Cyclical Part-Time Employment in an Estimated New Keynesian Model with Search Frictions*

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Abstract

This paper analyzes the dynamics of full-time employment and part-time employment over the business cycle. We first document basic macroeconomic facts on these employment stocks using the US data and decompose their cyclical dynamics into the contributions of different flows into and out of these stocks. Second, we develop and estimate a New Keynesian search-and-matching model with a segmented labor market to uncover the fundamental driving forces of the cyclical dynamics of employment stocks. We find the countercyclicality of the (net) flow from full-time to part-time employment is essential in accounting for countercyclical patterns of part-time employment.

Key Words: Part-time employment; Bayesian estimation; DSGE model; Search, matching, and bargaining.

JEL Classification: E24; E32.

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1 Introduction

Asymmetric roles of full-time and part-time workers in business cycles have attracted growing attention in recent years. In the US, the number of part-time workers increased dramatically in the process of recovery from the 2007–2009 Great Recession, while full-time employment increased only slightly.[1] In fact, the share of part-time workers in the total work force has become nearly 20% since 2010. It is, therefore, natural to infer that this heterogeneous behavior can play an important role in analyzing the business-cycle dynamics in the context of the aggregate labor market.[2]

This paper makes two contributions. The first contribution is empirical. We use the Current Population Survey (CPS) dataset to document the cyclical patterns of full-time and part-time employment in the US. Following the CPS definition, we classify individuals who work 35 hours or more per week as full-time workers and individuals who are employed but work fewer than 35 hours per week as part-time workers. Under this distinction, the full-time employment rate has a clear procyclical pattern, whereas the part-time employment rate exhibits less pronounced business-cycle dynamics during tranquil times and a sharply countercyclical pattern in deep recessions, such as the ones in the early 1980s and the Great Recession.

To uncover which labor market flows are responsible for the cyclical dynamics of employment stocks, we decompose the dynamics of stocks into the contributions of different flows into and out of the stocks. More specifically, using the rotated survey sample of the CPS, we separate the changes in population in the five distinct labor market states (full-time employment, part-time employment, unemployment looking for full-time work, unemployment looking for part-time work, and nonparticipation—or EF, EP, UF, UP, and O, respectively, in what follows) into different net flows. Because the consistent transition calculation is available only after 1996, we focus on one event: a sharp decline in EF and an increase in EP during the Great Recession, highlighting two features. First, transitions between employment and unemployment strongly contributed to the drop in EF, whereas they did not contribute to the increase in EP. Second, transitions between EF and EP were among the most important contributors to the increase in EP.[3]

[1] A similar asymmetry has also been observed in other countries. For example, Borowczyk-Martins (2017) documents that many of the major European countries saw increases in part-time employment in contrast to sharp declines in full-time employment in the aftermath of the Great Recession.

[2] In a seminal study, Blanchard and Diamond (1990) also emphasize the importance of considering the "primary" and "secondary" workers in understanding the cyclical behavior of the US labor market flows. Finegan, Penaloza, and Shintani (2008) confirm their findings using updated time-series data.

[3] As argued below, we also repeat the decomposition analysis, distinguishing between voluntary and involuntary part-time employment. We observe that during the Great Recession, transitions from full-

Our second contribution is to provide a new Dynamic Stochastic General Equilibrium (DSGE) framework. Earlier works on part-time employment in the DSGE context, such as Trigari (2009), typically consider the full-time and part-time distinction as the frictionless choice of the intensive-margin of employment, that is, the choice in hours for a given employed worker. However, some empirical facts suggest this "frictionless" assumption may not fully describe the nature of the part-time labor market.[4] First, conditional on finding a job, the majority of the unemployed looking for full-time work transition into full-time jobs, whereas the majority of the unemployed looking for part-time work transition into part-time jobs. This fact indicates the market for full-time jobs and the market for part-time jobs are segmented. Second, Canon et al. (2014) show the demographic and occupational characteristics of part-time workers are substantially different from those of full-time workers.[5] Part-time workers are relatively more concentrated in younger populations and non-routine manual types of jobs. Therefore, reallocating an employee from a part-time (full-time) position to a full-time (part-time) position is far from frictionless, given that the performed tasks and the skill levels required for different occupations tend to be different. Even when such a reallocation takes place within the same firm, assigning the right worker to the right job can require substantial costs. Unlike existing literature, our model explicitly takes these frictions into account.

Our model builds on the modeling strategy of Gertler, Sala, and Trigari (2008) (GST henceforth) who embed the Diamond-Mortensen-Pissarides (DMP) type of search-and-matching labor market frictions in a medium-scale DSGE framework. The most salient feature of our model, which differs from GST, is that it incorporates a segmented labor market (one for full-time work and the other for part-time work) and workers' transitions across different employment stocks. With this model framework, we consider all possible worker flows among EF, EP, UF, and UP.

The important characteristics of the model's dual labor market structure are as follows. First, part-time workers require fewer work hours and are assigned to less productive work than full-time workers, reflecting the CPS evidence that part-time workers work almost 50 % fewer hours on average and tend to perform lower-skilled jobs than full-time workers. Second, cyclical behaviors of unemployment, vacancies, and matches can differ between the two labor markets. Third, full-time and part-time jobs have different

time not only to involuntary part-time employment, but also to voluntary part-time employment, increased with a significant magnitude.

[4]The frictionless intensive margin adjustment is also at odds with the evidence in Borowczyk-Martins and Lalé (2019) showing that the workers flowing between EF and EP tend to make sizable adjustments in hours worked. In the CPS sample, such workers adjust weekly hours worked by 13 hours on average.

[5]See their Tables 2 and 3. They also claim that part-time workers, even those working part time for economic reasons, tend to be employed in lower-skilled occupations than full-time workers.

separation rates and different wage rigidities. Fourth, full-time employees may be involuntarily assigned to a part-time position by their employer, whereas part-time workers can move up to full-time employment without being unemployed in the meantime.

We estimate the model using the US quarterly series with standard Bayesian estimation methodology, as in Smets and Wouters (2007). We utilize the estimated model to uncover the fundamental driving forces of the cyclical dynamics of employment stocks and to quantitatively explore the US labor market dynamics over the business cycle. We find that, because of its rich features, the asymmetric dynamic responses of the two employment stocks are driven by various macroeconomic shocks that are typically analyzed in the quantitative DSGE literature. As a result, our model successfully generates empirically realistic cyclical fluctuations in both full-time and part-time employment.

Our model also does well in replicating the cyclical patterns of labor market flows that account for the cyclical fluctuations in the employment stocks. In particular, conducting the net-flow decomposition over the model-generated historical path, we find our model can reproduce the features of the US labor market dynamics in the Great Recession—countercyclical movement in transition from full-time to part-time employment plays a central role in increasing the number of part-time jobs in the recession. Furthermore, we counterfactually consider the case in which workers' transition from full-time to part-time employment is exogenously constant, and show that in such a case, the part-time employment rate counterfactually falls in the recession.

A number of studies in the recent DSGE literature consider labor market search and matching frictions along with wage bargaining between workers and firms.[6] To the best of our knowledge, however, our study is the first attempt to estimate a DSGE model that explicitly considers a dual labor market of full-time and part-time workers. In our dual labor market framework, an unemployed worker searches in a labor market for either full-time or part-time work. This framework is distinct from the model structure adopted in recent studies that analyze the business-cycle property of involuntary part-time employment. For example, Lariau (2017) incorporates the firm's choice of its part-time utilization margin into a model of single labor market search with heterogeneous workers, and Warren (2017) considers such a margin within a competitive search framework with heterogeneous firms.

[6]Merz (1995) and Andolfatto (1996) were among the first to integrate the DMP-style search frictions in the real business cycle model. Walsh (2005), Krause and Lubik (2007), Krause, López-Salido, and Lubik (2008), and Trigari (2009) incorporate labor market frictions into a New Keynesian DSGE model to study the link between labor market conditions and inflation dynamics. The more recent estimated DSGE literature allows for wage rigidities as well. For example, GST incorporate the staggered Nash bargaining setup of Gertler and Trigari (2009) and Christiano, Eichenbaum, and Trabandt (2016) incorporate the alternating-order bargaining protocol of Hall and Milgrom (2008).

As is mentioned above, an earlier work by Trigari (2009) considers both intensive and extensive margins of labor adjustment within a monetary DSGE model to study how the two margins influence inflation dynamics. Our model abstracts away the intensive-margin choice, instead considering full-time employment, part-time employment, and unemployment as distinct labor market states, motivated by the above-cited evidence suggesting full-time and part-time jobs differ in nature. In our model, the average hours per worker change through the change in the composition of the two employment stocks. This feature is in line with the recent empirical evidence of Borowczyk-Martins and Lalé (2019), which shows that changes in the composition of full-time and part-time employment, rather than changes in hours within full-time and part-time work, are more important drivers of the cyclical movements in hours per worker.

Finally, our work is related to the literature on the cyclical dynamics of employment-to-employment transitions. In the context of DSGE models, several studies (e.g., Krause and Lubik (2006, 2010); Van Zandweghe (2010); Tüzemen (2017)) incorporate such transitions into the DMP framework by allowing workers to conduct on-the-job search. Our analysis differ from this literature in focusing on endogenous transitions from full-time jobs to part-time jobs. In existing studies, job transitions are voluntary in that the worker typically moves to a better job. In our model, job-to-job transitions can be involuntary. The decision of how many workers to retain in full-time jobs is conducted by the firm, and the worker may be forced to move to a worse position.

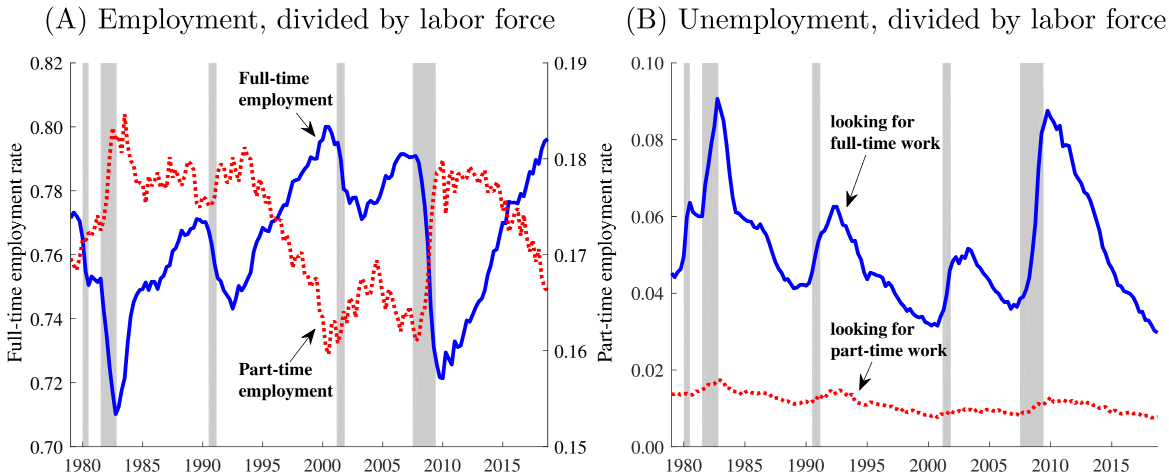
The remainder of this paper is organized as follows. Section 2 presents stylized facts regarding full-time and part-time labor markets. In Section 3, we introduce our model of two labor markets with an endogenous transition within employment stocks. In Section 4, we explain quantitative performance of the model and then conduct a counterfactual experiment. We conclude in Section 5.

2 Part-time employment and the US labor market

This section empirically analyzes the behavior of full-time and part-time employment over recent years. Our data source is the CPS, the primary source of labor force statistics in the US.[7] The statistics in this section cover all civilians 16 years old and over.

[7] The time series for full-time and part-time employment stocks are available at CPS Databases in the U.S. Bureau of Labor Statistics. See Online Appendix D.1 for the descriptions of the necessary adjustments to the time series due to the 1994 CPS redesign. See also Valletta and Bengali (2013).

Figure 1: Full-time and part-time employment rate and unemployment rate



Note: Panel (A) plots the full-time employment rate (solid line, left scale) and the part-time employment rate (dashed line, right scale). Panel (B) plots the labor-force share of the unemployed looking for full-time work (solid line) and the labor-force share of the unemployed looking for part-time work (dashed line). Shaded areas correspond to NBER recessions.

2.1 Cyclical patterns of employment stocks

As is well known, the unemployment rate is strongly countercyclical. The employment rate, the employment stock divided by the labor force, is therefore procyclical. In analyzing the dynamics of employment, we divide the total employment into full-time and part-time employment, following the CPS distinction: full-time workers are individuals who *usually* work 35 hours or more per week, and part-time workers are those who *usually* work fewer than 35 hours per week. As can seen from Figure 1, Panel (A), the full-time employment rate, full-time employment divided by the labor force, is clearly procyclical. By contrast, the part-time employment rate exhibits a less pronounced cyclical pattern during tranquil times, and a clear countercyclical pattern in deep recessions, such as one in the early 1980s and the Great Recession.[8] Another notable fact is that in the aftermath of the Great Recession, the part-time employment rate has stayed high even when the unemployment rate kept falling.

2.2 Cyclical patterns of unemployment stocks

On the unemployment side, the CPS contains a question about what type of jobs the unemployed workers are looking for. Although the answer to that question does not

[8]The correlation of the full-time employment rate with the output gap (based on HP-filtered per capita real GDP with a smoothing parameter of 1,600) is 0.549, while the correlation of the part-time employment rate with the output gap is −0.366 over the sample period of 1979-2018.

restrict the worker's actual behavior (e.g., a worker who is looking for full-time work can transition into a part-time job), it does provide some information for analyzing why part-time employment exhibits a countercyclical pattern. Note this distinction is indeed informative in light of eventual behavior: in Online Appendix E.1, we show that, conditional on finding a job, a majority (around two thirds) of UF transition into EF, whereas a majority (around four fifths) of UP transition into EP. These patterns imply that for the majority of workers flowing from the unemployment state, the full-time and the part-time jobs are *different types of jobs*. This observation motivates us to consider the part-time labor market and the full-time labor market as separate markets in Section 3. This feature distinguishes our study from the existing models of (single) labor market search with a flexible intensive margin, such as Trigari (2009), or those with part-time utilization by firms, such as Lariau (2017) and Warren (2017).

Turning to the unemployment stocks (plotted in Figure 1, Panel (B)), we find both series are countercylical, and UF exhibits stronger cyclicality than UP. This pattern indicates the main reason that part-time employment increases in recessions (especially compared to full-time employment) is *not* because so many unemployed workers are looking for part-time work in recessions. For this reason, it is important to investigate all possible flows that can change the level of the part-time employment stock.

2.3 Which labor market flows are responsible for the cyclical dynamics of employment stocks?

To further investigate the cyclical dynamics of full-time and part-time employment, we decompose the changes in employment stocks into the contributions of different flows into and out of the stocks, using the panel structure of the CPS.[9] Due to the limitations of data availability, we look into the particular case of the Great Recession era in detail.[10]

We first describe the steps to obtain our decomposition formula. Let S_t^j be the stock of labor market state $j \in \mathbb{J} = \{EF, EP, UF, UP, O\}$ in month $t = 0, \ldots, T$, and let $F_{t,t+1}^{ij}$

[9] In calculating monthly transition across the labor market states, we use the IPUMS-CPS database (available at `https://doi.org/10.18128/D030.V6.0`) provided by the Minnesota Population Center (Flood et al. (2018)). The sample period is limited to 1996-2018 by data availability for the coherent transition calculation among the five labor market states. We exclude unpaid family workers and non-incorporated self-employed workers because their usual hours tend not to be fixed. The full-time and part-time distinction of the individuals who report that their usual hours vary is based on their actual hours and the additional CPS questions to such respondents to determine if they usually work full time. The time series for the monthly transition probabilities are reported in Online Appendix E.1.

[10] Recent literature argues that, despite its severeness, the labor market dynamics in the Great Recession are qualitatively very similar to the previous recessions, except for the behavior of long-term unemployment. See Elsby et al. (2011).

be the *net* flow from state i to state j between month t and the next month $t+1$. We begin with the identity that the change in a stock is the sum of the net inflow to the stock:

$$S_{t+1}^{j} - S_{t}^{j} = \sum_{i \in \mathbb{J}} F_{t,t+1}^{ij} \quad \text{for all } j \in \mathbb{J} \quad \text{and} \quad t = 0, \dots, T-1. \tag{1}$$

We denote the (monthly) *rate of change* in stock j between month t and $t+1$ by $r_{t,t+1}^{j} = (S_{t+1}^{j} - S_{t}^{j})/S_{t}^{j}$ and normalize the net flows as $f_{t,t+1}^{ij} = F_{t,t+1}^{ij}/S_{t}^{j}$, which we call the (monthly) *net flow rate*. Furthermore, we denote the long-run time-series average of $r_{t,t+1}^{j}$ and $f_{t,t+1}^{ij}$ by $\overline{r}^{j} = T^{-1}\sum_{t=0}^{T-1} r_{t,t+1}^{j}$ and $\overline{f}^{ij} = T^{-1}\sum_{t=0}^{T-1} f_{t,t+1}^{ij}$, respectively. Then, transforming (1) delivers

$$r_{t,t+1}^{j} - \overline{r}^{j} = \sum_{i \in \mathbb{J}} \left(f_{t,t+1}^{ij} - \overline{f}^{ij} \right). \tag{2}$$

This formula shows the deviation of the rate of change in stock from the long-run average is decomposed into the deviations of the net flow rates.

Now, let T_1 and T_2 be the month when the Great Recession started and ended, respectively ($0 < T_1 < T_2 < T$). We denote the average value of the monthly rate of change in stocks and the net flow rates during the recession period by $\overline{r}_{GR}^{j} = (T_2 - T_1)^{-1}\sum_{t=T_1}^{T_2-1} r_{t,t+1}^{j}$ and $\overline{f}_{GR}^{ij} = (T_2 - T_1)^{-1}\sum_{t=T_1}^{T_2-1} f_{t,t+1}^{ij}$, respectively. Taking the time-series average of both sides of (2) during the era, we obtain our decomposition formula:

$$\overline{r}_{GR}^{j} - \overline{r}^{j} = \sum_{i \in \mathbb{J}} \left(\overline{f}_{GR}^{ij} - \overline{f}^{ij} \right). \tag{3}$$

This formula decomposes the average deviation of the monthly rate of change in stock during the Great Recession era into the average deviation of the monthly net flow rates during the era. To our knowledge, this decomposition formula is novel. The formula is widely applicable to various other situations.

Table 1 applies the decomposition (3) for EF and EP.[11] Here, to avoid seasonality issues, the Great Recession period is set to be the two years starting from December 2007.[12] The first row is the change in stock. During the recession, the share of full-time employment stock in the population over 15 years old fell by 0.48% per month, relative to the average over the entire sample period. In total, it declined about 12% during

[11]Because the CPS rotates the survey sample, a discrepancy exists between the sample used to calculate workers' flows (the right-hand side of (3)) and changes in stock (the left-hand side of (3)). We correct this margin error using the method employed by Elsby, Hobijn, and Şahin (2015). See Online Appendix D.2 for the detailed description.

[12]The corresponding recession dated by the NBER is a period between December 2007 and June 2009.

Table 1: Net flow decomposition of employment stocks over the Great Recession period

	$j = EF$	$j = EP$
The rate of change in stock of state j	-0.48	0.33
Net flow rate from state i to state j		
$i = EF$	–	0.74
$i = EP$	-0.16	–
$i = UF$	-0.22	-0.09
$i = UP$	-0.01	-0.13
$i = O$	-0.09	-0.19

Note: Average monthly flow (%) over the Great Recession period (December 2007 to November 2009), compared to the long-run average over the entire period (January 1996 to December 2018).

this two-year span.[13] The part-time employment stock, by contrast, *increased* at the rate of 0.33% per month (once again, this magnitude is consistent with Figure 1). The second to sixth rows are net flow components. For the full-time employment stock, the largest contributors are the net flows from UF and EP. Note the flow from UP has almost no contribution. For the part-time employment stock, the main contributor is the net flow from EF. These results suggest that the cyclical behavior of flows between EF and EP (shown in Online Appendix E.1 in detail) is a crucial driving force behind the cyclical pattern of both EF and EP. Some recent papers analyze the cyclicality of part-time work, distinguishing between voluntary part-time employment (EVP) and involuntary part-time employment (EIP). For example, Warren (2017) finds the transition probabilities between EIP and EF are more cyclical than the ones between EIP and unemployment. Lariau (2017) finds the countercyclical movement in the transition probability from EF to EIP employment is the key driving force of the fluctuations in EIP. Canon et al. (2014) and Borowczyk-Martins and Lalé (2018) find a similar pattern, and Borowczyk-Martins and Lalé (2019) also report related evidences. Our results are consistent with these studies. Online Appendix E.2 repeats our analysis with the distinction between EVP and EIP. Empirically, the novel contributions of our study are twofolds: (i) developing a novel decomposition method; and (ii) distinguishing between UF and UP.

In sum, to analyze the cyclical behavior of EF and EP, it is essential to (i) explicitly incorporate the flow between the two employment stocks and to (ii) separately model the labor market dynamics of workers for full-time work and part-time work. These two facts motivate our model formulation in the next section.

[13] This number is calculated by $0.48\% \times 24(\text{months}) = 11.52\%$. This magnitude can also be seen from Figure 1, where the full-time employment rate fell from 79% to 72%.

3 Model

We set up a DSGE model with cyclical part-time employment. The model is a variant of the medium-scale DSGE with frictional labor markets developed by GST, augmented to incorporate part-time employment. An important modeling decision here is that we model separate labor markets for full-time jobs and part-time jobs. This decision is motivated by the CPS evidence shown in Section 2.2 that suggests labor market segmentation between full-time and part-time jobs.[14] It is also motivated by the finding in Table 1 that the net flow from and to UF plays an prominent role in accounting for the cyclical movement of EF, whereas UP has almost no influence on the cyclicality of EP.

Another important ingredient of our model is the endogenous transitions between EF and EP. In Table 1, we find the transitions between the two employments are an important contributor to the cyclicality of both EF and EP.

The model consists of households, wholesale firms, retail firms, and the government. Households consume, invest, rent the capital stock, and supply both full-time and part-time labor. Each wholesale firm has two internal divisions (full-time and part-time division), and each division produces intermediate goods using capital and either full-time or part-time labor. Retail firms process the intermediate goods into differentiated retail goods, which are combined into the final good. The government conducts monetary and fiscal policy based on pre-specified rules.

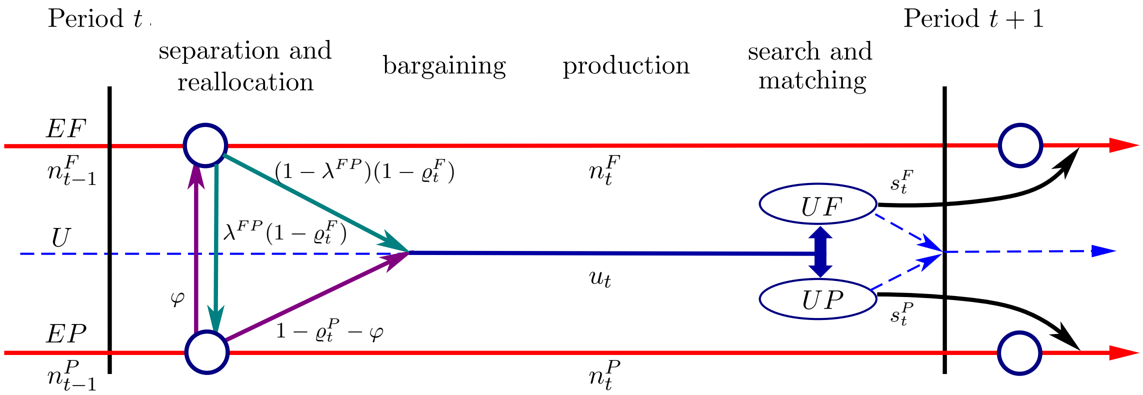
3.1 Unemployment, matching, and labor market dynamics

We consider the dynamics of workers' flows among the four employment states: EF, EP, UF, and UP. The total labor force is normalized to one. In what follows, the superscript F represents full-time work and P represents part-time work.

A summary of the timing assumptions is presented in Figure 2. At the beginning of the period, all aggregate shocks are revealed. Then, idiosyncratic employment shocks realize. There are three types of movements: (i) Unemployed workers from the previous period can move to new jobs (following the matching activity at the end of previous period), (ii) employed workers from the last period may lose jobs, and (iii) there can be movements between full-time and part-time jobs (detailed in Section 3.2). Subse-

[14] Some existing papers emphasize the distinction between EVP and EIP, detailed in Online Appendix E.2. Here we did not distinguish between these two states, so that (i) we can maintain the model to be parsimonious and (ii) the model distinction is based on simple observables (hours per week in this case). Online Appendix E.2 details the similarities and difference between EVP and EIP, as well as the importance of both stocks in relation to the flows that involve EF and UP.

Figure 2: Timing of the model



quently, wages are determined by negotiation between the workers and their employer (the wholesale firm). At the time of production, employed workers work at the wholesale firm, whereas unemployed workers receive the flow value b_t from non-market activities, denominated in units of consumption goods. Full-time work hours are normalized to one ($\mu_b^F = 1$) and part-time work hours are fixed at $\mu_b^P \in (0, 1)$. Outside working hours, part-time workers collect the flow value of $(1-\mu_b^P)b_t^P$, where the per hour flow value b_t^P is allowed to differ from b_t. At the end of the period, job seekers and vacancy meet through matching functions. With our timing assumptions, workers who lost their employment at the beginning of the period can start seeking a new job immediately, whereas job seekers who meet a vacant job at the end of period are allowed to work from the subsequent period.

Let n_t^F and n_t^P be the mass of employed full-time and part-time workers, respectively, *at the time of production* in period t. With our timing assumptions, there are $u_t = 1-n_t^F-n_t^P$ unemployed workers (job seekers) at the end of period t. From the perspective of unemployed workers, the labor market is separated for full-time work and part-time work, where the job seekers can frictionlessly choose whichever labor market they prefer to search. Therefore, the mass of those seeking a full-time job u_t^F and the mass of those seeking a part-time job u_t^P are determined to be such that seeking full-time work and seeking part-time work provide the same present-value expected utility.

Let v_t^F and v_t^P be the mass of vacant full-time and part-time jobs, respectively, posted by wholesale firms. The matching process at the two segmented labor markets is described by the matching function $m_t^\ell = m(u_t^\ell, v_t^\ell)$ for $\ell = F, P$. Specifically, we adopt the form employed by den Haan, Ramey, and Watson (2000):

$$m(u, v) = \frac{uv}{(u^\sigma + v^\sigma)^{1/\sigma}} \tag{4}$$

with $\sigma > 0$, assumed to be common between the two labor markets. Due to the constant returns-to-scale property of the matching function, the job-finding probability $s_t^\ell = m_t^\ell / u_t^\ell$ and the vacancy-filling probability $q_t^\ell = m_t^\ell / v_t^\ell$ in each market are expressed as functions of the labor market tightness $\theta_t^\ell = v_t^\ell / u_t^\ell$, that is, $s_t^\ell = s(\theta_t^\ell)$ and $q_t^\ell = q(\theta_t^\ell)$, satisfying $s(\theta_t^\ell) = \theta_t^\ell q(\theta_t^\ell)$ for $\ell = F, P$. Because θ_t^ℓ is different between $\ell = F$ and $\ell = P$, the dynamic behavior of the two labor markets can be different, even with a common matching curvature parameter σ.[15]

3.2 Wholesale firms

There is a continuum $[0, 1]$ of identical wholesale firms, each of which has two internal *divisions*: full-time and part-time.[16] In addition, within each division, there is a continuum of subdivisions with mass one. All the subdivisions within the same division are ex-ante identical. Each subdivision's employment may differ ex post, due to the difference in the occasions of wage adjustment.

Let $\mathcal{J}^F$ and $\mathcal{J}^P$ denote the set of the full-time subdivisions (i.e., the subdivisions in the full-time division) and the part-time subdivisions, respectively. At the beginning of period t, there is a mass $n_{j,t-1}^F$ of full-time employees in subdivision $j \in \mathcal{J}^F$, whereas there is a mass $n_{j,t-1}^P$ of part-time employees in subdivision $j \in \mathcal{J}^P$.

At the beginning of the period, each full-time subdivision adjusts the number of workers. In particular, it decides the fraction of full-time workers it retains (denoted by $\varrho_{j,t}^F$). Because the workers are identical, the separation is random from the viewpoint of a worker. Among the separated workers (the fraction $(1 - \varrho_{j,t}^F)$ of the existing workers), the fraction λ^{FP} is reassigned to a part-time position in the firm, and the remaining workers (the $(1 - \lambda^{FP})$ fraction) move to unemployment. In this study, we assume the fraction of the partial job separation (λ^{FP}) is time invariant, reflecting the CPS evidence presented in Online Appendix E. During the Great Recession period (when the outflow from EF increased dramatically), the relative proportions of workers flowing out of EF into EP and UF were stable.

The gross flow from EF to EP turns out to be the key determinant of how the part-time stock moves over the business cycle. Because the decision of retention is done by the firm, moving from EF to EP is involuntary for most of the workers, in the sense that their utility is higher if they stay as full-time employment. This model

[15] Another favorable feature of the functional form is that s_t^ℓ and q_t^ℓ are bounded between 0 and 1.

[16] An alternative interpretation of the model is that some firms that use only full-time workers and some firms use only part-time workers. We employ the current interpretation because the existing literature suggests transitions among full-time and part-time employment occur at the same employer in large part. (Borowczyk-Martins and Lalé (2019) and Warren (2017)).

structure, therefore, is consistent with the recent studies that emphasize the cyclicality of involuntary part-time employment.

In each part-time subdivision, a constant fraction $\varphi \in (0,1)$ of the existing part-time workers move to a full-time position within the same firm. Each part-time subdivision then chooses the retention probability $\varrho^P_{j,t}$. The remaining fraction $1-\varphi-\varrho^P_{j,t}$ of the existing part-time employees move to unemployment. As a consequence, in each wholesale firm, $\lambda^{FP}\int_{\mathcal{J}^F}(1-\varrho^F_{j',t})n^F_{j',t-1}dj'$ workers are flowing from EF to EP and $\varphi\int_{\mathcal{J}^P} n^P_{j',t-1}dj'$ workers are flowing from EP to EF.[17]

Each period, each subdivision can post vacancy: $v^F_{j,t}\geq 0$ is the mass of vacant full-time jobs posted by subdivision $j\in\mathcal{J}^F$, and $v^P_{j,t}\geq 0$ is the mass of vacant part-time jobs posted by subdivision $j\in\mathcal{J}^P$. When the labor markets open at the end of the period, given the vacancy-filling probabilities q^F_t and q^P_t in the respective labor markets, subdivision $j\in\mathcal{J}^F$ hires $q^F_t v^F_{j,t}$ new employees and subdivision $j\in\mathcal{J}^P$ hires $q^P_t v^P_{j,t}$ new employees who will start working from the subsequent period. In sum, the employment stock in each subdivision evolves according to the following transition equations[18]:

$$n^F_{j,t} = \varrho^F_{j,t}n^F_{j,t-1} + q^F_{t-1}v^F_{j,t-1} + \varphi n^P_{t-1} \quad \text{for} \quad j\in\mathcal{J}^F \tag{5}$$

and

$$n^P_{j,t} = \varrho^P_{j,t}n^P_{j,t-1} + q^P_{t-1}v^P_{j,t-1} + \lambda^{FP}\int_{\mathcal{J}^F}(1-\varrho^F_{j',t})n^F_{j',t-1}dj' \quad \text{for} \quad j\in\mathcal{J}^P. \tag{6}$$

At the time of production, each subdivision uses labor and capital to produce an intermediate good. Full-time and part-time divisions produce different goods, whereas within a division, the goods produced are homogeneous. In what follows, we label the intermediate good produced in the full-time division a "full-time intermediate good" and that produced in the part-time division a "part-time intermediate good".

Let $y^F_{j,t}$ and $y^P_{j,t}$ denote the output in subdivision $j\in\mathcal{J}^F$ and subdivision $j\in\mathcal{J}^P$ in period t, respectively. Their production function takes the form

$$y^F_{j,t} = (k^F_{j,t})^\alpha(z_t n^F_{j,t})^{1-\alpha} \qquad \text{for } j\in\mathcal{J}^F$$

and

$$y^P_{j,t} = (k^P_{j,t})^\alpha(z_t\varepsilon^\phi_t\mu^P_b n^P_{j,t})^{1-\alpha} \qquad \text{for } j\in\mathcal{J}^P \tag{7}$$

[17] Reallocating the exiting workforce across subdivisions within the same division is not allowed.

[18] The workers who transitioned across divisions are uniformly assigned to subdivisions in the new division. In fact, the distributional assumption for such workers do not matter in all model equations after being log-linearized, as long as the density is positive for all the subdivisions.

with $\alpha \in (0, 1)$. The gross growth rate of (economy-wide) labor-augmenting productivity $\varepsilon_t^z = z_t/z_{t-1}$ is assumed to follow a stationary first-order autoregressive (AR(1)) process in logs with an i.i.d. normal innovation term:

$$\log(\varepsilon_t^z) = (1-\rho_z)\log(\gamma_z) + \rho_z \log(\varepsilon_{t-1}^z) + \sigma_z \varsigma_t^z,$$

where $|\rho_z| < 1$, $\gamma_z > 0$ is the gross balanced-growth rate, $\sigma_z > 0$ is the standard deviation of innovations, and ς_t^z is a standard Gaussian random variable.[19] In (7), ε_t^ϕ is the *per hour* productivity of jobs performed by part-time workers relative to full-time workers, assumed to evolve as an AR(1) process in logs with an i.i.d. normal innovation term:

$$\log(\varepsilon_t^\phi) = (1-\rho_\phi)\log(\varepsilon^\phi) + \rho_\phi \log(\varepsilon_{t-1}^\phi) + \sigma_\phi \varsigma_t^\phi,$$

where $\varepsilon^\phi > 0$ is its steady-state value.[20]

We turn to the wholesale firm's optimization problem. Let p_t^F and p_t^P be the real sales price of the full-time and part-time intermediate goods, respectively. As in GST, we define $x_{j,t}^F \equiv q_t^F v_{j,t}^F / n_{j,t}^F$ and $x_{j,t}^P \equiv q_t^P v_{j,t}^P / n_{j,t}^P$ to be the hiring rate. Each period, each subdivision gains profit

$$\pi_{j,t}^F = p_t^F y_{j,t}^F - \frac{w_{j,t}^{Fn}}{p_t} n_{j,t}^F - r_t^k k_{j,t}^F - z_t \left(\mathcal{K}^F(x_{j,t-1}^F) + \mathcal{A}^F(\varrho_{j,t}^F)\right) n_{j,t-1}^F \quad \text{for} \quad j \in \mathcal{J}^F$$

and

$$\pi_{j,t}^P = p_t^P y_{j,t}^P - \frac{w_{j,t}^{Pn}}{p_t} (\mu_b^P n_{j,t}^P) - r_t^k k_{j,t}^P - z_t \left(\mathcal{K}^P(x_{j,t-1}^P) + \mathcal{A}^P(\varrho_{j,t}^P)\right) n_{j,t-1}^P \quad \text{for} \quad j \in \mathcal{J}^P,$$

where $w_{j,t}^{Fn}$ and $w_{j,t}^{Pn}$ denote the nominal *hourly* wages paid to the employees in the subdivision, p_t denotes the price level of the final good, and r_t^k denotes the rental rate of capital stock. Two types of cost are incurred: costs for hiring new employees, denoted by $\mathcal{K}^F(x_{j,t-1}^F) n_{j,t-1}^F$ and $\mathcal{K}^P(x_{j,t-1}^P) n_{j,t-1}^P$, and costs for retaining the exiting employees, denoted by $\mathcal{A}^F(\varrho_{j,t}^F) n_{j,t-1}^F$ and $\mathcal{A}^P(\varrho_{j,t}^P) n_{j,t-1}^P$.[21] For the hiring cost functions, following GST, we posit quadratic forms:

$$\mathcal{K}^F(x) = \frac{\kappa^F}{2} x^2 \quad \text{and} \quad \mathcal{K}^P(x) = \frac{\kappa^P}{2} x^2,$$

[19] Throughout this paper, we assume all other shocks also follow the autoregressive models of order 1 in logs with the same stationarity condition and distributional assumption on innovation.

[20] It is allowed that $\varepsilon^\phi \neq 1$. It is plausible, for example, that $\varepsilon^\phi < 1$; in this case, part-time workers are less efficient in their capacity *per hour* than full-time workers.

[21] To maintain the balanced growth, the costs are scaled by the level of technology z_t.

with $\kappa^F > 0$ and $\kappa^P > 0$. For the retention cost functions, we consider the following power functions:

$$\mathcal{A}^F(\varrho) = A^F \frac{(\varrho)^{\zeta^F+1}}{1+\zeta^F} \quad \text{and} \quad \mathcal{A}^P(\varrho) = A^P \frac{(\varrho)^{\zeta^P+1}}{1+\zeta^P}$$

with $A^F > 0$, $A^P > 0$, $\zeta^F > 0$, and $\zeta^P > 0$, ensuring $\mathcal{A}^F(\cdot)$ and $\mathcal{A}^P(\cdot)$ are strictly increasing and strictly convex.

The firms discount one-period ahead payoffs using the stochastic discount factor $\Lambda_{t,t+1}$, which is equal to the household's intertemporal marginal rate of substitution. Each subdivision's value function is given by

$$\mathcal{F}_t^F(n_{j,t-1}^F, v_{j,t-1}^F; w_{j,t}^F) = \max_{\varrho_{j,t}^F, v_{j,t}^F, n_{j,t}^F, k_{j,t}^F} \pi_{j,t}^F + \mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{F}_{t+1}^F(n_{j,t}^F, v_{j,t}^F; w_{j,t+1}^F) \right] \quad \text{for } \ j \in \mathcal{J}^F$$

and

$$\mathcal{F}_t^P(n_{j,t-1}^P, v_{j,t-1}^P; w_{j,t}^P) = \max_{\varrho_{j,t}^P, v_{j,t}^P, n_{j,t}^P, k_{j,t}^P} \pi_{j,t}^P + \mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{F}_{t+1}^P(n_{j,t}^P, v_{j,t}^P; w_{j,t+1}^P) \right] \quad \text{for } \ j \in \mathcal{J}^P,$$

where $\mathbb{E}_t$ denotes the expectations operator conditional on the information in period t.

Due to the constant-returns assumption and the free mobility of capital, all subdivisions in the same division choose the same capital-labor ratio, regardless of their wages. The same capital-labor ratio implies the same marginal product of labor: $a_{j,t}^F \equiv \partial y_{j,t}^F / \partial n_{j,t}^F = a_t^F$ for all $j \in \mathcal{J}^F$ and $a_{j,t}^P \equiv \partial y_{j,t}^P / \partial n_{j,t}^P = a_t^P$ for all $j \in \mathcal{J}^P$. Let $J_{j,t}^F$ and $J_{j,t}^P$ be the value of hiring an additional full-time worker and hiring an additional part-time worker, respectively. The interior optimality conditions associated with the firm's problem are[22]

$$z_t A^F (\varrho_{j,t}^F)^{\zeta^F} = J_{j,t}^F, \tag{8}$$

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \left(J_{j,t+1}^F - z_{t+1} \kappa^F x_{j,t}^F \right) \right] = 0,$$

$$J_{j,t}^F = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \left(z_{t+1} \left(\mathcal{K}^F(x_{j,t}^F) - \mathcal{A}^F(\varrho_{j,t+1}^F) \right) + \varrho_{j,t+1}^F J_{j,t+1}^F \right) \right],$$

$$z_t A^P (\varrho_{j,t}^P)^{\zeta^P} = J_{j,t}^P, \tag{9}$$

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \left(J_{j,t+1}^P - z_{t+1} \kappa^P x_{j,t}^P \right) \right] = 0,$$

[22] See Online Appendix F.1 for explicit derivation of the optimality conditions. In this study, we focus on the fluctuation around the interior balanced-growth steady state.

and

$$J_{j,t}^{P} = p_t^P a_t^P - \frac{w_{j,t}^{Pn}}{p_t}\mu_b^P + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(z_{t+1}\left(\mathcal{K}^P(x_{j,t}^P) - \mathcal{A}^P(\varrho_{j,t+1}^P)\right) + \varrho_{j,t+1}^P J_{j,t+1}^P\right)\right].$$

As a result, we can denote the value of the job as a function of the wage: $J_{i,t}^{\ell} = J_t^{\ell}(w_{j,t}^{\ell n})$. We use this expression in the wage bargaining. As shown in (8) and (9), the curvature parameters in the retention cost function (ζ^F and ζ^P) are the reciprocal of elasticity of the retention rates to the marginal values of a worker. Therefore, our specification nests the case in which the retention rate (equivalently job-separation rate) is exogenously constant, which is the case as the curvature parameter approaches infinity (with an appropriate adjustment to the proportional parameter).

3.3 Workers

We present the worker's value in respective employment states. Let $V_t^F(w_{j,t}^{Fn})$, $V_t^P(w_{j,t}^{Pn})$, and U_t be the worker's value of employment in a full-time job with nominal wage $w_{j,t}^{Fn}$, employment in a part-time job with nominal wage $w_{j,t}^{Pn}$, and unemployment at the time of production in period t, respectively. To set up the value of unemployment, we denote the average value of new employment in a full-time job by $V_{x,t}^F \equiv \int_{\mathcal{J}^F} V_t^F(w_{j,t}^{Fn})\hat{f}_{j,t}^F dj$, where $\hat{f}_{j,t}^F$ is the share of new full-time employees in subdivision j to the firm's total new full-time employees: $\hat{f}_{j,t}^F \equiv x_{j,t}^F n_{j,t-1}^F / \int_{\mathcal{J}^F} x_{j,t}^F n_{j,t-1}^F dj$. Analogously, we denote the average value of new employment in a part-time job by $V_{x,t}^P$.

The unemployed workers can frictionlessly choose to seek either full-time or part-time employment. The numbers of job seekers in the respective labor markets are determined so that seeking a full-time job or a part-time job provide the same expected utility:

$$s_t^F \mathbb{E}_t[\Lambda_{t,t+1}V_{x,t+1}^F] + (1-s_t^F)\mathbb{E}_t[\Lambda_{t,t+1}U_{t+1}] = s_t^P \mathbb{E}_t[\Lambda_{t,t+1}V_{x,t+1}^P] + (1-s_t^P)\mathbb{E}_t[\Lambda_{t,t+1}U_{t+1}].$$

As a result, U_t is given by

$$U_t = b_t + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(U_{t+1} + \max\{s_t^F(V_{x,t+1}^F - U_{t+1}), s_t^P(V_{x,t+1}^P - U_{t+1})\}\right)\right].$$

We build the employed worker's value functions. Recall that full-time workers in subdivision $j \in \mathcal{J}^F$ have probability $\varrho_{j,t+1}^F$ of being retained in the same position, probability $\lambda^{FP}(1-\varrho_{j,t+1}^F)$ of being reassigned to the part-time division, and probability

$(1-\lambda^{FP})(1-\varrho^F_{j,t+1})$ of being laid off. Hence, $V^F_t(w^{Fn}_{j,t})$ is given by

$$V^F_t(w^{Fn}_{j,t}) = \frac{w^{Fn}_{j,t}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\begin{array}{l} \varrho^F_{j,t+1} V^F_{t+1}(w^{Fn}_{j,t+1}) + \lambda^{FP}(1-\varrho^F_{j,t+1}) \int_{\mathcal{J}^P} V^P_{t+1}(w^{Pn}_{j,t+1}) dj \\ +(1-\lambda^{FP})(1-\varrho^F_{j,t+1}) U_{t+1} \end{array} \right) \right].$$

Part-time workers in subdivision $j \in \mathcal{J}^P$ have probability $\varrho^P_{j,t+1}$ of being retained at the same position, probability φ of moving up to a full-time position, and probability $1-\varphi-\varrho^P_{j,t+1}$ of being laid off. Hence, $V^P_t(w^{Pn}_{j,t})$ is given by

$$\begin{aligned} V^P_t(w^{Pn}_{j,t}) &= \frac{w^{Pn}_{j,t}}{p_t} \mu^P_b + b^P_t (1-\mu^P_b) \\ &+ \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\varrho^P_{j,t+1} V^P_{t+1}(w^{Pn}_{j,t+1}) + \varphi \int_{\mathcal{J}^F} V^F_{t+1}(w^{Fn}_{j,t+1}) dj + (1-\varphi-\varrho^P_{j,t+1}) U_{t+1} \right) \right]. \end{aligned}$$

As in GST, we assume the flow values from non-market activities, b_t and b^P_t, evolve proportionally to physical capital: $b_t = b k^p_t$ with $b > 0$ and $b^P_t = b^P k^p_t$ with $b^P > 0$.

3.4 Wage dynamics

The (nominal) wages are determined by a bargaining between the workers and the wholesale firm. To allow for rigidity in the wages, we adopt staggered multi-period Nash bargaining contracting à la Gertler and Trigari (2009), where wage renegotiations take place only periodically. We assume that, for each period, workers in a full-time subdivision renegotiate and recontract their wage with a constant probability $(1-\vartheta^F_w) \in (0,1]$. For a part-time subdivision, the probability is given by $(1-\vartheta^P_w) \in (0,1]$ so that the degree of wage rigidities is allowed to be different between full-time and part-time workers. In the non-negotiation subdivisions, the nominal wage is automatically adjusted following an indexation rule, $w^{\ell n}_{j,t} = \gamma_z \pi^{1-\iota_w} {\pi_{t-1}}^{\iota_w} w^{\ell n}_{j,t-1}$ (for $\ell = F, P$), where $\pi_t \equiv p_t/p_{t-1}$ is the gross inflation, π is its steady-state value, and $\iota_w \in [0,1]$ is the degree of indexation.

Each period, the newly contracted (nominal) wage in full-time division w^{*Fn}_t is chosen to maximize the Nash product

$$(V^F_t(w^{Fn}_{j,t}) - U_t)^{\eta^F_t} J^F_t(w^{Fn}_{j,t})^{1-\eta^F_t}$$

subject to

$$w^{Fn}_{j,t+k} = \begin{cases} \gamma_z \pi^{1-\iota_w} {\pi_{t+k-1}}^{\iota_w} w^{Fn}_{j,t+k-1} & \text{with probability } \vartheta^F_w \\ w^{*Fn}_{t+k} & \text{with probability } 1-\vartheta^F_w, \end{cases}$$

for all $k \geq 1$. The bargaining weight satisfies $\eta_t^F = \eta^F \varepsilon_t^{\eta,F}$, where $\eta^F \in (0,1)$ is its steady-state value and $\varepsilon_t^{\eta,F}$ is an exogenous shock that follows an AR(1) process in logs with an i.i.d. normal innovation term: $\log(\varepsilon_t^{\eta,F}) = \rho_\eta^F \log(\varepsilon_{t-1}^{\eta,F}) + \sigma_\eta^F \varsigma_t^{\eta,F}$.

Analogously, in the part-time division, the newly contracted nominal wage w_t^{*Pn} maximizes $(V_t^P(w_{j,t}^{Pn}) - U_t)^{\eta_t^P} J_t^P(w_{j,t}^{Pn})^{1-\eta_t^P}$ subject to the aforementioned staggered wage-setting process. The bargaining weight satisfies $\eta_t^P = \eta^P \varepsilon_t^{\eta,P}$, where $\eta^P \in (0,1)$ is its steady-state value and $\varepsilon_t^{\eta,P}$ is an exogenous shock that follows an AR(1) process in logs with an i.i.d. normal innovation term: $\log(\varepsilon_t^{\eta,P}) = \rho_\eta^P \log(\varepsilon_{t-1}^{\eta,P}) + \sigma_\eta^P \varsigma_t^{\eta,P}$.

With the above formulations, the dynamic equations for the average real wages for full-time and part-time workers, denoted by w_t^F and w_t^P, respectively, (after being log-linearized) can be written as

$$\tilde{w}_t^\ell = \omega_b^\ell \left(\tilde{w}_{t-1}^\ell - \tilde{\pi}_t + \iota_w \tilde{\pi}_{t-1} - \tilde{\varepsilon}_t^z\right) + \omega_o^\ell \tilde{w}_t^{o,\ell} + \omega_f^\ell \mathbb{E}_t \left[\tilde{w}_{t+1}^\ell + \tilde{\pi}_{t+1} - \iota_w \tilde{\pi}_t + \tilde{\varepsilon}_{t+1}^z\right], \quad (10)$$

with $\omega_b^\ell + \omega_o^\ell + \omega_f^\ell = 1$ and $w_t^{o,\ell}$ being the wage that would be chosen if the wages were negotiated period by period for $\ell = F, P$. The variables with tilde $(\tilde{\ })$ here are log deviations from the steady-state values (see Online Appendix F.2 for the derivations of (10)).

3.5 Retailers

There is a continuum of monopolistically competitive retailers with mass one. Retailer $i \in [0,1]$ purchases $y_{i,t}^F$ units of a full-time intermediate good at price p_t^F and $y_{i,t}^P$ units of a part-time intermediate good at price p_t^P from the wholesale firms. Then, it transforms them into a differentiated retail good using the production function

$$y_{i,t} = \left[(\Omega^F)^{\frac{1}{\xi}} (y_{i,t}^F)^{\frac{\xi-1}{\xi}} + (1-\Omega^F)^{\frac{1}{\xi}} (y_{i,t}^P)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

with $\Omega^F \in (0,1)$ and $\xi > 0$ being the elasticity of substitution across the two intermediate goods. Retailers have pricing power. However, following a Calvo rule, a randomly chosen fraction $(1-\vartheta_p) \in (0,1]$ of them are allowed to reoptimize their price, whereas non-reoptimizing retailers fully index their price to a mixture of trend inflation and lagged inflation: $p_{i,t} = \pi^{1-\iota_p} {\pi_{t-1}}^{\iota_p} p_{i,t-1}$, with $\iota_p \in [0,1)$ being the degree of price indexation.

We adopt the Kimball (1995) formulation of product differentiation, which is a generalization of the Dixit-Stiglitz formulation. Let $y_{i,t}$ be the quantity sold by retailer i. The composite final goods, whose quantity is y_t, is implicitly defined by $\int_0^1 \mathcal{G}\left(y_{i,t}/y_t; \varepsilon_t^p\right) di = 1$, where the demand aggregator function $\mathcal{G}(\cdot)$ satisfies $\mathcal{G}(1) = 1$,

$\mathcal{G}'(\cdot) > 0$, and $\mathcal{G}''(\cdot) < 0$. The markup shock ε_t^p follows an AR(1) process in logs with an i.i.d. normal innovation term: $\log(\varepsilon_t^p) = (1-\rho_p)\log(\varepsilon^p) + \rho_p \log(\varepsilon_{t-1}^p) + \sigma_p \varsigma_t^p$, where $\varepsilon^p > 1$ is the steady-state price markup.

3.6 Households

We consider a representative household that consists of a continuum of infinitely-lived workers with a total mass of one. The workers are either employed full time (a fraction n_t^F), employed part time (a fraction n_t^P), or unemployed (a fraction $1-n_t^F-n_t^P$). There is a perfect insurance for consumption within a household, as in Merz (1995) and Andolfatto (1996). In our setting, the household members pool their income and maximize the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^b \log(c_t - h_c c_{t-1}), \tag{11}$$

where c_t is the household's real consumption spending in period t, $\beta \in (0,1)$ is the discount factor, and $h_c \in [0,1)$ is the degree of habit persistence. The intertemporal preference shock ε_t^b follows an AR(1) process in logs with an i.i.d. normal innovation term: $\log(\varepsilon_t^b) = \rho_b \log(\varepsilon_{t-1}^b) + \sigma_b \varsigma_t^b$.

Let d_t denote the sum of the household's real labor income and flow value from non-market activity in period t: $d_t = \int_{\mathcal{J}^F} w_{j,t}^F n_{j,t}^F dj + \int_{\mathcal{J}^P} w_{j,t}^P \mu_b^P n_{j,t}^P dj + (1 - n_t^F - \hat{\mu}_b^P n_t^P) b_t$ with $\hat{\mu}_b^P \equiv 1 - (1-\mu_b^P) b^P / b$. The household faces a dynamic budget constraint

$$c_t + i_t + \frac{B_t}{r_t^n p_t} = d_t + T_t + r_t^k k_t - \mathcal{C}(\nu_t) k_{t-1}^p + \frac{B_{t-1}}{p_t}, \tag{12}$$

where i_t is real investment expenditure, B_t is nominal bond holdings, r_t^n is the gross one-period nominal risk-free rate, T_t is the lump-sum transfer (net of taxes) plus the firms' profit, k_{t-1}^p is physical capital stock available at period t, ν_t is capital utilization, and $k_t = \nu_t k_{t-1}^p$ is the effective capital used for production in period t. Capital utilization incurs a cost $\mathcal{C}(\nu_t)$ per unit of capital. We assume that, in a steady state, $\nu_t = 1$, $\mathcal{C}(1) = 0$, and $\mathcal{C}'(1)/\mathcal{C}''(1) > 0$ hold. The evolution equation of physical capital is given by

$$k_t^p = (1-\delta) k_{t-1}^p + \varepsilon_t^i \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right] i_t, \tag{13}$$

with $\delta \in (0,1)$ being the depreciation rate. The investment adjustment costs function $\mathcal{S}(\cdot)$ satisfies $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and $\mathcal{S}''(\gamma_z) > 0$. The investment-specific technology shock ε_t^i follows an AR(1) process in logs with an i.i.d. normal innovation term: $\log(\varepsilon_t^i) = \rho_i \log(\varepsilon_{t-1}^i) + \sigma_i \varsigma_t^i$. The household maximizes (11) subject to (12) and (13) by choosing

c_t, B_t, i_t, k_t^p, and ν_t, taking the prices p_t, r_t^n, and r_t^k as given.

3.7 Government

The fiscal authority sets the government spending g_t as a time-varying fraction υ_t of aggregate output: $g_t = \upsilon_t y_t$. The government spending shock defined by $\varepsilon_t^g = 1/(1-\upsilon_t)$ follows an AR(1) process in logs with an i.i.d. normal innovation term: $\log(\varepsilon_t^g) = (1-\rho_g)\log(\varepsilon^g) + \rho_g \log(\varepsilon_{t-1}^g) + \sigma_g \varsigma_t^g$ with $\varepsilon^g = 1/(1-\upsilon)$. The monetary authority sets the nominal interest rate following a Taylor-type policy rule:

$$\frac{r_t^n}{r^n} = \left(\frac{r_{t-1}^n}{r^n}\right)^{\phi_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{nt}}\right)^{\phi_y}\right]^{(1-\phi_r)} \varepsilon_t^r,$$

where y_{nt} is the natural level of output, r^n is the steady-state nominal interest rate and ϕ_r, ϕ_π, and ϕ_y are positive policy parameters. The monetary policy shock ε_t^r follows an AR(1) process in logs with an i.i.d. normal innovation term: $\log(\varepsilon_t^r) = \rho_r \log(\varepsilon_{t-1}^r) + \sigma_r \varsigma_t^r$.

3.8 Evolution of the employment stocks

The aggregate full-time and part-time employment stocks evolve following

$$n_t^F = \int_{\mathcal{J}^F} \varrho_{j,t}^F n_{j,t-1}^F dj + s_{t-1}^F u_{t-1}^F + \varphi n_{t-1}^P \tag{14}$$

and

$$n_t^P = \int_{\mathcal{J}^P} \varrho_{j,t}^P n_{j,t-1}^P dj + s_{t-1}^P u_{t-1}^P + \lambda^{FP} \int_{\mathcal{J}^F} (1-\varrho_{j,t}^F) n_{j,t-1}^F dj, \tag{15}$$

respectively.[23] The first terms in the right-hand side of both equations are the workers who stay at the same employment states, the second terms are new inflows from the unemployment pool, and the third terms are inflows from the other employment state.

4 Quantitative experiments

In this section, we estimate the model using Bayesian methods and investigate the key factor that accounts for labor market fluctuations.

[23] The evolution equations (14) and (15) are obtained by integrating (5) and (6), which describe the law of motion of the employment stock in each subdivision, and using the equilibrium conditions $q_t^F v_t^F = s_t^F u_t^F$ and $q_t^P v_t^P = s_t^P u_t^P$, where $v_t^F = \int_{\mathcal{J}^F} v_{j,t}^F dj$ and $v_t^P = \int_{\mathcal{J}^P} v_{j,t}^P dj$ are aggregate vacancy in the respective labor markets.

Table 2: Fixed parameters

α	δ	Ξ	υ	μ_b^P	ϱ^F	ϱ^P	λ^{FP}	φ	s^F	s^P	σ	ζ^F
0.333	0.025	10	0.24	0.501	0.847	0.423	0.504	0.361	0.812	0.923	1.245	7.0

4.1 Estimation procedure

To estimate the model in the linear state space form, we convert the model equations into the linear equations in terms of the log deviations from the deterministic steady state. In the log-linearized equations, the coefficients depend not only on the primitive parameters, but also on the steady-state values of endogenous variables (see Online Appendices G.1 and G.2 for the lists of the steady-state conditions and the log-linearized equations).

We use the following nine quarterly series for the sample period from 1979:Q1 to 2018:Q4: (i) per capita real GDP, (ii) per capita real consumption, (iii) per capita real investment, (iv) the real average wage, (v) inflation, (vi) the employment rate, (vii) the nominal interest rate, (viii) the ratio of full-time employment to part-time employment, and (ix) the relative real hourly wages for full-time and part-time workers.[24] Note that time-series information on labor market flows is *not* used, because labor market transition data are not consistently available before the 1994 CPS redesign.

Table 2 lists the values of the parameters fixed in the estimation. We take the model period as one quarter and set the following: capital intensity $\alpha = 1/3$; capital depreciation rate $\delta = 0.025$; the curvature parameter of the Kimball aggregator $\Xi = 10$; the steady-state ratio of government spending to output $\upsilon = 0.24$, which is the sample average of the GDP share of external demands; hours worked by part-time workers $\mu_b^P = 0.501$, being consistent with the CPS evidence that the median hours worked per week for part-time workers is 50.1% of that for full-time workers.

We fix the parameters governing the steady-state labor-market transition probabilities. Let $Q_{i,j}$ denote the time-series average of the quarterly transition probability from labor market state i to j measured from the CPS over 1996-2018.[25] The steady-state employment retention rates are set to $\varrho^F = Q_{EF,EF} = 0.847$ and $\varrho^P = Q_{EP,EP} = 0.423$, the fraction of separated full-time workers who are reassigned to a part-time position is set to $\lambda^{FP} = Q_{EF,EP}/(1 - Q_{EF,EF}) = 0.504$, and the fraction of part-time workers who transition into full-time employment is set to $\varphi = Q_{EP,EF} = 0.361$. The steady-state job-finding probabilities are chosen to satisfy the following two conditions: (i) The

[24] See Appendix A for the detailed data description and the corresponding observation equations.

[25] The quarterly transition probability matrix is obtained by the matrix multiplication of the monthly transition probability matrices of three consecutive months.

ratio of the probability with which an unemployed worker fails to find a job in the full-time labor market to that in the part-time labor market, $(1-s^F)/(1-s^P)$, equals $(Q_{UF,UF}+Q_{UF,UP})/(Q_{UP,UF}+Q_{UP,UP}) = 2.410$, and (ii) the aggregate job-finding probability, defined as the average of s^F and s^P weighted by the workers' share in the two labor markets, is consistent with Shimer's (2005) estimate of the average monthly job-finding probability, or $(u^F/u)s^F + (u^P/u)s^P = 1-(1-0.45)^3 = 0.834$, where $u = u^F + u^P$. As a result, we set $s^F = 0.812$ and $s^P = 0.923$. The matching curvature parameter σ and the steady-state labor market tightness $\theta^F = v^F/u^F$, $\theta^P \equiv v^P/u^P$ are set as follows. With the matching function (4), s^F and θ^F satisfy $s^F = \theta^F/(1+(\theta^F)^\sigma)^{\frac{1}{\sigma}}$ and $\theta^F = s^F/q^F$. Then, following Hagedorn and Manovskii (2008), we use the steady-state monthly vacancy-filling probability of 0.71 to obtain the target level of $\theta^F = 0.45/0.71 = 0.634$. This procedure results in $\sigma = 1.245$. For the part-time market, we assume σ is the same as for the full-time market.

For the curvature parameters in the retention cost functions, considering the observation that full-time workers have more cyclical separations than part-time workers (see Figure E.1), we focus on the case where the retention rates are endogenous only for full-time employment. We assign a high value to ζ^P so that the retention rate of part-time employment becomes practically constant. We set $\zeta^F = 7.0$, by targeting the standard deviation of the retention rate of 0.005.[26]

We next describe the parameters for estimation. A summary of the prior distribution is reported in the third to sixth columns in Table 3. Our prior choice for the non-labor-market parameters and the parameters for exogenous-shock processes is in accordance with the literature on DSGE model estimation.[27] Below, we detail our choice of prior for eight new labor market parameters ($1/\xi$, η^F, η^P, $\bar{b}^F$, ε^ϕ, ϑ_w^F, ϑ_w^P, ι_w) that arise from the introduction of the dual labor market structure.

We use a Gamma prior with a mean of 0.25 for the reciprocal of the elasticity of substitution between full-time and part-time intermediate goods $1/\xi$, allowing imperfect substitutability between the two labors. For the workers' surplus shares in wage bargaining (η^F, η^P), we use a Beta prior with a mean of 0.5, a value often used in the literature. $\bar{b}^F$ is a new parameter defined as $\bar{b}^F = \bar{b}/[p^F\bar{a}^F + \beta(\mathcal{K}^F(x^F) + \mathcal{A}^F(\varrho^F))]$, where $\bar{b}$ and $\bar{a}^F$ are the steady-state values of the de-trended variables b_t/z_t and a_t^F/z_t, respectively. Con-

[26]In the CPS data over 1996-2018, the standard deviation of the corresponding rate is 0.0075. We target a smaller value with the consideration that the CPS value contains considerable measurement errors.

[27]For the steady-state capital utilization elasticity $\mathcal{C}'(1)/\mathcal{C}''(1)$, we estimate the parameter $\psi_\nu = 1/(1+(\mathcal{C}'(1)/\mathcal{C}''(1))) \in (0,1)$ instead. The priors for γ_z, r^n, and π are centered around the sample average of the growth rate of per capita output, the nominal interest rate, and the inflation rate, respectively. The subjective discount factor β is set to satisfy the steady-state condition $\gamma_z = \beta(r^n/\pi)$.

Table 3: Prior and posterior distributions

Parameters		Prior distribution Shape	Support	Mean	Std.	Posterior distribution Mean	90% interval
			Structural parameters				
Preferences and technology parameters							
ψ_ν	Elasticity in utilization rate	Beta	[0.0 , 1.0]	0.50	0.15	0.63	[0.49 , 0.79]
$\mathcal{S}''(\gamma_z)$	Capital adjustment cost elasticity	Normal	$\mathbb{R}_+$	4.00	1.50	3.22	[2.43 , 4.04]
h_c	Habit persistence in consumption	Beta	[0.0 , 1.0]	0.70	0.15	0.86	[0.81 , 0.91]
Labor market parameters							
$1/\xi$	Product substitutability	Gamma	$\mathbb{R}_+$	0.25	0.25	0.25	[0.17 , 0.33]
η^F	Bargaining power: FT worker	Beta	[0.0 , 1.0]	0.50	0.15	0.77	[0.66 , 0.87]
η^P	Bargaining power: PT worker	Beta	[0.0 , 1.0]	0.50	0.15	0.79	[0.68 , 0.89]
$\bar{b}^F$	Relative flow value of unemployment	Beta	[0.0 , 1.0]	0.70	0.05	0.71	[0.64 , 0.79]
ε^ϕ	Relative PT labor productivity	Uniform	[0.0 , 1.0]	0.50	0.29	0.69	[0.65 , 0.73]
ϑ_w^F	Calvo wage parameter: FT worker	Beta	[0.0 , 1.0]	0.50	0.15	0.37	[0.27 , 0.47]
ϑ_w^P	Calvo wage parameter: PT worker	Beta	[0.0 , 1.0]	0.50	0.15	0.10	[0.05 , 0.16]
ι_w	Wage indexation	Beta	[0.0 , 1.0]	0.50	0.15	0.47	[0.28 , 0.66]
Price setting and monetary policy parameters							
ϑ_p	Calvo price parameter	Beta	[0.0 , 1.0]	0.50	0.15	0.85	[0.81 , 0.89]
ι_p	Price indexation	Beta	[0.0 , 1.0]	0.50	0.15	0.58	[0.40 , 0.75]
ε^p	Steady-state price markup	Normal	[1.0 , Inf)	1.15	0.05	1.41	[1.36 , 1.47]
ϕ_π	Taylor rule response to inflation	Gamma	[1.0 , Inf)	1.50	0.30	1.27	[1.03 , 1.48]
ϕ_y	Taylor rule response to output gap	Gamma	$\mathbb{R}_+$	0.15	0.10	0.07	[0.04 , 0.11]
ϕ_r	Monetary policy smoothing	Beta	[0.0 , 1.0]	0.75	0.15	0.72	[0.67 , 0.77]
Trend and steady-state values							
$100\log(\gamma_z)$	Growth rate in balanced growth path	Gamma	$\mathbb{R}_+$	0.34	0.05	0.28	[0.22 , 0.34]
$100\log(r^n)$	Steady-state nominal interest rate	Gamma	$\mathbb{R}_+$	1.22	0.05	1.28	[1.20 , 1.35]
$100\log(\pi)$	Steady-state inflation rate	Gamma	$\mathbb{R}_+$	0.69	0.05	0.67	[0.61 , 0.73]
			Exogenous-processes parameters				
Autoregressive coefficient							
ρ_z	Technology	Beta	[0.0 , 1.0]	0.50	0.20	0.09	[0.02 , 0.16]
ρ_b	Intertemporal preference	Beta	[0.0 , 1.0]	0.50	0.20	0.41	[0.26 , 0.55]
ρ_i	Investment-specific technology	Beta	[0.0 , 1.0]	0.50	0.20	0.88	[0.84 , 0.91]
ρ_p	Price markup	Beta	[0.0 , 1.0]	0.50	0.20	0.19	[0.03 , 0.35]
ρ_g	Government spending	Beta	[0.0 , 1.0]	0.50	0.20	0.99	[0.98 , 1.00]
ρ_r	Monetary policy	Beta	[0.0 , 1.0]	0.50	0.20	0.21	[0.10 , 0.32]
ρ_w^F	Wage specific to FT work	Beta	[0.0 , 1.0]	0.50	0.20	0.12	[0.03 , 0.20]
ρ_w^P	Wage specific to PT work	Beta	[0.0 , 1.0]	0.50	0.20	0.91	[0.87 , 0.95]
ρ_ϕ	Part-time job productivity	Beta	[0.0 , 1.0]	0.50	0.20	0.93	[0.89 , 0.97]
Standard deviation							
σ_z	Technology	IG	$\mathbb{R}_+$	0.50	1.50	0.87	[0.79 , 0.95]
σ_b	Intertemporal preference	IG	$\mathbb{R}_+$	0.50	1.50	2.97	[2.00 , 3.91]
σ_i	Investment-specific technology	IG	$\mathbb{R}_+$	0.50	1.50	2.83	[2.28 , 3.37]
σ_p	Price markup	IG	$\mathbb{R}_+$	0.50	1.50	0.12	[0.10 , 0.14]
σ_g	Government spending	IG	$\mathbb{R}_+$	0.50	1.50	0.42	[0.38 , 0.46]
σ_r	Monetary policy	IG	$\mathbb{R}_+$	0.50	1.50	0.23	[0.21 , 0.26]
σ_w^F	Wage specific to FT work	IG	$\mathbb{R}_+$	0.50	1.50	3.54	[2.05 , 4.98]
σ_w^P	Wage specific to PT work	IG	$\mathbb{R}_+$	0.50	1.50	2.56	[1.62 , 3.50]
σ_ϕ	Part-time job productivity	IG	$\mathbb{R}_+$	0.50	1.50	2.10	[1.68 , 2.50]

Note: IG denotes the Inverse Gamma distribution. FT and PT denote full time and part time, respectively. The 90% interval reports the range between the 5th and 95th percentiles of the draws from the posterior distribution.

sidering that $\bar{b}^F$ corresponds to the value of non-market activity relative to the market activity of a full-time worker, we use a Beta prior with a mean of 0.7, close to the value proposed by Hall and Milgrom (2008), who argue it should reflect non-monetary benefits from nonworking as well as monetary benefits. For the relative per hour productivity of part-time work ε^{ϕ}, we use a uniform prior over the unit interval because evidence that suggests the appropriate value of this parameter is scant. We use a Beta prior with a mean of 0.5 for the Calvo wage parameters (ϑ_w^F, ϑ_w^P) and the wage indexing parameter ι_w, reflecting accumulated evidence that suggests a substantial degree of wage rigidity from estimated New Keynesian DSGE models (e.g., Christiano, Eichenbaum, and Evans (2005); Smets and Wouters (2007); GST).

4.2 Estimation results

The posterior distribution is numerically obtained with Markov chain Monte Carlo methods.[28] We report the mean and the 90% credible interval (i.e., the 5th-95th percentile range) of the posterior parameter distribution in the seventh to eighth columns in Table 3. Our estimates imply a substantial degree of adjustment costs in investment, habit persistence in consumption, and price stickiness in line with previous studies (e.g., Smets and Wouters (2007); Justiniano, Primiceri, and Tambalotti (2010)).

With regard to the labor market parameters, the posterior mean estimate of $1/\xi$ is 0.25, implying full-time and part-time work are considerably substitutable. The estimates of the wage-bargaining parameters are $\eta^F = 0.77$ and $\eta^P = 0.79$, which are in between the values often used in calibration studies (0.5 to 0.7) and GST's estimate (0.91). The estimate of $\bar{b}^F$ is 0.71, in line with existing macroeconomic evidence from estimated DSGE models.[29] The estimate of ε^{ϕ} is 0.69, indicating hourly productivity of a part-time worker is lower by about 30% relative to that of a full-time worker. This result is consistent with the fact that part-time workers tend to be concentrated in low-skilled occupations than full-time workers (Canon et al. (2014)). The estimates of the Calvo wage parameters are $\vartheta_w^F = 0.37$ and $\vartheta_w^P = 0.10$. These values indicate full-time workers face more rigidity in wages than part-time workers, supporting the assumptions on wage rigidity often used in analyses of calibrated DSGE models with part-time employment.[30]

To assess how the model fits the data, we display the 90% posterior interval of the

[28]We generate 700,000 draws from the posterior distribution using the random walk Metropolis-Hastings algorithm and discard the first 350,000 draws as burn-in replications.

[29]For example, GST estimate this parameter as 0.73 under a single labor market model, and Christiano, Eichenbaum, and Trabandt (2016) estimate it as 0.88 under the alternating-offers wage-bargaining framework.

[30]For example, Lariau (2017) allows for wage rigidity only for full-time workers.

Table 4: Cyclical property of the employment stocks

Full-time employment rate			Part-time employment rate		
Model		US data	Model		US data
0.642	$[0.479, 0.799]$	0.549	-0.525	$[-0.692, -0.350]$	-0.366

Note: This table reports the correlation of the employments rates (full-time employment rate on the left-hand side and part-time employment rate on the right-hand side) with the GDP gap (based on HP-filtered real output with a smoothing parameter of $1,600$) in the estimated model (the mean and the 90% interval over $1,200$ draws from the posterior parameter distribution and 200 simulated samples of 160 observations for each draw) and the US data.

model's autocovariance function along with the empirical autocovariance function in Figure C.1 in Appendix C.[31] This figure shows the empirical autocovariance function falls within the 90% posterior interval for the most part, implying that the model does well in reproducing the joint dynamics of output, consumption, wage, inflation, and employment observed in the data.

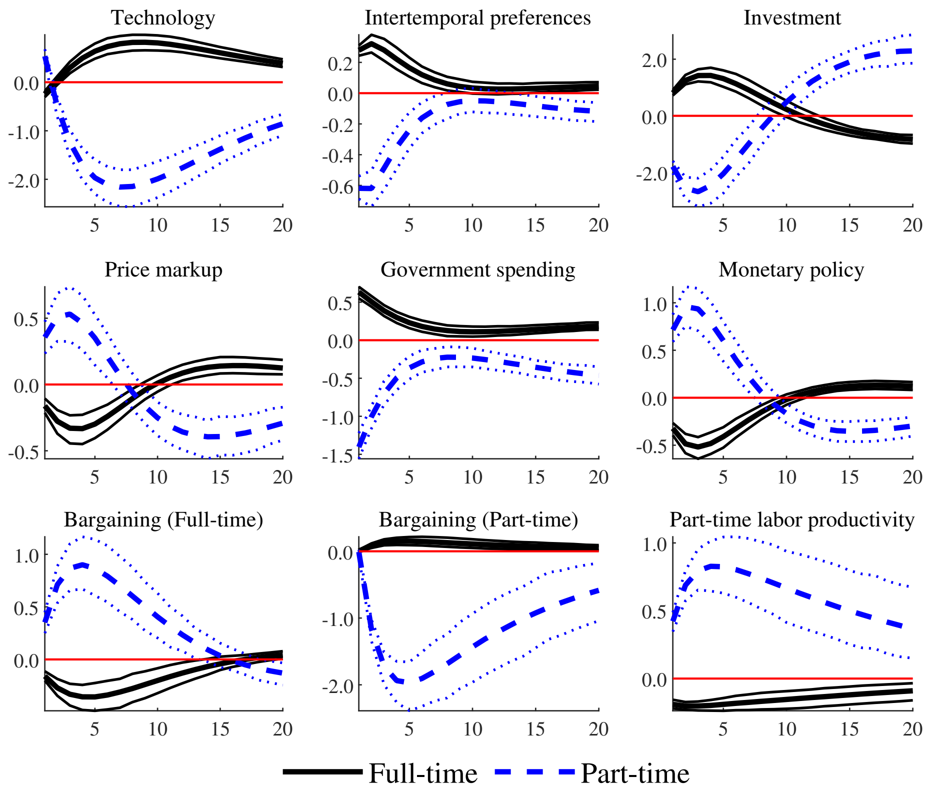
4.3 Asymmetric dynamic responses of full-time and part-time employment

Figure 3 displays the posterior impulse response functions of the full-time and part-time employment rates to nine shocks in the model. We find our model produces asymmetric responses of these employment rates to a large variety of the shocks. Note that, as can be seen from the top and middle rows, such asymmetric responses are the results of the macroeconomic shocks that are typically considered in many quantitative DSGE models. For example, following a positive technology shock and a positive investment-specific technology shock, the full-time employment rate increases, while the part-time employment rate falls. Following a contractionary monetary-policy shock, the full-time employment rate falls, while the part-time employment rate increases.

Such asymmetric dynamic responses result in asymmetric cyclical behavior of the two employment stocks. Table 4 compares their correlations with the cyclical component of real GDP obtained from the estimated model (the mean and the 90% interval over the draws from the posterior distribution) with those correlations measured in the data. The table demonstrates that the estimated model is able to generate empirically realistic cyclical patterns of the full-time and part-time employment rates.

[31]The model's autocovariances are computed on simulated data for 160 periods generated by $1,200$ draws from the posterior parameter distribution and 200 simulated samples for each draw. The 90% posterior interval reports the range between their 5th and 95th percentiles.

Figure 3: Posterior impulse-response functions of full-time and part-time employment



Note: Each panel shows the posterior median and the 90% posterior interval of the impulse responses of the full-time employment rate (solid lines) and the part-time employment rate (broken lines) to a one-standard-deviation shock as percentage deviations from the steady state. 'Investment' in the upper-right panel denotes investment-specific technology shock.

In Appendix C, we report the responses of other key macroeconomic variables to the technology shock (in Figure C.2) and the investment-specific technology shock (Figure C.3). The shapes of the impulse responses to both shocks, except for the aggregate wage, are similar to those estimated by GST using a single labor market model with wage rigidity. The dynamic response of the aggregate wage in our model is more responsive than GST's estimate, largely because of the change in the composition of the employment stocks. As shown above, both shocks increase the share of full-time employment to total employment. Because the average wage for the full-time workers is higher than that of the part-time workers, the increase in the share of full-time employment increases the aggregate wage. This force makes the aggregate wage more responsive to the shocks.

4.4 Cyclical dynamics of labor market flows

Next, we turn to the labor market flows that account for the fluctuations in the employment stocks. Here, we apply the decomposition method we used for the empirical analysis in Section 2 to the model variables.

For this purpose, we rearrange the terms in the evolution equations of the employments stocks (14) and (15) and express the rate of change in the employment stock ($r^{\ell}_{t-1,t} = (n^{\ell}_t - n^{\ell}_{t-1})/n^{\ell}_{t-1}$ for $\ell = F, P$) in terms of the net flow rate from the other employment stock and the net flow rate from the unemployment stock.[32] Specifically, for the full-time employment rate, we obtain

$$r^{F}_{t-1,t} = f^{PF}_{t-1,t} + f^{UF}_{t-1,t}, \tag{16}$$

where $f^{PF}_{t-1,t}$ and $f^{UF}_{t-1,t}$ represent the net flow rate from EP to EF and the net flow rate from the unemployment stock to EF, respectively, between $t-1$ and t (see Appendix B for the explicit expressions of $f^{PF}_{t-1,t}$ and $f^{UF}_{t-1,t}$). On the other hand, for the part-time employment rate, we obtain

$$r^{P}_{t-1,t} = f^{FP}_{t-1,t} + f^{UP}_{t-1,t}, \tag{17}$$

where $f^{FP}_{t-1,t}$ and $f^{UP}_{t-1,t}$ represent the net flow rate from EF to EP and the net flow rate from the unemployment stock to EP, respectively, between $t-1$ and t (again, see Appendix B for the explicit expressions of $f^{FP}_{t-1,t}$ and $f^{UP}_{t-1,t}$).

Then, log-linearizing (16) and (17) and iterating backward, we obtain the decomposition formula for the full-time employment rate:

$$\tilde{n}^{F}_{h} - \tilde{n}^{F}_{0} = \sum_{s=0}^{h} f^{PF} \tilde{f}^{PF}_{s-1,s} + \sum_{s=0}^{h} f^{UF} \tilde{f}^{UF}_{s-1,s} \tag{18}$$

and that for the part-time employment rate:

$$\tilde{n}^{P}_{h} - \tilde{n}^{P}_{0} = \sum_{s=0}^{h} f^{FP} \tilde{f}^{FP}_{s-1,s} + \sum_{s=0}^{h} f^{UP} \tilde{f}^{UP}_{s-1,s} \tag{19}$$

with $h \geq 0$ being the number of periods from the reference time. Note the variables without time subscript denote the steady-state values of the net flow rates (e.g., f^{PF} is the steady-state value of $f^{PF}_{t-1,t}$) and the variables with tilde ($\tilde{}$) denote log deviations from the steady-state values, and therefore this formula is the log-linear version of the decomposition formula in Section 2.

In Table 5, we apply the formulas to the impulse responses to a (positive) investment-specific technology shock, the most important shock driving the business cycle.[33] As

[32] The net flow rate from state X to state Y between $t-1$ and t is defined as the number of the net flow from state X to state Y between $t-1$ and t divided by the number of workers in state Y at $t-1$.

[33] The variance decomposition shows the investment-specific technology shocks account for about 40%

Table 5: Decomposed impulse responses: A positive investment-specific technology shock

	Full-time employment rate ($\ell = F$)				Part-time employment rate ($\ell = P$)			
Horizons	on impact	1 quarters	2 quarters	one year	on impact	1 quarters	2 quarters	one year
$\tilde{n}_t^\ell$	0.80	1.25	1.42	1.30	−1.78	−2.55	−2.66	−2.01
From EF	–	–	–	–	−1.78	−1.82	−1.20	0.74
From EP	0.40	0.40	0.24	−0.24	–	–	–	–
From U	0.40	0.85	1.17	1.54	0.00	−0.72	−1.46	−2.75

Note: The third row shows the responses of the employment rates at the respective horizons, and the fourth-sixth rows show the contributions of the response of the respective labor market flows according to the decomposition formulas (18) and (19). The parameter values are fixed at the posterior means.

Table 6: Net flow decomposition of the employment stocks: 2008:Q1-2009:Q4

	Full-time employment	Part-time employment
The rate of change in employment stock	−9.18	10.44
Net flow from EF	–	17.86
Net flow from EP	−4.17	–
Net flow from U	−5.02	−7.42

Note: This table shows the net flow decomposition of the full-time employment rate and the part-time employment rate over the model-generated historical path in the Great Recession period.

shown in the third row, the full-time employment rate responds positively, whereas the part-time employment rate responds negatively for more than a year following the shock. The fourth to sixth rows show the contributions of each (net) flow into the employment stock. We find the flows across full-time and part-time employment have significant contributions to the responses of both stocks, especially in the short run.

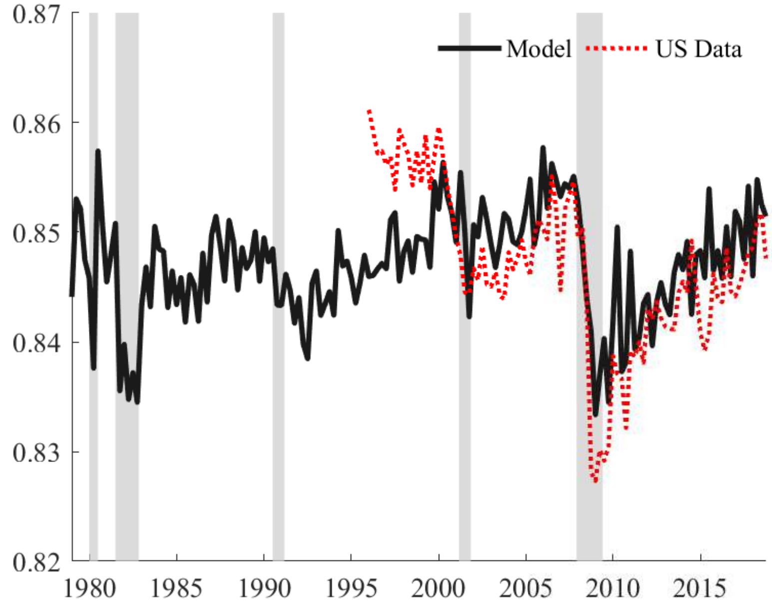
Next, we aim to conduct the net flow decomposition over the model-generated historical path of labor market flows, as we did over the CPS data in Table 1. To this end, we estimate the historical path of the unobserved state variables over the entire estimation period using the Kalman smoother, and then apply the decomposition formula to the estimates.[34] Table 6 reports the results for the full-time employment rate and the part-time employment rate in the Great Recession period (2008:Q1-2009:Q4).[35] We

of the variation in the growth rate of output and about 60% of the variation in the unemployment rate.

[34]See, for example, Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016) for the description of the Kalman-smoothing algorithm for the linear Gaussian state-space model.

[35]Unfortunately, comparing directly the numbers in Table 1 and Table 6 is not possible due to the following two reasons: (i) All the employment stocks are measured as a share of the labor force in the model (Table 6), whereas they are measured as a share of the population aged over 15 in the empirical analysis (Table 1), and (ii) the definition of the Great Recession period in Table 1 (2007:M12-2009:M11) is not applicable in the model analysis, because our model is built on a quarterly basis.

Figure 4: Historical path of the retention rate of full-time employment



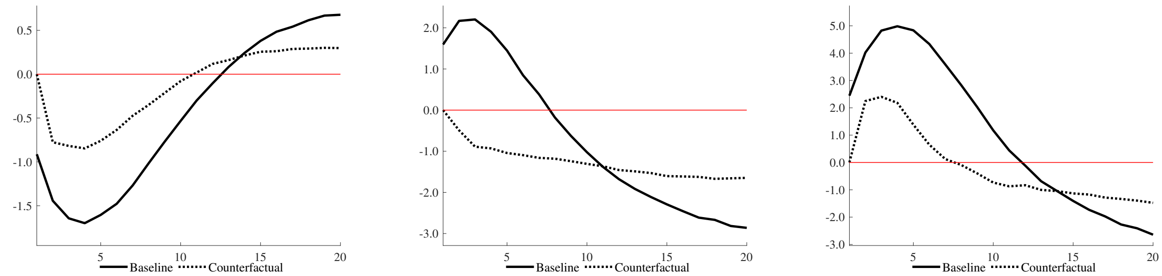
Note: The solid line shows the posterior mean of the historical path of the retention rate of full-time employment in the model. The dashed line shows that in the US data (the probability of keeping full-time employment measured from the CPS). Shaded areas correspond NBER recessions.

find that the patterns of labor market flows in the recession in our model share salient properties of the CPS data reported in Table 1: (i) The flows between employment and unemployment strongly contributed to the drop in full-time employment, but not to the increase in part-time employment, whereas (ii) the flows between full-time and part-time employment were among the most important contributors to the increase in part-time employment.

To see why our model is able to reproduce empirically realistic patterns of labor market flows, we plot the historical path of the retention rate of full-time employment (i.e., the fraction of full-time workers who remain full-time employed after one quarter) obtained from the estimated model and the empirical counterpart measured from the CPS in Figure 4. We find that, although time-series information on the labor market transitions is not directly used in the estimation process (the only relevant information used is the standard deviation of the retention rate, which is used to calibrate ζ^F), the estimated model can replicate the movements in the retention rate, in particular the sharp decline during the Great Recession. We therefore conclude that our model is well-designed to account for the US labor market dynamics.

Figure 5: Impulse-response functions to a negative investment-specific technology shock

(A) Full-time employment (B) Part-time employment (C) Unemployment rate



Note: The posterior medians of impulse responses of the full-time employment rate (Panel (A)), the part-time employment rate (Panel (B)), and the unemployment rate (Panel (C)) to a negative investment-specific technology shock of one standard deviation in the baseline model (solid lines) and the alternative model with counterfactual constant retention rate (dashed lines). All figures represent percentage deviations from the steady state.

4.5 A counterfactual experiment

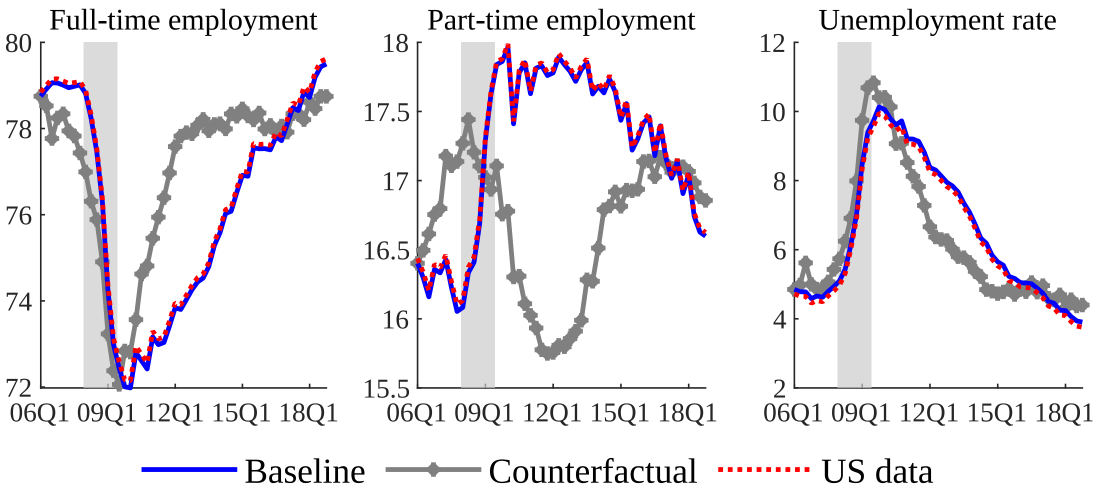
An important mechanism we highlight that has been absent in the existing literature is the transition from full-time jobs to part-time jobs. The key variable in our model is the retention rate $\varrho^F_{j,t}$.

In this section, we conduct a counterfactual analysis to further investigate the role of countercyclical fluctuations in $\varrho^F_{j,t}$. More specifically, we demonstrate how the dynamics of the employment stocks would behave in the absence of cyclical fluctuations in $\varrho^F_{j,t}$. To this end, we consider an alternative model specification in which the curvature parameter ζ^F and the scale parameter A^F of the retention cost function are set so that $\varrho^F_{j,t}$ is constant at its steady-state value. Note this experiment mutes the cyclicality of all worker flows out of full-time employment, including the flow into unemployment.

In Figure 5, the impulse responses of the full-time employment rate, the part-time employment rate, and the unemployment rate to a negative investment-specific technology shock, which is the largest contributor to the increase in the unemployment rate during the Great Recession, are compared between the two cases. As shown in Panels (A) and (B), the responses of both employment stocks are substantially smaller in the counterfactual case (dashed lines). In particular, the initial increase in the part-time employment rate disappears in the counterfactual. In Panel (C), the response of unemployment is also smaller.

To place the counterfactual experiment in a concrete context, we analyze how the labor market dynamics would have behaved around the Great Recession period in the counterfactual case. To this end, after recovering the realization of the unobservable shocks through the Kalman smoother, we simulate the historical paths of the full-time

Figure 6: Counterfactual simulation on the recent US labor dynamics



Note: The figure shows the path of the full-time employment rate (%), the part-time employment rate (%), and the unemployment rate (%) after 2006:Q1 simulated from the baseline model (solid lines without marks), the alternative model with counterfactual constant retention rate (solid lines with marks), and in the US data (dashed lines). Shaded areas correspond to NBER recessions.

employment rate, the part-time employment rate, and the unemployment rate in both scenarios. In Figure 6, we display the simulated historical paths after 2006:Q1 along with the data (dashed lines), taking the estimate of the endogenous variables in the baseline model at 2006:Q1 as the initial values of the simulation.

The simulation reveals that, starting from the onset of the Great Recession, the full-time employment rate sharply decreases in both scenarios. By contrast, whereas the part-time employment rate increases in the baseline, it decreases in the counterfactual scenario. This result is consistent with Table 6, which shows the increase in the net flows from EF to EP strongly contributes to increasing the part-time employment in the recession.

Also note the difference in the pace of recovery from the recession between the two scenarios. During the jobless recovery after the Great Recession, the pace of full-time employment growth continued to be slow, and the unemployment rate and the part-time employment rate stayed persistently high. In comparison, in the counterfactual case, the full-time employment rate recovers and the unemployment rate declines at a faster pace.

4.6 Hours, employment, and hours per worker

By construction, total hours worked is the product of hours worked per employed worker and total employment:

$$\underbrace{n_t^F \mu_b^F + n_t^P \mu_b^P}_{\text{total hours worked}} = \underbrace{\left[\frac{n_t^F}{n_t^F + n_t^P}\mu_b^F + \left(1 - \frac{n_t^F}{n_t^F + n_t^P}\right)\mu_b^P\right]}_{\text{hours worked per employed worker}} \times \underbrace{\left(n_t^F + n_t^P\right)}_{\text{total employment}} .$$

Hence, in our model, the average hours per worker fluctuate due to changes in the composition of the employment stocks. In this section, we argue our model can generate sizable fluctuations in the average hours per worker.

First, we find the patterns of model-generated dynamic responses of total hours worked, hours worked per worker, and total employment to a contractionary monetary-policy shock (displayed in Figure 7) are fully in line with those estimated by Trigari (2009) using a vector autoregressive methodology.[36] In response to the shock, both hours per worker and total employment fall, and the fall in total employment is larger in magnitude than the fall in hours per worker.

Second, we decompose the long-run variance in total hours worked into the variances in the average hours per worker and total employment (and their covariance). In the data, the variance in the average hours per worker accounts for 28% of the variance in total hours. In the model, it accounts for 14%.

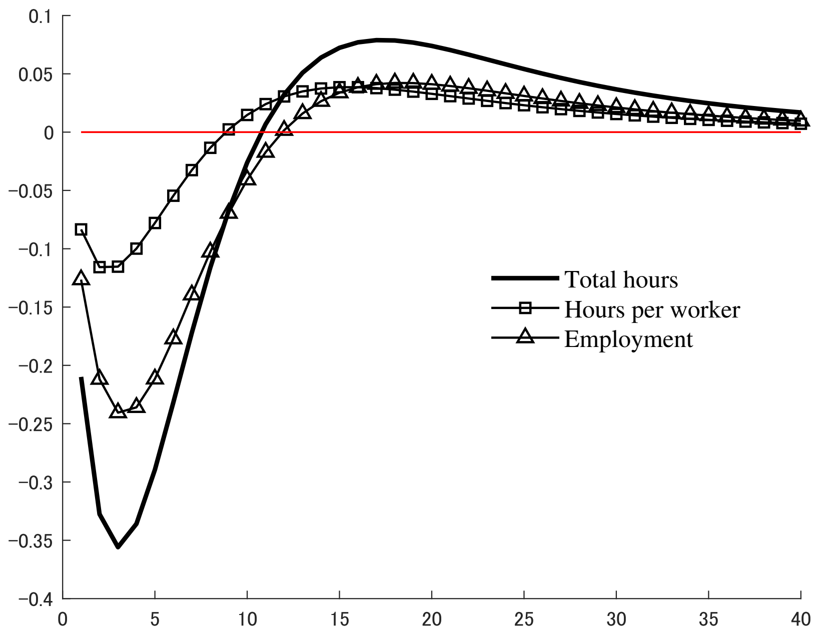
5 Conclusion

This paper studied the asymmetric roles of full-time employment and part-time employment over the business cycle. In the US data, the full-time employment rate is procyclical, whereas the part-time employment rate exhibits a countercyclical pattern, particularly in deep recessions. In the first part of the paper, we documented the macroeconomic facts on the labor market flows into and out of part-time employment, using the longitudinal data from the CPS.

We highlight the following two observations. First, conditional on finding jobs at all, the majority of workers in the "unemployed workers looking for full-time work" category transition into full-time jobs, whereas the majority of workers in the "unemployed workers looking for part-time work" category transition into part-time jobs. This finding, combined with the observation that full-time and part-time jobs differ in nature (e.g., in terms of occupations), suggests the labor market is segmented into full-time

[36] See her Figures 2 and 4.

Figure 7: Impulse-response functions of total hours worked, hours worked per worker, and total employment to a contractionary monetary-policy shock



Note: This figure shows the impulse-response functions of total hours worked, hours worked per worker, and total employment to a contractionary monetary-policy shock of one standard deviation. The values for the estimated parameters are fixed at their posterior mean.

and part-time work. Second, in the Great Recession, the flows between employment and unemployment strongly contributed to the fall in full-time employment, but not to the increase in part-time employment. The flows between full-time and part-time employment were among the most important contributors to the increase in part-time employment.

We built and estimated a DSGE model with frictional labor markets, featuring segmented labor markets for full-time and part-time workers. In the model, we allowed for endogenous reallocations from full-time to part-time employment. This flow is driven by the firm's decisions of retaining workers, implying some workers are involuntarily working part time. The estimated model performs well in accounting for the US labor market dynamics. We find the asymmetric dynamic responses of full-time and part-time employment can be replicated by the results of a variety of standard macroeconomic shocks. The counterfactual simulation reveals that cyclical transition out of full-time employment played an essential role in the increase in part-time employment during the Great Recession and the sluggish recovery of employment after the recession.

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Appendix

A The observation equations and the dataset

The observation equations in the state space representation of our model are given by

$$\begin{bmatrix} \Delta \log Y_t \\ \Delta \log C_t \\ \Delta \log I_t \\ \Delta \log W_t \\ \log \Pi_t \\ \log N_t \\ \log R_t^n \\ \log(N_t^F/N_t^P) \\ \log(W_t^F/W_t^P) \end{bmatrix} = \begin{bmatrix} \log(\gamma_z) \\ \log(\gamma_z) \\ \log(\gamma_z) \\ \log(\gamma_z) \\ \log(\pi) \\ \log(n) \\ \log(r^n) \\ \log(n^F/n^P) \\ \log(\bar{w}^F/\bar{w}^P) \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + z_t^z \\ \tilde{c}_t - \tilde{c}_{t-1} + z_t^z \\ \tilde{i}_t - \tilde{i}_{t-1} + z_t^z \\ \tilde{w}_t - \tilde{w}_{t-1} + z_t^z \\ \tilde{\pi}_t \\ \tilde{n}_t \\ \tilde{r}_t^n \\ \tilde{n}_t^F - \tilde{n}_t^P \\ \tilde{w}_t^F - \tilde{w}_t^P \end{bmatrix},$$

where $n_t = n_t^F + n_t^P$ and $\tilde{n}_t$ is the log deviation of n_t from its steady-state value n. The nine time series we use for the estimation include (i) the growth rate of per capita real GDP ($\Delta \log Y_t$); (ii) the growth rate of per capita real personal consumption expenditures of nondurables and services ($\Delta \log C_t$); (iii) the growth rate of per capita real investment ($\Delta \log I_t$); (iv) the growth rate of the real hourly compensation in nonfarm business sector ($\Delta \log W_t$); (v) the inflation rate using GDP deflator (Π_t); (vi) the employment rate (N_t); (vii) the federal funds rate (R_t^n); (viii) the ratio of the full-time employment rate to the part-time employment rate (N_t^F/N_t^P); (ix) the relative hourly wages for full-time and part-time workers (W_t^F/W_t^P). Note that the choice of the first seven time series corresponds to that of GST except for (vi) where hours worked has been replaced by the employment rate. Regarding (ix), we use the Merged Outgoing Rotation Group files of the CPS (the dataset is available at https://www.nber.org/data/morg.html#faq). The hourly wage is calculated by dividing earnings per week by the hours usually worked per week. We limit the sample to wage and salary workers, and top- and bottom-code the hourly wage at the 99th and 1st percentiles. Here, the hourly wage for full-time workers is the sample average of the hourly wages for the workers employed full time and the hourly wage for part-time workers is the sample average of the hourly wages for the workers employed part time.

B Net decomposition formulas

This section provides detailed derivation of (16) and (17). Equations (14) and (15) can be written as

$$\begin{aligned} n_t^F - n_{t-1}^F = &\left[\varphi n_{t-1}^P - \lambda^{FP} \int_{\mathcal{J}^F} (1-\varrho_{j,t}^F) n_{j,t-1}^F dj\right] \\ &+ \left[s_{t-1}^F u_{t-1}^F - (1-\lambda^{FP}) \int_{\mathcal{J}^F} (1-\varrho_{j,t}^F) n_{j,t-1}^F dj\right] \end{aligned} \tag{B.20}$$

and

$$\begin{aligned} n_t^P - n_{t-1}^P = &\left[\lambda^{FP} \int_{\mathcal{J}^F} (1-\varrho_{j,t}^F) n_{j,t-1}^F dj - \varphi n_{t-1}^P\right] \\ &+ \left[s_{t-1}^P u_{t-1}^P - \int_{\mathcal{J}^P} (1-\varrho_{j,t}^P - \varphi) n_{j,t-1}^P dj\right]. \end{aligned} \tag{B.21}$$

Note that, for both (B.20) and (B.21), the terms in the first square bracket represent the net flow from the other employment stock and the terms in the second square bracket represent the net flow from the unemployment stock.

Dividing both sides of (B.20) by n_{t-1}^F yields

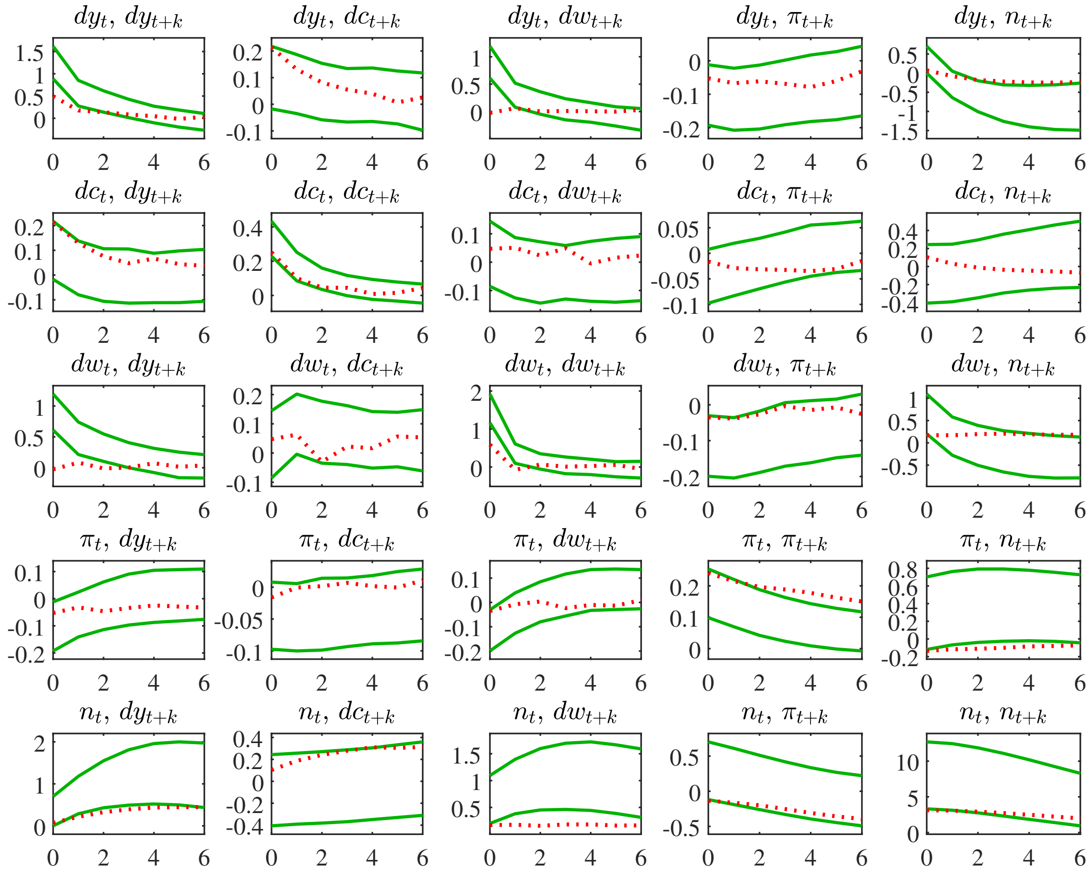
$$\begin{aligned} \underbrace{\frac{n_t^F - n_{t-1}^F}{n_{t-1}^F}}_{r_{t-1,t}^F} = &\underbrace{\left[\frac{\varphi n_{t-1}^P - \lambda^{FP} \int_{\mathcal{J}^F} (1-\varrho_{j,t}^F) n_{j,t-1}^F dj}{n_{t-1}^F}\right]}_{f_{t-1,t}^{PF}\text{: net (in)flow rate from } EP} \\ &+ \underbrace{\left[\frac{s_{t-1}^F u_{t-1}^F - (1-\lambda^{FP}) \int_{\mathcal{J}^F} (1-\varrho_{j,t}^F) n_{j,t-1}^F dj}{n_{t-1}^F}\right]}_{f_{t-1,t}^{UF}\text{: net (in)flow from } U}. \end{aligned}$$

Similarly, dividing both sides of (B.21) by n_{t-1}^P yields

$$\begin{aligned} \underbrace{\frac{n_t^P - n_{t-1}^P}{n_{t-1}^P}}_{r_{t-1,t}^P} = &\underbrace{\left[\frac{\lambda^{FP} \int_{\mathcal{J}^F} (1-\varrho_{j,t}^F) n_{j,t-1}^F dj - \varphi n_{t-1}^P}{n_{t-1}^P}\right]}_{f_{t-1,t}^{FP}\text{: net (in)flow rate from } EF} \\ &+ \underbrace{\left[\frac{s_{t-1}^P u_{t-1}^P - \int_{\mathcal{J}^P} (1-\varrho_{j,t}^P - \varphi) n_{j,t-1}^P dj}{n_{t-1}^P}\right]}_{f_{t-1,t}^{UP}\text{: net (in)flow rate from } U}. \end{aligned}$$

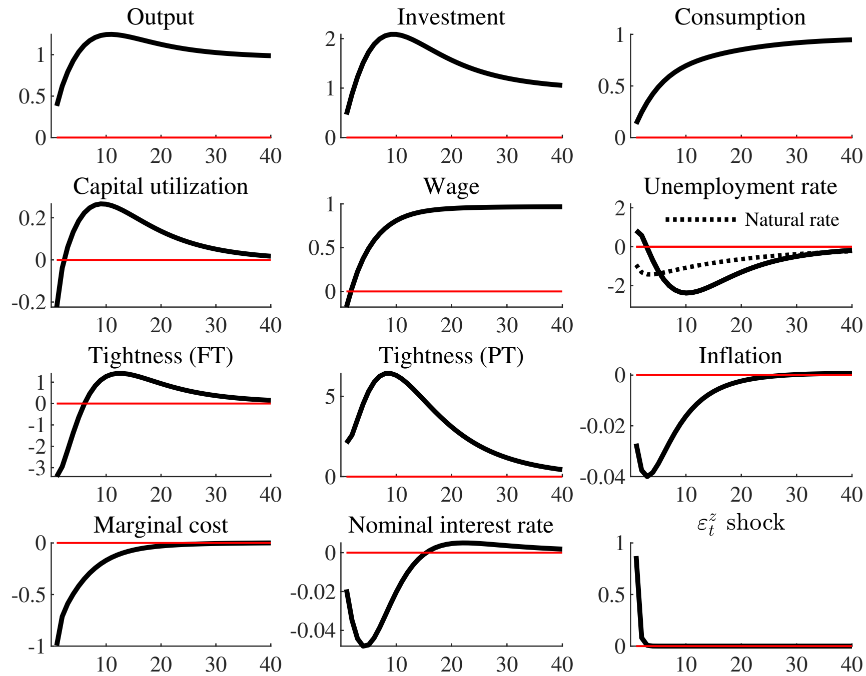
C Supplemental figures on the estimation result

Figure C.1: Autocovariance functions of selected variables - US data and estimated model



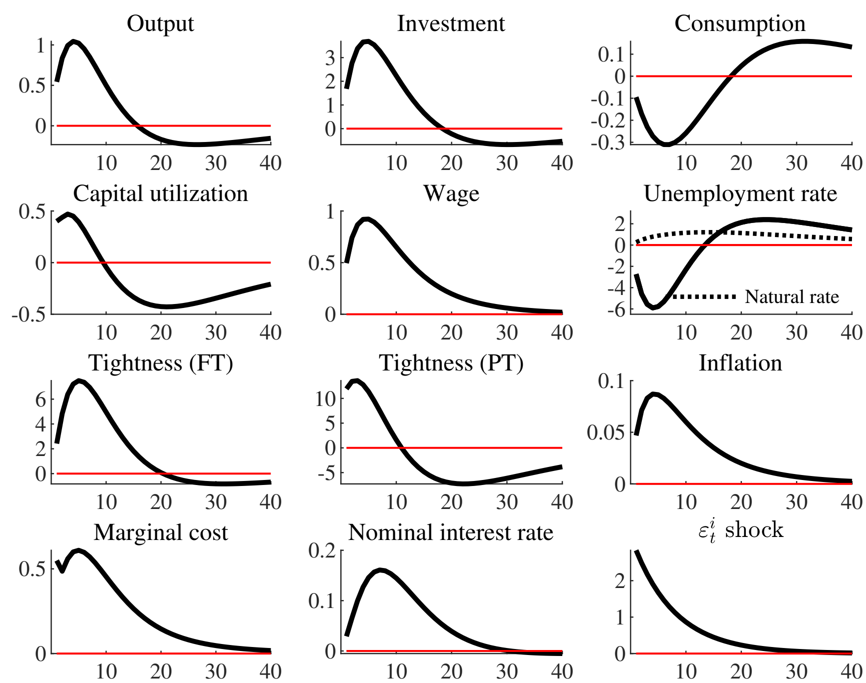
Note: This figure shows the autocovariance and cross-covariance function of the growth rates of real output, real consumption and the real wage, the inflation rate and the log of employment rate in the US data (dashed lines) and in the estimated model (solid lines, representing the 5th and 95th percentiles over 1,200 draws from the posterior parameter distribution and 200 simulated samples of 160 observations for each draw).

Figure C.2: Impulse responses to a positive technology shock



Note: This figure shows the impulse responses to to a positive technology shock of one standard deviation. The parameter values are set to the posterior mean estimates.

Figure C.3: Impulse responses to a positive investment-specific technology shock



Note: This figure shows the impulse responses to a positive investment-specific technology shock of one standard deviation. The parameter values are set to the posterior mean estimates.

Online Appendix

D CPS data

D.1 Adjustment for the CPS redesign in January 1994

In computing the (un)employment rate, we use the multiplicative factor constructed in Polivka and Miller (1998) to correct the break attributable to the CPS redesign in January 1994. The adjusted (un)employment rate equals the adjusted number of (un)employment divided by the adjusted number of labor force, where the adjusted number of employment is given by the adjusted employment-to-population rate times the civilian noninstitutional population and the adjusted number of labor force is given by the adjusted labor participation rate times the civilian noninstitutional population.

Similarly, in calculating the adjusted full-time and part-time employment rates plotted in Figure 1, we use the multiplicative factor for the ratio of part-time employment to total employment.

D.2 Margin adjustment

In order to correct margin errors, we employ the method proposed by Elsby, Hobijn, and Şahin (2015). Below, let E_t^F, E_t^P, U_t^F, U_t^P, and O_t be the number of workers in the labor market states EF, EP, UP, UP, and O, respectively, at the beginning of period t. We define a vector $\boldsymbol{\Delta}\mathbf{s}_t$ to be

$$\Delta\mathbf{s}_t = \mathbf{s}_t - \mathbf{s}_{t-1} = [E_t^F - E_{t-1}^F, E_t^P - E_{t-1}^P, U_t^F - U_{t-1}^F, U_t^P - U_{t-1}^P]'.$$

The identity that the change in the stock is the sum of the inflows out of the stock minus the outflows to the stock shows

$$\Delta\mathbf{s}_t = \mathbf{X}_{t-1}\mathbf{p},$$

where

$$\mathbf{X}_{t-1} =$$

$$\begin{bmatrix} -E_{t-1}^F & -E_{t-1}^F & -E_{t-1}^F & -E_{t-1}^F & E_{t-1}^P & 0 & 0 & 0 & U_{t-1}^F & 0 & 0 & 0 & U_{t-1}^P & 0 & 0 & 0 & O_{t-1} & 0 & 0 & 0 \\ E_{t-1}^F & 0 & 0 & 0 & -E_{t-1}^P & -E_{t-1}^P & -E_{t-1}^P & -E_{t-1}^P & 0 & U_{t-1}^F & 0 & 0 & 0 & U_{t-1}^P & 0 & 0 & 0 & O_{t-1} & 0 & 0 \\ 0 & E_{t-1}^F & 0 & 0 & 0 & E_{t-1}^P & 0 & 0 & -U_{t-1}^F & -U_{t-1}^F & -U_{t-1}^F & -U_{t-1}^F & 0 & 0 & U_{t-1}^P & 0 & 0 & 0 & O_{t-1} & 0 \\ 0 & 0 & E_{t-1}^F & 0 & 0 & 0 & E_{t-1}^P & 0 & 0 & 0 & U_{t-1}^F & 0 & -U_{t-1}^P & -U_{t-1}^P & -U_{t-1}^P & -U_{t-1}^P & 0 & 0 & 0 & O_{t-1} \end{bmatrix},$$

and

$$\mathbf{p} =$$

$$\begin{bmatrix} p_{E^FE^P} & p_{E^FU^F} & p_{E^FU^P} & p_{E^FO} & p_{E^PE^F} & p_{E^PU^F} & p_{E^PU^P} & p_{E^PO} & p_{U^FE^F} & p_{U^FE^P} & p_{U^FU^P} & p_{U^FO} & p_{U^PE^F} & p_{U^PE^P} & p_{U^PU^F} & p_{U^PO} & p_{OE^F} & p_{OE^P} & p_{OU^F} & p_{OU^P} \end{bmatrix}',$$

with the element p_{ij} denoting the transition probability from state i to state j.

Given the vector of the transition probabilities in data $\hat{\mathbf{p}}$, the vector of the change of the stocks in data $\Delta \mathbf{s}_t$ and the matrix $\mathbf{X}_{t-1}$, the vector of corrected transition probabilities is chosen so as to minimize

$$\frac{1}{2}(\mathbf{p} - \hat{\mathbf{p}})' \mathbf{W} (\mathbf{p} - \hat{\mathbf{p}})$$

subject to

$$\Delta \mathbf{s}_t = \mathbf{X}_{t-1} \mathbf{p},$$

where the weight matrix satisfies

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{EF} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{EP} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W}_{UF} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{UP} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W}_N \end{bmatrix}^{-1},$$

and

$$\mathbf{W}_{EF} = \begin{bmatrix} \frac{\hat{p}_{E^FE^P}(1-\hat{p}_{E^FE^P})}{E^F_{t-1}} & -\frac{\hat{p}_{E^FE^P}\hat{p}_{E^FU^F}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FE^P}\hat{p}_{E^FU^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FE^P}\hat{p}_{E^FN}}{E^F_{t-1}} \\ -\frac{\hat{p}_{E^FU^F}\hat{p}_{E^FE^P}}{E^F_{t-1}} & \frac{\hat{p}_{E^FU^F}(1-\hat{p}_{E^FU^F})}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^F}\hat{p}_{E^FU^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^F}\hat{p}_{E^FN}}{E^F_{t-1}} \\ -\frac{\hat{p}_{E^FU^P}\hat{p}_{E^FE^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^P}\hat{p}_{E^FU^F}}{E^F_{t-1}} & \frac{\hat{p}_{E^FU^P}(1-\hat{p}_{E^FU^P})}{E^F_{t-1}} & -\frac{\hat{p}_{E^FU^P}\hat{p}_{E^FN}}{E^F_{t-1}} \\ -\frac{\hat{p}_{E^FN}\hat{p}_{E^FE^P}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FN}\hat{p}_{E^FU^F}}{E^F_{t-1}} & -\frac{\hat{p}_{E^FN}\hat{p}_{E^FU^P}}{E^F_{t-1}} & \frac{\hat{p}_{E^FN}(1-\hat{p}_{E^FN})}{E^F_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{EP} = \begin{bmatrix} \frac{\hat{p}_{E^PE^F}(1-\hat{p}_{E^PE^F})}{E^P_{t-1}} & -\frac{\hat{p}_{E^PE^F}\hat{p}_{E^PU^F}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PE^F}\hat{p}_{E^PU^P}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PE^F}\hat{p}_{E^PN}}{E^P_{t-1}} \\ -\frac{\hat{p}_{E^PU^F}\hat{p}_{E^PE^P}}{E^P_{t-1}} & \frac{\hat{p}_{E^PU^F}(1-\hat{p}_{E^PU^F})}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^F}\hat{p}_{E^PU^P}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^F}\hat{p}_{E^PN}}{E^P_{t-1}} \\ -\frac{\hat{p}_{E^PU^P}\hat{p}_{E^PE^P}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^P}\hat{p}_{E^PU^F}}{E^P_{t-1}} & \frac{\hat{p}_{E^PU^P}(1-\hat{p}_{E^PU^P})}{E^P_{t-1}} & -\frac{\hat{p}_{E^PU^P}\hat{p}_{E^PN}}{E^P_{t-1}} \\ -\frac{\hat{p}_{E^PN}\hat{p}_{E^PE^F}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PN}\hat{p}_{E^PU^F}}{E^P_{t-1}} & -\frac{\hat{p}_{E^PN}\hat{p}_{E^PU^P}}{E^P_{t-1}} & \frac{\hat{p}_{E^PN}(1-\hat{p}_{E^PN})}{E^P_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{UF} = \begin{bmatrix} \frac{\hat{p}_{U^FE^F}(1-\hat{p}_{U^FE^F})}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^F}\hat{p}_{U^FE^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^F}\hat{p}_{U^FU^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^F}\hat{p}_{U^FN}}{U^F_{t-1}} \\ -\frac{\hat{p}_{U^FE^P}\hat{p}_{U^FE^F}}{U^F_{t-1}} & \frac{\hat{p}_{U^FE^P}(1-\hat{p}_{U^FE^P})}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^P}\hat{p}_{U^FU^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FE^P}\hat{p}_{U^FN}}{U^F_{t-1}} \\ -\frac{\hat{p}_{U^FU^P}\hat{p}_{U^FE^F}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FU^P}\hat{p}_{U^FE^P}}{U^F_{t-1}} & \frac{\hat{p}_{U^FU^P}(1-\hat{p}_{U^FU^P})}{U^F_{t-1}} & -\frac{\hat{p}_{U^FU^P}\hat{p}_{U^FN}}{U^F_{t-1}} \\ -\frac{\hat{p}_{U^FN}\hat{p}_{U^FE^F}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FN}\hat{p}_{U^FE^P}}{U^F_{t-1}} & -\frac{\hat{p}_{U^FN}\hat{p}_{U^FU^P}}{U^F_{t-1}} & \frac{\hat{p}_{U^FN}(1-\hat{p}_{U^FN})}{U^F_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{UP} = \begin{bmatrix} \frac{\hat{p}_{U^PE^F}(1-\hat{p}_{U^PE^F})}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^F}\hat{p}_{U^PE^P}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^F}\hat{p}_{U^PU^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^F}\hat{p}_{U^PN}}{U^P_{t-1}} \\ -\frac{\hat{p}_{U^PE^P}\hat{p}_{U^PE^F}}{U^P_{t-1}} & \frac{\hat{p}_{U^PE^P}(1-\hat{p}_{U^PE^P})}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^P}\hat{p}_{U^PU^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PE^P}\hat{p}_{U^PN}}{U^P_{t-1}} \\ -\frac{\hat{p}_{U^PU^F}\hat{p}_{U^PE^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PU^F}\hat{p}_{U^PE^P}}{U^P_{t-1}} & \frac{\hat{p}_{U^PU^F}(1-\hat{p}_{U^PU^F})}{U^P_{t-1}} & -\frac{\hat{p}_{U^PU^F}\hat{p}_{U^PN}}{U^P_{t-1}} \\ -\frac{\hat{p}_{U^PN}\hat{p}_{U^PE^F}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PN}\hat{p}_{U^PE^P}}{U^P_{t-1}} & -\frac{\hat{p}_{U^PN}\hat{p}_{U^PU^F}}{U^P_{t-1}} & \frac{\hat{p}_{U^PN}(1-\hat{p}_{U^PN})}{U^P_{t-1}} \end{bmatrix}$$

$$\mathbf{W}_{N} = \begin{bmatrix} \frac{\hat{p}_{NE^F}(1-\hat{p}_{NE^F})}{N_{t-1}} & -\frac{\hat{p}_{NE^F}\hat{p}_{NE^P}}{N_{t-1}} & -\frac{\hat{p}_{NE^F}\hat{p}_{NU^F}}{N_{t-1}} & -\frac{\hat{p}_{NE^F}\hat{p}_{NU^P}}{N_{t-1}} \\ -\frac{\hat{p}_{NE^P}\hat{p}_{NE^F}}{N_{t-1}} & \frac{\hat{p}_{NE^P}(1-\hat{p}_{NE^P})}{N_{t-1}} & -\frac{\hat{p}_{NE^P}\hat{p}_{NU^F}}{N_{t-1}} & -\frac{\hat{p}_{NE^P}\hat{p}_{NU^P}}{N_{t-1}} \\ -\frac{\hat{p}_{NU^F}\hat{p}_{NE^F}}{N_{t-1}} & -\frac{\hat{p}_{NU^F}\hat{p}_{NE^P}}{N_{t-1}} & \frac{\hat{p}_{NU^F}(1-\hat{p}_{NU^F})}{N_{t-1}} & -\frac{\hat{p}_{NU^F}\hat{p}_{NU^P}}{N_{t-1}} \\ -\frac{\hat{p}_{NU^P}\hat{p}_{NE^F}}{N_{t-1}} & -\frac{\hat{p}_{NU^P}\hat{p}_{NE^P}}{N_{t-1}} & -\frac{\hat{p}_{NU^P}\hat{p}_{NU^F}}{N_{t-1}} & \frac{\hat{p}_{NU^P}(1-\hat{p}_{NU^P})}{N_{t-1}} \end{bmatrix}.$$

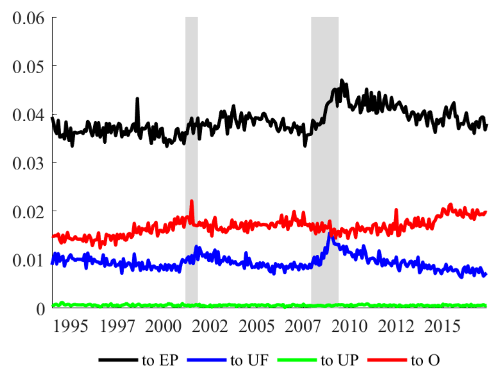
E Supplement on the CPS evidence

E.1 The monthly transition probabilities

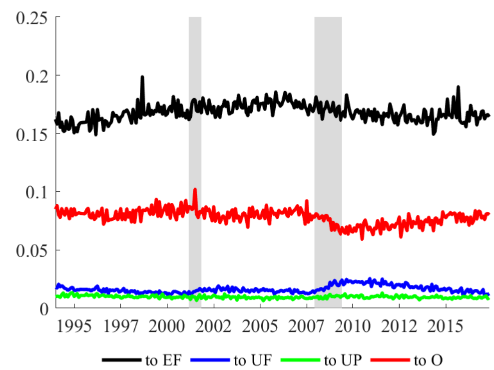
Figure E.1 displays the monthly transition probabilities from EF (Panel (A)), from EP (Panel (B)), from UF (Panel (C)), and from UP (Panel (D)) from 1994 to 2018.

Figure E.1: Transition probability

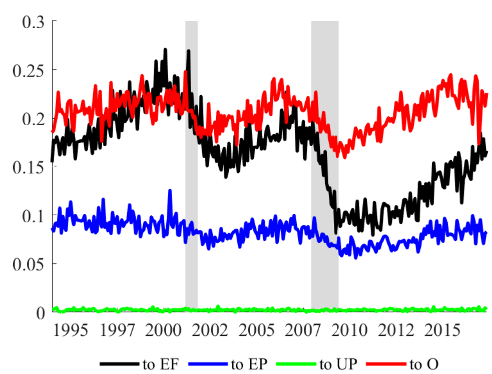
(A) Transition probability from EF



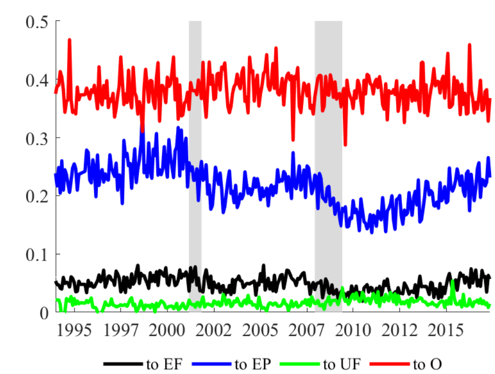
(B) Transition probability from EP



(C) Transition probability from UF



(D) Transition probability from UP



Note: All series are seasonally adjusted. Since there are missing observations in 1995 due to the failure of individual identifiers in the CPS, we use Tramo ("Time Series Regression with ARIMA Noise, Missing Observations, and Outliers")/Seats ("Signal Extraction in ARIMA Time Series") interface to interpolate the missing observations along with seasonal adjustment.

Table E.1 shows the average of the monthly transition probabilities. Due to the missing observations in 1995, we report the averages over 1996:M1–2018:M12. In this table, by construction, the sum of each row is 1. We find that about 24 percent of unemployed workers in full-time labor market find a job and 2/3 of them find a job in full-time position. Also, about 26 percent of unemployed workers in part-time labor market find a job and 4/5 of them find a job in part-time position.

Table E.1: Average transition probability (monthly)

		S_{t+1}^j				
		EF	EP	UF	UP	O
	EF	0.934	0.039	0.009	0.001	0.017
	EP	0.169	0.727	0.016	0.010	0.078
S_t^i	UF	0.163	0.081	0.548	0.002	0.206
	UP	0.049	0.215	0.016	0.340	0.380
	O	0.020	0.025	0.018	0.007	0.930

E.2 Involuntary part-time employment

In this section, we divide part-time employment further into voluntary part-time employment (EVP) and involuntary part-time employment (EIP) based on the CPS Questionnaire. Earlier work, such as Lariau (2017) and Warren (2017) emphasize this distinction. Table E.2 reproduces the transition matrix (Table E.1) with this distinction.

Table E.2: Average transition probability

		S_{t+1}^j					
		EF	EIP	EVP	UF	UP	O
	EF	0.945	0.007	0.021	0.009	0.001	0.017
	EIP	0.261	0.421	0.214	0.049	0.009	0.046
S_t^i	EVP	0.132	0.039	0.728	0.009	0.010	0.082
	UF	0.161	0.034	0.032	0.559	0.002	0.211
	UP	0.047	0.024	0.177	0.015	0.348	0.388
	O	0.018	0.002	0.018	0.017	0.007	0.938

Note: This table shows the average of the monthly transition probability between 1996 and 2018. EF is full-time employment, EIP is involuntary part-time employment, EVP is voluntary part-time employment, UF is the unemployed looking for a full-time job, UP is the unemployed looking for a part-time job, and O is out of labor force.

In the main text, we emphasize the gross flows between full time and part time. From Table E.2, one can see that a larger fraction of EIP moves to EF compared to EVP does. At the same time, in terms of the total number of people who flows into EF, EVP is also an important origin because the size of EVP stock is substantially larger than the size of EIP stock. (The average stock of EVP is about four times larger than the stock of EIP.) Therefore, both EVP and EIP are important sources of gross inflows into EF. For the inflow from UP, the largest employment destination is EVP. Note, however, UP is also an important source of inflow for EIP, given that the size of EIP is much smaller than EVP.

In our baseline model, we do not make distinction between EIP and EVP. The first reason is that the important counterparts of the gross flows are similar (EF and UP)

between these two. The second is the concern regarding the distinction: whereas there is a clear metric for distinction between full-time and part-time (usual hours of work), the difference between EIP and EVP is arguably more subjective. This subjective aspect, we suspect, is reflected in a large flow from EIP to EVP—in terms of economic intuitions, it is not clear why so many would change the status from "involuntary" to "voluntary" for each month.

Table E.3: Net flow decomposition of employment stocks over the Great Recession period

	$j = EF$	$j = EIP$	$j = EVP$
The rate of change in stock of state j	−0.49	3.37	−0.31
Net flow rate from state i to state j			
$i = EF$	–	2.94	0.38
$i = EIP$	−0.09	–	−0.26
$i = EVP$	−0.07	1.30	–
$i = UF$	−0.22	−0.62	−0.06
$i = UP$	−0.01	−0.26	−0.11
$i = O$	−0.09	0.01	−0.27

Note: Average monthly flow (%) during the Great Recession period (December 2007 to November 2009), compared to the long-run average between January 1996 and December 2018.

Table E.3 repeats the analysis of Table 1 with the distinction between EVP and EIP. Two results emerge. First, as is pointed out in previous studies such as Lariau (2017), Warren (2017), and Borowczyk-Martins and Lalé (2018) EIP stock is strongly countercyclical. Second, although EVP stock is procyclical, its flow components behave very similarly to the components of EIP (except for the net flow from O, which we ignore in the model). For example, the net flow from EF contributed positively to both EIP and EVP during the Great Recession. The decline in job finding from both UF and UP contribute strongly to both EIP and EVP stocks. This pattern is in contrast to EF stock, for which the UP contribution is almost zero. This similarity in the each component of the flows is another reason we put EIP and EVP together in the model specification.

F Derivation of model equations

F.1 The optimization problem of the wholesale firms

Since all the subdivisions in the full-time and part-time division face the analogous problem except that the evolution equations of the employment stock are differ depending on the division to which the subdivision belongs, we describe the optimization problem of the subdivision in the full-time division.

Each subdivision's value function in period t is given by

$$\mathcal{F}_t^F(n_{j,t-1}^F, v_{j,t-1}^F; w_{j,t}^F) = \max_{\varrho_{j,t}^F, v_{j,t}^F, n_{j,t}^F, k_{j,t}^F} \left(\begin{array}{r} p_t^F y_{j,t}^F - \frac{w_{j,t}^{Fn}}{p_t} n_{j,t}^F - r_t^k k_{j,t}^F - \left(\mathcal{K}_t^F \left(\frac{q_{t-1}^F v_{j,t-1}^F}{n_{j,t-1}^F} \right) + \mathcal{A}_t^F(\varrho_{j,t}^F) \right) n_{j,t-1}^F \\ +\mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{F}_{t+1}^F(n_{j,t}^F, v_{j,t}^F; w_{j,t+1}^F) \right] \end{array} \right)$$

subject to

$$n_{j,t}^F \leq \varrho_{j,t}^F n_{j,t-1}^F + q_{t-1}^F v_{j,t-1}^F + n_t^{PF}, \tag{F.1}$$

and

$$v_{j,t}^F \geq 0. \tag{F.2}$$

Let $J_{j,t}^F$ and $\Theta_{j,t}^{v,F}$ be the Lagrange multipliers for the constraints (F.1) and (F.2), respectively. The optimality condition for capital input implies

$$r_t^k = p_t^F \alpha \left(\frac{y_{j,t}^F}{k_{j,t}^F} \right) = p_t^F \alpha \left(\frac{y_t^F}{k_t^F} \right).$$

At the optimum, (F.1) holds with equality. The first-order necessary conditions for $n_{j,t}^F$, $\varrho_{j,t}^F$, and $v_{j,t}^F$ are respectively given by

$$p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\partial \mathcal{F}_{t+1}^F(n_{j,t}^F, v_{j,t}^F; w_{j,t+1}^F)}{\partial n_{j,t}^F} \right] = J_{j,t}^F, \tag{F.3}$$

$$\frac{\partial \mathcal{A}_t^F(\varrho_{j,t}^F)}{\partial \varrho_{j,t}^F} = J_{j,t}^F, \tag{F.4}$$

and

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\partial \mathcal{F}_{t+1}^F(n_{j,t}^F, v_{j,t}^F; w_{j,t+1}^F)}{\partial v_{j,t}^F} \right] + \Theta_{j,t}^{v,F} = 0, \tag{F.5}$$

with $v_{j,t}^F \Theta_{j,t}^{v,F} = 0$.

The envelop theorem shows

$$\frac{\partial \mathcal{F}_t^F(n_{j,t-1}^F, v_{j,t-1}^F; w_{j,t}^F)}{\partial n_{j,t-1}^F} = \frac{\partial \mathcal{K}_t^F(x_{j,t-1}^F)}{\partial x_{j,t-1}^F} x_{j,t-1}^F - \left(\mathcal{K}_t^F(x_{j,t-1}^F) + \mathcal{A}_t^F(\varrho_{j,t}^F)\right) + J_{j,t}^F \varrho_{j,t}^F, \quad \text{(F.6)}$$

and

$$\frac{\partial \mathcal{F}_t^F(n_{j,t-1}^F, v_{j,t-1}^F; w_{j,t}^F)}{\partial v_{j,t-1}^F} = q_{t-1}^F \left(J_{j,t}^F - \frac{\partial \mathcal{K}_t^F(x_{j,t-1}^F)}{\partial x_{j,t-1}^F} \right), \quad \text{(F.7)}$$

where

$$x_{j,t}^F = \frac{q_t^F v_{j,t}^F}{n_{j,t}^F}.$$

Here, we focus on the interior optimum, at which $\Theta_{j,t}^{v,F} = 0$ for all $j \in \mathcal{J}^F$. Then (F.5) and (F.7) imply that

$$\mathbb{E}_t \left[\Lambda_{t,t+1} q_t^F \left(J_{j,t+1}^F - \frac{\partial \mathcal{K}_{t+1}^F(x_{j,t}^F)}{\partial x_{j,t}^F} \right) \right] = 0.$$

Because $q_t^F > 0$, for any states, it must hold that

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \left(J_{j,t+1}^F - \frac{\partial \mathcal{K}_{t+1}^F(x_{j,t}^F)}{\partial x_{j,t}^F} \right) \right] = 0.$$

Since $\mathcal{K}^F(\cdot)$ is a quadratic function, substitute (F.6) into (F.3) delivers[37]

$$J_{j,t}^F = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\mathcal{K}_{t+1}^F(x_{j,t}^F) - \mathcal{A}_{t+1}^F(\varrho_{j,t+1}^F) + J_{j,t+1}^F \varrho_{j,t+1}^F \right) \right]. \quad \text{(F.8)}$$

Then, the above equation shows $J_{j,t}^F = J_t^F(w_{j,t}^{Fn})$, (F.4) shows $\varrho_{j,t}^F = \rho_t^F(w_{j,t}^{Fn})$, and, by construction, $x_{j,t}^F = x_t^F(w_{j,t}^{Fn})$ respectively. We can rewrite (F.8) as

$$J_t^F(w_{j,t}^{Fn}) = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} + \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\begin{array}{l} \mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) - \mathcal{A}_{t+1}^F(\rho_{t+1}^F(w_{j,t+1}^{Fn})) \\ +\rho_{t+1}^F(w_{j,t+1}^{Fn}) J_{t+1}^F(w_{j,t+1}^{Fn}) \end{array} \right) \right]. \quad \text{(F.9)}$$

Therefore, using

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) \right] = \mathbb{E}_t \left[\Lambda_{t,t+1} x_t^F(w_{j,t}^{Fn}) J_{t+1}^F(w_{j,t+1}^{Fn}) \right] - \mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) \right],$$

[37]When $\mathcal{K}^F(\cdot)$ is a quadratic function, it must be held that $-\mathcal{K}_t^F(x_{j,t}^F) + \frac{\partial \mathcal{K}_t^F(x_{j,t}^F)}{\partial x_{j,t}^F} x_{j,t}^F = \mathcal{K}_t^F(x_{j,t}^F)$.

we rewrite (F.9) as

$$J_t^F(w_{j,t}^{Fn}) = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\mathcal{K}_{t+1}^F(x_t^F(w_{j,t}^{Fn})) + \mathcal{A}_{t+1}^F(\rho_{t+1}^F(w_{j,t+1}^{Fn}))\right)\right]$$
$$+ \mathbb{E}_t\left[\Lambda_{t,t+1}(x_t^F(w_{j,t}^{Fn}) + \rho_{t+1}^F(w_{j,t+1}^{Fn}))J_{t+1}^F(w_{j,t+1}^{Fn})\right].$$

F.2 Wage functions

- Worker's surplus

  We define the worker's surplus from being employed as $H_t^F(w_{j,t}^{Fn}) = V_t^F(w_{j,t}^{Fn}) - U_t$ and $H_t^P(w_{j,t}^{Pn}) = V_t^P(w_{j,t}^{Pn}) - U_t$ and denote their average conditional on being a newly employed worker by $H_{x,t}^F = V_{x,t}^F - U_t$ and $H_{x,t}^P = V_{x,t}^P - U_t$. By construction, $H_t^F(w_{j,t}^{Fn})$ and $H_t^P(w_{j,t}^{Pn})$ respectively are give by

$$H_t^F(w_{j,t}^{Fn}) = \frac{w_{j,t}^{Fn}}{p_t} - b_t + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\begin{array}{l}\rho_{t+1}^F(w_{j,t+1}^{Fn})H_{t+1}^F(w_{j,t+1}^{Fn})\\ +\lambda^{FP}(1-\rho_{t+1}^F(w_{j,t+1}^{Fn}))H_{x,t+1}^P - s_{t+1}^F H_{x,t+1}^F\end{array}\right)\right].$$

  and

$$H_t^P(w_{j,t}^{Pn}) = \frac{w_{j,t}^{Pn}}{p_t}\mu_b^P - \hat{\mu}_b^P b_t + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\begin{array}{r}\rho_{t+1}^P(w_{j,t+1}^{Pn})H_{t+1}^P(w_{j,t+1}^{Pn}) + \varphi H_{x,t+1}^F)\\ -s_{t+1}^P H_{x,t+1}^P\end{array}\right)\right]$$

  where $\hat{\mu}_b^P = 1 - (1-\mu_b^P)b^P/b$.

- Firm's surplus

  The firm's surplus from hiring an additional employee is respectively given by

$$J_t^F(w_{j,t}^{Fn}) = p_t^F a_t^F - \frac{w_{j,t}^{Fn}}{p_t} - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{\kappa_{t+1}^F}{2}(x_t^F(w_{j,t}^{Fn}))^2 + \mathcal{A}_t(\varrho_{t+1}^F(w_{j,t+1}^{Fn}))\right)\right]$$
$$+ \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\rho_{t+1}^F(w_{j,t+1}^{Fn}) + x_t^F(w_{j,t}^{Fn})\right)J_{t+1}^F(w_{j,t+1}^{Fn})\right],$$

  and

$$J_t^P(w_{j,t}^{Pn}) = p_t^P a_t^P - \frac{w_{j,t}^{Pn}}{p_t}\mu_b^P + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{\kappa_{t+1}^P}{2}(x_t^P(w_{j,t}^{Pn}))^2 + \varrho^P J_{t+1}^P(w_{j,t+1}^{Pn})\right)\right].$$

- Optimality conditions

$$\frac{\partial \mathcal{A}_t^F(\rho_t^F(w_{j,t}^{Fn}))}{\partial \rho_t^F(w_{j,t}^{Fn})} = J_t^F(w_{j,t}^{Fn}), \tag{F.1}$$

$$\frac{\partial \mathcal{A}_t^P(\rho_t^P(w_{j,t}^{Pn}))}{\partial \rho_t^P(w_{j,t}^{Pn})} = J_t^P(w_{j,t}^{Pn}),$$

$$\kappa_{t+1}^F x_t^F(w_{j,t}^{Fn})\mathbb{E}_t\left[\Lambda_{t,t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1} J_{t+1}^F(w_{j,t+1}^{Fn})\right], \tag{F.2}$$

and

$$\kappa_{t+1}^P x_t^P(w_{j,t}^{Pn})\mathbb{E}_t\left[\Lambda_{t,t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1} J_{t+1}^P(w_{j,t+1}^{Pn})\right].$$

- De-trending

  We convert the non-stationary variables into the stationary variables: $\bar{\Lambda}_{t,t+1} = (z_{t+1}/z_t)\Lambda_{t,t+1}$, $\bar{a}_t^F = a_t^F/z_t$, $\bar{a}_t^P = a_t^P/z_t$, $\bar{b}_t = b_t/z_t$ $\bar{J}_t^F = J_t^F/z_t$, $\bar{J}_t^P = J_t^P/z_t$, $\bar{H}_t^F = H_t^F/z_t$, $\bar{H}_t^P = H_t^P/z_t$, $\bar{w}_{j,t}^{Fn} = w_{j,t}^{Fn}/z_t$, $\bar{w}_{j,t}^{Pn} = w_{j,t}^{Pn}/z_t$, $\mathcal{A}^F(\varrho_t^F) = \mathcal{A}_t^F(\varrho_t^F)/z_t$, $\mathcal{A}^P(\varrho_t^P) = \mathcal{A}_t^P(\varrho_t^P)/z_t$, $\kappa^F = \kappa_t^F/z_t$, and $\kappa^P = \kappa_t^P/z_t$.

- Some definitions and lemmas

  We define

$$\mathcal{E}_t^F(w) \equiv \frac{\partial_\rho \mathcal{A}^F(\varrho_t^F(w))}{\varrho_t^F(w)\partial_\rho^2 \mathcal{A}^F(\varrho_t^F(w))}.$$

  Recall that $\mathcal{A}^F(\cdot)$ takes a power function with an exponent ζ^F, so we have

$$\mathcal{E}_t^F(w) = \frac{1}{\zeta} \equiv \mathcal{E}^F.$$

  For later use, we present the following lemmas

  Lemma 1 *For any* $\lambda > 0$*,*

$$\frac{\partial \mathcal{A}^F(\rho_t^F(\lambda w))}{\partial w} = \lambda \mathcal{E}^F \varrho_t^F(\lambda w)\frac{\partial \bar{J}_t^F(\lambda w)}{\partial(\lambda w)},$$

  and

$$\frac{\partial \rho_t^F(\lambda w)\bar{J}_t^F(\lambda w)}{\partial w} = \lambda\left(\mathcal{E}^F + 1\right)\varrho_t^F(\lambda w)\frac{\partial \bar{J}_t^F(\lambda w)}{\partial(\lambda w)},$$

  We define the aggregate variables as follows:

$$w_t^\ell = \int_{\mathcal{J}^\ell} \frac{w_{j,t}^{\ell n}}{p_t}\left(\frac{n_{j,t}^F}{n_t^F}\right) dj, \qquad \text{for} \quad \ell = F, P \tag{F.3}$$

  In addition, we denote the average of the hiring rate and the retention rate weighted by the employment share by

$$x_t^\ell = \int_{\mathcal{J}^\ell} x_t^\ell(w_{j,t}^\ell) \left(\frac{n_{j,t}^\ell}{n_t^\ell} \right) dj, \qquad \text{for} \quad \ell = F, P \tag{F.4}$$

and

$$\rho_t^\ell = \int_{\mathcal{J}^\ell} \varrho_t^F(w_{j,t}^\ell) \left(\frac{n_{j,t}^\ell}{n_t^\ell} \right) dj \qquad \text{for} \quad \ell = F, P. \tag{F.5}$$

The law of large numbers implies the dynamics of the average nominal wage for full-time workers follows

$$w_t^F = (1-\vartheta_w^F) w_t^{*F} \int_0^1 \frac{n_{j,t}^F(w_t^{*Fn})}{n_t^F} dj + \vartheta_w^F \int_0^1 \frac{\bar{\iota}_{w,t-1}}{\pi_t} w_{j,t-1}^F \frac{n_{j,t}^F(\bar{\iota}_{w,t-1} w_{j,t-1}^{Fn})}{n_t^F} dj.$$

The log-linearized equation is given by

$$\tilde{\bar{w}}_t^F = (1-\vartheta_w^F)\tilde{\bar{w}}_t^{*F} + \vartheta_w^F \left(\iota_w \tilde{\pi}_{t-1} + \tilde{\bar{w}}_{t-1}^F - \tilde{\varepsilon}_t^z - \tilde{\pi}_t \right). \tag{F.6}$$

where $\bar{w}_t^F = w_t^F / z_t$.

Analogously, the (log-linearized) evolution equation of the average real wage for part-time workers is given by

$$\tilde{\bar{w}}_t^P = (1-\vartheta_w^P)\tilde{\bar{w}}_t^{*P} + \vartheta_w^P \left(\iota_w \tilde{\pi}_{t-1} + \tilde{\bar{w}}_{t-1}^P - \tilde{\varepsilon}_t^z - \tilde{\pi}_t \right), \tag{F.7}$$

where $\bar{w}_t^P = w_t^P / z_t$.

F.2.1 Wages for full-time workers

We aim to find the expression for $\tilde{\bar{w}}_t^{*F}$.

- The surplus sharing rule

  The renegotiated nominal wage w_t^{*Fn} satisfies the following surplus sharing rule:

$$\chi_t^F(w_t^{*Fn}) \bar{J}_t^F(w_t^{*Fn}) = [1 - \chi_t^F(w_t^{*Fn})] \bar{H}_t^F(w_t^{*Fn}), \tag{F.8}$$

where

$$\begin{aligned}\bar{J}^F_t(w^{Fn}_{j,t}) =& p^F_t \bar{a}^F_t - \frac{w^{Fn}_{j,t}}{p_t z_t} - \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\frac{\kappa^F}{2} (x^F_t(w^{Fn}_{j,t}))^2 + \mathcal{A}(\varrho^F_{t+1}(w^{*Fn}_{t+1})) \right) \right] \\ & + \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\rho^F_{t+1}(w^{*Fn}_{t+1}) + x^F_t(w^{Fn}_{j,t}) \right) \bar{J}^F_{t+1}(w^{*Fn}_{t+1}) \right] \\ & - \vartheta^F_w \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left[\mathcal{A}(\varrho^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t})) - \mathcal{A}(\varrho^F_{t+1}(w^{*Fn}_{t+1})) \right] \right] \\ & + \vartheta^F_w \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\begin{array}{l} \left(\rho^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) + x^F_t(\hat{\iota}_{w,t} w^{Fn}_{j,t}) \right) \bar{J}^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) \\ - \left(\rho^F_{t+1}(w^{*Fn}_{t+1}) + x^F_t(w^{*Fn}_{t+1}) \right) \bar{J}^F_{t+1}(w^{*Fn}_{t+1}) \end{array} \right) \right], \end{aligned} \tag{F.9}$$

and

$$\begin{aligned}\bar{H}^F_t(w^{Fn}_{j,t}) =& \frac{w^{Fn}_{j,t}}{p_t z_t} - \bar{b}_t + \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\begin{array}{l} \varrho^F_{t+1}(w^{*Fn}_{t+1}) \bar{H}^F_{t+1}(w^{*Fn}_{t+1}) \\ + \lambda^{FP}(1 - \rho^F_{t+1}(w^{*Fn}_{t+1})) \bar{H}^P_{x,t+1} - s^F_t \bar{H}^F_{x,t+1} \end{array} \right) \right] \\ & + \vartheta^F_w \mathbb{E}_t \left[\bar{\Lambda}_{t,t+1} \left(\begin{array}{l} \left(\varrho^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) \bar{H}^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \varrho^F_{t+1}(w^{*Fn}_{t+1}) \bar{H}^F_{t+1}(w^{*Fn}_{t+1}) \right) \\ -\lambda^{FP} \left(\varrho^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \varrho^F_{t+1}(w^{*Fn}_{t+1}) \right) \bar{H}^P_{x,t+1} \end{array} \right) \right], \end{aligned} \tag{F.10}$$

where $\hat{\iota}_{w,t} \equiv \gamma_z(\pi)^{1-\iota_w}(\pi_t)^{\iota_w}$.

- The effective bargaining power

  The effective workers' bargaining power satisfies

$$\chi^F_t(w^{*Fn}_t) = \frac{\eta^F_t}{\eta^F_t + (1-\eta^F_t)\mu^F_t(w^{*Fn}_t)/\epsilon^F_t(w^{*Fn}_t)} \tag{F.11}$$

  where

$$\epsilon^F_t(w^{Fn}_{j,t}) \equiv p_t \partial H^F_t(w^{Fn}_{j,t})/\partial w^{Fn}_{j,t},$$

$$\epsilon^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) \equiv (z_t/z_{t+1}) p_{t+1} \partial H^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t})/\partial(\hat{\iota}_{w,t} w^{Fn}_{j,t}),$$

$$\mu^F_t(w^{Fn}_{j,t}) \equiv -p_t \partial J^F_t(w^{Fn}_{j,t})/\partial w^{Fn}_{j,t},$$

  and

$$\mu^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) \equiv -(z_t/z_{t+1}) p_{t+1} \partial J^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t})/\partial(\hat{\iota}_{w,t} w^{Fn}_{j,t}).$$

- The recursive formulations for $\epsilon^F_t(w^{Fn}_{j,t})$ and $\mu^F_t(w^{Fn}_{j,t})$

  Using Lemma 1 and

$$\frac{\partial \bar{J}^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t})}{\partial(\hat{\iota}_{w,t} w^{Fn}_{j,t})} = \frac{1}{z_{t+1}} \frac{\partial J^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t})}{\partial(\hat{\iota}_{w,t} w^{Fn}_{j,t})},$$

$\epsilon_t^F(w_{j,t}^{Fn})$ and $\mu_t^F(w_{j,t}^{Fn})$ can be formulated recursively as

$$\epsilon_t^F(w_{j,t}^{Fn}) = 1+\vartheta_w^F\mathbb{E}_t\left[\begin{array}{l}\bar{\Lambda}_{t,t+1}\varrho_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})\frac{p_t}{p_{t+1}}\hat{\iota}_{w,t}\\ \times\left(\begin{array}{l}\epsilon_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})\\ -\mathcal{E}^F\mu_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})\frac{(\bar{H}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\lambda^{FP}\bar{H}_{x,t+1}^P)}{\bar{J}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})}\end{array}\right)\end{array}\right] \tag{F.12}$$

and

$$\mu_t^F(w_{j,t}^{Fn}) = 1+\vartheta_w^F\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})+x_t^F(w_{j,t}^{Fn})\right)\frac{p_t}{p_{t+1}}\hat{\iota}_{w,t}\mu_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})\right]. \tag{F.13}$$

Since $\vartheta_w^F\varrho^F\beta\in(0,1)$, we have in the balanced-growth steady state:

$$\epsilon^F = \frac{1-\vartheta_w^F\varrho^F\beta\mathcal{E}^F\mu^F\frac{\bar{H}^F-\lambda^{FP}\bar{H}^P}{\bar{J}^F}}{1-\vartheta_w^F\varrho^F\beta},$$

and

$$\mu^F = \frac{1}{1-\vartheta_w^F(\varrho^F+x^F)\beta}.$$

Below, we denote the real wage by $\bar{w}_{j,t}^F=\bar{w}_{j,t}^{Fn}/p_t$ and $\bar{w}_{j,t}^P=\bar{w}_{j,t}^{Pn}/p_t$.

- Log-linearization

  First, log-linearizing the optimality conditions (F.1) and (F.2) deliver

$$\tilde{\varrho}_t^F(w_{j,t}^{Fn})=\mathcal{E}^F\tilde{\bar{J}}_t^F(w_{j,t}^{Fn}), \tag{F.14}$$

  and

$$\tilde{x}_t^F(w_{j,t}^{Fn})=\mathbb{E}_t\left[\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn})\right]+\vartheta_w^F\mathbb{E}_t\left[\tilde{\bar{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn})\right], \tag{F.15}$$

  respectively.

  Then, combined with the steady-state condition $\kappa^Fx^F=\bar{J}^F$ and $\mathcal{A}^F(\varrho^F)/\partial_\rho\mathcal{A}^F(\varrho^F)=\varrho^F/(\zeta^F+1)$, the log-linearization of (F.8), (F.9) and (F.10) are respectively given by

$$\tilde{\bar{J}}_t^F(w_t^{*Fn})+(1-\chi^F)^{-1}\tilde{\chi}_t^F(w_t^{*Fn})=\tilde{\bar{H}}_t^F(w_t^{*Fn}), \tag{F.16}$$

where

$$\begin{aligned}\tilde{\tilde{J}}_t^F(w_{j,t}^{Fn}) =&\varkappa_a^F\left(\tilde{p}_t^F+\tilde{\tilde{a}}_t^F\right)-\varkappa_w^F\left(\tilde{w}_{j,t}^{Fn}-\tilde{p}_t-\tilde{z}_t\right)+\beta(\varrho^F+x^F)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})\\&+\left[(\varrho^F+x^F)-\left(x^F/2+\varrho^F/(\zeta+1)\right)\right]\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1}\\&+\vartheta_w^F(\varrho^F+x^F)\beta\mathbb{E}_t\left[\tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})\right],\end{aligned}\tag{F.17}$$

with $\varkappa_a^F=p^F\bar{a}^F/\bar{J}^F$ and $\varkappa_w^F=\bar{w}^F/\bar{J}^F$, and

$$\begin{aligned}\tilde{\tilde{H}}_t^F(w_{j,t}^{Fn}) =&\frac{\bar{w}^F}{\bar{H}^F}\left(\tilde{w}_{j,t}^{Fn}-\tilde{p}_t-\tilde{z}_t\right)-\frac{\bar{b}}{\bar{H}^F}\tilde{\tilde{b}}_t-\beta s^F\mathbb{E}_t\left[\tilde{s}_t^F+\tilde{\tilde{H}}_{x,t+1}^F\right]\\&+\beta\left(\varrho^F-s^F+\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}(1-\varrho^F)\right)\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1}\\&+\beta\varrho^F\mathcal{E}^F\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})+\beta\varrho^F\mathbb{E}_t\tilde{\tilde{H}}_{t+1}^F(w_{t+1}^{*Fn})\\&+\beta\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}(1-\varrho^F)\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^P\\&-\beta\varrho^F\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}\mathcal{E}^F\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})\\&+\vartheta_w^F\beta\varrho^F\mathcal{E}^F\mathbb{E}_t\left[\tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})\right]\\&+\vartheta_w^F\beta\varrho^F\mathbb{E}_t\left[\tilde{\tilde{H}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{H}}_{t+1}^F(w_{t+1}^{*Fn})\right]\\&-\vartheta_w^F\beta\varrho^F\mathcal{E}^F\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}\mathbb{E}_t\left[\tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})\right].\end{aligned}\tag{F.18}$$

- Find the expressions for $\mathbb{E}_t[\tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})]$ and $\mathbb{E}_t[\tilde{\tilde{H}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{H}}_{t+1}^F(w_{t+1}^{*Fn})]$

  With (F.17), it must be held that

$$\begin{aligned}&\mathbb{E}_t[\tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*Fn})]\\&\quad=-\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t+\tilde{w}_{j,t}^{Fn}-\tilde{w}_{t+1}^{*Fn}\right]\\&\quad\quad+\vartheta_w^F(\varrho^F+x^F)\beta\mathbb{E}_t[\tilde{\tilde{J}}_{t+2}^F(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t}w_{j,t}^{Fn})-\tilde{\tilde{J}}_{t+2}^F(\bar{\iota}_{w,t+1}w_{t+1}^{*Fn})]\\&\quad=-\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t+\tilde{\tilde{w}}_{j,t}^F-\tilde{\varepsilon}_{t+1}^z-\tilde{\pi}_{t+1}-\tilde{\tilde{w}}_{t+1}^{*F}\right]\\&\quad\quad-\vartheta_w^F(\varrho^F+x^F)\beta\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t+\tilde{\tilde{w}}_{j,t}^F-\tilde{\varepsilon}_{t+1}^z-\tilde{\pi}_{t+1}-\tilde{\tilde{w}}_{t+1}^{*F}\right]\\&\quad\quad-(\vartheta_w^F(\varrho^F+x^F)\beta)^2\varkappa_w^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t+\tilde{\tilde{w}}_{j,t}^F-\tilde{\varepsilon}_{t+1}^z-\tilde{\pi}_{t+1}-\tilde{\tilde{w}}_{t+1}^{*F}\right]+\cdots.\end{aligned}$$

  Recall that $\mu^F=(1-\vartheta_w^F(\varrho^F+x^F)\beta)^{-1}$. Iterating forward the above equation yields

$$\mathbb{E}_t[\tilde{\bar{J}}^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \tilde{\bar{J}}^F_{t+1}(w^{*Fn}_{t+1})] \\ = -(\varkappa^F_w \mu^F)\mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right]. \tag{F.19}$$

Similarly, with (F.18), it must be held that

$$\begin{aligned}
&\mathbb{E}_t[\tilde{\bar{H}}^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+1}(w^{*Fn}_{t+1})] \\
&= \frac{\bar{w}^F}{\bar{H}^F}\mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{w}^{Fn}_{j,t} - \tilde{w}^{*Fn}_{t+1}\right] \\
&\quad + \vartheta^F_w \beta \varrho^F \mathbb{E}_t[\tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1} w^{*Fn}_{t+1})] \\
&\quad + \frac{1}{\bar{H}^F}\vartheta^F_w \beta \varrho^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)\mathcal{E}^F \mathbb{E}_t[\tilde{\bar{J}}^F_{t+2}(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \tilde{\bar{J}}^F_{t+2}(\bar{\iota}_{w,t+1} w^{*Fn}_{t+1})] \\
&= \frac{\bar{w}^F}{\bar{H}^F}\left(1 - \vartheta^F_w \beta \varrho^F \mu^F \mathcal{E}^F \frac{\bar{H}^F - \lambda^{FP}\bar{H}^P}{\bar{J}^F}\right)\mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right] \\
&\quad + \vartheta^F_w \beta \varrho^F \mathbb{E}_t[\tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1}\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+2}(\bar{\iota}_{w,t+1} w^{*Fn}_{t+1})].
\end{aligned}$$

Iterating forward the above equation yields

$$\mathbb{E}_t[\tilde{\bar{H}}^F_{t+1}(\hat{\iota}_{w,t} w^{Fn}_{j,t}) - \tilde{\bar{H}}^F_{t+1}(w^{*Fn}_{t+1})] \\ = \frac{\bar{w}^F \epsilon^F}{\bar{H}^F}\mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right]. \tag{F.20}$$

- Find the expressions for $\tilde{\bar{J}}^F_t(w^F_{j,t})$ and $\tilde{\bar{H}}^F_t(w^F_{j,t})$

Substituting (F.19) into (F.17) delivers

$$\begin{aligned}
\tilde{\bar{J}}^F_t(w^F_{j,t}) &= \varkappa^F_a\left(\tilde{p}^F_t + \tilde{\bar{a}}^F_t\right) \\
&\quad - \varkappa^F_w\left(\tilde{\bar{w}}^F_{j,t} + \beta(\varrho^F + x^F)\vartheta^F_w \mu^F \mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}^F_{j,t} - \tilde{\varepsilon}^z_{t+1} - \tilde{\pi}_{t+1} - \tilde{\bar{w}}^{*F}_{t+1}\right]\right) \\
&\quad + \beta \mathbb{E}_t\left[x^F \tilde{\bar{J}}^F_{t+1}(w^{*F}_{t+1}) + X^F \tilde{\bar{\Lambda}}_{t,t+1}\right] + \beta \varrho^F \mathbb{E}_t\left[\tilde{\bar{J}}^F_{t+1}(w^{*F}_{t+1}) + \tilde{\bar{\Lambda}}_{t,t+1}\right].
\end{aligned} \tag{F.21}$$

with $X^F = (x^F + \varrho^F) - \varrho^F - \left(x^F/2 + \varrho^F/(\zeta^F + 1)\right) = x^F/2 - \varrho^F/(\zeta^F + 1)$.

Substituting (F.20) into (F.18) delivers

$$
\begin{aligned}
\tilde{\bar{H}}_t^F(w_{j,t}^F) =& \frac{\bar{w}^F}{\bar{H}^F}\left(\tilde{\bar{w}}_{j,t}^F + \vartheta_w^F\beta\varrho^F\hat{\epsilon}^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^F - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right]\right) \\
& - \frac{\bar{b}}{\bar{H}^F}\tilde{\bar{b}}_t - \beta s^F\mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\
& + \beta\varrho^F\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} + \beta\varrho^F\mathbb{E}_t\left[\mathcal{E}^F\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*Fn}) + \tilde{\bar{H}}_{t+1}^F(w_{t+1}^{*F})\right] \\
& + \beta\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}(1-\varrho^F)\mathbb{E}_t\left[\tilde{\bar{H}}_{x,t+1}^P + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\
& - \beta\varrho^F\frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F}\mathcal{E}^F\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}),
\end{aligned} \tag{F.22}
$$

where $\hat{\epsilon}^F = \epsilon^F - \mathcal{E}^F\mu^F(\bar{H}^F - \lambda^{FP}\bar{H}^P)/\bar{J}^F$.

- Find the expressions for $\tilde{\bar{w}}_t^{*F}$

  Substitute (F.21) and (F.22) into (F.16) and use the steady state condition $\chi^F\bar{J}^F = (1-\chi^F)\bar{H}^F$ to obtain

$$
\begin{aligned}
& \chi^F\frac{p^F\bar{a}^F}{\bar{w}^F}\left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F\right) - \chi^F\left(\tilde{\bar{w}}_t^{*F} + \beta\vartheta_w^F\mu^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_t^{*F} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right]\right) \\
& + \chi^F\beta x^F\frac{\bar{J}^F}{\bar{w}^F}\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) + \chi^F\beta X^F\frac{\bar{J}^F}{\bar{w}^F}\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} + \chi^F\beta\varrho^F\frac{\bar{J}^F}{\bar{w}^F}\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) \\
& = (1-\chi^F)\left(\tilde{\bar{w}}_t^{*F} + \vartheta_w^F\beta\varrho^F\hat{\epsilon}^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_t^{*F} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right]\right) \\
& - (1-\chi^F)\frac{\bar{b}}{\bar{w}^F}\tilde{\bar{b}}_t - (1-\chi^F)\beta s^F\frac{\bar{H}^F}{\bar{w}^F}\mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\
& + (1-\chi^F)\beta\varrho^F\frac{\bar{H}^F}{\bar{w}^F}\mathbb{E}_t\tilde{\bar{H}}_{t+1}^F(w_{t+1}^{*F}) + (1-\chi^F)\beta\frac{\lambda^{FP}\bar{H}^P}{\bar{w}^F}(1-\varrho^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\
& + (1-\chi^F)\beta\varrho^F\frac{\bar{H}^F - \lambda^{FP}\bar{H}^P}{\bar{w}^F}\mathcal{E}^F\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) - \chi^F(1-\chi^F)^{-1}\frac{\bar{J}^F}{\bar{w}^F}\tilde{\chi}_t^F(w_t^{*F}).
\end{aligned} \tag{F.23}
$$

  Collect the terms to simplify (F.23) as

$$
\begin{aligned}
& \varphi_a^F\left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F\right) + (\varphi_x^F - \varphi_\rho^F)\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) + \varphi_X^F\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} + \varphi_b^F\tilde{\bar{b}}_t \\
& + \varphi_s^F\mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_{\hat{\rho}}^F\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\
& + \varphi_\chi^F\left[\tilde{\chi}_t^F(w_t^{*F}) - \beta\varrho^F\mathbb{E}_t\tilde{\chi}_{t+1}^F(w_{t+1}^{*F})\right] \\
& = \tilde{w}_t^{*F} + \psi^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t + \tilde{\bar{w}}_t^{*F} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right]
\end{aligned} \tag{F.24}
$$

with

$$\varphi_a^F = \chi^F \frac{p^F \bar{a}^F}{\bar{w}^F}, \quad \varphi_x^F = \chi^F \beta x^F \frac{\bar{J}^F}{\bar{w}^F}, \quad \varphi_X^F = \chi^F \beta X^F \frac{\bar{J}^F}{\bar{w}^F}, \quad \varphi_b^F = (1-\chi^F)\frac{\bar{b}}{\bar{w}^F},$$
$$\varphi_s^F = (1-\chi^F)\beta s^F \frac{\bar{H}^F}{\bar{w}^F}, \quad \varphi_\chi^F = \chi^F(1-\chi^F)^{-1}\frac{\bar{J}^F}{\bar{w}^F}, \quad \varphi_{\hat{\rho}}^F = (1-\chi^F)\beta\frac{\lambda^{FP}\bar{H}^P}{\bar{w}^F}(1-\varrho^F),$$
$$\varphi_\rho^F = (1-\chi^F)\beta\frac{\bar{H}^F - \lambda^{FP}\bar{H}^P}{\bar{w}^F}\varrho^F\mathcal{E}^F, \quad \text{and} \quad \psi^F = (1-\chi^F)\vartheta_w^F\varrho^F\beta\hat{\epsilon}^F + \chi^F\beta\vartheta_w^F\mu^F.$$

Define $\tilde{\bar{w}}_t^o(w_t^{*F})$ as

$$\begin{aligned}\tilde{\bar{w}}_t^o(w_t^{*F}) =& \varphi_a^F\left(\tilde{p}_t^F + \tilde{\bar{a}}_t^F\right) + (\varphi_x^F - \varphi_\rho^F)\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F(w_{t+1}^{*F}) + \varphi_X^F\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} + \varphi_b^F\tilde{\bar{b}}_t \\ &+ \varphi_s^F\mathbb{E}_t\left[\tilde{s}_t^F + \tilde{\bar{H}}_{x,t+1}^F + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_{\hat{\rho}}^F\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\ &+ \varphi_\chi^F\left[\tilde{\chi}_t^F(w_t^{*F}) - \beta\varrho^F\mathbb{E}_t\tilde{\chi}_{t+1}^F(w_{t+1}^{*F})\right].\end{aligned} \tag{F.25}$$

Then, we simply (F.24) as

$$\tilde{\bar{w}}_t^{*F} = (1-\tau^F)\tilde{\bar{w}}_t^o(w_t^{*F}) - \tau^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*F}\right], \tag{F.26}$$

where

$$\tau^F = \psi^F/(1+\psi^F)$$

We note that if $\vartheta_w^F = 0$, then $\tilde{w}_t^{*F} = \tilde{\bar{w}}_t^o(w_t^{*F})$. Below, we will delve into the expression for $\tilde{w}_t^o(w_t^{*F})$ to express this in terms of difference between the newly-contracted wage w_t^{*F} and the aggregate wage w_t^F.

- Find the expression for $\tilde{\chi}_t^F(w_t^{*F})$

  We note that the expressions (F.19) and (F.20) hold in more general:

$$\tilde{\bar{J}}_t^F(w_t^F) - \tilde{\bar{J}}_t^F(w_t^{*F}) = -(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^F - \tilde{\bar{w}}_t^{*F}\right), \tag{F.27}$$

  and[38]

$$\tilde{\bar{H}}_t^F(w_t^F) - \tilde{\bar{H}}_t^F(w_t^{*F}) = (1-\eta^F)(\eta^F)^{-1}(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^F - \tilde{\bar{w}}_t^{*F}\right). \tag{F.28}$$

  Furthermore, (F.14), (F.15), (F.21), and (F.22) imply that[39]

$$\tilde{x}_t^F = \tilde{x}_t^F(w_t^F), \tag{F.29}$$

[38]Use the steady-state condition $(\bar{w}^F/\bar{H}^F)\epsilon^F = (1-\chi^F)(\chi^F)^{-1}\epsilon^F\varkappa_w^F = (1-\eta^F)(\eta^F)^{-1}\varkappa_w^F\mu^F$.
[39]See (F.3), (F.4). and (F.5) for the definition of w_t^F, x_t^F, and ϱ_t^F.

$$\tilde{\varrho}_t^F = \tilde{\varrho}_t^F(w_t^F) = \mathcal{E}^F \tilde{\tilde{J}}_t^F(w_t^F),$$

and

$$\tilde{\tilde{H}}_{x,t}^F = \tilde{\tilde{H}}_t^F(w_t^F). \tag{F.30}$$

Thus, we have

$$\mathbb{E}_t \tilde{\tilde{J}}_{t+1}^F(w_{t+1}^{*F}) = \mathbb{E}_t \tilde{\tilde{J}}_{t+1}^F - (\varkappa_w^F \mu^F)\mathbb{E}_t \left[\tilde{\tilde{w}}_{t+1}^{*F} - \tilde{\tilde{w}}_{t+1}^F\right]. \tag{F.31}$$

Now, we are ready to find the expression for $\tilde{\chi}_t^F(w_t^{*F})$. Log-linearizing (F.11), we express $\tilde{\chi}_t^F(w_t^{*F})$ in terms of $\tilde{\epsilon}_t^F(w_t^{*F})$ and $\tilde{\mu}_t^F(w_t^{*F})$ as follows:

$$\tilde{\chi}_t^F(w_t^{*F}) = (1-\chi^F)\left(\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\mu}_t^F(w_t^{*F})\right) + (1-\chi^F)(1-\eta^F)^{-1}\tilde{\varepsilon}_t^{\eta,F}.$$

Rearranging the terms in the above equation delivers

$$\begin{aligned}\tilde{\chi}_t^F(w_t^{*F}) =& (1-\chi^F)\left(\tilde{\epsilon}_t^F(w_t^F) - \tilde{\mu}_t^F(w_t^F)\right) + (1-\chi^F)(1-\eta^F)^{-1}\tilde{\varepsilon}_t^{\eta,F} \\ &+ (1-\chi^F)\left[(\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F(w_t^F)) - (\tilde{\mu}_t^F(w_t^{*F}) - \tilde{\mu}_t^F(w_t^F))\right].\end{aligned} \tag{F.32}$$

Let us define $\tilde{\epsilon}_t^F \equiv \tilde{\epsilon}_t^F(w_t^F)$, $\tilde{\epsilon}_{t+1}^F \equiv \tilde{\epsilon}_{t+1}^F(w_{t+1}^F)$, $\tilde{\mu}_t^F \equiv \tilde{\mu}_t^F(w_t^F)$, and $\tilde{\mu}_{t+1}^F \equiv \tilde{\mu}_{t+1}^F(w_{t+1}^F)$. We need to find the expressions for $\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F$ and $\tilde{\mu}_t^F(w_t^{*F}) - \tilde{\mu}_t^F$ and the dynamic equations for $\tilde{\epsilon}_t^F$ and $\tilde{\mu}_t^F$. For convince, define $\tilde{\Upsilon}_{t,t+1} \equiv \tilde{\tilde{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z$. Log-linearizing (F.12) yields

$$\begin{aligned}&\tilde{\epsilon}_t^F(w_t^{*F}) \\ &= \vartheta_w^F \varrho^F \beta \mathbb{E}_t \left[\begin{array}{l}(1-e_o^F)\left(\tilde{\Upsilon}_{t,t+1} + \tilde{\epsilon}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) + \mathcal{E}^F \tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F})\right) \\ -e_o^F \left(\begin{array}{l}\tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) \\ + \frac{\bar{H}^F}{\bar{H}^F - \lambda^{FP}\bar{H}^P}\tilde{\tilde{H}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \frac{\lambda^{FP}\bar{H}^P}{\bar{H}^F - \lambda^{FP}\bar{H}^P}\tilde{\tilde{H}}_{x,t+1}^P\end{array}\right)\end{array}\right]\end{aligned}$$

with $e_o^F \equiv \frac{1}{\epsilon^F}\mathcal{E}^F \mu^F \frac{\bar{H}^F - \lambda^{FP}\bar{H}^P}{\bar{J}^F}$. Log-linearizing (F.13) yields

$$\tilde{\mu}_t^F(w_t^{*F}) = \vartheta_w^F(\varrho^F + x^F)\beta\mathbb{E}_t\left[\tilde{\Upsilon}_{t,t+1} + \tilde{\mu}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) + m_o^F \tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F})\right] \tag{F.33}$$

with $m_o^F \equiv x^F + \varrho^F \mathcal{E}^F$. Then, substitute

$$\mathbb{E}_t\left[\tilde{\tilde{J}}_{t+1}^F(\hat{\iota}_{w,t} w_t^{*F}) - \tilde{\tilde{J}}_{t+1}^F(w_{t+1}^F)\right] = -(\varkappa_w^F \mu^F)\mathbb{E}_t\left[\iota_w \tilde{\pi}_t + \tilde{\tilde{w}}_t^{*F} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\tilde{w}}_{t+1}^F\right]$$

into (F.33) to obtain

$$
\begin{aligned}
\tilde{\mu}_t^F(w_t^{*F}) - \tilde{\mu}_t^F(w_t^F) =& - \vartheta_w^F(\varrho^F + x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \vartheta_w^F(\varrho^F + x^F)\beta\mathbb{E}_t\left[\tilde{\mu}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\mu}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)\right] \\
=& - \vartheta_w^F(\varrho^F + x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \vartheta_w^F(\varrho^F + x^F)\beta\left(-\vartheta_w^F(\varrho^F + x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right)\right) \\
&+ (\vartheta_w^F(\varrho^F + x^F)\beta)^2\left(-\vartheta_w^F(\varrho^F + x^F)\beta m_o^F(\varkappa_w^F\mu^F)\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right)\right) \\
&+ \cdots .
\end{aligned}
$$

Recall that $\vartheta_w^F(\varrho^F + x^F)\beta \in (0,1)$ and $\mu^F = (1 - \vartheta_w^F(\varrho^F + x^F)\beta)^{-1}$. Iterating forward the above equation delivers

$$
\tilde{\mu}_t^F(w_t^{*F}) - \tilde{\mu}_t^F(w_t^F) = -(\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right). \tag{F.34}
$$

As for $\tilde{\epsilon}_t^F(w_t^{*F})$, we have

$$
\begin{aligned}
&\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F(w_t^F) \\
&= \vartheta_w^F\varrho^F\beta\left((1 - e_o^F)\mathcal{E}^F + e_o^F\right)\mathbb{E}_t\left[\tilde{\bar{J}}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\bar{J}}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)\right] \\
&\quad - \vartheta_w^F\varrho^F\beta e_o^F\mathbb{E}_t\left[\tilde{\mu}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\mu}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)\right] \\
&\quad - \vartheta_w^F\varrho^F\beta e_o^F\frac{\bar{H}^F}{\bar{H}^F - \lambda^{FP}\bar{H}^P}\mathbb{E}_t\left[\tilde{\bar{H}}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\bar{H}}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)\right] \\
&\quad + \vartheta_w^F\varrho^F\beta(1 - e_o^F)\mathbb{E}_t\left[\tilde{\epsilon}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\epsilon}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)\right].
\end{aligned}
$$

Use (F.20) and (F.34) for $\mathbb{E}_t[\tilde{\bar{H}}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\bar{H}}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)]$ and $\mathbb{E}_t[\tilde{\mu}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\mu}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)]$ respectively to obtain

$$
\begin{aligned}
\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F(w_t^F) =& \mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \vartheta_w^F\varrho^F\beta(1 - e_o^F)\mathbb{E}_t\left[\tilde{\epsilon}_{t+1}^F(\hat{\imath}_{w,t}w_t^{*F}) - \tilde{\epsilon}_{t+1}^F(\hat{\imath}_{w,t}w_t^F)\right] \\
=& \mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \vartheta_w^F\varrho^F\beta(1 - e_o^F)\mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ (\vartheta_w^F\varrho^F\beta(1 - e_o^F))^2\mho\left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\
&+ \cdots
\end{aligned}
$$

with $\mho = -\vartheta_w^F\varrho^F\beta(\varkappa_w^F\mu^F)\left[(2 - e_o^F)\mathcal{E}^F + e_o^F - e_o^F(\vartheta_w^F\beta m_o^F)\mu^F\right]$. See Appendix F.2.2 for the derivation.

Lemma 2 *If it is satisfied that* $|\vartheta_w^F \varrho^F \beta(1-e_o^F)| < 1$*,*

$$1 + \vartheta_w^F \varrho^F \beta(1-e_o^F) + (\vartheta_w^F \varrho^F \beta(1-e_o^F))^2 + \cdots = \frac{1}{1-\vartheta_w^F \varrho^F \beta(1-e_o^F)} = \epsilon^F.$$

Under the condition that $|\vartheta_w^F \varrho^F \beta(1-e_o^F)| < 1$, applying Lemma 2 to above equation brings

$$\tilde{\epsilon}_t^F(w_t^{*F}) - \tilde{\epsilon}_t^F(w_t^F) = \epsilon^F \mho \left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right). \tag{F.35}$$

Now, substituting (F.34) and (F.35) into (F.32) delivers

$$\begin{aligned} \tilde{\chi}_t^F(w_t^{*F}) =& \tilde{\chi}_t^F + (1-\chi^F)[\epsilon^F \mho + (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F] \left(\tilde{\bar{w}}_t^{*F} - \tilde{\bar{w}}_t^F\right) \\ &+ (1-\chi^F)(1-\eta^F)^{-1} \tilde{\varepsilon}_t^{\eta,F}, \end{aligned} \tag{F.36}$$

where

$$\tilde{\chi}_t^F \equiv (1-\chi^F)(\tilde{\epsilon}_t^F - \tilde{\mu}_t^F). \tag{F.37}$$

Similarly,

$$\begin{aligned} \mathbb{E}_t \tilde{\chi}_{t+1}^F(w_{t+1}^{*F}) =& \mathbb{E}_t \tilde{\chi}_{t+1}^F + (1-\chi^F)[\epsilon^F \mho + (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F] \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^{*F} - \tilde{\bar{w}}_{t+1}^F\right] \\ &+ (1-\chi^F)(1-\eta^F)^{-1} \rho_\eta^F \tilde{\varepsilon}_t^{\eta,F}. \end{aligned}$$

- Find dynamic equations for for $\tilde{\epsilon}_t^F$ and $\tilde{\mu}_t^F$

  As shown in Appendix F.2.2, the dynamic equations for $\tilde{\epsilon}_t^F$ and $\tilde{\mu}_t^F$ are, respectively, given by

$$\begin{aligned} \tilde{\epsilon}_t^F =& \vartheta_w^F \varrho^F \beta(1-e_0^F) \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] - \vartheta_w^F \varrho^F \beta e_0^F \mathbb{E}_t \tilde{\mu}_{t+1}^F \\ &- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP} \bar{H}^P)^{-1} \mathbb{E}_t \left[\bar{H}^F \tilde{\bar{H}}_{x,t}^F - \lambda^{FP} \bar{H}^P \tilde{\bar{H}}_t^P\right] \\ &+ \vartheta_w^F \varrho^F \beta \left((1-e_o^F)\mathcal{E}^F + e_o^F\right) \mathbb{E}_t \tilde{\bar{J}}_{t+1}^F(w_{t+1}^F) \\ &+ \epsilon^F \mho \mathbb{E}_t \left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right]. \end{aligned} \tag{F.38}$$

  and

$$\begin{aligned} \tilde{\mu}_t^F =& (\vartheta_w^F \beta m_o^F) \mathbb{E}_t \tilde{x}_{t+1}^F + \vartheta_w^F \beta \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\mu}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\ &- (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F \mathbb{E}_t \left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right]. \end{aligned} \tag{F.39}$$

- Find the expression for $\tilde{\tilde{H}}^F_{x,t}$ Combining (F.28) and (F.30), we find that $\tilde{\tilde{H}}^F_{x,t}$ satisfies

$$\tilde{\tilde{H}}^F_{x,t} = \tilde{\tilde{H}}^F_t(w^{*F}_t) + (1-\eta^F)(\eta^F)^{-1}(\varkappa^F_w \mu^F)\left(\tilde{\tilde{w}}^F_t - \tilde{\tilde{w}}^{*F}_t\right).$$

Substituted into (F.16), $\tilde{\tilde{H}}^F_{x,t}$ is given by

$$\tilde{\tilde{H}}^F_{x,t} = \tilde{\tilde{J}}^F_t(w^{*F}_t) + (1-\chi^F)^{-1}\tilde{\chi}^F_t(w^{*F}_t) + (1-\eta^F)(\eta^F)^{-1}(\varkappa^F_w \mu^F)\left(\tilde{\tilde{w}}^F_t - \tilde{\tilde{w}}^{*F}_t\right).$$

Below, we abbreviate $\tilde{\tilde{J}}^F_t(w^F_t)$ to $\tilde{\tilde{J}}^F_t$. Using (F.6), (F.27), and (F.36), we obtain the expression for $\mathbb{E}_t\tilde{\tilde{H}}^F_{x,t+1}$:

$$\begin{aligned}\mathbb{E}_t\tilde{\tilde{H}}^F_{x,t+1} =& \mathbb{E}_t\tilde{\tilde{J}}^F_{t+1} + (1-\chi^F)^{-1}\mathbb{E}_t\tilde{\chi}^F_{t+1} + (1-\eta^F)^{-1}\mathbb{E}_t\tilde{\varepsilon}^{\eta,F}_{t+1} \\ &- \vartheta^F_w(1-\vartheta^F_w)^{-1}\Gamma^F\mathbb{E}_t\left[\tilde{\tilde{w}}^F_{t+1} - (\tilde{\tilde{w}}^F_t - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\varepsilon}^z_{t+1})\right],\end{aligned} \tag{F.40}$$

with $\Gamma^F = -\epsilon^F\mho + [1-\eta^F(\vartheta^F_w\beta m^F_o)\mu^F](\eta^F)^{-1}(\varkappa^F_w\mu^F)$.

- Find the expression for $\tilde{\tilde{w}}^o_t(w^{*F}_t)$

Hence, substitute (F.31), (F.36), and (F.40) into (F.25) and rearrange terms to derive

$$\tilde{\tilde{w}}^o_t(w^{*F}_t) = \tilde{\tilde{w}}^{o,F}_t + \frac{\tau^F_1}{1-\tau^F}\mathbb{E}_t\left[\tilde{\tilde{w}}^F_{t+1} - \tilde{\tilde{w}}^{*F}_{t+1}\right] + \frac{\tau^F_2}{1-\tau^F}(\tilde{\tilde{w}}^F_t - \tilde{\tilde{w}}^{*F}_t), \tag{F.41}$$

where

$$\begin{aligned}\tilde{\tilde{w}}^{o,F}_t =& \varphi^F_a\left(\tilde{p}^F_t + \tilde{\tilde{a}}^F_t\right) + (\varphi^F_x + \varphi^F_s - \varphi^F_\rho)\mathbb{E}_t\tilde{\tilde{J}}^F_{t+1} + (\varphi^F_s + \varphi^F_X - \varphi^F_{\hat{\rho}})\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1} \\ &+ \varphi^F_b\tilde{\tilde{b}}_t + \varphi^F_s\tilde{s}^F_t - \varphi^F_{\hat{\rho}}\mathbb{E}_t\tilde{\tilde{H}}^P_{x,t+1} + \varphi^F_\chi\left[\tilde{\chi}^F_t - \beta(\varrho^F - s^F)\mathbb{E}_t\tilde{\chi}^F_{t+1}\right] + \varphi^F_\eta\tilde{\varepsilon}^{\eta,F}_t,\end{aligned}$$

with

$$\tau^F_1 = (1-\tau^F)\left[(\varphi^F_x - \varphi^F_\rho)\varkappa^F_w\mu^F + \varphi^F_\chi(\varrho^F\beta)(1-\chi^F)[\epsilon^F\mho + (\vartheta^F_w\beta m^F_o)(\varkappa^F_w\mu^F)\mu^F] + \varphi_s\Gamma^F\right],$$

$$\tau^F_2 = -(1-\tau^F)\varphi^F_\chi(1-\chi^F)[\epsilon^F\mho + (\vartheta^F_w\beta m^F_o)(\varkappa^F_w\mu^F)\mu^F],$$

and[40]

$$\varphi^F_\eta = \frac{\varphi^F_\chi(1-\chi^F)(1-\beta\varrho^F\rho^F_\eta) + \varphi^F_s\rho^F_\eta}{1-\eta^F} = \frac{(1-\chi^F)\varphi^F_\chi[1-\beta(\varrho^F - s^F)\rho^F_\eta]}{1-\eta^F}.$$

Note that the dynamic equation for $\tilde{\chi}^F_t$ satisfies (F.51) and the dynamic equation

[40] We use $\varphi^F_s = (1-\chi^F)\varphi^F_\chi\beta s^F$ to simplify the coefficient of $\tilde{\varepsilon}^{\eta,F}_t$.

for $\tilde{x}_t^F$ satisfies[41]

$$\tilde{\tilde{J}}_t^F = \varkappa_a^F(\tilde{p}_t^F + \tilde{\tilde{a}}_t^F) - \varkappa_w^F \tilde{\tilde{w}}_t^F + \beta(\varrho^F + X^F)\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1} + \beta(1 + \varrho^F\mathcal{E}^F)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F.$$

- Derive the dynamic equation for w_t^F Substitute (F.41) into (F.26) to obtain

$$\tilde{\tilde{w}}_t^{*F} = (1-\tau^F)\tilde{\tilde{w}}_t^o + \tau_1^F\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^F - \tilde{\tilde{w}}_{t+1}^{*F}\right] + \tau_2^F(\tilde{\tilde{w}}_t^F - \tilde{\tilde{w}}_t^{*F}) - \tau^F\mathbb{E}_t\left[\iota_w\tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\tilde{w}}_{t+1}^{*F}\right].$$

Use (F.6) to express $\tilde{\tilde{w}}_t^{*F}$ in terms of aggregate variables. As a result, we have

$$\begin{aligned}
&\tilde{\tilde{w}}_t^F - \vartheta_w^F\left(\iota_w\tilde{\pi}_{t-1} + \tilde{\tilde{w}}_{t-1}^F - \tilde{\varepsilon}_t^z - \tilde{\pi}_t\right) \\
&= (1-\vartheta_w^F)(1-\tau^F)\tilde{\tilde{w}}_t^o + \vartheta_w^F\tau_1^F\left(\iota_w\tilde{\pi}_t + \tilde{\tilde{w}}_t^F - \mathbb{E}_t\tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\tilde{w}}_{t+1}^F\right) \\
&+ \vartheta_w^F\tau_2^F\left(\iota_w\tilde{\pi}_{t-1} + \tilde{\tilde{w}}_{t-1}^F - \tilde{\varepsilon}_t^z - \tilde{\pi}_t - \tilde{\tilde{w}}_t^F\right) \\
&- \tau^F\left(\iota_w\tilde{\pi}_t - \mathbb{E}_t\tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\tilde{w}}_{t+1}^F + \vartheta_w^F\tilde{\tilde{w}}_t^F\right).
\end{aligned}$$

Collecting the terms gives

$$\tilde{\tilde{w}}_t^F = \omega_b^F\left(\tilde{\tilde{w}}_{t-1}^F - \tilde{\pi}_t + \iota_w\tilde{\pi}_{t-1} - \tilde{\varepsilon}_t^z\right) + \omega_o^F\tilde{\tilde{w}}_t^{o,F} + \omega_f^F\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^F + \tilde{\pi}_{t+1} - \iota_w\tilde{\pi}_t + \tilde{\varepsilon}_{t+1}^z\right],$$

where $\omega_b^F = (1+\tau_2^F)/\Phi^F$, $\omega_o^F = \varsigma^F/\Phi^F$, $\omega_f^F = (\tau^F/\vartheta_w^F - \tau_1^F)/\Phi^F$, $\Phi^F = (1+\tau_2^F) + \varsigma^F + (\tau^F/\vartheta_w^F - \tau_1^F)$, and $\varsigma^F = (1-\vartheta_w^F)(1-\tau^F)/\vartheta_w^F$.

F.2.2 Derivations

- Derivation of $\mho$.

By construction of $\mho$:

$$\mho = \vartheta_w^F\varrho^F\beta\begin{bmatrix} -\left((1-e_o^F)\mathcal{E}^F + e_o^F\right)(\varkappa_w^F\mu^F) \\ -e_o^F(-(\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F) - e_o^F\frac{\bar{H}^F}{\bar{H}^F - \bar{H}^P}\frac{\bar{w}^F\epsilon^F}{\bar{H}^F}\end{bmatrix}$$

Recall that $e_o^F = (1/\epsilon^F)\mathcal{E}^F\mu^F(\bar{H}^F - \lambda^{FP}\bar{H}^P)/\bar{J}^F$ and then

$$e_o^F\frac{\bar{H}^F}{\bar{H}^F - \lambda^{FP}\bar{H}^P}\frac{\bar{w}^F\epsilon^F}{\bar{H}^F} = \mathcal{E}^F\mu^F\frac{\bar{w}^F}{\bar{J}^F} = \mathcal{E}^F\varkappa_w^F\mu^F.$$

Hence, we have

$$\mho = -\vartheta_w^F\varrho^F\beta(\varkappa_w^F\mu^F)\left[(2-e_o^F)\mathcal{E}^F + e_o^F - e_o^F(\vartheta_w^F\beta m_o^F)\mu^F\right].$$

[41] The dynamic equation for $\tilde{\tilde{J}}_t^F$ is derived from (F.21), (F.29), and (F.6).

- Derivation of $\tilde{\epsilon}_t^F$

$$
\begin{aligned}
\tilde{\epsilon}_t^F =& \vartheta_w^F \varrho^F \beta(1-e_0^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t + \tilde{\epsilon}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \vartheta_w^F \varrho^F \beta(1-e_0^F)\epsilon^F \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- \vartheta_w^F \varrho^F \beta e_0^F \mathbb{E}_t \tilde{\mu}_{t+1}^F \\
&+ \vartheta_w^F \varrho^F \beta e_0^F (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\mathbb{E}_t\left[\bar{H}^F \tilde{\bar{H}}_{x,t}^F - \lambda^{FP}\bar{H}^P \tilde{\bar{H}}_{x,t}^P\right] \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\bar{w}^F \epsilon^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \vartheta_w^F \varrho^F \beta \left((1-e_o^F)\mathcal{E}^F + e_o^F\right)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F \\
&- \vartheta_w^F \varrho^F \beta \left((1-e_o^F)\mathcal{E}^F + e_o^F\right)(\varkappa_w^F \mu^F)\mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
=& \vartheta_w^F \varrho^F \beta(1-e_0^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t + \tilde{\epsilon}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] - \vartheta_w^F \varrho^F \beta e_0^F \mathbb{E}_t \tilde{\mu}_{t+1}^F \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\mathbb{E}_t\left[\bar{H}^F \tilde{\bar{H}}_{x,t}^F - \lambda^{FP}\bar{H}^P \tilde{\bar{H}}_t^P\right] \\
&+ \vartheta_w^F \varrho^F \beta \left((1-e_o^F)\mathcal{E}^F + e_o^F\right)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F \\
&+ \vartheta_w^F \varrho^F \beta(1-e_0^F)\epsilon^F \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right].
\end{aligned}
$$

Recall that $1 + \vartheta_w^F \varrho^F \beta(1-e_0^F)\epsilon^F = \epsilon^F$ to simplify above equation into (F.38).

- Derivation of $\tilde{\mu}_t^F$

$$
\begin{aligned}
\tilde{\mu}_t^F =& (\vartheta_w^F \beta m_o^F)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F \\
&- (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&+ \vartheta_w^F \beta \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t + \tilde{\mu}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- \vartheta_w^F \beta(\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right].
\end{aligned}
$$

Recall that $1 + \vartheta_w^F \beta \mu^F = \mu^F$ to simplify above equation into (F.39).

F.2.3 Wages for part-time workers

We next find the expression for $\tilde{\bar{w}}_t^{*P}$. Since most process to derive it is analogous to the process to derive the expression for $\tilde{\bar{w}}_t^{*F}$, we just describe the setup and show the final outcomes.

- The renegotiated nominal wage for part-time workers w_t^{*Pn} satisfies the following wage sharing rule:

$$\chi_t^P(w_t^{*Pn})\bar{J}_t^P(w_t^{*Pn}) = [1-\chi_t^P(w_t^{*Pn})]\bar{H}_t^P(w_t^{*Pn}), \tag{F.42}$$

where

$$\begin{aligned}
\bar{J}_t^P(w_{j,t}^{Pn}) =& p_t^P\bar{a}_t^P - \frac{w_{j,t}^{Pn}}{p_t z_t} - \mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\frac{\kappa^P}{2}(x_t^P(w_{j,t}^{Pn}))^2\right] \\
&+ \mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P + x_t^P(w_{j,t}^{Pn})\right)\bar{J}_{t+1}^P(w_{t+1}^{*Pn})\right] \\
&+ \vartheta_w^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P + x_t^P(w_{j,t}^{Pn})\right)\left(\bar{J}_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) - \bar{J}_{t+1}^P(w_{t+1}^{*Pn})\right)\right],
\end{aligned}$$

and

$$\begin{aligned}
\bar{H}_t^P(w_{j,t}^{Pn}) =& \frac{w_{j,t}^{Pn}\mu_b^P}{p_t z_t} - \hat{\mu}_b^P\bar{b}_t + \mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P\bar{H}_{t+1}^P(w_{t+1}^{*Pn}) + \varphi\bar{H}_{x,t+1}^F\right) - s_{t+1}^P\bar{H}_{x,t+1}^P\right] \\
&+ \vartheta_w^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\varrho^P\left(\bar{H}_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) - \bar{H}_{t+1}^P(w_{t+1}^{*Pn})\right)\right].
\end{aligned}$$

- The effective bargaining power of the workers satisfies

$$\chi_t^P(w_t^{*Pn}) = \frac{\eta_t^P}{\eta_t^P + (1-\eta_t^P)\mu_t^P(w_t^{*Pn})/\varepsilon_t^F(w_t^{*Pn})}$$

where

$$\epsilon_t^P(w_{j,t}^{Pn}) \equiv p_t\partial H_t^P(w_{j,t}^{Pn})/\partial w_{j,t}^{Pn},$$

$$\epsilon_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) \equiv (z_t/z_{t+1})p_{t+1}\partial H_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})/\partial(\hat{\iota}_{w,t}w_{j,t}^{Pn}),$$

$$\mu_t^P(w_{j,t}^{Pn}) \equiv -p_t\partial J_t^P(w_{j,t}^{Pn})/\partial w_{j,t}^{Pn},$$

and

$$\mu_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn}) \equiv -(z_t/z_{t+1})p_{t+1}\partial J_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})/\partial(\hat{\iota}_{w,t}w_{j,t}^{Pn}).$$

Then, we have

$$\epsilon_t^P(w_{j,t}^{Pn}) = 1 + \vartheta_w^P\varrho^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\frac{p_t}{p_{t+1}}\hat{\iota}_{w,t}\epsilon_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})\right]$$

and

$$\mu_t^P(w_{j,t}^{Pn}) = 1 + \vartheta_w^P\mathbb{E}_t\left[\bar{\Lambda}_{t,t+1}\left(\varrho^P + x_t^P(w_{j,t}^{Pn})\right)\frac{p_t}{p_{t+1}}\hat{\iota}_{w,t}\mu_{t+1}^P(\hat{\iota}_{w,t}w_{j,t}^{Pn})\right].$$

In the balanced-growth steady state, ($\vartheta_w^P \varrho^P \beta \in (0,1)$ and $\vartheta_w^P(\varrho^P + x^P)\beta \in (0,1)$)

$$\epsilon^P = \frac{1}{1 - \vartheta_w^P \varrho^P \beta},$$

and

$$\mu^P = \frac{1}{1 - \vartheta_w^P(\varrho^P + x^P)\beta}.$$

- Log-linearization Log-linearizing the firm's surplus and the worker's surplus delivers

$$\begin{aligned}
\tilde{\bar{J}}_t^P(w_{j,t}^{Pn}) =& \varkappa_a^P(\tilde{p}_t^P + \tilde{\bar{a}}_t^P) \\
& - \varkappa_w^P \left(\tilde{\bar{w}}_{j,t}^P + \beta \vartheta_w^P \mu^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^P - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right]\right) \\
& + \beta x^P \mathbb{E}_t \left[\tilde{x}_t^P(w_{j,t}^{Pn}) + (1/2)\tilde{\bar{\Lambda}}_{t,t+1}\right] \\
& + \beta \varrho^P \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^P(w_{j,t+1}^{Pn}) + \tilde{\bar{\Lambda}}_{t,t+1}\right].
\end{aligned} \tag{F.43}$$

with $\varkappa_a^P = p^P \bar{a}^P / \bar{J}^P$ and $\varkappa_w^P = \bar{w}^P \mu_b^P / \bar{J}^P$ and

$$\begin{aligned}
\tilde{\bar{H}}_t^P(w_{j,t}^{Pn}) =& \frac{\bar{w}^P \mu_b^P}{\bar{H}^P} \left(\tilde{\bar{w}}_{j,t}^P + \beta \vartheta_w^P \varrho^P \epsilon^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_{j,t}^P - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right]\right) \\
& - \frac{\hat{\mu}_b^P \bar{b}}{\bar{H}^P} \tilde{\bar{b}}_t + \beta \varrho^P \mathbb{E}_t \left[\tilde{\bar{H}}_{t+1}^P + \tilde{\bar{\Lambda}}_{t,t+1}\right] \\
& - \beta s^P \mathbb{E}_t \left[\tilde{s}_t^P + \tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] + \frac{\beta \varphi \bar{H}^F}{\bar{H}^P} \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^F\right].
\end{aligned} \tag{F.44}$$

Log-linearize (F.42) to obtain

$$\tilde{\bar{J}}_t^P(w_t^{*Pn}) + (1 - \chi^F)^{-1} \tilde{\chi}_t^P(w_t^{*Pn}) = \tilde{\bar{H}}_t^P(w_t^{*Pn}), \tag{F.45}$$

Substitute (F.43) and (F.44) into (F.45) to obtain

$$\begin{aligned}
& \varphi_a^P \left(\tilde{p}_t^P + \tilde{\bar{a}}_t^P\right) + (\varphi_x^P - \varphi_\rho^P)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^P(w_{t+1}^{*P}) + (\varphi_x^P/2)\mathbb{E}_t \tilde{\bar{\Lambda}}_{t,t+1} + \varphi_b^P \tilde{\bar{b}}_t \\
& + \varphi_s^P \mathbb{E}_t \left[\tilde{s}_{t+1}^P + \tilde{\bar{H}}_{x,t+1}^P + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_\varphi \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^F\right] \\
& \varphi_\chi^P \left[\tilde{\chi}_t^P(w_t^{*P}) - \beta \varrho^P \mathbb{E}_t \tilde{\chi}_{t+1}^P(w_{t+1}^{*P})\right] \\
& = \tilde{w}_t^{*P} + \psi^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t + \tilde{\bar{w}}_t^{*P} - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right]
\end{aligned} \tag{F.46}$$

with

$$\begin{aligned}
\varphi_a^P =&\chi^P \frac{p^P \bar{a}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_x^P = \chi^P \frac{x^P \beta \bar{J}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_b^P = (1-\chi^P) \frac{\hat{\mu}_b^P \bar{b}}{\bar{w}^P \mu_b^P}, \\
\varphi_s^P =&(1-\chi^P) \frac{\beta s^P \bar{H}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_\chi^P = \frac{\bar{H}^P}{\bar{w}^P \mu_b^P}, \quad \varphi_\varphi = (1-\chi^P) \frac{\beta \varphi \bar{H}^F}{\bar{w}^P \mu_b^P}, \\
\bar{J}^P = \kappa^P x^P, \quad &\text{and} \quad \psi^P = (1-\chi^P) \vartheta_w^P \varrho^P \beta \epsilon^P + \chi^P \beta \vartheta_w^P \mu^P.
\end{aligned}$$

With $\tau^P = \psi^P/(1+\psi^P)$, we can simplify (F.46) into

$$\tilde{\bar{w}}_t^{*P} = (1-\tau^P)\tilde{\bar{w}}_t^o(w_t^{*P}) - \tau^P \mathbb{E}_t \left[\iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\bar{w}}_{t+1}^{*P}\right], \tag{F.47}$$

where

$$\begin{aligned}
\tilde{\bar{w}}_t^o(w_t^{*P}) =&\varphi_a^P \left(\tilde{p}_t^P + \tilde{\bar{a}}_t^P\right) + \varphi_x^P \mathbb{E}_t \left[\tilde{\bar{J}}_{t+1}^P(w_{t+1}^{*P}) + (1/2)\tilde{\bar{\Lambda}}_{t,t+1}\right] + \varphi_b^P \tilde{\bar{b}}_t \\
&+ \varphi_s^P \mathbb{E}_t \left[\tilde{s}_t^P + \tilde{\bar{H}}_{x,t+1}^P + \tilde{\bar{\Lambda}}_{t,t+1}\right] - \varphi_{\hat{\rho}}^P \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} + \tilde{\bar{H}}_{x,t+1}^P\right] \\
&- \varphi_\rho^P \mathbb{E}_t \tilde{x}_{t+1}^P(w_{t+1}^{*P}) + \varphi_\chi^P \left[\tilde{\chi}_t^P(w_t^{*P}) - \beta \varrho^P \mathbb{E}_t \tilde{\chi}_{t+1}^P(w_{t+1}^{*P})\right].
\end{aligned}$$

Taking the similar steps in the previous section, we obtain

$$\mathbb{E}_t \tilde{\bar{J}}_{t+1}^P(w_{t+1}^{*F}) = \mathbb{E}_t \tilde{\bar{J}}_{t+1}^P - (\varkappa_w^P \mu^P) \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^{*P} - \tilde{\bar{w}}_{t+1}^P\right], \tag{F.48}$$

and

$$\begin{aligned}
\mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^P =&\mathbb{E}_t \tilde{\bar{J}}_{t+1}^P + (1-\chi^P)^{-1} \mathbb{E}_t \tilde{\chi}_{t+1}^P + (1-\eta^P)^{-1} \rho_\eta^P \tilde{\varepsilon}_t^{\eta,P} \\
&- \vartheta_w^P (1-\vartheta_w^P)^{-1} \Gamma^P \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^P - (\tilde{\bar{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z)\right]
\end{aligned} \tag{F.49}$$

with $\Gamma^P = [1 - \eta^P(\vartheta_w^P \beta x^P)\mu^P](\eta^P)^{-1}(\varkappa_w^P \mu^P)$, and

$$\begin{aligned}
\tilde{\chi}_t^P(w_t^{*P}) =&\tilde{\chi}_t^P + (1-\chi^P)(\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P \left(\tilde{\bar{w}}_t^{*P} - \tilde{\bar{w}}_t^P\right) \\
&+ (1-\chi^P)(1-\eta^P)^{-1} \tilde{\varepsilon}_t^{\eta,P},
\end{aligned} \tag{F.50}$$

where

$$\tilde{\chi}_t^P \equiv (1-\chi^P)(\tilde{\epsilon}_t^P - \tilde{\mu}_t^P), \tag{F.51}$$

$$\tilde{\epsilon}_t^P = \varrho^P \vartheta_w^P \beta \mathbb{E}_t \left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right],$$

and

$$\begin{aligned}\tilde{\mu}_t^P =&(\vartheta_w^P\beta x^P)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^P + \beta\vartheta_w^P\mathbb{E}_t\left[\tilde{\tilde{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t + \tilde{\mu}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right]\\ &- (\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P\mathbb{E}_t\left[\tilde{\tilde{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w\tilde{\pi}_t - \tilde{\tilde{w}}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right].\end{aligned}$$

Similarly to (F.50)

$$\begin{aligned}\mathbb{E}_t\tilde{\chi}_{t+1}^P(w_{t+1}^{*P}) =&\mathbb{E}_t\tilde{\chi}_{t+1}^P + (1-\chi^P)(\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^{*P} - \tilde{\tilde{w}}_{t+1}^P\right]\\ &+ (1-\chi^P)(1-\eta^P)^{-1}\rho_\eta^P\tilde{\varepsilon}_t^{\eta,P}.\end{aligned}$$

Substitute (F.48), (F.50), and (F.49) into (F.47) and rearrange terms to derive

$$\tilde{\tilde{w}}_t^o(w_t^{*P}) = \tilde{\tilde{w}}_t^{o,P} + \frac{\tau_1^P}{1-\tau^P}\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^P - \tilde{\tilde{w}}_{t+1}^{*F}\right] + \frac{\tau_2^P}{1-\tau^P}(\tilde{\tilde{w}}_t^P - \tilde{\tilde{w}}_t^{*P}), \quad \text{(F.52)}$$

where

$$\begin{aligned}\tilde{\tilde{w}}_t^{o,P} =&\varphi_a^P(\tilde{p}_t^P + \tilde{\tilde{a}}_t^P) + (\varphi_s^P + \varphi_x^P)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^P + \varphi_s^P\tilde{s}_t^P + \varphi_b^P\tilde{\tilde{b}}_t\\ &+ (\varphi_s^P - \varphi_\varphi + \varphi_x^P/2)\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1} - \varphi_\varphi\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^F + \varphi_\eta^P\tilde{\varepsilon}_t^{\eta,P}\end{aligned}$$

with

$$\tau_1^P = (1-\tau^P)\left[\varphi_x^P\varkappa_w^P\mu^P + \varphi_\chi^P(1-\chi^P)(\varrho^P\beta)(\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P + \varphi_s\Gamma^P\right],$$

$$\tau_2^P = -(1-\tau^P)\varphi_\chi^P(1-\chi^P)(\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P,$$

and[42]

$$\varphi_\eta^P = (1-\eta^P)^{-1}(1-\chi^P)\varphi_\chi^P[1-\beta(\varrho^P - s^P)\rho_\eta^P].$$

See (F.40) for the expression for $\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^F$. Substitute (F.52) into (F.47) to obtain

$$\tilde{\tilde{w}}_t^{*P} = (1-\tau^P)\tilde{\tilde{w}}_t^o + \tau_1^P\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^P - \tilde{\tilde{w}}_{t+1}^{*P}\right] + \tau_2^P(\tilde{\tilde{w}}_t^P - \tilde{\tilde{w}}_t^{*P}) - \tau^P\mathbb{E}_t\left[\iota_w\tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z - \tilde{\pi}_{t+1} - \tilde{\tilde{w}}_{t+1}^{*P}\right].$$

Use (F.7) to express $\tilde{\tilde{w}}_t^{*P}$ in terms of aggregate variables and find that

$$\begin{aligned}&\tilde{\tilde{w}}_t^P - \vartheta_w^P\left(\iota_w\tilde{\pi}_{t-1} + \tilde{\tilde{w}}_{t-1}^P - \tilde{\varepsilon}_t^z - \tilde{\pi}_t\right)\\ &= (1-\vartheta_w^P)(1-\tau^P)\tilde{\tilde{w}}_t^o + \vartheta_w^P\tau_1^P\left(\iota_w\tilde{\pi}_t + \tilde{\tilde{w}}_t^P - \mathbb{E}_t\tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\tilde{w}}_{t+1}^P\right)\\ &+ \vartheta_w^P\tau_2^P\left(\iota_w\tilde{\pi}_{t-1} + \tilde{\tilde{w}}_{t-1}^P - \tilde{\varepsilon}_t^z - \tilde{\pi}_t - \tilde{\tilde{w}}_t^P\right)\\ &- \tau^P\left(\iota_w\tilde{\pi}_t - \mathbb{E}_t\tilde{\varepsilon}_{t+1}^z - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\tilde{w}}_{t+1}^P + \vartheta_w^P\tilde{\tilde{w}}_t^P\right).\end{aligned}$$

[42]We use $\varphi_s^P = (1-\chi^P)\varphi_r^P\chi\beta s^P$ to simplify the coefficient of $\tilde{\varepsilon}_t^{\eta,P}$.

Collecting the terms gives

$$\tilde{\bar{w}}_t^P = \omega_b^P \left(\tilde{\bar{w}}_{t-1}^P - \tilde{\pi}_t + \iota_w \tilde{\pi}_{t-1} - \tilde{\varepsilon}_t^z\right) + \omega_o^P \tilde{\bar{w}}_t^{o,P} + \omega_f^P \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^P + \tilde{\pi}_{t+1} - \iota_w \tilde{\pi}_t + \tilde{\varepsilon}_{t+1}^z\right],$$

where $\omega_b^P = (1+\tau_2^P)/\Phi^P$, $\omega_o^P = \varsigma^P/\Phi^P$, $\omega_f^P = (\tau^P/\vartheta_w^P - \tau_1^P)/\Phi^P$, $\Phi^P = (1+\tau_2^P) + \varsigma^P + (\tau^P/\vartheta_w^P - \tau_1^P)$, and $\varsigma^P = (1-\vartheta_w^P)(1-\tau^P)/\vartheta_w^P$.

F.2.4 Proof of Lemmas in Appendix F.2

Proof of Lemma 1. Taking derivative (F.1) with respect to w derives

$$\frac{\partial \mathcal{A}^{F'}(\rho_t^F(\lambda w))}{\partial w} = \frac{\partial \bar{J}_t^F(\lambda w)}{\partial w}, \qquad \Longrightarrow \qquad \mathcal{A}^{F''}(\rho_t^F(\lambda w))\rho_t^{F'}(\lambda w) = \bar{J}_t^{F'}(\lambda w),$$

and therefore

$$\rho_t^{F'}(\lambda w) = \frac{1}{\mathcal{A}^{F''}(\rho_t^F(\lambda w))}\bar{J}_t^{F'}(\lambda w) = \frac{\mathcal{A}^{F'}(\rho_t^F(\lambda w))}{\mathcal{A}^{F''}(\rho_t^F(\lambda w))}\frac{\bar{J}_t^{F'}(\lambda w)}{\bar{J}_t^F(\lambda w)} = \mathcal{E}_t^F(\lambda w)\frac{\varrho_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w)}{\bar{J}_t^F(\lambda w)}.$$

Hence,

$$\frac{\partial \mathcal{A}^F(\rho_t^F(\lambda w))}{\partial w} = \lambda \mathcal{A}^{\mathcal{F}'}(\rho_t^F(\lambda w))\rho_t^{F'}(\lambda w) = \lambda \varrho_t^F(\lambda w)\mathcal{E}_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w)$$

and

$$\begin{aligned}\frac{\partial \rho_t^F(\lambda w)\bar{J}_t^F(\lambda w)}{\partial w} &= \lambda \left(\rho_t^{F'}(\lambda w)\bar{J}_t^F(\lambda w) + \rho_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w)\right) \\ &= \lambda \left(\mathcal{E}_t^F(\lambda w) + 1\right)\varrho_t^F(\lambda w)\bar{J}_t^{F'}(\lambda w).\end{aligned}$$

■

Proof of Lemma 2. Let $A = \mathcal{E}^F\mu^F(\bar{H}^F - \bar{H}^P)/\bar{J}^F$ and $B = \vartheta_w^F\varrho^F\beta$.

$$\frac{1}{1-\vartheta_w^F\varrho^F\beta(1-e_o^F)} = \frac{1}{1-B(1-\frac{1-B}{1-AB}A)} = \frac{1-AB}{(1-AB)-B(1-A)} = \frac{1-AB}{1-B} = \epsilon^F.$$

■

G Model estimation

G.1 Steady-state conditions

- p^w, p^F, p^P, x^F, x^P, r^k, $\bar{k}^F$, $\bar{k}^P$, $\bar{a}^F$, $\bar{a}^P$, $\bar{y}^F/\bar{y}$, and $\bar{y}$ satisfy

$$p^w = \frac{1}{\varepsilon_p},$$

$$\frac{p^F}{p^P} = \left[\frac{\Omega^F}{1-\Omega^F}\frac{\varepsilon^\phi \mu_b^P n^P}{n^F}\right]^{\frac{1-\alpha}{\xi(1-\alpha)+\alpha}},$$

$$p^F = p^w\left(\Omega^F + (1-\Omega^F)\left(\frac{p^F}{p^P}\right)^{\xi-1}\right)^{\frac{1}{\xi-1}},$$

$$x^F = 1-(\varrho^F + \varphi n^P),$$

$$x^P = 1-(\varrho^P + \lambda^{FP}\varrho^F n^F),$$

$$A^F(\varrho^F)^{\zeta^F} = \kappa^F x^F,$$

$$A^P(\varrho^P)^{\zeta^P} = \kappa^P x^P,$$

$$r^k = \frac{\gamma_z}{\beta} - (1-\delta),$$

$$\frac{\bar{k}^F}{n^F} = \left(\frac{r^k}{p^F\alpha}\right)^{-1/(1-\alpha)},$$

$$\frac{\bar{k}^P}{\mu_b^P\varepsilon^\phi n^P} = \left(\frac{r^k}{p^P\alpha}\right)^{-1/(1-\alpha)},$$

$$\bar{a}^F = (1-\alpha)\left(\frac{\bar{k}^F}{n^F}\right)^\alpha,$$

$$\bar{a}^P = (1-\alpha)\mu_b^P\varepsilon^\phi\left(\frac{\bar{k}^P}{\mu_b^P\varepsilon^\phi n^P}\right)^\alpha,$$

$$\frac{\bar{y}^F}{\bar{y}} = \Omega^F\left(\frac{p^F}{p^w}\right)^{-\xi},$$

$$\bar{y} = \frac{1}{\Omega^F}\left(\frac{p^F}{p^w}\right)^\xi\left(\frac{\bar{k}^F}{n^F}\right)^\alpha n^F.$$

- The steady-state values for (n^F, n^P, u^F, u^P)

  The steady-state conditions for (14) and (15), the calibration target $n^F/n^P = 4.405$ and the condition that total labor force is unity induce the stationary distribution of workers' employment state:

$$\begin{pmatrix} 1-\varrho^F & -\varphi & -s^F & 0 \\ -\lambda_{FP}(1-\varrho^F) & 1-\varrho^P & 0 & -s^P \\ 1 & -4.405 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} n^F \\ n^P \\ u^F \\ u^P \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

- The steady-state values for the firm's surplus and worker's surplus

  The firm's surplus in the balanced-growth steady state is given by

$$\bar{J}^F = \frac{1}{1-\beta\varrho^F}\left[p^F\bar{a}^F - \bar{w}^F + \beta\left[\left(\frac{\kappa^F}{2}\right)(x^F)^2 + \vartheta^F\frac{(\varrho^F)^{1+\zeta^F}}{1+\zeta^F}\right]\right] \tag{G.1}$$

and

$$\bar{J}^P = \frac{1}{1-\beta\varrho^P}\left[p^P\bar{a}^P - \bar{w}^P\mu_b^P + \beta\left[\left(\frac{\kappa^P}{2}\right)(x^P)^2 + \vartheta^P\frac{(\varrho^P)^{1+\zeta^P}}{1+\zeta^P}\right]\right]. \tag{G.2}$$

The worker's surplus in the balanced-growth steady state is given by

$$\bar{H}^F = \frac{1}{1-\beta(\varrho^F - s^F C_\rho^F)}(\bar{w}^F - \bar{b}) \tag{G.3}$$

and

$$\bar{H}^P = \frac{1}{1-\beta(\varrho^P C_\varrho^P - s^P)}(\bar{w}^P\mu_b^P - \bar{\mu}_b^P\bar{b}), \tag{G.4}$$

where $C_\rho^F = 1 - \lambda^{FP}(1-\varrho^F)/s^P$ and $C_\varrho^P = 1 + (\varphi/\varrho^P)(s^P/s^F - 1)$.

- The steady-state conditions for wages

  Here, we describe how $\bar{w}^F$ and $\bar{w}^P$ are determined.

  Plug (G.1) and (G.2) into the definitions $\bar{J}^F = \kappa^F x^F$ and $\bar{J}^P = \kappa^P x^P$

$$\bar{w}^F = p^F\bar{a}^F - (1-\beta\varrho^F)\kappa^F x^F + \beta\kappa^F x^F\left(\frac{x^F}{2} + \frac{\varrho^F}{1+\zeta^F}\right) \tag{G.5}$$

and

$$\bar{w}^P\mu_b^P = p^P\bar{a}^P - (1-\beta\varrho^P)\kappa^P x^P + \beta\kappa^P x^P\left(\frac{x^P}{2} + \frac{\varrho^P}{1+\zeta^P}\right). \tag{G.6}$$

The (staggered) Nash bargaining solution requires

$$(1-\chi^F)\bar{H}^F = \chi^F\bar{J}^F, \tag{G.7}$$

and

$$(1-\chi^P)\bar{H}^P = \chi^P \bar{J}^P. \tag{G.8}$$

Plug (G.1), (G.2), (G.3), and (G.4), into (G.7) and (G.8)

$$(1-\chi^F)\bar{b} = \bar{w}^F - \chi^F \left[p^F \bar{a}^F + \beta \kappa^F x^F \left(\frac{x^F}{2} + \frac{\varrho^F}{1+\zeta^F} \right) + \beta s^F C_\rho^F \kappa^F x^F \right]. \tag{G.9}$$

and

$$(1-\chi^P)\bar{\mu}_b^P \bar{b} = \mu_b^P \bar{w}^P - \chi^P \left[\begin{array}{l} p^P \bar{a}^P + \beta \kappa^P x^P \left(\frac{x^P}{2} + \frac{\varrho^P}{1+\zeta^P} \right) \\ +\beta [s^P - \varrho^P (1 - C_\varrho^P)] \kappa^P x^P \end{array} \right]. \tag{G.10}$$

The steady-state condition $s^P \bar{H}^P = s^F \bar{H}^F$ induces

$$s^P \frac{\chi^P}{1-\chi^P} \kappa^P x^P = s^F \frac{\chi^F}{1-\chi^F} \kappa^F x^F. \tag{G.11}$$

This implies

$$\bar{H}^F - \lambda^{FP} \bar{H}^P = (1-\lambda^{FP} s^F/s^P) \bar{H}^F = (1-\lambda^{FP} s^F/s^P)((1-\chi^F)/\chi^F) \kappa^F x^F. \tag{G.12}$$

Solve χ^F and χ^P in terms of fixed and estimated parameters: χ^F is given by

$$\chi^F = \frac{\eta^F}{\eta^F + (1-\eta^F)\mu^F/\epsilon^F}, \tag{G.13}$$

where

$$\mu^F = \frac{1}{1-\beta(x^F + \varrho^F)\vartheta_w^F},$$

and

$$\epsilon^F = \frac{1 - \vartheta_w^F \varrho^F \beta \mathcal{E}^F \mu^F (\bar{H}^F - \lambda^{FP} \bar{H}^P)/(\kappa^F x^F)}{1 - \vartheta_w^F \varrho^F \beta}.$$

Substitute (G.12) into above expression to find

$$\epsilon^F = \frac{1 - \mu^F (\vartheta_w^F \varrho^F \beta) \mathcal{E}^F (1 - \lambda^{FP} s^F/s^P)(1-\chi^F)/\chi^F}{1 - \vartheta_w^F \varrho^F \beta},$$

Substituting this into (G.13) gives

$$\chi^F = \frac{\eta^F}{\eta^F + \mu^F \left[(\vartheta_w^F \varrho^F \beta) \mathcal{E}^F (1 - \lambda^{FP} s^F/s^P) \eta^F + (1 - \vartheta_w^F \varrho^F \beta)(1-\eta^F) \right]} \in (0,1),$$

and

$$\epsilon^F = \frac{1-\eta^F}{(\vartheta_w^F \varrho^F \beta)\mathcal{E}^F(1-\lambda^{FP}s^F/s^P)\eta^F + (1-\vartheta_w^F\varrho^F\beta)(1-\eta^F)}.$$

Next, χ^P is given by

$$\chi^P = \frac{\eta^P}{\eta^P + (1-\eta^P)\mu^P/\epsilon^P},$$

where

$$\mu^P = \frac{1}{1-\beta(x^P+\varrho^P)\vartheta_w^P},$$

and

$$\epsilon^P = \frac{1}{1-\vartheta_w^P\varrho^P\beta}.$$

The values for $(\kappa^F, \bar{w}^F, \bar{b})$ are solved out from (G.5), (G.9), and the estimate of $\bar{b}^F$,

$$\bar{b}^F = \frac{\bar{b}}{p^F\bar{a}^F + \beta\kappa^F x^F\left(x^F/2 + \varrho^F/(1+\zeta^F)\right)}.$$

The system of equations solves

$$\kappa^F = \frac{p^P\bar{a}^F(1-\bar{b}^F)(1-\chi^F)}{x^F[1-\beta(1-\bar{b}^F)(1-\chi^F)(x^F/2+\varrho^F/(1+\zeta^F)) - \beta(\varrho^F - s^F C_\rho^F\chi^F)]},$$

$$\bar{w}^F = \frac{p^F\bar{a}^F[\bar{b}^F(1-\chi^F)(1-\beta\varrho^F) - \chi^F(\beta\varrho^F - \beta s^F C_\rho^F - 1)]}{1-\beta(1-\bar{b}^F)(1-\chi^F)(x^F/2+\varrho^F/(1+\zeta^F)) - \beta(\varrho^F - s^F C_\rho^F\chi^F)},$$

and

$$\bar{b} = \frac{p^F\bar{a}^F\bar{b}^F[1-\beta(\varrho^F - s^F C_\rho^F\chi^F)]}{1-\beta(1-\bar{b}^F)(1-\chi^F)(x^F/2+\varrho^F/(1+\zeta^F)) - \beta(\varrho^F - s^F C_\rho^F\chi^F)}.$$

Given $(\bar{w}^F,\ \bar{b})$, the values for $(\kappa^P,\ \bar{w}^P,\ \bar{\mu}_b^P)$ are solved out from (G.6), (G.10), (G.11). From (G.11)

$$\kappa^P = \frac{s^F(1-\chi^P)}{x^P s^P\chi^P}\frac{\chi^F}{1-\chi^F}\kappa^F x^F,$$

and (G.6) can solve for $\bar{w}^P$. Then use (G.10) to solve out $\bar{\mu}_b^P$.

G.2 Log-linearized model equations

Consumption, Investment, and Production

$$(1-\beta h_z)\tilde{\lambda}_t = \hbar_{1,c}(\tilde{\bar{c}}_{t-1} - \tilde{\varepsilon}^z_t + \beta\mathbb{E}_t\tilde{\bar{c}}_{t+1} + \beta\mathbb{E}_t\tilde{\varepsilon}^z_{t+1}) - \hbar_{2,c}\tilde{\bar{c}}_t + \tilde{\varepsilon}^b_t - \beta h_z\mathbb{E}_t\tilde{\varepsilon}^b_{t+1},$$

$$\tilde{\lambda}_t = \tilde{r}^n_t + \mathbb{E}_t\tilde{\lambda}_{t+1} - \mathbb{E}_t\tilde{\pi}_{t+1} - \mathbb{E}_t\tilde{\varepsilon}^z_{t+1},$$

$$\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} = \mathbb{E}_t\tilde{\lambda}_{t+1} - \tilde{\lambda}_t,$$

$$\tilde{\bar{k}}^p_t = \delta_z(\tilde{\bar{k}}^p_{t-1} - \tilde{\varepsilon}^z_t) + (1-\delta_z)\tilde{\bar{i}}_t,$$

$$\tilde{\bar{k}}_t = \tilde{\nu}_t + \tilde{\bar{k}}^p_{t-1} - \tilde{\varepsilon}^z_t,$$

$$\tilde{\nu}_t = \eta_\nu\,\tilde{r}^k_t,$$

$$\tilde{Q}_t = \beta\delta_z\mathbb{E}_t\tilde{Q}_{t+1} + (1-\beta\delta_z)\,\mathbb{E}_t\tilde{r}^k_{t+1} - \tilde{r}^n_t + \mathbb{E}_t\tilde{\pi}_{t+1},$$

$$(1+\beta)\tilde{\bar{i}}_t = \tilde{\bar{i}}_{t-1} - \tilde{\varepsilon}^z_t + \tilde{\varepsilon}^i_t + [1/(\eta_k(\gamma_z)^2)]\tilde{Q}_t + \beta\mathbb{E}_t\left[\tilde{\bar{i}}_{t+1} + \tilde{\varepsilon}^z_{t+1} - \tilde{\varepsilon}^i_{t+1}\right],$$

$$\tilde{\bar{y}}^F_t = \alpha\tilde{\bar{k}}^F_t + (1-\alpha)\tilde{n}^F_t,$$

$$\tilde{\bar{y}}^P_t = \alpha\tilde{\bar{k}}^P_t + (1-\alpha)(\tilde{n}^P_t + \tilde{\varepsilon}^\phi_t),$$

$$\tilde{\bar{y}}_t = (\Omega^F)^{1/\xi}(\bar{y}^F/\bar{y})^{(\xi-1)/\xi}\tilde{\bar{y}}^F_t + (1-\Omega^F)^{1/\xi}(\bar{y}^P/\bar{y})^{(\xi-1)/\xi}\tilde{\bar{y}}^P_t,$$

$$\tilde{p}^w_t = \Omega^F(p^F/p^w)^{1-\xi}\tilde{p}^F_t + (1-\Omega^F)(p^P/p^w)^{1-\xi}\tilde{p}^P_t,$$

$$\tilde{\bar{y}}^F_t - \tilde{\bar{y}}^P_t = -\xi(\tilde{p}^F_t - \tilde{p}^P_t),$$

$$\tilde{r}^k_t = \tilde{p}^F_t + \tilde{\bar{y}}^F_t - \tilde{\bar{k}}^F_t,$$

$$\tilde{r}^k_t = \tilde{p}^P_t + \tilde{\bar{y}}^P_t - \tilde{\bar{k}}^P_t,$$

$$\tilde{\bar{k}}_t = (\bar{k}^F/\bar{k})\tilde{\bar{k}}^F_t + (\bar{k}^P/\bar{k})\tilde{\bar{k}}^P_t,$$

$$\begin{aligned}\tilde{\bar{y}}_t = y_c\tilde{\bar{c}}_t + y_i\tilde{\bar{i}}_t + \upsilon\tilde{\bar{g}}_t + y_\nu\tilde{\nu}_t + y^F_\rho\,\tilde{\varrho}^F_t + y^P_\varphi\tilde{n}^P_{t-1},\\ + y^F_x(2\tilde{x}^F_t + \tilde{n}^F_{t-1}) + y^P_x(2\tilde{x}^P_t + \tilde{n}^P_{t-1}),\end{aligned}$$

$$\tilde{\pi}_t = \iota_b\tilde{\pi}_{t-1} + \iota_o\tilde{p}^w_t + \iota_f\mathbb{E}_t\tilde{\pi}_{t+1} + \tilde{\varepsilon}^p_t,$$

where

$$\begin{aligned}
\hbar_{1,c} &= h_z/(1-h_z), \quad \hbar_{2,c} = (1+\beta(h_z)^2)/(1-h_z), \quad h_z = h_c/\gamma_z,\\
\delta_z &= (1-\delta)/\gamma_z, \quad \eta_\nu = (1-\psi_\nu)/\psi_\nu,\\
y_c &= 1-(y_i+\upsilon+y_\nu+y_\rho^F+y_x^F+y_x^P+y_\varphi^F), \quad y_i = (1-\delta_z)\gamma_z(\bar{k}/\bar{y}),\\
y_\nu &= r^k(\bar{k}/\bar{y}), \quad y_\rho^F = \varrho^F\kappa^F x^F n^F/\bar{y}, \quad y_x^\ell = (\kappa^\ell/2)[n^\ell(x^\ell)^2/\bar{y}],\\
y_\varphi^F &= (\zeta+1)^{-1}\varrho^P\kappa^F x^F n^F/\bar{y},\\
\iota_b &= \iota_p\phi_p, \quad \iota_o = [(1-\vartheta_p)(1-\beta\vartheta_p)/\vartheta_p][1+(\epsilon_p-1)\Xi]^{-1}\phi_p,\\
\iota_f &= \beta\phi_p, \quad \text{and} \quad \phi_p = 1/(1+\beta\iota_p).
\end{aligned}$$

Labor markets and employment dynamics

$$n^F\tilde{n}_t^F + n^P\tilde{n}_t^P + u^F\tilde{u}_t^F + u^P\tilde{u}_t^P = 0,$$

$$\tilde{n}_t^F = \varrho^F(\tilde{\varrho}_t^F + \tilde{n}_{t-1}^F) + x^F(\tilde{s}_{t-1}^F + \tilde{u}_{t-1}^F) + \varphi\varrho^P(n^P/n^F)\tilde{n}_{t-1}^P,$$

$$\tilde{n}_t^P = (1-\varphi)\varrho^P\tilde{n}_{t-1}^P + x^P(\tilde{s}_{t-1}^P + \tilde{u}_{t-1}^P) + \lambda^{FP}(n^F/n^P)((1-\varrho^F)\tilde{n}_{t-1}^F - \varrho^F\tilde{\varrho}_t^F),$$

$$\tilde{x}_t^F = \tilde{q}_t^F + \tilde{v}_t^F - \tilde{n}_t^F,$$

$$\tilde{x}_t^P = \tilde{q}_t^P + \tilde{v}_t^P - \tilde{n}_t^P,$$

$$\tilde{\varrho}_t^F = \mathcal{E}^F\tilde{\bar{J}}_t^F,$$

$$\tilde{x}_t^F = \mathbb{E}_t\tilde{\bar{J}}_{t+1}^F,$$

$$\tilde{\theta}_t^F = \tilde{v}_t^F - \tilde{u}_t^F,$$

$$\tilde{\theta}_t^P = \tilde{v}_t^P - \tilde{u}_t^P,$$

$$\tilde{q}_t^F = -\sigma_m^F\tilde{\theta}_t^F,$$

$$\tilde{q}_t^P = -\sigma_m^P\tilde{\theta}_t^P,$$

$$\tilde{s}_t^F = (1-\sigma_m^F)\tilde{\theta}_t^F,$$

$$\tilde{s}_t^P = (1-\sigma_m^P)\tilde{\theta}_t^P,$$

$$\tilde{\bar{J}}_t^F = \varkappa_a^F(\tilde{p}_t^F + \tilde{\bar{a}}_t^F) - \varkappa_w^F\tilde{\bar{w}}_t^F + \beta(\varrho^F + X^F)\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} + \beta(1+\varrho^F\mathcal{E}^F)\mathbb{E}_t\tilde{\bar{J}}_{t+1}^F,$$

$$\tilde{\bar{J}}_t^P = \varkappa_a^P(\tilde{p}_t^w + \tilde{\bar{a}}_t^P) - \varkappa_w^P\tilde{\bar{w}}_t^P + \varkappa_\lambda^P\mathbb{E}_t\tilde{\bar{\Lambda}}_{t,t+1} + \beta\mathbb{E}_t\tilde{\bar{J}}_{t+1}^P$$

$$\tilde{\bar{a}}_t^F = \tilde{\bar{y}}_t^F - \tilde{n}_t^F,$$

$$\tilde{\bar{a}}_t^P = \tilde{\bar{y}}_t^P - \tilde{n}_t^P,$$

$$\tilde{s}_t^F + \mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^F = \tilde{s}_t^P + \mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^P,$$

$$\begin{aligned}
\mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^F =& \mathbb{E}_t \tilde{\bar{J}}_{t+1}^F - \vartheta_w^F (1-\vartheta_w^F)^{-1} \Gamma^F \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^F - (\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z) \right] \\
&+ (1-\chi^F)^{-1} \mathbb{E}_t \tilde{\chi}_{t+1}^F + (1-\eta^F)^{-1} \rho_\eta^F \tilde{\varepsilon}_t^{\eta,F}, \\
\mathbb{E}_t \tilde{\bar{H}}_{x,t+1}^P =& \mathbb{E}_t \tilde{\bar{J}}_{t+1}^P - \vartheta_w^P (1-\vartheta_w^P)^{-1} \Gamma^P \mathbb{E}_t \left[\tilde{\bar{w}}_{t+1}^P - (\tilde{\bar{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\varepsilon}_{t+1}^z) \right] \\
&+ (1-\chi^P)^{-1} \mathbb{E}_t \tilde{\chi}_{t+1}^P + (1-\eta^P)^{-1} \rho_\eta^P \tilde{\varepsilon}_t^{\eta,P},
\end{aligned}$$

where

$$\begin{aligned}
&\mathcal{E}^F = \mathcal{A}^{F'}(\varrho^F)/(\varrho^F \mathcal{A}^{F''}(\varrho^F)) = 1/\zeta, \quad \sigma_m^\ell = (\theta^\ell)^\sigma/(1+(\theta^\ell)^\sigma), \\
&\varkappa_a^\ell = p^\ell \bar{a}^\ell/(\kappa^\ell x^\ell), \quad \varkappa_w^\ell = (\bar{w}^\ell \mu_b^\ell)/(\kappa^\ell x^\ell), \\
&X^F = 1 - \varrho^F - (x^F/2 + \varrho^F/(\zeta+1)), \quad \varkappa_\lambda^F = \beta(1+\varrho^F)/2, \quad \varkappa_\lambda^P = \beta(1+\varrho^P(1-\varphi))/2, \\
&\Gamma^F = -\epsilon^F \mho + [1 - \eta^F(\vartheta_w^F \beta m_o^F)\mu^F](\varkappa_w^F \mu^F)/\eta^F, \\
&\Gamma^P = [1 - \eta^P(\vartheta_w^P \beta x^P)\mu^P](\varkappa_w^P \mu^P)/\eta^P, \\
&\mho = -\vartheta_w^F \varrho^F \beta(\varkappa_w^F \mu^F)\left[(2-e_o^F)\mathcal{E}^F + e_o^F - e_o^F(\vartheta_w^F \beta m_o^F)\mu^F\right], \\
&e_o^F = (1/\epsilon^F)\mu^F \mathcal{E}^F(\bar{H}^F - \lambda^{FP}\bar{H}^P)/(\kappa^F x^F), \quad \text{and} \quad m_o^F = x^F + \varrho^F \mathcal{E}^F.
\end{aligned}$$

Wage dynamics

$$\tilde{\bar{b}}_t = \tilde{\bar{k}}_t^p,$$

$$\tilde{\chi}_t^F = -(1-\chi^F)(\tilde{\mu}_t^F - \tilde{\epsilon}_t^F),$$

$$\tilde{\chi}_t^P = -(1-\eta^P)(\tilde{\mu}_t^P - \tilde{\epsilon}_t^P),$$

$$\begin{aligned}
\tilde{\epsilon}_t^F =& \vartheta_w^F \varrho^F \beta(1-e_0^F)\mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] - \vartheta_w^F \varrho^F \beta e_0^F \mathbb{E}_t \tilde{\mu}_{t+1}^F \\
&- \vartheta_w^F \varrho^F \beta e_o^F (\bar{H}^F - \lambda^{FP}\bar{H}^P)^{-1}\mathbb{E}_t\left[\bar{H}^F \tilde{\bar{H}}_{x,t}^F - \lambda^{FP}\bar{H}^P \tilde{\bar{H}}_t^P\right] \\
&+ \vartheta_w^F \varrho^F \beta\left((1-e_o^F)\mathcal{E}^F + e_o^F\right)\mathbb{E}_t \tilde{\bar{J}}_{t+1}^F + \epsilon^F \mho \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right],
\end{aligned}$$

$$\tilde{\epsilon}_t^P = \varrho^P(1-\varphi)\vartheta_w^P \beta \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\epsilon}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right],$$

$$\begin{aligned}
\tilde{\mu}_t^F =& (\vartheta_w^F \beta m_o^F)\mathbb{E}_t \tilde{x}_{t+1}^F + \beta \vartheta_w^F \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\mu}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right] \\
&- (\vartheta_w^F \beta m_o^F)(\varkappa_w^F \mu^F)\mu^F \mathbb{E}_t\left[\tilde{\bar{w}}_t^F - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^F - \tilde{\varepsilon}_{t+1}^z\right],
\end{aligned}$$

$$\begin{aligned}
\tilde{\mu}_t^P =& (\vartheta_w^P \beta x^P)\mathbb{E}_t \tilde{x}_{t+1}^P + \beta \vartheta_w^P \mathbb{E}_t\left[\tilde{\bar{\Lambda}}_{t,t+1} - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t + \tilde{\mu}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right] \\
&- (\vartheta_w^P \beta x^P)(\varkappa_w^P \mu^P)\mu^P \mathbb{E}_t\left[\tilde{\bar{w}}_t^P - \tilde{\pi}_{t+1} + \iota_w \tilde{\pi}_t - \tilde{\bar{w}}_{t+1}^P - \tilde{\varepsilon}_{t+1}^z\right],
\end{aligned}$$

$$\begin{aligned}\tilde{\tilde{w}}_t^{o,F} =&\varphi_a^F\left(\tilde{p}_t^F+\tilde{\tilde{a}}_t^F\right)+(\varphi_x^F+\varphi_s^F-\varphi_\rho^F)\mathbb{E}_t\tilde{\tilde{J}}_{t+1}^F+\varphi_\lambda^F\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1}+\varphi_b^F\tilde{\tilde{b}}_t\\&+\varphi_s^F\mathbb{E}_t\tilde{s}_{t+1}^F-\varphi_{\hat{\rho}}^F\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^P+\varphi_\chi^F\left[\tilde{\chi}_t^F-\beta(\varrho^F-s^F)\mathbb{E}_t\tilde{\chi}_{t+1}^F\right]+\tilde{\varepsilon}_t^{w,F},\end{aligned}$$

$$\tilde{\varepsilon}_t^{w,F}=(1-\eta^F)^{-1}(1-\chi^F)\varphi_\chi^F[1-\beta(\varrho^F-s^F)\rho_\eta^F]\tilde{\varepsilon}_t^{\eta,F},$$

$$\begin{aligned}\tilde{\tilde{w}}_t^{o,P} =&\varphi_a^P(\tilde{p}_t^w+\tilde{\tilde{a}}_t^P)+(\varphi_s^P+\varphi_x^P)\mathbb{E}_t\tilde{x}_{t+1}^P+\varphi_b^P\tilde{\tilde{b}}_t+\varphi_\lambda^P\mathbb{E}_t\tilde{\tilde{\Lambda}}_{t,t+1}\\&+\varphi_s^P\mathbb{E}_t\tilde{s}_{t+1}^P-\varphi_\varphi\mathbb{E}_t\tilde{\tilde{H}}_{x,t+1}^F+\tilde{\varepsilon}_t^{w,P},\end{aligned}$$

$$\tilde{\varepsilon}_t^{w,P}=(1-\eta^P)^{-1}(1-\chi^P)\varphi_\chi^P[1-(1-x^P-s^P)\beta\varrho_\eta^P]\tilde{\varepsilon}_t^{\eta,P}$$

$$\tilde{\tilde{w}}_t^F=\omega_b^F(\tilde{\tilde{w}}_{t-1}^F-\tilde{\pi}_t+\iota_w\tilde{\pi}_{t-1}-\tilde{\varepsilon}_t^z)+\omega_o^F\tilde{\tilde{w}}_t^{o,F}+\omega_f^F\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^F+\tilde{\pi}_{t+1}-\iota_w\tilde{\pi}_t+\tilde{\varepsilon}_{t+1}^z\right],$$

$$\tilde{\tilde{w}}_t^P=\omega_b^P(\tilde{\tilde{w}}_{t-1}^P-\tilde{\pi}_t+\iota_w\tilde{\pi}_{t-1}-\tilde{\varepsilon}_t^z)+\omega_o^P\tilde{\tilde{w}}_t^{o,P}+\omega_f^P\mathbb{E}_t\left[\tilde{\tilde{w}}_{t+1}^P+\tilde{\pi}_{t+1}-\iota_w\tilde{\pi}_t+\tilde{\varepsilon}_{t+1}^z\right],$$

where

$$\begin{aligned}
&\varphi_a^\ell=\chi^\ell p^\ell\bar{a}^\ell(\bar{w}^\ell\mu_b^\ell)^{-1},\quad \varphi_x^\ell=\chi^\ell\beta\kappa^\ell(x^\ell)^2(\bar{w}^\ell\mu_b^\ell)^{-1},\quad \varphi_s^\ell=(1-\chi^\ell)s^\ell\beta\bar{H}^\ell(\bar{w}^\ell\mu_b^\ell)^{-1},\\
&\varphi_b^F=(1-\chi^F)\bar{b}(\bar{w}^F\mu_b^F)^{-1},\quad \varphi_b^P=(1-\chi^P)\bar{\mu}_b^P\bar{b}(\bar{w}^P\mu_b^P)^{-1},\\
&\varphi_\chi^\ell=\chi^\ell\kappa^\ell x^\ell[(1-\chi^\ell)\bar{w}^\ell\mu_b^\ell]^{-1},\quad \varphi_\varphi=\beta\varrho^P(1-\chi^P)\bar{H}^F(\bar{w}^P\mu_b^P)^{-1},\\
&\varphi_\lambda^F=\varphi_s^F+\varphi_X^F-\varphi_{\hat{\rho}}^F,\quad \varphi_\lambda^P=\varphi_s^P-\varphi_\varphi+\varphi_x^P/2,\quad \varphi_X^\ell=\chi^\ell\beta X^\ell\bar{J}^\ell(\bar{w}^\ell\mu_b^\ell)^{-1}\\
&\varphi_{\hat{\rho}}^F=(1-\chi^F)\beta(\lambda^{FP}\bar{H}^P/\bar{w}^F)(1-\varrho^F),\qquad \varphi_\rho^F=(1-\chi^F)\beta((\bar{H}^F-\lambda^{FP}\bar{H}^P)/\bar{w}^F)\varrho^F\mathcal{E}^F,\\
&\gamma_b^\ell=(1+\tau_2^\ell)/\Phi^\ell,\quad \gamma_o^\ell=\varsigma^\ell/\Phi^\ell,\quad \gamma_f^\ell=(\tau^\ell/\vartheta_w^\ell-\tau_1^\ell)/\Phi^\ell,\\
&\Phi^\ell=(1+\tau_2^\ell)+\varsigma^\ell+(\tau^\ell/\vartheta_w^\ell-\tau_1^\ell),\quad \varsigma^\ell=(1-\vartheta_w^\ell)(1-\tau^\ell)/\vartheta_w^\ell,\quad \tau^\ell=\psi^\ell(1+\psi^\ell)^{-1},\\
&\tau_1^F=(1-\tau^F)\left[\varkappa_w^F\mu^F(\varphi_x^F-\varphi_\rho^F)+\varphi_\chi^F(\varrho^F\beta)(1-\chi^F)[\epsilon^F\mho+(\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F]+\varphi_s\Gamma^F\right],\\
&\tau_2^F=-(1-\tau^F)\varphi_\chi^F(1-\chi^F)[\epsilon^F\mho+(\vartheta_w^F\beta m_o^F)(\varkappa_w^F\mu^F)\mu^F],\\
&\tau_1^P=(1-\tau^P)\left[\varphi_x^P\varkappa_w^P\mu^P+\varphi_\chi^P(1-\chi^P)(\varrho^P\beta)(\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P+\varphi_s\Gamma^P\right],\\
&\tau_2^P=-(1-\tau^P)\varphi_\chi^P(1-\chi^P)(\vartheta_w^P\beta x^P)(\varkappa_w^P\mu^P)\mu^P,\\
&\psi^F=(1-\chi^F)\vartheta_w^F\varrho^F\beta\hat{\epsilon}^F+\chi^F\beta\vartheta_w^F\mu^F,\quad \hat{\epsilon}^F=\epsilon^F-\mathcal{E}^F\mu^F(\bar{H}^F-\lambda^{FP}\bar{H}^P)/(\kappa^F x^F)\quad\text{and}\\
&\psi^P=(1-\chi^P)\vartheta_w^P\varrho^P\beta\epsilon^P+\chi^P\beta\vartheta_w^P\mu^P,
\end{aligned}$$

Monetary policy and Government spending

$$\tilde{r}_t^n=\phi_r\tilde{r}_{t-1}^n+(1-\phi_r)[\phi_\pi\tilde{\pi}_t+\phi_y(\tilde{y}_t-\tilde{y}_{nt})]+\tilde{\varepsilon}_t^r,$$

$$\tilde{\tilde{g}}_t=\tilde{\tilde{y}}_t+((1-\upsilon)/\upsilon)\,\tilde{\varepsilon}_t^g.$$