A Model of Entry, Exit, and Plant-level Dynamics over the Business Cycle*

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Abstract

We build a general equilibrium model of plant-level dynamics to quantitatively analyze the entry and exit of manufacturing plants over the business cycle. The baseline model, which is a direct extension of Hopenhayn and Rogerson's (1993) model of industry dynamics, can account for the macroeconomic rates of entry and exit over the business cycle observed in the U.S. data. It cannot, however, account for the microeconomic characteristics of entering plants. Assuming that entry costs are cyclical can resolve this inconsistency between the model and the data. We also show that this pattern of entry costs can arise from an endogenous mechanism. The business cycle results are robust when we extend the model to incorporate plant life cycle features.

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1 Introduction

Recent macroeconomic studies have made substantial progress in understanding the role of heterogeneity at the microeconomic level on aggregate dynamics. In this paper, we explore the implications of plant-level heterogeneity for macroeconomic dynamics. In particular, we focus on the plant-level dynamics over the business cycle. Our aim for this paper is to interpret the cyclical patterns of U.S. businesses documented in the previous studies (e.g., Lee and Mukoyama, 2015a) through the lens of the standard firm dynamics model.

To this end, we build a dynamic general equilibrium model with heterogeneous plants. Our model extends the standard Hopenhayn (1992)-style general equilibrium industry dynamics model of Hopenhayn and Rogerson (1993) by incorporating aggregate productivity shocks. To account for the cyclical changes in entrants' productivity and size over the business cycle, we explicitly incorporate self-selection in the entry process.

We find that the baseline model with aggregate productivity shocks performs well in replicating the *macroeconomic facts*: cyclicality of entry rates and exit rates observed in the U.S. manufacturing data. In the data, the entry rates are strongly procyclical while the exit rates are almost acyclical. The baseline model, however, fails in replicating the *microeconomic facts*: the cyclical patterns of size and productivity of entrants in the model are not consistent with the data. While the properties of the entrants vary significantly over the cycle in the data, the model exhibits almost no variations. This lack of variation arises for the following two reasons. First, when entry costs are constant over the cycle, the effect of aggregate productivity shocks on entrants' size and productivity is almost completely offset by the general equilibrium effect (i.e., cyclical changes in wages). That is, while the increase in aggregate productivity makes entry attractive even for low-productivity plants, this effect is completely negated by the increase in wages. Second, even when the wage effect is shut down, the aggregate productivity shocks measured by Solow residuals do not have a sufficiently strong effect to generate a *quantitatively* significant amount of selection.

This inconsistency between the data and the standard model poses a puzzle: a model that performs well in absence of aggregate shocks does not generate sufficient microeconomic volatility in the characteristic of entrants in the context of business cycle analysis. Facing this puzzle, we explore the role of entry costs in the aggregate patterns of entry. We find that with a particular combination of the cyclicality in entry costs, we can replicate the microeconomic facts in addition to the macroeconomic patterns. Then we show that these patterns of entry costs can be endogenized.

Some other recent papers extend a similar style of an industry dynamics model to incorporate business cycles. The main difference between this paper's model and those of the previous studies is that in ours both entry and exit are endogenously determined. Veracierto (2002, 2008) was among the first to incorporate aggregate productivity shocks to the Hopenhayn-style industry dynamics model.[1] However, Veracierto (2002, 2008) assumes exogenous entry and exit, and therefore these models are not directly suitable to explain the cyclical patterns of entry and exit observed in the data (e.g., selection of entrants over the cycle). Comin and Gertler (2006) build a model with endogenous innovation and technology adoption over the 'medium-term' business cycle. In their intermediate-goods sector, entry (innovation and adoption) is endogenous but exit (obsolescence of technology) is exogenous. In their final-goods sector, profit is a function of the *total* number of firms, and the zero-profit condition determines the total number of firms in equilibrium. Therefore, their final-goods sector does not have separate gross flows of entry and exit.[2] A recent work by Bilbiie et al. (2012) also constructs a model of business cycles with endogenous entry, but their model features firms with homogeneous technology and exogenous exit.

Samaniego (2008) constructs a general equilibrium model of industry dynamics with endogenous entry and exit. Instead of solving a model with aggregate shocks, he characterizes the (deterministic) transition path after a change in the aggregate productivity. He finds

[1]As in Veracierto (2001), the models in Veracierto (2002, 2008) also incorporate saving and the capital stock, which are absent in our model.

[2]Similarly, in other papers that employ monopolistic competition models to explore markup dynamics (e.g., Chatterjee and Cooper, 2014; Devereux et al. 1996; and Jaimovich and Floetotto, 2008), the zero-profit condition in each period determines the total number of firms. In these frameworks, it is not possible to address the issues of gross entry and exit separately.

that both entry and exit respond very little to the change in the aggregate productivity. This outcome is in contrast to our result—in our model, the entry rate responds strongly to the aggregate productivity shock. A large part of this difference comes from the specification of the entry cost. Samaniego assumes that the marginal cost of building a plant increases with the number of newly created plants, while we assume a constant marginal cost in our benchmark model.[3] The empirical evidence presented in Section 2 suggests that the constant marginal cost specification achieves an outcome closer to the behavior of the U.S. manufacturing plants in terms of the entry rates.

Since an earlier version of this paper (Lee and Mukoyama, 2008) was first circulated,[4] a number of follow-up papers have been written. One notable extension of our model is carried out by Clementi and Palazzo (2016), who incorporate capital stock into our model. They show that, in a model with capital stock and capital adjustment costs, fluctuations in entry can serve as an important propagation mechanism.[5] Clementi et al. (2014) extend Clementi and Palazzo's (2016) model to a general equilibrium framework and examine the cyclical implication of endogenous entry and exit when a capital adjustment cost is explicitly considered. Woo (2016) introduces imperfect information to the model with non-convex adjustment costs in both capital and labor. He shows that a model with imperfect information can explain the higher volatility of job flows observed among young plants. While most studies depend on aggregate shock to drive the business cycle, Carvalho and Grassi (2015) show that the standard firm dynamics model with no aggregate shock is able to generate sizable aggregate fluctuations once the traditional assumption of a continuum of firms is dropped.

One important development in the literature during the recent few years is the emphasis on firm life cycle. Since the influential work of Haltiwanger et al. (2013), researchers became

[3]A modified version of our model, explored in Appendix F, can be viewed as an extreme case of increasing marginal cost.

[4]The first version of the Cleveland Fed Working Paper was published in 2007.

[5]In Section 4 (especially, Figure 1), we show that when the entry cost is constant, entry fluctuations can serve as a propagation mechanism even when the model does not have capital stocks. The propagation is weakened when the entry cost is assumed to be cyclical.

aware that young firms have distinct labor-market dynamics compared to old firms. Sedláček (2015) documents that young firms, whose share in total employment is relatively small at only 16 percent, account for about 40 percent of aggregate employment fluctuations. His finding suggests that an exogenous drop in firm entry (as observed in the Great Recession) may slow down recovery and thus affect the long run growth by changing the firm age distribution over time. This is also consistent with our theoretical results of the stabilizing effect of entry subsidy. Sedláček and Sterk (2014) show that by shaping the composition of the cohorts, macroeconomic conditions in the year of birth have long-lasting effects on the aggregate employment fluctuations. In a study utilizing a different micro-level dataset from ours, Moreira (2015) finds that the business cycle affects the cohort of entrants over time: establishments born during recessions are not only smaller than those born during booms, but also remain smaller over long periods of time.

This new evidence strengthens the case for carefully analyzing the firm entry and exit. We show (in Section 6.1) that the life-cycle pattern at the establishment level is similar to firm-level evidence, documented in recent papers: young establishments behave very differently from old establishments. Thus, it is of interest to examine whether incorporating this property would change our main conclusions. In Section 6.2, we show that our main results are robust to adding the life-cycle considerations into the baseline model.

Following up on the conclusion of Lee and Mukoyama (2008) in terms of the importance of cyclicality of entry costs, a growing number of researchers have explicitly incorporated financial frictions into a general equilibrium firm dynamics model. Based on the empirical evidence in the confidential firm-level dataset from the of Bureau of Labor Statistics, Siemer (2014) builds a heterogeneous firm dynamics model and shows that a large financial shock may generate a persistent recession through a substantial decrease in firm entry. Zhang (2016) builds a model with firms with a borrowing constraint, and finds that the financial leverage and decentralized bargaining of wages is important in matching the job reallocation patterns in the data. In a related work, Zhang (2013) studies how the capital structure and financial frictions affect firms' entry and exit behavior over the cycle. Macnamara (2012) examines the importance of credit market conditions in explaining cyclical changes in entry and exit rates as well as in output and hours.

The paper is organized as follows. In the next section, we summarize the empirical finding of Lee and Mukoyama (2015a). In Section 3, we build a general equilibrium model of plant-level dynamics and calibrates it. In Section 4, we analyze the cyclical dynamics of the model. Section 5 explores the role of cyclical entry costs in the aggregate dynamics. Section 6 documents the life cycle pattern of establishments and extends our model to explicitly incorporate the plant life cycle. Section 7 concludes.

2 Empirical evidence on employment and productivity dynamics

In this section, we summarize the empirical findings of our companion paper (Lee and Mukoyama, 2015a). In that paper, we use the Annual Survey of Manufactures (from 1972 through 1997), which is constructed by the U.S. Census Bureau. See Lee and Mukoyama (2015a) for the details of the measurement of the entry and exit patterns.

To analyze the business cycle patterns, sample years are divided into two categories, good and bad, based on the growth rate of manufacturing output. If the growth rate of manufacturing output from year $t-1$ to t is above average, year t is called a good year; if it is below average, year t is called a bad year.[6] Table 1 exhibits the entry and exit rates of plants in good and bad years. The entry (exit) rate is measured by the number of entering (exiting) establishments as a percentage of the total number of establishments in each period. The entry rate is much higher during booms than during recessions. In contrast, exit rates are similar throughout the business cycle.[7]

[6]Good years are '72, '73, '76, '77, '78, '83, '87, '88, '92, '93, ('94,) '95, '96, and '97 and bad years are ('74,) '75, ('79,) '80, '81, '82, ('84,) '85, '86, ('89,) '90, and '91. The years in parentheses are not used because of the ASM panel rotation.

[7]Huynh et al. (2008) find a similar pattern in a firm-level dataset from Canada. Woo (2015) also finds that the establishment entry rate is strongly procyclical but the exit rate is acyclical in the Business Dynamics Statistics (BDS) for the sample period between 1980 and 2012. Broda and Weinstein (2010) find that the

Table 1: Entry and exit rates

	Good	Bad	Total average
Entry (birth)	8.1%	3.4%	6.2%
Exit (death)	5.8%	5.1%	5.5%

Note: Entry (exit) rate is measured by the number of entering (exiting) establishments as a percentage of the total number of establishments in each period. Source: Lee and Mukoyama (2015a)

Table 2: Average, relative size (employment), and relative size of continuing, entering, and exiting plants

	Good	Bad	Average
Average size, continuing	85.4	89.5	87.5
Average size, entering	45.1	59.2	50.3
Average size, exiting	34.9	35.9	35.3
Relative size, entering	0.53	0.70	0.60
Relative size, exiting	0.50	0.46	0.49
Relative productivity, entering	0.69	0.85	0.75
Relative productivity, exiting	0.65	0.65	0.65

Note: Each column represents the average of each variable during good years, bad years, and the entire period. The relative size is obtained by dividing the average size of entering (exiting) establishments by the average size of continuing establishments in the same four-digit SIC industry. Relative productivity of the entering (exiting) plant is obtained by dividing the productivity of the entering (exiting) plant by the average productivity of continuing plants in the same four-digit SIC industry. Source: Lee and Mukoyama (2015a)

The first three rows of Table 2 show the average size (employment) of continuing, entering, and exiting plants during booms and recessions. Overall, the average size is larger during recessions. Exiting plants are of a similar size across different phases of the business cycle, but the average size of entering plants dramatically changes. Entering plants in recessions start with about 30% more workers compared to entering plants in booms. The fourth and fifth rows report the size of entering and exiting plants, respectively, relative to continuing plants in the same four-digit SIC industry. The entering plants are about 25% larger in

procyclicality of product creation is quantitatively more pronounced than the countercyclicality of product destruction.

recessions than in booms.

The sixth and seventh row are the relative productivity of entering and exiting plants, respectively. Here, we consider the specification of the production function $y_t = s_t n_t^{\theta}$, where y_t by value added, s_t is productivity, and n_t is labour input. This is the specification that is used in the model presented in the next section.[8] Then s_t can be measured from the growth accounting equation

$$\ln(s_t) = \ln(y_t) - \theta \ln(n_t).$$

As in the case of the size, the relative productivity of exiting plants is similar over the business cycle, whereas the relative productivity of entering plants is substantially different during booms and recessions. In recessions, the relative productivity of entering plants is about 20% higher than that of entering plants in booms.

3 Model

In this section, we set up a dynamic general equilibrium model of plant employment, entry, and exit. We base our model on Hopenhayn and Rogerson (1993), departing from their model in four respects.

First, we add aggregate shocks to the economy. This ingredient is essential in analyzing the business cycle implications of the model. As in the standard real business cycle literature, we consider aggregate productivity shocks.

Second, we assume that there is a positive (and stochastic) value of exiting. This modification is necessary for the model to match the exit pattern observed in the data. In Hopenhayn and Rogerson's model (in their benchmark case), plants compare the value of staying with the value of exiting, which is zero. In their model, there is a threshold idiosyncratic productivity, s^*: when the productivity is higher than s^*, the plant stays; if the productivity is lower than s^*, the plant exits. Since productivity and employment have a one-to-one relationship without frictions and adjustment costs, it means that the exit rate is 100% for plants that

[8] The outcome is qualitatively similar with different specifications. See Lee and Mukoyama (2015a) for details.

are smaller than a certain threshold and is zero for plants that are larger than the threshold. The data do not exhibit this type of pattern: even large plants with more than 250 employees have an exit rate of over 1% (see Table 5 later). Furthermore, with Hopenhayn and Rogerson's formulation, an annual exit rate of 5.5% (see Table 1) implies that only very unproductive and small plants exit, which is at odds with the employment and productivity evidence in Table 2.[9]

Third, we consider entry in two steps—to enter, one first must pay some cost and come up with an 'idea.' Then, after observing the quality of the idea, one decides whether to pay an additional cost to actually enter the market. This 'two-step' process introduces the endogenous selection of the entering plants.[10] In the data, we observe that the productivity of entering plants is very different across booms and recessions, as we discussed in the previous section. In Hopenhayn and Rogerson's model, the entering plants receive a productivity draw after their decision to enter, so that the productivity distribution of entrants in the economy is always the same and is given exogenously. Our modification enables the productivity distribution of entrants to vary endogenously across booms and recessions.

Finally, we introduce the cost of adjusting employment. The estimation of the employment process by Cooper et al. (2004) strongly indicates that there are important adjustment costs in the employment process. It turns out that adding a moderate amount of adjustment cost dramatically changes the number of job reallocations.

3.1 Plants

The model consists of two kinds of entities: plants and consumers. Plants use labor to produce output. Consumers own plants, supply labor, and consume. There is only one type of good, which is used for entry costs and consumption; we use it as the numeraire. In our

[9] Samaniego (2006) is the first to point out this problem. Samaniego (2006, 2008) assumes a stochastic continuation value, rather than a stochastic exit value that we employ, to cope with this problem. We have also experimented with a model that assumes a stochastic continuation value. The results are essentially the same.

[10] Melitz (2003) employs a similar selection process in entry. Here, the interpretation of this process is slightly different from Melitz (2003).

model, the only price we have to keep track of is the wage of the workers. We assume that the plants have to pay adjustment costs and the firing tax when labor input is adjusted. The specifics of the adjustment costs and the firing tax are explained later.

Here, we describe the decision of the plants. First, we outline the behavior of the incumbent plants. Then we illustrate the entrant's behavior.

The timing of events for an incumbent plant at period t is as follows. In the beginning of the period, all plants observe the current aggregate state, z_t. An incumbent plant starts a period with the individual state (s_{t-1}, n_{t-1}). s_{t-1} is the individual plant's productivity level at period $t-1$. n_{t-1} is the employment level at period $t-1$. The value function of a plant at this stage is denoted as $W(s_{t-1}, n_{t-1}; z_t)$. Then, it observes its (stochastic) exit value, x_t. Here, x_t can be interpreted as the scrap value of its capital (and owned land), although we do not explicitly model capital stock or land.[11] After observing the exit value, the plant decides whether to stay or exit. If it exits, it must pay the firing tax, since it must adjust the employment level from n_{t-1} to zero. If it decides to stay, it observes this period's individual productivity (i.e., idiosyncratic shock), s_t. The value function at this point is denoted as $V^c(s_t, n_{t-1}; z_t)$. Then, it decides the amount of employment in the current period, n_t, and produces. The production function is $z_t f(n_t, s_t)$, where the function $f(n_t, s_t)$ is increasing and concave in n_t.[12] If $n_t \neq n_{t-1}$, it pays adjustment costs (and a firing tax, if $n_t < n_{t-1}$).[13] This concludes the period.

The timing for entrants is as follows. In the beginning of the period, everyone observes z_t. To enter, the first step is to come up with an idea. To come up with an idea, one must pay the cost c_q (we will refer to this as an 'idea cost') and receive a random number q_t (quality of the idea). A large q_t indicates that productivity after the entry is high. We call the people with ideas 'potential entrants.' We denote the expected value of having an idea, before knowing

[11] The entry cost that is introduced later can be interpreted as (partially sunk) investment in new capital and land.

[12] We abstract from the capital stock (besides the entry cost) in the production function. This abstraction makes the computation of the model easier, and this formulation is consistent with our measurement of the plant-level productivity.

[13] The details of the adjustment costs are explained later.

q_t, as $V^p(z_t)$. We denote the value of a potential entrant after paying c_q and receiving q_t as $V^e(q_t; z_t)$. Given q_t, a potential entrant decides whether to enter. To enter, an additional entry cost c_e (we will refer to this portion as an 'implementation cost') is paid. We interpret c_e as (partially sunk) investment in plants. A potential entrant compares $V^e(q_t; z_t)$ and c_e in deciding whether to enter. After the entry, the decision is the same as for the incumbent, except that the productivity s_t would depend on q_t instead of s_{t-1}. The plant observes s_t (the value function is $V^c(s_t, 0; z_t)$ for a new plant). Then, it decides the employment n_t, pays the adjustment costs, and produces.

An incumbent's value at the beginning of the period is described by the Bellman equation

$$W(s_{t-1}, n_{t-1}; z_t) = \int \max\langle E_s[V^c(s_t, n_{t-1}; z_t)|s_{t-1}], x_t - g(0, n_{t-1})\rangle d\xi(x_t).$$

Here, $g(n_t, n_{t-1})$ is the firing tax. In the $\max\langle\cdot,\cdot\rangle$, the plant compares the value of staying (the first term) and that of exiting (the second term). $E_s[\cdot|s_{t-1}]$ denotes the expectation regarding s_t, conditional on s_{t-1}. We assume that the exit value x_t follows an i.i.d. distribution $\xi(x_t)$[14] and that the exit value distribution does not vary over the business cycle. As we will see in Section 4, our model can match the exit pattern in the data without relying on the cyclical exit values. $E_s[V^c(s_t, n_{t-1}; z_t)|s_{t-1}]$ is the expected value of a continuing plant $V^c(s_t, n_{t-1}; z_t)$ and is calculated as

$$E_s[V^c(s_t, n_{t-1}; z_t)|s_{t-1}] = \int V^c(s_t, n_{t-1}; z_t) d\psi(s_t|s_{t-1}),$$

where

$$V^c(s_t, n_{t-1}; z_t) = \max\langle V^a(s_t, n_{t-1}; z_t), V^n(s_t, n_{t-1}; z_t)\rangle,$$

and $\psi(s_t|s_{t-1})$ is the distribution of s_t given s_{t-1}. Here, $V^a(s_t, n_{t-1}; z_t)$ is the value function when the plant adjusts employment, and $V^n(s_t, n_{t-1}; z_t)$ is the value function when it does not adjust employment.

[14]Formulating the exit decision using i.i.d. stochastic scrap values is popular in the empirical industrial organization literature—see, for example, Doraszelski and Pakes (2007) and Weintraub et al. (2008). Ramey and Shapiro (2001) analyze the resale prices of displaced capital. The resale prices vary substantially—see their Figure 2.

If the plant decides to adjust employment, the current period profit is

$$\pi^a(s_t, n_{t-1}, n_t; z_t) \equiv \lambda z_t f(n_t, s_t) - w_t n_t - g(n_t, n_{t-1}),$$

where $\lambda < 1$ represents the 'disruption cost' type of adjustment cost, emphasized by Cooper et al. (2004). This represents the cost of slowing down the production process when employment is adjusted. In Cooper et al.'s (2004) estimation, this cost turns out to be the most important type of adjustment cost in explaining employment dynamics observed at the plant level.

If the plant does not adjust employment, the current period profit is

$$\pi^n(s_t, n_{t-1}; z_t) \equiv z_t f(n_{t-1}, s_t) - w_t n_{t-1}.$$

Therefore,

$$V^a(s_t, n_{t-1}; z_t) = \max_{n_t} \pi^a(s_t, n_{t-1}, n_t; z_t) + \beta E_z[W(s_t, n_t; z_{t+1})|z_t],$$

and

$$V^n(s_t, n_{t-1}; z_t) = \pi^n(s_t, n_{t-1}; z_t) + \beta E_z[W(s_t, n_{t-1}; z_{t+1})|z_t].$$

Here, $E_z[\cdot|z_t]$ takes the expectation regarding z_{t+1}, conditional on z_t.

The entrant's value function is

$$V^e(q_t; z_t) = \int V^c(s_t, 0; z_t) d\eta(s_t|q_t),$$

where $\eta(s_t|q_t)$ is the distribution of s_t given q_t. Only the potential entrants with sufficiently high values of q_t would enter. There is a threshold value of q_t, q_t^*, which is determined by

$$V^e(q_t^*; z_t) = c_e. \tag{1}$$

A potential entrant would enter if and only if $q_t \geq q_t^*$. A potential entrant's value function is

$$V^p(z_t) = \int \max\langle V^e(q_t; z_t) - c_e, 0\rangle d\nu(q_t),$$

where $\nu(q_t)$ is the distribution of ideas. We impose a free-entry condition for becoming a potential entrant:

$$V^p(z_t) = c_q. \tag{2}$$

3.2 Consumers

The representative consumer maximizes the expected utility:

$$\mathbf{U} = E\left[\sum_{t=0}^{\infty} \beta^t [C_t + Av(1 - L_t)]\right],$$

where $v(\cdot)$ is an increasing and concave utility function for leisure, C_t is the consumption level, L_t is the employment level, $\beta \in (0, 1)$ is the discount factor, and A is a parameter. Here, for simplicity, we consider linear utility for consumption.[15] This simplification enables us to discount the firm's profit by the discount factor β. Since we consider the adjustment of L_t at the extensive margin, the appropriate interpretation of the $v(\cdot)$ function is that it is the result of an aggregation of many consumers who have different preferences over consumption and leisure. The budget constraint in each period is

$$C_t = w_t L_t + \Pi_t + R_t, \tag{3}$$

where w_t is the wage rate, Π_t is the firm's profit, and R_t is the transfer from the government. The government transfers the firing tax to the consumer in a lump-sum manner in every period. We assume that there is no saving. The first-order condition in each period is

$$Av'(1 - L_t) = w_t. \tag{4}$$

3.3 General equilibrium

Now, we analyze the general equilibrium of the model. The general equilibrium is defined as a situation where (i) consumers and firms (plants) optimize and (ii) the markets clear. For (ii), it is sufficient to ensure that the labor market clears.

First, consider a situation where z_t is constant. We use the solution of this steady-state situation for the purpose of calibration later. In this case, the definition of the stationary equilibrium is similar to Hopenhayn and Rogerson (1993). In our model, the general equilibrium can be summarized in the labor market. The free-entry condition (2) characterizes the

[15] Hopenhayn and Rogerson (1993) assume a period utility function that is concave in consumption and linear in leisure.

demand side of the labor market. The quantity of the labour demand in the steady state is given by

$$L^d = N \int \phi(s', n) d\mu(s', n), \tag{5}$$

where $\mu(s', n)$ is the stationary distribution of the plants with the state (s', n) when we assume that the mass of potential entry in each period is one. $\phi(s', n)$ is the labor demand for a plant with the state (s', n). (Here, s' is the plant-level productivity at the current period and n is the plant-level employment one period before.) N is the actual mass of potential entry at each period.

The consumer's first-order condition (4) characterizes the labor supply side. The labor demand side, in effect, determines the wage level at w^* (with the free-entry condition (2)). Combined with the labor-supply curve (4), the equilibrium level of labor, L^*, is determined. Once L^* is determined, the equilibrium level of N, N^*, is determined by (5).

When we introduce an aggregate shock, L^* and N^* move over time. The labor demand is now characterized by

$$L^d_t = L^d_{it} + N_t L^d_{et}, \tag{6}$$

where L^d_{it} is the labor demand from incumbents at period t and L^d_{et} is the labor demand from the entrant when the mass of potential entry is assumed to be one. The determination of the equilibrium is similar: the free-entry condition (2) determines the wage; the labor-supply equation (4) determines L; and the labor-demand equation (6) determines N.

Aggregate profit is given by

$$\Pi_t = Y_t - w_t L_t - R_t - N_t c_q - M_t c_e + X_t,$$

where Y_t is aggregate output, N_t is the number of potential entrants, M_t is the number of actual entrants, and X_t is the total value of exiting. Therefore, combining this with (3), in equilibrium (where labour demand equals labour supply)

$$C_t = Y_t - N_t c_q - M_t c_e + X_t \tag{7}$$

holds.

3.4 Calibration

Our strategy is to use the steady state of the model with constant z (we set $z = 1$) as the benchmark for calibration, and to add the aggregate shocks later on. A large part of our calibration is based on the statistics presented in Section 2. We set one period as one year. Following Hopenhayn and Rogerson (1993), we normalize the wage rate, w, in the benchmark to 1. As in Hopenhayn and Rogerson (1993), the model exhibits a homogeneity property in the sense that given prices, all of the aggregate variables (quantities) are proportional to the number of potential entrants, N. We pin down the benchmark value of N by setting aggregate employment, L, to 0.6 (approximate employment rate in the U.S.). The value of A is backed out from (4) and the fact that $w = 1$ and $L = 0.6$ in the benchmark. We set $\beta = 0.94$ and $\theta = 0.7$.

The process for idiosyncratic productivity, s, is chosen so that the model generates the employment process observed in the data. Lee and Mukoyama (2015b) estimate the employment process with the ASM dataset. First, the process is assumed to be

$$\ln(s') = a_s + \rho_s \ln(s) + \varepsilon_s, \tag{8}$$

where

$$\varepsilon_s \sim N(0, \sigma_s^2). \tag{9}$$

Then, this process is approximated by a Markov process by using Tauchen's (1986) method to obtain $\psi(s'|s)$. We set 30 evenly spaced grids on $\ln(s)$ over the interval $[a_s/(1-\rho_s) - 3\sqrt{\sigma_s^2/(1-\rho_s^2)}, a_s/(1-\rho_s) + 3\sqrt{\sigma_s^2/(1-\rho_s^2)}]$. The constant a_s is set so that the average value of employment matches the data. The value ρ_s is set to 0.97, which matches the autocorrelation parameter for the AR(1) process for employment (simulated in the model) to the empirical value of 0.97.[16] σ_s is set so that the variance of the growth rate of n is close

[16] In Lee and Mukoyama (2015b), it is shown that the parameter estimates for the employment AR(1) process can differ depending on which econometric method is employed. In the model, we also experimented with lower values of ρ_s. The problem with lower values of ρ_s is that it is impossible to replicate the steady-state distribution of plant size (in particular, there are too few large plants). One remedy for this would be to incorporate a plant-level fixed effect in s_t, reflecting the heterogeneity in the 'planned size' of the plants. We did not explore this direction due to computational complexity.

to the empirical value of 0.14. The resulting values are $a_s = 0.04$ and $\sigma_s = 0.11$.

The adjustment factor λ is set at 0.983, following Cooper et al. (2004).[17] As mentioned above, Cooper et al. (2004) show that this form of adjustment cost is empirically most relevant. In our model, including a large quadratic adjustment cost makes the size of entering plants unrealistically small, and a small quadratic adjustment cost does not alter the quantitative predictions along other dimensions. Thus, we include only the 'disruption cost' type of adjustment cost.

The exit value is assumed to be zero with probability x_0. With probability $(1 - x_0)$, the exit value is uniformly distributed over $[0, \bar{x}]$. We set x_0 and $\bar{x}$ so that the exit rate and the size of the exiting plants are similar to the empirical values. We choose $x_0 = 0.9$ and $\bar{x} = 2500$. As we see in Table 5 below, with this assumption, we obtain the model exit rates for different sizes of plants, which are closely matched to the exit rates in the data. We assume the entry transition function to be identical to the transition function for the incumbents: $\eta(s'|q) = \psi(s'|s)$. The entry costs, c_q and c_e, are backed out from the model. Given the value function $V^c(s', n)$, conditions (1) and (2) determine the values of c_q and c_e, given $\nu(q)$ and the equilibrium value of q^* that we target. We assume that $\nu(q)$ follows $\nu(q) = B\exp(-q)$ over the lower part of the grids on s (B is the scale parameter to make $\nu(q)$ sum up to one).[18] We select the value of c_e so that the target value of $\ln(q^*)$ is 0.5. As we see below in Table 5, this choice of $\nu(q)$ and q^* brings the size distribution of young plants close to the data. In the benchmark, we set the firing cost, $g(n', n)$, to zero. For the $v(\cdot)$ function, we use $\ln(\cdot)$. Table 3 lists the main parameter values for the benchmark case. Note that c_e and c_q are measured in annual wages because we normalized the wage so that $w = 1$ in equilibrium.

[17] This is their point estimate with a small quadratic adjustment cost. Their point estimate for λ with no other adjustment cost is 0.988. Although we do not have a quadratic adjustment cost in our model, we prefer the former number because it produces a more reasonable job reallocation rate.

[18] We set 200 grids on x and 25 grids on q. The results do not change when we increase the number of x grids to 1000 or when we use interpolation to approximate the continuous x distribution.

Table 3: Benchmark parameters

β	θ	a_s	ρ_s	σ_s	λ	c_e	c_q
0.94	0.7	0.04	0.97	0.11	0.983	872.9	103.1

Table 4: Data and model statistics in the steady-state

	Data	Model
Average size of continuing plants	87.5	87.6
Average size of entering plants	50.3	49.7
Average size of exiting plants	35.0	35.8
Entry rate	6.2%	5.4%
Exit rate	5.5%	5.4%
AR(1) coefficient ρ for employment	0.97	0.97
Variance of growth rate for n	0.14	0.14
Job reallocation rate	19.4%	23.0%

3.5 Steady-state results

First, we compute the model without aggregate shocks to establish the steady-state behavior of the model. The details of the computation of the steady-state model are described in Appendix A. Table 4 compares the output of our model to the data.[19] Everything except the job reallocation rate is our 'target' for calibration, and we can see that we are able to match the empirical values closely. The job reallocation rate is also close to the value in the data.[20]

Table 5 evaluates the distributional performance of the model. The first two columns summarize the distribution of plant size by the fraction of plants in each size class. Although there are a few cells that do not exhibit a perfect match with the data, overall the size distribution of the model matches the data quite well. The third and fourth columns describe the employment share by the size class. They also exhibit a good match of model statistics to the data.

[19] The job reallocation rate is taken from Davis et al. (1996, Table 2.1).

[20] If we assume that $\lambda = 1.0$, the job reallocation rate increases to over 30%.

Table 5: Size distribution of plants, data and model statistics

	Number		Employment		Number (Young)		Exit rate	
	Data	Model	Data	Model	Data	Model	Data	Model
1-19	0.457	0.478	0.049	0.038	0.712	0.612	0.080	0.067
20-49	0.239	0.187	0.090	0.064	0.156	0.151	0.039	0.062
50-99	0.131	0.107	0.109	0.077	0.068	0.079	0.031	0.056
100-249	0.106	0.155	0.194	0.284	0.044	0.117	0.025	0.039
250+	0.067	0.072	0.559	0.536	0.020	0.041	0.015	0.004

Note: The first two columns report the number of plants in each size class. Each cell is the fraction of the total number of plants (each column adds up to one). The third and fourth columns report employment share by size class. Each cell represents the share of employment for each size category in total employment (each column adds up to one). The fifth and sixth columns report the number of young plants within each size class. The last two columns report exit rates for each size class.

Because much of our focus is on entry and exit behavior, it is critical that the model's properties for entering and exiting plants match the data. The fifth and sixth columns show the distribution of young plants in the model and the data, which are reasonably similar.[21] The last two columns show that the exit rates of the model also exhibit a good match to the data.

4 Aggregate fluctuations: the entry selection puzzle

To analyze business cycles, we assume that z_t fluctuates between two values.[22] We assume that z_t takes either 1.01 or 0.99. This results in a 1% standard deviation in z_t.[23] z_t follows a symmetric Markov process. We calibrate the transition probabilities so that the average duration of each state is three years.[24]

[21]The definition of young plants follow Davis et al. (1996) for the data. In the model, young plants are defined as those that are 0, 1, or 2 years old.

[22]Appendix B shows that the results below are robust to having more than two possible values of z_t.

[23]Appendix B shows that the estimated unconditional standard deviation of $\ln(z_t)$ during our sample period (derived from the AR(1) regression of the Solow residuals) is about 1.1%.

[24]The average duration of the post-war (1945-2009) NBER contraction (peak to trough) is 11 months, and NBER expansion (trough to peak) is 59 months. Thus, overall, the average duration of each state is 35 months.

Table 6: Results with aggregate shocks

	Good	Bad
Wage	1.014	0.986
q^*	0.5000	0.5000
Entry rate	6.7%	4.0%
Exit rate	5.3%	5.4%
Average size of all plants	84.6	86.5
Relative size of entrants	0.57	0.57
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.85	0.85
Relative productivity of exiting plants	0.84	0.84

The computation turns out to be much simpler than for standard heterogeneous-agent models, such as Krusell and Smith (1998), since the wage depends only on z (this is because of the utility function that is linear in consumption and the free entry assumption).[25] From this property, we can perform the optimization by plants and determine $w(z)$ without considering the labor-market equilibrium. After $w(z)$ is determined, the labor-market equilibrium determines the equilibrium quantities, particularly the mass of entrants, N. The details of the computation are in Appendix C.

The results of the model with aggregate shocks are summarized in Table 6. Here, 'Good' corresponds to the periods with $z_t = 1.01$, and 'Bad' corresponds to the periods with $z_t = 0.99$.[26] First, notice that the wage fluctuates substantially. While the cyclicality of wages is empirically controversial, in Cooley and Prescott (1995) the wages are procyclical and have a standard deviation of less than 1% (see their Table 1.1).[27] In our model, a procyclical wage is necessary to make employment procyclical—in (4), for L to increase when A stays the same, we need w to increase. Somewhat surprisingly, in Table 6, the equilibrium value

[25]In the language of Menzio and Shi (2011), our model is block recursive. Similar to the case with Kaas and Kircher (2015), free entry of plants is key in obtaining this tractability.

[26]In Appendix D, we consider a different method of dividing good times and bad times—there, we categorize the good times as the times when output growth rates are more than 0.1%, and the bad times as the times when output growth rates are less than −0.1%. The following results are robust to this alternative categorization.

[27]Similarly, Gertler and Trigari (2009) report that the standard deviation of wages is about half of the standard deviation of the aggregate output.

of q^* does not change with the change in z. One reason for this result is that the effect of z and the effect of w offset each other. If z increases and w does not change, it is profitable for low-q plants to enter. Therefore, q^* decreases. However, since w increases, the profitability of entry decreases. It turns out that these two effects offset each other almost exactly. Since q^* is the same across the two states, the plants that enter are similar across different aggregate states. This is reflected in the similar relative sizes and relative productivities of entrants. This pattern is at odds with the data—as is shown in Section 2, the data show a strong cyclicality in the selection of the entrants.

In the model, exiting plants compare the value of staying with the value of exiting when making exit decisions. Since the distributions of the value of staying and the value of exiting are both considerably dispersed, a 1% difference in z does not make a large difference for this comparison.[28] Thus the exit rate and the size and productivity of exiting plants are similar throughout the business cycle in Table 6. This fits well with the pattern observed in the data.

The entry rate fluctuates significantly in Table 6, as we see in the data. The mechanism here is simple: since the wage increases during booms, the average size of incumbent plants shrinks (this is consistent with what we observe in the data: see Table 2). The labor that is released from the incumbents can be hired by the entrants. Simultaneously, the labor supply increases because of the wage increase. Therefore, the entry rate increases. Here, again, the procyclicality of wages plays an important role.

In summary, we find that the model is successful in matching patterns in the data in some respects. The two characteristics at odds with the data are the large fluctuations in wages and the lack of selection in entry. These two are (qualitatively) related in the sense that if the wage fluctuates less, we expect that some selection effect would emerge, although it turns out that this effect is quantitatively not very large.

Thus, it seems natural to consider a modification of the model that reduces the fluctua-

[28] If both values are concentrated around one value and there are many 'marginal' plants around that value, it is possible that these plants exit with a small change in z.

tions of the wages. To find out a way to moderate the fluctuation of wages, it is useful to first understand why wages are very volatile in the model. As is argued above, the equilibrium wage is determined from the free-entry condition (2), which equates the idea cost to the present value of the profit associated with coming up with an idea. If the idea cost stays the same, the present value of the profit must stay the same. Since an increase in z would increase the profit, w must increase to offset the increase in the profit.[29]

Therefore, it is possible to reduce the volatility of wages in the model by introducing another force that counteracts the change in the profit. In the above model, we assumed that the entry costs, c_q and c_e, are constant. If either c_q or c_e is cyclical, the fluctuation of wages can be smaller. In the following, we explain how changing the entry costs affects the aggregate performance of the model.[30]

5 A solution to the puzzle: cyclical entry costs

In this section, we explore the role of cyclical entry costs on the dynamics of entry. Below, we show that a particular combination of the cyclical entry costs can resolve the puzzle—the model generates an outcome that is consistent with both microeconomic and macroeconomic pattern of entry. We then show that the cyclical entry costs can be endogenized.

5.1 Exogenous fluctuations in entry costs

As a diagnostic step to resolving this puzzle, we first explore how various exogenous changes in entry costs can affect the aggregate dynamics of entry. Because we have two entry costs, c_q and c_e, we can consider various combinations of the cyclicality. From the external evidence,

[29] By using a simple static version of the model, Appendix E explains the intuition of why these two effects offset each other.

[30] An alternative modeling strategy is to remove the free-entry assumption. Veracierto (2001) and Samaniego (2006) consider a convex cost of producing new plants. Appendix F explores the opposite extreme of free entry along this line: the pool of potential entrants is fixed at each period. (A similar assumption is employed by Clementi and Palazzo (2016) who extend our model and incorporate capital stock.) This can be seen as the extreme case of convex cost for becoming a potential entrant—the cost is zero until a fixed amount $\bar{N}$, and infinity after $\bar{N}$. It turns out that, while this modification reduces the volatility of wages, this variant of the model is also incapable of generating a sufficient degree of selection over the business cycle.

Table 7: The case of a procyclical c_q

	Good	Bad
Wage	1.010	0.990
q^*	0.4997	0.5003
Entry rate	6.0%	4.7%
Exit rate	5.3%	5.4%
Average size of all plants	85.3	85.9
Relative size of entrants	0.57	0.57
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.85	0.85
Relative productivity of exiting plants	0.84	0.84

it is a natural first step to consider that c_q is procyclical—it is more expensive to conduct R&D and come up with ideas in booms. Then we proceed to allow c_e also to be cyclical so that a quantitatively strong selection is achieved.

Thus first, we consider a procyclical c_q. The interpretation of c_q is the cost for producing new ideas. In reality, c_q can be represented by the cost of R&D for innovation. Idea creation is a human capital intensive process. The cost of hiring a good inventor is particularly higher during booms, partly because the wages for these workers are higher then.[31] In this sense, a procyclical c_q can be considered as a stand-in for the labor market of R&D workers. In addition, there are more entries and idea creations during booms, and the idea creation process may suffer from decreasing returns (due to, for example, the 'fishing out' effect: over-exploitation of the existing ideas).

Table 7 shows the result. Here, c_q is 0.165% larger during booms and 0.165% smaller during recessions, targeting 1% cyclicality of the wages. We can see that this generates a *qualitatively* successful result. The selection goes in the right direction—q^* is countercyclical. It does not, however, generate a selection effect *quantitatively* large enough to match the data.

Next, we explore how we can generate a stronger selection effect. From the potential

[31] The National Science Foundation (NSF) collects various data on R&D expenditures and costs. On average, the cost per R&D scientist or engineer in companies performing R&D was about 8.6% higher during booms (good times) than during recessions (bad times).

entrant's entry condition (1), we know that c_e has a direct effect on the selection process—a large c_e makes the actual entry harder. Then, a countercyclical c_e would help to generate a larger selection effect.[32] In fact, empirical studies on investment costs suggest that c_e may move in a countercyclical direction. Recall that c_e can be thought of as the cost of actual entry—in particular the sunk investment in equipment and structures at entry. It is known that the price of investment goods tends to be lower during booms (see Fisher, 2006), and this evidence suggests that c_e may be lower during booms. In terms of the model, this can be treated as an exogenous shock to the value of c_e, which is negatively correlated with the variation in z. Furthermore, financial costs may depend on the aggregate state of the economy. If financing new plants is more difficult during recessions than during booms, higher financial costs will cause c_e to rise during these periods. Although we do not explicitly model the financial intermediation process, this may be an important factor when we consider the role of financial frictions in business cycle propagation.[33]

Together, the above intuitions suggest that if we combine a countercyclical c_e with a procyclical c_q (which would counteract the counterfactual wage effect of a countercyclical c_e—we target the wage to have 1% standard deviation), we may be able to generate a larger selection effect because both a countercyclical c_e and a procyclical c_q make the selection effect work in the right direction. Table 8 describes the result of an experiment where c_e is 0.8% higher during recessions and 0.8% lower during booms, and c_q is 3.3% lower during recessions and 4.1% higher during booms. This generates a large selection effect, and the differences in the relative size and productivity of entrants in booms and recessions are comparable to the data.

Figure 1 draws simulated sample paths of aggregate output. The solid line is the path for the current case: the values of c_e and c_q vary with the business cycle. The dotted line

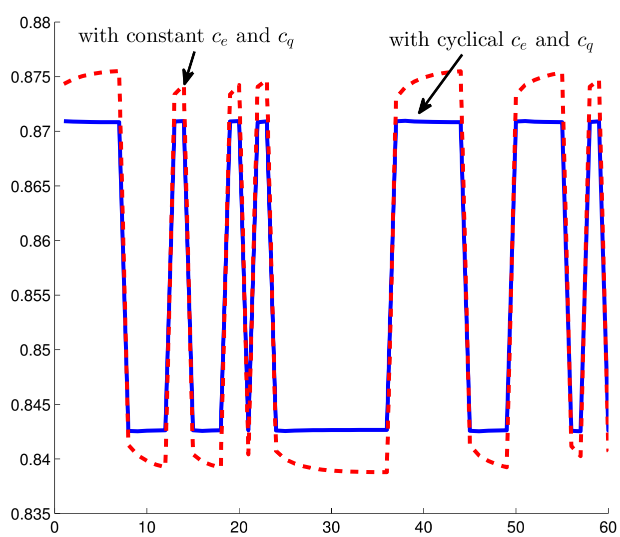
[32] We also experimented with procyclical c_e. Even though the fluctuation in w is smaller, the selection effect goes in the opposite direction: it is more difficult for low-productivity plants to enter during booms.

[33] See, for example, Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Kiyotaki and Moore (1997). Recently, a number of researchers have explicitly incorporated financial frictions into a firm dynamics model (See Siemer (2014), Zhang (2013, 2016), and Macnamara (2012)).

Table 8: The case of a countercyclical c_e and procyclical c_q

	Good	Bad
Wage	1.010	0.990
q^*	0.3047	0.6335
Entry rate	7.0%	3.9%
Exit rate	5.3%	5.5%
Average size of all plants	79.5	82.4
Relative size of entrants	0.48	0.69
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.78	0.94
Relative productivity of exiting plants	0.84	0.84

Figure 1: Sample paths of aggregate output



corresponds to the case with constant c_e and c_q, as in Table 6. One can see that most changes in output occur during periods where the aggregate state switches. The case with constant c_e and c_q exhibits significant propagation after the switch. Recall that the wage w is a function of only z, and the labor supply is a function of only wage. Therefore, L is constant for a given aggregate state z in both cases. The change in output thus comes only from the change in overall productivity. For a given z, the dynamics of productivity is dictated by the composition of the idiosyncratic productivity at different plants. The propagation with constant c_e and c_q mainly comes from an increase in the number of plants for a given L increasing average productivity; this is because a plant's production function is subject to decreasing returns to scale. Since the number of entrants is above average during a boom, this increases the number of total plants, and average productivity increases. There is an offsetting effect, however: the entering plant is less productive than incumbents. This effect is pronounced with business cycles in the case with cyclical c_e and c_q: entrants are very inefficient during a boom, and thus, the distribution of plants' productivity worsens as new plants are added during the boom. We can see that this effect almost offsets the effect of having more plants in the case of cyclical c_e and c_q, providing an almost flat profile of output for a given z.[34] This analysis underscores the importance of properly modeling entry in analyzing the propagation of aggregate shocks in an environment with heterogeneous firms.

5.2 Endogenizing cyclical entry costs

Now we briefly show that this particular combination of entry costs, procyclical c_q and countercyclical c_e, can be generated from an endogenous mechanism. As can be seen below, the assumptions we make are simple and straightforward; the detailed modeling of the endogenous mechanism is beyond the scope of this paper. Rather, the point of this section is to show that the main results above are not an artifact of the exogeneity of the cyclicality of the costs. We demonstrate this point by making a small departure from our baseline model

[34] However, from a closer look, we can see that there is a hump-shaped dynamics that is similar to Chang et al. (2002). See Figures 2 and 3 of Lee and Mukoyama (2008).

while maintaining tractability.

As we discussed earlier, there are many possible channels that make c_e cyclical. The primal culprits are cyclical prices of investment goods and cyclical financial frictions. Here, we do not rely on either of them, while we believe that these elements are also very important. A recent paper by Vardishvili (2018) shows that a model that allows the potential entrants to delay the timing of entry can generate significant fluctuations in entrants' selection. The reason is that having the option of delayed entry implies that the net benefit of immediate entry is small even for the potential entrants with very good ideas, and therefore their behavior is sensitive even to small shocks. Although her mechanism itself is too involved to be adopted here, below we incorporate her insight in a simple manner by assuming that c_e depends on idea quality and denote it $c_e(q)$. Recall that our interpretation of c_e is the capital cost of building a plant. In our context, it is sensible to think that a good idea, one that brings in higher productivity and profit in future, is more costly to implement—for example, firms may want to build a large plant to cash in from a good idea. Therefore, we assume that $c_e(q)$ is increasing in q. We use the following specification for $c_e(q)$:

$$c_e(q) = \gamma \bar{c}_e + (1-\gamma)\bar{V}^e(q),$$

where $\bar{V}^e(q)$ is the value function of the potential entrant in the economy without aggregate shocks, and $\bar{c}_e$ is defined as $\bar{c}_e \equiv \bar{V}^e(\bar{q}^*)$. Here, $\bar{q}^*$ is the threshold value of q for entry when there are no aggregate shocks, which is set at 0.5 in the previous section. This means that $c_e(0.5) = \bar{c}_e = \bar{V}^e(0.5)$.

We set $\gamma = 0.0015$. The parameter values in this section are meant to be illustrative rather than rigorously estimated values. Once again, the goal of this section is to show that the cyclical entry costs in the previous section can be endogenized; investigating the proper mechanism is left for future research. The parameters other than γ are set in the same manner in the baseline case. Note that, among the endogenously calibrated variables, only c_q changes from the baseline case. The value of c_q becomes lower, because of a higher value of $c_e(q)$ when q is larger than the threshold. The value of c_q is 0.15 in the current case.

Table 9: The case with q-dependent c_e and constant c_q

	Good	Bad
Wage	1.014	0.986
q^*	0.5006	0.5021
Entry rate	6.7%	4.0%
Exit rate	5.3%	5.4%
Average size of all plants	84.6	86.5
Relative size of entrants	0.57	0.57
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.85	0.85
Relative productivity of exiting plants	0.84	0.84

Table 9 summarizes the outcome when c_q is assumed as constant. It turns out that assuming that c_e is increasing in q is not sufficient in generating significant selection. The intuition is related to the logic in the last section. Because c_q is constant, the value of becoming a potential entrant must be invariant. This implies that in booms, the wage increase offsets the benefit of having a higher z. Because of this, $V_e(q; z)$ is almost invariant over the cycle. Because q^* is constant over the cycle, $c_e(q^*)$ is also constant.

Therefore, as in the last section, c_q also must vary over the cycle. We have argued that c_q is likely to be procyclical because the R&D costs, in large part the cost of R&D workers, are procyclcial. Given that the purpose of this section is to simply show that the cyclical costs can be endogenized, it is beyond the scope of our paper to explicitly incorporate the R&D sector to the model. Instead, we make a simple assumption that embodies this mechanism: c_q is increasing in the labor cost w.[35] In particular, we assume that

$$c_q(w) = \bar{c}_q w^\eta,$$

where $\bar{c}_q$ is the steady-state value of c_q (that is, when $w = 1$). The parameter η measures how responsive the R&D costs are to the wages. Table 10 displays the outcome for the case $\eta = 80$. This value of η is chosen so that the wage fluctuates by a similar amount as in

[35] An alternative assumption is that c_q depends on N. This formulation, however, is significantly less tractable for model computation.

Table 10: The case with q-dependent c_e and w-dependent c_q

	Good	Bad
Wage	1.010	0.989
q^*	0.2941	0.5406
Entry rate	7.0%	4.0%
Exit rate	5.4%	5.5%
Average size of all plants	76.3	78.9
Relative size of entrants	0.49	0.65
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.78	0.91
Relative productivity of exiting plants	0.84	0.84

the Table 15. The formulation for c_e is the same as in Table 9. It can be seen that this formulation resolves the puzzle: a significant difference in the selection of entrants over the business cycle is achieved. The entry costs are endogenously cyclical with this formulation: $c_q(w)$ moves procyclically because w is procyclical, and $c_e(q^*)$ moves countercyclically because q^* is countercyclical.

6 An extension: establishment life cycle

As is discussed in the Introduction, one important recent development in the firm dynamics literature is the emphasis on the firm life cycle. In particular, young firms tend to grow faster and have a higher likelihood to exit than old firms. In the following, we incorporate these features into the baseline model. A simple extension of the model can generate a reasonable fit for the observed life-cycle patterns in the data. It turns out that the business cycle properties generated by the baseline model are robust to this extension.

6.1 Life-cycle statistics

In this section, we describe the life-cycle pattern of establishment-level dynamics.[36] We use the Business Dynamics Statistics (BDS) from the Census Bureau, which is a public use

[36]We use the language of 'establishments' instead of 'plants' in this section, as 'plants' are usually reserved for manufacturing establishments, and here our dataset includes sectors other than manufacturing.

Table 11: Establishment-level life cycle statistics

age	JC rate	JD rate	exit rate	average size
1	1.47	1.81	2.11	0.64
2	1.07	1.60	1.61	0.69
3	0.95	1.41	1.39	0.74
4	0.88	1.33	1.25	0.78
5	0.83	1.26	1.17	0.82
6–10	0.73	1.11	0.95	0.90
11+	0.51	0.80	0.68	1.41
all	16.0%	14.5%	10.4%	17.2

Note: JC: job creation; JD: job destruction. Size refers to the number of employees. For each age group, the statistics are relative to those for all age groups together. Source: BDS.

version of the Longitudinal Business Database.[37] This is a different dataset from Lee and Mukoyama (2015a), on which Section 2 is based. Unlike in Section 2, here we need not examine the micro-level properties of establishments in detail (e.g. productivity), and it is sufficient to use aggregate information presented in the BDS. The advantages of using the BDS over the ASM are (i) it has a wider coverage in terms of sectors, age, and size, encompassing the entire private economy and (ii) the data covers a longer period of time (we use 1977–2013). The second advantage is particularly important here, as we would lose the significant initial part of the sample period in order to construct the statistics by age. In our case, the category for the oldest is 11 years and older, which allows us to use the data starting 1988.[38]

Table 11 summarizes the establishment-level summary statistics over the life cycle. The first two columns are job creation and job destruction rates by age, respectively, relative to

[37]See Haltiwanger et al. (2013), Sedláček and Sterk (2014), and the BDS website for the detailed description of the data (http://www.census.gov/ces/dataproducts/bds/index.html) for more details on BDS.

[38]The BDS provides more detailed age breakdown for older firms (e.g., 16 to 20, 21 to 25, and 26+) but only for more recent years. Because using more detailed age categories limits the sample years, we choose to collapse all older firms into a single category of 'age 11 and older' and use the sample starting 1988. This category also includes establishments in the 'Left Censored' category (i.e., establishments that existed before 1977). This choice of sample years naturally drops the initial two years, the quality of which was questioned in Moscarini and Postel-Vinay (2012). The results are similar when we expanded the age categories in more detail.

the rates of all plants reported in the last row. The third column is the exit rate by age, again relative to the rate for all. The fourth column is the average size in each age group, relative to the average size of the entire population. As in the case of the firm dynamics (Haltiwanger et al. 2013), the establishment-level dynamics exhibits a strong life cycle pattern. In particular, all rates are monotonically decreasing in age, and the average size is monotonically increasing with age. Our baseline model does not capture this pattern quantitatively, largely because the stochastic processes for the productivity and the outside options are assumed to be independent of age. In the next subsection, we extend the baseline model and show that it can accommodate these patterns.

6.2 Model

To explicitly incorporate establishment life cycle, we make the following changes to our baseline model. First, as in the Blanchard-Yaari overlapping generations model, we assume that an establishment ages stochastically. An establishment starts as a 'young' establishment, and every period it faces a probability μ of becoming 'old.' Second, we assume that the stochastic process for the productivity is different between young and old establishments. While we continue assuming that the stochastic process follows (8) and (9), we allow σ_s to be different across the young and the old. Third, we assume that a young establishment faces a *failure risk*: the probability δ of exogenous exit. Below we show that this extension, with three additional parameters (μ, δ, and two values for σ_s instead of one) can successfully replicate the patterns exhibited in Table 11.

We assume that the transition probability μ is 0.5. The value of δ is set so that the exit rate of establishment with age 1 is similar to the data. This results in $\delta = 0.06$. The relative value of σ_s for young establishments compared to the old establishments is determined so that the age 1 job creation rate is similar to the data. We set σ_s for young as 1.9 times larger than σ_s for old. The target for old establishments' σ_s is the same as that in the baseline model: the variance of establishment growth rate being 0.14. This leads to $\sigma_s = 0.105$ for old establishments. Several other parameters are adjusted so that the model outcome hits

Table 12: Data and model statistics in the steady-state

	Data	Model
Average size of continuing plants	87.5	87.6
Average size of entering plants	50.3	51.6
Average size of exiting plants	35.0	35.3
Entry rate	6.2%	5.7%
Exit rate	5.5%	5.7%
AR(1) coefficient ρ for employment	0.97	0.97
Variance of growth rate for n	0.14	0.14
Job reallocation rate	19.4%	23.6%

Table 13: Model statistics

age	JC rate	JD rate	exit rate	average size
1	1.44	1.60	2.09	0.66
2	1.11	1.35	1.53	0.70
3	0.89	1.16	1.24	0.73
4	0.78	1.07	1.10	0.75
5	0.73	1.02	1.03	0.77
6–10	0.69	0.98	0.97	0.83
11+	0.66	0.97	0.88	1.20

Note: JC: job creation; JD: job destruction. For each age, the entries are relative to all age groups.

the same targets as in the baseline model. The upper bound for the outside options, $\bar{x}$, is set at 2000 so that the average size of exiting establishments is similar to the data. The value of a_s is adjusted so that the average size of all establishments is comparable to the data. Table 12 corresponds to Table 4 for the baseline model. The model outcome here is similar to the baseline model.

Table 13 shows the model statistics that correspond to Table 11 in the data. We can see that the model captures the pattern we described in Section 6.1. In particular, all rates (job creation rate, job destruction rate, and the exit rate) are decreasing with age, and the average size is increasing in age.

In the following, we conduct the same business cycle exercises as in the baseline model.

Table 14: Results with aggregate shocks

	Good	Bad
Wage	1.014	0.986
q^*	0.5000	0.5000
Entry rate	7.1%	4.4%
Exit rate	5.8%	5.7%
Average size of all plants	84.7	86.5
Relative size of entrants	0.58	0.59
Relative size of exiting plants	0.40	0.40
Relative productivity of entrants	0.84	0.84
Relative productivity of exiting plants	0.73	0.73

Table 15: The case of a countercyclical c_e and procyclical c_q

	Good	Bad
Wage	1.010	0.990
q^*	0.3714	0.5971
Entry rate	7.1%	4.5%
Exit rate	5.8%	5.7%
Average size of all plants	82.3	84.6
Relative size of entrants	0.53	0.67
Relative size of exiting plants	0.40	0.40
Relative productivity of entrants	0.79	0.90
Relative productivity of exiting plants	0.73	0.73

Table 14 corresponds to Table 6 in Section 3. We find that the overall results remain the same as in the baseline case. While the business cycle responses of entry rate and exit rate are in line with the data, the selection threshold q^* does not vary over the cycle, and therefore, the micro-level properties of entrants do not vary over the cycle.

To replicate the cyclical changes in the size and productivity entrants, we allow the entry costs to be cyclical in a similar way to the baseline case. As in Section 3, we increase c_e by 0.8% in recessions and decrease it by 0.8% in booms. To target the same magnitude of fluctuations in wages as in Table 8, we decrease c_q by 2.93% in recessions and increase it by 2.63% in booms. Table 15 shows that the resulting patterns over the business cycle are very similar to the baseline case in Section 3 (Table 8). With cyclical c_e and c_q, our model can match the micro-level properties of entrants as well as the macro-level behavior of entry and exit rates. We conclude that our baseline results in Section 3 are robust to incorporating the life cycle of the plants. Once again, our model underscores the importance of understanding the cyclical changes in the environment for new business entry. It is an important future research agenda to investigate why the cost for entry varies over the business cycle.

7 Conclusion

This paper analyzed the business-cycle implications of plant-level dynamics, particularly the entry and exit behavior of plants. We constructed a general equilibrium model of plant dynamics by extending Hopenhanyn and Rogerson's (1993) model. Our model reveals that a simple extension, with aggregate productivity shocks, can account for the pattern of entry and exit in terms of the aggregate rates; it fails, however, in achieving the micro-level fact that there are considerable differences in the selection of entrants over the business cycle. We found that a countercyclical implementation cost and a procyclical idea cost can achieve this micro-level fact. These model properties remain the same when we extend the model to incorporate the plant life cycle.

We have also shown that the cyclical entry costs can be endogenized. Our assumptions are still very primitive and capture only one channel for each cost. An important research topic for the future is to further uncover the nature of these costs theoretically (by modeling the microeconomic foundations of these costs) and empirically (by looking into the microeconomic process of entry).[39] Some recent papers mentioned in Introduction have already made some progress in this direction.

In this paper, we employed a stationary model and abstracted from the secular productivity growth. Foster et al. (2001) show that a significant part of the productivity growth in the U.S. manufacturing sector comes from entry and exit of plants. As a promising fu-

[39] Explicitly modeling the limited enforceability of contracts, as in Cooley et al. (2004), is one possible direction.

ture project, extending our analysis to incorporate secular productivity growth might open up a new possible link between aggregate fluctuations and aggregate growth, as in Barlevy (2004).[40]

[40] One possible direction is to extend an R&D-based growth model that incorporates a realistic firm dynamics, such as Mukoyama and Osotimehin (2016), to allow for aggregate fluctuations.

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Appendix

A Computation of the steady state

This section outlines the computation of the model without aggregate shocks. We omit the notation on z since it is constant here.

1. Set discrete grids on n and s. Set the Markov transition matrix for s. Set the distribution of the exit value x.

2. Optimization loop. Objects: $W(s,n)$, $V^a(s',n)$, $V^n(s',n)$,$V^c(s',n)$, $Z(s,n)$, $\phi^a(s',n)$, $\phi(s',n)$, $\zeta(s',n)$, and $\chi(s,n)$. (These functions are defined in the following.)

   (a) Give the initial value for $W(s,n)$, where s and n are the realizations in the last period. This is the beginning-of-period value for an incumbent.

   (b) Calculate $V^a(s',n)$ and $V^n(s',n)$ by

   $$V^a(s',n) = \max_{n'} \pi^a(s',n,n') + \beta W(s',n')$$

   and

   $$V^n(s',n) = \pi^n(s',n) + \beta W(s',n),$$

   where

   $$\pi^a(s',n,n') = \lambda z f(n',s') - wn' - g(n',n)$$

   and

   $$\pi^n(s',n) = z f(n,s') - wn.$$

   Record the decision rule of n' when adjusted: $\phi^a(s',n)$.

   (c) Calculate $V^c(s',n)$ by

   $$V^c(s',n) = \max\langle V^a(s',n), V^n(s',n)\rangle.$$

   Record the decision rule. $\zeta(s',n) = 1$ if adjust, and $\zeta(s',n) = 0$ if not. $\phi(s',n) = \phi^a(s',n)$ if $\zeta(s',n) = 1$ and $\phi(s',n) = n$ if $\zeta(s',n) = 0$.

(d) Calculate $Z(s,n) = E_{s'}[V^c(s',n)|s]$ by

$$Z(s,n) = \int V^c(s',n) d\psi(s'|s).$$

(e) Calculate $W(s,n)$ by

$$W(s,n) = \int \max\langle Z(s,n), x - g(0,n)\rangle d\xi(x).$$

Thus, the ratio of plants that exit with the state (s,n) is

$$\chi(s,n) = \int_{Z(s,n)+g(0,n)}^{\infty} d\xi(x).$$

(f) Update and repeat.

3. Now we can calculate

$$V^e(q) = \int V^c(s',0) d\eta(s'|q)$$

for each q.

We can set the cut-off for q, q^*, and find c_e by

$$V^e(q^*) = c_e. \tag{10}$$

Then we find V^p by

$$V^p = \int \max\langle V^e(q) - c_e, 0\rangle d\nu(q)$$

and find c_q by the free-entry condition:

$$V_p = c_q. \tag{11}$$

4. Calculate the stationary measure of plants (survivors from the last period plus this period's entrant, after receiving this period's shock), $\mu(s',n)$, given $N = 1$. From linear homogeneity, the actual measure of survivors will be $N\mu$.

Note that M is a function of N:

$$M = N \int_{q^*}^{\infty} d\nu(q).$$

5. Obtain N by solving

$$Av'(1 - L(N)) = w. \tag{12}$$

In the benchmark, we choose A so that $L = 0.6$ when $w = 1$. Thus, when $v(x) = \ln(x)$,

$$A = w - wL(N) = 1 - 1 \times 0.6 = 0.4.$$

Here, $L(N)$ is

$$L(N) = N \int \phi(s', n) d\mu(s', n)$$

so that N is calculated by

$$N = \frac{0.6}{\int \phi(s', n) d\mu(s', n)}.$$

The total output can be calculated by:

$$Y(N) = N \int [zf(\phi(s', n), s') - (1 - \lambda)\zeta(s', n)zf(\phi(s', n), s')] d\mu(s', n).$$

The total exit value $X(N)$ is calculated by

$$X(N) = N \int \int_{Z(s',\phi(s',n))+g(0,\phi(s',n))}^{\infty} x d\xi(x) d\mu(s', n).$$

B Results with more than two points of z

In the main text, we assumed that the aggregate productivity z_t follows a two-point Markov chain. In this appendix, we check the robustness of the results in terms of this assumption. Our strategy is to estimate an AR(1) process for Solow residual as in the standard Real Business Cycles literature, and we approximate it by a 10-point Markov chain using Tauchen's (1986) method.

The AR(1) regression is

$$\ln(z_{t+1}) = a_0 + a_1 t + \rho \ln(z_t) + \varepsilon_{t+1},$$

where $\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$. z_t here is computed from the growth accounting equation in annual frequency

$$\ln(z_t) = \ln(Y_t) - 0.7 \ln(L_t),$$

Table 16: Results with aggregate shocks

	Good	Bad
Wage	1.015	0.986
q^*	0.5000	0.5000
Entry rate	6.1%	4.6%
Exit rate	5.3%	5.4%
Average size of all plants	84.2	86.8
Relative size of entrants	0.57	0.57
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.85	0.85
Relative productivity of exiting plants	0.84	0.84

where Y_t is the real GDP, L_t is the index of aggregate weekly hours for production and nonsupervisory employees in total private industries. When the data from $t+1 = 1966$ to $t+1 = 2010$ is used, the point estimate for ρ is calculated as $\hat{\rho} = 0.754$, and the point estimate for σ_ε is calculated as $\hat{\sigma}_\varepsilon = 0.0094$.[41] We put 10 equally-spaced grids on $\ln(z_t)$ over the interval $[-1.5\hat{\sigma}_\varepsilon/\sqrt{1-\hat{\rho}^2}, 1.5\hat{\sigma}_\varepsilon/\sqrt{1-\hat{\rho}^2}]$. Then we construct the Markov transition matrix following Tauchen (1986).

Table 16 is the benchmark result (that is, when c_e and c_q are constant). Here, "Good" time is the period t in which z_t is above average, and "Bad" time is the period t in which z_t is below average. The results are very similar (also in the experiment later) when we define "Good" time as the period t in which the output increased by more than 0.1% from $t-1$ to t and define "Bad" time as the period t in which the output decreased by more (in absolute value) than 0.1% from $t-1$ to t. The message is the same as the body of the paper: the model successfully replicates the patterns of the entry rate and the exit rate but fails to generate any fluctuations in the selection of the entrant plants.

Table 17 is the outcome with countercyclical c_e (1.4% smaller than the steady-state value when z_t is the largest and 1.4% larger than the steady-state value when z_t is the smallest) and

[41]This implies that implied unconditional standard deviation of $\ln(z)$, $\sigma_\varepsilon/\sqrt{1-\hat{\rho}^2}$, is 0.014. When the data from $t+1 = 1972$ to $t+1 = 1997$ is used, the corresponding value is 0.011. These are in line with the value of the standard deviation of the process used in the body of the paper (0.01).

Table 17: The case of a countercyclical c_e and procyclical c_q

	Good	Bad
Wage	1.010	0.992
q^*	0.3287	0.6082
Entry rate	6.1%	4.1%
Exit rate	5.5%	5.5%
Average size of all plants	74.8	81.2
Relative size of entrants	0.54	0.69
Relative size of exiting plants	0.40	0.41
Relative productivity of entrants	0.82	0.94
Relative productivity of exiting plants	0.83	0.84

procyclical c_q (7% larger than the steady-state value when z_t is the largest and 7% smaller than the steady-state value when z_t is the smallest). c_e and c_q move together with z_t, and the grids for c_e and c_q are placed with the equal spacing. Again, the message is the same as the body of the paper: with countercyclical c_e and procyclical c_q, the model can replicate the behavior of the plant entry and exit in the data, including the size and productivity patterns of the entrants over the business cycle.

C Computation of the model with aggregate shocks

1. Set discrete grids on n and s. Set the Markov transition matrix for s. Set the distribution of the exit value x.

2. Guess w as a function of z.

3. Optimization loop. Objects: $W(s,n;z)$, $V^a(s',n;z)$, $V^n(s',n;z)$,$V^c(s',n;z)$, $Z(s,n;z)$, $\phi^a(s',n;z)$, $\phi(s',n;z)$, $\zeta(s',n;z)$, and $\chi(s,n;z)$.

   (a) Give the initial value for $W(s,n;z)$, where s and n are the realizations in the last period. This is the beginning-of-period value for an incumbent.

(b) Calculate $V^a(s', n; z)$ and $V^n(s', n; z)$ by

$$V^a(s', n; z) = \max_{n'} \pi^a(s', n, n'; z) + \beta E_{z'}[W(s', n'; z')|z]$$

and

$$V^n(s', n; z) = \pi^n(s', n; z) + \beta E_{z'}[W(s', n; z')|z],$$

where

$$\pi^a(s', n, n'; z) = \lambda f(n', s', z) - w(z)n' - g(n', n)$$

and

$$\pi^n(s', n; w, z) = f(n, s', z) - w(z)n.$$

Record the decision rule of n' when adjusted: $\phi^a(s', n; z)$.

(c) Calculate $V^c(s', n; z)$ by

$$V^c(s', n; z) = \max\langle V^a(s', n; z), V^n(s', n; z)\rangle.$$

Record the decision rule. $\zeta(s', n; z) = 1$ if adjust, and $\zeta(s', n; z) = 0$ if not. $\phi(s', n; z) = \phi^a(s', n; z)$ if $\zeta(s', n; z) = 1$ and $\phi(s', n; z) = n$ if $\zeta(s', n; z) = 0$.

(d) Calculate $Z(s, n; z) = E_{s'}[V^c(s', n; z)|s]$ by

$$Z(s, n; z) = \int V^c(s', n; z) d\psi(s'|s).$$

(e) Calculate $W(s, n; z)$ by

$$W(s, n; z) = \int \max\langle Z(s, n; z), x - g(0, n)\rangle d\xi(x).$$

Thus, the ratio of plants that exit with the state $(s, n; z)$ is

$$\chi(s, n; z) = \int_{Z(s,n;z)+g(0,n)}^{\infty} d\xi(x).$$

(f) Update and repeat.

4. Now we can calculate

$$V^e(q;z) = \int V^c(s',0;z)d\eta(s'|q)$$

for each q.

We can set the cut-off for q, $q^*(z)$, and find c_e by $V^e(q^*;z) = c_e(z)$.

Then we find $V^p(z)$ by

$$V^p(z) = \int \max\langle V^e(q;z) - c_e(z), 0\rangle d\nu(q).$$

5. Check if $V_p(z) = c_q$ is satisfied. If not, revise $w(z)$. Repeat until convergence.

6. Simulation.

   (a) Let $\delta_t(s,n)$ be the total measure of incumbents at the beginning of period t, before entry and exit.

   (b) After observing z, incumbents decide whether to exit. The measure of survivors is

   $$\sigma_t(s,n;z) = (1 - \chi(s,n;z))\,\delta_t(s,n).$$

   The total measure of survivors, with new shocks, is

   $$\vartheta_t(s',n;z) = \int \psi(s'|s)\sigma_t(ds,n;z).$$

   (c) The measure of entrants for unit N (after observing z) is

   $$\varrho_t(s';z) = \int_{q^*(z)}^{\infty} \eta(s'|q)d\nu(q)$$

   (d) Thus the total measure is

   $$\mu_t(s',n;z) = N\varrho_t(s';z) + \vartheta_t(s',n;z).$$

   (e) We must solve for N in order to actually calculate the total measure. The labor market equilibrium condition is (from the representative consumer's first-order condition)

   $$w(z) = A\frac{1}{1-L}.$$

L is the sum of the incumbent's employment L^i, which can be calculated from $\theta_t(s', n; z)$, and the entrant's employment, NL^e, where L^e can be calculated from $\varrho_t(s'; z)$. Therefore,

$$N = \frac{(w(z) - A)/w(z) - L^i}{L^e}.$$

(f) The beginning-of-next-period measure can be found by

$$\delta_{t+1}(s', n') = \int_{n'=\phi(s',n;z)} \mu_t(s', dn; z).$$

(g) Repeat.

D Results for significant positive and negative growth states

In this section (Tables 18, 19, and 20), we report the results of the experiments originally reported in Tables 6 to 8, when we use a different categorization of good and bad times. Here, we categorize the good times as the times when output growth rates are more than 0.1%, and the bad times as the times when output growth rates are less than −0.1%.[42] This is closer to the spirit of the categorization in the data, which is based on the output growth rate. The main properties of the results are unaltered from the main text, although the difference between good and bad states is more pronounced quantitatively.

E A static model

This section builds a simple static model to develop intuitions for why the aggregate productivity shock does not cause a significant response in the selection margin. Consider the following static model of two-step plant entry. Each plant has production function $y = zsn^\theta$, where y is output, z is aggregate productivity (common to all firms), s is the idiosyncratic productivity, n is employment, and $\theta \in (0, 1)$ is a parameter. As in the main text, entry proceeds in two steps: first, upon observing z, one can pay $c_q > 0$ to receive an idea q (which is a random value following the distribution $\nu(q)$) and can become a potential entrant. Idea is

[42] We do not set zero as a threshold, so that we do not capture the small movements of output.

Table 18: Results with aggregate shocks

	Good	Bad
Wage	1.014	0.986
q^*	0.5000	0.5000
Entry rate	8.9%	1.7%
Exit rate	5.2%	5.6%
Average size of all plants	84.8	86.2
Relative size of entrants	0.56	0.58
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.85	0.85
Relative productivity of exiting plants	0.84	0.84

Table 19: The case of a procyclical c_q

	Good	Bad
Wage	1.010	0.990
q^*	0.4997	0.5003
Entry rate	7.4%	3.3%
Exit rate	5.2%	5.5%
Average size of all plants	85.2	86.0
Relative size of entrants	0.57	0.57
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.85	0.85
Relative productivity of exiting plants	0.84	0.84

positively linked to future productivity, so a potential entrant with a higher q has a stronger incentive to enter. In particular, an entrant with q larger than a threshold value q^* enters. Here, for simplicity, assume that $q = s$. A potential entrant can pay $c_e > 0$ to enter.

Normalizing the output price to 1 and denoting the wage to be w, a plant with productivity s solves

$$\max_n \quad zsn^\theta - wn. \tag{13}$$

Let the resulting profit be $\pi(z, s, w)$. The idea threshold q^* is determined by the condition

$$c_e = \pi(z, q^*, w). \tag{14}$$

Table 20: The case of a countercyclical c_e and procyclical c_q

	Good	Bad
Wage	1.010	0.990
q^*	0.3047	0.6335
Entry rate	8.8%	2.6%
Exit rate	5.2%	5.6%
Average size of all plants	80.4	80.8
Relative size of entrants	0.47	0.71
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.77	0.95
Relative productivity of exiting plants	0.84	0.84

The free-entry condition

$$c_q = \int_{q^*}^{\infty} (\pi(z, q, w) - c_e) d\nu(q) \tag{15}$$

determines the equilibrium wage w. In this environment, one can show the following.

Proposition 1 *When z increases by 1%, the equilibrium w increases by $1/\theta$%, and the equilibrium q^* is unchanged.*

Proof. Suppose that z goes up by 1%. We conjecture that in the new equilibrium w increases by $1/\theta$%. Then we verify that all equilibrium conditions (14) and (15) are satisfied with the same value of q^*.

Consider the maximization problem (13). The solution is

$$\pi(z, s, w) = (1 - \theta)\theta^{\frac{\theta}{1-\theta}} (zs)^{\frac{1}{1-\theta}} w^{-\frac{\theta}{1-\theta}}.$$

Thus, for a given s, if z goes up by 1% and w goes up by $1/\theta$%, π remains constant. Then, (14) and (15) hold with the same value of q^*. ■

The intuition is that, for the free-entry condition (15) to hold, w has to increase sufficiently in order to counteract the effect of z on the expected profit upon entry. This exactly offsets two forces that influence q^*: an increase in z would decrease q^* while an increase in w would increase q^*.

In the model of the main text, there are many more elements so that the effect of w does not offset the effect of the aggregate shock perfectly, but the quantitative outcome turned out to be very close to the result in this static model. Thus, the result here is useful for understanding the outcome in the model of the main text as well.

F An alternative entry assumption: fixed pool of potential entrants

Here, we consider a model with alternative entry process. In this model, we assume that the number of potential entrants in each period is fixed at a level $\bar{N}$. That is, in every period there are a fixed number of potential entrants, each with its own q_t. Because the distribution of q_t is unchanged, entry and selection are inversely related—when q_t^* is high, there are more stringent selections into entry (the average productivity of entrants is high), and the entry rate is low. When q_t^* is low, the average productivity of entrants is low and the entry rate is high.

This model is more difficult to analyze when compared to the model in Section 3. (Hereafter, we call the model with constant entry costs in Section 3 "the basic model.") This is because the wage w_t is now simultaneously determined with L_t in labor market equilibrium. Here, we cannot separate the determination of w_t and q^* from the determination of equilibrium quantities (which we could do in Section 3). This, in turn, implies that the plants which are making dynamic decisions have to predict what will happen to the future equilibrium prices and quantities. Because the future equilibrium quantities depend on the distribution of plants in the current period, the entire distribution of the plants enters into the set of relevant information for the plant's decision problem. We utilize a variant of Krusell and Smith's (1998) method to solve for the general equilibrium of this model.

F.1 Model

The model primitives are almost identical to the basic model. The only difference is the entry process. Instead of free entry with a fixed payment c_q, here we assume that there are a fixed number of potential entrants with an idea every period. In the beginning of each period, these potential entrants decide whether to enter by paying c_e. All other timings and aspects are the same as in the basic model.

The consumer's optimization problem is identical to the basic model. Equation (4) characterizes the labor supply.

The plant side of the model becomes entirely different as a consequence of the modification in the entry process. In particular, because the wage is now determined by the labor market equilibrium, to predict the future wage, each plant owner must predict where the labor demand curve lies. Because the future labor demand depends on the current productivity distribution of plants, the plant owner must incorporate more information when making decisions in the current period.

An incumbent plant's value at the beginning of the period is described by the Bellman equation (the firing tax is omitted here)

$$W(s_{t-1}, n_{t-1}; z_t, \Omega_{t-1}) = \int \max\langle E_s[V^c(s_t, n_{t-1}; z_t, \Omega_{t-1})|s_{t-1}], x_t\rangle d\xi(x_t).$$

Ω_{t-1} is the information that the plant owner possesses at the end of period $t-1$ and includes the distribution of plants over productivity and employment in period $t-1$. The first term on the right hand side is calculated as

$$E_s[V^c(s_t, n_{t-1}; z_t, \Omega_{t-1})|s_{t-1}] = \int V^c(s_t, n_{t-1}; z_t, \Omega_{t-1}) d\psi(s_t|s_{t-1}),$$

where

$$V^c(s_t, n_{t-1}; z_t, \Omega_{t-1}) = \max\langle V^a(s_t; z_t, \Omega_{t-1}), V^n(s_t, n_{t-1}; z_t, \Omega_{t-1})\rangle.$$

The current period profit of a plant for which employment is adjusted is

$$\pi^a(s_t, n_t; z_t, L_t) \equiv \lambda z_t f(n_t, s_t) - w(L_t)n_t.$$

Note that now we denote the wage w_t as $w(L_t)$, using the relationship (4).

If employment is not adjusted, the current period profit is

$$\pi^n(s_t, n_{t-1}; z_t, L_t) \equiv z_t f(n_{t-1}, s_t) - w(L_t)n_{t-1}.$$

Therefore, the value functions are

$$V^a(s_t; z_t, \Omega_{t-1}) = \max_{n_t} \pi^a(s_t, n_t; z_t, L_t) + \beta E_z[W(s_t, n_t; z_{t+1}, \Omega_t)|z_t],$$

and

$$V^n(s_t, n_{t-1}; z_t, \Omega_{t-1}) = \pi^n(s_t, n_{t-1}; z_t, L_t) + \beta E_z[W(s_t, n_{t-1}; z_{t+1}, \Omega_t)|z_t].$$

Here, $E_z[\cdot|z_t]$ takes the expectation regarding z_{t+1}, conditional on z_t. Ω_t evolves following the law of motion: $\Omega_t = \Gamma(z_t, \Omega_{t-1})$. Employment L_t is the sum of n_t over all plants; therefore, it is a part of Ω_t.

The entrant's value function is

$$V^e(q_t; z_t, \Omega_{t-1}) = \int V^c(s_t, 0; z_t, \Omega_{t-1}) d\eta(s_t|q_t).$$

As in Model 1, only potential entrants with sufficiently high q_t actually enter. There is a threshold value of q_t, q_t^*, which is determined by

$$V^e(q_t^*; z_t, \Omega_{t-1}) = c_e.$$

A potential entrant will enter if and only if $q_t \geq q_t^*$. The total mass of potential entrants is fixed at $\bar{N}$.

In equilibrium, the combination (w_t, L_t) must clear the labor market. The labor supply is given by (4), and the labor demand is given by (6) with $N_t = \bar{N}$. The equilibrium (w_t, L_t) is determined by these two equations. Once (w_t, L_t) is given, the values of the other variables can be determined in the same manner as in the basic model. The only difference is that, in (7), $N_t c_q$ does not exist. This difference only affects the value of consumption.

F.2 Computation

The computation of this model is potentially much more complex than the basic model, since the optimization (potentially) involves many more state variables. To overcome this difficulty, we follow Krusell and Smith (1998) in using limited information instead of the entire state variables to perform optimization and then checking whether the "forecast" using this limited information is accurate by simulation. In particular, what is necessary for optimization is to forecast the value of L_t. In the following, we omit the time subscript and represent the $t-1$ variable by subscript -1 and $t+1$ variable by a prime ($'$).

1. Set discrete grids on n and s. Set the Markov transition matrices for s and z. Set the distribution of the exit value x.

2. Guess the "prediction rule" for L. After some experimentation, we found that the following formulation works:

$$\log(L) = a_0 + a_1 \log(L_{-1}) + a_2 \log(z) + a_3 \log(z) \times \log(L_{-1}) + a_4 \mathcal{I}(z \neq z_{-1}). \quad (16)$$

   $\mathcal{I}(\cdot)$ is an indicator function, which takes the value 1 if the statement in the parenthesis is true and 0 if it is false. By adopting this formulation, we are reducing the aggregate state variable from (z, Ω_{-1}) to (z, z_{-1}, L_{-1}). Let $\Upsilon \equiv (z, z_{-1}, L_{-1})$. Make a guess on the coefficients $(a_0, a_1, a_2, a_3, a_4)$.

3. Optimization loop. Objects: $W(s, n; \Upsilon)$, $V^a(s'; \Upsilon)$, $V^n(s', n; \Upsilon)$,$V^c(s', n; \Upsilon)$, $Z(s, n; \Upsilon)$, $\phi^a(s'; \Upsilon)$, $\phi(s', n; \Upsilon)$, $\zeta(s', n; \Upsilon)$, and $\chi(s, n; \Upsilon)$.

   (a) Give the initial value for $W(s, n; \Upsilon)$, where s and n are the realizations in the last period. This is the beginning-of-period value for an incumbent.

   (b) Calculate $V^a(s'; \Upsilon)$ and $V^n(s', n; \Upsilon)$ by

$$V^a(s'; \Upsilon) = \max_{n'} \pi^a(s', n'; \Upsilon) + \beta E_{z'}[W(s', n'; \Upsilon')|\Upsilon]$$

and

$$V^n(s', n; \Upsilon) = \pi^n(s', n; \Upsilon) + \beta E_{z'}[W(s', n; \Upsilon')|\Upsilon],$$

where

$$\pi^a(s', n'; \Upsilon) = \lambda f(n', s', z) - w(L)n'$$

and

$$\pi^n(s', n; w, \Upsilon) = f(n, s', z) - w(L)n,$$

Where $w(L)$ function is from (4), and L is calculate by the forecasting rule (16). Record the decision rule of n' when adjusted: $\phi^a(s'; \Upsilon)$.

(c) Calculate $V^c(s', n; \Upsilon)$ by

$$V^c(s', n; \Upsilon) = \max\langle V^a(s'; \Upsilon), V^n(s', n; \Upsilon)\rangle.$$

Record the decision rule. $\zeta(s', n; \Upsilon) = 1$ if adjusted, and $\zeta(s', n; \Upsilon) = 0$ if not. $\phi(s', n; \Upsilon) = \phi^a(s'; \Upsilon)$ if $\zeta(s', n; \Upsilon) = 1$ and $\phi(s', n; \Upsilon) = n$ if $\zeta(s', n; \Upsilon) = 0$.

(d) Calculate $Z(s, n; \Upsilon) = E_{s'}[V^c(s', n; \Upsilon)|s]$ by

$$Z(s, n; \Upsilon) = \int V^c(s', n; \Upsilon) d\psi(s'|s).$$

(e) Calculate $W(s, n; \Upsilon)$ by

$$W(s, n; \Upsilon) = \int \max\langle Z(s, n; \Upsilon), x\rangle d\xi(x).$$

Thus, the ratio of plants that exit with the state $(s, n; \Upsilon)$ is

$$\chi(s, n; \Upsilon) = \int_{Z(s,n;\Upsilon)}^{\infty} d\xi(x).$$

(f) Update and repeat.

4. Now we can calculate

$$V^e(q; \Upsilon) = \int V^c(s', 0; \Upsilon) d\eta(s'|q)$$

for each q.

We can calculate the cut-off for q, $q^*(\Upsilon)$ by $V^e(q^*(\Upsilon); \Upsilon) = c_e$.

5. Simulation.

   (a) Let $\delta_t(s,n)$ be the total measure of incumbents at the beginning of period t, before entry and exit. Give initial conditions.

   (b) Draw z randomly. After observing z, incumbents decide whether to exit. The measure of survivors is

$$\sigma_t(s,n;\Upsilon) = (1-\chi(s,n;\Upsilon))\,\delta_t(s,n).$$

   The total measure of survivors, with new shocks, is

$$\vartheta_t(s',n;\Upsilon) = \int \psi(s'|s)\sigma_t(ds,n;\Upsilon).$$

   (c) The measure of entrants for unit N (after observing z) is

$$\varrho_t(s';\Upsilon) = \int_{q^*(\Upsilon)}^{\infty} \eta(s'|q)d\nu(q)$$

   (d) Thus the total measure is

$$\mu_t(s',n;\Upsilon) = \bar{N}\varrho_t(s';\Upsilon) + \vartheta_t(s',n;\Upsilon).$$

   (e) L is the sum of the incumbent's employment L^i, which can be calculated from $\theta_t(s',n;\Upsilon)$, and the entrant's employment, $\bar{N}L^e$, where L^e can be calculated from $\varrho_t(s';\Upsilon)$.

   The wage can be solved from

$$w(L) = A\frac{1}{1-L}.$$

   (f) The beginning-of-next-period measure can be found by

$$\delta_{t+1}(s',n') = \int_{n'=\phi(s',n;\Upsilon)} \mu_t(s',dn;\Upsilon).$$

   (g) Repeat.

Table 21: Results with aggregate shocks

	Good	Bad
Wage	1.004	0.996
q^*	0.4994	0.5004
Entry rate	5.4%	5.4%
Exit rate	5.4%	5.4%
Average size of all plants	85.7	85.2
Relative size of entrants	0.57	0.57
Relative size of exiting plants	0.41	0.41
Relative productivity of entrants	0.85	0.85
Relative productivity of exiting plants	0.84	0.84

6. From simulation, we have the time series data of z_t and L_t. Check if the initial guess on the coefficients of (16) is correct. If not, adjust these coefficients and go back to Step 3.

The resulting forecast rule is

$$\log(L) = -0.31 + 0.39\log(L_{-1}) - 3.44\log(z) - 7.19\log(z) \times \log(L_{-1}) + 0.06\mathcal{I}(z \neq z_{-1}),$$

and

$$R^2 = 0.968.$$

Thus, this forecasting rule is fairly accurate.

F.3 Results

Here, we use the same steady-state model as in the basic model, and treat the value of N as fixed ($\bar{N}$ in the current model) instead of treating it as endogenously arising from free entry. The only necessary modification is to remove the resource cost c_q. As is mentioned earlier, this affects only the value of consumption in (7).

The results are summarized in Table 21. There are four notable differences from the basic model. First, the wage fluctuates much less than in the benchmark outcome in the main body of the paper (with constant c_e and c_q). This magnitude is in line with the data. Second, there are some fluctuations in q^*. In particular, q^* is larger during the recession—that is, there is more selection of entrants during recessions. Third, the fluctuations in entry rates are very small. Fourth, average plant size is now procyclical.

The wage fluctuates less because the wage determination mechanism is different. Now, the labor demand function is downward-sloping, because there is only a limited number of potential entrants that can enter the market. In considering the shift in the labor demand curve, now the labor demand from all plants (not only entrants) matters. As a result, the equilibrium wage does not fluctuate as much as in the basic model. Because the wage does not fluctuate significantly, there is less offsetting force against the selection of entrants. This generates a more cyclical q^* in this model. The magnitude of the selection, however, is too small to produce the fluctuations in the size and productivity of entrants that we see in the data. In fact, the relative size and productivity of entrants are still very similar across good times and bad times.

The small fluctuations in wage have a direct consequence on the average size of the plants. Now, the direct effect of z on the plant size is stronger than the effect of the wage, and the average plant size is procyclical. This, in turn, reduces the quantity of labor that is released for the entrants. As a result, the fluctuations in the entry rate are very small.

In this modified model, the exit rate and the characteristics of exiting plants still remain acyclical. This feature of the model is robust to the change in the entry process.