Gross Worker Flows over the Life Cycle*

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Abstract

This paper constructs a quantitative general equilibrium model which allows us to analyze the gross worker flows over the workers' life cycle and how various policies interact with these flows. We first document the life-cycle patterns of flows across different labor market states (employment, unemployment, and not in the labor force), as well as job-to-job transitions, in the US. Then we build a model of the aggregate labor market that incorporates the life cycle of workers, consumption-saving decisions, and labor market frictions. We estimate the model with the US data and find that the model fits the data patterns very well. Through the lens of the model, we uncover the fundamental forces that drive the life-cycle patterns. Finally, we use the estimated model to investigate the effects of policies on aggregate labor market outcomes, such as unemployment rate and the labor-force participation rate. In particular, we analyze a taxes-and-transfers policy and an unemployment insurance policy.

Keywords: worker flows, life cycle, taxes and transfers, unemployment insurance

JEL Classification: E24, J21, J22, J64, J65

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1 Introduction

Over the last several decades, the study of the aggregate labor market has made significant progress by analyzing the gross job flows and the gross worker flows. By investigating beyond the net changes in labor market stocks, such as the unemployment rate and employment-population ratio, our understanding of the dynamics of the labor market and the effect of labor market policies has deepened substantially.

This paper contributes to this strand of literature. We analyze worker flows across three different labor market states, *employment* (E), *unemployment* (U), and *not in the labor force* (or *nonparticipation*) (N), over the workers' life cycle. These gross flows influence the policy-relevant labor market stocks, such as the employment-population ratio, unemployment rate, and labor-force participation rate. In addition, we consider an important worker flow: the flow of employed workers across different jobs. Various studies have found such *job-to-job transitions* play an important role in macroeconomic outcome by reallocating workers to appropriate jobs. For example, Topel and Ward (1992) attribute about 40% of wage growth for young workers to job transitions. More recently, Engbom (2020) argues the patterns of job-to-job transitions, combined with human capital accumulation, can explain a large part of the differences in life-cycle wage growth patterns across OECD countries.[1] We document the empirical patterns of these flows, and conduct a quantitative-theoretic analysis based on these observations.

The particularly novel element in our analysis is the life cycle of workers. Various empirical studies have documented the flows and stocks in the labor market vary substantially with age. For example, the unemployment rate for young workers is known to be higher than for prime-age workers, and young workers tend to experience more frequent job-to-job transitions than older workers. All gross flows, including the ones involving the participation margin, are important in shaping the heterogeneous outcomes in the labor market across different age groups. A recent accounting exercise by Choi et al. (2015) reveals, for example, that low movements from the N state to the E state and from the N state to the U state (we will call them NE flow and NU flow) account for a large part of the low participation and unemployment rates for old workers.

In this paper, we build a quantitative general equilibrium model that replicates the behavior of individuals we focus on, run policy experiments using the model, and interpret the mechanisms. Using the framework, we ask how labor market policies affect the flows and stocks in the labor market for different age groups of workers. We particularly focus on two policies: the first policy involves taxes and transfers and the second is unemployment insurance (UI).

Our model features frictional labor market with an operative labor-supply margin, based on Krusell et al. (2010, 2011, 2017). Krusell et al. (2010), in particular, studied the effects

[1] Barlevy (2002) and Mukoyama (2014) analyze the effect of job-to-job transitions on aggregate productivity. Their model analyses imply the effect of job-to-job transitions on the aggregate productivity can be sizable.

of taxes and transfers using an infinite-horizon model, and found important interactions between frictions and the labor-supply margin in this class of models. Our departure from their analysis is that we explicitly consider the worker life cycle. This departure is important because (i) the heterogeneity in worker flows across different age groups is so significant that analyzing the policy effects with explicit treatment of this heterogeneity is itself very important, (ii) this framework is the first that features labor market frictions and the operative labor-supply margin in a life-cycle context, and this framework can be applied to many other policy experiments, and (iii) quantitatively matching the model to data is quite challenging because fitting six *life-cycle profiles* (plus the job-to-job flow rate and the wage profile) is significantly more difficult than fitting six *numbers* (corresponding flow rates in aggregate). As can be seen below, substantial extensions of the model, compared to Krusell et al. (2010, 2011, 2017), are necessary for the model to replicate salient life-cycle patterns of worker flows in the data.

The estimated model fits the data patterns very well. Through the lens of the model, we uncover the fundamental forces that drive the life-cycle pattern. We find that, across different age groups, the magnitude of heterogeneity in the opportunities for a new job is relatively small compared to the observed heterogeneity in the corresponding flow rates. The outcome highlights the importance of worker choices and how they change over the life cycle. To properly consider the heterogeneity in the opportunities that is relevant for policy analyses, it is essential to utilize an economic model that incorporates the workers' economic choices, rather than a mechanical accounting model.

The two policy experiments we conduct reveal the heterogeneous effects of the policies on the worker flows and stocks for different stages of the life cycle. For the taxes-and-transfers policy, we find the heterogeneity in the changes in employment largely reflects the pattern of the changes in labor-force participation, underscoring the importance of the endogenous participation margin. Only the young workers' unemployment stock significantly changes with the policy. By explicitly analyzing the gross flows, we can pin down which flows are responsible for these policy responses in stocks. For the UI policy, the heterogeneity of policy outcome over the life cycle reflects the different time horizons across workers. Unemployment rate changes more significantly for older workers.

The main contribution of this paper is theoretical; we provide a framework that can replicate the salient features of life-cycle worker flows, and this framework can be used for various policy analyses. To illustrate the usefulness of our framework, we conduct policy exercises that have been analyzed extensively in the macroeconomic literature. Our model features (i) the worker life cycle, (ii) the frictional labor market with heterogeneous jobs, and (iii) the operative labor-supply margin with concave utility and self-insurance. The model is able to fit the quantitative features of the life-cycle patterns of labor market flows and stocks, allowing us to analyze the effect of policies on the labor market outcomes of different age groups of workers. We intentionally keep the model parsimonious so that the main mechanisms remain transparent despite the quantitative nature of the policy experiments.

In particular, as in Krusell et al. (2010, 2011, 2017), the labor-market frictions are modeled using a simple "island" structure, because the most important channel for our experiment is operative labor supply.

The paper is related to several strands of literature. First, several recent papers have analyzed life-cycle worker flows in a frictional labor market. The contributions include Chéron et al. (2013), Esteban-Pretel and Fujimoto (2014), Menzio et al. (2016), and Jung and Kuhn (2019). None of these papers, however, explicitly model the endogenous participation margin, through which the labor-supply channel works. Above papers instead emphasize the labor-demand side, by incorporating (variants of) the Diamond-Mortensen-Pissarides (DMP) type matching process. As we see later in detail, the operative labor-supply channel is essential for the policy experiments in this paper. Our model features a very good fit to the observed life-cycle patterns of worker flows. Fitting six flows (plus the job-to-job flow and the wage series) as functions of age is significantly more challenging than fitting six numbers (as Krusell et al. (2011, 2017) have done), and we view that constructing a framework that can replicate the data pattern as one of our significant contributions.

There are two recent papers that feature worker flows across three states. Lalé and Tarasonis (2020) describe the life-cycle pattern of worker flows in European countries, and construct a three-state life-cycle model. Goensch et al. (2021) extend Menzio et al.'s (2016) model and add a search decision of workers. In contrast to our study, both papers feature linear utility, and abstract from the wealth effect that plays an important role in our policy experiments. Instead, these models have active labor demand side in the form of firms' vacancy posting. We abstract from the vacancy posting by firms to focus on the analysis of the labor supply side under incomplete markets, and in that sense, these studies are complementary to our work.

Second, the policies we consider have been extensively analyzed in the macroeconomic literature. For the taxes-and-transfers policy, examples include Prescott (2004), Ohanian et al. (2008), Alonso-Ortiz and Rogerson (2010), and Krusell et al. (2010). Compared to these studies, this paper is novel in that we explicitly consider life-cycle elements in a framework that features incomplete asset markets and labor market frictions. Incorporating life-cycle elements is important because patterns of transitions across labor-market stocks are markedly heterogeneous over the life cycle. Incorporating frictions enables us to talk about the effect of taxes and transfers on unemployment. The structure of incomplete asset markets with concave utility allows us to consider each consumer's asset-accumulation behavior and life-cycle behavior, particularly how the wealth effect operates, in a natural manner. An important interaction also exists between self-insurance and precautionary saving in that transfers can act as insurance against employment shocks. For this experiment, Ljungqvist and Sargent (2008) is closest to ours in this literature. Similarly to the current paper, they analyze a general equilibrium incomplete-market life-cycle model with indivisible labor and search decision. The largest difference is that they do not explicitly analyze gross worker flows. A recent paper by Pizzo (2020) analyzes the effect of progressive taxation in Krusell et al.'s

(2010) framework, while abstracting from the labor supply margin.

A large literature exists on the analysis of UI policy under incomplete markets. Alvarez and Veracierto (1999) employ a similar market structure as ours, that is, an "island" search model with workers' search decisions and competitive factor markets. Similarly to us, they focus on the workers' search decision by abstracting from firms' search. They do not explicitly consider the gross worker flows and life cycle. A complementary literature focus on the firms' search (vacancy posting) decisions, employing Diamond-Mortensen-Pissarides search and matching model. Examples include Krusell et al. (2010), Mukoyama (2013), Mitman and Rabinovich (2015), Jung and Kuester (2015), Landais et al. (2018), and Setty and Yedid-Levi (2021). These papers do not consider the participation decision by workers. Their models also abstract from life-cycle considerations.

Third, an extensive macroeconomic literature that considers life-cycle labor supply exists. Examples include Rogerson and Wallenius (2009), Low et al. (2010), and Erosa et al. (2016). These studies do not explicitly match the patterns of gross worker flows observed in the data. The explicit analysis of gross worker flows allows us to relate the effect of the policy on stocks to the patterns of reallocation in an economy with heterogeneous agents.

Finally, from a modeling perspective, our model has the Bewley-Huggett-Aiyagari (BHA) structure with a frictional labor market and operative labor supply with indivisible labor. Thus, our model shares many features with Chang and Kim (2006). Compared to Chang and Kim (2006), our model incorporates worker life cycle and frictional labor market.

One significant advantage of employing the BHA framework is that we can explicitly incorporate the precautionary wealth holding. Our model outcome fits the life-cycle pattern of wealth holding in the US data reasonably well. Explicitly analyzing the individual wealth holding is important for three reasons. First, the wealth effect is an important determinant of the individual labor supply. For example, Cesarini et al. (2017) document that lottery winners reduce their labor supply immediately and persistently. Second, as Krusell et al. (2017) show, the labor market flows are closely associated with the individual wealth level. Third, to evaluate the effect of policies like UI, it is essential to explicitly consider the degree of self-insurance. A cost of employing the BHA structure is the model complexity. Incorporating labor market frictions in a BHA model is extremely challenging. As Krusell et al. (2010) and Mukoyama (2013) show, incorporating the DMP-style labor demand side can make the model significantly complex even without life cycle and the participation margin. Some studies, such as Griffy (2021), achieve simplifications by adopting directed search assumption, but in our knowledge, no existing papers have successfully incorporated all elements (gross worker flows across three states with operative participation margin, life cycle, and incomplete asset markets) in a tractable general equilibrium model. We have decided to focus on the labor supply response on policies, given that the labor demand side is already extensively analyzed in the complementary literature (cited above) using the DMP structure,

The paper is organized as follows. The next section summarizes the empirical patterns of the gross worker flows over the life cycle. Section 3 sets up the model, and we calibrate the model in Section 4. Section 5 conducts policy experiments. Section 6 concludes.

2 Empirical observations

In this section, we briefly summarize the life-cycle patterns of worker flows and stocks in the US data. The patterns we observe in the data will be used for quantifying the model in the next section.

2.1 Data

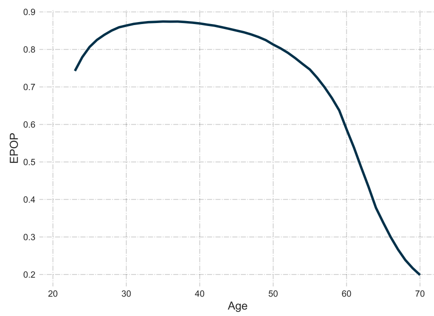
We use the monthly files of the Current Population Survey (CPS) from 1994 to 2017. Because our preliminary analysis found notable differences in labor market flows between men and women (likely related to decisions to stay at home and take care of children, which are more common for women and are beyond the scope of our model analysis), we decided to limit the sample to the population of men. This sample selection, of course, does not mean incorporating women's labor-supply behavior is not important—the analysis of this paper should be viewed as merely a first step. In Appendix C, we calibrate our model using data from all workers and repeat the tax-and-transfer exercise.

To calculate transition rates between different labor market states, we longitudinally match observations over two consecutive months by using data on household and person identification variables, as well as data on sex, race, and age, as is standard in the literature. Additionally, we correct for transitions that are plausibly spurious by using the deNUNifying procedure (purging the temporary appearance of U state by, for example, replacing N-U-N with N-N-N) as described in Elsby et al. (2015). Life-cycle profiles are obtained by estimating weighted OLS regressions of each labor market stock and flow on a set of age dummies.

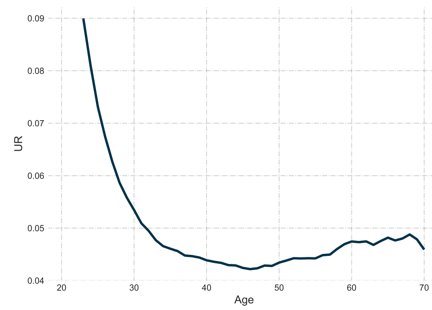
2.2 Labor market stocks

First, we describe the life-cycle patterns of stocks in the labor market. In this study, we focus on male workers ages 23 to 70. All the data figures are means of six-year-moving windows, and the horizontal-axis labels are the mid-points of the windows.[2] Figure 1 plots the age profile of employment. The employment-to-population ratio exhibits an inverted-U shape: smaller fractions of young and old workers are employed than middle-aged workers. As we can see from the comparison between panel (a) and panel (c), this pattern of employment mostly mimics the pattern of labor-force participation. The unemployment rate also exhibits a strong life-cycle pattern, although the pattern is markedly different from the one associated with labor-force participation. Young workers below 30 years old experience significantly higher unemployment rate than other age groups, while the unemployment rate is slightly

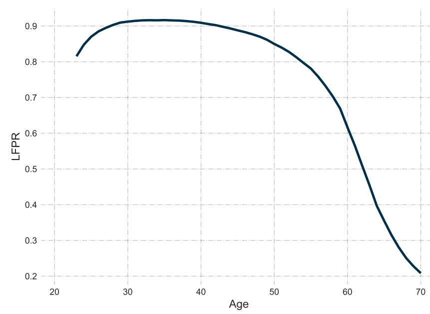
[2]We calculate our data moments from age 16 onward, take the rolling means, but report only age 23 and above, which is the age group we focus on.



(a) Employment-to-population ratio



(b) Unemployment rate



(c) Labor-force participation rate

Figure 1: Labor market ratios in the data

increasing past the age 40. This pattern of unemployment also contributes non-trivially to the low employment-to-population ratio, especially for young workers.

2.3 Labor market flows

The main innovation of this paper is to provide a model analysis for gross worker flows. The patterns in the data have previously been described by Choi et al. (2015), for example; thus, our summary here will be brief.[3] Figure 2 plots the monthly gross worker flow rates over the life cycle. The notations are conventional: with E for employment, U for unemployment, and N for nonparticipation, the flow rate ij represents the movement of the worker from state i to state j. The EE flow rate represents the job-to-job transition rate. The flow rate ij is computed by dividing the number of workers who moved from state i to state j between time t to time $t+1$, divided by the stock of the state i at time t.

All gross flow rates have strong life-cycle patterns. Overall, young workers tend to have higher mobility across states (and across jobs) compared to other age groups, although the very old workers have a strong tendency to move into the N state, likely because of their retirement.

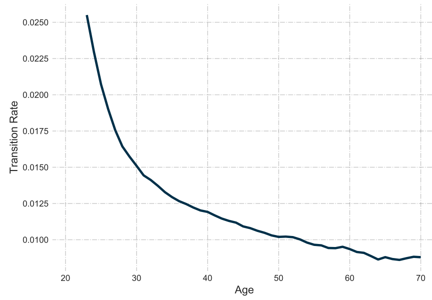
By comparing the patterns of gross flow rates with the stocks in the previous section, Figure 2 shows the large inflows into N (panels (b) and (d)) for the young and very old contribute to the inverted-U pattern of the labor-force participation rate, although the outflow rates (panels (e) and (f)) have offsetting effects for young workers. For the unemployment stock, the high inflow rates from E and N (panels (a) and (f)) contribute to high unemployment rates of young workers, although the outflow rates (panels (c) and (d)) have offsetting effects. Thus, overall, to explain the patterns of labor-force participation, accounting for the particularly strong life-cycle pattern of the inflow into N is important. For the unemployment rate, the large flow into U is the key to understanding the high unemployment rate of young workers. Because the employment-population ratio can be represented as

$$\frac{E}{E+U+N} = (1-u)p,$$

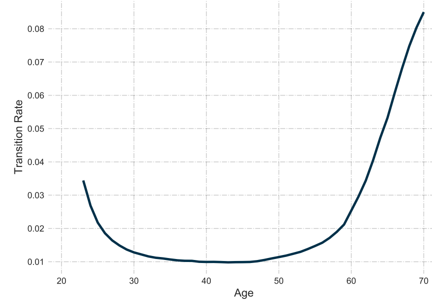
where $u \equiv U/(E+U)$ is the unemployment rate and $p \equiv (E+U)/(E+U+N)$ is the labor force participation rate, analyzing the life-cycle behavior of employment-population ratio requires explicit analysis of gross flows involving both U and N.

The behavior of flows and stocks in the steady state does not necessarily directly speak to their reactions to the policies. However, they provide an important guideline to construct and quantify the relevant model. In the next section, we construct a model that contains all relevant elements and is sufficiently flexible to match the data patterns. The results of the

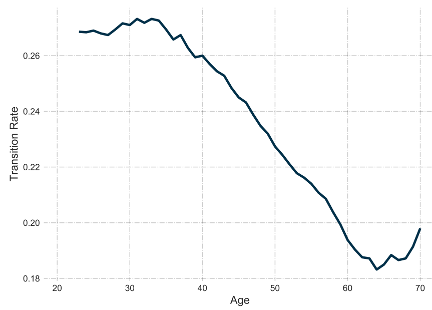
[3] Although Choi et al. (2015) use data from 1976 to 2013, our empirical patterns are essentially identical to theirs. One difference is that our NU and UN flow rates have a touch lower levels due to the deNUNifying procedure, but lifecycle patterns are nevertheless very similar.



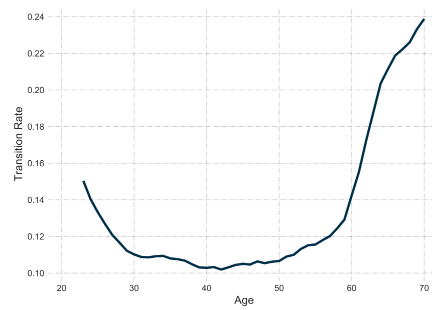
(a) EU



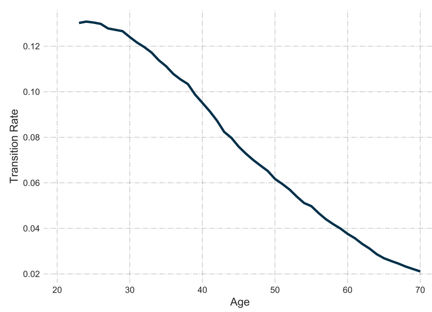
(b) EN



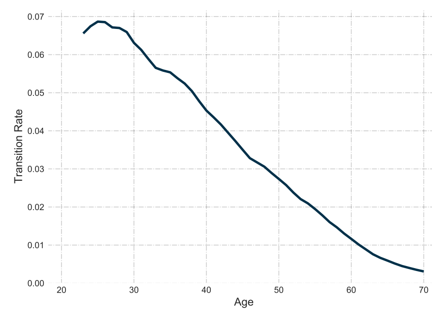
(c) UE



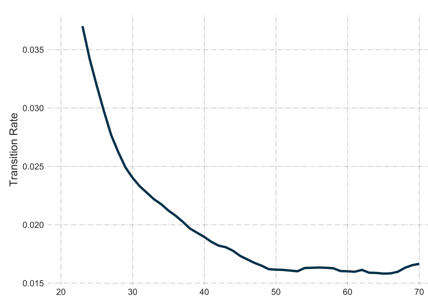
(d) UN



(e) NE



(f) NU



(g) EE

Figure 2: Gross flow rates in the data

policy experiments that come out of the model analysis are credible in the sense that the model itself is consistent with the life-cycle patterns we observe in the data.

A separate, yet interesting question is to investigate the reasons that young workers' flow rates are in general so high. The model in the next section, which incorporates both individuals' voluntary movements across states (as a reaction to changes in productivity and wealth) and labor market frictions, has a potential to provide insights for the origins of young workers' high mobility.

3 Model

Our model extends Krusell et al. (2010, 2011, 2017) to a life-cycle setting. In addition to worker life cycle, the model features a frictional labor market with heterogeneous jobs and operative labor-supply margin with concave utility and self-insurance. Thus, the model has the BHA structure with labor market frictions and operative labor supply. An attractive feature of this type of model is that the individuals in the model behave consistently with the permanent-income hypothesis, which has been extensively studied in the consumption-saving literature. Krusell et al. (2011, 2017) have already shown (the infinite-horizon version of) the model is consistent with overall behavior of the gross flows in the economy, including the duration of each state, flow rates for wealth quintiles, and business-cycle properties. The details of the model computation are presented in Appendix B.

Similar to Krusell et al. (2010), the model features a general equilibrium in that the prices depend on the aggregate capital (which the workers accumulate) and the aggregate labor. One important caveat (shared by Krusell et al. (2010, 2011, 2017)) is that the labor market frictions are exogenous and assumed to be policy invariant. This modeling decision reflects our focus on the labor-supply margin in the policy experiments.

3.1 Overall model structure

Three types of agents—workers, firms, and the government—exist in the economy. The workers supply labor and rent capital out to the firms. The total worker population is normalized to 1. Using capital and labor, firms produce the final good that can be used for consumption and investment. The government taxes labor and transfers taxes back to all workers in a lump-sum manner. All markets are perfectly competitive. Both the rental market for capital and the final-good market are frictionless as in the standard BHA model. As in the BHA model, the financial market is incomplete. The workers can self-insure by accumulating capital stock.

In the labor market, the worker's labor supply is indivisible in the sense that she can supply either zero or one unit of labor each period. The labor market is frictional. For the frictional labor market to be compatible with perfect competition, we consider the following arrangement, similar to Krusell et al. (2017).

The economy has two islands, *work island* and *leisure island*. All firms are located in the work island. All workers in the work island are employed by firms and receive wages. The work island is divided into many (continuum of) *districts*, and each worker who lives in the district works for one of the firms that are located in that district. The total measure of districts is normalized to 1.

Each worker's productivity has three components: the age component, the general productivity, and the match-specific productivity. The general productivity applies to the worker when working with any firm, whereas the match-specific productivity applies when working in the firm that is located in that district. In other words, the match-specific productivity is specific to the district-worker match. Because many firms exist in the district, the wages are still determined competitively even though the match-specific component exists. All workers in the leisure island do not work.

The mobility of workers across islands is limited, and this lack of mobility is a source of the labor market frictions. Workers in the leisure island receive an opportunity to move to a randomly drawn district every period. The frequency of this job opportunity depends on the search effort of the worker; if the worker searches, in which case she is categorized as *unemployed*, she receives job opportunities more frequently than when she does not search, in which case she is categorized as *not in the labor force*. Within the work island, moving across different districts is limited; every period, an employed worker may receive an opportunity to move to a different district (an "outside job offer") with some probability. We assume that the worker doesn't move across firms within a district (therefore no job-to-job transitions occur within a district), given that, in equilibrium, the worker would receive an identical wage from any firm within the same district. With some probability, an employed worker receives a separation shock and is forced to move to the leisure island. Employed workers can voluntarily move to the leisure island anytime they want to.

Note that the labor market structure with similar spatial frictions (the "island model") has a long tradition following Lucas and Prescott (1974). In contrast to an alternative modeling strategy, following Diamond-Mortensen-Pissarides framework (Pissarides, 1985), the model abstracts from the firms' vacancy-posting activity. The island model is especially suitable for the analysis of policies where the labor supply margin is operative. Therefore, later we demonstrate the usefulness of the model using two policies where labor supply margin is known to be important.

3.2 Workers

A worker is characterized by (i) her labor market state: employed (has a job), unemployed (not employed but actively searching for a job), not in the labor force (not employed and not searching for a job), (ii) her wealth (in capital stock), a, (iii) her idiosyncratic general productivity, z, (iv) her match-specific productivity (if employed), μ, and (v) her age, j. Let

s_j be the survival probability of a worker from age j to $j+1$. Then each worker maximizes:

$$\mathbf{U}_w = \sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log(c_j) - d_j],$$

where c_j is the consumption at age $j \in \{1, ..., J\}$ and d_j is the disutility of working or searching, which are detailed below. $E_0[\cdot]$ represents the expected value taken at age 0.

The log of idiosyncratic general productivity, $\log(z)$, is stochastic and follows an AR(1) process. The job-offer probabilities, which are age dependent, are denoted as $\lambda_u(j)$, $\lambda_n(j)$, and $\lambda_e(j)$ for unemployed, not in the labor force, and employed workers at age j. An unemployed worker incurs a search cost of ψ for active searching. An employed worker with general productivity z, match-specific productivity μ, and age j receives wage

$$\omega_j(\mu, z) \equiv g(j)\mu z \tilde{\omega},$$

where the function $g(j)$ is the deterministic age component of market productivity and $\tilde{\omega}$ is the wage per efficiency unit of labor. While working in a firm, $\log(\mu)$ follows an AR(1) process. At the end of a period, a match is destroyed with a probability, σ_j, depending on age of the worker. A worker in the leisure island receives b units of the final goods from home production.

Upon being matched, the worker draws the match-specific component of productivity μ. We assume the true quality of the match may not be immediately revealed with a probability ζ. In each period, if the match quality is unknown, it remains unknown with probability ζ. In that case, the value of μ is assumed to be $\bar{\mu}$. The wage worker receives is also based on $\bar{\mu}$, and therefore there is no learning from wages. With probability $1-\zeta$, the true quality is revealed. This gradual learning of match quality is necessary to make the job-to-job transition process in the model match the data. Without such a mechanism, young workers learn their match quality too quickly, and the job-to-job transition rate declines too rapidly with age. Similar formulations are used by Esteban-Pretel and Fujimoto (2014), Gorry (2016), and Menzio et al. (2016).

We assume the true match-quality shocks for the newly matched are drawn independently from a Pareto distribution with parameters (μ_1, α), where μ_1 is the lower bound of the support of the match-quality distribution, and α determines the rate at which the density of the distribution decreases (note M denotes the random variable and μ denotes its realization):

$$\Pr[M > \mu] = \begin{cases} \left(\dfrac{\mu_1}{\mu}\right)^{\alpha} & \text{for } \mu \geq \mu_1, \\ 1 & \text{for } \mu < \mu_1. \end{cases}$$

The new match quality for an employed worker who obtains an outside job offer is drawn from the same distribution.

The timing within a period is the following. First, idiosyncratic general productivity shocks and match-specific productivity shocks for already-employed workers realize. Second, some nonemployed workers find jobs, and the initial match-specific shocks for new jobs are drawn. Some employed workers receive an opportunity to move to another district, with a new match-specific shock realization. Third, nonemployed workers with job opportunities decide whether to accept the match, and employed workers with moving opportunities decide whether to move. Then, production and consumption take place. At the end of the period, a possible death and the separation shock occur.

Let the value function of an employed worker at age j be $W_j(a,z,\mu)$, the value function of an unemployed worker be $U_j(a,z)$, and the value function of a worker who is not in the labor force be $N_j(a,z)$.

The Bellman equation for the employed is:

$$
\begin{aligned}
W_j(a,z,\mu) = \max_{c_j,a'} \Big\{ u(c_j) - \psi\gamma + \beta s_j E_{\mu',z'}[\,&(1-\sigma_j)(1-\lambda_e(j))T_{j+1}(a',z',\mu') \\
&+(1-\sigma_j)\lambda_e(j)S_{j+1}(a',z',\mu') \\
&+\sigma_j(1-\lambda_e(j))O_{j+1}(a',z') \\
&+\sigma_j\lambda_e(j)F_{j+1}(a',z')] \Big\},
\end{aligned}
$$

subject to

$$c_j + a' = (1+r)a + (1-\tau)\omega_j(\mu,z) + \mathbf{T}$$

and

$$a' \geq 0,$$

where

$$T_{j+1}(a',z',\mu') = \max\{W_{j+1}(a',z',\mu'), O_{j+1}(a',z')\},$$

$$S_{j+1}(a',z',\mu') = \int_{\underline{\mu}}^{\bar{\mu}} \max\{T_{j+1}(a',z',\mu'), W_{j+1}(a',z',\hat{\mu})\} dG(\hat{\mu}),$$

$$O_{j+1}(a',z') = \max\{U_{j+1}(a',z'), N_{j+1}(a',z')\},$$

and

$$F_{j+1}(a',z') = \int_{\underline{\mu}}^{\bar{\mu}} \max\{W_{j+1}(a',z',\mu), O_{j+1}(a',z')\} dG(\mu).$$

Here, r is the real interest rate (rental rate of capital), τ is the labor income tax rate, and $\mathbf{T}$ is the lump-sum government transfer. Each employed worker faces four possible scenarios in the next period: (i) not receiving a separation shock (σ_j) or an outside job offer, in which case she needs to decide between continuing with employment or becoming nonemployed (the value function T), (ii) not receiving a separation shock, but receiving an outside job offer, in which case she additionally needs to decide whether to switch jobs (the value function S), where $G(\cdot)$ is the outside wage-offer distribution, (iii) receiving a separation shock and no outside offer, in which case she becomes nonemployed and needs to decide whether to search (the value function O), or (iv) receiving a separation shock and an outside job offer, in which case she can move directly to another firm (the value function F). While employed, a worker faces disutility of work equal to ψ times γ.

The Bellman equation for the unemployed is:

$$U_j(a,z) = \max_{a',c_j} \left\{ u(c_j) - \psi + \beta s_j E_{z'}[\lambda_u(j) F_{j+1}(a',z') + (1-\lambda_u(j)) O_{j+1}(a',z')] \right\},$$

subject to

$$c_j + a' = (1+r)a + b + \mathbf{T}$$

and

$$a' \geq 0,$$

where b is home production and ψ is the disutility of active search effort.

Those not in the labor force are not subject to the disutility of active search, but their job-offer probability will be different (lower), as explained later:

$$N_j(a,z) = \max_{a',c_j} \left\{ u(c_j) + \beta s_j E_{z'}[\lambda_n(j) F_{j+1}(a',z') + (1-\lambda_n(j)) O_{j+1}(a',z')] \right\}.$$

subject to

$$c_j + a' = (1+r)a + b + \mathbf{T}$$

and

$$a' \geq 0.$$

3.3 Firms

In each district k of the work island, competitive firms with a constant-returns-to-scale production function operate. The production function for the representative firm in district k takes the Cobb-Douglas form,

$$Y_k = A K_k^{\theta} L_k^{1-\theta},$$

where $\theta \in (0,1)$, and A is productivity. The inputs K_k and L_k are the capital and labor (in efficiency units) demands. Capital is freely mobile across districts, although labor mobility is restricted. Total capital and labor demand in the economy are

$$K = \int_0^1 K_k dk$$

and

$$L = \int_0^1 L_k dk.$$

We assume that capital is freely mobile across districts. Thus the rental rate under competitive market,

$$r = A\theta \left(\frac{K_k}{L_k}\right)^{\theta-1},$$

is common across districts. This equalization implies that the capital-labor ratio, K_k/L_k, is common across districts. Therefore, the wage per efficiency unit of labor,

$$\tilde{\omega} = A(1-\theta)\left(\frac{K_k}{L_k}\right)^{\theta},$$

is also equalized. We assume firms within a district are homogeneous and the allocation of workers to the districts is entirely random. With the law of large numbers, each district's L_k becomes the same in a stationary equilibrium. Therefore,

$$K = K_k$$

and

$$L = L_k$$

hold in a stationary equilibrium. Capital stock depreciates at a rate δ.

3.4 Government

The government collects tax on labor income and also confiscates assets of the deceased individuals in the economy. It redistributes all revenue to individuals in the economy uniformly while running a balanced budget. Thus, the government budget constraint is

$$\mathbf{T} = \tau \int e(i)\omega_{j(i)}(i)(\mu(i), z(i))di + \int a(i)(1-s(i))di, \tag{1}$$

where i is the index for each individual. Here, $e(i)$ is the employment status of individual i with 1 for employed and 0 for not employed, and $s(i)$ is the survival status of individual i, 1 for surviving individuals, and 0 for deceased individuals.

3.5 Equilibrium

We solve for a stationary equilibrium in which the real interest rate and wage profile are constant over time. After all new matching opportunities realize (with new idiosyncratic productivity and match-specific shocks), workers make the following decisions.

(i) A nonemployed worker at age j, wealth a, idiosyncratic productivity z, and who has an offer of match-specific productivity μ accepts the offer and becomes employed if and only if

$$W_j(a, z, \mu) \geq O_j(a, z).$$

(ii) A nonemployed worker at age j, wealth a, and idiosyncratic productivity z who rejected a job offer or did not receive a job offer decides to be in the labor force if and only if

$$U_j(a, z) \geq N_j(a, z).$$

(iii) An employed worker at age j, wealth a, idiosyncratic productvity z, and current match-specific productivity μ, who does not have an outside job offer stays in her job if only if

$$W_j(a, z, \mu) \geq O_j(a, z).$$

(iv) An employed worker at age j, wealth a, idiosyncratic productvity z, current match-specific productivity μ, and outside offer μ' switches jobs if and only if

$$W_j(a, z, \mu') > T_j(a, z, \mu).$$

(v) Each worker makes optimal consumption and investment decisions according to the Bellman equations described in Section 3.2.

Capital and labor markets clear.

(i) Total assets supplied are equal to total capital demand,

$$\int a_i di = K.$$

(ii) Labor supply in efficiency units is equal to labor demand,

$$\int e(i) z_i \mu_i g_i di = L.$$

As described in Section 3.4, the government runs a balanced budget, represented by the constraint (1): the total lump-sum transfer is equal to the sum of labor income tax revenue and wealth of the deceased agents.

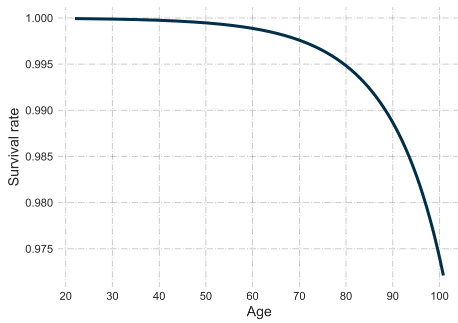
4 Calibration

In quantifying the model, first, a subset of parameters are calibrated using external information. Then the remaining parameter values are estimated so that the distance between the model outcome and the data are minimized.

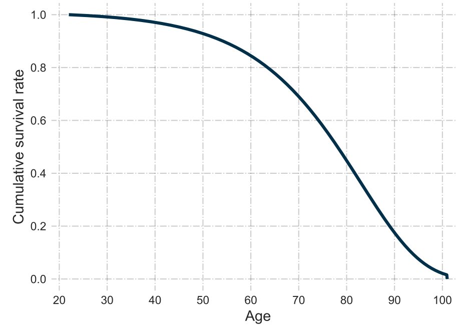
Each period corresponds to one month.[4] Following Krusell et al. (2010), we consider $\tau = 0.30$ as the benchmark. On the production side, θ is set at 0.3. The death probabilities at each age are taken from life tables at the Social Security Administration.[5] The calibrated survival rates are plotted in Figure 3. The relative disutility of working compared to search,

[4] We assume model age $j = 1$ corresponds to an annual age of 22. The monthly age after which everyone dies for sure is $J = 947$, which corresponds to an annual age of (one month before) 101. However, during calibration we only consider workers aged between 23 and 70.

[5] Our calibrated survival rate is given by the following function: $s_j = (1-(0.000149 \exp(0.0751((j-1)/12 + 22)))^{1/12}$ for j in $1, 2, ...946$, and $s_j = 0$ for $j > 946$. Essentially, workers at age 947 die for sure.



(a) Survival rate



(b) Cumulative survival rate

Figure 3: Survival rate

γ, is set to $\gamma = 40/3.5$, which corresponds to the ratio of the average hours worked by the workers to the average hours the unemployed actively search for a job, taken from Mukoyama et al. (2018). The persistence parameter of the monthly AR(1) idiosyncratic productivity (z) process is set to $\rho_z = 0.97$ and the persistence parameter of the monthly AR(1) match-specific productivity (μ) process is set to $\rho_\mu = 0.98$. We assume match-specific productivity of matches with unrevealed quality is equal to median productivity, $\bar{\mu} = 1.0$.

The interest rate, r, is targeted to be equal to 0.00327 in equilibrium, which corresponds to a 4% annual compound interest rate. A is set to 0.49 to normalize $\tilde{\omega}$ to 1 in equilibrium. Investment to GDP ratio is targeted to be equal to 20%.

For age-dependent parameters, we allow them to be a simple function of age. Specifically, let the age component of market productivity, $g(j)$, the logarithms[6] of job-offer arrival rates, $\log \lambda_e(j)$, $\log \lambda_u(j)$, $\log \lambda_n(j)$, and the logarithm of exogenous job separation rate be characterized as second-degree polynomials of age, j:

$$\lambda_e(j) = \exp(\lambda_{e,2} j^2 + \lambda_{e,1} j + \lambda_{e,0}),$$

$$\lambda_u(j) = \exp(\lambda_{u,2} j^2 + \lambda_{u,1} j + \lambda_{u,0}),$$

$$\lambda_n(j) = \exp(\lambda_{n,2} j^2 + \lambda_{n,1} j + \lambda_{n,0}),$$

$$\sigma(j) = \exp(\sigma_2 j^2 + \sigma_1 j + \sigma_0),$$

and

$$g(j) = g_2 j^2 + g_1 j + g_0.$$

The remaining parameters that need to be calibrated are

$$\boldsymbol{\xi} \equiv \{\beta, \delta, \lambda_{e,2}, \lambda_{e,1}, \lambda_{e,0}, \lambda_{u,2}, \lambda_{u,1}, \lambda_{u,0}, \lambda_{n,2}, \lambda_{n,1}, \lambda_{n,0}, \sigma_2, \sigma_1, \sigma_0, g_2, g_1, g_0, \psi, \sigma_\mu, \sigma_z, b, \zeta, \alpha\},$$

[6]Throughout this text, when we write "logarithm" we mean natural logarithm.

where δ is the depreciation rate of the capital stock, σ_z and σ_μ are the standard deviations of AR(1) shocks of idiosyncratic productivity and match-specific productivity. To estimate these parameters, we minimize the sum of the squared log distance between (i) gross worker flows, average market wage, the interest rate target, and the investment to GDP ratio target and (ii) the corresponding moments from the model simulations. More precisely, for a given $\boldsymbol{\xi}$, we solve for the value functions and decision rules recursively and simulate the model according to the decision rules. To simulate the model, we need to make assumptions about the initial distribution of workers' state variables. We assume each worker begins life at the leisure island with no assets. Then, we calculate the monthly transition rates from one state to another state as follows, using the employment-to-unemployment transition (EU) as an example:

$$EU^{model}(\boldsymbol{\xi}, j) = \frac{\text{Measure of the employed at age } j \text{ moving to unemployment the next period}}{\text{Measure of the employed at age } j}.$$

Because our model is stationary and no aggregate shocks occur, we drop the time index. We calculate age-specific transition rates between labor market states by taking the mean of the monthly transition rates. Continuing with the EU transition as an example,

$$EU^{model}(\boldsymbol{\xi}, j^a) = \frac{1}{12} \sum_{j=12(j^a-22)+1}^{12(j^a-21)} EU^{model}(\boldsymbol{\xi}, j),$$

where j^a is the (annual) age. We assume model-age $j = 1$ corresponds to age 22 in the data. We calculate the monthly wage rate by taking the average of wages of same-aged workers. Then, we convert the monthly wage rate to the age-specific wage as above. However, we normalize the average wage at age 42 to 1:

$$\bar{\omega}(\boldsymbol{\xi}, j^a) = \frac{1}{12} \sum_{j=12(j^a-22)+1}^{12(j^a-21)} \omega^{model}(\boldsymbol{\xi}, j),$$

$$\omega^{model}(\boldsymbol{\xi}, j^a) = \frac{\bar{\omega}(\boldsymbol{\xi}, j^a)}{\bar{\omega}(\boldsymbol{\xi}, 42)}.$$

Let $X^{model}(\boldsymbol{\xi})$ be the collection of transition rates among worker states, normalized average wages, the interest rate, and investment to GDP ratio (x) for the workers (annual) aged between 23 and 70,[7]

$$\begin{aligned} X^{model}(\boldsymbol{\xi}) = \quad & \text{vec}\bigg(\Big\{EU^{model}(\boldsymbol{\xi}, j^a), EN^{model}(\boldsymbol{\xi}, j^a), EE^{model}(\boldsymbol{\xi}, j^a), \\ & NU^{model}(\boldsymbol{\xi}, j^a), NE^{model}(\boldsymbol{\xi}, j^a), UN^{model}(\boldsymbol{\xi}, j^a), \\ & UE^{model}(\boldsymbol{\xi}, j^a), \omega^{model}(\boldsymbol{\xi}, j^a)\Big\}_{j^a=23}^{70}, r(\boldsymbol{\xi}), x(\boldsymbol{\xi})\bigg), \end{aligned}$$

[7] Recall that our model starts with annual age of 22 but we only consider ages between 23 and 70 in the calibration.

and let X^{data} be the collection of the six-year rolling average of transition rates, normalized average wages[8] of males observed in the data, the interest rate target, and the investment to GDP ratio target for the workers (annual) aged between 23 and 70,

$$\begin{aligned} X^{data} = \quad & \mathrm{vec}\Bigg(\Big\{EU^{data}(j^a), EN^{data}(j^a), EE^{data}(j^a), \\ & NU^{data}(j^a), NE^{data}(j^a), UN^{data}(j^a), \\ & UE^{data}(j^a), \omega^{data}(j^a)\Big\}_{j^a=23}^{70}, r, x\Bigg). \end{aligned}$$

In the numerical solution of the model, we discretize idiosyncratic and match-quality AR(1) processes using the Tauchen method. In our calibration exercise, we minimize the sum of the squared log distance between $X^{model}(\boldsymbol{\xi})$ and X^{data} by choosing (ξ) :

$$\min_{\boldsymbol{\xi}} |(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})|' W |(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})|,$$

where W is a diagonal weighting matrix. We set all but the last two elements of the diagonal of W to 1, and the last two elements are set to 100. We chose this specification to put higher weights on age-independent moments.

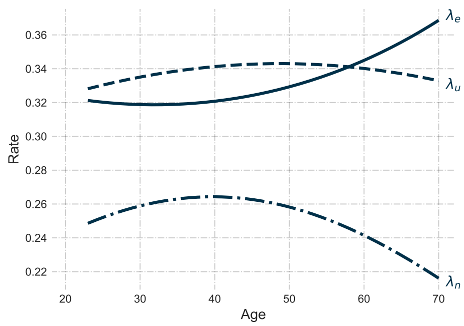
The calibrated parameters that do not have an age component are shown in Table 1. The estimated coefficients are in Appendix A. Figure 4 visualizes the calibrated outcome in graphs. As we discussed in the introduction, these results are of independent interest—they reveal the fundamental forces that shapes the life-cycle patterns of gross worker flow. In particular, these results uncover the nature of frictions for different age groups of individuals, which we cannot observe directly in the data.

Panel (a) of Figure 4 shows job-offer arrival rates over the life cycle for the unemployed (dashed line), the employed (solid line), and the nonparticipant (dot-dash line). For nonemployed workers, job-offer arrival rates increase until they reach prime age, and then decreases. The decrease in the job-offer arrival rate is sharper for the non-participant. As expected, the job-offer arrival rate for the unemployed is greater than that of the non-participant, highlighting the active-job-search trade-off: active job search is costly but results in a higher probability of receiving an offer.

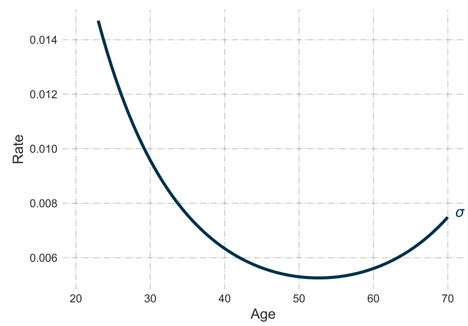
The job-offer arrival rate for the employed has three notable features. First, the overall level of λ_e is similar to λ_u, despite that the corresponding flows (EE and UE flows) have significantly different levels. This result is reminiscent of Tobin's (1972) argument that no evidence exists that employed workers are less efficient in job search than nonemployed workers.[9] In fact, employed workers appear to be more efficient in search than nonemployed

[8] Average wages at each age is expressed relative to average wage of 42-year-old male workers.

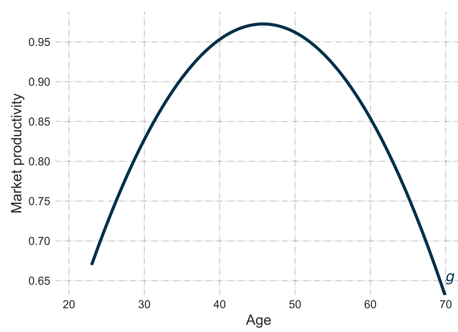
[9] Mukoyama (2014) reports a similar outcome with a simple job-ladder model when the separation rate strongly depends on match quality.



(a) Job offer arrival rate



(b) Exogenous job separation rate



(c) Age component of market productivity

Figure 4: Age dependent parameters

Table 1: Age independent parameters

Parameter	Definition	Value
β	Discount factor	0.997
θ	Elasticity of output w.r.t. capital	0.3
δ	Depreciation rate	0.0088
ψ	Disutility of active job search	0.045
A	Total factor productivity	0.490
ρ_μ	Persistence parameter of monthly AR(1) match-specific productivity	0.98
σ_μ	Std. dev. of innovations in match specific productivity	0.107
ρ_z	Persistence parameter of monthly AR(1) idiosyncratic productivity	0.97
σ_z	Std. dev. of innovations in idiosyncratic productivity	0.091
b	Home productivity	0.132
ζ	Unknown match quality probability	0.263
α	Shape parameter of Pareto distribution (new job wage offers)	7.235
$\bar{\mu}$	Match quality for unrevealed matches	1.0
γ	Disutility of work over disutility of active job search	11.4
J	Monthly age at which everyone dies	947

workers when they are old. This finding, of course, is consistent with the fact that job-to-job transitions are less frequent than UE transitions, because the employed workers tend to be choosier because of their outside options. Second, unlike λ_u and λ_n, λ_e exhibits an increasing pattern after the 40s, after remaining flat during the younger years. This pattern could, for example, reflect that employed workers can build a better network as they become older. Although our model is too stylized to investigate this point further, it seems to be an interesting hypothesis for future inquiry. Third, the overall life-cycle profile of λ_e, λ_u, λ_n are relatively flat, compared to the corresponding flows (EE, UE, and NE flows). The difference, of course, comes from the fact that workers choose whether to accept the job, and the "choosiness" depends on the stage in the life cycle. This contrast highlights the importance of analyzing an *economic model* as opposed to an *accounting model*.

Panel (b) of Figure 4 shows that exogenous job separation decreases over the life cycle of an individual with a slight increase after age 60. The age component of market productivity in Panel (c) displays an inverse-U shape. Market productivity increases until middle age and then decrease toward the end of an agent's working life. This pattern is largely consistent with the results from direct measurements from microeconomic data, widely used in quantitative public finance literature.[10]

[10] For example, Conesa et al. (2009) use the measurement from Hansen (1993).

Several results from Figure 4 are surprising when compared with the actual worker flows. First, despite the strong life-cycle pattern of UE flows, the job-offer probability of unemployed individuals, λ_u, is almost flat over the life cycle. Second, although the EE flow rates decline over the life cycle, the offer probability, λ_e, increases over the age. These results, once again, caution against identifying the patterns of actual worker flows with the patterns of the opportunities that workers face.

Comparing panels (a) and (b), one can conclude the large U stock for young workers is mostly the result of a large separation shock σ. Investigating why σ exhibits such a pattern is beyond the scope of this paper, but it is an important future research topic. In the context of Mortensen and Pissarides's (1994) model, one can interpret the σ shock as an event where the job-worker match suffers from a large negative productivity shock. The matches involving young workers, not having as much information on the strengths and weaknesses of the individuals, may be subject to these shocks more frequently.

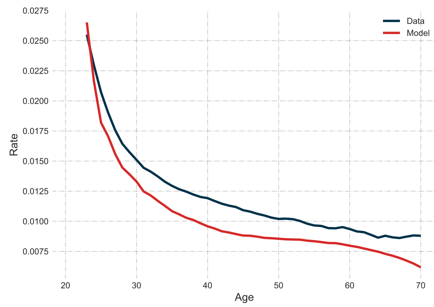
Our model outcomes against the targeted data moments are plotted on Figure 5. We are able to match qualitative features of the flows rates by age quite well. For some flow rates, EU, EN, and EE, we are able to match the entire life-cycle dynamics almost perfectly. We would like to emphasize the challenge of obtaining such a good fit. The model is quite parsimonious and most of the assumptions are standard in the life-cycle literature. However, the computational burden is quite high and the model has to fit six gross flows (plus the job-to-job flow and the wage profile) as functions of age. Fitting six *functions* is substantially more difficult than fitting six *numbers* that Krusell et al. (2011, 2017) achieve. No previous papers have accomplished such good fit in a model where all flows are endogenous, and we view that finding a framework that fits these life-cycle patterns as one of the important contributions of this paper.

5 Policy experiments

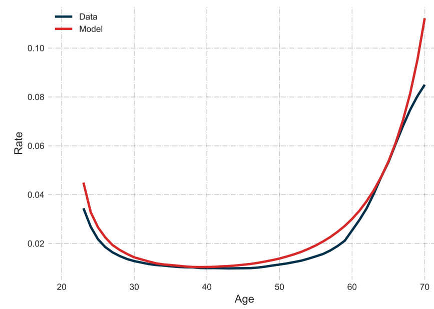
In this section, we utilize the above framework to conduct policy experiments. We examine two different policies. The first is the taxes-and-transfers policy. Given that our baseline model highlights the role of labor supply margin, this model suits the analysis of the policy that directly affects the labor supply incentives. The second policy is the UI policy. UI policy affects both the job search incentive and the incentives for taking up a new job. Our model features these two choices as important determinants of the gross worker flows.

5.1 Taxes and transfers

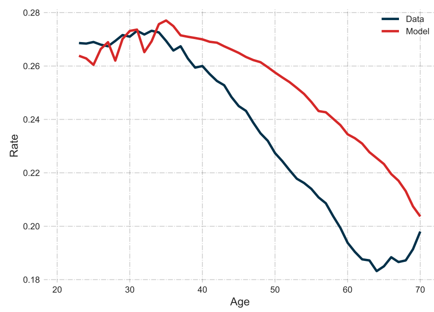
First, we examine the effect of an increase in labor tax. In his influential work, Prescott (2004) argues the difference in total hours between the US and the continental Europe can largely be explained by the difference in tax system. Although various studies have followed up on Prescott's (2004) study, none has explicitly analyzed a model with gross worker flows



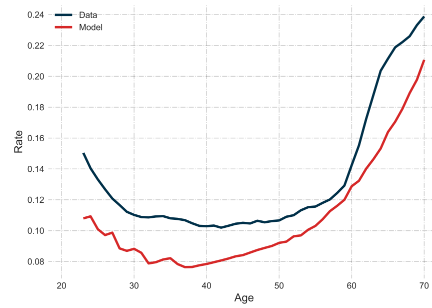
(a) EU



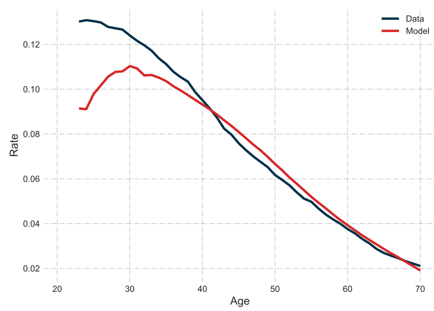
(b) EN



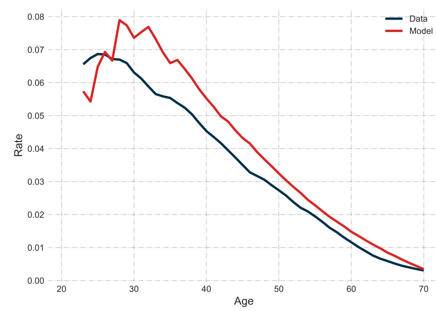
(c) UE



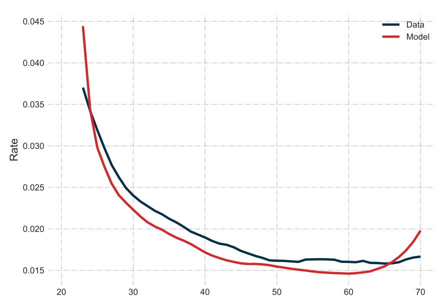
(d) UN



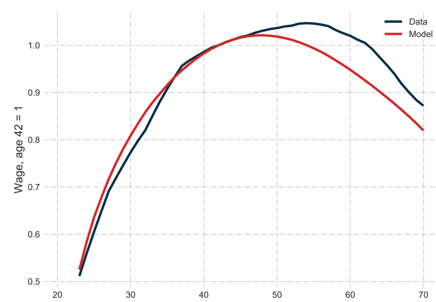
(e) NE



(f) NU



(g) EE



(h) Wage

Figure 5: Model moments and calibration targets

Table 2: Aggregate statistics from the experiment

Tax	N	E	U	u	$lfpr$	Labor (L)	Efficiency	K/L	Welfare Gain
0.30	0.349	0.619	0.033	0.050	0.651	0.616	0.995	35.498	N/A
0.45	0.516	0.461	0.024	0.049	0.484	0.488	1.060	34.293	-8.1%

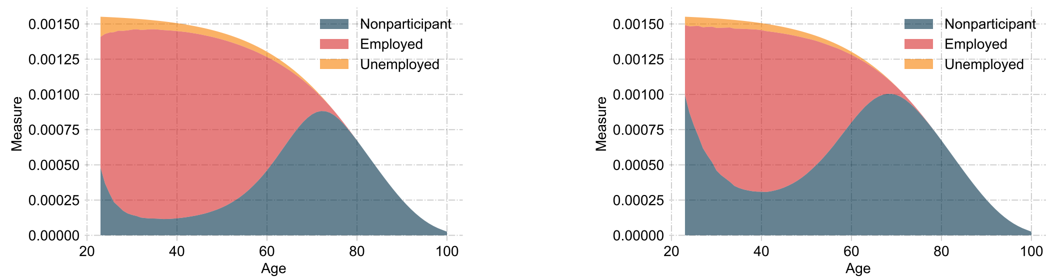
in a life-cycle economy. Our model reveals two novel effects of the tax and transfer: the effects on reallocation (worker flows) over the life cycle and the decomposition of effects on nonemployment into the one on unemployment and the one on nonparticipation.

Following Krusell et al. (2010), we consider an experiment of raising the labor tax rate τ from 0.30 to 0.45. Table 2 summarizes the results at the aggregate level, where *Efficiency* is defined as the labor in efficiency units (L) over the number of employed workers (E):

$$\text{Efficiency} = \frac{L}{E},$$

representing the average productivity of employed workers. The magnitude of decline in aggregate employment is somewhat stronger than the infinite-horizon economy in Krusell et al. (2010); here, E declines by $0.461/0.619 = 0.74$, whereas in Krusell et al. (2010), the corresponding value is $0.488/0.633 = 0.77$. One factor that increases the impact of tax in the life-cycle economy is the heterogeneity of responses across different age groups. Figure 6 draws the composition of the labor market states at each age, for both the benchmark (30% tax) and the experiment (45% tax). Although employment clearly decreases and the nonparticipation increases in all ages, the decline in participation is particularly strong in young workers. Because young workers tend to be less productive than the prime-aged workers (see panel (c) of Figure 4), the changes in young workers' employment has less impact on the efficiency units, and thus on wages. Therefore, for the same change in total efficiency units of labor and in wages, the change in aggregate E appears larger when the impact is skewed to young workers.

The unemployment rate in Table 2 slightly declines with the tax increase. This finding is in contrast to the baseline case in Krusell et al. (2010), where the unemployment rate increases with a higher tax rate. Figure 7 compares the stocks in Figure 6 one by one. In panel (b), the unemployment stock for young workers declines dramatically. The rates comparable to Figure 1 for the data are plotted in Figure 8. Somewhat surprisingly, the life-cycle profile of the unemployment rate changes very little for all ages. Even for the very young workers, where the total U stock changes significantly in Figure 7, the change in the unemployment rate is relatively small because E also falls significantly. For middle-aged workers, the employment decline, driven by the participation margin, is larger than the unemployment decline and the unemployment rate increases. In total, the middle-aged workers' effect dominates, and the total unemployment rate increases. The heterogeneous



(a) Benchmark

(b) 45% tax

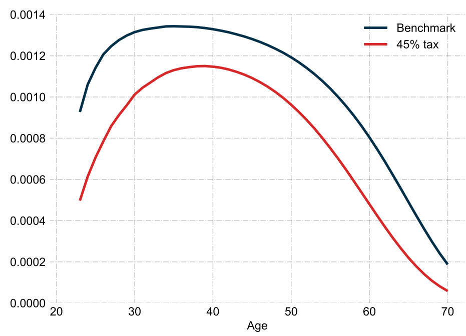
Figure 6: Composition of the labor market states over the lifecycle

responses across different age groups add a complexity in considering the aggregate outcome, compared to the infinite-horizon model of Krusell et al. (2010).

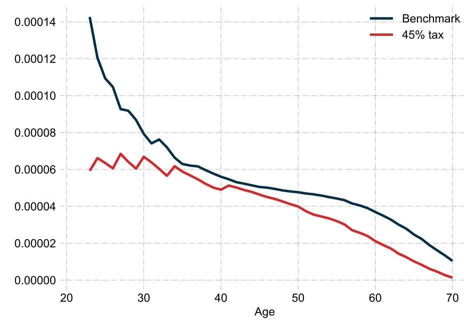
Table 2 also shows the welfare effect of the tax increase. The "Welfare Gain" entry measures by what percentage do we have to increase consumption (at each period and state) in the benchmark economy to make the worker indifferent to being born in the 45-percent-tax economy (see Appendix E). Increasing the tax rate to 45 percent reduces the newborn's present-value welfare by 8.1 percent. (That is, we have to decrease the consumption by 8.1 percent from the benchmark economy to make the worker indifferent to being born in the higher-tax economy.) Notice that, as shown in Table 2, the capital-labor ratio goes down after the tax increase. The level of capital stock K is even lower. Being born in an economy with low capital stock implies a lower future income for workers. This effect is one of the reasons that the welfare decline is relatively large.

Now we investigate the gross worker flows. Figure 9 draws each labor market transition rate for the benchmark and 45% tax case. First, we investigate the cause of the decrease in U stocks in Figure 7. Among the flows that involve the U state, two flows show a strong impact on young workers. The first is the EU flow. Because only high-productivity workers participate when the tax is high, the likelihood of moving from E to U when the match quality becomes worse is lower in a high-tax situation. The second is NU flow. Two (potential) reasons exist for moving from N to U: (i) running down assets (the wealth effect), and (ii) improvement of the idiosyncratic productivity. The reduction in labor income and the increase in the lump-sum transfer implies the individuals in the N state do not (have to) run down assets while nonemployed as quickly when the labor tax is high. In other words, the income is smoother across states, and thus reducing the precautionary saving (and precautionary work) motive for the individuals. The impact of a lump-sum transfer is larger for a young worker, who tends to have lower labor income and a lower level of assets. Thus, in explaining the decrease in U for young workers, (i) the selection of employed workers and (ii) the improved opportunities for consumption smoothing play important roles.

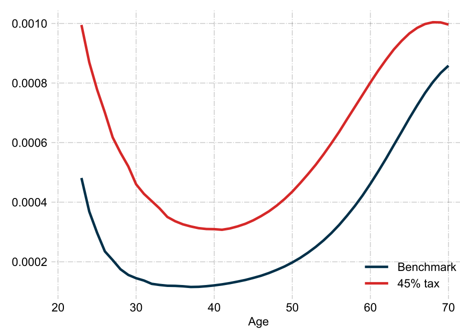
Second, in relation to labor-force participation rates, both NE and NU flow shift sub-



(a) Employed

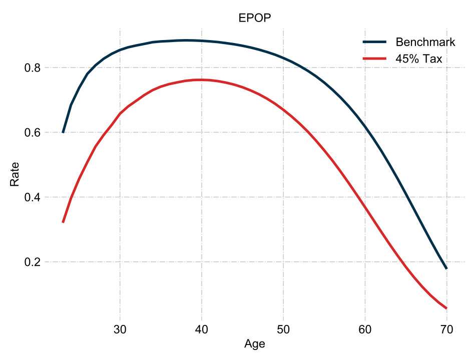


(b) Unemployed

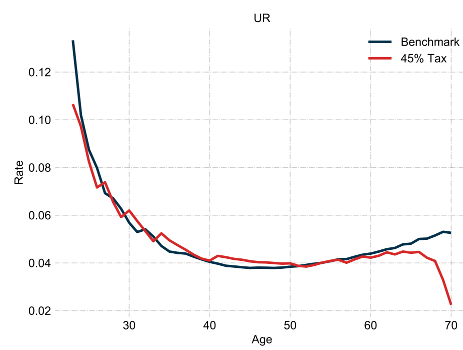


(c) Not in the labor force

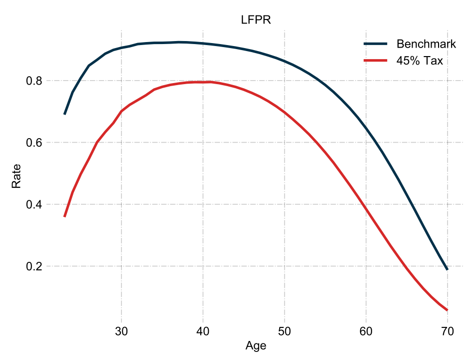
Figure 7: Labor market stocks after a tax hike



(a) Employment to population ratio

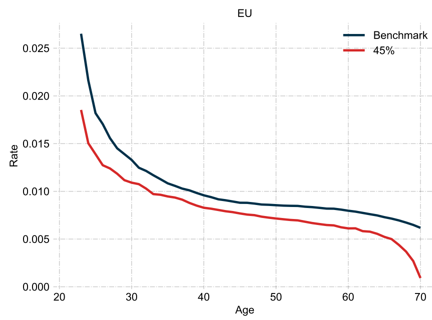


(b) Unemployment rate

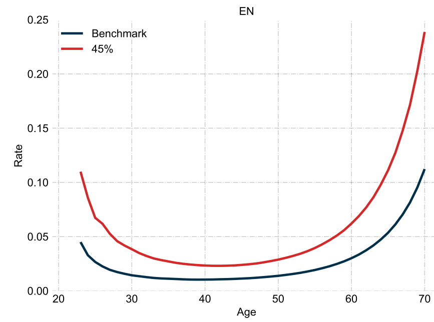


(c) Labor force participation rate

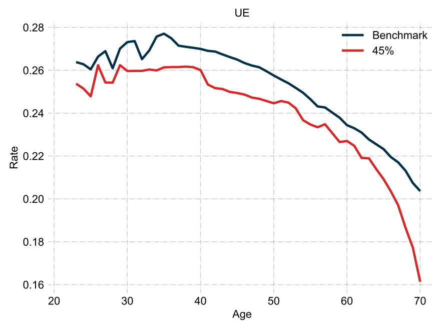
Figure 8: Labor market ratios after a tax hike



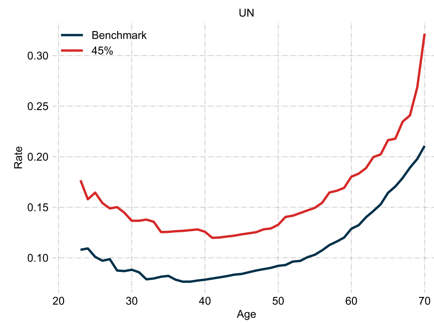
(a) EU



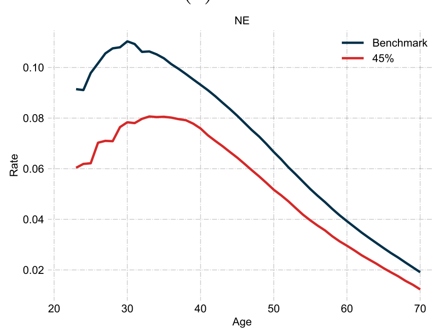
(b) EN



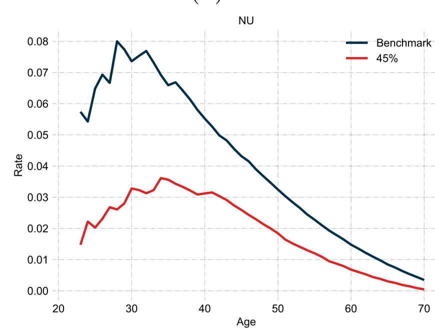
(c) UE



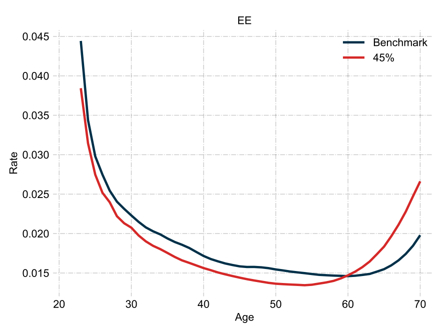
(d) UN



(e) NE



(f) NU



(g) EE

Figure 9: Gross worker flow rates after a tax hike

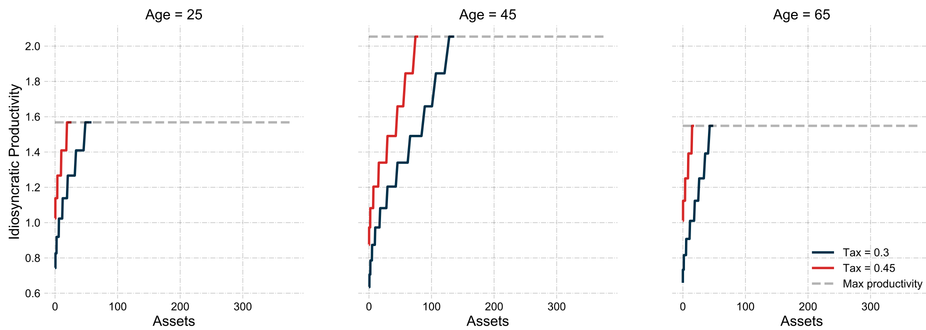
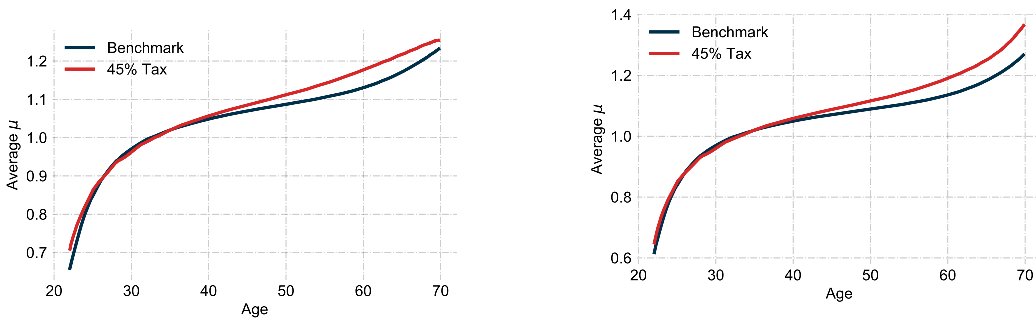


Figure 10: Age-adjusted idiosyncratic productivity cutoffs for labor force participation



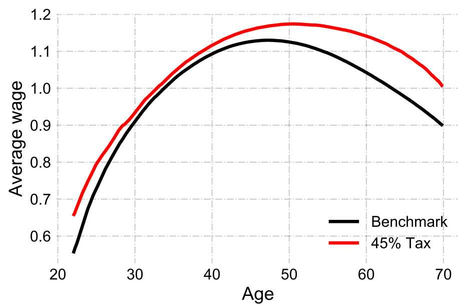
(a) All employed

(b) Employed with revealed match-specific productivity

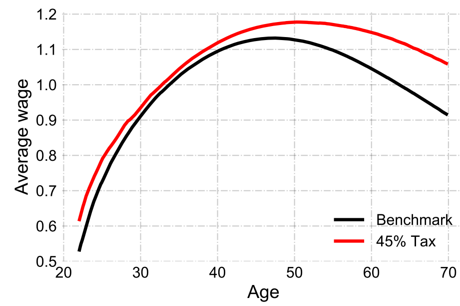
Figure 11: Mean match-specific productivity

stantially more for young workers. This finding is in contrast to the shifts of the opposite-direction flows, EN and UN, which are fairly uniform across all ages. Combined with the fact that the employment response is largely coming from the participation margin, we conclude that the outflow from non-participation is the key to generating the life-cycle pattern of the employment response to the taxes.

Analyzing more deeply at the micro level, Figure 10 plots the cutoff levels of the age-adjusted idiosyncratic productivity ($g(j) \times z$) for given assets. Above the cutoff level, a non-employed worker decides to participate in the labor market. Three panels for different ages (25, 45, and 65 years old) compare the cutoffs for the baseline ($\tau = 0.3$) and the experiment ($\tau = 0.45$). The amount of the shift of the cutoffs turns out to be not too different across different ages. Note that the aggregate responses are affected by the combinations of the change in the cutoffs and the location of the distributions of the state variables (in particular, the joint distributions of asset and productivity), as well as the change in distributions by the policy. Overall, younger workers exhibit more action in the aggregate participation margin, largely because they tend to have a lower level of wealth (where taxes have a larger impact) and more workers tend to be in the neighborhood of the cutoff lines.



(a) All employed



(b) Employed with revealed match-specific productivity

Figure 12: Mean wages

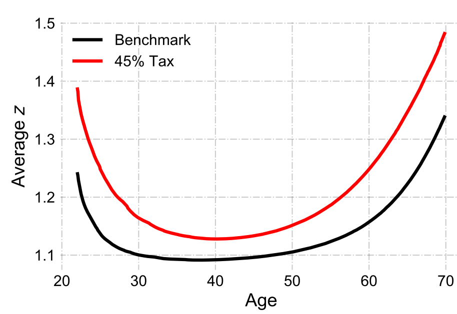


Figure 13: Mean idiosyncratic productivity

In concluding this experiment, we show three more consequences of the labor tax. First, Figure 11 plots the average values of μ for each age (including (a) and excluding (b) the "unrevealed" matches). As one would expect, the case with a higher tax exhibits a higher level of μ, because the workers are choosier. However, the difference becomes only visible after 40 years of age. One reason is that, with a higher tax, workers have fewer opportunities to climb the job ladder, as they stay employed for a shorter duration. Young workers spend more time in U and N states, and therefore, do not experience as many job-to-job transitions when the taxes are high. Even conditional on employment, the EE transitions are less frequent—see panel (g) of Figure 9. This type of reallocation effect is absent in past analyses, such as Prescott (2004), and highlights the importance of explicitly incorporating worker flows in the analysis of taxes and transfers.

Second, the wages before tax are plotted in Figure 12. Wages have three components (aside from the age component); μ, z, and $\tilde{\omega}$. The first two components increase, as seen in Figure 11 and in Figure 13 with the increase in labor tax. Both μ and z are higher on average, because of the selection. The base wage $\tilde{\omega}$ decreases because K/L is lower, which is expected. Workers save less because of a reduction in the precautionary saving motive. The overall impact of the tax hike on mean wages is positive. The before-tax wages go up more for old workers, and as a result, after tax wage of the old goes down less than that of the middle-aged workers. Younger workers (below 40 years old) do not experience a significant increase in before-tax wages. Therefore, from the (static) welfare standpoint, individuals in different age groups experience the effect of the tax increase very differently, even if these individuals are employed in both regimes.

Finally, Figure 14 plots the effect of taxes on wealth distribution across age groups and compares the distribution with wealth distribution in the data, tabulated by Kuhn et al. (2016).[11] In this figure, the average wealth of each age group relative to the average wealth of the entire population is plotted. Using the Survey of Consumer Finances (SCF),[12] Kuhn et al. (2016) provide the average wealth of different age groups in different years. Data in the figure is constructed by first calculating the relative average wealth in each year (see Figure 24 in Appendix D) and then taking the average over the years. As in the data, workers in the model gradually accumulate wealth until they come close to retirement and then decumulate capital. Overall the model matches the data very well, except for the very old age. This discrepancy is due to the model assumption that the individuals do not receive any utility from leaving bequests.

The tax increase in this exercise reduces the average asset holdings of every age group. However, the reduction in asset holdings is larger (in percentage terms) for the young and the old than the middle-aged; therefore, the relative average asset holdings of the middle-aged go up. Because the transfer payments are larger now, nonparticipants have less incentive to

[11] Data is updated using 2019 Survey of Consumer Finances and is accessed from https://sites.google.com/site/kuhnecon/home/us-inequality.

[12] See https://www.federalreserve.gov/econres/scfindex.htm for more information.

hold large wealth and old individuals can consume more and run down their wealth quickly. Appendix D presents other dimensions of the data and the model outcomes related to the wealth distribution.

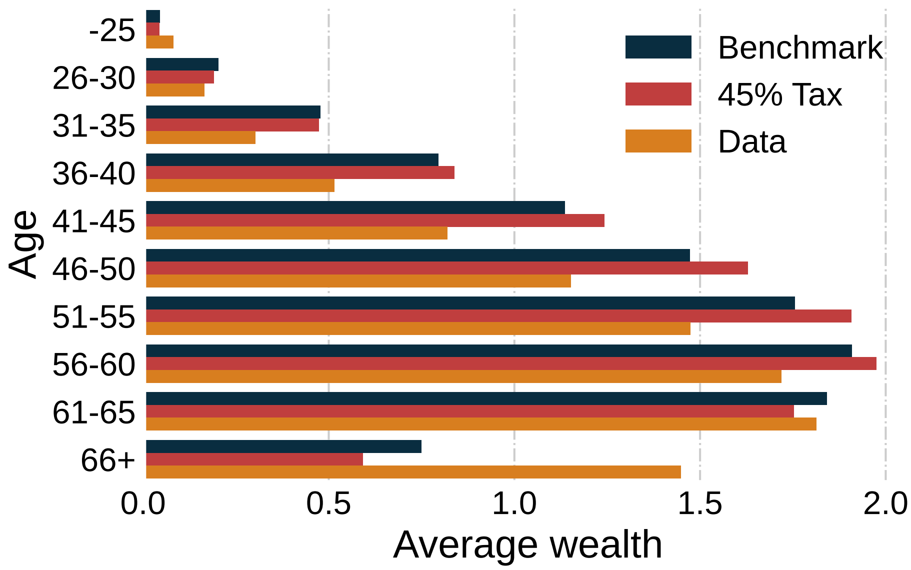


Figure 14: Average wealth from model and data.

Data: averages across years from Kuhn et al. (2016). Accessed from `https://sites.google.com/site/kuhnecon/home/us-inequality`. All values are relative to the average wealth in the entire population.

5.2 UI

In this section, we assume that the transfer is dependent on the worker's state. In addition to the lump-sum transfer described in Section 4, some unemployed workers receive an extra amount as UI. Those non-employed workers who lost their jobs due to an exogenous job separation shock, and are actively seeking a job receive a transfer proportional to their productivity for a limited duration. Therefore, the employed, nonparticipants, the unemployed who quit their jobs, and the unemployed whose benefits have expired cannot receive this benefit, which mimics the unemployment insurance payments in the US. Let $b(z, \mu, j) = b_0 \tilde{\omega} z \mu g(j)$ be the UI that an eligible unemployed worker aged-j receives, where z is the current idiosyncratic productivity of the worker, μ is the match-specific productivity the worker had in his last position, $g(j)$ is the market productivity, and b_0 is the unemployment replacement rate. Therefore, the payment that an eligible unemployed worker receives is proportional to the wage he would have gotten if he had kept his position. To account for the limited duration of unemployment benefits, we assume that an eligible worker loses his

Table 3: Aggregate statistics from the UI experiment

Tax	Replacement	N	E	U	u	$lfpr$	Labor (L)	Efficiency	K/L	Welfare Gain
0.300	0	0.349	0.619	0.033	0.050	0.651	0.616	0.995	35.498	N/A
0.304	0.10	0.340	0.615	0.045	0.068	0.660	0.612	0.996	35.437	-0.17%
0.313	0.23	0.342	0.607	0.051	0.077	0.658	0.605	0.997	35.336	-0.51%

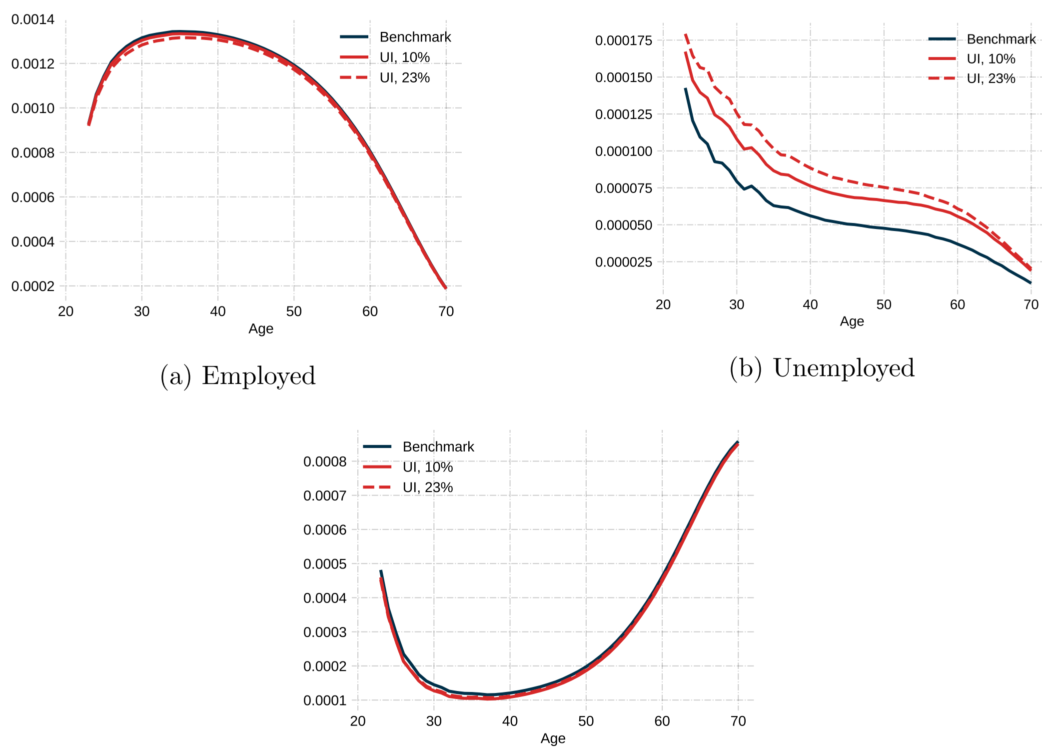
benefits with a probability equal to $\eta = 1/6$.

We conduct an UI exercise with two different values of replacement rates, $b_0 = 0.10$ and $b_0 = 0.23$. A 10% replacement rate might seem low, but in our benchmark calibration there is no UI and everyone receives a lump-sum transfer payment. Hence, giving unemployment workers an extra 10% of their potential wages is an economically meaningful experiment. A 23% replacement rate, on the other hand, is closer to the estimates of the UI system in the US (Krusell et al., 2017). In each case, all workers receive the benchmark transfer payments and eligible unemployed workers receive extra unemployment insurance benefits. UI and transfer payments are financed with linear tax on labor earnings. With a 10 percent replacement rate, tax rate needs to go up to 30.4%, whereas with 23% replacement rate the tax rate increases to 31.3%. Effects of providing unemployment benefits to the unemployed on aggregate variables are reported in Table 3.

Introducing a UI with 10 percent replacement rate leads to 1.2 percentage points increase in the stock of unemployed compared to the zero-UI baseline. The stock of non-participants decreases by 0.9 percentage points and the stock of employed goes down by 0.4 percentage points. Unemployment rate (u) increases from 5.0 percent to 6.8 percent. Labor force participation rate also increases from 65.1 percent to 66.0 percent. Because unemployed workers becomes choosier in moving into employment, average productivity goes up from 0.995 to 0.996. The welfare, measured by the expected present-value utility for newborns, declines with the introduction of the UI. The magnitude of welfare change is -0.17% in consumption equivalence when the benefit goes up by 10 percentage points. Although the UI helps smoothing the workers' consumption, this positive effect is dominated by the negative effect of distortions created by the UI. Here, modeling the asset accumulation by the individuals is important for the quantitative analysis, because it dictates the degree of self-insurance. A higher replacement rate of 23 percent leads to a larger increase in the unemployment rate and a larger decrease in the labor force participation rate.

Figure 15 shows that the response of the stock is relatively uniform across different age groups. Because the employment level is small for very old workers, the response of unemployment rate in Figure 16 is more pronounced for very old workers. A relatively large response of the unemployment rate is in contrast to the tax-and-transfer experiment.

The patterns of gross flows in Figure 17 exhibit several notable properties. First, two flows between E and U exhibit large responses. UE flow declines with a larger size of UI, reflecting the workers becoming more selective. The response is somewhat larger for older

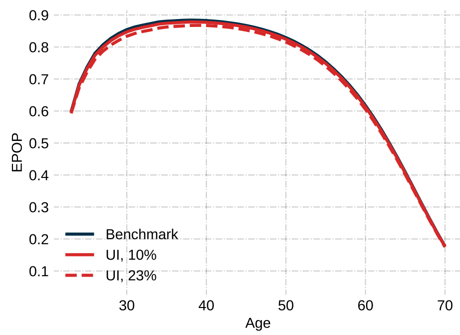


(a) Employed

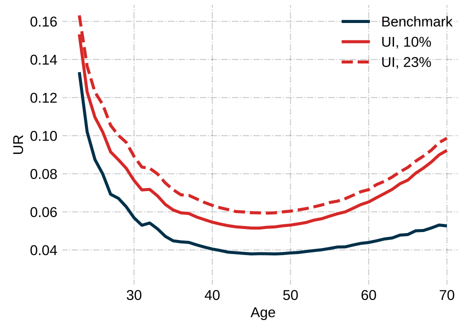
(b) Unemployed

(c) Not in the labor force

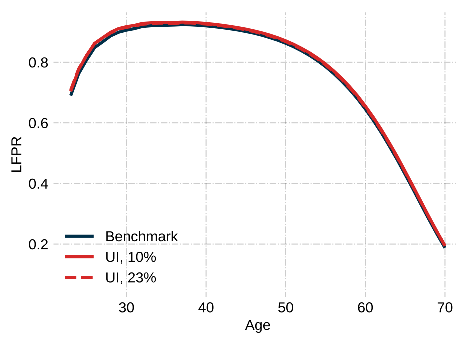
Figure 15: Labor market stocks after introducing UI benefits



(a) Employment to population ratio

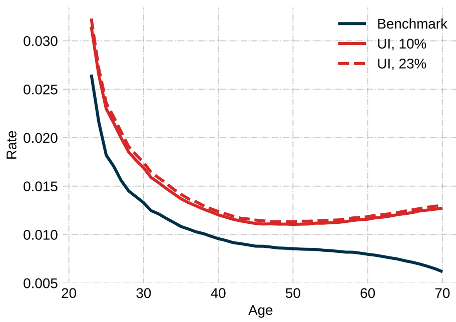


(b) Unemployment rate

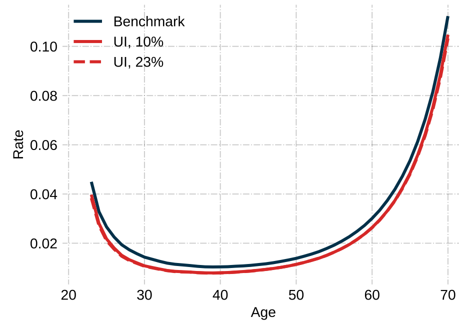


(c) Labor force participation rate

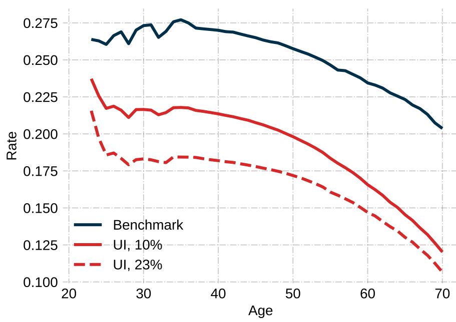
Figure 16: Labor market ratios after introducing UI benefits



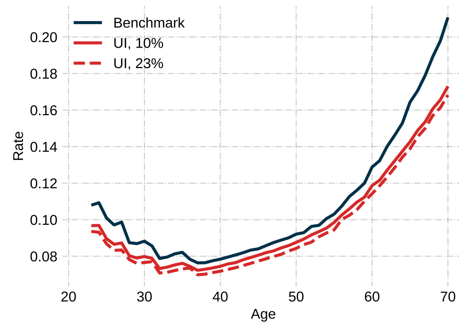
(a) EU



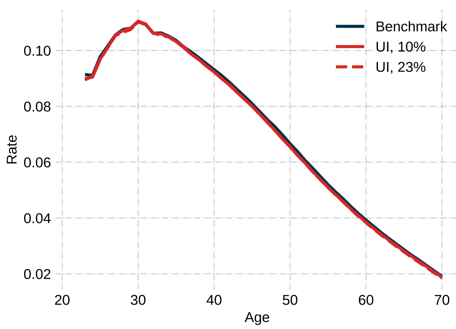
(b) EN



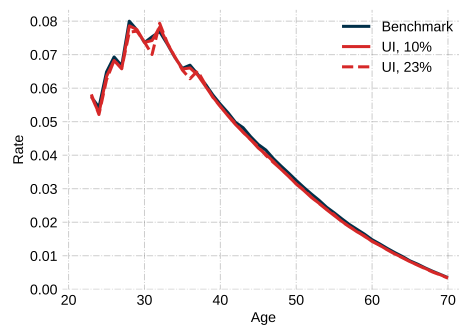
(c) UE



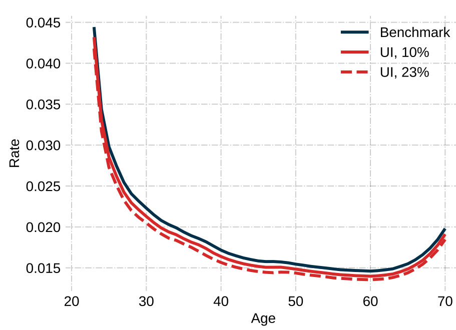
(d) UN



(e) NE



(f) NU



(g) EE

Figure 17: Gross worker flow rates after after introducing UI benefits

workers. The response is nonlinear in the level of UI: changes in the gross flow rates are smaller when the replacement rate increases from 10% to 23%, compared to the change from 0% to 10%. EU flow increases. Note that this policy outcome does not mean that the UI is inducing the workers to quit, because the quitters do not qualify for UI. This outcome rather comes from a substitution between EU flow and EN flow. Some workers who would have gone to the N state upon separation shock now go into U because the UI is tied to the search activity. Similarly, UN flow decreases by a small amount due to the search requirement for receiving the UI benefit. All these effects are somewhat stronger for older workers. One reason is that UI is in limited duration, and thus it is a temporary income. Because older workers have a shorter time horizon, temporary income has a relatively stronger effect on the incentives of older workers. This effect cannot be captured in a model with infinite-horizon, and is one of new insights in this experiment.

6 Conclusion

This paper developed a general equilibrium framework to analyze the gross worker flows over the life cycle. Our model features life-cycle permanent-income consumers who can self-insure from various shocks by accumulating assets. In the labor market, individuals can make labor-market participation decisions under labor-market frictions.

The calibrated model can match the salient features of the life-cycle patterns of the gross worker flows in the data. The estimated parameter values reveal how the nature of frictions vary across the worker's life cycle. The pattern of the job-separation shock has an important impact on the life-cycle behavior of unemployment stocks.

With the calibrated model, we ran two policy experiments. First, we experimented with taxes-and-transfers policy. An increase in labor tax decreases employment and labor-force participation for all age groups, although the changes are larger for younger workers. Unemployment stock decreases significantly only for young workers. The analysis of gross worker flows finds the changes in EU flow and NU flow (inflow into unemployment) are the main causes of the age heterogeneity in the unemployment response. For the N state, the outflow from N is of prominent importance. Overall, young workers move less into the U state and also leave less from the N state when the labor tax is high. The changes in gross flows also have an impact on productivity and wages, highlighting the importance of explicitly considering effects on reallocation in the analysis of taxes and transfers. The reallocation effects are heterogeneous across age groups.

Second, we introduced a realistic UI system. An increase in UI increases the unemployment rate, accompanied with a smaller increase in the labor force participation rate. Large changes occur in the gross flows between UE and EU flow, due to the increased selectiveness of workers. In general, older workers respond more to the UI incentives, because UI is a temporary income and older workers have a shorter time horizon.

Although we view our study as a significant progress compared to the existing literature, much room remains for future research. The model of this paper, as in the case with Krusell et al. (2017), does not address the endogenous response of the frictions to the change in taxes. The DMP framework, for example, would suggest a change in the labor tax can affect the firms' vacancy-posting behavior and eventually alter the frequency of workers receiving job offers. The omission here is by two reasons. First, our focus is the labor-supply response, which has been the focus of the taxes-and-transfers literature starting from Prescott (2004) and also of many UI papers that focus on workers' search effort. Second, as we discussed in the introduction, incorporating such a mechanism into a BHA-style model is extremely challenging. Incorporating such an effect is outside the scope of this paper, but it is an important future research agenda.

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Appendix

A Age coefficients on frictions and productivity

The age component of market productivity, $g(j)$, the logarithms of job-offer arrival rates, $\log\lambda_e(j), \log\lambda_u(j), \log\lambda_n(j)$, and the logarithm of exogenous job-separation rate are characterized as second degree polynomials of age, j:

$$\begin{aligned}
\lambda_e(j) &= \exp(\lambda_{e,2}j^2 + \lambda_{e,1}j + \lambda_{e,0}), \\
\lambda_u(j) &= \exp(\lambda_{u,2}j^2 + \lambda_{u,1}j + \lambda_{u,0}), \\
\lambda_n(j) &= \exp(\lambda_{n,2}j^2 + \lambda_{n,1}j + \lambda_{n,0}), \\
\sigma(j) &= \exp(\sigma_2 j^2 + \sigma_1 j + \sigma_0), \\
g(j) &= g_2 j^2 + g_1 j + g_0.
\end{aligned}$$

The calibrated coefficients are shown in Table 4.

Table 4: Coefficients of polynomials

Parameter	**Definition**	**Value**
$\lambda_{u,2}$	Log of the job offer rate of the unemployed	-1.115
$\lambda_{u,1}$		2.877E-04
$\lambda_{u,0}$		-4.641E-07
$\lambda_{e,2}$	Log of the job offer rate of the employed	-1.135
$\lambda_{e,1}$		-1.508E-04
$\lambda_{e,0}$		6.996E-07
$\lambda_{n,2}$	Log of the job offer rate of the non-participant	-1.393
$\lambda_{n,1}$		6.149E-04
$\lambda_{n,0}$		-1.528E-06
σ_2	Log of the job separation rate	-4.2141
σ_1		-5.810E-03
σ_0		8.163E-06
g_2	Age component of market productivity	6.675E-01
g_1		2.229E-03
g_0		-4.074E-06

B Details of computation

In the model, a worker is characterized by (i) his (note that in the baseline we match the data only with males) labor market state: employed (with a job), unemployed (not employed but actively searching for a job), not in the labor force (not employed and not searching for a job), (ii) his wealth, a, (iii) his idiosyncratic general productivity, z, (iv) his match-specific productivity (if employed), μ, and (v) his age, j. Age in the model is monthly and ranges from 1 to 947. Model age $j = 1$ corresponds to annual age 22 in the data. At each age j, an s_j fraction of workers survive to age $j+1$. At age 947, which corresponds to one month before age 101 in the data, everyone dies for sure.

To solve the model numerically, we discretize the state space. For assets, a, we create a log-spaced grid of 100 points between 0 and 380. The discretization of asset space is independent of other model parameters. We also discretize the state space of z and μ. However, discretization of $z-$space and $\mu-$space depend on other model parameters and are explained in the later paragraphs.

Our model is in a general equilibrium. We assume competitive markets, and thus i) marginal product of capital minus depreciation rate must be equal to the interest rate in the household problem and ii) marginal product of labor must be equal to the wage in efficiency units. Moreover, the government budget constraint must hold: total labor tax collection plus the assets of deceased workers must be equal to total transfer payments. Since we normalize wage in the efficiency unit to 1, we achieve condition (ii) by setting A to a value that ensures marginal product of labor is equal to 1.

Normally, one would use a standard nested algorithm in which the inner loop calculates the general equilibrium for each set of parameter values and the outer loop minimizes the distance between the data and model moments. Due to substantial computational complexity, we solve the minimization and general equilibrium together, by adding the capital market equilibrium condition and government budget constraint as additional targets to be minimized. This procedure speeds up the calibration process substantially.

Recall that $\boldsymbol{\xi} \equiv \{\beta, \delta, \lambda_{e,2}, \lambda_{e,1}, \lambda_{e,0}, \lambda_{u,2}, \lambda_{u,1}, \lambda_{u,0}, \lambda_{n,2}, \lambda_{n,1}, \lambda_{n,0}, \sigma_2, \sigma_1, \sigma_0, g_2, g_1, g_0, \psi, \sigma_\mu, \sigma_z, b, \zeta, \alpha\}$ is the set of parameters to be calibrated within the model. Let $\boldsymbol{\xi^o} \equiv \{\{s_j\}_{j=1}^{947}, \rho_\mu, \rho_z\}$ be the set of parameters calibrated externally.

We solve our model as follows:

1. Given parameter values $\boldsymbol{\xi}$ and $\boldsymbol{\xi^o}$, we first create the discrete state space to solve the model numerically.

   (a) We discretize the AR(1) (log-) idiosyncratic productivity process using the Tauchen method. We create a grid of idiosyncratic productivity, z-grid, consisting of 15 points. The Tauchen method also generates transition probabilities from current idiosyncratic productivity, z, to the next period's idiosyncratic productivity, z': $P_{z,z'}$ for z and z' in z-grid.

(b) Similarly, we discretize the AR(1) (log-) match-specific productivity process of the worker-firm pair using the Tauchen method. μ-grid consists of 15 points. The probability of a match-specific productivity, μ, becoming μ' in the next period if the worker-firm pair survives is denoted as $P_{\mu,\mu'}$ for μ and μ' in μ-grid.

(c) Match-specific productivity for a new job (for the workers in the leisure islands, and for the workers who have a job but receive an outside offer) is drawn from a Pareto distribution:

$$\Pr[M > \mu] = \begin{cases} \left(\dfrac{\mu_1}{\mu}\right)^{\alpha} & \text{for } \mu \geq \mu_1, \\ 1 & \text{for } \mu < \mu_1, \end{cases}$$

where μ_1 is the lowest point in the μ-grid. Note that M is the random variable and μ is its realization. Let μ_k be the k-th lowest point in the μ-grid. Then, probability of receiving an outside offer with match-specific productivity of μ_k is equal to:

$$S(\mu_k) = \begin{cases} \left(\dfrac{\mu_1}{\mu_{k-1}}\right)^{\alpha} - \left(\dfrac{\mu_1}{\mu_k}\right)^{\alpha} & \text{for } k > 1, \\ 0 & \text{for } k = 1. \end{cases}$$

(d) Recall that with a probability, ζ, match-specific productivity is unrevealed (or unkown). To account for the unknown state, we add one more grid point to $\mu-$grid, which now consists of 16 points. We assume that if the match-specific productivity is not known, workers are paid as if they have the median match-specific productivity. Let $P^{ext}_{\mu,\mu'}$ represent transition probabilities in the extended μ grid:

$$P^{ext}_{\mu,\mu'} = \begin{cases} P_{\mu,\mu'} & \text{if both } \mu \text{ and } \mu' \text{ are known,} \\ 0 & \text{if } \mu \text{ is known but } \mu' \text{ is unknown,} \\ (1-\zeta)S(\mu') & \text{if } \mu \text{ is unknown but } \mu' \text{ is known,} \\ \zeta & \text{if both } \mu \text{ and } \mu' \text{ are unknown.} \end{cases} \tag{2}$$

Similarly, let $S^{ext}(\mu_k)$ be the probability of receiving an outside job offer with match-specific productivity, μ_k, while taking into account that match quality might be unknown.

$$S^{ext}(\mu_k) = \begin{cases} (1-\zeta)S(\mu_k) & \text{if } \mu_k \text{ is known,} \\ \zeta & \text{if } \mu_k \text{ is unknown.} \end{cases}$$

2. Given parameter values $\boldsymbol{\xi}$ and $\boldsymbol{\xi^o}$, a guess for the interest rate r, normalized wage rate, $\tilde{\omega} = 1$, and a guess for the government transfer to households, $\mathbf{T}$, we recursively

solve for the value functions of the workers: $W_j(a, z, \mu)$, $U_j(a, z)$, $N_j(a, z)$, $T_j(a, z, \mu)$, $S_j(a, z, \mu)$, $O_j(a, z)$, and $F_j(a, z)$. Starting from age $j = 947$, when the continuation value is equal to 0, we solve for the consumption/saving decision of age $j = 947$ workers and calculate the value function as described in section 3.2. We iterate this process until we reach age $j = 1$. We linearly interpolate the continuation value and solve for the optimal saving decision using the golden-section method.

3. Using the value functions from step 2, we generate the decision rules for labor force participation, accepting/rejecting an offer, and switching jobs as described in section 3.5.

4. Starting from age $j = 1$, we simulate the model using the decision rules from 3. We assume all the workers are born at the leisure island with no assets and no wage offer at hand.

5. After observing gross worker flows between three labor market states and job-to-job flows, we calculate the transition rates as described in section 4.

6. We solve for a stationary equilibrium in which the interest rate and wage rate are constant and the distribution of agents over the state space is stationary. Hence, given the model parameters, interest rate, and wage rate, total capital supply in the economy is equal to $K = \int a_i di$ and total labor supply in efficiency units is equal to $\int e(i) z_i \mu_i g_i di = L$, where i represents a worker and $e(i)$ is the employment status of the workers with $e(i) = 1$ if i is employed and 0 otherwise. The integration is over all the workers (both in the work island and leisure island) in the model.

7. The next step is to ensure general equilibrium labor-market-clearing condition. Having solved for K and L in step 6, we choose A such that $(1 - \theta)A(K/L)^{\theta} = 1$

8. We calculate i) the log difference between the interest rate and marginal product of capital minus depreciation, $\log(r) - \log\left(\theta A(K/L)^{\theta-1} - \delta\right)$, and ii) the log difference between tax collection plus assets of the deceased workers and transfer payments.

9. We repeat steps 1 to 8 with new r and T values until the sum of square of the two log differences from step 8 gets close to 0.

B.1 Calibration

To calibrate the model, we use the following algorithm.

1. Set $\boldsymbol{\xi^o}$ to their respected values as described in section 4.

2. For an initial $\boldsymbol{\xi}$, $\mathbf{T}$, and r, we solve and simulate the model from steps 1 to 8 and calculate the gross worker flows from the simulated data. Instead of doing step 9 of the

model solution algorithm in every iteration, we combine step 9 of the model solution algorithm with the calibration algorithm as described in the following points. In this step, we also calculate the average wage for each age and normalize average wage at (annual) age 42 to 1. Hence, average wages in both data and simulated data are expressed relative to average wage at age 42.

3. In section 4, we characterized the loss function as follows:

$$\mathcal{L} \equiv |(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})|' W |(\log X^{model}(\boldsymbol{\xi}) - \log X^{data})|,$$

where $X^{model}(\boldsymbol{\xi})$ is the collection of transition rates among worker states, normalized average wages, the interest rate, and investment to GDP ratio (x).

$$\begin{aligned} X^{model}(\boldsymbol{\xi}) = \quad & \mathrm{vec}\Bigg(\Bigg\{ EU^{model}(\boldsymbol{\xi}, j^a), EN^{model}(\boldsymbol{\xi}, j^a), EE^{model}(\boldsymbol{\xi}, j^a), \\ & NU^{model}(\boldsymbol{\xi}, j^a), NE^{model}(\boldsymbol{\xi}, j^a), UN^{model}(\boldsymbol{\xi}, j^a), \\ & UE^{model}(\boldsymbol{\xi}, j^a), \omega^{model}(\boldsymbol{\xi}, j^a) \Bigg\}_{j^a=23}^{70}, r(\boldsymbol{\xi}), x(\boldsymbol{\xi}) \Bigg). \end{aligned}$$

X^{data} is the collection of the six-year rolling average of transition rates, normalized average wages of males observed in the data, the interest rate target, and the investment to GDP ratio target,

$$\begin{aligned} X^{data} = \quad & \mathrm{vec}\Bigg(\Bigg\{ EU^{data}(j^a), EN^{data}(j^a), EE^{data}(j^a), \\ & NU^{data}(j^a), NE^{data}(j^a), UN^{data}(j^a), \\ & UE^{data}(j^a), \omega^{data}(j^a) \Bigg\}_{j^a=23}^{70}, r, x \Bigg), \end{aligned}$$

and W is a diagonal weighting matrix. We set all but the last two elements of the diagonal of W to be equal to 1, and the last two elements to be equal to 100.[13]

We modify the loss function, $\mathcal{L}$, by adding the capital market equilibrium condition and the government budget condition to $\mathcal{L}$. We define the modified loss function, $\tilde{\mathcal{L}}$, that we are minimizing as

$$\begin{aligned} \tilde{\mathcal{L}} \quad = \quad & \mathcal{L} + \tilde{W}\Bigg[\log(\theta A (K/L)^{\theta-1} - \delta) - \log(r(\boldsymbol{\xi})) \\ + \quad & \log(\mathbf{T}) - \log\left(\tau \int e(i)\omega_{j(i)}(i)(\mu(i), z(i))di + \int a(i)(1-s(i))di\right)\Bigg], \end{aligned}$$

[13] We chose 100 as a moderately large number to give aggregate variables (interest rate and investment to GDP ratio) higher weights than a flow rate at a particular age in our minimization algorithm.

where $e(i)$ is the employment status of agent i with 1 for employed and 0 for not employed, $s(i)$ is the survival status of agent i, 1 for surviving agents, and 0 for deceased agents, and $\tilde{W}$ is a large number in order to ensure that the market clearing condition and the government budget conditions are always satisfied at optimum.[14]

4. We repeat steps 2 and 3 for different $\boldsymbol{\xi}$, $\mathbf{T}$ and r values until the the modified loss function is minimized. We use the Powell method from Scipy minimization sub-package as our minimizer.

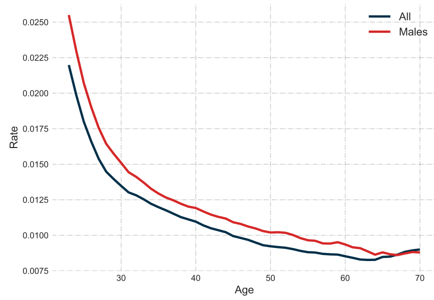
C Calibration with all population

In this section, we perform a robustness exercise in which we calibrate the model using worker flows of all workers, not just males. Figure 18 shows gross worker flows of all workers and male workers. E to E transition rates of all workers are very close to that of males. Similarly, transitions out of employment (EU and EN) are quantitatively and qualitatively similar across the two groups: males and all workers. However, transitions into employment (UE and NE) until prime ages differ considerably between the two groups. Transitions into employment of female workers at child-rearing ages are substantially lower than male workers' transition into employment. Comparably, UN flows are higher with all workers and NU flows are higher with male workers.

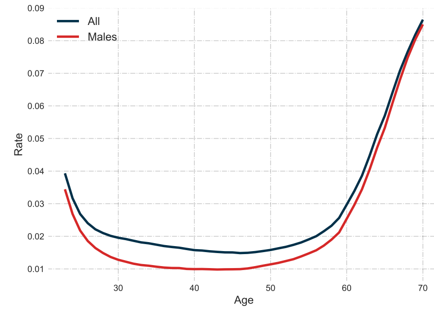
Our calibration method in this section is as in Section 4 but, the current calibration uses data from all workers, not just from male workers. Table 5 reports estimates of age-independent parameters. Figure 19 plots estimates of age-dependent parameters. Figure 20 depicts gross workers from the data and from the calibrated model. Our calibrated model does mostly well in replicating qualitative features of gross flows. However, the model fails to generate decreasing gross flows into labor force (NE and NU) early in the lifecycle. In our model, NE and NU flows increases until 30s and then decrease gradually. Our model is able to replicate the qualitative features of EE flows, but it generates EE flows rates considerably higher than the data when workers in their 20s.

As in Section 5.1, we perform a tax and transfer exercise in which we increase labor tax rate to 45% from 30% and rebate the tax collection to households in a lump-sum manner. The results are reported in Table 6 (aggregate statistics), in Figures 21 (labor market stocks), 22 (labor market ratios), and 23 (gross worker flows). The results with this particular calibration are very similar to the results in Section 5.1.

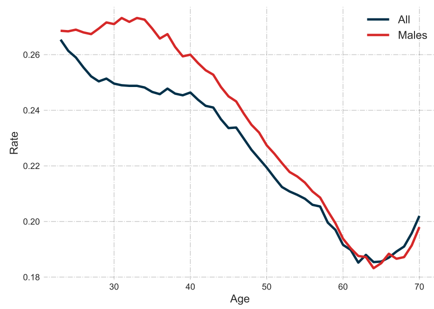
[14] In the solution we set $\tilde{W}$ equal to 38400, a large number to ensure equilibrium conditions are always satisfied in our minimization algorithm. Remember that we combine equilibrium conditions and distance between data and model moments in a single loop in our minimization algorithm to keep the running time of the code at manageable levels. Separating the two would increase already very long running time to unmanageable levels.



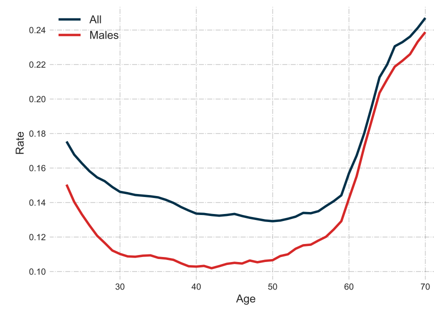
(a) EU



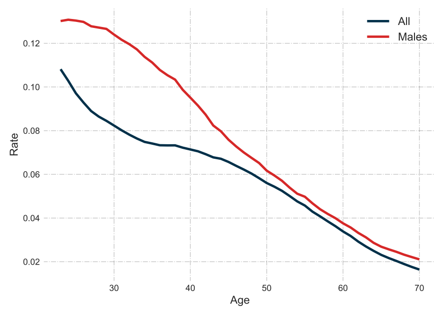
(b) EN



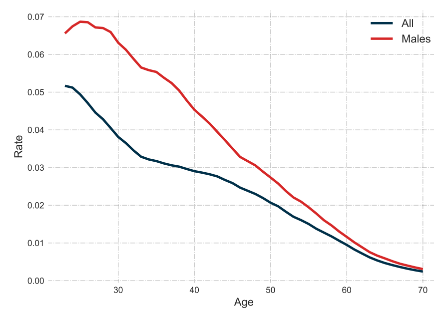
(c) UE



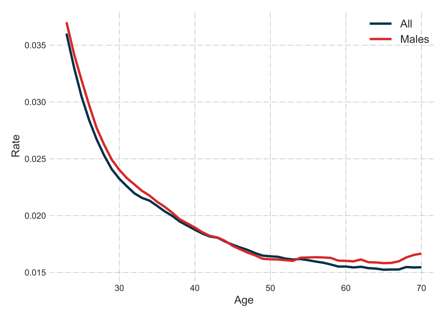
(d) UN



(e) NE



(f) NU



(g) EE

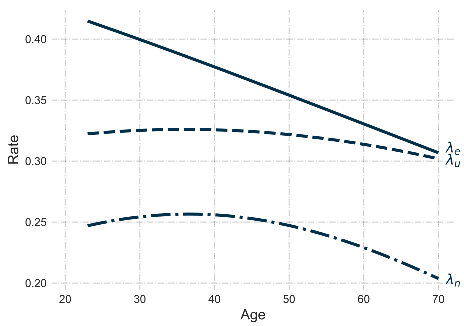
Figure 18: Gross flows in the data, all workers

Table 5: Age independent parameters

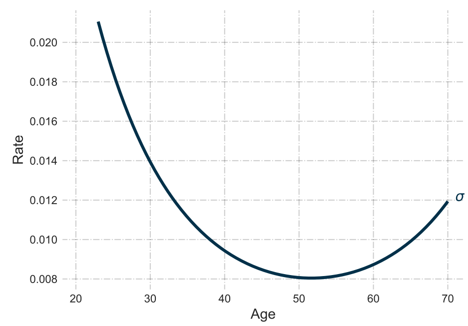
Parameter	Definition	Value
β	Discount factor	0.997
θ	Elasticity of output w.r.t. capital	0.3
δ	Depreciation rate	0.008
ψ	Disutility of active job search	0.045
A	Total factor productivity	0.485
ρ_μ	Persistence parameter of monthly AR(1) match-specific productivity	0.98
σ_μ	Std. dev. of innovations in match specific productivity	0.106
ρ_z	Persistence parameter of monthly AR(1) idiosyncratic productivity	0.97
σ_z	Std. dev. of innovations in idiosyncratic productivity	0.090
b	Home productivity	0.166
ζ	Unknown match quality probability	0.183
α	Shape parameter of Pareto distribution (new job wage offers)	6.231
$\bar{\mu}$	Match quality for unrevealed matches	1.0
γ	Disutility of work over disutility of active job search	11.4
J	Monthly age at which everyone dies	947

Table 6: Aggregate statistics from the experiment

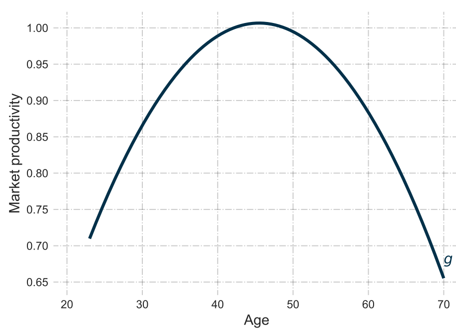
Tax	N	E	U	u	$lfpr$	Labor	Efficiency
0.30	0.421	0.548	0.03	0.053	0.579	0.573	1.045
0.45	0.594	0.386	0.02	0.050	0.406	0.434	1.125



(a) Job offer arrival rate

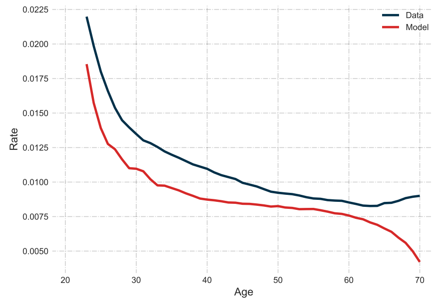


(b) Exogenous job separation rate

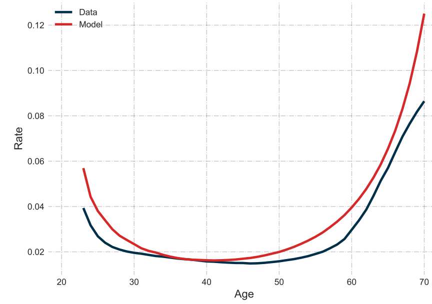


(c) Age component of market productivity

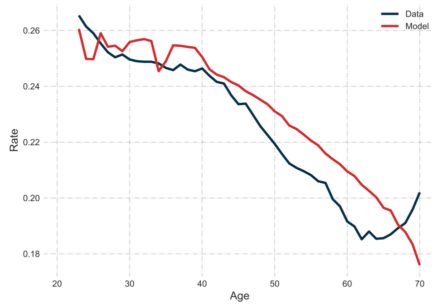
Figure 19: Age dependent parameters, all workers



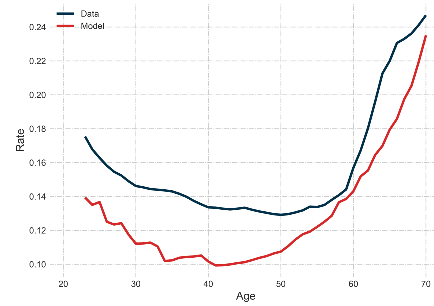
(a) EU



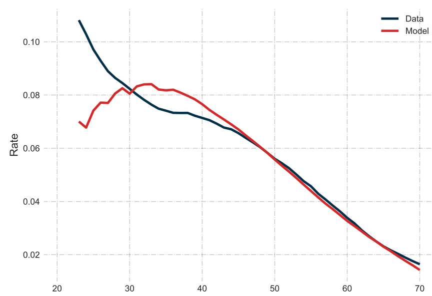
(b) EN



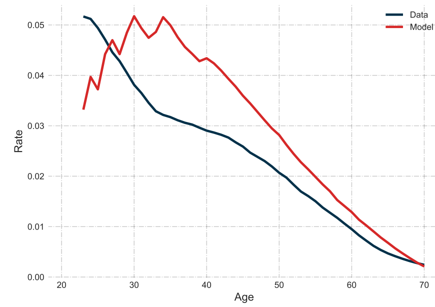
(c) UE



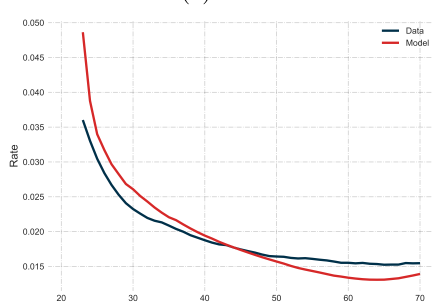
(d) UN



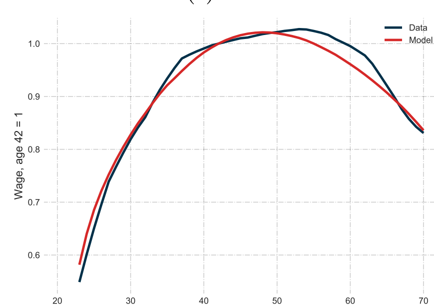
(e) NE



(f) NU

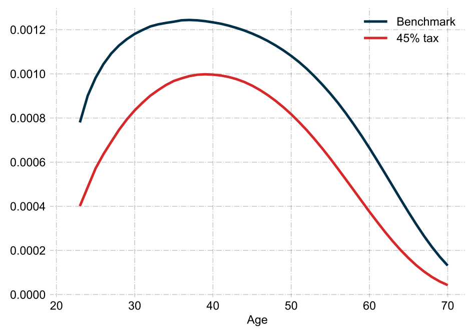


(g) EE

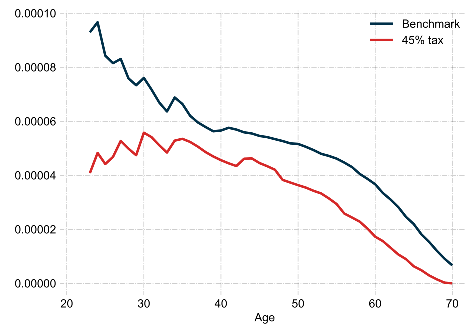


(h) Wage

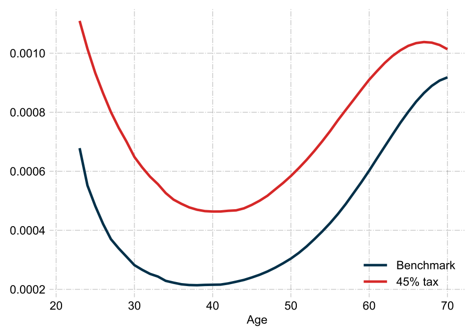
Figure 20: Model moments and calibration targets, all workers



(a) Employed

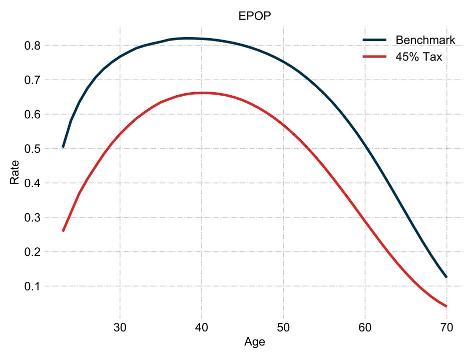


(b) Unemployed

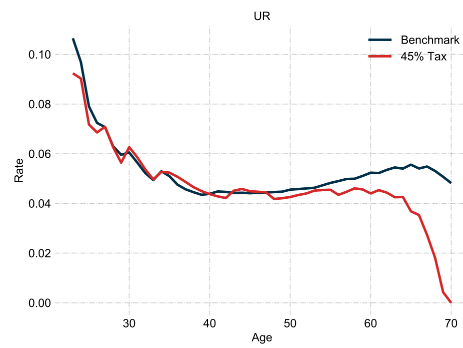


(c) Not in the labor force

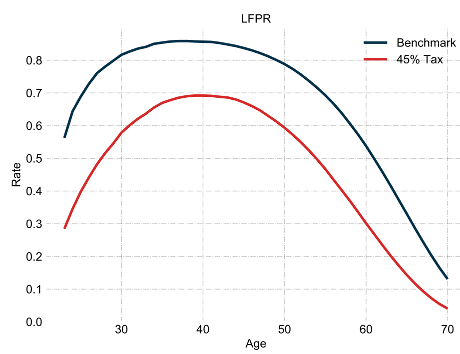
Figure 21: Labor market stocks after a tax hike, all workers



(a) Employment to population ratio

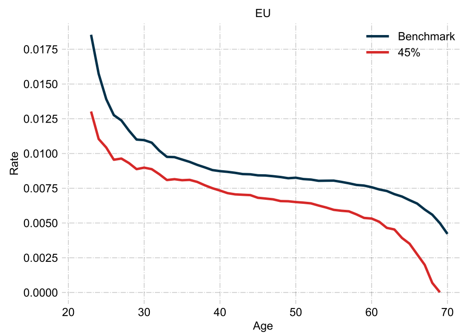


(b) Unemployment rate

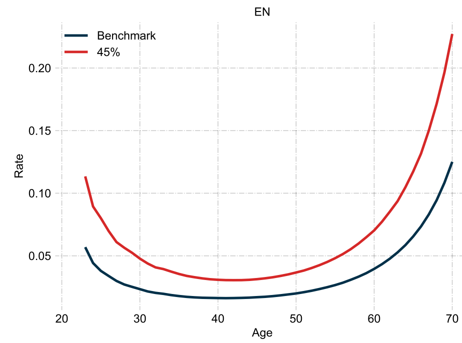


(c) Labor force participation rate

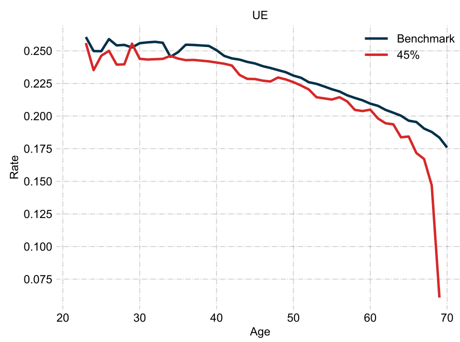
Figure 22: Labor market ratios after a tax hike, all workers



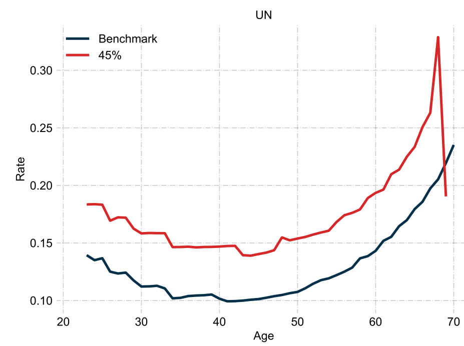
(a) EU



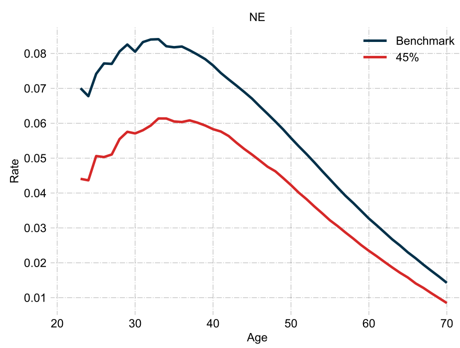
(b) EN



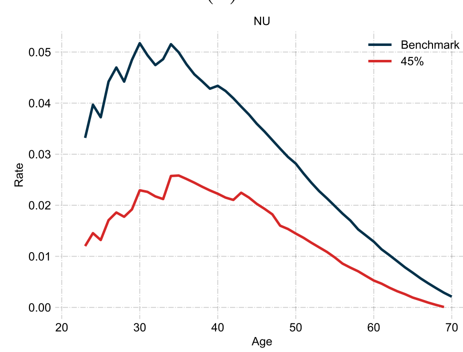
(c) UE



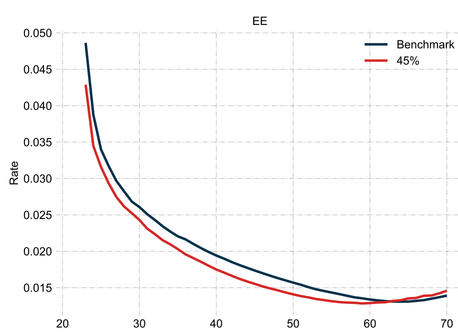
(d) UN



(e) NE



(f) NU



(g) EE

Figure 23: Gross worker flow rates after a tax hike, all workers

D Wealth

This section compares wealth distribution in the model with the data. We rely on Kuhn et al. (2016) for wealth statistics in the US, calculated from the Survey of Consumer Finances (SCF). We normalize the average wealth in each group by the average wealth in the entire population. Figure 24 shows the average wealth for each age group in different years.

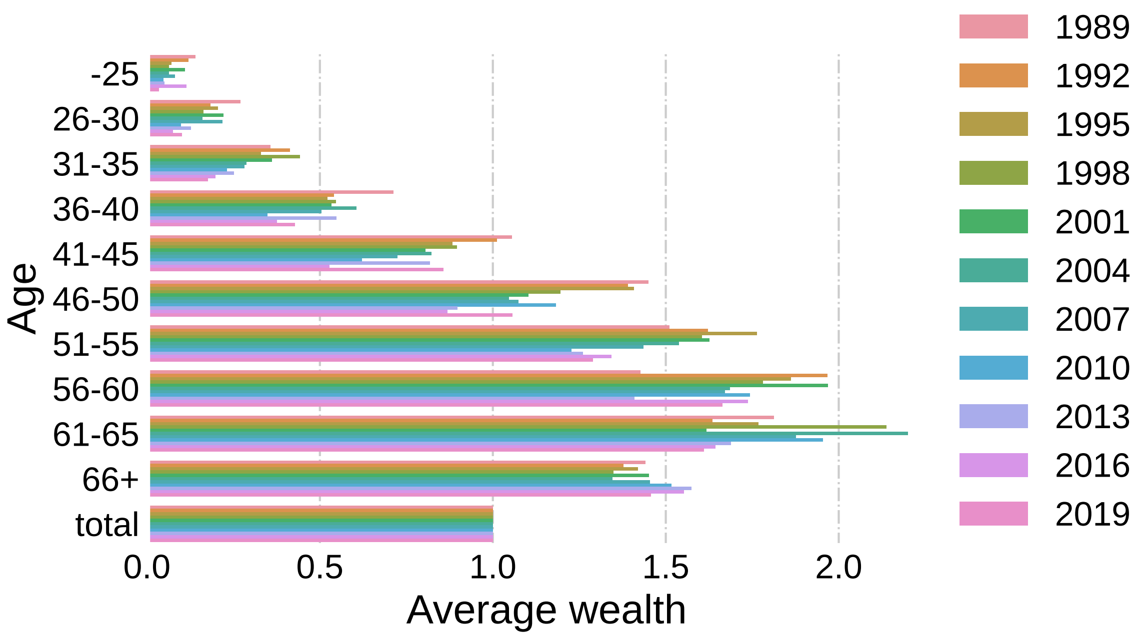


Figure 24: Average wealth from data

Data from Kuhn et al. (2016). Accessed from `https://sites.google.com/site/kuhnecon/home/us-inequality`. All values are relative to the average wealth in the entire population.

Figure 25 compares the Lorenz curves for the benchmark case (30% tax) and for the 45% tax case. The Lorenz curve for the 45% tax case lies below, and and the Gini coefficient increases slightly. This outcome reflects the increase in relative wealth for the middle-aged workers, as discussed in the main text. Figures 26 and 27 decomposes these Lorenz curves into different labor market states for the individuals. One can see that the nonparticipant holds a larger share of wealth in the 45% tax case, reflecting the larger population of nonparticipants.

Table 7 shows the relative wealth level of individuals in each category of age and labor market state. The comparison across different tax rates is not straightforward given the productivity distribution in each state is different, but it can be seen that (i) the wealth level of nonparticipants is lower when the tax rate is higher and (ii) old individuals run down wealth more quickly with higher taxes.

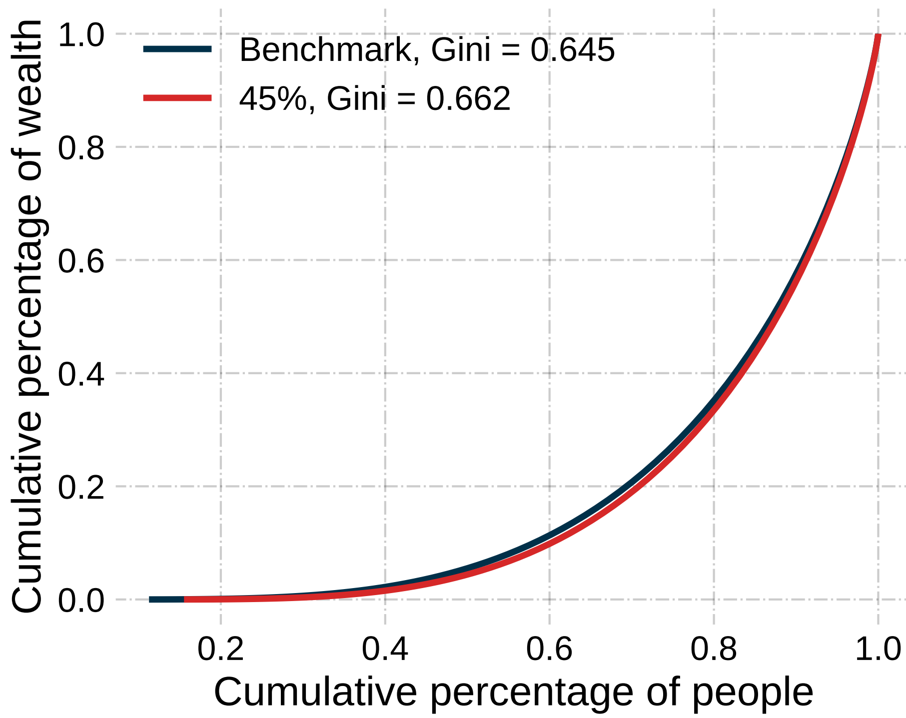


Figure 25: Lorenz curve of wealth in the model

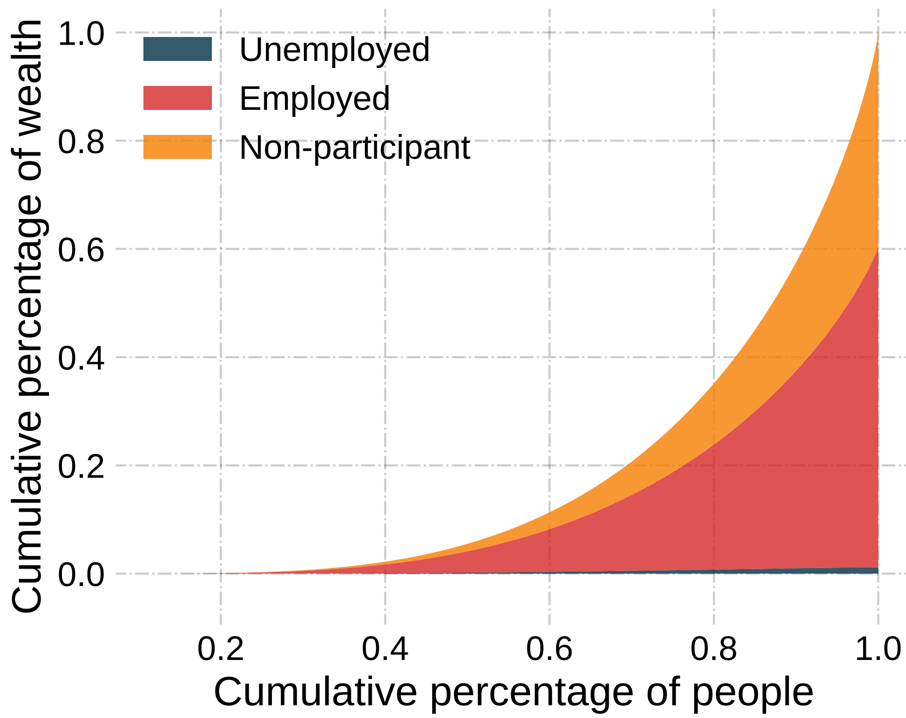


Figure 26: Lorenz curve of wealth by worker state in the benchmark model

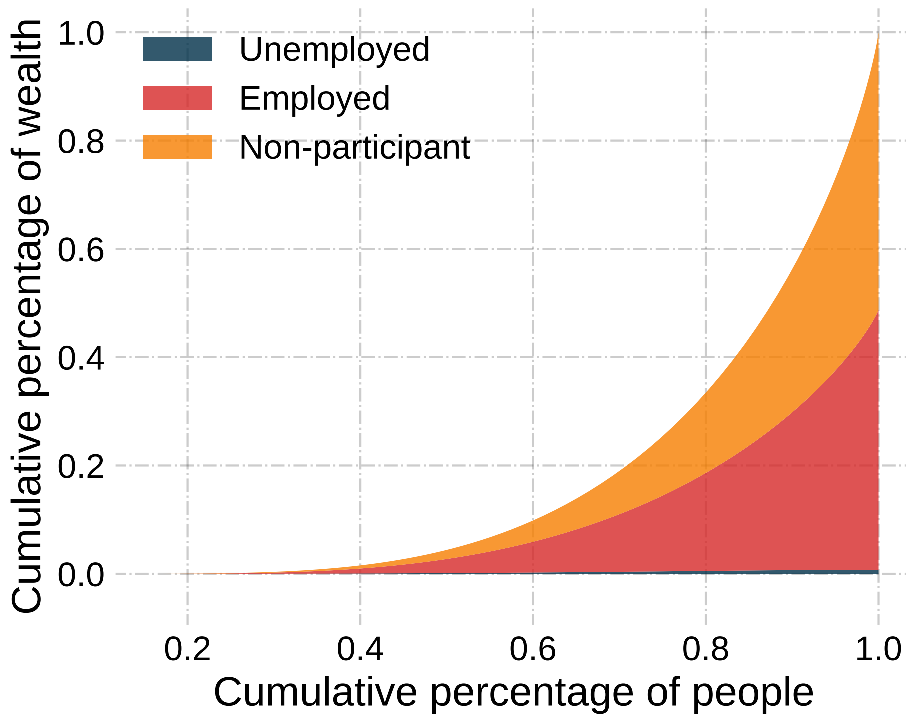


Figure 27: Lorenz curve of wealth by worker state in the model with 45% tax

Benchmark			
Age	Employed	Nonparticipant	Unemployed
23-25	0.06	0.01	0.02
26-30	0.23	0.10	0.07
31-35	0.49	0.51	0.18
36-40	0.79	1.11	0.32
41-45	1.10	1.76	0.49
46-50	1.38	2.36	0.65
51-55	1.57	2.75	0.74
56-60	1.61	2.73	0.73
61-65	1.47	2.29	0.60
66+	1.10	0.71	0.32
45% Tax			
Age	Employed	Nonparticipant	Unemployed
23-25	0.09	0.01	0.03
26-30	0.28	0.07	0.07
31-35	0.55	0.30	0.16
36-40	0.88	0.77	0.30
41-45	1.23	1.41	0.43
46-50	1.52	2.02	0.56
51-55	1.69	2.34	0.63
56-60	1.68	2.27	0.57
61-65	1.47	1.87	0.39
66+	1.03	0.58	0.15

Table 7: Average wealth in age group-worker state pairs relative to the average wealth in the entire economy

Figure 28 compares average wealth in the model (Benchmark model (30% tax), UI (10% replacement rate) and UI (23% replacement rate)) with average wealth in the data. Data represents mean of relative wealth in each age group across years. Somewhat surprisingly, the response of wealth distribution to the change in UI is very small.

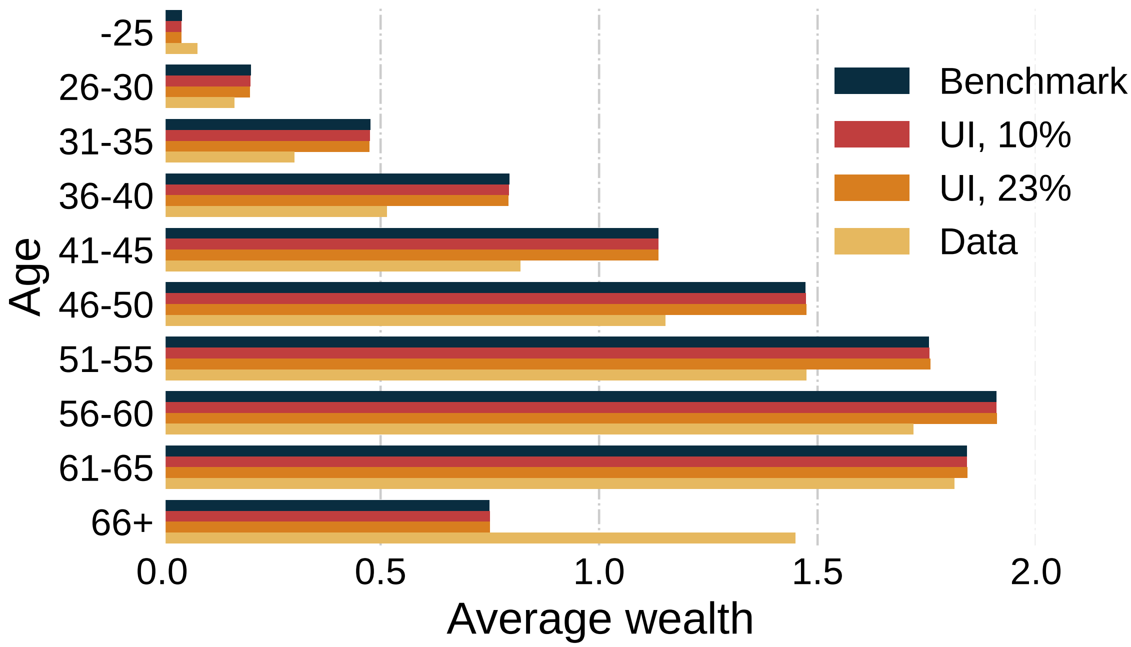


Figure 28: Average wealth from model and data

Data: averages across years from Kuhn et al. (2016). Accessed from `https://sites.google.com/site/kuhnecon/home/us-inequality`. All values are relative to the average wealth in the entire population.

E Welfare Gain

Recall we define utility as:

$$\mathbf{U}_\omega = \sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log c_j - d_j].$$

Here, $\omega \in \{U, N\}$ is the initial labor market state (a newborn starts from nonemployment). We calculate the welfare gain as follows:

1. Solve for the indirect utility of a newborn with zero wealth and idiosyncratic productivity z when he is unemployed and out of labor force in the benchmark economy: $\mathbf{U}_U(a = 0, z, \tau = 0.3)$ and $\mathbf{U}_N(a = 0, z, \tau = 0.3)$

2. Similarly, solve for the indirect utility when the tax rate is 45 percent: $\mathbf{U}_U(a = 0, z, \tau = 0.45)$ and $\mathbf{U}_N(a = 0, z, \tau = 0.45)$

3. Find the extra consumption (in every period) a worker in the benchmark economy at age 1 with 0 assets and idiosyncratic productivity z asks to make him indifferent between being in the benchmark economy and being in the 45-percent-tax economy. Call this function $\xi_\omega(z)$:

$$\sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log \xi_\omega(z) c_j^b - d_j^b] = \sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log c_j^{45} - d_j^{45}],$$

where superscripts b and 45 represent benchmark and 45-percent-tax economy.

Then,

$$\sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) \log \xi_\omega(z) + \sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log c_j^b - d_j^b] = \sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log c_j^{45} - d_j^{45}]$$

Then substitute the indirect utilities. For example, when $\omega = U$,

$$\sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) \log \xi_\omega(z) + \mathbf{U}_U(a = 0, z, \tau = 0.3) = \mathbf{U}_U(a = 0, z, \tau = 0.45)$$

Then

$$\log \xi_U(z) = \frac{\mathbf{U}_U(a = 0, z, \tau = 0.45) - \mathbf{U}_U(a = 0, z, \tau = 0.3)}{\sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right)}$$

4. Then calculate the expected welfare gain of a newborn with 0 wealth before his idiosyncratic productivity is revealed and he made a decision to participate in the labor force:

$$\tilde{\xi} \equiv \sum_{z=1}^{Z} S_z \Big((\xi_U(z) - 1)\, 1_p(a = 0, z) + (\xi_N(z) - 1)\, (1 - 1_p(a = 0, z)) \Big),$$

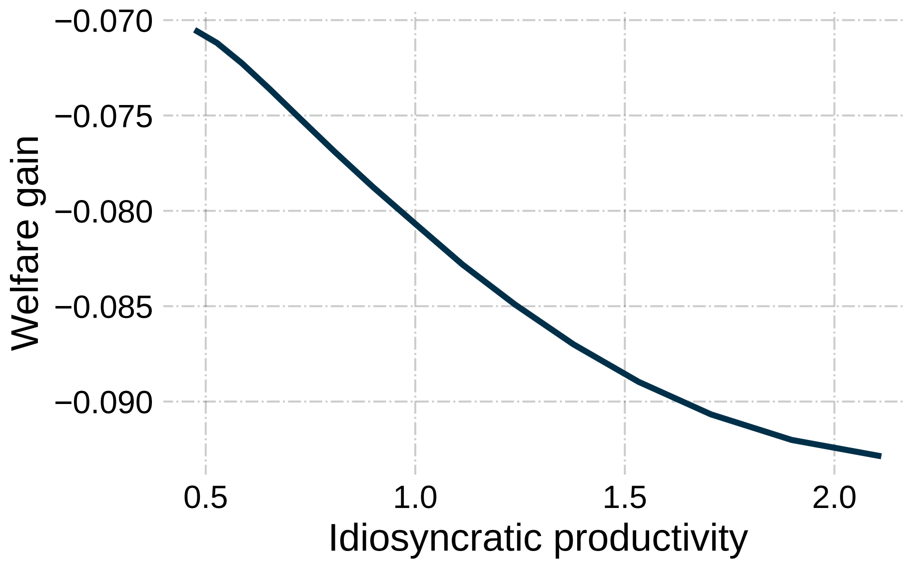


Figure 29: Welfare gain as a function of z

where S_z is the unconditional probability of drawing an idiosyncratic productivity z, and $1_p(a = 0, z)$ is the participation decision of an age 1 individual with 0 assets and productivity z.

Figure 29 plots the welfare gain of moving from the benchmark economy to 45-percent-tax economy for workers with different idiosyncratic productivity at age 1 after the workers made their participation decisions. As seen in the figure, the welfare gain is decreasing with productivity. Notice that capital-labor ratio goes down after an increase in the tax rates, which leads to an increase in the capital rental rate and reduction in the wage rate. Consequently, the value of employment goes down. Since higher productivity individuals tend to work, the reduction in the value of employment has larger effects on them. Therefore, the welfare loss of a tax increase is larger for the high productivity workers.

The expected welfare gain of a worker before his productivity is revealed, $\tilde{\xi}$, is equal to -8.1%. Overall, this policy results in a welfare loss. On the one hand, increased rental rate of capital and transfers increase the welfare. On the other hand, reduction in pre- and after-tax wage rates reduces the benefits of the employment. Since all individuals start with 0 wealth, the benefit of increase in the rental rate of capital is limited. In this quantitative exercise, the welfare loss due to wage reductions dominates.