Firm Dynamics and the Macroeconomy: Basics

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From today, we focus on L

- If the capital is fixed,

$$A_i(K_i^{\alpha}L_i^{1-\alpha})^{\gamma} = B_i L_i^{\beta}$$

where

$$B_i = A_i K_i^{\alpha\gamma}$$

and

$$\beta = \gamma(1-\alpha) \in (0,1).$$

From today, we focus on L

- If the rental market for capital is perfectly competitive,

$$\max_{K_i} \; A_i(K_i^{\alpha} L_i^{1-\alpha})^{\gamma} - rK_i$$

imply

$$K_i = \left(\frac{\alpha\gamma}{r}\right)^{\frac{1}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}} L_i^{\frac{\gamma-\alpha\gamma}{1-\alpha\gamma}}$$

Plugging this solution into the production function

$$A_i(K_i^{\alpha} L_i^{1-\alpha})^{\gamma} = \left(\frac{\alpha\gamma}{r}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}} L_i^{\frac{\gamma-\alpha\gamma}{1-\alpha\gamma}}$$

Thus we can write a new production function

$$B_i L_i^{\beta}$$

where

$$B_i = \left(\frac{\alpha\gamma}{r}\right)^{\frac{\alpha\gamma}{1-\alpha\gamma}} A_i^{\frac{1}{1-\alpha\gamma}}$$

and

$$\beta = \frac{\gamma - \alpha\gamma}{1-\alpha\gamma} \in (0,1)$$

Misallocation

First, continuing with the discussion in the last class, let us talk about the misallocation. An example:

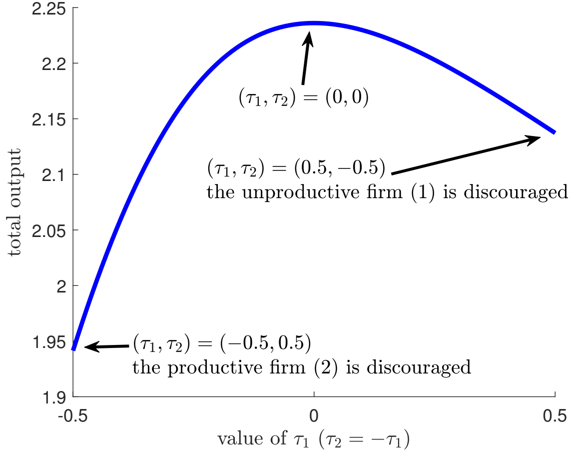
- There are two firms, firm 1: $Y_1 = A_1 L_1^\alpha$ and firm 2: $Y_2 = A_2 L_2^\alpha$ where $\alpha \in (0,1)$.
- Let $A_1 = 1$ and $A_2 = 2$. $\alpha = 1/2$.
- Assume that there are firm-specific distortions τ_i $(i = 1, 2)$. We can think of τ_i as a tax.
- Firm i maximizes the profit

  $$(1 - \tau_i) A_i L_i^\alpha - w L_i.$$

  Because the actual output is $A_i L_i^\alpha$, the firm's decision problem is distorted.
- Assume that the total labor is fixed at 1. Thus w is deterimined by $L_1 + L_2 = 1$.
- The total output is computed as

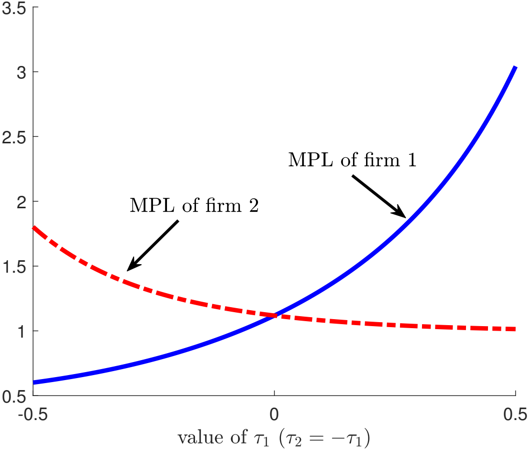
  $$Y = Y_1 + Y_2 = A_1 L_1^\alpha + A_2 L_2^\alpha$$

Misallocation



- The total output is reduced the most with positive correlation between the distortion (discouragement) and productivity (Restuccia and Rogerson, 2008)

Misallocation



- MPL ($\alpha A_i L_i^{\alpha-1}$) dispersion is the source of the productivity loss.

Situations where misallocations can occur: Examples

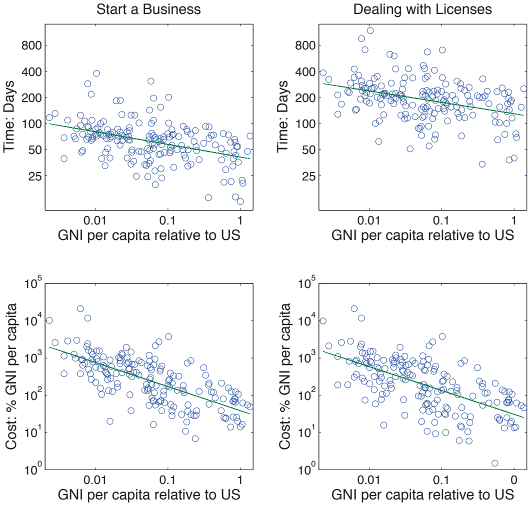
- Tax rates are different depending on firm identity/characteristics
- Regulations that depend on firm size
- Firing/hiring taxes/subsidies
- Entry/exit taxes/subsidies, some other frictions
- Financial frictions
- Contract enforcement

Notes:

- The cost of reduced entry depends on the post-entry importance of entrants.
- The importance of financial frictions depends on the persistence of shocks (whether the firm can overcome the friction by self-financing); see Moll (2014). The shock does seem to be persistent; see Lee and Mukoyama (2015). There still can be effects for young firms and potential entrants.
- The importance of contract enforcement at the industry level positively correlates with industry productivity (Mukoyama and Popov, 2020)

Entry barriers

From Moscoso Boedo and Mukoyama (2012)



- Why? Political economy considerations (Mukoyama and Popov, 2014)

Misallocation as a theory of TFP

- Misallocation can change the measured TFP (measured by Y/L^{α}, for example) without changing A_1 and A_2.
- The effect can be sizable but not as much as 10-folds differences between rich and poor countries.
- For the development questions, the determination of A_i (growth of productivity at the firm level) is still important.

Firm growth

On firm growth

Two small points first:

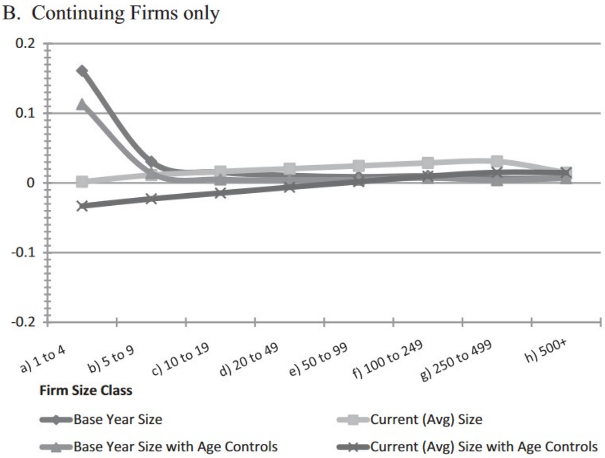
- First, note that individual firm growth is not necessary or sufficient for aggregate growth.
- Second, the loss from missing entry can be large if we take firm growth into account.

An example:

- Labor supply is elastic (employment is demand-determined). One firm hires one worker.
- The production of a firm who enters at time τ and age a (today is $t = a + \tau$) is $A_\tau e^{\gamma a}$. $\gamma > 0$ is the firm growth rate. Assume that $A_\tau = A_0 e^{g\tau}$.
- The surviving firms at age a is $e^{-\delta a}$. Assume $\delta > \gamma$.
- The mass of entrants is 1.
- Outcome: The total employment is $\int_0^\infty e^{-\delta a} da = 1/\delta$. The aggregate production is $A_0 e^{gt}/(\delta + g - \gamma)$.
- If Δ units of entrants are lost, the immediate loss is $\Delta A_\tau dt$ but the present value of loss is $\Delta A_\tau/(\rho + \delta - \gamma)$, where ρ is the discount rate.

Some facts on firm growth

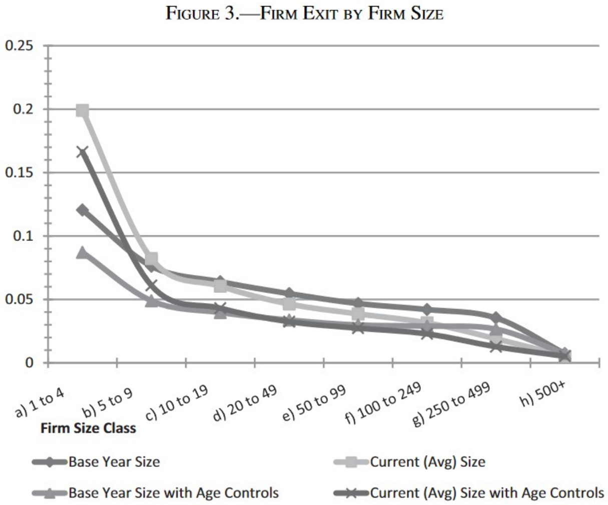
Figures from Haltiwanger et al. (2013)



- The growth rate of a firm is independent of size: "Gibrat's Law" (mixed supports in the data)

Some facts on firm growth

Figures from Haltiwanger et al. (2013)



Some facts on firm growth

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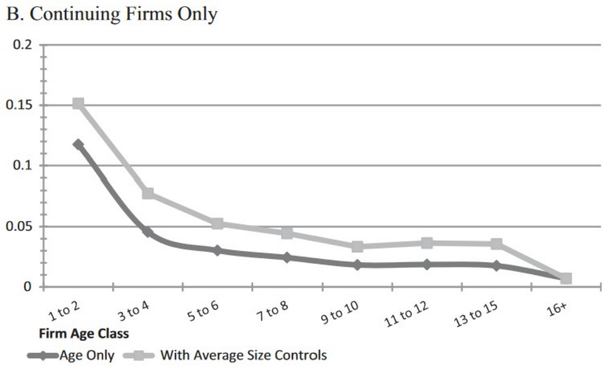
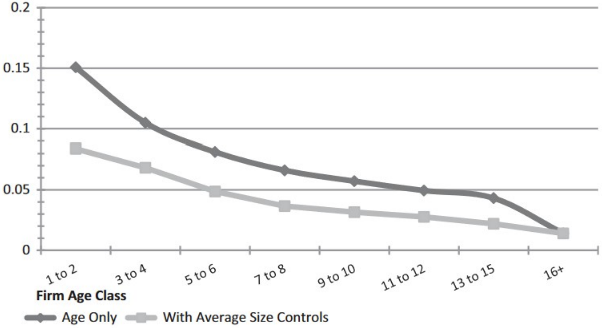


Some facts on firm growth

Figures from Haltiwanger et al. (2013)

FIGURE 5.—FIRM EXIT BY FIRM AGE



Gibrat's law implies a Pareto tail

Let us go back to the previous example, but with growth in terms of size.

- A firm is born with size $A > 0$.
- It grows at the rate γ.
- It exits at the rate δ.
- At the stationary distribution, for any size $x > A$, the density $s(x)$ has to satisfy

  $$s(xe^{\gamma dt})\Delta e^{\gamma dt} = e^{-\delta dt} s(x)\Delta$$

  for small dt and Δ.
- Guess that the distribution is Pareto: $s(x) = Fx^{-(\kappa+1)}$ (for $x \geq A$), where $F > 0$ and κ is the shape parameter. Then

  $$F(xe^{\gamma dt})^{-(\kappa+1)} e^{\gamma dt} = e^{-\delta dt} F x^{-(\kappa+1)}$$

  and therefore

  $$\kappa = \frac{\delta}{\gamma}.$$

  A large γ or a small δ implies a small κ (thick tail).

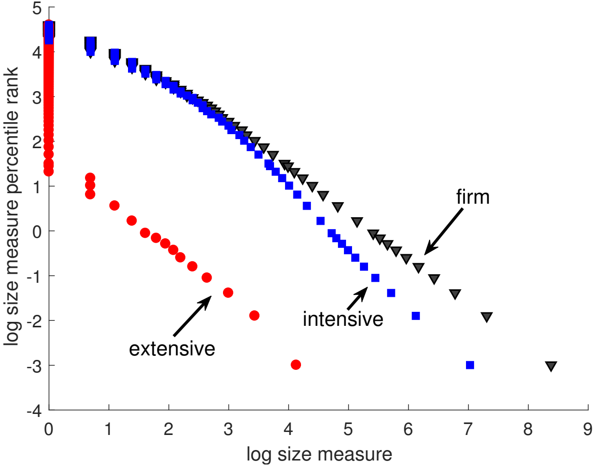
Gibrat's law implies a Pareto tail

$$s(xe^{\gamma dt})\Delta e^{\gamma dt} = e^{-\delta dt}s(x)\Delta$$

The distributions in the US, from QCEW

From Cao et al. (2020):

- Intensive margin: average employment per establishment in each firm
- Extensive margin: number of establishments in each firm



US versus Japan

Tables from Mukoyama (2009)

Table 2 Entry and Exit Rates

Annual, percent

	United States	Japan
Entry rate	11.6	4.4
Exit rate	10.2	4.4

Table 4 Establishments in the United States and Japan: Average Sizes

	United States	Japan
Average size of all establishments	17.6	9.4
Average size of opening establishments	8.3	9.6
Average size of closing establishments	9.0	7.9

- Low average size in Japan (despite a large entrant size) with low exit rates: lack of growth in establishments.

A simplified version of Mukoyama and Osotimehin (2019)

One way of looking at this paper is Hopenhayn and Rogerson (1993) with endogenous productivity shocks (and growth).

- Representative consumer:

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t [\log(C_t) - \xi L_t],$$

  where $\beta \in (0, 1)$ and $\xi > 0$.

- Final good (used for consumption and R&D):

$$Y_t = \left(\int_{\mathcal{N}_t} {q_{jt}}^{\psi} {y_{jt}}^{1-\psi} dj \right)^{\frac{1}{1-\psi}}.$$

- Quality q_{jt} can be improved by incumbent intermediate producer's innovation or entrants' creative destruction ("quality ladders").
- Aggregate quality (productivity) index: $Q_t \equiv \bar{q}_t^{\frac{\psi}{1-\psi}}$.

Intermediate-good firms (monopolistic competition)

- Good j only produced by the cutting-edge producer (monopoly).
- Produced only by labor.

$$y_{jt} = \ell_{jt}.$$

- Exit if entrant innovates on product j or if hit by an exogenous shock δ.
- Firing costs
  - Tax for each worker fired τw.
  - The tax is transferred lump-sum to the consumer.
- Incumbents can innovate on their own products.
- Entrants innovate randomly across different products.
- Innovation is stochastic.

$$q_{jt} = \begin{cases} (1+\lambda_i) q_{j,t-1} & \text{if innovates} \\ q_{j,t-1} & \text{if does not innovate} \end{cases}$$

where $i = I, E$.

Quality ladders

Model: innovation

- Incumbents
  - Improves the quality of its own product by R&D (in final goods).
  - Probability of successful innovation: x_{Ijt}
  - Innovation cost:

$$\mathbf{r}_{Ijt} = \theta_I Q_t \frac{q_{jt}}{\bar{q}_t} {x_{Ijt}}^{\gamma}.$$

- Entrants
  - First pay the entry cost ϕQ_t (become a potential entrant) and then conduct R&D
  - Probability of successful innovation: x_{Et}
  - Innovation cost:

$$\mathbf{r}_{Et} = \theta_E Q_t {x_{Et}}^{\gamma}.$$

  - Cost is increasing in the aggregate productivity Q_t.
  - Creative destruction rate: $\mu = m x_E$, where m is the number of potential entrants.

Model: firm's problem

$$
\begin{aligned}
& V_t(q_t, \ell_{t-1}) \\
& \quad = \max_{\ell_t, x_{It}} \quad \Pi_t(q_t, \ell_{t-1}, \ell_t, x_{It}) \\
& \qquad + \frac{1}{1+r} \Big\{ (1-\mu_t) \left[(1-x_{It}) Z_{t+1}(q_t, \ell_t) \right. \\
& \qquad\qquad \left. + x_{It} Z_{t+1}((1+\lambda_I) q_t, \ell_t) \right] \\
& \qquad\qquad - \mu_t \tau w_{t+1} \ell_t \Big\}
\end{aligned}
$$

where

$$
Z_t(q_t, \ell_{t-1}) = (1-\delta) V_t(q_t, \ell_{t-1}) - \delta \tau w_t \ell_{t-1}
$$

and

$$
\begin{aligned}
& \Pi_t(q_t, \ell_{t-1}, \ell_t, x_{It}) \\
& \qquad = (p_t - w_t)\ell_t - \theta_I Q_t \frac{q_t}{\bar{q}_t} {x_{It}}^{\gamma} - \tau w_t \max\langle 0, \ell_{t-1} - \ell_t \rangle,
\end{aligned}
$$

with $p_t = q_t^{\psi} {y_{jt}}^{-\psi} {Y_t}^{\psi}$.

Model: entry

- Free-entry condition

$$\max_{x_{E_t}} \left\{ -\theta_E Q_t {x_{Et}}^{\gamma} - \phi Q_t + \frac{1}{1+r} x_{Et} \bar{V}_{E,t+1} \right\} = 0,$$

- $\bar{V}_E$ is the expected value of entry
- Creative destruction rate $\mu = m x_E$

Model: solving for the stationary equilibrium

1. Normalized model: $\hat{Y} \equiv Y_t/Q_t$, $\hat{w} \equiv w_t/Q_t$, $\hat{q} \equiv q_t/\bar{q}_t$,....
2. Given g_q, μ, $\hat{Y}$, $\hat{w}$
   - Compute value functions and decision functions
   - Stationary distribution of firms over $\hat{q}$, α and ℓ_{-1}
3. Stationary GE conditions: find g_q, μ, $\hat{Y}$, $\hat{w}$ such that
   (i) $\hat{Y}$ consistent with firms' output decision
   (ii) $\hat{\bar{V}}_E$ satisfies the free entry condition
   (iii) $\hat{Y} = \hat{C} + \hat{R}$
   (iv) $\frac{1}{N}\int\int \hat{q} f(\hat{q}, \ell) d\ell dq = 1$

The steps are very similar to the standard firm dynamics model (Hopenhayn and Rogerson, 1993) and the standard heterogeneous-agent models (Bewley-Huggett-Aiyagari)

Model: stationarized problem

$$\hat{V}(\hat{q},\ell) = \max_{\ell' \geq 0, x_I} \hat{\Pi}(\hat{q},\ell,\ell',x_I) + \beta\left((1-\mu)\hat{S}(x_I,\hat{q}/(1+g_q),\ell') - \mu\tau\hat{w}\ell'\right),$$

where

$$\hat{S}(x_I,\hat{q}/(1+g_q),\ell') = (1-x_I\hat{Z}(\hat{q}/(1+g_q),\ell') + x_I\hat{Z}((1+\lambda)\hat{q}/(1+g_q),\ell').$$

$$\hat{\Pi}(q,\alpha,\ell,\ell',x_I) = (\hat{q}^{\psi}\ell'^{-\psi}\hat{Y}^{\psi} - \hat{w})\ell' - \theta_I\hat{q}{x_I}^{\gamma} - \tau\hat{w}\max\langle 0,\ell-\ell'\rangle.$$

- The frictionless case can be solved analytically. (Next slide)
- For the case with $\tau > 0$, there is one more step to computation: rewrite ℓ as the deviation from the frictionless level of ℓ (which can be computed from static optimization). This step is important because ℓ can have a very long tail.

Model: frictionless benchmark

- The value function $\hat{Z}$ is linear in productivity q.
- The innovation decision is independent of q ($\rightarrow$ Gibrat's law)
- Note: the normalized productivity next period is $(1+\lambda_I)\hat{q}/(1+g_q)$ with successful innovation, but without innovation $\hat{q}/(1+g_q) < \hat{q}$. $\rightarrow$ the firm has to contract if it does not innovate.
- Right tail of the productivity distribution is Pareto

$$F(\hat{q} > u) \propto u^{-\kappa},$$

  where κ is the solution to:

$$1 = (1-\delta)\left[(\mu + (1-\mu)x_I)\gamma_i^\kappa + (1-\mu-(1-\mu)x_I)\gamma_n^\kappa\right].$$

- Growth rate of $\bar{q}$:

$$1 + g_q = (1-\delta)[(1+\lambda_I x_I)(1-\mu) + (1+\lambda_E)\mu] + \delta(1+\lambda_E)\bar{q}^h$$

Results: the effect of a higher firing tax

Innovation rate of entrants is lower.

- period profit (tax payment/distortion/wages) $(-)$

Innovation rate of incumbents can be higher or lower.

- period profit (tax payment/distortion/wages) $(-)$
- creative destruction effect (lower μ) $(+)$
- tax-escaping effect (escape tax payment by innovating) $(+)$

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