Gross Worker Flows over the Life Cycle*

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Abstract

This paper constructs a quantitative general equilibrium model that allows us to analyze the gross worker flows over the workers' life cycle and the interaction of various policies with these flows. We first document the life-cycle patterns of flows across different labor market states (employment, unemployment, and not in the labor force), as well as job-to-job transitions in the US. Then we build a model of the aggregate labor market that incorporates the life cycle of workers, consumption-saving decisions, and labor market frictions. We estimate the model with the US data and find the model fits the data patterns very well. Through the lens of the model, we uncover the fundamental forces that drive the life-cycle patterns. Finally, we use the estimated model to investigate the effects of policies on aggregate labor market outcomes, such as the unemployment rate and the labor force participation rate. In particular, we analyze a taxes-and-transfers policy and an unemployment insurance policy.

Keywords: worker flows, life cycle, taxes and transfers, unemployment insurance

JEL Classification: E24, J21, J22, J64, J65

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1 Introduction

Over the last several decades, the study of the aggregate labor market has made significant progress by analyzing the gross job flows and the gross worker flows. By investigating beyond the net changes in labor market stocks, such as the unemployment rate and employment-population ratio, our understanding of the labor market dynamics and the effects of labor market policies has deepened substantially.

This paper contributes to this literature. We analyze worker flows across three different labor market states—*employment* (E), *unemployment* (U), and *not in the labor force* (or *nonparticipation*) (N)—over the workers' life cycle. These gross flows influence the policy-relevant labor market stocks, such as the employment-population ratio, the unemployment rate, and the labor force participation rate. In addition, we consider an important worker flow: the flow of employed workers across different jobs. Various studies have found such *job-to-job transitions* play an important role in macroeconomic outcomes by reallocating workers to appropriate jobs. For example, Topel and Ward (1992) attribute about 40% of wage growth for young workers to job transitions. More recently, Engbom (2020) argues the patterns of job-to-job transitions, combined with human capital accumulation, can explain a large part of the differences in life-cycle wage growth patterns across OECD countries.[1] We document the empirical patterns of these flows and conduct a quantitative-theoretic analysis based on these observations.

The particularly novel element in our analysis is the life cycle of workers. Various empirical studies have documented that the flows and stocks in the labor market vary substantially with age. For example, the unemployment rate for young workers is known to be higher than for prime-age workers, and young workers tend to experience more frequent job-to-job transitions than older workers. All gross flows, including the ones involving the participation margin, are important in shaping the heterogeneous outcomes in the labor market across different age groups. The accounting exercise by Choi et al. (2015) reveals, for example, that movements from the N state to the E state and from the N state to the U state (we will call them NE flow and NU flow) account for a large part of the lower participation and unemployment rates for old workers.

In this paper, we build a quantitative general equilibrium model that replicates the behavior of the individuals we focus on, run policy experiments using the model, and interpret the mechanisms. Using the framework, we ask how labor market policies affect the flows and stocks in the labor market for different age groups of workers. We focus on two policies: the first policy involves taxes and transfers, and the second is unemployment insurance (UI).

Our model features a frictional labor market with an operative labor supply margin, based on Krusell et al. (2010, 2011, 2017). Krusell et al. (2010), in particular, study the effects of taxes and transfers using an infinite-horizon model and find the existence of frictions

[1] Barlevy (2002) and Mukoyama (2014) analyze the effect of job-to-job transitions on aggregate productivity. Their model analyses imply the effect of job-to-job transitions on aggregate productivity can be sizable.

influences the behavior of the labor supply margin in this class of models. Our departure from their analysis is that we explicitly consider the worker life cycle. This departure is essential because (i) the heterogeneity in worker flows across different age groups is so significant that analyzing the policy effects with an explicit treatment of this heterogeneity is itself very important, (ii) this framework is the first that features labor market frictions and the operative labor supply margin in a life-cycle context, and this framework can be applied to many other policy experiments, and (iii) quantitatively matching the model to data is quite challenging because fitting six *life-cycle profiles* (plus the job-to-job flow rate and the wage profile) is significantly more difficult than fitting six *numbers* (corresponding flow rates in aggregate). As can be seen below, substantial extensions of the model, compared with Krusell et al. (2010, 2011, 2017), are necessary for the model to replicate salient life-cycle patterns of worker flows in the data.

The estimated model fits the data patterns very well. Through the lens of the model, we uncover the fundamental forces that drive the life-cycle pattern. We find that across different age groups, the magnitude of heterogeneity in the opportunities for a new job is relatively small compared with the observed heterogeneity in the corresponding flow rates. The outcome highlights the importance of worker choices and how they change over the life cycle. To properly consider the heterogeneity relevant for policy analyses, it is essential to utilize an economic model that incorporates the workers' economic choices rather than a mechanical accounting model. The two policy experiments we conduct reveal the heterogeneous effects of the policies on worker flows and stocks for different life-cycle stages.

The main contribution of this paper is theoretical: we provide a framework that can replicate the salient features of life-cycle worker flows, and this framework can be used for various policy analyses. To illustrate the usefulness of our framework, we conduct policy exercises that have been analyzed extensively in the macroeconomic literature. Our model features (i) the worker life cycle, (ii) the frictional labor market with heterogeneous jobs, and (iii) the operative labor supply margin with concave utility and self-insurance. The model can fit the quantitative features of the life-cycle patterns of labor market flows and stocks, allowing us to analyze the effect of policies on the labor market outcomes of different age groups of workers. We intentionally keep the model parsimonious so that the mechanisms remain transparent despite the quantitative nature of the policy experiments. In particular, as in Krusell et al. (2010, 2011, 2017), the labor market frictions are modeled using a simple "island" structure, because the most important channel for our experiment is operative labor supply.

The paper is related to several strands of literature. First, several recent papers have analyzed life-cycle worker flows in a frictional labor market. The contributions include Chéron et al. (2013), Esteban-Pretel and Fujimoto (2014), Menzio et al. (2016), and Jung and Kuhn (2019). None of these papers, however, explicitly model the endogenous participation margin. The above papers instead emphasize the labor demand side by incorporating (variants of) the Diamond-Mortensen-Pissarides (DMP)-type matching process. As we see later in detail, the operative labor supply channel is essential for the policy experiments in this paper. Our model features a very good fit to the observed life-cycle patterns of worker flows. Fitting six flows (plus the job-to-job flow and the wage series) as functions of age is significantly more challenging than fitting six numbers (as Krusell et al. (2011, 2017) do), and constructing a framework that can replicate the data pattern is one of our significant contributions.

Two recent papers feature worker flows across three states in a life-cycle setting. Lalé and Tarasonis (2020) describe the life-cycle pattern of worker flows in European countries and construct a three-state life-cycle model. Goensch et al. (2021) extend Menzio et al.'s (2016) model and add a search decision of workers. In contrast to our study, both papers feature linear utility and abstract the wealth effect that plays an important role in our policy experiments. Instead, these models have an active labor demand side in the form of firms' vacancy postings. We abstract the vacancy posting by firms to focus on analyzing the labor supply side under incomplete markets, and in that sense, these studies are complementary to our work.

Second, the policies we consider have been extensively analyzed in the macroeconomic literature. For the taxes-and-transfers policy, examples include Prescott (2004), Ohanian et al. (2008), Alonso-Ortiz and Rogerson (2010), and Krusell et al. (2010). Compared with these studies, this paper is novel in that we explicitly consider life-cycle elements in a framework that features incomplete asset markets and labor market frictions. Incorporating life-cycle elements is important because patterns of transitions across labor market stocks are markedly heterogeneous over the life cycle. Incorporating frictions enables us to talk about the effect of taxes and transfers on unemployment. The structure of incomplete asset markets with concave utility allows us to consider each consumer's asset accumulation and life-cycle behavior, particularly how the wealth effect operates. An important interaction also exists between self-insurance and precautionary saving, in that transfers can act as insurance against employment shocks. For this experiment, Ljungqvist and Sargent (2008) is closest to ours in this literature. Similar to our paper, they analyze a general equilibrium incomplete-market life-cycle model with indivisible labor and search decision. The largest difference is that they do not explicitly analyze gross worker flows. A recent paper by Pizzo (2020) analyzes the effect of progressive taxation in Krusell et al.'s (2010) framework, while abstracting the labor supply margin.

A large literature exists on the analysis of UI policy under incomplete markets. Alvarez and Veracierto (1999) employ a similar market structure to ours, that is, an "island" search model with workers' search decisions and competitive factor markets. Like our paper, their paper focuses on the workers' search decision by abstracting from firms' search. They do not explicitly consider the gross worker flows and life cycle. A complementary literature focuses on the firms' search (vacancy posting) decisions, employing the DMP search and matching model. Examples include Krusell et al. (2010), Mukoyama (2013), Mitman and Rabinovich (2015), Jung and Kuester (2015), Landais et al. (2018), and Setty and Yedid-Levi (2021). These papers do not consider the participation decision by workers, and their models also abstract from life-cycle considerations.

Third, an extensive macroeconomic literature analyzes the life-cycle labor supply. Examples include Rogerson and Wallenius (2009), Low et al. (2010), and Erosa et al. (2016). These studies do not explicitly match the patterns of gross worker flows observed in the data. The explicit analysis of gross worker flows allows us to relate the effect of the policy on stocks to the patterns of reallocation in an economy with heterogeneous agents.

Finally, from a modeling perspective, our model has the Bewley-Huggett-Aiyagari (BHA) structure with a frictional labor market and operative labor supply with indivisible labor. Thus, our model shares many features with Chang and Kim (2006). Compared with Chang and Kim (2006), our model incorporates the worker life cycle and frictional labor market.

One significant advantage of employing the BHA framework is that we can explicitly incorporate precautionary wealth holding. Our model outcome fits the life-cycle pattern of wealth holding in the US data reasonably well. Explicitly analyzing individual wealth holding is important for three reasons. First, the wealth effect is a critical determinant of the individual labor supply. For example, Cesarini et al. (2017) document that lottery winners reduce their labor supply immediately and persistently. Second, as Krusell et al. (2017) show, the labor market flows are closely associated with the individual wealth level. Third, to evaluate the effect of policies like UI, it is essential to consider the degree of self-insurance explicitly. A cost of employing the BHA structure is the model complexity, and incorporating labor market frictions in a BHA model is extremely challenging. As Krusell et al. (2010) and Mukoyama (2013) show, incorporating the DMP-style labor demand side can make the model significantly complex even without a life cycle and the participation margin. Some studies, such as Griffy (2021), achieve simplifications by adopting a directed search assumption, but to our knowledge, no existing papers have successfully incorporated all elements (gross worker flows across three states with an operative participation margin, life cycle, and incomplete asset markets) in a tractable general equilibrium model. We have decided to focus on the labor supply response to policies, given that the labor demand side is already extensively analyzed in the complementary literature (cited above) using the DMP structure.

The paper is organized as follows. The following section summarizes the empirical patterns of the gross worker flows over the life cycle. Section 3 sets up the model, and we calibrate the model in Section 4. Section 5 conducts policy experiments. Section 6 concludes.

2 Empirical observations

This section briefly summarizes the life-cycle patterns of worker flows and stocks in the US data. We use these data patterns for quantifying the model in the next section.

2.1 Data

We use the monthly files of the Current Population Survey (CPS) from 1994 to 2017. Because our preliminary analysis found notable differences in labor market flows between men and women (likely related to decisions to stay at home and take care of children, which are more common for women and are beyond the scope of our model analysis), we decided to limit the sample to the population of men. This sample selection, of course, does not mean incorporating women's labor supply behavior is not important—the analysis of this paper should be viewed as merely a first step.

To calculate transition rates between different labor market states, we longitudinally match observations over two consecutive months using data on household and person identification variables and sex, race, and age, as is standard in the literature. Additionally, we correct for transitions that are plausibly spurious by using the deNUNifying procedure (purging the temporary appearance of U state by, for example, replacing N-U-N with N-N-N) as described in Elsby et al. (2015). Life-cycle profiles are obtained by estimating weighted OLS regressions of each labor market stock and flow on a set of age dummies.

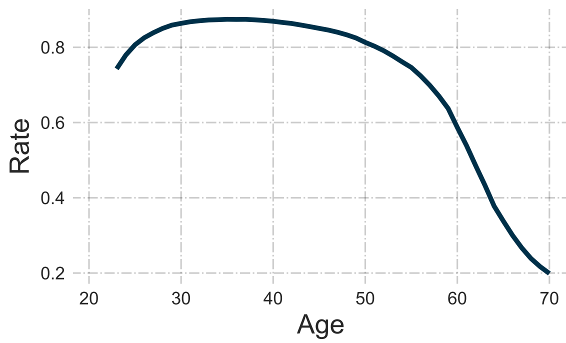
2.2 Labor market stocks

First, we describe the life-cycle patterns of stocks in the labor market. In this study, we focus on male workers ages 23 to 70. All the data figures are means of six-year-moving windows over age, and the horizontal-axis labels are the midpoints of the windows.[2] Figure 1 plots the age profile of employment. The employment-population ratio exhibits an inverted-U shape: smaller fractions of young and old workers are employed than middle-aged workers. Because the employment-population ratio can be represented as

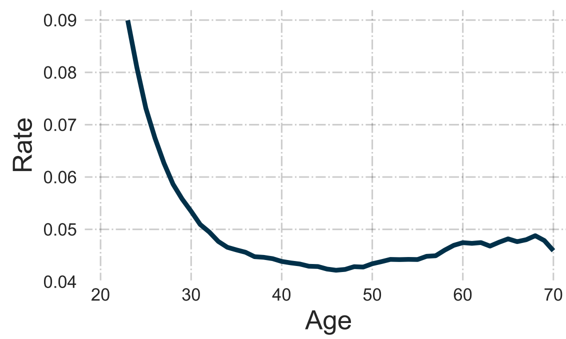
$$\frac{E}{E+U+N} = (1-u)p,$$

where $u \equiv U/(E+U)$ is the unemployment rate and $p \equiv (E+U)/(E+U+N)$ is the labor force participation rate, analyzing the life-cycle behavior of the employment-population ratio requires explicit analysis of gross flows involving both U and N. As we can see from the comparison between panel (a) and panel (c), the pattern of the employment-population ratio mostly mimics the pattern of labor force participation. The unemployment rate also exhibits a strong life-cycle pattern, although the pattern is markedly different from the one associated with labor force participation. Young workers below 30 years old experience a significantly higher unemployment rate than other age groups, whereas the unemployment rate slightly increases past the age of 40. This pattern of unemployment also contributes non-trivially to the low employment-population ratio, especially for young workers.

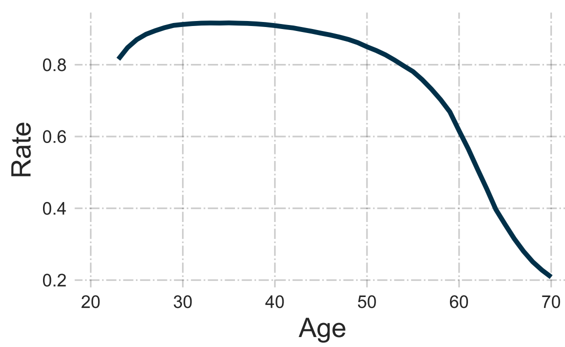
[2] We calculate our data moments from age 16 onward, take the rolling means, but report only age 23 and above, which is the age group we focus on.



(a) Employment-population ratio



(b) Unemployment rate



(c) Labor force participation rate

Figure 1: Labor market ratios in the data

2.3 Labor market flows

The main innovation of this paper is to provide a model analysis for gross worker flows. The patterns in the data have previously been described by Choi et al. (2015), for example; thus, our summary here is brief.[3] Figure 2 plots the monthly gross worker flow rates over the life cycle. The notations are conventional: with E for employment, U for unemployment, and N for nonparticipation, the flow rate ij represents the worker's movement from state i to state j. The EE flow rate represents the job-to-job transition rate. The flow rate ij is computed by dividing the number of workers who moved from state i to state j between time t to time $t+1$, divided by the stock of the state i at time t.

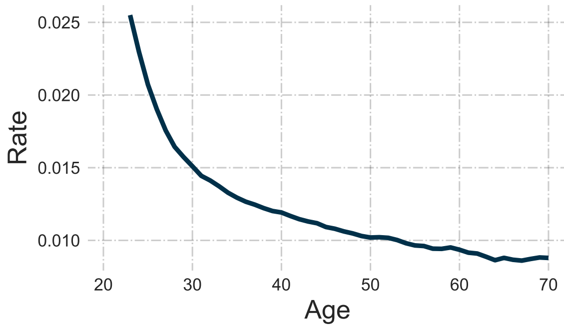
All gross flow rates have clear life-cycle patterns. Overall, young workers tend to have higher mobility across states (and across jobs) than other age groups. Very old workers have a strong tendency to move into the N state, likely because of their retirement.

By comparing the patterns of gross flow rates with the stocks in the previous section, Figure 2 shows the large inflows into N (panels (b) and (d)) for the young and very old contribute to the inverted-U pattern of the labor force participation rate, although the outflow rates (panels (e) and (f)) have offsetting effects for young workers. For the unemployment stock, the high inflow rates from E and N (panels (a) and (f)) contribute to high unemployment rates of young workers, although the outflow rates (panels (c) and (d)) have offsetting effects. Thus, overall, to explain the patterns of labor force participation, accounting for the particularly strong life-cycle pattern of the inflow into N is important. For the unemployment rate, the large flow into U is key to understanding the high unemployment rate of young workers.

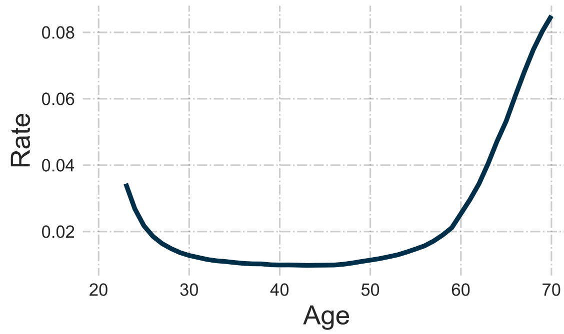
The behavior of flows and stocks in the steady state, described here, does not necessarily directly speak to their reactions to the policies. However, they provide an important guideline to construct and quantify the relevant model. In the next section, we build a model that contains all relevant elements and is sufficiently flexible to match the data patterns. The results of the policy experiments that come out of the model analysis are credible in the sense that the model itself is consistent with the life-cycle patterns we observe in the data.

A separate yet interesting question is why young workers' flow rates are generally so high. The model in the next section, which incorporates both individuals' voluntary movements across states (as a reaction to changes in productivity and wealth) and labor market frictions, has the potential to provide insights into the origins of young workers' high mobility.

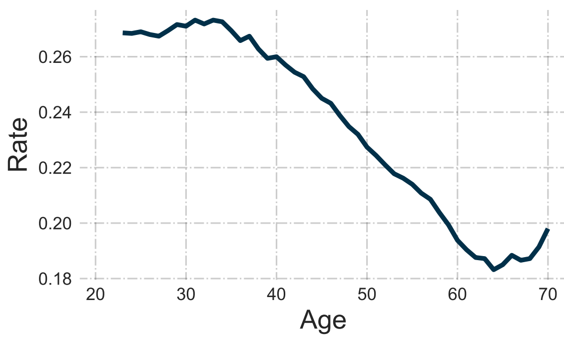
[3] Although Choi et al. (2015) use data from 1976 to 2013, our empirical patterns are essentially identical to theirs. One difference is that our NU and UN flow rates have slightly lower levels due to the deNUNifying procedure, but life-cycle patterns are nevertheless very similar.



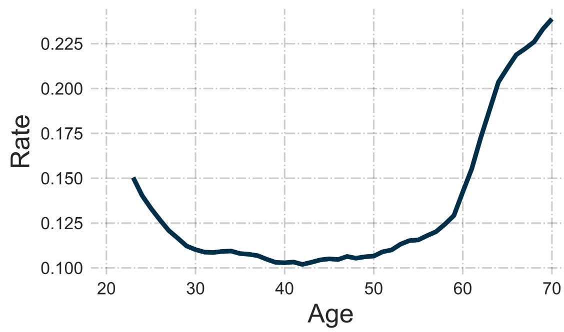
(a) EU



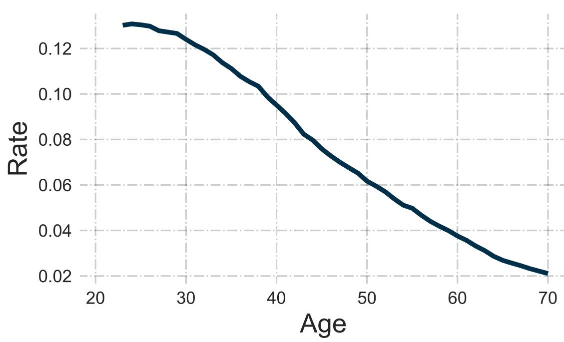
(b) EN



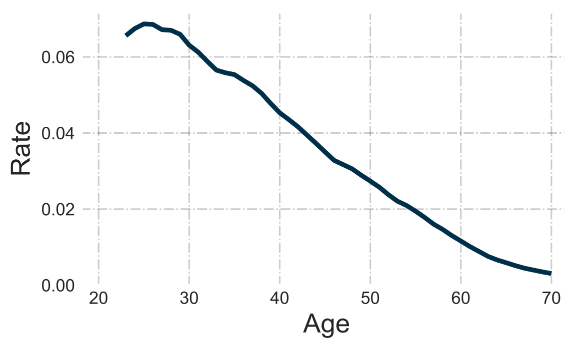
(c) UE



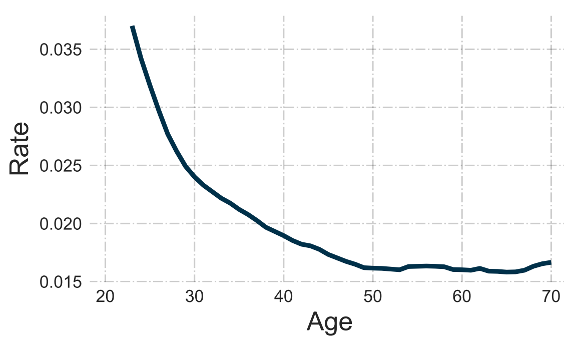
(d) UN



(e) NE



(f) NU



(g) EE

Figure 2: Gross flow rates in the data

3 Model

Our model extends Krusell et al. (2010, 2011, 2017) to a life-cycle setting. In addition to the worker life cycle, the model features a frictional labor market with heterogeneous jobs and an operative labor supply margin with concave utility and self-insurance. Thus, the model has the BHA structure with labor market frictions and operative labor supply. An attractive feature of this type of model is that the individuals in the model behave consistently with the permanent income hypothesis, which has been extensively studied in the consumption-saving literature. Krusell et al. (2011, 2017) have already shown that (the infinite-horizon version of) the model is consistent with the overall behavior of the gross flows in the economy, including the duration of each state, flow rates for wealth quintiles, and business-cycle properties. The details of the model computation are presented in Online Appendix A.

Similar to Krusell et al. (2010), the model features a general equilibrium in that the prices depend on the aggregate capital (which the workers accumulate) and the aggregate labor. One important caveat (shared by Krusell et al. (2010, 2011, 2017)) is that the labor market frictions are exogenous and assumed to be policy invariant. This modeling decision reflects our focus on the labor supply margin in the policy experiments.

3.1 Overall model structure

Three types of agents—workers, firms, and the government—exist in the economy. The workers supply labor and rent capital out to the firms. The total worker population is normalized to 1. Using capital and labor, firms produce the final good that can be used for consumption and investment. The government taxes labor and UI payments and transfers taxes back to all workers in a lump-sum manner. All markets are perfectly competitive. The rental market for capital and the final-good market are frictionless, as in the standard BHA model. As in the BHA model, the financial market is incomplete. The workers can self-insure by accumulating capital stock.[4]

In the labor market, the worker's labor supply is indivisible in the sense that she can supply either zero or one unit of labor each period. The labor market is frictional. For the frictional labor market to be compatible with perfect competition, we consider the following arrangement, similar to Krusell et al. (2017).

The economy has two islands, *work island* and *leisure island*. All firms are located on the work island. Workers on the work island are employed by firms and receive wages. The work island is divided into many (continuum of) *districts*, and each worker works for one of the firms located in the district she lives in. The total measure of districts is normalized to 1.

Each worker's productivity has three components: the age component, general productivity, the match-specific productivity. General productivity applies to the worker when working

[4]We do not include the transaction costs for asset accumulation, and thus, the asset in this model is all "liquid assets," although we do not make explicit the distinction below.

with any firm, whereas match-specific productivity applies when working in a firm located in that district. In other words, the match-specific productivity is specific to the district-worker match. Because many firms exist in the district, the wages are still determined competitively even though the match-specific component exists. All workers on the leisure island do not work.

The mobility of workers across islands is limited, and this lack of mobility is a source of the labor market frictions. Workers on the leisure island receive an opportunity to move to a randomly drawn district every period. The frequency of this job opportunity depends on the search effort of the worker; if the worker searches, in which case she is categorized as *unemployed*, she receives job opportunities more frequently than when she does not search, in which case she is categorized as *not in the labor force*. On the work island, moving across different districts is limited; every period, an employed worker may receive an opportunity to move to another district (an "outside job offer") with some probability. We assume the worker doesn't move across firms within a district (therefore, no job-to-job transitions occur within a district), given that, in equilibrium, the worker would receive an equal wage from any firm within the same district. With some probability, an employed worker receives a separation shock and is forced to move to the leisure island. Employed workers can voluntarily move to the leisure island anytime they want to.

Note the labor market structure with similar spatial frictions (the "island model") has a long tradition following Lucas and Prescott (1974). In contrast to an alternative modeling strategy, following the Diamond-Mortensen-Pissarides framework (Pissarides, 1985), the model abstracts from the firms' vacancy-posting activity. The island model is especially suitable for analyzing policies for which the labor supply margin is operative. Therefore, we later demonstrate the model's usefulness using two policies for which the labor supply margin is essential.

3.2 Workers

A worker is characterized by the following: (i) her labor market state: employed (has a job), unemployed (not employed but actively searching for a job), not in the labor force (not employed and not searching for a job); (ii) her wealth (in capital stock), a; (iii) her idiosyncratic general productivity, z; (iv) her match-specific productivity (if employed), μ, and (v) her age, j. Let s_j be the survival probability of a worker from age j to $j+1$. Then each worker maximizes:

$$\mathbf{U}_w = \sum_{j=1}^{J} \left(\beta^j \prod_{t=1}^{j} s_t \right) E_0[\log(c_j) - d_j],$$

where c_j is the consumption at age $j \in \{1, ..., J\}$ and d_j is the disutility of working or searching, which are detailed below. $E_0[\cdot]$ represents the expected value taken at age 0. The discount factor is $\beta \in (0,1)$.

The log of idiosyncratic general productivity, $\log(z)$, is stochastic and follows an AR(1) process. The job-offer probabilities, which are age-dependent, are denoted as $\lambda_u(j)$, $\lambda_n(j)$, and $\lambda_e(j)$ for unemployed, not in the labor force, and employed workers at age j. An unemployed worker incurs a search cost of ψ for active searching. An employed worker with general productivity z, match-specific productivity μ, and age j receives a wage

$$\omega_j(\mu, z) \equiv g(j)\mu z\tilde{\omega},$$

where the function $g(j)$ is the deterministic age component of market productivity and $\tilde{\omega}$ is the wage per efficiency unit of labor. While working in a firm, $\log(\mu)$ follows an AR(1) process. At the end of a period, a match is destroyed with a probability σ_j, depending on the worker's age. A worker on the leisure island receives h units of the final goods from home production.

Upon being matched, the worker draws the match-specific component of productivity μ. We assume the true quality of the match is not revealed immediately with a probability ζ. In each period, if the match quality is unknown, it remains unknown with probability ζ. In that case, the value of μ is assumed to be $\bar{\mu}$. The wage is also based on $\bar{\mu}$, and therefore no learning from wages occurs. With probability $1-\zeta$, the true quality is revealed. This gradual learning of match quality is necessary to make the job-to-job transition process in the model match the data. Without such a mechanism, young workers learn their match quality too quickly, and the job-to-job transition rate declines too rapidly with age. Esteban-Pretel and Fujimoto (2014), Gorry (2016), and Menzio et al. (2016) use similar formulations.

We assume the true match-quality shocks for the newly matched are drawn independently from a Pareto distribution with parameters (μ_1, α), where μ_1 is the lower bound of the support of the match-quality distribution, and α determines the rate at which the density of the distribution decreases (note M denotes the random variable and μ denotes its realization):

$$\Pr[M > \mu] = \begin{cases} \left(\dfrac{\mu_1}{\mu}\right)^{\alpha} & \text{for } \mu \geq \mu_1, \\ 1 & \text{for } \mu < \mu_1. \end{cases}$$

The new match quality for an employed worker who obtains an outside job offer is drawn from the same distribution.

Those nonemployed workers who lost their jobs due to an exogenous job separation shock and are actively seeking a job receive a transfer proportional to their productivity for a limited duration. Therefore, the employed, the nonparticipants, the unemployed who quit their jobs, and the unemployed whose benefits have expired cannot receive this benefit, which mimics the UI payments in the US. Let $b(z, \mu, j) = \min\{b_0\tilde{\omega}z\mu g(j), \bar{b}\}$ be the UI that an eligible unemployed age-j worker receives, where z is the current idiosyncratic productivity of the worker, μ is the match-specific productivity the worker had in her last position, $g(j)$ is the market productivity, and b_0 is the UI replacement rate. Therefore, the payment that an eligible unemployed worker receives is proportional to the wage she would have gotten if she had kept her position. The payment has the cap $\bar{b}$. Auray et al. (2019) show that, in the US from 1989 to 2012, only 77% of eligible workers collected their benefits. Thus, we assume the (randomly selected) fraction $\chi = 0.77$ of the above workers are actually eligible.

The timing within a period is the following. First, idiosyncratic general productivity shocks and match-specific productivity shocks for already-employed workers realize. Second, some nonemployed workers find jobs, and the initial match-specific shocks for new jobs are drawn. Some employed workers receive an opportunity to move to another district with a new match-specific shock realization. Third, nonemployed workers with job opportunities decide whether to accept the match, and employed workers with moving opportunities decide whether to move. Then, production and consumption take place. At the end of the period, possible death and separation shock occur.

Let the value function of an employed worker at age j be $W_j(a, z, \mu)$, the value function of a UI-ineligible unemployed worker be $U_j(a, z)$, the value function of a UI-eligible unemployed worker be $\tilde{U}_j(a, z, \mu)$, and the value function of a worker who is not in the labor force be $N_j(a, z)$.

The Bellman equation for the employed is

$$
\begin{aligned}
W_j(a, z, \mu) = \max_{c_j, a'} \Bigg\{ & u(c_j) - \psi\gamma + \beta s_j E_{\mu', z'}[(1-\sigma_j)(1-\lambda_e(j))T_{j+1}(a', z', \mu') \\
& + (1-\sigma_j)\lambda_e(j)S_{j+1}(a', z', \mu') \\
& + \sigma_j(1-\lambda_e(j))\chi\tilde{O}_{j+1}(a', z', \mu) + \sigma_j(1-\lambda_e(j))(1-\chi)O_{j+1}(a', z') \\
& + \sigma_j\lambda_e(j)\chi\tilde{F}_{j+1}(a', z', \mu) + \sigma_j\lambda_e(j)(1-\chi)F_{j+1}(a', z')] \Bigg\},
\end{aligned}
$$

subject to

$$
c_j + a' = (1+r)a + (1-\tau)\omega_j(\mu, z) + \mathbf{T}
$$

and

$$
a' \geq 0,
$$

where

$$
T_{j+1}(a', z', \mu') = \max\{W_{j+1}(a', z', \mu'), O_{j+1}(a', z')\},
$$

$$
S_{j+1}(a', z', \mu') = \int_{\underline{\mu}}^{\bar{\mu}} \max\{T_{j+1}(a', z', \mu'), W_{j+1}(a', z', \hat{\mu})\} dG(\hat{\mu}),
$$

$$
O_{j+1}(a', z') = \max\{U_{j+1}(a', z'), N_{j+1}(a', z')\},
$$

and

$$
F_{j+1}(a', z') = \int_{\underline{\mu}}^{\bar{\mu}} \max\{W_{j+1}(a', z', \mu), O_{j+1}(a', z')\} dG(\mu).
$$

$\tilde{X}$, where $X = O, F$, represents the value function of a worker who faces the same choices as in X, but she is also eligible for UI if she chooses to be unemployed. Here, r is the real interest rate (rental rate of capital), τ is the labor income tax rate (and UI payment tax rate), and

$\mathbf{T}$ is the lump-sum government transfer. Each employed worker faces four possible scenarios in the next period: (i) not receiving a separation shock (σ_j) or an outside job offer, in which case she needs to decide between continuing with employment or becoming nonemployed (the value function T); (ii) not receiving a separation shock, but receiving an outside job offer, in which case she additionally needs to decide whether to switch jobs (the value function S), where $G(\cdot)$ is the outside wage-offer distribution; (iii) receiving a separation shock and no outside offer, in which case she becomes nonemployed and needs to decide whether to search (the value function O); or (iv) receiving a separation shock and an outside job offer, in which case she can move directly to another firm (the value function F). Remember that a fraction χ of UI-eligible workers actually receive UI, which is reflected in scenarios (iii) and (iv). While employed, a worker faces disutility of work equal to ψ times γ, where γ represents the relative disutility of working over an active job search.

The Bellman equation for the UI-ineligible unemployed is

$$U_j(a,z) = \max_{a',c_j} \left\{ u(c_j) - \psi + \beta s_j E_{z'}[\lambda_u(j) F_{j+1}(a',z') + (1-\lambda_u(j)) O_{j+1}(a',z')] \right\},$$

subject to

$$c_j + a' = (1+r)a + h + \mathbf{T}$$

and

$$a' \geq 0,$$

where h is the home production. The parameter ψ is the disutility of active search effort.

The Bellman equation for the UI-eligible unemployed is

$$\begin{aligned}\tilde{U}_j(a,z,\mu) = \max_{a',c_j} \Big\{ & u(c_j) - \psi + \beta s_j E_{z'}[\lambda_u(j)\eta F_{j+1}(a',z') + (1-\lambda_u(j))\eta O_{j+1}(a',z') \\ & + \lambda_u(j)(1-\eta)\tilde{F}_{j+1}(a',z',\mu) + (1-\lambda_u(j))(1-\eta)\tilde{O}_{j+1}(a',z',\mu)] \Big\},\end{aligned}$$

subject to

$$c_j + a' = (1+r)a + h + (1-\tau)b(z,\mu,j) + \mathbf{T}$$

and

$$a' \geq 0,$$

where b is the UI as defined above and η is the probability with which a UI-eligible unemployed loses her UI benefits.

Workers not in the labor force are not subject to the disutility of active search, but their job-offer probability will be different (lower), as explained later:

$$N_j(a,z) = \max_{a',c_j} \left\{ u(c_j) + \beta s_j E_{z'}[\lambda_n(j) F_{j+1}(a',z') + (1-\lambda_n(j)) O_{j+1}(a',z')] \right\},$$

subject to

$$c_j + a' = (1+r)a + h + \mathbf{T}$$

and

$$a' \geq 0.$$

3.3 Firms

In each district k of the work island, competitive firms with a constant-returns-to-scale production function operate. The production function for the representative firm in district k takes the Cobb-Douglas form,

$$Y_k = AK_k^{\theta} L_k^{1-\theta},$$

where $\theta \in (0, 1)$ and $A > 0$ is a parameter. The inputs K_k and L_k are the capital and labor (in efficiency units) demands. Capital is freely mobile across districts, although labor mobility is restricted. Total capital and labor demand in the economy are

$$K = \int_0^1 K_k dk$$

and

$$L = \int_0^1 L_k dk.$$

Because we assume capital is freely mobile (within and) across districts, the rental rate under a competitive market,

$$r = A\theta \left(\frac{K_k}{L_k} \right)^{\theta - 1} - \delta,$$

is common across districts. This equalization implies the capital-labor ratio, K_k/L_k, is common across districts. Therefore, the wage per efficiency unit of labor,

$$\tilde{\omega} = A(1-\theta) \left(\frac{K_k}{L_k} \right)^{\theta},$$

is also equalized. We assume firms within a district are homogeneous, and the allocation of workers to the districts is entirely random. With the law of large numbers, each district's L_k becomes the same in stationary equilibrium. Therefore,

$$K = K_k$$

and

$$L = L_k$$

hold in stationary equilibrium. Capital stock depreciates at a rate δ.

3.4 Government

The government collects tax on labor income and UI payments, and also confiscates assets of the deceased individuals in the economy. It redistributes all revenue to individuals in the economy uniformly while running a balanced budget.

Thus, the government budget constraint is

$$\mathbf{T} = \tau \int e(i)\omega_{j(i)}(i)(\mu(i), z(i))di + \int a(i)(1 - s(i))di - \int (1 - \tau)b(i)di, \tag{1}$$

where i is the index for each individual. Here, $e(i)$ is the employment status of individual i, with 1 for employed and 0 for not employed, $j(i)$ is the age of individual i, and $s(i)$ is the survival status of individual i, with 1 for surviving individuals and 0 for deceased individuals. $b(i)$ represents the UI benefits to worker i (0 for the workers who are not collecting the UI).

3.5 Equilibrium

We solve for a stationary equilibrium in which the real interest rate and wage profile are constant over time. After all new matching opportunities realize (with new idiosyncratic productivity and match-specific shocks), workers make the following decisions.

(i) A nonemployed worker at age j, wealth a, idiosyncratic productivity z, and has an offer of match-specific productivity μ accepts the offer and becomes employed if and only if

$$W_j(a, z, \mu) \geq O_j(a, z).$$

(ii) A nonemployed worker at age j, wealth a, and idiosyncratic productivity z who rejected a job offer or did not receive a job offer decides to be in the labor force if and only if

$$U_j(a, z) \geq N_j(a, z).$$

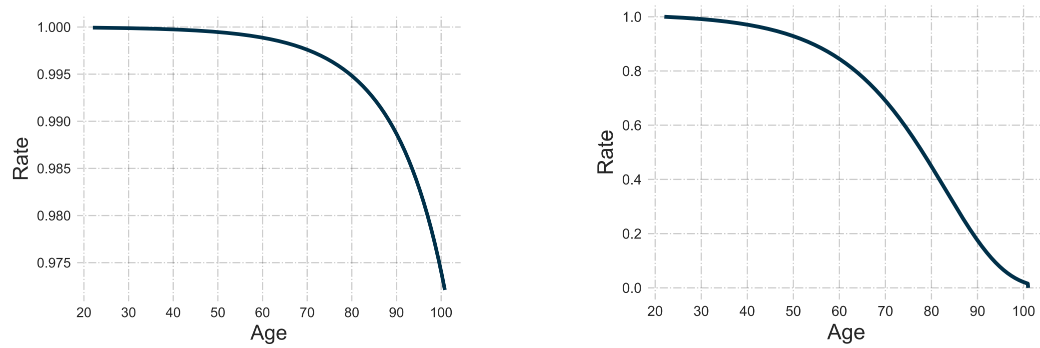
(iii) An employed worker at age j, wealth a, idiosyncratic productivity z, and current match-specific productivity μ, who does not have an outside job offer stays in her job if only if

$$W_j(a, z, \mu) \geq O_j(a, z).$$

(iv) An employed worker at age j, wealth a, idiosyncratic productivity z, current match-specific productivity μ, and outside offer μ' switches jobs if and only if

$$W_j(a, z, \mu') > T_j(a, z, \mu).$$

(v) Each worker makes optimal consumption and investment decisions according to the Bellman equations described in Section 3.2.



(a) Survival rate

(b) Cumulative survival rate

Figure 3: Survival rate

Without loss of generality, the above decision rules are written for the UI-ineligible workers. O and U could be replaced with $\tilde{O}$ and $\tilde{U}$ when the worker is eligible for UI.

Capital and labor markets clear.

(i) Total assets supplied are equal to total capital demand,

$$\int a_i di = K.$$

(ii) Labor supply in efficiency units is equal to labor demand,

$$\int e(i) z_i \mu_i g_i di = L.$$

As described in Section 3.4, the government runs a balanced budget, represented by the constraint (1): the total lump-sum transfer is equal to the sum of labor income tax revenue and wealth of the deceased agents minus after-tax UI payments.

4 Calibration

In quantifying the model, first, a subset of parameters is calibrated using external information. Then the remaining parameter values are estimated so that the distance between the model outcome and the data is minimized.

Each period corresponds to one month.[5] Following Krusell et al. (2010), we consider $\tau = 0.30$ as the benchmark. On the production side, θ is set at 0.3. The death probabilities

[5]We assume model age $j = 1$ corresponds to an annual age of 22. The monthly age after which everyone dies for sure is $J = 947$, which corresponds to an annual age of (one month before) 101. However, during calibration, we only consider workers at ages between 23 and 70.

at each age are taken from life tables at the Social Security Administration.[6] The calibrated survival rates are plotted in Figure 3. The persistence parameter of the monthly AR(1) idiosyncratic productivity (the logarithm of z) process is set to $\rho_z = 0.97$ and the persistence parameter of the monthly AR(1) match-specific productivity (the logarithm of μ) process is set to $\rho_\mu = 0.98$, broadly consistent with estimates in Balke and Lamadon (2022).[7] We assume match-specific productivity of matches with unrevealed quality is equal to median productivity, $\bar{\mu} = 1.0$.

The interest rate, r, is targeted to be equal to 0.00327 in equilibrium, which corresponds to a 4% annual compound interest rate. A is set to 0.49 to normalize $\tilde{\omega}$ to 1 in equilibrium. The investment-to-GDP ratio is targeted to be equal to 20%. Following Krusell et al. (2017), we set the replacement rate b_0 so that the total UI payments are 0.75% of total earnings. The cap $\bar{b}$ is set at 50% of the average pre-tax wage in the economy. To account for the limited duration of unemployment benefits, we assume an eligible worker loses her benefits with a probability equal to $\eta = 1/6$.

For age-dependent parameters, we allow them to be a simple function of age. Specifically, let the age component of market productivity, $g(j)$, the natural logarithms of job-offer arrival rates, $\log \lambda_e(j)$, $\log \lambda_u(j)$, $\log \lambda_n(j)$, and the logarithm of exogenous job separation rate be characterized as second-degree polynomials of age j:

$$\lambda_e(j) = \exp(\lambda_{e,2} j^2 + \lambda_{e,1} j + \lambda_{e,0}), \tag{2}$$

$$\lambda_u(j) = \exp(\lambda_{u,2} j^2 + \lambda_{u,1} j + \lambda_{u,0}), \tag{3}$$

$$\lambda_n(j) = \exp(\lambda_{n,2} j^2 + \lambda_{n,1} j + \lambda_{n,0}), \tag{4}$$

$$\sigma(j) = \exp(\sigma_2 j^2 + \sigma_1 j + \sigma_0), \tag{5}$$

and

$$g(j) = g_2 j^2 + g_1 j + g_0. \tag{6}$$

The remaining parameters that need to be calibrated are

$$\boldsymbol{\xi} \equiv \{\beta, \delta, \lambda_{e,2}, \lambda_{e,1}, \lambda_{e,0}, \lambda_{u,2}, \lambda_{u,1}, \lambda_{u,0}, \lambda_{n,2}, \lambda_{n,1}, \lambda_{n,0}, \sigma_2, \sigma_1, \sigma_0, g_2, g_1, g_0, \psi, \gamma, \sigma_\mu, \sigma_z, h, \zeta, \alpha, b_0, \bar{b}\},$$

where σ_z and σ_μ are the standard deviations of AR(1) shocks of idiosyncratic productivity and match-specific productivity. To estimate these parameters, we minimize the sum of the squared log distance between (i) gross worker flows, labor market ratios,[8] and average market wage over the life cycle, UI-cap-to-average-wage ratio target, UI-payments-to-earnings ratio target, the interest rate target, and the investment-to-GDP ratio target and (ii) the

[6] Our calibrated survival rate is given by the following function: $s_j = (1-(0.000149\exp(0.0751((j-1)/12+22)))^{1/12}$ for j in $1, 2, \ldots 946$, and $s_j = 0$ for $j > 946$. Essentially, workers at age 947 die for sure.

[7] Online Appendix B conducts the robustness check regarding the values of ρ_z and ρ_μ.

[8] Labor market ratios: the unemployment rate, the employment-population ratio, and the labor force participation rate over the life cycle.

corresponding moments from the model simulations. More precisely, for a given $\boldsymbol{\xi}$, we solve for the value functions and decision rules recursively, simulate the model according to the decision rules, and calculate monthly gross worker flows, labor market ratios, and calibration moments.

To simulate the model, we need to make assumptions about the initial distribution of workers' state variables. We assume each worker begins life at the leisure island with no assets. The idiosyncratic general productivity of a newborn worker is drawn from the long-run distribution of idiosyncratic productivity. In the numerical solution of the model, we discretize idiosyncratic and match-quality AR(1) processes using the Tauchen method. In our calibration exercise, we minimize the sum of the squared log distance between data moments and model moments. See Online Appendix A for the details of the numerical solution and calibration.

4.1 Case 1: Age-independent parameters

Our baseline calibration allows the job offer rate, job separation rate, and market productivity to be age-dependent. However, before considering all of these parameters to be flexible, we start our calibration exercise with a hypothetical case where we restrict all of these parameters to be age-independent. We conduct the estimation procedure above under the assumption that $\lambda_e(j)$, $\lambda_u(j)$, $\lambda_n(j)$, $\sigma(j)$, and $g(j)$ are constant across different j.

The calibrated parameters and results are detailed in Online Appendices C and D. The results show some of the life-cycle patterns of stocks and flows cannot be replicated with this specification. For stocks, the employment-population ratio is significantly flatter than what we see in the data. This result reflects the flat profile of both the labor force participation rate and the unemployment rate. In the data, the labor force participation rate exhibits an inverted-U shape, and the unemployment rate is significantly larger for young workers than for old workers. Neither pattern emerges from this specification. The flow statistics that are directly linked to job-finding (EE, UE, and NE) perform relatively well, as does NU flow. The model performs poorly for the flows that are linked to separation (EU) or labor supply (EN and UN).

4.2 Case 2: Age-dependent productivity

Now, to evaluate the effect of the age-dependent productivity, we allow the productivity profile $g(j)$ to follow (6), while keeping $\lambda_e(j)$, $\lambda_u(j)$, $\lambda_n(j)$, and $\sigma(j)$ constant over age. Once again, the results are in Online Appendices C and D. The estimated $g(j)$ exhibits an inverted-U shape.

The low productivity in young and old age induces a reduction in labor supply for these age groups. This effect is sufficient in matching the stocks (employment rate, labor force participation rate, and unemployment rate). Somewhat surprisingly, the flow rates also achieve a good fit to the data.

Notation	Definition	Value
β	Discount factor	0.997
θ	Elasticity of output with respect to capital	0.300
δ	Depreciation rate	0.009
ψ	Disutility of active job search	0.058
A	Total factor productivity	0.489
ρ_μ	Persistence parameter of monthly AR(1) match-specific productivity	0.980
σ_μ	Std. dev. of innovations in match-specific productivity	0.107
ρ_z	Persistence parameter of monthly AR(1) idiosyncratic productivity	0.970
σ_z	Std. dev. of innovations in idiosyncratic productivity	0.093
h	Home productivity	0.120
ζ	Unknown match quality probability	0.275
α	Shape parameter of Pareto distribution	6.934
$\bar{\mu}$	Match quality for unrevealed matches	1
γ	Disutility of work over disutility of active job search	8.850
$\mathbf{T}$	Transfer	0.207
b_0	UI replacement rate	0.419
$\bar{b}$	UI payment cap	0.498
χ	Initial UI takeup rate	0.770
η	The probability of losing UI benefits	0.167
J	Monthly age at which everyone dies	947

Table 1: Age-independent parameters

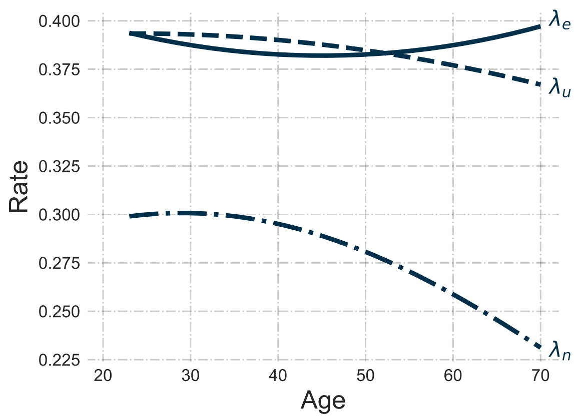
4.3 Case 3: Age-dependent job offer rates

What happens if the productivity profile is flat but the job-offer rates are allowed to vary across ages? Specifically, we allow $\lambda_e(j)$, $\lambda_u(j)$, and $\lambda_n(j)$ to follow (2), (3), and (4), whereas $\sigma(j)$ and $g(j)$ are constant over the life cycle. The results are in Online Appendices C and D. The estimated $\lambda_e(j)$, $\lambda_u(j)$, and $\lambda_n(j)$ are close to flat over the life cycle, and thus the fit to the stocks and the flows in the data does not improve significantly from Case 1 above.

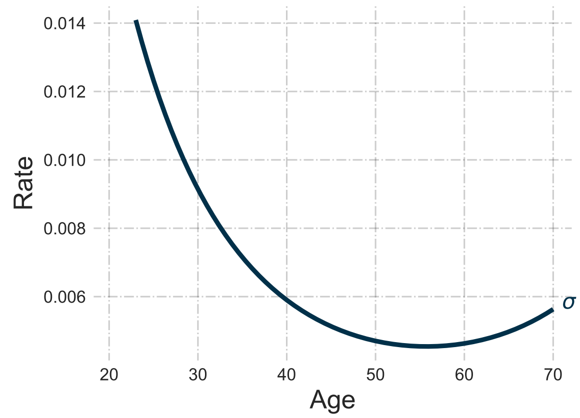
4.4 Case 4: The baseline calibration

Our baseline specification allows all $\lambda_e(j)$, $\lambda_u(j)$, $\lambda_n(j)$, $\sigma(j)$, and $g(j)$ to be flexible and follow (2), (3), (4), (5), and (6). The estimated coefficients are in Online Appendix D. Figure 4 visualizes the calibrated outcome in graphs. The calibrated parameters that do not have an age component are shown in Table 1.

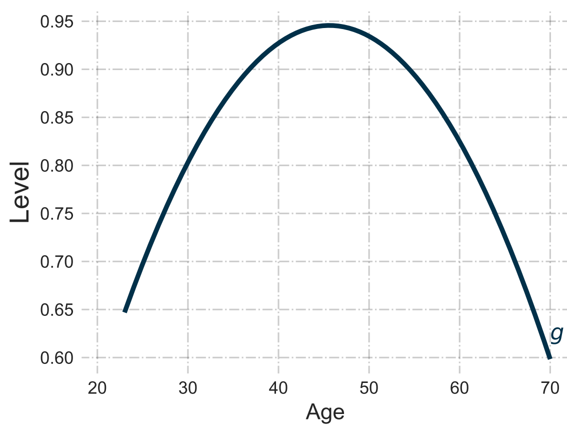
Panel (a) of Figure 4 shows job-offer arrival rates over the life cycle for the unemployed



(a) Job offer arrival rate



(b) Exogenous job separation rate



(c) Age component of market productivity

Figure 4: Age dependent parameters

(dashed line), the employed (solid line), and the nonparticipant (dot-dash line). For nonemployed workers, job-offer arrival rates increase until they reach prime age and then decrease. The decrease in the job-offer arrival rate is sharper for the nonparticipant. As expected, the job-offer arrival rate for the unemployed is greater than that for the nonparticipant, highlighting the active-job-search trade-off: the active job search is costly but results in a higher probability of receiving an offer.

The job-offer arrival rate for the employed has three notable features. First, the overall level of λ_e is similar to λ_u, despite the corresponding flows (EE and UE flows) having significantly different levels. This result is reminiscent of Tobin's (1972) argument that no evidence exists that employed workers are less efficient in a job search than nonemployed workers.[9] In fact, employed workers appear to be more efficient in the search than nonemployed workers when they are old. This finding is not inconsistent with the fact that job-to-job transitions are less frequent than UE transitions, because employed workers tend to be choosier because of their outside options. Second, unlike λ_u and λ_n, λ_e exhibits an increasing pattern after the 40s, after remaining flat during the younger years. This pattern could, for example, reflect that employed workers can build a better network as they become older. Although our model is too stylized to investigate this point further, it seems to be an interesting hypothesis for future inquiry. Third, the overall life-cycle profiles of λ_e, λ_u, and λ_n are relatively flat, compared with the corresponding flows (EE, UE, and NE flows). The difference comes from the fact that workers choose whether to accept the job, and the "choosiness" depends on the stage in the life cycle. This contrast highlights the importance of analyzing an *economic model* as opposed to an *accounting model*. This result is consistent with the result in Section 4.2, where the model with flat profiles of λ_e, λ_u, λ_n, and σ can fit the data fairly well.

Panel (b) of Figure 4 shows exogenous job separation decreases over the life cycle of an individual with a slight increase after age 60. The age component of market productivity in Panel (c) displays an inverted-U shape. Market productivity increases until middle age and decreases toward the end of an agent's working life. This life-cycle pattern of market productivity is largely consistent with the results from direct measurements from microeconomic data, widely used in the quantitative public finance literature.[10] Note the model in Section 4.2, with the flat profile of σ over the life cycle, already achieves a fairly good fit to the data.

[9] Mukoyama (2014) reports a similar outcome with a simple job-ladder model when the separation rate strongly depends on match quality. Comparing this result with existing estimates is difficult, given that we assume all employed workers can potentially receive job offers (with the same distribution of μ as the unemployed), whereas, in reality, many potential job opportunities for the employed workers never materialize as formal job offers. In Faberman et al.'s (2022) survey data (Table V), unemployed workers receive about four times more formal job offers than employed workers. Because our model's job offers include many offers that are well below the acceptance threshold, it is natural to think these offers never materialize as formal job offers. In light of Faberman et al.'s (2022) evidence, the natural interpretation of our result is that if the quality of job offers is identical between the employed and the unemployed, three-quarters of the potential job opportunities for the employed workers never materialize as formal job offers, because they are far less desirable than the current job they hold.

[10] For example, Conesa et al. (2009) use the measurement from Hansen (1993).

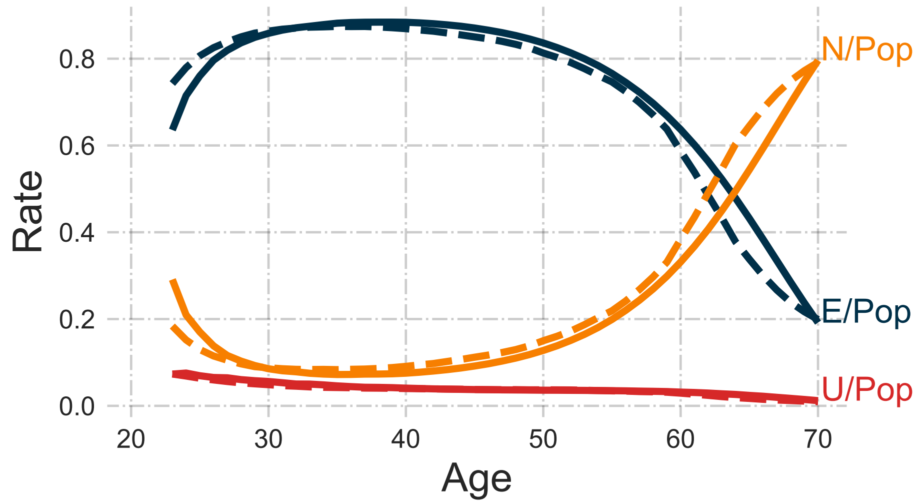


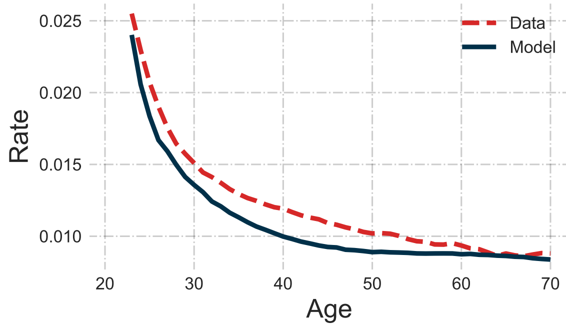
Figure 5: Labor market stocks as a fraction of population. Solid lines represent the model and dashed lines represent the data.

Thus, the life-cycle variation of σ has less impact on the resulting flows and stocks.

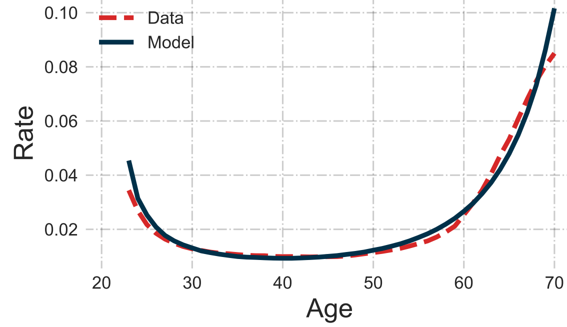
Our model outcomes against the targeted data moments are plotted in Figures 5 and 6. In Figure 5, each line represents the fraction of each stock within the corresponding age group. The solid line is the model outcome and the dashed line is the corresponding data outcome. One can see the model fits the data very well. In particular, the inverted U-shape pattern of the E stock, the U-shape pattern of the N stock, and the declining pattern of the U stock are all consistent between the model and the data.

In Figure 6, we are able to match qualitative features of the flow rates by age quite well. For some flow rates, EU, EN, and EE, we are able to match the entire life-cycle dynamics almost perfectly. We would like to emphasize the challenge of obtaining such a good fit. The model is quite parsimonious, and most assumptions are standard in the life-cycle literature. However, the computational burden is quite high, and the model has to fit six gross flows (plus the job-to-job flow and the wage profile) as functions of age. Fitting six *functions* is substantially more difficult than fitting six *numbers* that Krusell et al. (2011, 2017) achieve. No previous papers have accomplished such a good fit in a model in which all flows are endogenous. We view finding a framework that fits these life-cycle patterns as one of the important contributions of this paper.

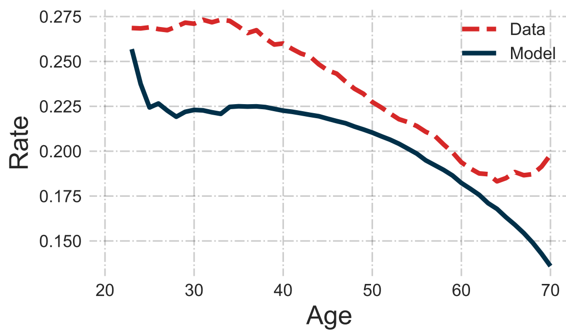
Online Appendix E compares the model outcome with the US data in the wealth-income ratio. One advantage of the BHA structure is that we can make such a comparison. The model turns out to have an excellent fit in terms of the life-cycle profile of the average wealth-income ratio. Online Appendix E also presents the changes in the wealth-income ratio profile with the counterfactual policy experiments in the next section.



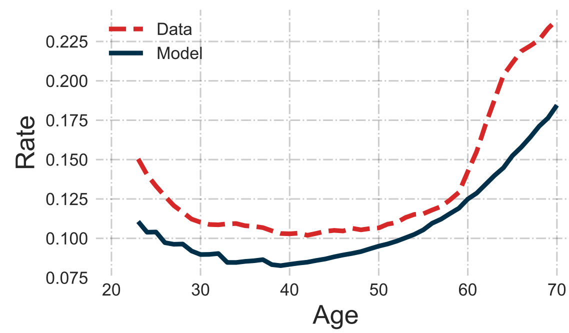
(a) EU



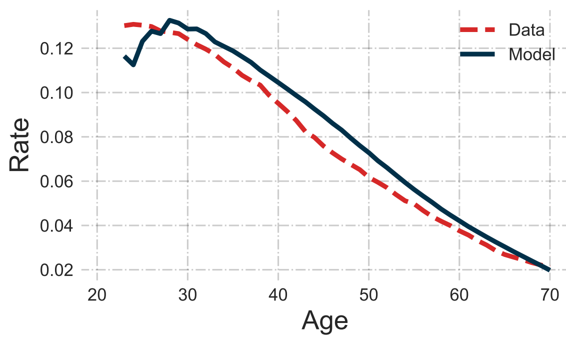
(b) EN



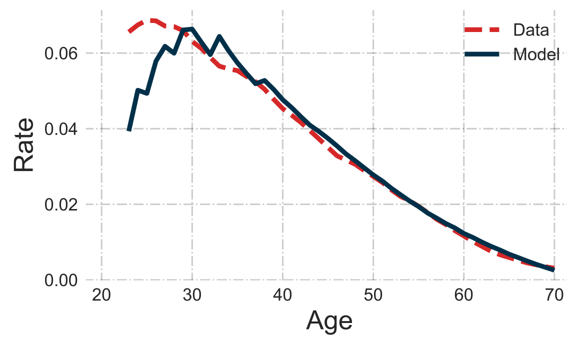
(c) UE



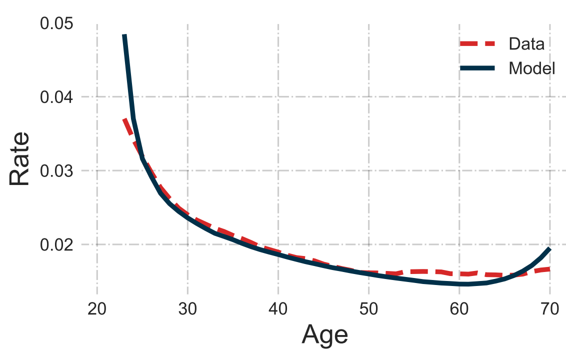
(d) UN



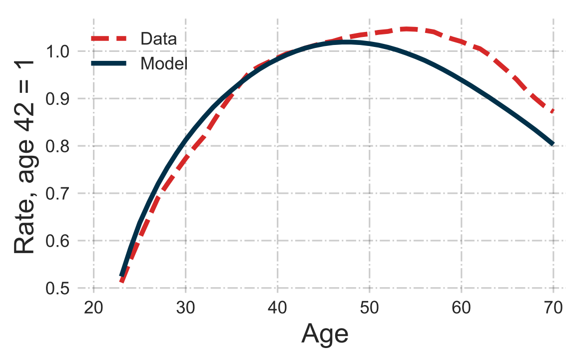
(e) NE



(f) NU



(g) EE



(h) Wage

Figure 6: Model moments and calibration targets

Table 2: Aggregate statistics from the experiment

Tax	N	E	U	u	$lfpr$	Labor (L)	Efficiency	K/L	Welfare Gain
0.30	0.337	0.628	0.035	0.052	0.663	0.619	0.986	35.746	N/A
0.45	0.498	0.475	0.026	0.053	0.502	0.499	1.049	34.581	−7.73%

5 Policy experiments

In this section, we utilize the above framework to conduct policy experiments. We examine two different policies. The first is the taxes-and-transfers policy. Given that our baseline model highlights the role of the labor supply margin, this model suits the analysis of policies that directly affect labor supply incentives. The second policy is the UI policy. UI policy affects both the job search incentive and the incentives for taking up a new job. Our model features these two choices as important determinants of the gross worker flows.

5.1 Taxes and transfers

First, we examine the effect of an increase in labor tax. In his influential work, Prescott (2004) argues the difference in total hours between the US and continental Europe can largely be explained by the difference in the tax system. Although various studies have followed up on Prescott's (2004) study, none has explicitly analyzed a model with gross worker flows in a life-cycle economy. Our model reveals two novel effects of the tax and transfer: reallocation (worker flows) over the life cycle and the decomposition of effects on nonemployment into unemployment and nonparticipation.

Following Krusell et al. (2010), we consider an experiment of raising the labor tax rate τ from 0.30 to 0.45. Table 2 summarizes the results at the aggregate level, where *Efficiency* is defined as the labor in efficiency units (L) over the number of employed workers (E):

$$\text{Efficiency} = \frac{L}{E},$$

representing the average productivity of employed workers. The magnitude of the decline in aggregate employment is essentially the same as the infinite-horizon economy in Krusell et al. (2010); here, E declines by $0.475/0.628 = 0.76$, whereas in Krusell et al. (2010), the corresponding value is $0.488/0.633 = 0.77$. One factor that alters the impact of the tax in the life-cycle economy is the heterogeneity of responses across different age groups. The unemployment rate in Table 2 slightly increases with the tax increase. This finding is in line with the baseline case in Krusell et al. (2010).

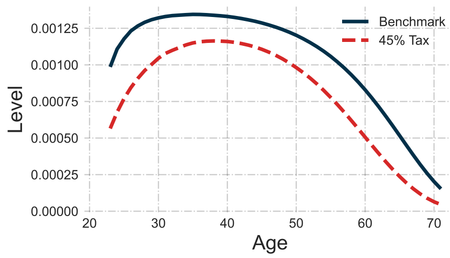
Figure 7 compares the stocks one by one. The stocks are represented as fractions of the total population (which is normalized to 1). Although employment decreases and nonparticipation increases in all ages, the decline in participation is particularly strong in young workers. Because young workers tend to be less productive than prime-aged workers (see panel (c) of Figure 4), the changes in young workers' employment have less impact on the efficiency units and thus on wages. Therefore, for the same change in total efficiency units of labor and in wages, the change in aggregate E appears more significant when the impact is skewed toward young workers.

Prescott et al. (2009) make a comparison of hours profile between the US and several European countries. They observe that the difference in hours mainly comes from the 20s and over the 50s. For the workers between 30 and 50 years old, the hours are very similar between the US and Europe. Thus, the current model is not consistent with the US-Europe comparison in hours across different age groups. One possibility is that we are missing some factors in the model, such as the nonlinearity of return from work, as suggested by Rogerson and Wallenius (2009). Another possibility is that the US-Europe discrepancy is (at least partly) due to factors other than the difference in taxes and transfers.

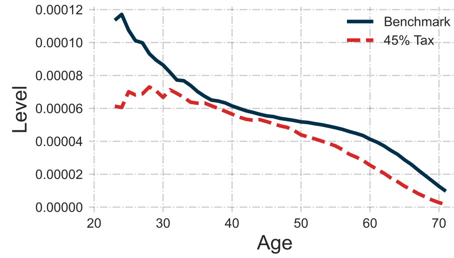
In panel (b), the unemployment stock for young workers declines dramatically. The rates comparable to Figure 1 for the data are plotted in Figure 8. Somewhat surprisingly, the life-cycle profile of the unemployment rate changes very little for all ages. Even for the very young workers, where the total U stock changes significantly in Figure 7, the change in the unemployment rate is relatively small because E also falls significantly. For middle-aged workers, the employment decline, driven by the participation margin, is larger than the unemployment decline, and as a result, the unemployment rate increases. In total, the middle-aged workers' effect dominates, and the total unemployment rate increases. The heterogeneous responses across different age groups add complexity in considering the aggregate outcome, compared with the infinite-horizon model of Krusell et al. (2010).

Table 2 also shows the welfare effect of the tax increase. The "Welfare Gain" entry measures the percentage by which we have to increase consumption (at each period and state) in the benchmark economy to make the worker indifferent to being born in the 45% tax economy (see Online Appendix F). Increasing the tax rate to 45% reduces the newborn's present-value welfare by 7.7%. (That is, we have to decrease consumption by 7.7% from the benchmark economy to make the worker indifferent to being born in the higher-tax economy.) Notice that, as shown in Table 2, the capital-labor ratio goes down after the tax increase. The level of capital stock K is even lower. Being born in an economy with low capital stock implies a lower future income for workers. This effect is one of the reasons that the welfare decline is relatively large.

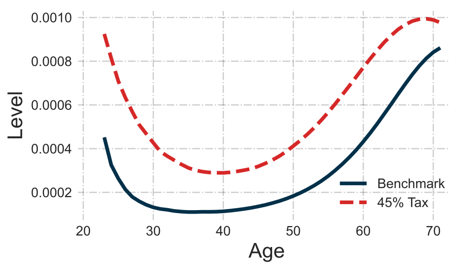
Now we investigate the gross worker flows. Figure 9 draws each labor market transition rate for the benchmark and 45% tax case. First, we investigate the cause of the decrease in U stocks in Figure 7. Among the flows involving the U state, two flows strongly impact young workers. The first is the EU flow. Because only high-productivity workers participate when the tax is high, the likelihood of moving from E to U when the match quality becomes worse is lower in a high-tax situation. The second is NU flow. Two (potential) reasons exist for moving from N to U: (i) running down assets (the wealth effect) and (ii) improvement of



(a) Employed

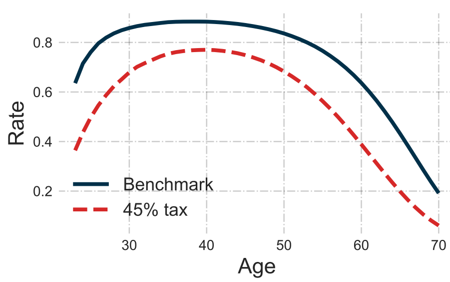


(b) Unemployed

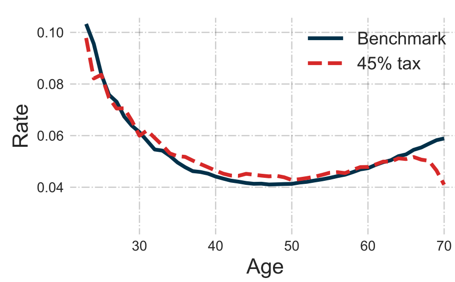


(c) Not in the labor force

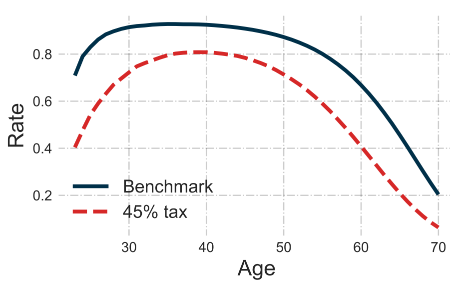
Figure 7: Labor market stocks after a tax hike



(a) Employment-population ratio

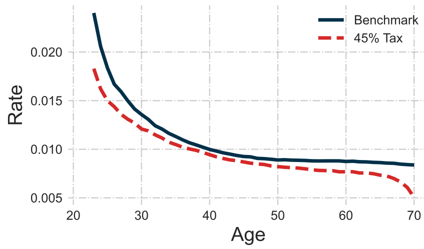


(b) Unemployment rate

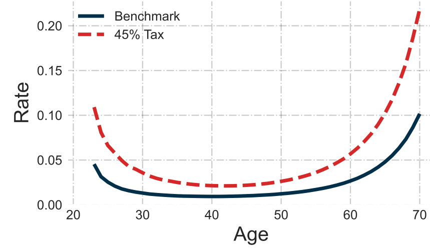


(c) Labor force participation rate

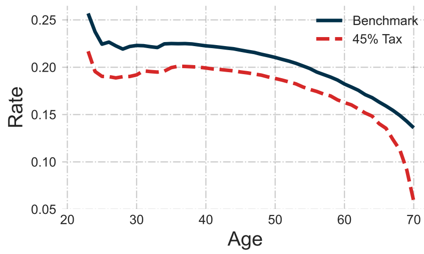
Figure 8: Labor market ratios after a tax increase



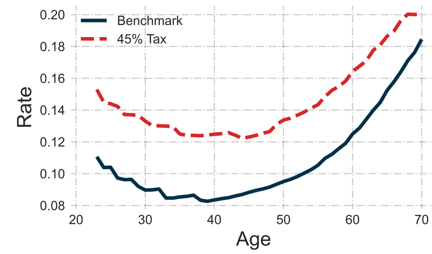
(a) EU



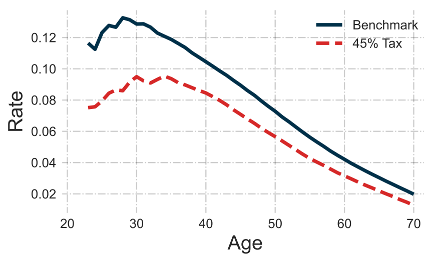
(b) EN



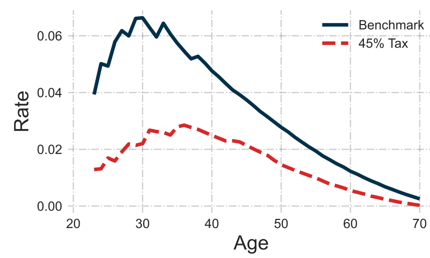
(c) UE



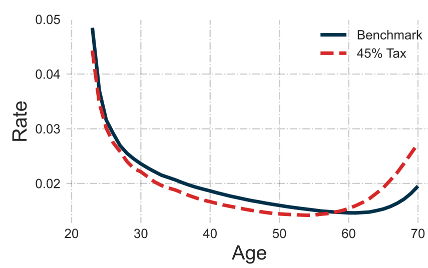
(d) UN



(e) NE



(f) NU



(g) EE

Figure 9: Gross worker flow rates after a tax hike

the idiosyncratic productivity. The reduction in labor income and the increase in the lump-sum transfer implies the individuals in the N state do not (have to) run down assets while nonemployed as quickly when the labor tax is high. In other words, the income is smoother across states, thus reducing the individuals' precautionary saving (and precautionary work) motive. The impact of a lump-sum transfer is larger for a young worker, who tends to have lower labor income and a lower level of assets. Thus, in explaining the decrease in U for young workers, (i) the selection of employed workers and (ii) the improved opportunities for consumption smoothing play important roles.

Second, concerning labor force participation rates, both NE and NU flow shift substantially more for young workers. This finding contrasts the shifts of the opposite-direction flows, EN and UN, which are fairly uniform across all ages. Combined with the fact that the employment response is largely coming from the participation margin, we conclude that the outflow from nonparticipation is the key to generating the life-cycle pattern of the employment response to the taxes.

Analyzing more deeply at the micro-level, Figure 10 plots the cutoff levels of the age-adjusted idiosyncratic productivity ($g(j) \times z$) for given assets and past match-specific productivity (if eligible for UI). Above the cutoff level, a nonemployed worker participates in the labor market. Three panels for different ages (25, 45, and 65 years old) compare the cutoffs for the baseline ($\tau = 0.3$) and the experiment ($\tau = 0.45$). The amount of the shift of the cutoffs turns out to be not too different across different ages. Note the aggregate responses are affected by the combinations of the change in the cutoffs and the distributions of the state variables (in particular, the joint distribution of asset and productivity), as well as the change in the distributions by the policy. Overall, younger workers exhibit more action in the aggregate participation margin, largely because they tend to have a lower level of wealth (where taxes have a larger impact), and more workers tend to be in the neighborhood of the cutoff lines. A comparison of the rows in Figure 10 reveals the effects of UI eligibility on participation decisions. Nonemployed with no UI eligibility or with lower UI payments are less inclined to participate than nonemployed with higher UI payments.

5.2 Unemployment Insurance (UI)

In this section, we analyze the UI policy change. The calibrated replacement rate is $b_0 = 0.419$. Although it is endogenously determined by the targets in Section 3.4, this value is in line with the literature (e.g., Shimer (2005) sets the average replacement rate to approximately 40% in his quantitative exercise). We consider an experiment of increasing the replacement rate to 60%. One could think of this value as representing the continental European welfare state. The quantitative work by Ljungqvist and Sargent (1998) considers the replacement rate of 0.7 as representing the welfare state economy.

The details of the results are described in Online Appendix G. In contrast to the taxes-and-transfers experiment, the increase in the UI benefit mainly affects the unemployment

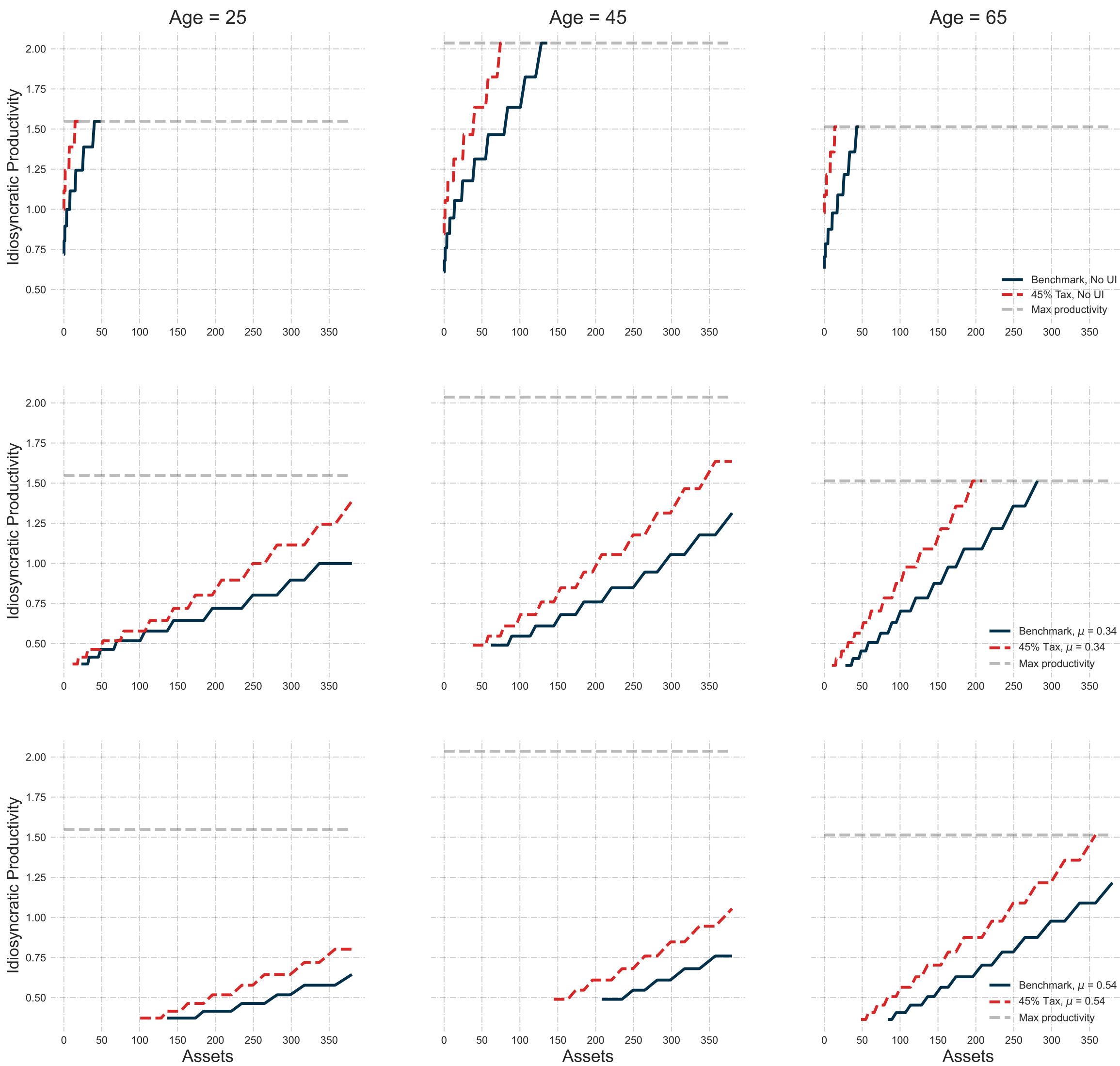


Figure 10: Age-adjusted idiosyncratic productivity cutoffs for labor force participation

rate without affecting the labor force participation rate. The main channel is the decline of UE flow. The life cycle also matters: the effect on the unemployment rate is larger for younger workers.

6 Conclusion

This paper developed a general equilibrium framework to analyze the gross worker flows over the life cycle. Our model features life-cycle permanent-income consumers who can self-insure from various shocks by accumulating assets. In the labor market, individuals can make labor market participation decisions under labor market frictions.

The calibrated model can match the salient features of the life-cycle patterns of the gross worker flows in the data. The estimated parameter values reveal how frictions vary across the worker's life cycle. The job-finding frictions are remarkably flat over the life cycle, compared with the behavior of gross flows, highlighting the importance of the individual decision.

With the calibrated model, we ran two policy experiments. First, we experimented with the taxes-and-transfers policy. An increase in labor tax decreases employment and labor force participation for all age groups, although the changes are more significant for younger workers. The unemployment stock decreases significantly only for young workers. The analysis of gross worker flows finds the changes in EU flow and NU flow (inflow into unemployment) are the main causes of age heterogeneity in the unemployment response. For the N state, the outflow from N is of prominent importance. Overall, young workers move less into the U state and leave less from the N state when the labor tax is high. The changes in gross flows also affect productivity and wages, highlighting the importance of explicitly considering effects on reallocation in the analysis of taxes and transfers. The reallocation effects are heterogeneous across age groups. Second, we introduced a realistic UI system. An increase in UI increases the unemployment rate, especially for young workers, with little change in the labor force participation rate.

Although we view our study as significant progress compared with the existing literature, much room remains for future research. The model of this paper, as in the case with Krusell et al. (2017), does not address the endogenous response of the frictions to the change in taxes. The DMP framework, for example, would suggest a change in the labor tax can affect the firms' vacancy-posting behavior and eventually alter the frequency of workers receiving job offers. The omission here is for two reasons. First, our focus is the labor supply response, which has been the focus of the taxes-and-transfers literature since Prescott (2004), as well as of many UI papers that focus on workers' search efforts. Second, incorporating such a mechanism into a general equilibrium BHA-style model is technically challenging in both random search and directed search frameworks. Incorporating such an effect is outside the scope of this paper, but it is an important future research agenda.

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