# MIT Shock Implies Market Incompleteness*

Toshihiko Mukoyama
Georgetown University
tm1309@georgetown.edu

May 2020

**Abstract**

This paper shows, with an analytically tractable example, that the allocation after an unanticipated event (often called an "MIT shock") is different from the allocation of a corresponding complete-market model that explicitly considers the possibility of the shock, even when the probability of the event approaches zero. Instead, the MIT-shock outcome is equivalent to a probability-zero limit of a model with incomplete asset markets. Thus, an MIT-shock analysis of an event implicitly assumes (i) the probability of the event is small and (ii) asset markets are incomplete.

Keywords: MIT shock, incomplete markets

JEL Classification: D52, E32, E60

*I thank Per Krusell and Latchezar Popov for comments. Bobbie Goettler provided editorial help. All errors are mine.

# 1 Introduction

Many theoretical macroeconomic studies analyze a one-time change in a parameter, starting from a steady-state of a model economy, followed by a deterministic transition path. The applications have been broad, including policy analysis, business cycles, and other shocks.[1] The change in the parameter is interpreted as an unanticipated "shock," called the "MIT shock" in the recent literature, and the transition dynamics are interpreted as a response to the shock. Behind this interpretation is a presumption that if we write a model with explicit uncertainty regarding the same event and take a limit that the probability of the event approaches to zero, the equilibrium allocation would also converge to the outcome from the MIT-shock experiment.

This paper, by way of a simple example, demonstrates this presumption is not necessarily correct. In particular, if the initial economy with uncertainty has a complete asset market, the equilibrium allocation does not converge to the MIT-shock allocation (i.e., the response to a deterministic parameter change) as the shock probability approaches to zero. The intuition is simple. The complete-market case features a perfect risk-sharing; the marginal rates of substitution across states are equal among consumers. By contrast, the MIT-shock case has no mechanism that equates two consumers' marginal rates of substitutions between two states, one of which involves the MIT shock and the other does not. The market structure of the MIT-shock analysis, in fact, better resembles an incomplete-market model in which the asset markets between the states with the MIT shock and the states without are missing. In this paper's example, the MIT shock outcome is replicated by the limit of an economy with a bond (self-insurance) and an economy without one of the Arrow securities.

This paper shows that the allocation after the MIT shock, therefore, is an outcome of two assumptions: (i) the (vanishingly) small probability of the shock event, and (ii) an incomplete asset market. The discrepancy of the outcome between the complete-market limit and the MIT-shock outcome results in the different wealth distribution after the shock. An MIT shock, when it is truly unanticipated, creates changes in individual wealth that does not exist in the complete-market counterpart. When one thinks of the complete-market case as a baseline, the MIT shock creates an implicit wealth transfer. Failing to recognize this transfer can have an important consequence when (i) the individual welfare effects of the shocks are evaluated and (ii) the aggregate outcome depends on the wealth distribution upon the occurrence of the shock.

The paper is organized as follows. The next section sets up the model. Section 3 solves the model for the allocation with the MIT shock, and Sections 4 and 5 solves it with complete and incomplete-asset-market structures. Section 6 considers the possible generalizations, and Section 7 discusses the implication of the results. Section 8 concludes.

[1]Earlier examples are Abel and Blanchard (1983), Auerbach and Kotlikoff (1983), and Judd (1985). Recent examples include Kaplan et al. (2018), Boppart et al. (2018), and Guerrieri et al. (2020).

## 2 Model setup

Consider a two-period endowment economy with two types of consumers, Type I and Type II. Each type has a continuum of population 1. Both types are price-takers and maximize the utility

$$u(c_1) + E[u(c_2)],$$

where $c_1$ is the consumption in period 1 and $c_2$ is the consumption in period 2. The expected value $E[\cdot]$ is taken in period 1. In period 1, both types receive the endowment 1. In period 2, uncertainty exists. In a *regular state*, which occurs with probability $(1-\pi)$, where $\pi \in [0, 1]$, both types receive endowment 1. In an *irregular state*, which occurs with probability $\pi$, Type I receives $(1-\tau)$, where $\tau \in (0, 1)$, and Type II receives $(1+\tau)$. Thus, in the irregular state, a transfer occurs from Type I to Type II. The function $u(\cdot)$ is strictly increasing, strictly concave, and continuously differentiable.

## 3 MIT shock

Suppose that the irregular state is not anticipated ($\pi$ is considered zero) in period 1. Then, the problem for a Type-I consumer is

$$\max_{c_1, c_2, s} \; u(c_1) + u(c_2)$$

subject to

$$c_1 + qs = 1$$

and

$$c_2 = 1 + s,$$

where $q$ is the price of a bond that pays out one unit of the consumption good in period 2. The consumptions in period 1 and 2 are denoted as $c_1$ and $c_2$, and the bond holding is represented by $s$. Here, the saving/borrowing vehicle is assumed to be a bond. Given that no uncertainty exists, this asset structure is equivalent to assuming the saving vehicle is an Arrow security, and $q$ is a price of the Arrow security that pays out in the regular state.[2]

The Type-II consumer faces the identical problem:

$$\max_{c_1', c_2', s'} \; u(c_1') + u(c_2')$$

subject to

$$c_1' + qs' = 1$$

[2] The resulting allocation, even after the MIT shock, would also be the same with these two alternative assumptions on the saving vehicle, because the equilibrium savings of each consumer is zero in this economy. However, as I discuss in Section 6, the post-MIT-shock allocation would be different when the equilibrium saving is nonzero.

and

$$c_2' = 1 + s',$$

where prime ($'$) denotes variables for the Type-II consumers.

It is straightforward to show the unique competitive equilibrium is with $q = 1$, $s = 0$, and $c_1 = c_1' = c_2 = c_2' = 1$. Now suppose the MIT shock hits the economy. Then, the ex-post allocation will be $\tilde{c}_2 = 1 - \tau$ and $\tilde{c}_2' = 1 + \tau$. I add a tilde ($\tilde{}$) to denote the irregular state. Thus, the entire ex-post allocation ends up with

$$(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau). \tag{1}$$

# 4 Complete market

In the next two sections, we treat the uncertainty explicitly. Suppose the presence of a complete set of Arrow securities that spans all possible states. In the current example, two Arrow securities exist, and each one pays one unit of consumption good when each state takes place. Let the price of the Arrow security that pays out in the regular state be $p$ and the price of the security that pays out in the irregular state be $\tilde{p}$.

A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, a, \tilde{a}} \ u(c_1) + (1 - \pi)u(c_2) + \pi u(\tilde{c}_2)$$

subject to

$$c_1 + pa + \tilde{p}\tilde{a} = 1,$$

$$c_2 = 1 + a,$$

and

$$\tilde{c}_2 = 1 - \tau + \tilde{a},$$

where the other notations are identical to the previous section except that $a$ and $\tilde{a}$ denote the holding of Arrow securities.

A Type-II consumer's problem is (with the same notation convention as the last section)

$$\max_{c_1', c_2', \tilde{c}_2', a', \tilde{a}'} \ u(c_1') + (1 - \pi)u(c_2') + \pi u(\tilde{c}_2')$$

subject to

$$c_1' + pa' + \tilde{p}\tilde{a}' = 1,$$

$$c_2' = 1 + a',$$

and

$$\tilde{c}_2' = 1 + \tau + \tilde{a}'.$$

One can confirm the competitive equilibrium, where the consumers' first-order conditions are satisfied and the market-clearing conditions

$$a + a' = 0$$

and

$$\tilde{a} + \tilde{a}' = 0$$

has the following solution:

$$p = 1 - \pi$$

and

$$\tilde{p} = \pi$$

with

$$(a, a', \tilde{a}, \tilde{a}') = \left( -\frac{\pi}{2}\tau, \frac{\pi}{2}\tau, \frac{2-\pi}{2}\tau, -\frac{2-\pi}{2}\tau \right).$$

The resulting consumptions are:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left( 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau \right),$$

that is, each consumer can smooth consumption across time and state.

To see which allocation corresponds to the MIT-shock situation, take $\pi \to 0$ and look at the consumption when the irregular shock hits. The result is

$$\lim_{\pi \to 0} (c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1, 1).$$

This finding is in contrast to (1). The ex-post allocation of the complete-market outcome, even when $\pi$ approaches zero, does not approximate the ex-post allocation with the MIT shock. The intuition is simple. The Arrow security for the irregular state becomes cheaper and cheaper as $\pi \to 0$, and thus the Type-I consumers still demand it to hedge against the irregular state even if the state rarely occurs. The Type-II consumers are willing to sell the security at a cheap price because the probability of the state is low.

# 5 Incomplete market

This section considers two incomplete-market asset structures. The first is an economy with only bonds and the second considers a missing Arrow security.

## 5.1 Bond economy

Here, for the ease of computation, I assume the utility function is quadratic:

$$u(c) = \alpha c - \frac{\gamma}{2} c^2, \tag{2}$$

where $\alpha > 0$ and $\gamma > 0$. The value of $\alpha$ is assumed to be sufficiently large so that utility is increasing in $c$ over the relevant range.

A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, b} \; u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2)$$

subject to

$$c_1 + qb = 1,$$

$$c_2 = 1 + b,$$

and

$$\tilde{c}_2 = 1 - \tau + b,$$

where $q$ is the bond price and $b$ is the bond holding. After solving for the first-order condition, the bond demand of Type-I consumers is

$$b = \frac{q(\gamma - \alpha) + \alpha - \gamma(1 - \pi\tau)}{\gamma(q^2 + 1)}.$$

Similarly, the Type-II consumer's problem is

$$\max_{c_1', c_2', \tilde{c}_2', b'} \; u(c_1') + (1-\pi)u(c_2') + \pi u(\tilde{c}_2')$$

subject to

$$c_1' + qb' = 1,$$

$$c_2' = 1 + b',$$

and

$$\tilde{c}_2' = 1 + \tau + b',$$

The bond demand of the Type-II consumers is

$$b = \frac{q(\gamma - \alpha) + \alpha - \gamma(1 + \pi\tau)}{\gamma(q^2 + 1)}.$$

The bond price $q$ is set so that the excess bond demand is zero:

$$b + b' = 0.$$

It is straightforward to derive that, in equilibrium,

$$q = 1$$

and

$$(b, b') = \left(\frac{\pi}{2}\tau, -\frac{\pi}{2}\tau\right)$$

hold. The resulting consumptions are:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \left(\frac{\pi}{2} - 1\right)\tau, 1 + \left(1 - \frac{\pi}{2}\right)\tau\right),$$

which achieves some but not perfect consumption smoothing.

In the limit of $\pi \to 1$, the consumption profile when the irregular state takes place in period 2 would approach

$$\lim_{\pi \to 0} (c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau),$$

which is identical to (1). In the current example, the incomplete-market structure that delivers the equivalence to the MIT-shock outcome is not unique. The following section looks at the economy with one Arrow security.

## 5.2 Lack of Arrow security

Let us go back to the general utility. Suppose the Arrow security does not exist for the irregular state. A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, a} \quad u(c_1) + (1 - \pi)u(c_2) + \pi u(\tilde{c}_2)$$

subject to

$$c_1 + pa = 1,$$
$$c_2 = 1 + a,$$

and

$$\tilde{c}_2 = 1 - \tau.$$

For Type-II consumers,

$$\max_{c_1', c_2', \tilde{c}_2', a'} \quad u(c_1') + (1 - \pi)u(c_2') + \pi u(\tilde{c}_2')$$

subject to

$$c_1' + pa' = 1,$$
$$c_2' = 1 + a',$$

and

$$\tilde{c}_2' = 1 + \tau.$$

The competitive equilibrium is $p = 1 - \tau$, $a = a' = 0$. The resulting consumptions are:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1, 1, 1 - \tau, 1 + \tau).$$

Thus, in the limit of $\pi \to 1$,

$$\lim_{\pi \to 0} (c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau),$$

which is identical to (1).

## 6 Discussions on generalizations

The above logic can be generalized. In particular, the discrepancy between the MIT-shock outcome (Section 3) and the complete-market outcome (Section 4) would generalize to broader situations. The logic is simple. The MIT-shock outcome, by construction, has no structure that equates the marginal rate of substitution between a state that involves the MIT shock and a state that does not across consumers. The complete-market outcome has this equivalence built in, as long as the solutions for the consumers' problems are interior, no matter how small the shock probabilities are.

The equivalence results in Section 5 requires some qualifications in a more general setting. One can generally find an incomplete-market setting that delivers the same outcome as the MIT-shock allocation in the probability-zero limit. However, what kind of market incompleteness corresponds to the MIT-shock outcome depends on what kind of asset structure is (implicitly) assumed in the MIT-shock setting.

In the example in Section 5, the equilibrium saving is zero in the MIT-shock case, which is why both the bond economy in Section 5.1 and the economy with one Arrow security in Section 5.2 have the same allocation as the MIT-shock case in the probability-zero limit. If the saving is nonzero in the Section 3 setting, the initial wealth levels of consumers after the MIT shock would differ depending on whether the saving is in a non-state-contingent bond or an Arrow security for the regular state. If it is the former, when the MIT shock hits, the consumer would hold the same level of wealth as in the regular state (and the allocation is equivalent to the bond economy limit), whereas if it is the latter, the consumers' wealth levels are zero after the MIT shock (and the allocation is equivalent to the limit of the one-Arrow-security economy).

## 7 Implications

In the context of policy design, if one believes a particular event is truly unanticipated, the MIT-shock analysis of Section 3 can be a useful benchmark.[3] In this case, the above analysis has some practical implications. In such an environment, because the benchmark MIT-shock economy *is* an incomplete-market economy, the allocation suffers from an inherent inefficiency, even if the economy appears efficient absent MIT shocks. Ex-post government interventions, such as transfers to the most affected sectors and consumers (e.g., injecting capital to banks after financial shock, providing shelters for people who lost houses in a storm, and subsidizing people who lost jobs when a virus strikes the economy) can be justified from an ex-ante Pareto-efficiency perspective, instead of appealing to a particular social welfare function or humanitarian considerations. The fact that these people do not insure themselves against such rare shocks indicates a problem of missing markets ex ante. The government

[3] Alternatively, one can imagine a situation where writing a contract with many contingencies is costly, as in Dye (1985).

acting as an insurance company can be justified because the consumers would be able to buy such insurance at essentially no cost if the private insurance market worked perfectly.

From a purely theoretical perspective, the lesson is that we have to be careful when conducting an analysis of the behavior of the economy after an MIT shock. The outcomes after the MIT shock and the small-probability limit of a complete-market model are different. As the above analysis shows, the outcome can be meaningfully different at the individual level. Given that the effect is purely distributional, the (positive) aggregate consequences are often unaffected by the underlying assumptions on the asset market if Gorman preferences are assumed.[4] Even with Gorman preferences, however, in an environment where the markets have other imperfections, such as market power, sticky prices, and search frictions, the positive predictions can be affected. Many macroeconomic analyses consider a complete-market outcome as a benchmark, and one has to be aware that the analysis of the MIT shock may not deliver an approximate solution to this benchmark scenario even when the probability is small. Even with an analysis that explicitly treats market incompleteness, the modeling decision of what asset can be carried into the shock state would have a nontrivial effect on outcome.[5]

In conducting normative analysis, the asset distribution after the shock can have a profound effect on the conclusions concerning the welfare effects of the shocks to individuals. In the example in Sections 3 and 4, having an Arrow security to carry into the after-the-shock state would change the individual welfare ex-post.[6] As was argued in Section 6, even with an assumption of incomplete markets, the limit result can differ depending on what kind of assets can be carried into the state after the shock.

# 8 Conclusion

This paper showed, using an analytically tractable example, that an "unanticipated MIT shock" analysis implicitly assumes not only that the probability of the event occurring is very small, but also that the asset market is incomplete. Because of the implicit market incompleteness, the distribution of wealth upon the occurrence of the shock can be different from the limit of the complete-market counterpart. Therefore, MIT-shock analysis does

[4]This is the case with the complete-market analysis of Guerrieri et al. (2020).

[5]This decision is particularly important in a model with multiple assets. In Mukoyama (2013), it is assumed that, before the MIT shock, consumers hold capital stock and equity in the same fraction across consumers (without this assumption, the individual portfolio is indeterminate when shocks are absent). Because the equity suffers from a capital loss with the MIT shock and the capital stock does not, the welfare effect on individuals is non-trivially affected by this assumption. In Kaplan et al. (2018), the consumers maintain the value of liquid and illiquid assets into the post-shock state. In Boppart et al. (2018), the consumers maintain the total value of assets. In contrast to Mukoyama (2013), both papers do not consider a capital loss.

[6]In Mukoyama (2010), the general results are established allowing for the distribution of wealth after the shock to be arbitrary. However, in the example given in Mukoyama (2010), it is assumed that the consumers are allowed to bring in the asset ("trees") into the post-shock state, while no other securities are allowed to be brought in. In that sense, a particular market incompleteness is implicitly assumed in that example.

not necessarily replicate the complete-market dynamics, especially when the initial wealth distribution matters. One also has to be careful in conducting the analysis of the individual welfare effects of these shocks.

The result that the MIT-shock allocation diverges from the complete-market allocation also poses a question on the analysis of optimal policy design, especially when the policy involves distributional effects. Typically, the switch to the optimal policy is considered an unanticipated permanent change. What if, instead, the agents in the model anticipate that with some probability, the government "wakes up" and starts imposing the optimal policy, and the agents are allowed to trade securities for this event? This type of scenario seems to be closer to the spirit of the rational expectations hypothesis, because it allows the agents in the model to be as smart as the model solvers who can figure out the optimal policy. What does the optimal policy that is consistent with the agents' anticipation ("expectations-consistent" optimal policy) look like?[7] Is the expectations-consistent optimal policy similar to the outcome of the traditional optimal-policy analysis? This question seems to be an important topic for future research.

[7]The problem of finding this type of policy will be a nontrivial fixed-point problem, as the ex-ante behavior of the agents affects the nature of the optimal policy.

## References

Abel, A. B. and O. J. Blanchard (1983). An Intertemporal Model of Saving and Investment. *Econometrica 51*, 675–692.

Auerbach, A. J. and L. J. Kotlikoff (1983). National Savings, Economic Welfare, and the Structure of Taxation. In M. Feldstein (Ed.), *Behavioral Simulation Methods in Tax Policy Analysis*, pp. 459–498. University of Chicago Press.

Boppart, T., P. Krusell, and K. Mitman (2018). Exploiting MIT Shocks in Heterogeneous-Agent Economies: the Impulse Response as a Numerical Derivative. *Journal of Economic Dynamics and Control 89*, 68–92.

Dye, R. A. (1985). Costly Contract Contingencies. *International Economic Review 26*, 233–350.

Guerrieri, V., G. Lorenzoni, L. Straub, and I. Werning (2020). Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages? NBER Working Paper 26918.

Judd, K. L. (1985). Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model. *Journal of Political Economy 93*, 298–319.

Kaplan, G., B. Moll, and G. L. Violante (2018). Monetary Policy According to HANK. *American Economic Review 108*, 697–743.

Mukoyama, T. (2010). Welfare Effects of Unanticipated Policy Changes with Complete Asset Markets. *Economics Letters 109*, 134–138.

Mukoyama, T. (2013). Understanding the Welfare Effects of Unemployment Insurance Policy in General Equilibrium. *Journa of Macroeconomics 38*, 347–368.