^F)]$ respectively to obtain

$$\begin{aligned}
\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F(w_t^F) =& \mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \vartheta_w^F \varrho^F \beta (1-e_o^F) \mathbb{E}_t \left[\tilde{\epsilon}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\epsilon}_{t+1}^F(\hat{\iota}_{w,t} w_t^F)\right] \\
=& \mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \vartheta_w^F \varrho^F \beta (1-e_o^F) \mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ (\vartheta_w^F \varrho^F \beta (1-e_o^F))^2 \mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \cdots
\end{aligned}$$

with $\mho = -\vartheta_w^F \varrho^F \beta (\varkappa_w^F \mu^F)\left[(2-e_o^F)\mathcal{E}^F + e_o^F - e_o^F(\vartheta_w^F \beta m_o^F)\mu^F\right]$. See Appendix F.2.2 for the derivation.

Lemma 2 *If it is satisfied that* $|\vartheta_w^F \varrho^F \beta (1-e_o^F)| < 1$,

$$1 + \vartheta_w^F \varrho^F \beta (1-e_o^F) + (\vartheta_w^F \varrho^F \beta (1-e_o^F))^2 + \cdots = \frac{1}{1 - \vartheta_w^F \varrho^F \beta (1-e_o^F)} = \epsilon^F.$$

Under the condition that $|\vartheta_w^F \varrho^F \beta (1-e_o^F)| < 1$, applying Lemma 2 to above equation brings

$$\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F(w_t^F) = \epsilon^F \mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right). \tag{F.35}$$

Now, substituting (F.34) and (F.35) into (F.32) delivers

$$\begin{aligned}
\tilde{\chi}_t^F(w_t^{*F}) =& \tilde{\chi}_t^F + (1-\chi^F)[\epsilon^F \mho + (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F]\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ (1-\chi^F)(1-\eta^F)^{-1} \tilde{\varepsilon}_t^{\eta,F},
\end{aligned} \tag{F.36}$$

where

$$\tilde{\chi}_t^F \equiv (1-\chi^F)(\tilde{\epsilon}_t^F - \tilde{\mu}_t^F). \tag{F.37}$$

Similarly,

$$\mathbb{E}_t\tilde{\chi}^F_{t+1}(w^{*F}_{t+1}) = \mathbb{E}_t\tilde{\chi}^F_{t+1} + (1-\chi^F)[\epsilon^F\mho + (\vartheta^F_w\beta m^F_o)(\varkappa^F_w\mu^F)\mu^F]\mathbb{E}_t\left[\tilde{\bar{w}}^{*F}_{t+1} - \tilde{\bar{w}}^F_{t+1}\right] \\ + (1-\chi^F)(1-\eta^F)^{-1}\rho^F_\eta\tilde{\varepsilon}^{\eta,F}_t.$$

- Find dynamic equations for for $\tilde{\epsilon}^F_t$ and $\tilde{\mu}^F_t$

  As shown in Appendix F.2.2, the dynamic equations for $\tilde{\epsilon}^F_t$ and $\tilde{\mu}^F_t$ are, respectively, given by

$$\begin{aligned}\tilde{\epsilon}^F_t =& \vartheta^F_w\varrho^F\beta(1-e^F_0)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t + \tilde{\epsilon}^F_{t+1} - \tilde{\varepsilon}^z_{t+1}\right] - \vartheta^F_w\varrho^F\beta e^F_0\mathbb{E}_t\tilde{\mu}^F_{t+1} \\ & - \vartheta^F_w\varrho^F\beta e^F_o(\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\mathbb{E}_t\left[\bar{H}^F\tilde{\bar{H}}^F_{x,t} - \lambda^{FP}\bar{H}^P\tilde{\bar{H}}^P_t\right] \\ & + \vartheta^F_w\varrho^F\beta\left((1-e^F_o)\mathcal{E}^F + e^F_o\right)\mathbb{E}_t\tilde{\bar{J}}^F_{t+1}(w^F_{t+1}) \\ & + \epsilon^F\mho\mathbb{E}_t\left[\tilde{\bar{w}}^F_t - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}^F_{t+1} - \tilde{\varepsilon}^z_{t+1}\right].\end{aligned} \tag{F.38}$$

  and

$$\begin{aligned}\tilde{\mu}^F_t =& (\vartheta^F_w\beta m^F_o)\mathbb{E}_t\tilde{x}^F_{t+1} + \vartheta^F_w\beta\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t + \tilde{\mu}^F_{t+1} - \tilde{\varepsilon}^z_{t+1}\right] \\ & - (\vartheta^F_w\beta m^F_o)(\varkappa^F_w\mu^F)\mu^F\mathbb{E}_t\left[\tilde{\bar{w}}^F_t - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}^F_{t+1} - \tilde{\varepsilon}^z_{t+1}\right].\end{aligned} \tag{F.39}$$

- Find the expression for $\tilde{\bar{H}}^F_{x,t}$ Combining (F.28) and (F.30), we find that $\tilde{\bar{H}}^F_{x,t}$ satisfies

$$\tilde{\bar{H}}^F_{x,t} = \tilde{\bar{H}}^F_t(w^{*F}_t) + (1-\eta^F)(\eta^F)^{-1}(\varkappa^F_w\mu^F)\left(\tilde{\bar{w}}^F_t - \tilde{\bar{w}}^{*F}_t\right).$$

  Substituted into (F.16), $\tilde{\bar{H}}^F_{x,t}$ is given by

$$\tilde{\bar{H}}^F_{x,t} = \tilde{\bar{J}}^F_t(w^{*F}_t) + (1-\chi^F)^{-1}\tilde{\chi}^F_t(w^{*F}_t) + (1-\eta^F)(\eta^F)^{-1}(\varkappa^F_w\mu^F)\left(\tilde{\bar{w}}^F_t - \tilde{\bar{w}}^{*F}_t\right).$$

  Below, we abbreviate $\tilde{\bar{J}}^F_t(w^F_t)$ to $\tilde{\bar{J}}^F_t$. Using (F.6), (F.27), and (F.36), we obtain the expression for $\mathbb{E}_t\tilde{\bar{H}}^F_{x,t+1}$:

$$\begin{aligned}\mathbb{E}_t\tilde{\bar{H}}^F_{x,t+1} =& \mathbb{E}_t\tilde{\bar{J}}^F_{t+1} + (1-\chi^F)^{-1}\mathbb{E}_t\tilde{\chi}^F_{t+1} + (1-\eta^F)^{-1}\mathbb{E}_t\tilde{\varepsilon}^{\eta,F}_{t+1} \\ & - \vartheta^F_w(1-\vartheta^F_w)^{-1}\Gamma^F\mathbb{E}_t\left[\tilde{\bar{w}}^F_{t+1} - (\tilde{\bar{w}}^F_t - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\varepsilon}^z_{t+1})\right],\end{aligned} \tag{F.40}$$

  with $\Gamma^F = -\epsilon^F\mho + [1-\eta^F(\vartheta^F_w\beta m^F_o)\mu^F](\eta^F)^{-1}(\varkappa^F_w\mu^F)$.

- Find the expression for $\tilde{\bar{w}}^o_t(w^{*F}_t)$

  Hence, substitute (F.31), (F.36), and (F.40) into (F.25) and rearrange terms to derive

$$\tilde{\tilde{w}}_t^o(w_t^{*F}) = \tilde{\tilde{w}}_t^{o,F} + \frac{\tau_1^F}{1-\tau^F}\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^F - \tilde{\tilde{w}}_{t+1}^{*F}\right] + \frac{\tau_2^F}{1-\tau^F}(\tilde{\tilde{w}}_t^F - \tilde{\tilde{w}}_t^{*F}), \quad \text{(F.41)}$$

where

$$\begin{aligned}\tilde{\tilde{w}}_t^{o,F} =& \varphi_a^F\left(\tilde{p}_t^F + \tilde{\tilde{a}}_t^F\right) + (\varphi_x^F + \varphi_s^F - \varphi_\rho^F)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F + (\varphi_s^F + \varphi_X^F - \varphi_{\hat{\rho}}^F)\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1} \\ &+ \varphi_b^F\tilde{\tilde{b}}_t + \varphi_s^F\tilde{s}_t^F - \varphi_{\hat{\rho}}^F\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^P + \varphi_\chi^F\left[\tilde{\chi}_t^F - \beta(\varrho^F - s^F)\mathbb{E}_t\tilde{\chi}_{t+1}^F\right] + \varphi_\eta^F\tilde{\varepsilon}_t^{\eta,F},\end{aligned}$$

with

$$\tau_1^F = (1-\tau^F)\left[(\varphi_x^F - \varphi_\rho^F)\varkappa_w^F\mu^F + \varphi_\chi^F(\varrho^F\beta)(1-\chi^F)[\epsilon^F\mho + (\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F] + \varphi_s\Gamma^F\right],$$

$$\tau_2^F = -(1-\tau^F)\varphi_\chi^F(1-\chi^F)[\epsilon^F\mho + (\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F],$$

and[46]

$$\varphi_\eta^F = \frac{\varphi_\chi^F(1-\chi^F)(1-\beta\varrho^F\rho_\eta^F) + \varphi_s^F\rho_\eta^F}{1-\eta^F} = \frac{(1-\chi^F)\varphi_\chi^F[1-\beta(\varrho^F - s^F)\rho_\eta^F]}{1-\eta^F}.$$

Below, we write

$$\tilde{\varepsilon}_t^{wF} = \varphi_\eta^F\tilde{\varepsilon}_t^{\eta,F}.$$

Note that the dynamic equation for $\tilde{\chi}_t^F$ satisfies (F.51) and the dynamic equation for $\tilde{x}_t^F$ satisfies[47]

$$\tilde{\tilde{J}}_t^F = \varkappa_a^F(\tilde{p}_t^F + \tilde{\tilde{a}}_t^F) - \varkappa_w^F\tilde{\tilde{w}}_t^F + \beta(\varrho^F + X^F)\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1} + \beta(1+\varrho^F\mathcal{E}^F)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F.$$

- Derive the dynamic equation for w_t^F Substitute (F.41) into (F.26) to obtain

$$\tilde{\tilde{w}}_t^{*F} = (1-\tau^F)\tilde{\tilde{w}}_t^o + \tau_1^F\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^F - \tilde{\tilde{w}}_{t+1}^{*F}\right] + \tau_2^F(\tilde{\tilde{w}}_t^F - \tilde{\tilde{w}}_t^{*F}) - \tau^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\tilde{w}}_{t+1}^{*F}\right].$$

Use (F.6) to express $\tilde{\tilde{w}}_t^{*F}$ in terms of aggregate variables. As a result, we have

$$\begin{aligned}&\tilde{\tilde{w}}_t^F - \vartheta_w^F\left(\iota_w\tilde{\pi}_{t-1} + \tilde{\tilde{w}}_{t-1}^F - \tilde{\varepsilon}_t^z - \tilde{\pi}_t\right) \\ &= (1-\vartheta_w^F)(1-\tau^F)\tilde{\tilde{w}}_t^o + \vartheta_w^F\tau_1^F\left(\iota_w\tilde{\pi}_t + \tilde{\tilde{w}}_t^F - \mathbb{E}_t\tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\tilde{w}}_{t+1}^F\right) \\ &+ \vartheta_w^F\tau_2^F\left(\iota_w\tilde{\pi}_{t-1} + \tilde{\tilde{w}}_{t-1}^F - \tilde{\varepsilon}_t^z - \tilde{\pi}_t - \tilde{\tilde{w}}_t^F\right) \\ &- \tau^F\left(\iota_w\tilde{\pi}_t - \mathbb{E}_t\tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\tilde{w}}_{t+1}^F + \vartheta_w^F\tilde{\tilde{w}}_t^F\right).\end{aligned}$$

[46] We use $\varphi_s^F = (1-\chi^F)\varphi_\chi^F\beta s^F$ to simplify the coefficient of $\tilde{\varepsilon}_t^{\eta,F}$.

[47] The dynamic equation for $\tilde{\tilde{J}}_t^F$ is derived from (F.21), (F.29), and (F.6).

Collecting the terms gives

$$\tilde{\tilde{w}}_t^F = \omega_b^F \left(\tilde{\tilde{w}}_{t-1}^F - \tilde{\pi}_t + \iota_w \tilde{\pi}_{t-1} - \tilde{\varepsilon}_t^z\right) + \omega_o^F \tilde{\tilde{w}}_t^{o,F} + \omega_f^F \mathbb{E}_t \left[\tilde{\tilde{w}}_{t+1}^F + \tilde{\pi}_{t+1} - \iota_w \tilde{\pi}_t + \tilde{\varepsilon}_{t+1}^z\right],$$

where $\omega_b^F = (1+\tau_2^F)/\Phi^F$, $\omega_o^F = \varsigma^F/\Phi^F$, $\omega_f^F = (\tau^F/\vartheta_w^F - \tau_1^F)/\Phi^F$, $\Phi^F = (1+\tau_2^F) + \varsigma^F + (\tau^F/\vartheta_w^F - \tau_1^F)$, and $\varsigma^F = (1-\vartheta_w^F)(1-\tau^F)/\vartheta_w^F$.

F.2.2 Derivations

- Derivation of $\mho$.

  By construction of $\mho$:

$$\mho = \vartheta_w^F \varrho^F \beta \left[\begin{array}{l} -\left((1-e_o^F)\mathcal{E}^F + e_o^F\right)(\varkappa_w^F \mu^F) \\ -e_o^F(-(\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F) - e_o^F \frac{\bar{H}^F}{\bar{H}^F - \bar{H}^P} \frac{\bar{w}^F \epsilon^F}{\bar{H}^F} \end{array} \right]$$

  Recall that $e_o^F = (1/\epsilon^F)\mathcal{E}^F \mu^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)/\bar{J}^F$ and then

$$e_o^F \frac{\bar{H}^F}{\bar{H}^F - \lambda^{FP}\bar{H}^P} \frac{\bar{w}^F \epsilon^F}{\bar{H}^F} = \mathcal{E}^F \mu^F \frac{\bar{w}^F}{\bar{J}^F} = \mathcal{E}^F \varkappa_w^F \mu^F.$$

  Hence, we have

$$\mho = -\vartheta_w^F \varrho^F \beta (\varkappa_w^F \mu^F) \left[(2-e_o^F)\mathcal{E}^F + e_o^F - e_o^F(\vartheta_w^F \beta m_o^F)\mu^F\right].$$

- Derivation of $\tilde{\epsilon}_t^F$

$$\begin{aligned}
\tilde{\epsilon}_t^F =& \vartheta_w^F \varrho^F \beta(1-e_0^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \vartheta_w^F \varrho^F \beta(1-e_0^F)\epsilon^F \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- \vartheta_w^F \varrho^F \beta e_0^F \mathbb{E}_t \tilde{\mu}_{t+1}^F \\
&+ \vartheta_w^F \varrho^F \beta e_0^F (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\mathbb{E}_t\left[\bar{H}^F \tilde{\bar{H}}_{x,t}^F - \lambda^{FP}\bar{H}^P \tilde{\bar{H}}_{x,t}^P\right] \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\bar{w}^F \epsilon^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \vartheta_w^F \varrho^F \beta\left((1-e_o^F)\mathcal{E}^F + e_o^F\right)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F \\
&- \vartheta_w^F \varrho^F \beta\left((1-e_o^F)\mathcal{E}^F + e_o^F\right)(\varkappa_w^F \mu^F)\mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
=& \vartheta_w^F \varrho^F \beta(1-e_0^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] - \vartheta_w^F \varrho^F \beta e_0^F \mathbb{E}_t \tilde{\mu}_{t+1}^F \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\mathbb{E}_t\left[\bar{H}^F \tilde{\bar{H}}_{x,t}^F - \lambda^{FP}\bar{H}^P \tilde{\bar{H}}_t^P\right] \\
&+ \vartheta_w^F \varrho^F \beta\left((1-e_o^F)\mathcal{E}^F + e_o^F\right)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F \\
&+ \vartheta_w^F \varrho^F \beta(1-e_0^F)\epsilon^F \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right].
\end{aligned}$$

Recall that $1 + \vartheta_w^F \varrho^F \beta(1-e_0^F)\epsilon^F = \epsilon^F$ to simplify above equation into (F.38).

- Derivation of $\tilde{\mu}_t^F$

$$\begin{aligned}
\tilde{\mu}_t^F =& (\vartheta_w^F \beta m_o^F)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F \\
&- (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \vartheta_w^F \beta \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\mu}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- \vartheta_w^F \beta(\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right].
\end{aligned}$$

Recall that $1 + \vartheta_w^F \beta \mu^F = \mu^F$ to simplify above equation into (F.39).

F.2.3 Wages for part-time workers

We next find the expression for $\tilde{\bar{w}}_t^{*P}$. Since most process to derive it is analogous to the process to derive the expression for $\tilde{\bar{w}}_t^{*F}$, we just describe the setup and show the final outcomes.

- The renegotiated nominal wage for part-time workers w_t^{*Pn} satisfies the following wage sharing rule:

$$\chi_t^P(w_t^{*Pn})\bar{J}_t^P(w_t^{*Pn}) = [1-\chi_t^P(w_t^{*Pn})]\bar{H}_t^P(w_t^{*Pn}), \tag{F.42}$$

where

$$\begin{aligned}\bar{J}_t^P(w_{j,t}^{Pn}) =& p_t^P\bar{a}_t^P - \frac{w_{j,t}^{Pn}}{p_t z_t} - \mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\frac{\kappa^P}{2}(x_t^P(w_{j,t}^{Pn}))^2\right] \\ &+ \mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P + x_t^P(w_{j,t}^{Pn})\right)\bar{J}_{t+1}^P(w_{t+1}^{*Pn})\right] \\ &+ \vartheta_w^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P + x_t^P(w_{j,t}^{Pn})\right)\left(\bar{J}_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) - \bar{J}_{t+1}^P(w_{t+1}^{*Pn})\right)\right],\end{aligned}$$

and

$$\begin{aligned}\bar{H}_t^P(w_{j,t}^{Pn}) =& \frac{w_{j,t}^{Pn}\mu_b^P}{p_t z_t} - \hat{\mu}_b^P\bar{b}_t + \mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P\bar{H}_{t+1}^P(w_{t+1}^{*Pn}) + \varphi\bar{H}_{x,t+1}^F\right) - s_{t+1}^P\bar{H}_{x,t+1}^P\right] \\ &+ \vartheta_w^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\varrho^P\left(\bar{H}_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) - \bar{H}_{t+1}^P(w_{t+1}^{*Pn})\right)\right].\end{aligned}$$

- The effective bargaining power of the workers satisfies

$$\chi_t^P(w_t^{*Pn}) = \frac{\eta_t^P}{\eta_t^P + (1-\eta_t^P)\mu_t^P(w_t^{*Pn})/\varepsilon_t^F(w_t^{*Pn})}$$

where

$$\epsilon_t^P(w_{j,t}^{Pn}) \equiv p_t\partial H_t^P(w_{j,t}^{Pn})/\partial w_{j,t}^{Pn},$$

$$\epsilon_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) \equiv (z_t/z_{t+1})p_{t+1}\partial H_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})/\partial(\hat{\iota}_{w,t}w_{j,t}^{Pn}),$$

$$\mu_t^P(w_{j,t}^{Pn}) \equiv -p_t\partial J_t^P(w_{j,t}^{Pn})/\partial w_{j,t}^{Pn},$$

and

$$\mu_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) \equiv -(z_t/z_{t+1})p_{t+1}\partial J_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})/\partial(\hat{\iota}_{w,t}w_{j,t}^{Pn}).$$

Then, we have

$$\epsilon_t^P(w_{j,t}^{Pn}) = 1 + \vartheta_w^P\varrho^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\frac{p_t}{p_{t+1}}\hat{\iota}_{w,t}\epsilon_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})\right]$$

and

$$\mu_t^P(w_{j,t}^{Pn}) = 1 + \vartheta_w^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P + x_t^P(w_{j,t}^{Pn})\right)\frac{p_t}{p_{t+1}}\hat{\iota}_{w,t}\mu_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})\right].$$

In the balanced-growth steady state, ($\vartheta_w^P \varrho^P \beta \in (0,1)$ and $\vartheta_w^P(\varrho^P + x^P)\beta \in (0,1)$)

$$\epsilon^P = \frac{1}{1 - \vartheta_w^P \varrho^P \beta},$$

and

$$\mu^P = \frac{1}{1 - \vartheta_w^P(\varrho^P + x^P)\beta}.$$

- Log-linearization Log-linearizing the firm's surplus and the worker's surplus delivers

$$\begin{aligned}\tilde{\bar{J}}_t^P(w_{j,t}^{Pn}) =& \varkappa_a^P(\tilde{p}_t^P + \tilde{\bar{a}}_t^P) \\ &- \varkappa_w^P \left(\tilde{\bar{w}}_{j,t}^P + \beta\vartheta_w^P \mu^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^P - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right]\right) \\ &+ \beta x^P \mathbb{E}_t \left[\tilde{x}_t^P(w_{j,t}^{Pn}) + (1/2)\tilde{\bar{\Lambda}}_{t,t+1}\right] \\ &+ \beta \varrho^P \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^P(w_{j,t+1}^{Pn}) + \tilde{\bar{\Lambda}}_{t,t+1}\right].\end{aligned} \tag{F.43}$$

with $\varkappa_a^P = p^P \bar{a}^P / \bar{J}^P$ and $\varkappa_w^P = \bar{w}^P \mu_b^P / \bar{J}^P$ and

$$\begin{aligned}\tilde{\bar{H}}_t^P(w_{j,t}^{Pn}) =& \frac{\bar{w}^P \mu_b^P}{\bar{H}^P} \left(\tilde{\bar{w}}_{j,t}^P + \beta\vartheta_w^P \varrho^P \epsilon^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^P - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right]\right) \\ &- \frac{\hat{\mu}_b^P \bar{b}}{\bar{H}^P} \tilde{\bar{b}}_t + \beta\varrho^P \mathbb{E}_t \left[\tilde{\bar{H}}_{t+1}^P + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\ &- \beta s^P \mathbb{E}_t \left[\tilde{s}_t^P + \tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] + \frac{\beta\varphi \bar{H}^F}{\bar{H}^P} \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^F\right].\end{aligned} \tag{F.44}$$

Log-linearize (F.42) to obtain

$$\tilde{\bar{J}}_t^P(w_t^{*Pn}) + (1-\chi^F)^{-1}\tilde{\chi}_t^P(w_t^{*Pn}) = \tilde{\bar{H}}_t^P(w_t^{*Pn}), \tag{F.45}$$

Substitute (F.43) and (F.44) into (F.45) to obtain

$$\begin{aligned}&\varphi_a^P \left(\tilde{p}_t^P + \tilde{\bar{a}}_t^P\right) + (\varphi_x^P - \varphi_\rho^P)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^P(w_{t+1}^{*P}) + (\varphi_x^P/2)\mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} + \varphi_b^P \tilde{\bar{b}}_t \\ &+ \varphi_s^P \mathbb{E}_t \left[\tilde{s}_{t+1}^P + \tilde{\bar{H}}_{x,t+1}^P + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_\varphi \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^F\right] \\ &\varphi_\chi^P \left[\tilde{\chi}_t^P(w_t^{*P}) - \beta\varrho^P \mathbb{E}_t \tilde{\chi}_{t+1}^P(w_{t+1}^{*P})\right] \\ &= \tilde{w}_t^{*P} + \psi^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_t^{*P} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right]\end{aligned} \tag{F.46}$$

with

$$
\begin{aligned}
\varphi_a^P =&\chi^P \frac{p^P \bar{a}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_x^P = \chi^P \frac{x^P \beta \bar{J}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_b^P = (1-\chi^P)\frac{\hat{\mu}_b^P \bar{b}}{\bar{w}^P \mu_b^P}, \\
\varphi_s^P =&(1-\chi^P)\frac{\beta s^P \bar{H}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_\chi^P = \frac{\bar{H}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_\varphi = (1-\chi^P)\frac{\beta \varphi \bar{H}^F}{\bar{w}^P \mu_b^P}, \\
&\bar{J}^P = \kappa^P x^P, \quad \text{and} \quad \psi^P = (1-\chi^P)\vartheta_w^P \varrho^P \beta \epsilon^P + \chi^P \beta \vartheta_w^P \mu^P.
\end{aligned}
$$

With $\tau^P = \psi^P/(1+\psi^P)$, we can simplify (F.46) into

$$
\tilde{\bar{w}}_t^{*P} = (1-\tau^P)\tilde{\bar{w}}_t^o(w_t^{*P}) - \tau^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right], \tag{F.47}
$$

where

$$
\begin{aligned}
\tilde{\bar{w}}_t^o(w_t^{*P}) =&\varphi_a^P \left(\tilde{p}_t^P + \tilde{\bar{a}}_t^P\right) + \varphi_x^P \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^P(w_{t+1}^{*P}) + (1/2)\tilde{\bar{\Lambda}}_{t,t+1}\right] + \varphi_b^P \tilde{\bar{b}}_t \\
&+ \varphi_s^P \mathbb{E}_t \left[\tilde{s}_t^P + \tilde{\bar{H}}_{x,t+1}^P + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_{\hat{\rho}}^P \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\
&- \varphi_\rho^P \mathbb{E}_t \tilde{x}_{t+1}^P(w_{t+1}^{*P}) + \varphi_\chi^P \left[\tilde{\chi}_t^P(w_t^{*P}) - \beta \varrho^P \mathbb{E}_t \tilde{\chi}_{t+1}^P(w_{t+1}^{*P})\right].
\end{aligned}
$$

Taking the similar steps in the previous section, we obtain

$$
\mathbb{E}_t \tilde{\bar{J}}_{t+1}^P(w_{t+1}^{*F}) = \mathbb{E}_t \tilde{\bar{J}}_{t+1}^P - (\varkappa_w^P \mu^P)\mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^{*P} - \tilde{\bar{w}}_{t+1}^P\right], \tag{F.48}
$$

and

$$
\begin{aligned}
\mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^P =&\mathbb{E}_t \tilde{\bar{J}}_{t+1}^P + (1-\chi^P)^{-1}\mathbb{E}_t \tilde{\chi}_{t+1}^P + (1-\eta^P)^{-1}\rho_\eta^P \tilde{\varepsilon}_t^{\eta,P} \\
&- \vartheta_w^P (1-\vartheta_w^P)^{-1}\Gamma^P \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^P - (\tilde{\bar{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z)\right]
\end{aligned} \tag{F.49}
$$

with $\Gamma^P = [1-\eta^P(\vartheta_w^P \beta x^P)\mu^P](\eta^P)^{-1}(\varkappa_w^P \mu^P)$, and

$$
\begin{aligned}
\tilde{\chi}_t^P(w_t^{*P}) =&\tilde{\chi}_t^P + (1-\chi^P)(\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P \left(\tilde{\bar{w}}_t^{*P} - \tilde{\bar{w}}_t^P\right) \\
&+ (1-\chi^P)(1-\eta^P)^{-1}\tilde{\varepsilon}_t^{\eta,P},
\end{aligned} \tag{F.50}
$$

where

$$
\tilde{\chi}_t^P \equiv (1-\chi^P)(\tilde{\epsilon}_t^P - \tilde{\mu}_t^P), \tag{F.51}
$$

$$
\tilde{\epsilon}_t^P = \varrho^P \vartheta_w^P \beta \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right],
$$

and

$$\begin{aligned}\tilde{\mu}_t^P =&(\vartheta_w^P \beta x^P)\mathbb{E}_t \tilde{\tilde{J}}_{t+1}^P + \beta\vartheta_w^P \mathbb{E}_t \left[\tilde{\tilde{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\mu}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right] \\ &- (\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P \mathbb{E}_t \left[\tilde{\tilde{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\tilde{w}}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right].\end{aligned}$$

Similarly to (F.50)

$$\begin{aligned}\mathbb{E}_t \tilde{\chi}_{t+1}^P (w_{t+1}^{*P}) =&\mathbb{E}_t \tilde{\chi}_{t+1}^P + (1-\chi^P)(\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P \mathbb{E}_t \left[\tilde{\tilde{w}}_{t+1}^{*P} - \tilde{\tilde{w}}_{t+1}^P\right] \\ &+ (1-\chi^P)(1-\eta^P)^{-1}\rho_\eta^P \tilde{\varepsilon}_t^{\eta,P}.\end{aligned}$$

Substitute (F.48), (F.50), and (F.49) into (F.47) and rearrange terms to derive

$$\tilde{\tilde{w}}_t^o (w_t^{*P}) = \tilde{\tilde{w}}_t^{o,P} + \frac{\tau_1^P}{1-\tau^P}\mathbb{E}_t \left[\tilde{\tilde{w}}_{t+1}^P - \tilde{\tilde{w}}_{t+1}^{*F}\right] + \frac{\tau_2^P}{1-\tau^P}(\tilde{\tilde{w}}_t^P - \tilde{\tilde{w}}_t^{*P}), \quad \text{(F.52)}$$

where

$$\begin{aligned}\tilde{\tilde{w}}_t^{o,P} =&\varphi_a^P(\tilde{p}_t^P + \tilde{\tilde{a}}_t^P) + (\varphi_s^P + \varphi_x^P)\mathbb{E}_t \tilde{\tilde{J}}_{t+1}^P + \varphi_s^P \tilde{s}_t^P + \varphi_b^P \tilde{\tilde{b}}_t \\ &+ (\varphi_s^P - \varphi_\varphi + \varphi_x^P/2)\mathbb{E}_t \tilde{\tilde{\Lambda}}_{t,t+1} - \varphi_\varphi \mathbb{E}_t \tilde{\tilde{H}}_{x,t+1}^F + \varphi_\eta^P \tilde{\varepsilon}_t^{\eta,P}\end{aligned}$$

with

$$\tau_1^P = (1-\tau^P)\left[\varphi_x^P \varkappa_w^P \mu^P + \varphi_\chi^P (1-\chi^P)(\varrho^P \beta)(\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P + \varphi_s \Gamma^P\right],$$

$$\tau_2^P = -(1-\tau^P)\varphi_\chi^P (1-\chi^P)(\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P,$$

and[48]

$$\varphi_\eta^P = (1-\eta^P)^{-1}(1-\chi^P)\varphi_\chi^P[1-\beta(\varrho^P - s^P)\rho_\eta^P].$$

Below, we write

$$\tilde{\varepsilon}_t^{wP} = \varphi_\eta^P \tilde{\varepsilon}_t^{\eta,P}.$$

See (F.40) for the expression for $\mathbb{E}_t \tilde{\tilde{H}}_{x,t+1}^F$. Substitute (F.52) into (F.47) to obtain

$$\tilde{\tilde{w}}_t^{*P} = (1-\tau^P)\tilde{\tilde{w}}_t^o + \tau_1^P \mathbb{E}_t \left[\tilde{\tilde{w}}_{t+1}^P - \tilde{\tilde{w}}_{t+1}^{*P}\right] + \tau_2^P(\tilde{\tilde{w}}_t^P - \tilde{\tilde{w}}_t^{*P}) - \tau^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\tilde{w}}_{t+1}^{*P}\right].$$

[48] We use $\varphi_s^P = (1-\chi^P)\varphi_r^P \chi\beta s^P$ to simplify the coefficient of $\tilde{\varepsilon}_t^{\eta,P}$.

Use (F.7) to express $\tilde{\bar{w}}_t^{*P}$ in terms of aggregate variables and find that

$$
\begin{aligned}
&\tilde{\bar{w}}_t^P - \vartheta_w^P \left(\iota_w \tilde{\pi}_{t-1} + \tilde{\bar{w}}_{t-1}^P - \tilde{\varepsilon}_t^z - \tilde{\pi}_t\right) \\
&= (1-\vartheta_w^P)(1-\tau^P)\tilde{\bar{w}}_t^o + \vartheta_w^P \tau_1^P \left(\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_t^P - \mathbb{E}_t \tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t \tilde{\pi}_{t+1} - \mathbb{E}_t \tilde{\bar{w}}_{t+1}^P\right) \\
&+ \vartheta_w^P \tau_2^P \left(\iota_w \tilde{\pi}_{t-1} + \tilde{\bar{w}}_{t-1}^P - \tilde{\varepsilon}_t^z - \tilde{\pi}_t - \tilde{\bar{w}}_t^P\right) \\
&- \tau^P \left(\iota_w \tilde{\pi}_t - \mathbb{E}_t \tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t \tilde{\pi}_{t+1} - \mathbb{E}_t \tilde{\bar{w}}_{t+1}^P + \vartheta_w^P \tilde{\bar{w}}_t^P\right).
\end{aligned}
$$

Collecting the terms gives

$$
\tilde{\bar{w}}_t^P = \omega_b^P \left(\tilde{\bar{w}}_{t-1}^P - \tilde{\pi}_t + \iota_w \tilde{\pi}_{t-1} - \tilde{\varepsilon}_t^z\right) + \omega_o^P \tilde{\bar{w}}_t^{o,P} + \omega_f^P \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^P + \tilde{\pi}_{t+1} - \iota_w \tilde{\pi}_t + \tilde{\varepsilon}_{t+1}^z\right],
$$

where $\omega_b^P = (1+\tau_2^P)/\Phi^P$, $\omega_o^P = \varsigma^P/\Phi^P$, $\omega_f^P = (\tau^P/\vartheta_w^P - \tau_1^P)/\Phi^P$, $\Phi^P = (1+\tau_2^P) + \varsigma^P + (\tau^P/\vartheta_w^P - \tau_1^P)$, and $\varsigma^P = (1-\vartheta_w^P)(1-\tau^P)/\vartheta_w^P$.

F.2.4 Proof of Lemmas in Appendix F.2

Proof of Lemma 1. Taking derivative (F.1) with respect to w derives

$$
\frac{\partial \mathcal{A}^{F'}(\rho_t^F(\lambda w))}{\partial w} = \frac{\partial \bar{J}_t^F(\lambda w)}{\partial w}, \qquad \Longrightarrow \qquad \mathcal{A}^{F''}(\rho_t^F(\lambda w))\rho_t^{F'}(\lambda w) = \bar{J}_t^{F'}(\lambda w),
$$

and therefore

$$
\rho_t^{F'}(\lambda w) = \frac{1}{\mathcal{A}^{F''}(\rho_t^F(\lambda w))}\bar{J}_t^{F'}(\lambda w) = \frac{\mathcal{A}^{F'}(\rho_t^F(\lambda w))}{\mathcal{A}^{F''}(\rho_t^F(\lambda w))}\frac{\bar{J}_t^{F'}(\lambda w)}{\bar{J}_t^F(\lambda w)} = \mathcal{E}_t^F(\lambda w)\frac{\varrho_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w)}{\bar{J}_t^F(\lambda w)}.
$$

Hence,

$$
\frac{\partial \mathcal{A}^F(\rho_t^F(\lambda w))}{\partial w} = \lambda \mathcal{A}^{\mathcal{F}'}(\rho_t^F(\lambda w))\rho_t^{F'}(\lambda w) = \lambda \varrho_t^F(\lambda w)\mathcal{E}_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w)
$$

and

$$
\begin{aligned}
\frac{\partial \rho_t^F(\lambda w)\bar{J}_t^F(\lambda w)}{\partial w} &= \lambda \left(\rho_t^{F'}(\lambda w)\bar{J}_t^F(\lambda w) + \rho_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w)\right) \\
&= \lambda \left(\mathcal{E}_t^F(\lambda w) + 1\right)\varrho_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w).
\end{aligned}
$$

■

Proof of Lemma 2. Let $A = \mathcal{E}^F \mu^F (\bar{H}^F - \bar{H}^P)/\bar{J}^F$ and $B = \vartheta_w^F \varrho^F \beta$.

$$
\frac{1}{1-\vartheta_w^F \varrho^F \beta(1-e_o^F)} = \frac{1}{1-B(1-\frac{1-B}{1-AB}A)} = \frac{1-AB}{(1-AB)-B(1-A)} = \frac{1-AB}{1-B} = \epsilon^F.
$$

■

G Model estimation

G.1 Steady-state conditions

- p^w, p^F, p^P, x^F, x^P, r^k, $\bar{k}^F$, $\bar{k}^P$, $\bar{a}^F$, $\bar{a}^P$, $\bar{y}^F/\bar{y}$, and $\bar{y}$ satisfy

$$p^w = \frac{1}{\varepsilon_p},$$

$$\frac{p^F}{p^P} = \left[\frac{\Omega^F}{1-\Omega^F}\frac{\varepsilon^\phi \mu_b^P n^P}{n^F}\right]^{\frac{1-\alpha}{\xi(1-\alpha)+\alpha}},$$

$$p^F = p^w \left(\Omega^F + (1-\Omega^F)\left(\frac{p^F}{p^P}\right)^{\xi-1}\right)^{\frac{1}{\xi-1}},$$

$$x^F = 1 - (\varrho^F + \varphi n^P),$$

$$x^P = 1 - (\varrho^P + \lambda^{FP}\varrho^F n^F),$$

$$A^F(\varrho^F)^{\zeta^F} = \kappa^F x^F,$$

$$A^P(\varrho^P)^{\zeta^P} = \kappa^P x^P,$$

$$r^k = \frac{\gamma_z}{\beta} - (1-\delta),$$

$$\frac{\bar{k}^F}{n^F} = \left(\frac{r^k}{p^F\alpha}\right)^{-1/(1-\alpha)},$$

$$\frac{\bar{k}^P}{\mu_b^P\varepsilon^\phi n^P} = \left(\frac{r^k}{p^P\alpha}\right)^{-1/(1-\alpha)},$$

$$\bar{a}^F = (1-\alpha)\left(\frac{\bar{k}^F}{n^F}\right)^\alpha,$$

$$\bar{a}^P = (1-\alpha)\mu_b^P\varepsilon^\phi\left(\frac{\bar{k}^P}{\mu_b^P\varepsilon^\phi n^P}\right)^\alpha,$$

$$\frac{\bar{y}^F}{\bar{y}} = \Omega^F\left(\frac{p^F}{p^w}\right)^{-\xi},$$

$$\bar{y} = \frac{1}{\Omega^F}\left(\frac{p^F}{p^w}\right)^{\xi}\left(\frac{\bar{k}^F}{n^F}\right)^\alpha n^F.$$

- The steady-state values for (n^F, n^P, u^F, u^P)

  The steady-state conditions for (14) and (15), the calibration target $n^F/n^P = 4.405$ and the condition that total labor force is unity induce the stationary distribution of workers' employment state:

$$\begin{pmatrix} 1-\varrho^F & -\varphi & -s^F & 0 \\ -\lambda_{FP}(1-\varrho^F) & 1-\varrho^P & 0 & -s^P \\ 1 & -4.405 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} n^F \\ n^P \\ u^F \\ u^P \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

- The steady-state values for the firm's surplus and worker's surplus

The firm's surplus in the balanced-growth steady state is given by

$$\bar{J}^F = \frac{1}{1-\beta\varrho^F}\left[p^F\bar{a}^F - \bar{w}^F + \beta\left[\left(\frac{\kappa^F}{2}\right)(x^F)^2 + \vartheta^F\frac{(\varrho^F)^{1+\zeta^F}}{1+\zeta^F}\right]\right] \tag{G.1}$$

and

$$\bar{J}^P = \frac{1}{1-\beta\varrho^P}\left[p^P\bar{a}^P - \bar{w}^P\mu_b^P + \beta\left[\left(\frac{\kappa^P}{2}\right)(x^P)^2 + \vartheta^P\frac{(\varrho^P)^{1+\zeta^P}}{1+\zeta^P}\right]\right]. \tag{G.2}$$

The worker's surplus in the balanced-growth steady state is given by

$$\bar{H}^F = \frac{1}{1-\beta(\varrho^F - s^F C_\rho^F)}(\bar{w}^F - \bar{b}) \tag{G.3}$$

and

$$\bar{H}^P = \frac{1}{1-\beta(\varrho^P C_\varrho^P - s^P)}(\bar{w}^P\mu_b^P - \bar{\mu}_b^P\bar{b}), \tag{G.4}$$

where $C_\rho^F = 1-\lambda^{FP}(1-\varrho^F)/s^P$ and $C_\varrho^P = 1+(\varphi/\varrho^P)(s^P/s^F-1)$.

- The steady-state conditions for wages

Here, we describe how $\bar{w}^F$ and $\bar{w}^P$ are determined.

Plug (G.1) and (G.2) into the definitions $\bar{J}^F = \kappa^F x^F$ and $\bar{J}^P = \kappa^P x^P$

$$\bar{w}^F = p^F\bar{a}^F - (1-\beta\varrho^F)\kappa^F x^F + \beta\kappa^F x^F\left(\frac{x^F}{2} + \frac{\varrho^F}{1+\zeta^F}\right) \tag{G.5}$$

and

$$\bar{w}^P\mu_b^P = p^P\bar{a}^P - (1-\beta\varrho^P)\kappa^P x^P + \beta\kappa^P x^P\left(\frac{x^P}{2} + \frac{\varrho^P}{1+\zeta^P}\right). \tag{G.6}$$

The (staggered) Nash bargaining solution requires

$$(1-\chi^F)\bar{H}^F = \chi^F\bar{J}^F, \tag{G.7}$$

and

$$(1-\chi^P)\bar{H}^P = \chi^P \bar{J}^P. \tag{G.8}$$

Plug (G.1), (G.2), (G.3), and (G.4), into (G.7) and (G.8)

$$(1-\chi^F)\bar{b} = \bar{w}^F - \chi^F \left[p^F \bar{a}^F + \beta\kappa^F x^F \left(\frac{x^F}{2} + \frac{\varrho^F}{1+\zeta^F} \right) + \beta s^F C_\rho^F \kappa^F x^F \right]. \tag{G.9}$$

and

$$(1-\chi^P)\bar{\mu}_b^P \bar{b} = \mu_b^P \bar{w}^P - \chi^P \left[\begin{array}{c} p^P \bar{a}^P + \beta\kappa^P x^P \left(\frac{x^P}{2} + \frac{\varrho^P}{1+\zeta^P} \right) \\ +\beta[s^P - \varrho^P(1-C_\varrho^P)]\kappa^P x^P \end{array} \right]. \tag{G.10}$$

The steady-state condition $s^P \bar{H}^P = s^F \bar{H}^F$ induces

$$s^P \frac{\chi^P}{1-\chi^P}\kappa^P x^P = s^F \frac{\chi^F}{1-\chi^F}\kappa^F x^F. \tag{G.11}$$

This implies

$$\bar{H}^F - \lambda^{FP}\bar{H}^P = (1-\lambda^{FP}s^F/s^P)\bar{H}^F = (1-\lambda^{FP}s^F/s^P)((1-\chi^F)/\chi^F)\kappa^F x^F. \tag{G.12}$$

Solve χ^F and χ^P in terms of fixed and estimated parameters: χ^F is given by

$$\chi^F = \frac{\eta^F}{\eta^F + (1-\eta^F)\mu^F/\epsilon^F}, \tag{G.13}$$

where

$$\mu^F = \frac{1}{1-\beta(x^F+\varrho^F)\vartheta_w^F},$$

and

$$\epsilon^F = \frac{1-\vartheta_w^F \varrho^F \beta \mathcal{E}^F \mu^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)/(\kappa^F x^F)}{1-\vartheta_w^F \varrho^F \beta}.$$

Substitute (G.12) into above expression to find

$$\epsilon^F = \frac{1-\mu^F(\vartheta_w^F \varrho^F \beta)\mathcal{E}^F(1-\lambda^{FP}s^F/s^P)(1-\chi^F)/\chi^F}{1-\vartheta_w^F \varrho^F \beta},$$

Substituting this into (G.13) gives

$$\chi^F = \frac{\eta^F}{\eta^F + \mu^F\left[(\vartheta_w^F \varrho^F \beta)\mathcal{E}^F(1-\lambda^{FP}s^F/s^P)\eta^F + (1-\vartheta_w^F \varrho^F \beta)(1-\eta^F)\right]} \in (0,1),$$

and

$$\epsilon^F = \frac{1-\eta^F}{(\vartheta_w^F \varrho^F \beta)\mathcal{E}^F(1-\lambda^{FP}s^F/s^P)\eta^F + (1-\vartheta_w^F\varrho^F\beta)(1-\eta^F)}.$$

Next, χ^P is given by

$$\chi^P = \frac{\eta^P}{\eta^P + (1-\eta^P)\mu^P/\epsilon^P},$$

where

$$\mu^P = \frac{1}{1-\beta(x^P+\varrho^P)\vartheta_w^P},$$

and

$$\epsilon^P = \frac{1}{1-\vartheta_w^P\varrho^P\beta}.$$

The values for $(\kappa^F, \bar{w}^F, \bar{b})$ are solved out from (G.5), (G.9), and the estimate of $\bar{b}^F$,

$$\bar{b}^F = \frac{\bar{b}}{p^F\bar{a}^F + \beta\kappa^F x^F\left(x^F/2 + \varrho^F/(1+\zeta^F)\right)}.$$

The system of equations solves

$$\kappa^F = \frac{p^P\bar{a}^F(1-\bar{b}^F)(1-\chi^F)}{x^F[1-\beta(1-\bar{b}^F)(1-\chi^F)(x^F/2+\varrho^F/(1+\zeta^F)) - \beta(\varrho^F - s^F C_\rho^F\chi^F)]},$$

$$\bar{w}^F = \frac{p^F\bar{a}^F[\bar{b}^F(1-\chi^F)(1-\beta\varrho^F) - \chi^F(\beta\varrho^F - \beta s^F C_\rho^F - 1)]}{1-\beta(1-\bar{b}^F)(1-\chi^F)(x^F/2+\varrho^F/(1+\zeta^F)) - \beta(\varrho^F - s^F C_\rho^F\chi^F)},$$

and

$$\bar{b} = \frac{p^F\bar{a}^F\bar{b}^F[1-\beta(\varrho^F - s^F C_\rho^F\chi^F)]}{1-\beta(1-\bar{b}^F)(1-\chi^F)(x^F/2+\varrho^F/(1+\zeta^F)) - \beta(\varrho^F - s^F C_\rho^F\chi^F)}.$$

Given $(\bar{w}^F,\ \bar{b})$, the values for $(\kappa^P,\ \bar{w}^P,\ \bar{\mu}_b^P)$ are solved out from (G.6), (G.10), (G.11). From (G.11)

$$\kappa^P = \frac{s^F(1-\chi^P)}{x^P s^P \chi^P}\frac{\chi^F}{1-\chi^F}\kappa^F x^F,$$

and (G.6) can solve for $\bar{w}^P$. Then use (G.10) to solve out $\bar{\mu}_b^P$.

G.2 Log-linearized model equations

Consumption, Investment, and Production

$$(1-\beta h_z)\tilde{\lambda}_t = \hbar_{1,c}(\tilde{\tilde{c}}_{t-1} - \tilde{\varepsilon}_t^z + \beta\mathbb{E}_t\tilde{\tilde{c}}_{t+1} + \beta\mathbb{E}_t\tilde{\varepsilon}_{t+1}^z) - \hbar_{2,c}\tilde{\tilde{c}}_t + \tilde{\varepsilon}_t^b - \beta h_z\mathbb{E}_t\tilde{\varepsilon}_{t+1}^b,$$

$$\tilde{\lambda}_t = \tilde{r}^n_t + \mathbb{E}_t\tilde{\lambda}_{t+1} - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\varepsilon}^z_{t+1},$$

$$\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} = \mathbb{E}_t\tilde{\lambda}_{t+1} - \tilde{\lambda}_t,$$

$$\tilde{\bar{k}}^p_t = \delta_z(\tilde{\bar{k}}^p_{t-1} - \tilde{\varepsilon}^z_t) + (1-\delta_z)\tilde{\bar{i}}_t,$$

$$\tilde{\bar{k}}_t = \tilde{\nu}_t + \tilde{\bar{k}}^p_{t-1} - \tilde{\varepsilon}^z_t,$$

$$\tilde{\nu}_t = \eta_\nu\,\tilde{r}^k_t,$$

$$\tilde{Q}_t = \beta\delta_z\mathbb{E}_t\tilde{Q}_{t+1} + (1-\beta\delta_z)\,\mathbb{E}_t\tilde{r}^k_{t+1} - \tilde{r}^n_t + \mathbb{E}_t\tilde{\pi}_{t+1},$$

$$(1+\beta)\tilde{\bar{i}}_t = \tilde{\bar{i}}_{t-1} - \tilde{\varepsilon}^z_t + \tilde{\varepsilon}^i_t + [1/(\eta_k(\gamma_z)^2)]\tilde{Q}_t + \beta\mathbb{E}_t\left[\tilde{\bar{i}}_{t+1} + \tilde{\varepsilon}^z_{t+1} - \tilde{\varepsilon}^i_{t+1}\right],$$

$$\tilde{\bar{y}}^F_t = \alpha\tilde{\bar{k}}^F_t + (1-\alpha)\tilde{n}^F_t,$$

$$\tilde{\bar{y}}^P_t = \alpha\tilde{\bar{k}}^P_t + (1-\alpha)(\tilde{n}^P_t + \tilde{\varepsilon}^\phi_t),$$

$$\tilde{\bar{y}}_t = (\Omega^F)^{1/\xi}(\bar{y}^F/\bar{y})^{(\xi-1)/\xi}\tilde{\bar{y}}^F_t + (1-\Omega^F)^{1/\xi}(\bar{y}^P/\bar{y})^{(\xi-1)/\xi}\tilde{\bar{y}}^P_t,$$

$$\tilde{p}^w_t = \Omega^F(p^F/p^w)^{1-\xi}\tilde{p}^F_t + (1-\Omega^F)(p^P/p^w)^{1-\xi}\tilde{p}^P_t,$$

$$\tilde{\bar{y}}^F_t - \tilde{\bar{y}}^P_t = -\xi(\tilde{p}^F_t - \tilde{p}^P_t),$$

$$\tilde{r}^k_t = \tilde{p}^F_t + \tilde{\bar{y}}^F_t - \tilde{\bar{k}}^F_t,$$

$$\tilde{r}^k_t = \tilde{p}^P_t + \tilde{\bar{y}}^P_t - \tilde{\bar{k}}^P_t,$$

$$\tilde{\bar{k}}_t = (\bar{k}^F/\bar{k})\tilde{\bar{k}}^F_t + (\bar{k}^P/\bar{k})\tilde{\bar{k}}^P_t,$$

$$\tilde{\bar{y}}_t = y_c\tilde{\bar{c}}_t + y_i\tilde{\bar{i}}_t + \upsilon\tilde{\bar{g}}_t + y_\nu\tilde{\nu}_t + y^F_\rho\,\tilde{\varrho}^F_t + y^P_\varphi\tilde{n}^P_{t-1},$$
$$+ y^F_x(2\tilde{x}^F_t + \tilde{n}^F_{t-1}) + y^P_x(2\tilde{x}^P_t + \tilde{n}^P_{t-1}),$$

$$\tilde{\pi}_t = \iota_b\tilde{\pi}_{t-1} + \iota_o\tilde{p}^w_t + \iota_f\mathbb{E}_t\tilde{\pi}_{t+1} + \tilde{\varepsilon}^p_t,$$

where

$$\begin{aligned}
&\hbar_{1,c} = h_z/(1-h_z), \quad \hbar_{2,c} = (1+\beta(h_z)^2)/(1-h_z), \quad h_z = h_c/\gamma_z,\\
&\delta_z = (1-\delta)/\gamma_z, \quad \eta_\nu = (1-\psi_\nu)/\psi_\nu,\\
&y_c = 1 - (y_i + \upsilon + y_\nu + y^F_\rho + y^F_x + y^P_x + y^F_\varphi), \quad y_i = (1-\delta_z)\gamma_z(\bar{k}/\bar{y}),\\
&y_\nu = r^k(\bar{k}/\bar{y}), \quad y^F_\rho = \varrho^F\kappa^F x^F n^F/\bar{y}, \quad y^\ell_x = (\kappa^\ell/2)[n^\ell(x^\ell)^2/\bar{y}],\\
&y^F_\varphi = (\zeta+1)^{-1}\varrho^P\kappa^F x^F n^F/\bar{y},\\
&\iota_b = \iota_p\phi_p, \quad \iota_o = [(1-\vartheta_p)(1-\beta\vartheta_p)/\vartheta_p][1+(\epsilon_p-1)\Xi]^{-1}\phi_p,\\
&\iota_f = \beta\phi_p, \quad \text{and} \quad \phi_p = 1/(1+\beta\iota_p).
\end{aligned}$$

Labor markets and employment dynamics

$$n^F\tilde{n}_t^F + n^P\tilde{n}_t^P + u^F\tilde{u}_t^F + u^P\tilde{u}_t^P = 0,$$

$$\tilde{n}_t^F = \varrho^F(\tilde{\varrho}_t^F + \tilde{n}_{t-1}^F) + x^F(\tilde{s}_{t-1}^F + \tilde{u}_{t-1}^F) + \varphi\varrho^P(n^P/n^F)\tilde{n}_{t-1}^P,$$

$$\tilde{n}_t^P = (1-\varphi)\varrho^P\tilde{n}_{t-1}^P + x^P(\tilde{s}_{t-1}^P + \tilde{u}_{t-1}^P) + \lambda^{FP}(n^F/n^P)((1-\varrho^F)\tilde{n}_{t-1}^F - \varrho^F\tilde{\varrho}_t^F),$$

$$\tilde{x}_t^F = \tilde{q}_t^F + \tilde{v}_t^F - \tilde{n}_t^F,$$

$$\tilde{x}_t^P = \tilde{q}_t^P + \tilde{v}_t^P - \tilde{n}_t^P,$$

$$\tilde{\varrho}_t^F = \mathcal{E}^F\tilde{\tilde{J}}_t^F,$$

$$\tilde{x}_t^F = \mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F,$$

$$\tilde{\theta}_t^F = \tilde{v}_t^F - \tilde{u}_t^F,$$

$$\tilde{\theta}_t^P = \tilde{v}_t^P - \tilde{u}_t^P,$$

$$\tilde{q}_t^F = -\sigma_m^F\tilde{\theta}_t^F,$$

$$\tilde{q}_t^P = -\sigma_m^P\tilde{\theta}_t^P,$$

$$\tilde{s}_t^F = (1-\sigma_m^F)\tilde{\theta}_t^F,$$

$$\tilde{s}_t^P = (1-\sigma_m^P)\tilde{\theta}_t^P,$$

$$\tilde{\tilde{J}}_t^F = \varkappa_a^F(\tilde{p}_t^F + \tilde{\tilde{a}}_t^F) - \varkappa_w^F\tilde{\tilde{w}}_t^F + \beta(\varrho^F + X^F)\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1} + \beta(1+\varrho^F\mathcal{E}^F)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F,$$

$$\tilde{\tilde{J}}_t^P = \varkappa_a^P(\tilde{p}_t^w + \tilde{\tilde{a}}_t^P) - \varkappa_w^P\tilde{\tilde{w}}_t^P + \varkappa_\lambda^P\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1} + \beta\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^P$$

$$\tilde{\tilde{a}}_t^F = \tilde{\tilde{y}}_t^F - \tilde{n}_t^F,$$

$$\tilde{\tilde{a}}_t^P = \tilde{\tilde{y}}_t^P - \tilde{n}_t^P,$$

$$\tilde{s}_t^F + \mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^F = \tilde{s}_t^P + \mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^P,$$

$$\begin{aligned}\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^F =& \mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F - \vartheta_w^F(1-\vartheta_w^F)^{-1}\Gamma^F\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^F - (\tilde{\tilde{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z)\right]\\ &+ (1-\chi^F)^{-1}\mathbb{E}_t\tilde{\chi}_{t+1}^F + (1-\eta^F)^{-1}\rho_\eta^F\tilde{\varepsilon}_t^{\eta,F},\end{aligned}$$

$$\begin{aligned}\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^P =& \mathbb{E}_t\tilde{\tilde{J}}_{t+1}^P - \vartheta_w^P(1-\vartheta_w^P)^{-1}\Gamma^P\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^P - (\tilde{\tilde{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z)\right]\\ &+ (1-\chi^P)^{-1}\mathbb{E}_t\tilde{\chi}_{t+1}^P + (1-\eta^P)^{-1}\rho_\eta^P\tilde{\varepsilon}_t^{\eta,P},\end{aligned}$$

where

$$\mathcal{E}^F = \mathcal{A}^{F'}(\varrho^F)/(\varrho^F \mathcal{A}^{F''}(\varrho^F)) = 1/\zeta, \quad \sigma_m^\ell = (\theta^\ell)^\sigma/(1+(\theta^\ell)^\sigma),$$
$$\varkappa_a^\ell = p^\ell \bar{a}^\ell/(\kappa^\ell x^\ell), \quad \varkappa_w^\ell = (\bar{w}^\ell \mu_b^\ell)/(\kappa^\ell x^\ell),$$
$$X^F = 1-\varrho^F - (x^F/2 + \varrho^F/(\zeta+1)), \quad \varkappa_\lambda^F = \beta(1+\varrho^F)/2, \quad \varkappa_\lambda^P = \beta(1+\varrho^P(1-\varphi))/2,$$
$$\Gamma^F = -\epsilon^F \mho + [1-\eta^F(\vartheta_w^F \beta m_o^F)\mu^F](\varkappa_w^F \mu^F)/\eta^F,$$
$$\Gamma^P = [1-\eta^P(\vartheta_w^P \beta x^P)\mu^P](\varkappa_w^P \mu^P)/\eta^P,$$
$$\mho = -\vartheta_w^F \varrho^F \beta(\varkappa_w^F \mu^F)\left[(2-e_o^F)\mathcal{E}^F + e_o^F - e_o^F(\vartheta_w^F \beta m_o^F)\mu^F\right],$$
$$e_o^F = (1/\epsilon^F)\mu^F \mathcal{E}^F(\bar{H}^F - \lambda^{FP}\bar{H}^P)/(\kappa^F x^F), \quad \text{and} \quad m_o^F = x^F + \varrho^F \mathcal{E}^F.$$

Wage dynamics

$$\tilde{\bar{b}}_t = \tilde{\bar{k}}_t^p,$$
$$\tilde{\chi}_t^F = -(1-\chi^F)(\tilde{\mu}_t^F - \tilde{\epsilon}_t^F),$$
$$\tilde{\chi}_t^P = -(1-\eta^P)(\tilde{\mu}_t^P - \tilde{\epsilon}_t^P),$$

$$\begin{aligned}
\tilde{\epsilon}_t^F =& \vartheta_w^F \varrho^F \beta(1-e_0^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] - \vartheta_w^F \varrho^F \beta e_0^F \mathbb{E}_t \tilde{\mu}_{t+1}^F \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\mathbb{E}_t\left[\bar{H}^F \tilde{\bar{H}}_{x,t}^F - \lambda^{FP}\bar{H}^P \tilde{\bar{H}}_t^P\right] \\
&+ \vartheta_w^F \varrho^F \beta\left((1-e_o^F)\mathcal{E}^F + e_o^F\right)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F + \epsilon^F \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right],
\end{aligned}$$

$$\tilde{\epsilon}_t^P = \varrho^P(1-\varphi)\vartheta_w^P \beta \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right],$$

$$\begin{aligned}
\tilde{\mu}_t^F =& (\vartheta_w^F \beta m_o^F)\mathbb{E}_t \tilde{x}_{t+1}^F + \beta \vartheta_w^F \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\mu}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right],
\end{aligned}$$

$$\begin{aligned}
\tilde{\mu}_t^P =& (\vartheta_w^P \beta x^P)\mathbb{E}_t \tilde{x}_{t+1}^P + \beta \vartheta_w^P \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\mu}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right] \\
&- (\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P \mathbb{E}_t\left[\tilde{\bar{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right],
\end{aligned}$$

$$\begin{aligned}
\tilde{\bar{w}}_t^{o,F} =& \varphi_a^F\left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F\right) + (\varphi_x^F + \varphi_s^F - \varphi_\rho^F)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F + \varphi_\lambda^F \mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} + \varphi_b^F \tilde{\bar{b}}_t \\
&+ \varphi_s^F \mathbb{E}_t \tilde{s}_{t+1}^F - \varphi_{\hat{\rho}}^F \mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^P + \varphi_\chi^F\left[\tilde{\chi}_t^F - \beta(\varrho^F - s^F)\mathbb{E}_t \tilde{\chi}_{t+1}^F\right] + \tilde{\varepsilon}_t^{wF},
\end{aligned}$$

$$\tilde{\varepsilon}_t^{wF} = (1-\eta^F)^{-1}(1-\chi^F)\varphi_\chi^F[1-\beta(\varrho^F - s^F)\rho_\eta^F]\tilde{\varepsilon}_t^{\eta,F},$$

$$\begin{aligned}
\tilde{\bar{w}}_t^{o,P} =& \varphi_a^P(\tilde{p}_t^w + \tilde{\bar{a}}_t^P) + (\varphi_s^P + \varphi_x^P)\mathbb{E}_t \tilde{x}_{t+1}^P + \varphi_b^P \tilde{\bar{b}}_t + \varphi_\lambda^P \mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} \\
&+ \varphi_s^P \mathbb{E}_t \tilde{s}_{t+1}^P - \varphi_\varphi \mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^F + \tilde{\varepsilon}_t^{wP},
\end{aligned}$$

$$\tilde{\varepsilon}_t^{wP} = (1-\eta^P)^{-1}(1-\chi^P)\varphi_\chi^P[1-(1-x^P-s^P)\beta\varrho_\eta^P]\tilde{\varepsilon}_t^{\eta,P},$$

$$\tilde{\bar{w}}_t^F = \omega_b^F(\tilde{\bar{w}}_{t-1}^F - \tilde{\pi}_t + \iota_w\tilde{\pi}_{t-1} - \tilde{\varepsilon}_t^z) + \omega_o^F\tilde{\bar{w}}_t^{o,F} + \omega_f^F\mathbb{E}_t\left[\tilde{\bar{w}}_{t+1}^F + \tilde{\pi}_{t+1} - \iota_w\tilde{\pi}_t + \tilde{\varepsilon}_{t+1}^z\right],$$
$$\tilde{\bar{w}}_t^P = \omega_b^P(\tilde{\bar{w}}_{t-1}^P - \tilde{\pi}_t + \iota_w\tilde{\pi}_{t-1} - \tilde{\varepsilon}_t^z) + \omega_o^P\tilde{\bar{w}}_t^{o,P} + \omega_f^P\mathbb{E}_t\left[\tilde{\bar{w}}_{t+1}^P + \tilde{\pi}_{t+1} - \iota_w\tilde{\pi}_t + \tilde{\varepsilon}_{t+1}^z\right],$$

where

$$\varphi_a^\ell = \chi^\ell p^\ell \bar{a}^\ell(\bar{w}^\ell\mu_b^\ell)^{-1}, \quad \varphi_x^\ell = \chi^\ell\beta\kappa^\ell(x^\ell)^2(\bar{w}^\ell\mu_b^\ell)^{-1}, \quad \varphi_s^\ell = (1-\chi^\ell)s^\ell\beta\bar{H}^\ell(\bar{w}^\ell\mu_b^\ell)^{-1},$$
$$\varphi_b^F = (1-\chi^F)\bar{b}(\bar{w}^F\mu_b^F)^{-1}, \quad \varphi_b^P = (1-\chi^P)\bar{\mu}_b^P\bar{b}(\bar{w}^P\mu_b^P)^{-1},$$
$$\varphi_\chi^\ell = \chi^\ell\kappa^\ell x^\ell[(1-\chi^\ell)\bar{w}^\ell\mu_b^\ell]^{-1}, \quad \varphi_\varphi = \beta\varrho^P(1-\chi^P)\bar{H}^F(\bar{w}^P\mu_b^P)^{-1},$$
$$\varphi_\lambda^F = \varphi_s^F + \varphi_X^F - \varphi_{\hat{\rho}}^F, \quad \varphi_\lambda^P = \varphi_s^P - \varphi_\varphi + \varphi_x^P/2, \quad \varphi_X^\ell = \chi^\ell\beta X^\ell\bar{J}^\ell(\bar{w}^\ell\mu_b^\ell)^{-1}$$
$$\varphi_{\hat{\rho}}^F = (1-\chi^F)\beta(\lambda^{FP}\bar{H}^P/\bar{w}^F)(1-\varrho^F), \qquad \varphi_\rho^F = (1-\chi^F)\beta((\bar{H}^F - \lambda^{FP}\bar{H}^P)/\bar{w}^F)\varrho^F\mathcal{E}^F,$$
$$\gamma_b^\ell = (1+\tau_2^\ell)/\Phi^\ell, \quad \gamma_o^\ell = \varsigma^\ell/\Phi^\ell, \quad \gamma_f^\ell = (\tau^\ell/\vartheta_w^\ell - \tau_1^\ell)/\Phi^\ell,$$
$$\Phi^\ell = (1+\tau_2^\ell) + \varsigma^\ell + (\tau^\ell/\vartheta_w^\ell - \tau_1^\ell), \quad \varsigma^\ell = (1-\vartheta_w^\ell)(1-\tau^\ell)/\vartheta_w^\ell, \quad \tau^\ell = \psi^\ell(1+\psi^\ell)^{-1},$$
$$\tau_1^F = (1-\tau^F)\left[\varkappa_w^F\mu^F(\varphi_x^F - \varphi_\rho^F) + \varphi_\chi^F(\varrho^F\beta)(1-\chi^F)[\epsilon^F\mho + (\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F] + \varphi_s\Gamma^F\right],$$
$$\tau_2^F = -(1-\tau^F)\varphi_\chi^F(1-\chi^F)[\epsilon^F\mho + (\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F],$$
$$\tau_1^P = (1-\tau^P)\left[\varphi_x^P\varkappa_w^P\mu^P + \varphi_\chi^P(1-\chi^P)(\varrho^P\beta)(\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P + \varphi_s\Gamma^P\right],$$
$$\tau_2^P = -(1-\tau^P)\varphi_\chi^P(1-\chi^P)(\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P,$$
$$\psi^F = (1-\chi^F)\vartheta_w^F\varrho^F\beta\hat{\epsilon}^F + \chi^F\beta\vartheta_w^F\mu^F, \quad \hat{\epsilon}^F = \epsilon^F - \mathcal{E}^F\mu^F(\bar{H}^F - \lambda^{FP}\bar{H}^P)/(\kappa^F x^F) \quad \text{and}$$
$$\psi^P = (1-\chi^P)\vartheta_w^P\varrho^P\beta\epsilon^P + \chi^P\beta\vartheta_w^P\mu^P,$$

Monetary policy and Government spending

$$\tilde{r}_t^n = \phi_r\tilde{r}_{t-1}^n + (1-\phi_r)[\phi_\pi\tilde{\pi}_t + \phi_y(\tilde{y}_t - \tilde{y}_{nt})] + \tilde{\varepsilon}_t^r,$$

$$\tilde{\bar{g}}_t = \tilde{\bar{y}}_t + ((1-\upsilon)/\upsilon)\,\tilde{\varepsilon}_t^g.$$