Occupational Reallocation Within and Across Firms: Implications for Labor-Market Polarization*

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Abstract

This study analyzes how labor-market frictions interact with firms' decisions to reallocate workers across different occupations during labor-market polarization. We compare the patterns of occupational reallocation within and across firms in the US and Germany in recent years. We find that within-firm reallocation contributes significantly to the decline in employment in routine occupations in Germany, but much less in the US. We construct a general equilibrium model of firm dynamics and find that the model with different firing taxes can replicate the difference in firm-level adjustment patterns across these countries. We conduct two counterfactual experiments for each country, highlighting the different roles played by the within-firm cost of reorganizing occupational mix and across-firm frictions created by firing taxes. The results suggest that the latter plays a more significant role in labor market polarization. Higher firing cost leads to greater and faster polarization in the US.

Keywords: occupational reallocation, firing costs, labor-market polarization

JEL Classifications: E24, J24, J62

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1 Introduction

In recent years, many advanced economies have experienced significant declines in employment in middle-skilled routine occupations. This phenomenon, often referred to as "labor-market polarization," has received considerable attention in the macroeconomic and labor economics literature.[1] The polarization is often attributed to technological change, which allows firms to automate routine tasks by substituting workers with machines.

From a firm's perspective, automation requires occupational reallocation: reducing employment in occupations that are substituted by automation and increasing employment in occupations that complement automation. Given the heterogeneity in technology adoption and various (potentially time-varying) factors across firms, the transformation of the occupational mix likely accompanies reallocation of workers across firms.

How do firms reallocate workers across occupations under different labor-market environments? In particular, do firms change occupational mix by hiring and firing different workers or by changing workers' tasks within the firm? We ask this question with a particular focus on differences in labor-market institutions. Several decades of research have shown that the US economy and continental European economies have very different labor-market institutions. One specific difference with extensive research attention is the ease of firing. In our context as well, across-firm occupational reallocation may be costlier in a labor market when firing is difficult.

Using micro-level panel datasets from the US and Germany, we develop a novel decomposition method to compare the contributions of within-firm occupational reallocation to labor-market polarization in both countries. We show that within-firm reallocation contributes more to the decline of routine occupation employment in Germany than in the US.

Motivated by the empirical observations, we build a dynamic general equilibrium model with heterogeneous workers, extending the standard firm-dynamics framework by Hopenhayn (1992) and Hopenhayn and Rogerson (1993). Our framework departs from the standard model by considering three different occupations and firms' endogenous decisions of automation. When the firm decides to automate, it optimally adjusts the occupational mix, given the costs of adjustments within and across firms.

Our theoretical framework includes two distinct variables that determine the firm-level productivity. The first variable affects productivity in a Hicks-neutral manner. This variable is formulated

[1] See, for example, Autor, Katz, and Kearney (2006) for the US, Goos and Manning (2007) for the UK, and Goos, Manning, and Salomons (2009) for 16 European countries. Acemoglu and Autor (2011) survey the literature.

as an exogenous idiosyncratic shock. This type of shock is commonly employed in the Hopenhayn (1992)-type standard firm dynamics models. Essentially, this shock symmetrically affects the demand for all occupations. The second variable represents "automation productivity," which influences the marginal products of different tasks differently. Automation productivity is chosen by the firm and is subject to a cost. We interpret the improvement in automation productivity as the costly adoption of new technology. Because the adoption induces changes in the occupational mix, the adoption decision is also affected by labor market frictions.

We calibrate the model to the German economy and replicate the differences in the patterns of occupational reallocation between the US and Germany. We then conduct two counterfactual experiments for each country to assess how the frictions of occupational reallocation affect the degree and the speed of labor market polarization. In the first experiment, we impose a firing tax at the German level on the US. Next, we evaluate the impact of firms' reorganization cost, an adjustment cost for within-firm occupational reallocation in the US. Then, we move our focus to Germany and see whether the reorganization cost has different implications from the previous experiment. In the last experiment, we reduce firms' firing costs in Germany.

We find that the within-firm reorganization cost has a small impact on the degree of polarization, whereas the firing cost has a significant impact on polarization in the US. In particular, we find that the firing tax makes the labor market *more* polarized in the US: the level of routine employment is higher without the firing taxes, whereas the level of cognitive employment is lower. Individual firms adjust the composition of occupational employment faster when the firing tax is larger. The reason for this seemingly counterintuitive result is that the firms are forward-looking. In the model, firms are constantly hit by idiosyncratic productivity shocks. Thus, when there is a firing tax, firms that are likely to adopt automation technology will reduce routine hires when they suffer a negative idiosyncratic shock, seeing it as an opportunity to prepare for future automation adoption. On the other hand, without a firing tax, the firm is more likely to keep the routine workers because the firm can easily adjust the occupational composition in the future. This result has important implications for predicting how policies like the firing tax affect the polarizing labor market.

Our work is motivated by the recent empirical literature, which documents sizable within-firm occupational reallocation in several European countries during labor-market polarization. Using French establishment data, Behaghel, Caroli, and Walkowiak (2012) is one of the earliest papers to find within-firm occupational reallocation following a firm's adoption of information and communication technologies (ICT). Battisti, Dustmann, and Schönberg (2017) and Dauth, Findeisen, Suedekum, and Woessner (2018) report similar evidence using German establishment data after ICT or industrial robot-exposure shocks. Our empirical analysis of the US and Germany builds on these empirical findings and finds patterns consistent with these studies. We further evaluate the role of labor-market institutions on the intra- and inter-firm reallocation of workers.

Several recent macroeconomic studies build general equilibrium models and quantitatively analyze the process of labor-market polarization. For example, Eden and Gaggl (2018), vom Lehn (2020), and Jaimovich, Saporta-Eksten, Siu, and Yedid-Levi (2021) analyze the polarizing labor market using dynamic general equilibrium models. With a representative-firm assumption, the first two studies focus on accounting for the changes in occupational employment shares in the aggregate, whereas the last study analyzes the adverse effects of automation on workers and labor market policies. In contrast, we study worker reallocation across occupations *and* firms, explicitly considering firm heterogeneity. We construct a novel theoretical framework, which is a natural extension of the standard heterogeneous-firm model a la Hopenhayn (1992) and Hopenhayn and Rogerson (1993). This framework enables us to analyze heterogeneous firm dynamics during the process of job polarization.

In recent papers, Humlum (2021) and Rodrigo (2021) analyze the firm-level adjustment after adopting robots. These authors quantitatively analyze heterogeneous-firm models using micro-level datasets (Humlum (2021) uses Danish data, and Rodrigo (2021) uses Brazilian data), considering endogenous robot adoption and labor market responses. Meanwhile, we conduct a cross-country comparison of the US and Germany and focus on the role of labor market adjustment costs. Neither of these studies considers across-firm labor adjustment costs (firing taxes), which is the main focus of our study.

Finally, various studies have examined the interactions between institutions (policies) and shocks (technologies). Blanchard and Wolfers (2000) emphasize the interaction between shocks, such as the decline in productivity growth, and institutions, such as labor market policies, in explaining the increase in unemployment in Europe from the 1960s to the 1990s. Some studies explicitly consider the comparison between the US and Europe in the labor market institutions. Ljungqvist and Sargent (1998) argue that in European welfare states, characterized, for example, by more generous unemployment insurance, shocks to human capital depreciation upon unemployment translate into a high unemployment rate more strongly. Mortensen and Pissarides (1999) analyze how skill-biased shocks, interacting with different policy regimes, explain the rise of unemployment in Europe. Using the quantitative general equilibrium model, Hornstein, Krusell, and Violante (2007) demonstrate that the labor-market response to capital-embodied technological change can be different depending on the labor market institutions. While our focus is on labor-market polarization and occupational reallocation, the motivations are similar to these earlier studies: different labor-market institutions can result in different responses to technology shocks.

The remainder of this article is organized as follows. Section 2 conducts the empirical analysis. Section 3 constructs a general equilibrium model of firm dynamics. Section 4 analyzes the model quantitatively and compares it with the data. Section 5 conducts counterfactual experiments using the calibrated model. Section 6 presents the conclusions of this study.

2 Empirical findings

Here, we document the patterns of occupational reallocation in the US and Germany. Both countries have experienced significant changes in the occupational composition of their labor markets in the past decades.[2] The patterns of reallocation, however, are markedly different across these two countries, as we show. We start by describing the data and then present the empirical results for the patterns of occupational reallocations.

2.1 Data

For the US, we use the Survey of Income and Program Participation (SIPP), a household survey dataset that provides detailed information on individuals' labor market activities.[3] For Germany, we use the Sample of Integrated Labor Market Biographies (SIAB), an administrative dataset that contains employment records for a 2% sample of the German labor market. The details of the datasets and data cleaning procedures are described in Appendix A.

2.1.1 US: SIPP

SIPP is a household-based panel survey dataset administered by the US Census Bureau. We use the following seven panels of the SIPP for our analysis: 1990, 1991, 1992, 1993, 1996, 2001, and 2004. We do not use the panels before 1990 as no reliable job IDs are provided and thus cannot identify job switches. We do not use the 2008 panel or later because the new data cleaning procedure

[2] See Acemoglu and Autor (2011) for the US and Böhm, von Gaudecker, and Schran (2024) for Germany.

[3] An alternative dataset often used in the literature on occupational reallocation is the Current Population Survey (CPS). We use SIPP for our main analysis for two reasons. First, the 1994 redesign of the CPS and the associated introduction of dependent interviewing appears to have created a significant discrepancy between the periods before and after 1994, and thus, we cannot go back beyond 1994 for reliable data. Second, as discussed in Kambourov and Manovskii (2013), there appear to be some data problems in the CPS even after the introduction of its dependent interviewing after 1994. We limit the use of the CPS to the robustness check in Appendix C.1.

that the US Census introduced in 2004 removed the significant number of within-job occupational changes for the 2008 panel.[4] These panels have a sample of 14,000 to 52,000 individuals. We select observations where an individual is between ages 23 and 55. We drop observations where an individual works in the public sector or is self-employed.[5] Following Kambourov and Manovskii (2009), we also exclude managerial occupations from our analysis.[6]

Following the literature of the task-based approach (Acemoglu and Autor, 2011), we identify occupational switches when a worker changes their occupations across three broad occupational groups, *Cognitive*, *Routine*, and *Manual*, based on the nature of tasks performed in an occupation. These occupation groups are listed in Appendix A.4. Among those occupational switches, we further identify *within-firm occupational switches*, those that involve employer changes, and *across-firm occupational switches*, those that do not involve employer changes, by examining changes in job IDs in SIPP. We identify these within-job and between-job occupational switches on an annual basis.

2.1.2 Germany: SIAB

SIAB is a dataset based on the administrative employment records in Germany (Antoni, Graf, Grießemer, Kaimer, Köhler, Lehnert, Oertel, Schmucker, Seth, Seysen, and vom Berge, 2019). The data are provided by the Institute for Employment Research (IAB) for research. The dataset has a 2% sample of employment histories from the entire German employment records for the period 1975–2017.[7] It includes employees covered by social security, marginal part-time employment (since 1999), unemployment insurance benefit recipients, and individuals who are officially registered as job-seeking or who are participating in programs of active labor market policies. The dataset excludes the self-employed, civil servants, individuals performing military service, and those not in the labor force. It contains information on the starting and ending dates of each employment spell with an employer identification number and occupation classification code. Similar to SIPP, we select the sample of individuals between ages 23 and 55 and exclude managerial occupations.

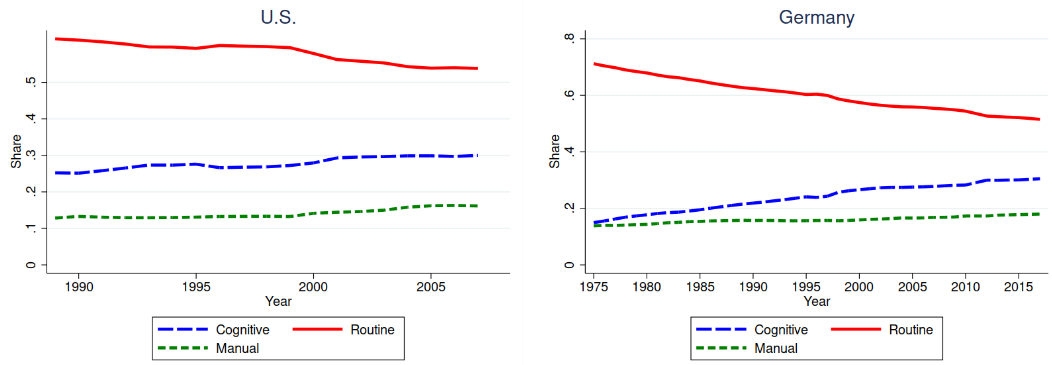
[4] This new data cleaning procedure pulls forward the occupation reported last time within the same job. This procedure allowed only a few of within-job occupation changes for the 2008 Panel.

[5] Further details of the dataset, data cleaning procedures, and the sample selection criteria are in Appendix A.1.

[6] This paper focuses on the effect of technological change on the horizontal reallocation of workers across occupations, not on career progression. Lee and Shin (2017) analyze the effect of technological changes on both workers (horizontal polarization) and managers (vertical polarization). Once the managerial occupation is included, the within-firm reallocation increases for both the US and German datasets.

[7] During the period, Germany experienced a series of labor market reforms (Hartz reforms). The most relevant one for our analysis is Hartz IV. Starting in January 2005, the Hartz IV reform restructured the unemployment benefit system by reducing the level of benefits and shortening the duration of benefit receipt. Although these changes can potentially affect the worker flows, we do not observe a large change in the patterns of internal-external reallocation before and after the reform.

Figure 1: Occupational Employment Share in the US (Left) and Germany (Right)



Data Source: SIPP (US); SIAB (Germany)

With the occupation information, we first create three broad occupation classifications (Cognitive, Routine, and Manual) similar to those for the US, following Böhm, von Gaudecker, and Schran (2024).[8] We then identify job and occupation switches at the annual frequency and document within-firm (within-establishment) and across-firm (across-establishment) patterns of occupational reallocation, similar to those in the US.[9]

2.2 Time-series patterns of the occupational shares

Figure 1 plots the share of employment across occupations for the US from 1989 to 2007 and for Germany from 1975 to 2017. As is commonly observed in the literature (see Acemoglu and Autor, 2011), the share of routine-occupation employment has declined both in the US and Germany. In contrast, cognitive and manual occupations have gained employment shares. This phenomenon is often referred to as the labor-market polarization.[10]

2.3 Decomposition of the occupational employment share changes

Next, we investigate how firms reallocate workers behind the change in the stocks of occupational shares by analyzing the flows in and out of these stocks. To quantify the role of occupational switches within and across firms in the process of labor-market polarization, we decompose the change in each occupational employment share into contributions of the net flows of the internal and external occupational changes. *Internal* occupational changes occur when workers change

[8] Böhm, von Gaudecker, and Schran (2024) created task-based occupational groups for Germany that are comparable to those in Acemoglu and Autor (2011). The details of the occupational groups are in Appendix A.4.

[9] The SIAB dataset only provides establishment IDs, and thus, only the establishment switches can be identified. If a within-firm occupational switch is recognized as an across-firm one because the worker switches establishments but not an employer, our results for the within-firm occupational reallocation for Germany could be underestimated. Therefore, our result on the U.S.-German gap in within-firm occupational reallocation would be a lower bound.

[10] For the US, we confirm the same pattern with the CPS in Figure 28 in Appendix C.1.

occupations but remain with the same employer. *External* occupational changes occur when the workers switch both their employers and occupations. Previous studies, such as Moscarini and Thomsson (2007), have shown that there are substantial internal occupation changes in the US. However, how the internal and external occupation changes separately contribute to the changes in occupation stocks has not been studied.

Let ℓ_{it} be the stock of employment in occupation i at time t. The index i takes c, r, or m, where c represents cognitive, r represents routine, and m represents manual occupations. Further, let

$$E_t \equiv \sum_{i=c,r,m} \ell_{it}$$

be the total employment.

Now, we employ the following decomposition formula to quantify the contributions of different (net) flows to the change in the occupational stock. Let the employment share at time t for occupation i be ℓ_{it}/E_t. We decompose the change in the (log) employment share of occupation i from period t to period $t+T$:

$$\begin{aligned}
&\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) \\
&\qquad \approx \Bigg[\underbrace{\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,s}_{t+\tau,t+\tau+1} - f^{ij,s}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{internal net flow}} + \underbrace{\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,d}_{t+\tau,t+\tau+1} - f^{ij,d}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{external } EE \text{ net flow}} \\
&\qquad\qquad + \underbrace{\sum_{\tau=0}^{T-1}\frac{f^{Ui}_{t+\tau,t+\tau+1} - f^{iU}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{external net flow from/to unemployment and OLF}} - \underbrace{\sum_{\tau=0}^{T-1}\Delta^{E}_{t+\tau,t+\tau+1}}_{\text{total employment effect}}\Bigg].
\end{aligned} \tag{1}$$

The derivation of equation (1) is in Appendix B. The equation shows that the cumulative change in employment share is decomposed into four components on the right-hand side. The first term (labeled as "internal net flow") is the contribution of within-firm occupational switches. The notation $f^{ji,s}_{t+\tau,t+\tau+1}$ is the gross worker flow from occupation j to occupation i between time $t+\tau$ and $t+\tau+1$, conditional on staying with the same employer (s for "the same employer"). The term $f^{ij,s}_{t+\tau,t+\tau+1}$ is the worker flow in the opposite direction. Therefore, $\sum_{j\neq i} f^{ji,s}_{t+\tau,t+\tau+1} - \sum_{j\neq i} f^{ij,s}_{t+\tau,t+\tau+1}$ is the sum of the total inflow minus the sum of the total outflow for occupation i. Thus, this term is the net inflow due to the internal occupational switches. Similarly, the second term (labeled as "external EE net flow") is the contribution of between-firm occupational switches. $f^{ji,d}_{t+\tau,t+\tau+1}$ represents the gross worker flow from occupation j to occupation i between time $t+\tau$ and $t+\tau+1$,

Table 1: Decompositions of Occupational Employment Share Changes for the US and Germany

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
US	1989	2007	log (Δshare)	Internal	External
Cognitive	0.25	0.30	0.17	0.01	0.17
Routine	0.62	0.54	−0.14	0.00	−0.14
Manual	0.13	0.16	0.23	−0.01	0.24
Germany	1975	2017	log (Δshare)	Internal	External
Cognitive	0.15	0.30	0.71	0.17	0.54
Routine	0.71	0.52	−0.32	−0.05	−0.27
Manual	0.14	0.18	0.26	−0.04	0.30

Data Source: SIPP (US); SIAB (Germany)

conditional on workers switching to different employers (d represents "different employers"). The third term (labeled "external net from/to unemployment and OLF," where OLF means "out of labor force") represents the net inflow from unemployment and out of the labor force, where $f^{Ui}_{t+\tau,t+\tau+1}$ is the flow from U to occupation i employment and $f^{iU}_{t+\tau,t+\tau+1}$ is the opposite flow. Finally, the fourth term (labeled as "total employment effect") is the change in occupational employment share due to the change in total employment.

We call the first term on the right-hand side as *internal flow*, and the sum of the second to fourth terms as *external flow*. We do not distinguish between the second to fourth terms largely for the purpose of comparability.[11] In particular, it is difficult to make comparable distinctions between the third and fourth terms across the US (SIPP) and German (SIAB) datasets because survey data, like SIPP, are often affected by sample attrition, which creates a spurious effect in the fourth term.[12] Given that our focus is on the internal flow, the most important task is to distinguish the internal flow and other occupational switches.

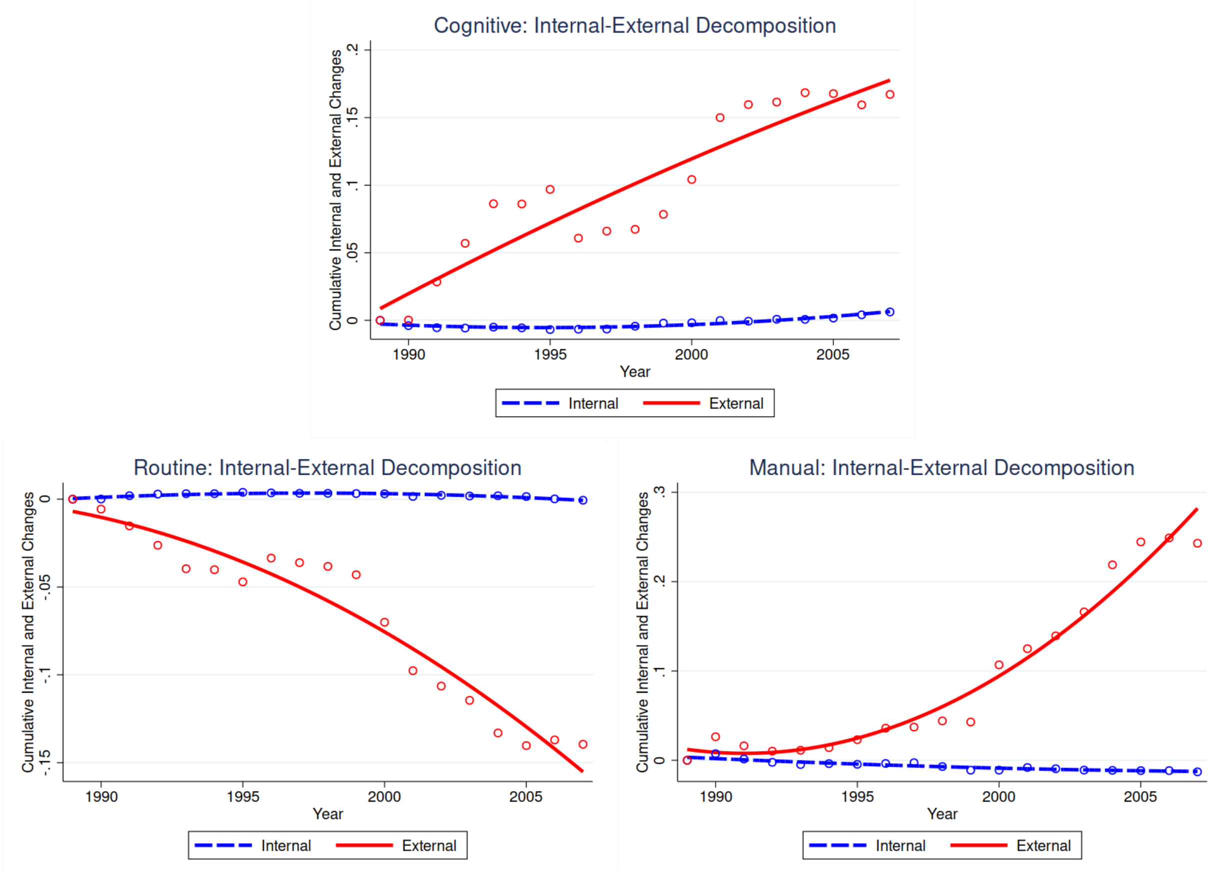
Table 1 implements the decomposition equation (1) to SIPP for the US and SIAB for Germany from the periods 1989–2007 and 1975–2019, respectively. The frequency is annual.

We observe striking differences between the US and Germany. As Columns (4) and (5) show, internal switches play almost no role in explaining the rise of cognitive employment and decline in routine employment in the US. This result indicates that most of the workers' gross occupational movements offset each other. By contrast, internal switches make non-negligible contributions to

[11] Appendix C.2 provides further decomposition.

[12] To check how the sample attrition affects our results, we create and analyze the balanced panels of SIPP in Appendix C.4.

Figure 2: Cumulative Changes in Occupational Employment in the US, SIPP, 1989–2007



Note: The data are from the SIPP. The circular dots indicate data points, and the lines indicate quadratic fits to the data points.

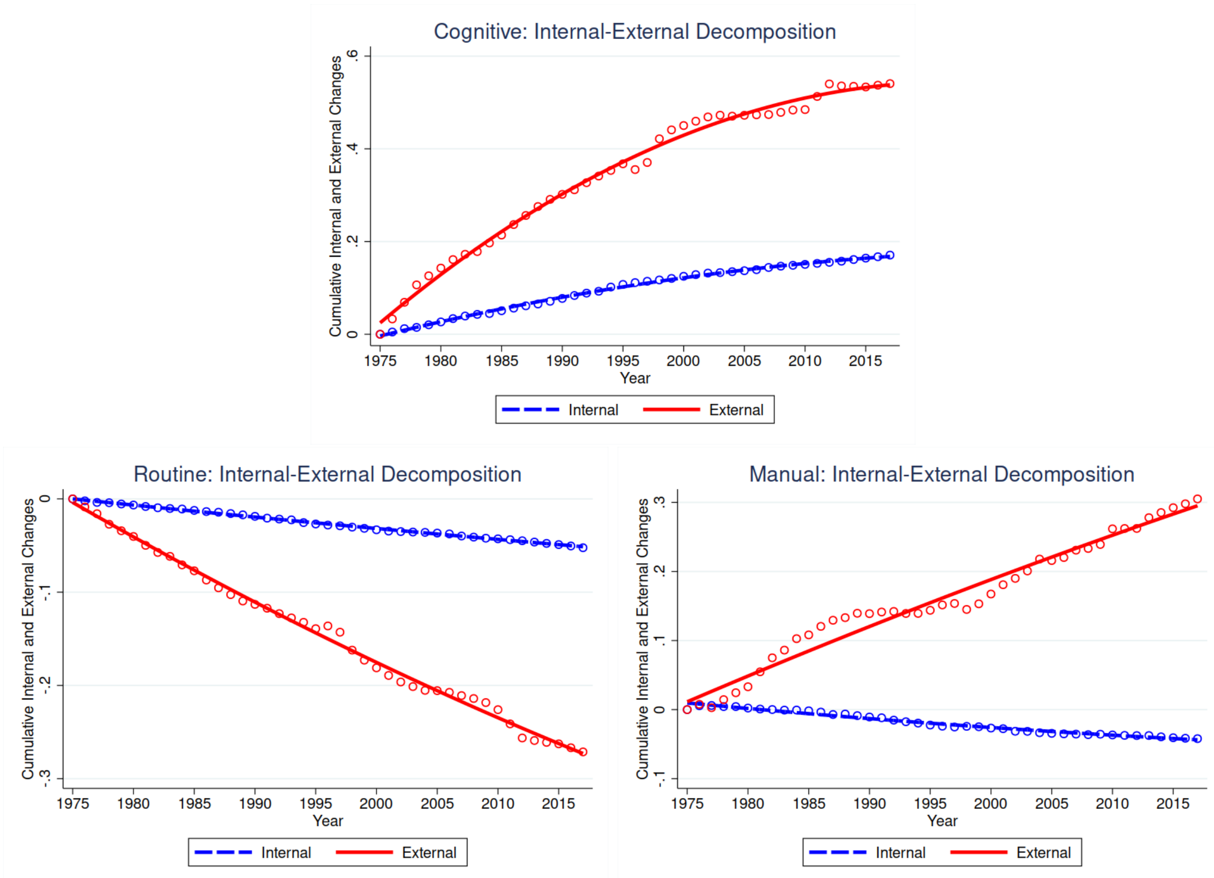
the changes in occupational employment in Germany.[13] The internal contribution for the manual share goes in the opposite direction for both the US and Germany. Given that the magnitude of the internal switches is small for both countries and that the change in the share is substantially smaller than the other two occupations, below we will mainly focus on the changes in the cognitive and routine shares.

We plot the cumulative contributions of net flows to occupational employment changes over time in Figures 2 and 3.[14] The figures show the dynamics that correspond to those in Columns (4) and (5) in Table 1.

[13] Appendix C.3 examines whether demographic composition plays any role in this difference between the US and Germany. We find the educational composition has some explanatory power for the differences in internal flows to cognitive occupations, although the difference in educational composition does not explain the entire difference between the US and Germany.

[14] As is usually the case with datasets based on surveys, the SIPP dataset is subject to sample attrition, which can potentially generate biases in the decomposition results. Therefore, we run a robustness check by conducting the decomposition with balanced panels of the SIPP in Appendix C.4. The results are essentially the same.

Figure 3: Cumulative Changes in Occupational Employment in Germany, SIAB, 1975–2017



Note: The data are from the SIAB. The circular dots indicate data points, and the lines indicate quadratic fits to the data points.

What is the cause of the different patterns between the US and Germany in Table 1? In the next two sections, we construct a model of heterogeneous firms to investigate the role of labor-market policies in the process of labor-market polarization.

3 Model

This section constructs a dynamic model with heterogeneous firms to examine the interaction between labor market policies and the process of polarization. Our model builds on Hopenhayn (1992) and Hopenhayn and Rogerson (1993), and features a CES production structure with three broad types of occupations (cognitive, routine, and manual) and two firm-level productivity variables. As in Hopenhayn (1992) and Hopenhayn and Rogerson (1993), the first variable is an exogenous Hicks-neutral productivity shock. The second variable, which represents firm-level automation, is an endogenous choice variable for each firm. Automation affects different types of occupational labor demand differently. This differential labor demand drives the labor market polarization.[15]

3.1 Setup

Time is discrete. We assume that an infinitely-lived representative consumer exists. The consumer supplies labor and receives wage income. They also own the firms and receive the profit. The consumer is a price-taker and maximizes utility

$$\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$C_t = w_t N_t + \Pi_t + R_t.$$

Here, $\beta \in (0,1)$, $U(\cdot,\cdot)$ is the period utility function, C_t is consumption at period t, and N_t is the labor supply. On the income side, w_t is the wage rate and Π_t is the profit from production in the

[15] Although there are many potential causes of labor market polarization (for example, the effect of offshoring has often been pointed out), in this model, we focus on the effect of automation. Many researchers find that automation is linked to the occupational switching of workers. Rodrigo (2021) examine Brazilian data. His idea is that, in Brazil, robots are all imported, and from the customs data, one can tell the location where robots are sent. Through this information, we can identify the region where the robots are introduced. Comparing the regions with and without robot adoption, he finds that the introduction of robots induces occupational switching (see Section 4 of that paper). Restrepo (2023) writes a survey paper that contains some recent empirical papers that relate automation to changes in occupational structure. The discussion of our paper, published in this volume, also points out another potential factor of polarization, that is, the change in the composition of the labor force. In the main text, we do not consider heterogeneous labor. In Appendix D, we construct a simple model with heterogeneous labor and show that the model (qualitatively) behaves similarly to our baseline model as long as the margin of the ex-ante skill choice by workers is operative.

firm. Firms pay firing taxes to the government, which is lump-sum rebated to the consumer as R_t. Below, we assume a quasi-linear period utility

$$U(C_t, N_t) = C_t - \xi \frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}},$$

where $\xi > 0$ and $\eta > 0$ are parameters. This specification implies that the equilibrium return to saving has to be equal to $1/\beta - 1$. To simplify the notation below, we adopt the recursive formulation, where the next-period variable is denoted by prime ($'$).

There is a unit mass of firms, and we abstract from entry and exit for simplicity. Firms produce the consumption goods using labor. They act competitively in both the product and labor markets. The production process involves three different tasks (which correspond to three different occupations): manual (m), cognitive (c), and routine (r). The production function also features two additional variables that affect worker productivity. The first is the standard Hicks-neutral total factor productivity (TFP) shock, denoted by s_h, which is exogenous and acts similarly to the standard firm dynamics model by Hopenhayn (1992) and Hopenhayn and Rogerson (1993). The second, denoted by s_a, is a choice variable for the firm's automation productivity. It represents the degree of introduction of new technology (machines). A firm can choose the next period s_a, denoted by s'_a, subject to cost (denote it $\Gamma(s_a, s'_a)$). Here, we specify the $\Gamma(s_a, s'_a)$ function as follows. We assume that each firm's s_a can take two possible values, $\underline{s}_a$ and $\overline{s}_a$, where $\underline{s}_a < \overline{s}_a$. The interpretation of $\underline{s}_a$ is "before automation" and $\overline{s}_a$ is "after automation." The transition between these two values is one direction: from $\underline{s}_a$ to $\overline{s}_a$. Therefore, $\overline{s}_a$ is the absorbing state. The cost of transition is assumed to be $\bar{c}_a$. The cost is zero when the value of s_a does not change. Because $\overline{s}_a$ provides a higher productivity than $\underline{s}_a$, when the additional (present) value surpasses $\bar{c}_a$, firms prefer to pay the cost and transition from $\underline{s}_a$ to $\overline{s}_a$. A firm with $s_a = \underline{s}_a$ has an opportunity to automate (by paying the cost $\bar{c}_a$) with i.i.d. probability p. The value of p governs the aggregate speed of automation.[16]

After observing the s_h shock, a firm makes hiring decisions (as well as the automation decision), where employment at task $i \in \{m, c, r\}$ is denoted as n_i. Note that, in the model, we build in a mechanism where the labor market polarization is driven by the firm's automation choice. Various existing studies have investigated the fundamental cause of labor market polarization and suggested several potential causes, including automation. Here, we solely focus on the effect of automation.

[16]Appendix E explores an alternative formulation of the automation cost. There, the automation opportunity is open to all firms, and the automation cost falls deterministically. The overall results are very similar to the specifications in the main text.

In particular, what is important here (by calling s_a as "automation") is that a change in s_a causes differential responses to the demands for different tasks (occupations).

The production function is specified as

$$f(\mathbf{n}, \mathbf{s}) = s_h \mathbf{F}^{\alpha},$$

where $\alpha \in (0, 1)$ is the returns-to-scale parameter,

$$\mathbf{F}(n_m, \mathbf{G}) = \left(\mu_m n_m^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \mu_m) \mathbf{G}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}},$$

where $\sigma_m \geq 0$ is the elasticity of substitution parameter and n_m is the manual labor,

$$\mathbf{G}(n_c, \mathbf{M}) = \left(\mu_c n_c^{\frac{\sigma_c - 1}{\sigma_c}} + (1 - \mu_c) \mathbf{M}^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}},$$

where $\sigma_c \geq 0$ is the elasticity of substitution parameter and n_m is the cognitive labor,

$$\mathbf{M}(n_r, s_a) = \left(\mu_r n_r^{\frac{\sigma_r - 1}{\sigma_r}} + (1 - \mu_r) s_a^{\frac{\sigma_r - 1}{\sigma_r}} \right)^{\frac{\sigma_r}{\sigma_r - 1}},$$

where $\sigma_r \geq 0$ is the elasticity of substitution parameter and n_r is the routine labor. One can interpret s_a as the (automation) capital stock. This specification of the production function is in line with the existing literature on labor-market polarization. For example, $\sigma_r = \infty$ corresponds to Cortes, Jaimovich, and Siu (2017) and $\sigma_c = 1$ case corresponds to Autor and Dorn (2013). Using the same specification as above, vom Lehn (2020) estimates the values of σ_i and μ_i. For simplicity, we assume that the workers are (ex-ante) homogeneous, and thus, each occupation pays the identical wage.[17]

Changing occupational employment from one period to the next may require the firm to pay certain costs for adjustment. To describe these costs, we first introduce new notations. Let us denote the current period's employment in occupation $i \in \{m, c, r\}$ as n'_i.[18] The previous period's employment in occupation i is denoted as n_i. Firms decide on the vector $\mathbf{n}' \equiv \{n'_m, n'_c, n'_r\}$ for

[17] In Appendix D, we formulate and solve a simple model where the worker skills are heterogeneous. There, we also assume that the workers can make the skill decision ex-ante. We find that qualitative results are essentially the same as in the baseline. The intuition is that, in the heterogeneous-skill labor market, the changing labor demand for different occupations translates into changing skill premium, and the skill premium influences the skill acquisition decision of workers. Thus, the skill-decision margin can act as a channel to move the equilibrium occupational mix as automation occurs. The mechanism that firing taxes moves the composition of internal versus external adjustment is the same as in the baseline model. When firing taxes exist, the firm has an incentive to reassign (retrain) some workers internally instead of through firing and hiring. Therefore, the essential economic mechanism is the same in this heterogeneous worker economy as in the baseline model.

[18] The convention of using $'$ for the current period employment follows Hopenhayn and Rogerson (1993).

given $\mathbf{s} = \{s_h, s_a\}$ and $\mathbf{n} \equiv \{n'_m, n'_c, n'_r\}$. That is, $\mathbf{s}$ and $\mathbf{n}$ are the state variables for the firm's employment decision $\mathbf{n}'$.

While increasing the number of occupational hires ($n'_i > n_i$), the firm has to bring in new workers into that occupation from inside or outside the firm. Define $\tilde{n}'_i \in [0, n'_i]$ as the internal workers (from any occupation but from the same firm) who now work in occupation i this period. Then, $\tilde{n}'_i - n_i$ is the number of internally-moved workers and $n'_i - \tilde{n}'_i$ is the number of workers who are brought from outside. Furthermore, define $\hat{n}'_i \in [0, \min\{n_i, \tilde{n}'_i\}]$ as internal workers who stayed in the same occupation i (that is, the same firm and occupation) from the previous period. Then, $\tilde{n}'_i - \hat{n}'_i$ is the number of workers who are internally brought into that occupation (from another occupation) within the firm.[19] Let $\tilde{\mathbf{n}}'$ be the vector of $\tilde{n}'_i$ and $\hat{\mathbf{n}}'$ be the vector of $\hat{n}'_i$.

In summary, the firm makes three layers of employment decisions: (i) how many people to hire this period $\mathbf{n}'$; (ii) within $\mathbf{n}'$, how many people come from the same firm ($\tilde{\mathbf{n}}'$), and (iii) how many people in $\tilde{\mathbf{n}}'$ are the ones from the same occupation ($\hat{\mathbf{n}}$). Clearly, $\hat{n}_i$ cannot exceed n_i and the sum of $\tilde{n}'_i$ must be less than the sum of n_i.

We assume two types of costs for employment adjustment. The first is the *firing taxes* imposed by the government. We denote it as $g(\mathbf{n}, \tilde{\mathbf{n}}')$ and assume that it takes the form

$$g(\mathbf{n}, \tilde{\mathbf{n}}') = \tau \left(\sum_{i=m,c,r} n_i - \sum_{i=m,c,r} \tilde{n}'_i \right),$$

where $\tau \geq 0$ is the tax rate. We also assume that a firm has to incur a *reorganization cost*, $h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}')$, which takes the form

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=m,c,r} H_i(\tilde{n}'_i - \hat{n}'_i),$$

where $H_i(\cdot)$ is an increasing function. In the quantitative analysis below, we consider a quadratic form of H_i function:

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=0}^{k} \kappa_i (\max\{\tilde{n}'_i - \hat{n}'_i, 0\})^2,$$

where $\kappa_i \geq 0$.

[19] Note that we implicitly assume that the firm keeps the workers in the firm and in the same occupation whenever possible. This assumption can be justified by, for example, having an infinitesimally small amount of cost of moving workers across firms and occupations.

Formally, the firm's problem is

$$V(\mathbf{n},\mathbf{s};t) = \max_{\mathbf{n}',\tilde{\mathbf{n}}',\hat{\mathbf{n}}',s_a'} f(\mathbf{n}',\mathbf{s}) - w\mathbf{1}\cdot\mathbf{n} - g(\mathbf{n},\tilde{\mathbf{n}}') - h(\tilde{\mathbf{n}}',\hat{\mathbf{n}}')$$
$$+ \beta E_{s_h'}[p\{V(\mathbf{n}',s_h',s_a';t+1) - \Gamma(s_a,s_a')\} + (1-p)V(\mathbf{n}',s_h',s_a;t+1)],$$

subject to

$$\sum_{i=m,c,r} \tilde{n}_i' \leq \sum_{i=m,c,r} n_i,$$

where

$$g(\mathbf{n},\tilde{\mathbf{n}}') = \tau\left(\sum_{i=m,c,r} n_i - \sum_{i=m,c,r} \tilde{n}_i'\right),$$

and

$$h(\tilde{\mathbf{n}}',\hat{\mathbf{n}}') = \sum_{i=m,c,r} H_i(\tilde{n}_i' - \hat{n}_i').$$

Here, we use the fact that the quasi-linear utility of consumers implies the firm's discount factor to be β.

In the competitive equilibrium of this economy, the wage w_t is determined by the labor market clearing condition. Next, we examine the transitional dynamics of the aggregate economy from one steady state to another.

4 Quantitative analysis

The empirical analysis in Section 2 highlights a significant difference in how firms react to the labor market polarization process. Motivated by this outcome, we quantitatively assess how labor market institutions affect the reallocation of workers across occupations and firms during the transition dynamics.

4.1 The transition economy

We consider the transition process of automation. The economy starts from the steady state where all firms have $s_a = \underline{s}_a$. In each period, some firms (who have an opportunity) choose to transition into $s_a = \overline{s}_a$ and the economy eventually converges to the new steady state where $s_a = \overline{s}_a$ for all firms. This process leads to gradual changes in the aggregate demand for each occupation. Therefore, the transition process entails labor-market polarization.

Below, we further impose a restriction that the *within-firm* occupation reallocation occurs only from the routine occupation to the cognitive occupation. This assumption simplifies the computation of the model dramatically and is motivated by the analysis of the German data in Section 2, where the within-firm reallocation contributes to the polarization mainly through the transition from routine to cognitive occupation.

With this assumption, $\hat{n}'_m = \tilde{n}'_m = \min\{n_m, n'_m\}$ holds because there are no internal movements (both in and out) of workers for the manual occupation. For routine workers, suppose that $x' \geq 0$ number of workers move to the cognitive occupation. If $n'_r > n_r - x'$, $\hat{n}_r = \tilde{n}_r = n_r - x'$ and the remaining workers $(n'_r - (n_r - x'))$ must be brought in from outside the firm. If $n_r - x' \geq n'_r$, $\hat{n}_r = \tilde{n}_r = n'_r$ and the excess workers $((n_r - x') - n'_r = n_r - (n'_r + x'))$ must be fired. For cognitive workers, $n'_c - x'$ number of workers have to come from either previously cognitive workers or outside the firm. If $n'_c - x' > n_c$, $\hat{n}_c = n_c$, $\tilde{n}_c = n_c + x'$, and $(n'_c - x') - n_c$ workers must be brought in from outside. If $n_c \geq n'_c - x'$, $\hat{n}_c = n'_c - x'$, $\tilde{n}_c = n'_c$, and the excess workers $(n_c - (n'_c - x'))$ must be fired. In short, the firm chooses four numbers (n'_m, n'_r, n'_c, x'), the firing tax is $\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'), 0\} + \max\{n_r - (n'_r + x'), 0\})$, and the reorganization cost is $\kappa x'^2$. The computational details are described in Appendix F.

4.2 Calibration

The calibration of parameters is summarized in Table 2. The main calibration target is set in Germany in order to determine a coherent set of parameters, including those governing frictions. First, τ^{DE} is based on the reference formula for the severance payment in the German Protection against Dismissal Act (Kündigungsschutzgesetz), which is 0.5 times monthly wage times years of employment, and the average tenure of 10 years in Germany.[20] Therefore, the ratio of average severance payment to annual wage is calculated as $\tau^{DE} = 0.5 \times (1/12) \times 10$. Note that here we measure the firing tax as severance payment, which do not necessarily have the same effects depending on the model structure. Here, we are implicitly disallowing the contract that can "undo" the severance payment by lowering wages.[21] Next, the disutility for work ξ is set to clear the labor market at the initial steady state with $\tau = \tau^{DE}$ and $w = 1$. The Frisch elasticity η is in the range of standard values to calibrate macroeconomic models. The return-to-scale parameter α^{DE} is based on Bachmann and Bayer (2013). The initial level of s_a, $\underline{s}_a$, is set to unity. Meanwhile, the final level of s_a, $\overline{s}_a$, is calibrated jointly with the parameters of the production function except

[20] See Goerkea and Pannenberg (2010).

[21] See Garibaldi and Violante (2005) for the detailed analysis in the context of the model environment where these two have different effects.

σ_m and σ_r. Those parameters are determined so that the initial steady-state values of shares of tasks and the labor share under τ^{DE} for firms hit their counterparts of Germany in 1975, and the final steady-state values of shares of tasks hit their counterparts in Germany in 2017. Then, σ_m is set to unity to reduce it to Cobb-Douglas. The remaining parameter of production function σ_r is assumed to make the automation capital stock a perfect substitute for the routine task. This assumption is also employed in Cortes, Jaimovich, and Siu (2017). The discount factor β is set to be consistent with the annual safe interest rate of four percent. Regarding the idiosyncratic TFP shock s_h, we assume that $\log(s_h)$ follows an AR(1) process

$$\log(s_h') = \rho_h \log(s_h) + \epsilon_h,$$

where

$$\epsilon_h \sim N(0, \sigma_h^2)$$

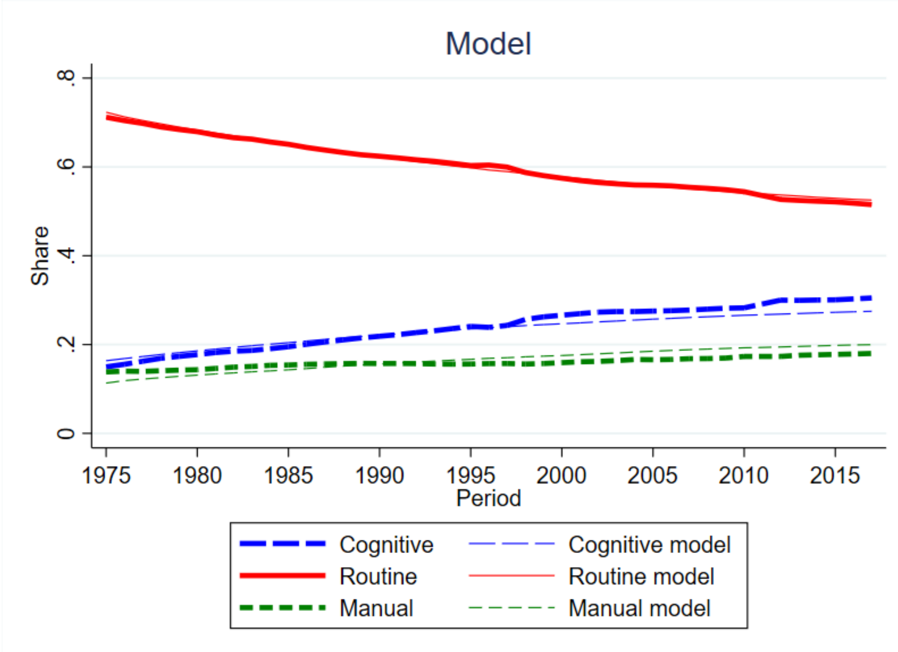
and the parameter values for ρ_h^{DE} and σ_h^{DE} in Germany are taken from Bachmann and Bayer (2013). The value for the cost of transition $\bar{c}_a$ is set to the highest value with which the conversion immediately starts at $t = 0$ with $\tau = \tau^{DE}$. The probability p that a firm can make a transition decision is set so that the changes in shares of tasks in the transition match the changes in the counterparts of Germany. Finally, the reorganization cost κ is set to match the internal reallocation flow from the routine occupation in Germany.

After calibrating the German baseline economy, the US is considered to be the case with $\tau = 0$, α^{US} from Veracierto (2001) and ρ_h^{US} and σ_h^{US} from Lee and Mukoyama (2015).

Table 2: Calibrated Parameters

Parameter	Value	Description
τ^{DE}	0.417	Based on German Protection against Dismissal Act
ξ	4.136	To match $w = 1$ at the steady state with $s_a = \underline{s}_a$ and $\tau = 0$
η	2	Standard value
α^{DE}	0.764	Bachmann and Bayer (2013)
$\underline{s}_a$	1	Normalized to one
$\bar{s}_a$	3.093	
μ_m	0.091	
μ_c	2.273×10^{-9}	Jointly determined to target the shares of tasks and labor share of Germany
μ_r	0.975	
σ_c	0.097	
σ_m	1	Normalization
σ_r	∞	Cortes, Jaimovich, and Siu (2017)
β	0.962	Annual safe interest rate of 4%
ρ_h^{DE}	0.950	Bachmann and Bayer (2013)
σ_h^{DE}	0.0905	Bachmann and Bayer (2013)
$\bar{c}_a$	1.230	Highest value with which the conversion immediately starts at $t = 0$
p	0.04	To hit the change in the share of tasks in Germany
κ	520	To match the internal reallocation from the routine occupation in Germany
τ^{US}	0	Frictionless
α^{US}	0.830	Veracierto (2001), Bachmann and Bayer (2013)
ρ_h^{US}	0.969	Lee and Mukoyama (2015)
σ_h^{US}	0.282	Lee and Mukoyama (2015)

Figure 4: Occupation Share in Data versus Model: Germany



4.3 Model fit

We simulate the model's general equilibrium separately for Germany and the US. Note that the computation of the US is substantially simpler than the German case because $\tau = 0$ implies that x' is always zero. This result follows because when $\tau = 0$, it is always cheaper to adjust the occupational composition through hiring and firing rather than internal reallocation.

Figure 4 plots the German data (SIAB), presented in Section 2.2, and the model simulation side-by-side. The model captures the main features of the data pattern very well: the routine share falls over time, whereas the manual and cognitive shares increase. This pattern of labor market polarization is driven by the endogenous automation (s_a moving from $\underline{s}_a$ to $\overline{s}_a$) of individual firms. Existing macroeconomic studies, such as Eden and Gaggl (2018) and vom Lehn (2020), generate similar patterns in the representative-firm framework. In our model, heterogeneous firms make the automation decision at different timings from each other.

Figure 5 plots the corresponding figures for the US. Note that we do not target any moments of the US data. Except for some level differences, the patterns of the labor market polarization in the US are also captured well by the model.

Now, we move to the net flows. The model is targeted to match the cumulative change in the share of routine occupation via internal reallocation in Germany. Figures 6 through 11 compare the actual (presented in Section 2.3) and model-simulated data in terms of the cumulative change

Figure 5: Occupation Share in Data versus Model: US

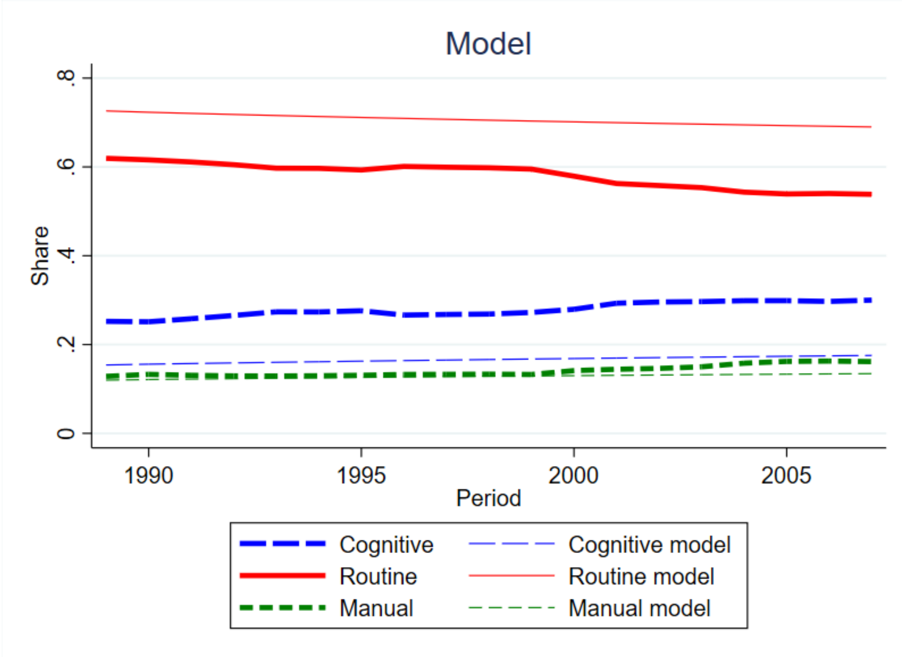


Figure 6: Cumulative Share Changes of Cognitive in Data versus Model: Germany

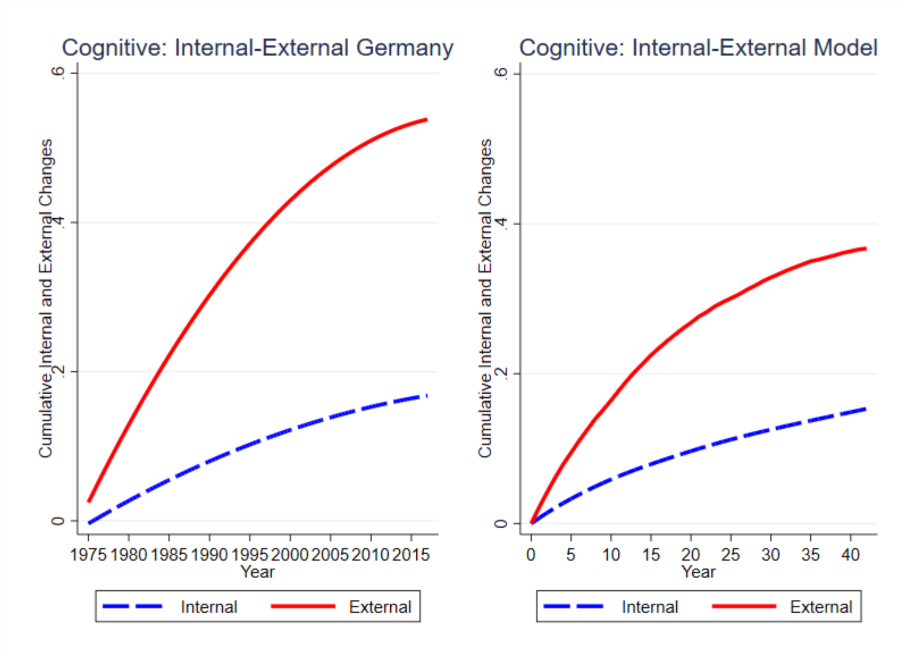


Figure 7: Cumulative Share Changes of Cognitive in Data versus Model: US

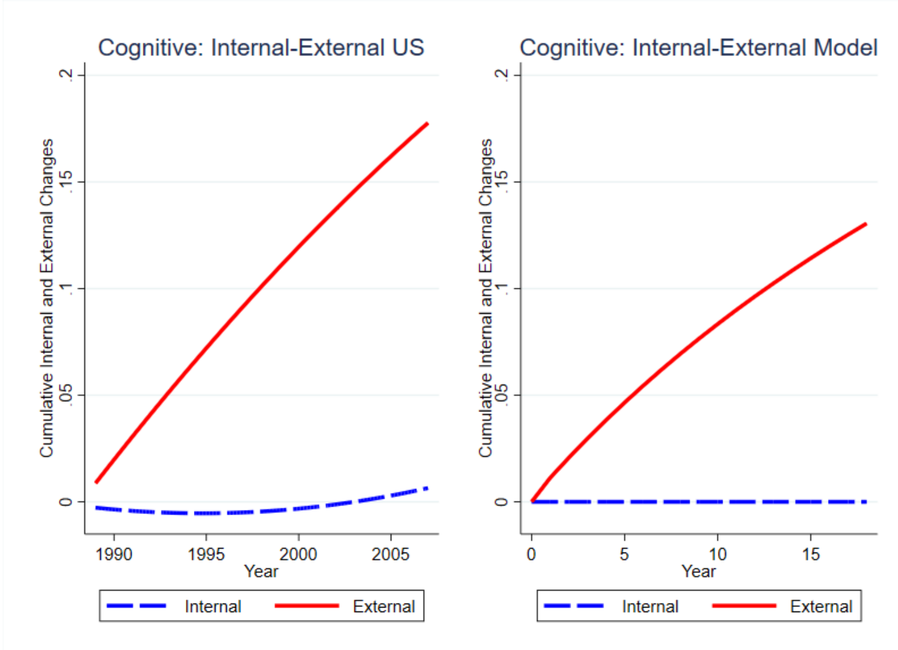


Figure 8: Cumulative Share Changes of Routine in Data versus Model: Germany

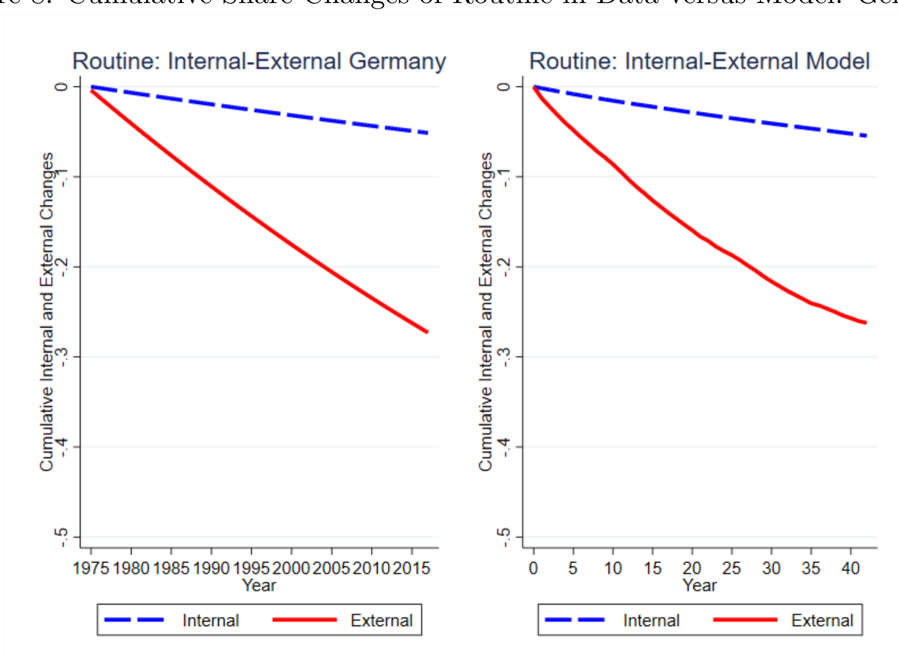


Figure 9: Cumulative Share Changes of Routine in Data versus Model: US

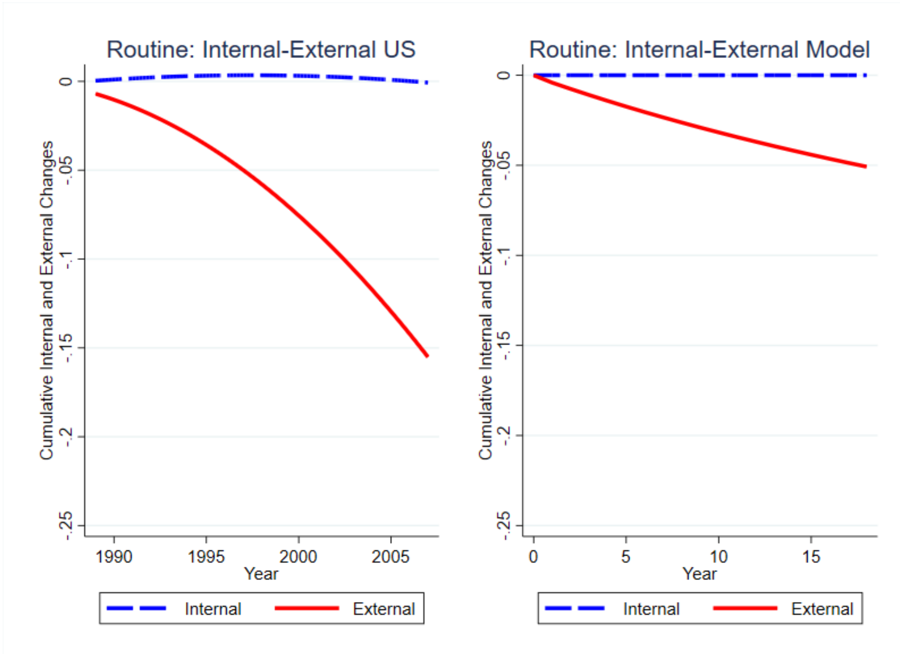


Figure 10: Cumulative Share Changes of Manual in Data versus Model: Germany

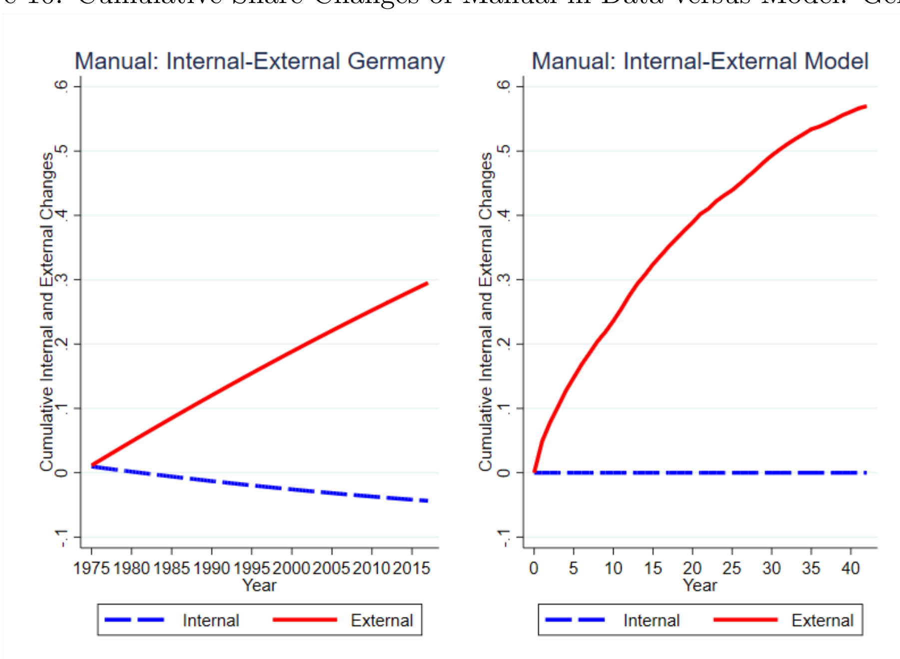
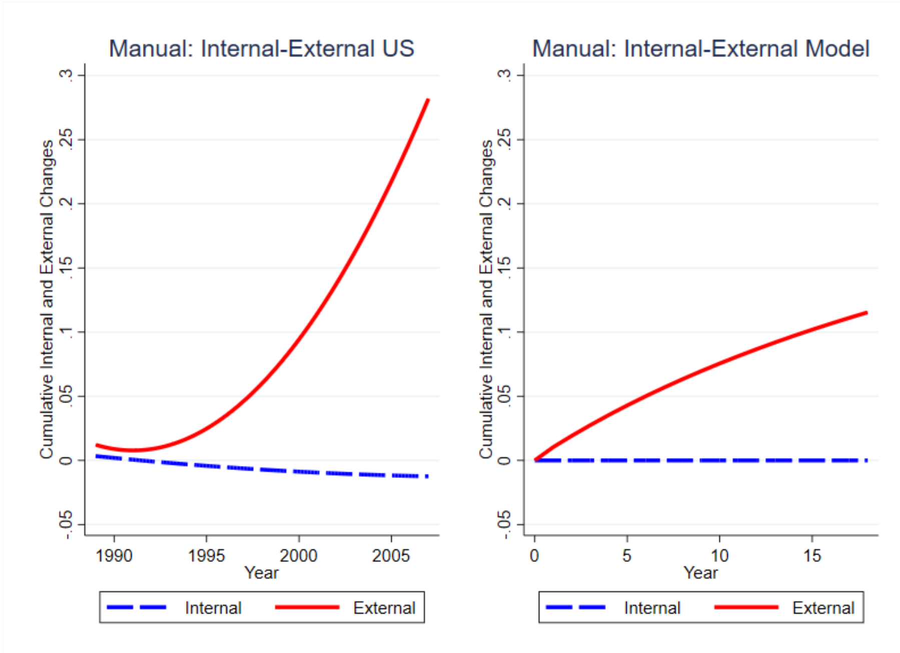


Figure 11: Cumulative Share Changes of Manual in Data versus Model: US



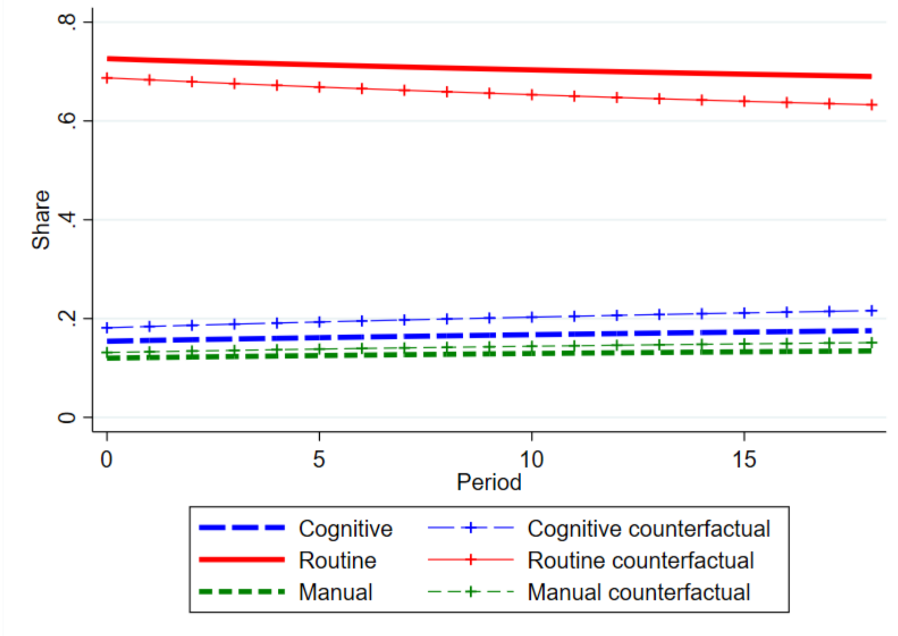
in the share of occupations. Again, the model does well in recovering the features of the data for other non-targeted components.

Finally, the summary table for model fit is provided as Table 3.

Table 3: Comparing Data and Model for Germany and the US

	Occupational employment share			Decomposed contributions	
Germany: Data	1975	2017	log (Δshare)	Internal	External
Cognitive	0.15	0.30	0.71	0.17	0.54
Routine	0.71	0.52	−0.32	−0.05	−0.27
Manual	0.14	0.18	0.26	−0.04	0.30
Germany: Model	1975	2017	log (Δshare)	Internal	External
Cognitive	0.16	0.27	0.52	0.15	0.37
Routine	0.72	0.53	−0.32	−0.05	−0.26
Manual	0.11	0.20	0.57	0.00	0.57
US: Data	1989	2007	log (Δshare)	Internal	External
Cognitive	0.25	0.30	0.17	0.01	0.17
Routine	0.62	0.54	−0.14	0.00	−0.14
Manual	0.13	0.16	0.23	−0.01	0.24
US: Model	1989	2007	log (Δshare)	Internal	External
Cognitive	0.15	0.18	0.13	0.00	0.13
Routine	0.73	0.69	−0.05	0.00	−0.05
Manual	0.12	0.13	0.12	0.00	0.12

Figure 12: Counterfactual on the Occupation Share: Introducing $\tau > 0$ in the US



5 Counterfactual experiments

The previous section demonstrated that the model can match the observed patterns of the labor market adjustment during the polarization.

Here, we further investigate the effects of these two parameters τ and κ by conducting counterfactual experiments substituting different constant values to those parameters. In the counterfactual experiments, the parameter τ (and/or κ) is set at a different value, and the economy starts from the steady state with these counterfactual parameter values. These parameter values are kept constant over time.

We present four experiments. In the first experiment, we introduce the firing tax τ in the US-calibrated economy. Next, we evaluate whether changing κ has an impact on the results of the first experiment. In the third experiment, we reduce the value of κ to half in Germany to see if the results differ from the previous experiment. In the final experiment, we reduce the value of the firing tax τ to half in Germany. For all experiments, we highlight outcomes from two separate questions: How is the speed of labor market polarization affected by the change in these parameters? How is the margin of adjustments, internal or external, affected by these parameters?

5.1 Introducing $\tau > 0$ in the US-calibrated economy

Our first experiment imposes a Germany-level firing tax ($\tau = 0.417$) on the US-calibrated economy. As in the benchmark, the reorganization cost κ is set at the same level as Germany. Figure 12 shows the path of stocks. The thick lines are the baseline US case, and the thin lines are from the counterfactual economy with $\tau = 0.417$. The result indicates that the firing tax makes the labor market more polarized: the level of routine employment is higher without the firing taxes, whereas the level of cognitive employment is lower. The speed of polarization during the transition (indicated by the absolute value of the slopes for cognitive and routine occupations) is also faster. Individual firms adjust the composition of labor *faster* when the firing tax is larger.

This result may sound counterintuitive, given that a larger τ implies greater labor market frictions. The intuition here is that the firms are forward-looking. In a more frictional economy, firms adjust their occupational composition *before* they change s_a. Firms are constantly hit by the s_h shocks and adjust employment in each occupation in response to these shocks. The timing when a positive shock to s_h hits the firm is an opportunity to expand cognitive employment. When a negative shock to s_h hits the firm, the firm has to reduce its employment (by firing taxes). It uses this occasion as an opportunity to readjust the occupational composition. A firm that is likely to adopt automation technology reduces routine employment at that time, even though s_a is not yet upgraded. Therefore, the occupational composition under frictions is naturally *more biased*.

It turns out that, although the polarization speed changes with τ, the speed of automation is almost identical between $\tau = 0.417$ and $\tau = 0$. On the one hand, the firing tax makes the reward of automation smaller and thus slows down the speed of automation. On the other hand, facing a higher firing cost, a large and unproductive firm has a large incentive to automate so that it can utilize a large employment.[22] These two forces are almost exactly offset with each other. Therefore, the difference in the speed of polarization in Figure 12 is almost entirely due to the firm's forward-looking ("precautionary") adjustment of labor given the path of automation.

Figures 13, 14, and 15 show the impact on the flow dimension. With the German level of τ and κ, the external adjustment increases. Note that with $\tau > 0$, some adjustment is done internally. It is not visible in the graph because the magnitude is still small.

Table 4 presents the impacts of introducing τ on the US aggregate variables. We compare the four variables, aggregate consumption, aggregate output, aggregate labor, and labor productivity (output divided by labor) in the end period of the above graphs. The baseline US results are

[22] A similar intuition appear in Mukoyama and Osotimehin (2019) in a model of innovation and growth.

Figure 13: Counterfactual Flow of Cognitive: Introducing $\tau > 0$ in the US

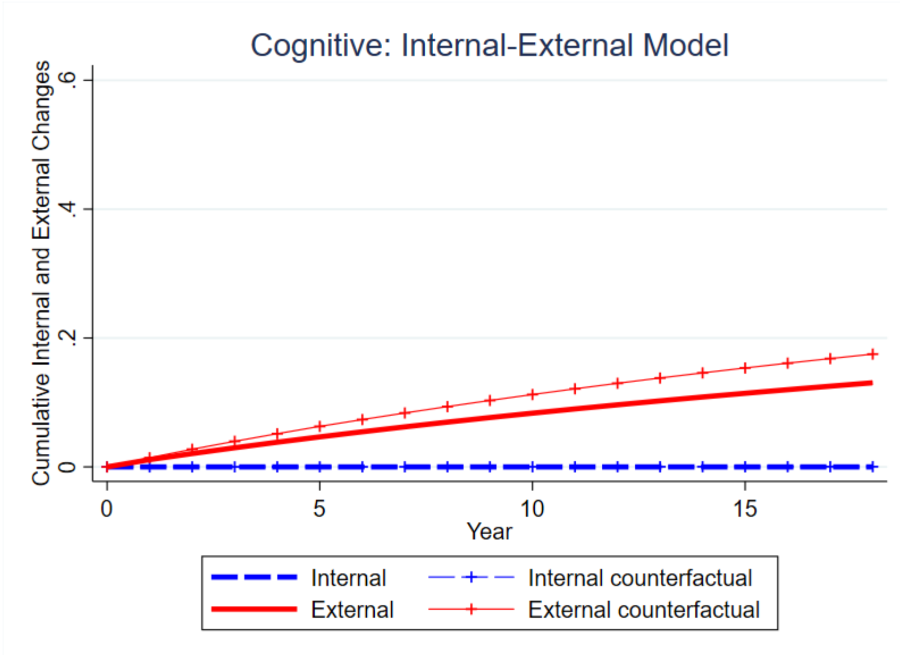


Figure 14: Counterfactual Flow of Routine: Introducing $\tau > 0$ in the US

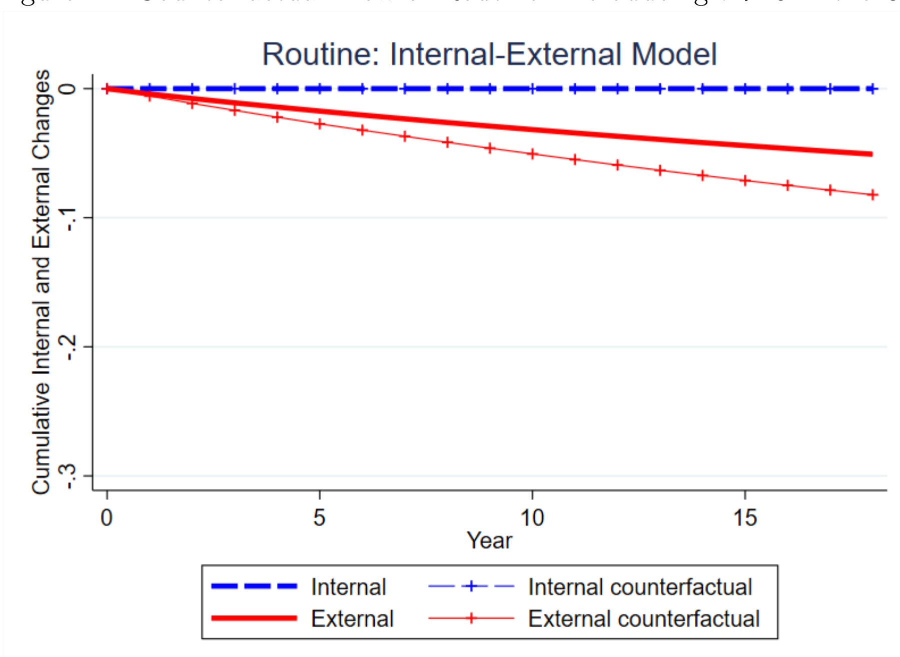
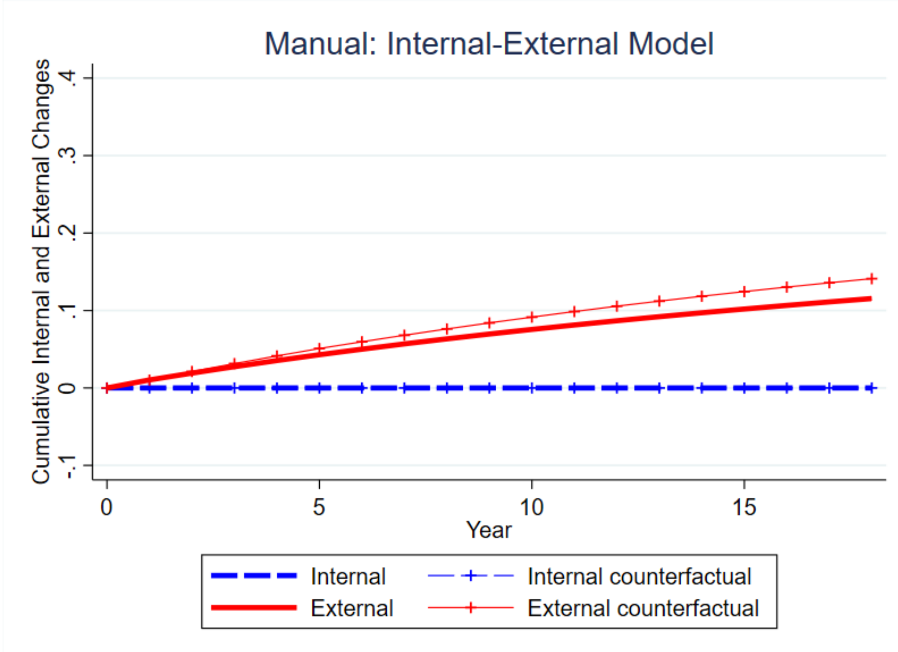


Table 4: Counterfactual Aggregate Variables: Introducing $\tau > 0$ in the US

Variable	Baseline	Counterfactual
Aggregate consumption	1.000	0.744
Aggregate output	1.000	0.744
Aggregate labor	1.000	0.603
Labor productivity	1.000	1.233

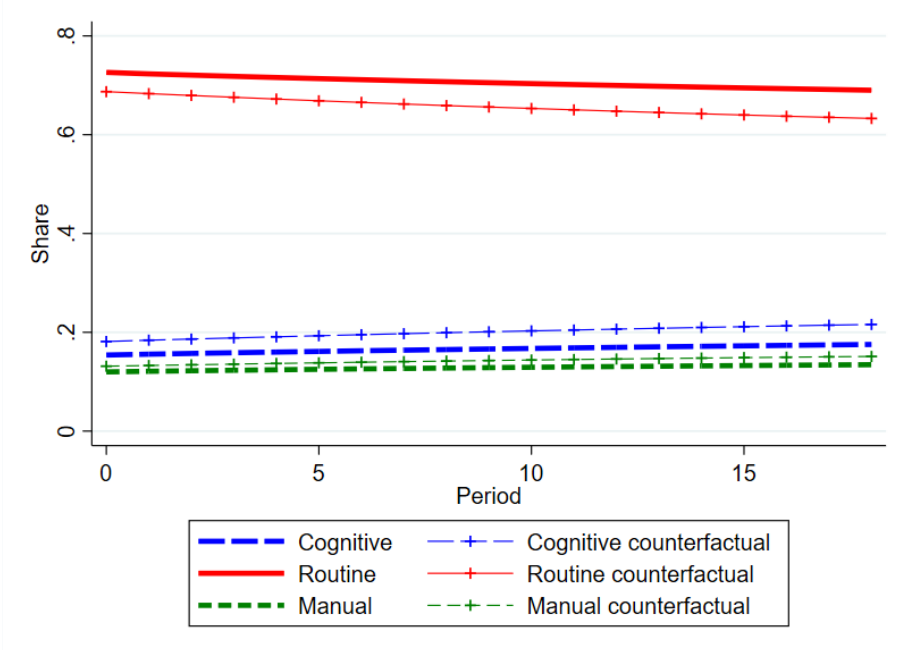
Figure 15: Counterfactual Flow of Manual: Introducing $\tau > 0$ in the US



normalized to one. As in Hopenhayn and Rogerson (1993), whether the aggregate labor input increases or decreases when τ changes is a quantitative question. There are two counteracting forces: On the one hand, the firing tax discourages firing and thus increases employment. On the other hand, the firms are forward-looking, and the future possibility of firing tax payments makes firms reluctant to hire. Overall, in our model, the latter effect is stronger in the US, and the aggregate labor input is larger with a lower firing tax. The increase in aggregate output largely reflects this result. The results thus far are similar to Hopenhayn and Rogerson (1993).

In contrast with Hopenhayn and Rogerson (1993), here the labor productivity increases with the firing tax. This result is somewhat puzzling, given the reduced across-firm misallocation with a lower τ. There are several factors that contribute to this result. One obvious factor is the decreasing returns to scale. We have a fixed number of firms with decreasing returns to scale, and a larger labor input implies lower average productivity. Second, compared to Hopenhayn and Rogerson (1993), our model is different in endogenous technology s_a and intra-firm reallocation frictions. A forward-looking reallocation in a high-τ economy implies that labor is more effectively concentrated in high-productivity firms during the transition. In this economy, the marginal product of labor is not infinity for a near zero value of labor input, given the existence of s_a, and thus, sometimes it is not worthwhile to allocate any labor to low-s_h firms. In the simulation, we observe that the high-τ economy effectively shuts down many low-s_h firms and concentrates more resources on

Figure 16: Counterfactual Occupation Share: Reducing κ by Half with $\tau > 0$



high-productivity firms.

5.2 Reducing κ in the US-calibrated economy with $\tau > 0$

The second experiment starts from the first experiment (i.e., set $\tau = 0.417$) and reduces the reorganization cost parameter κ by half ($\kappa = 260$). Figure 16 shows the time series of stocks. The figure is almost identical to Figure 12—the change in reorganization cost κ has little impact on the path of stocks. This result implies that the timing of adjustment in individual firms is not significantly affected by the value of κ in the US.

Figures 17, 18, and 19 show the effect on the margins of adjustment. Again, the thick lines are the baseline US case and thin lines represent the counterfactual economy where $\kappa = 260$. The results are virtually identical to the first experiment. Even with reduced κ, the firms are almost solely dependent on external reallocation—the amount of internal reallocation is not zero, but it is very small.

Because the occupational stock and the timing of automation are nearly identical in the first and second experiments, aggregate variables such as aggregate consumption, aggregate output, aggregate labor input, and labor productivity are also nearly identical in both experiments, and we do not report them here.

Figure 17: Counterfactual Flow of Cognitive: Reducing κ by Half in the US

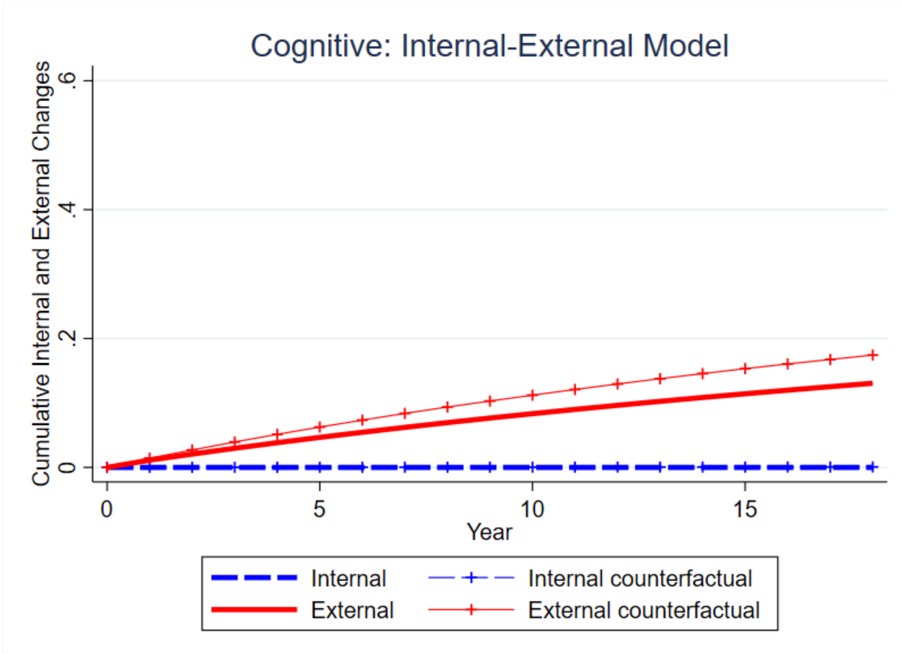


Figure 18: Counterfactual Flow of Routine: Reducing κ by Half in the US

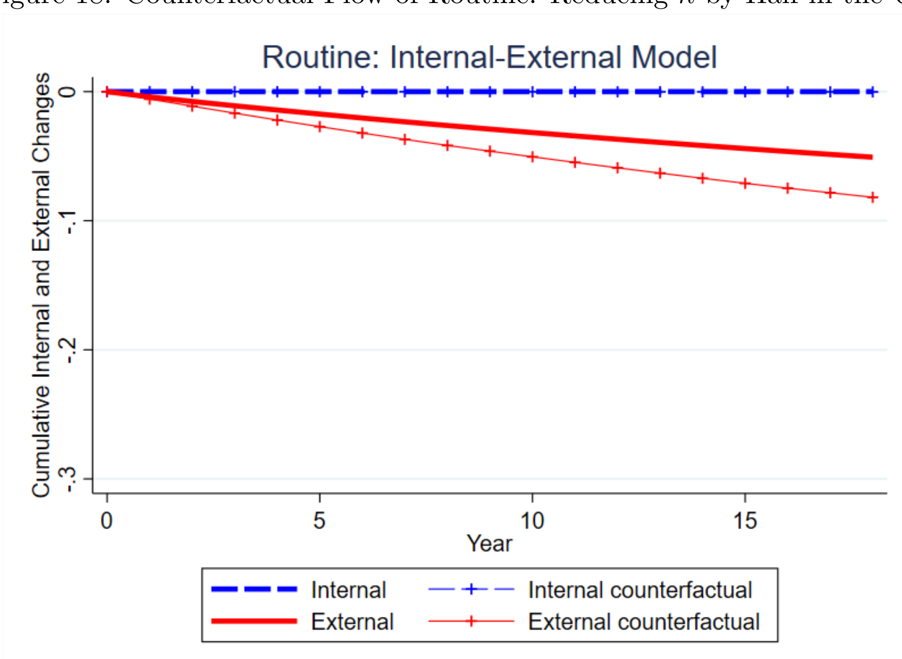
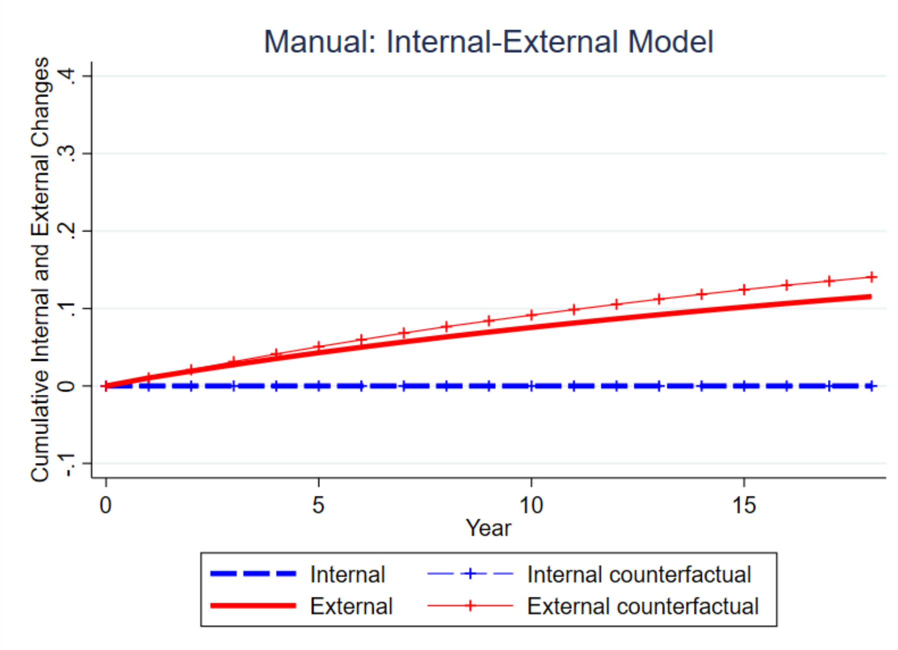


Figure 19: Counterfactual Flow of Manual: Reducing κ by Half in the US



5.3 Reducing κ in the Germany-calibrated economy

The third and fourth experiments work with the Germany-calibrated baseline. Here, we start from the German baseline and reduce the reorganization cost parameter κ by half ($\kappa = 260$). Figure 20 shows the time series of stocks. As in the second experiment for the US, the reorganization cost has little impact on the path of stocks. Because the actual German data and counterfactual are nearly identical, as the thick (baseline) and thin lines (counterfactual) overlap and are not visible separately. This result implies that the timing of adjustment in individual firms is not significantly affected by the value of κ in the Germany-calibrated model, similar to the second experiment. Overall, the effect of κ on the polarization of the labor market is limited.

The results regarding flows are very different in this experiment. Figures 21, 22, and 23 show the effect on the margins of adjustment. The thick lines are the baseline case, and the thin lines represent the counterfactual economy where $\kappa = 260$. In contrast to the US case, the effect of κ on the internal and external adjustment is both large and visible. When κ is small, the firm can shift a substantial part of the adjustment to internal worker movement. Therefore, this experiment reveals that the cost of internal adjustment plays an important role in *how* the labor market adjusts to the process of labor market polarization in the Germany-calibrated economy.

Because the occupational stock and the timing of automation are nearly identical in the baseline and the counterfactual, aggregate variables such as aggregate consumption, aggregate output,

Figure 20: Counterfactual Occupation Share: Reducing κ by Half in Germany

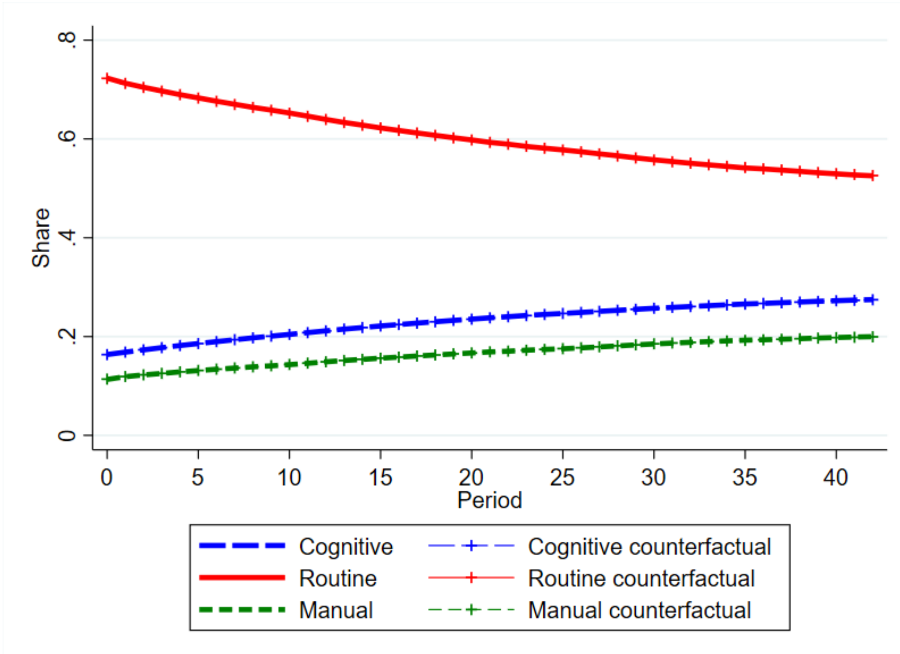


Figure 21: Counterfactual Flow of Cognitive: Reducing κ by Half in Germany

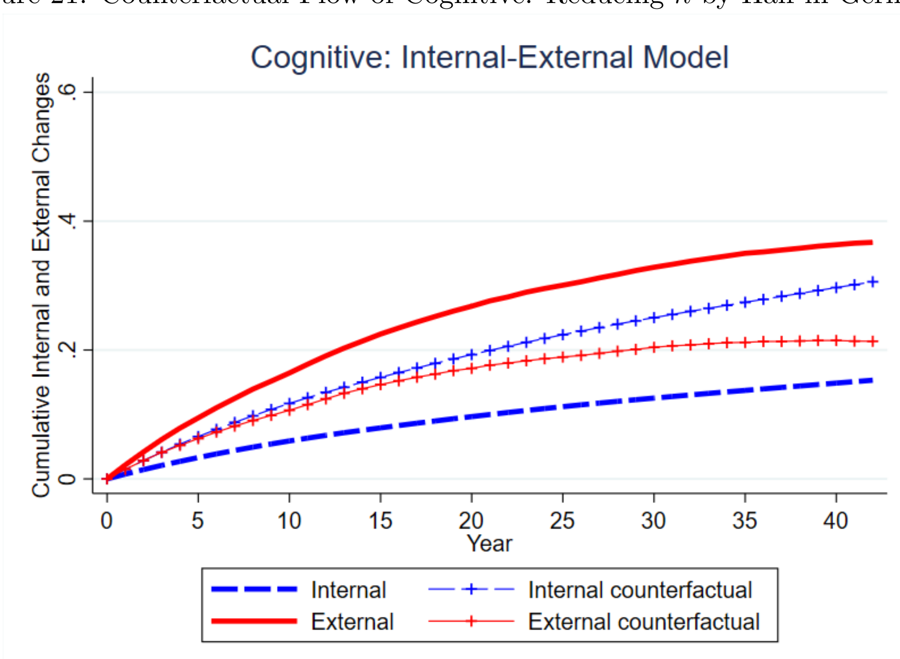


Figure 22: Counterfactual Flow of Routine: Reducing κ by Half in Germany

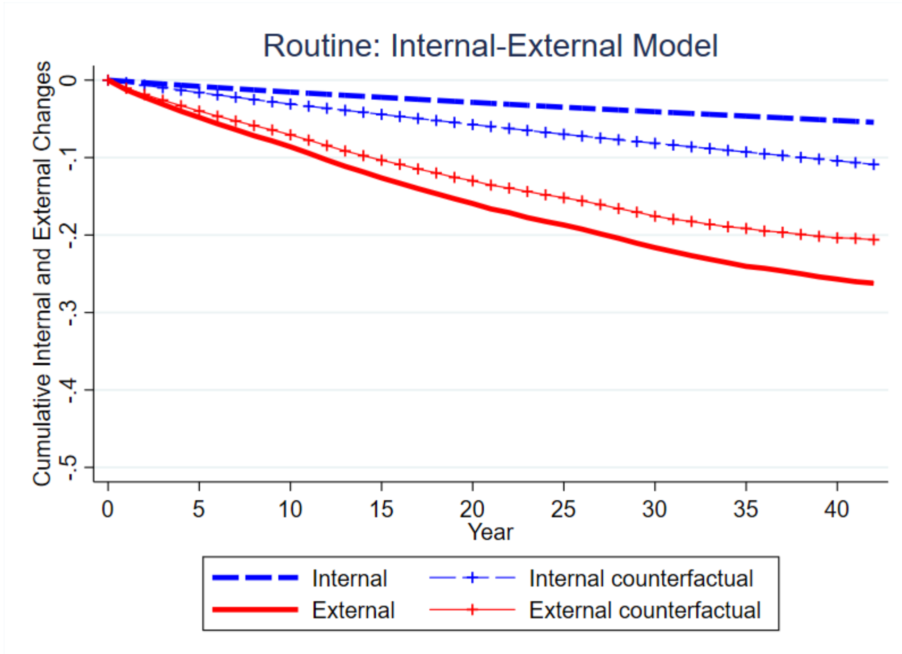


Figure 23: Counterfactual Flow of Manual: Reducing κ by Half in Germany

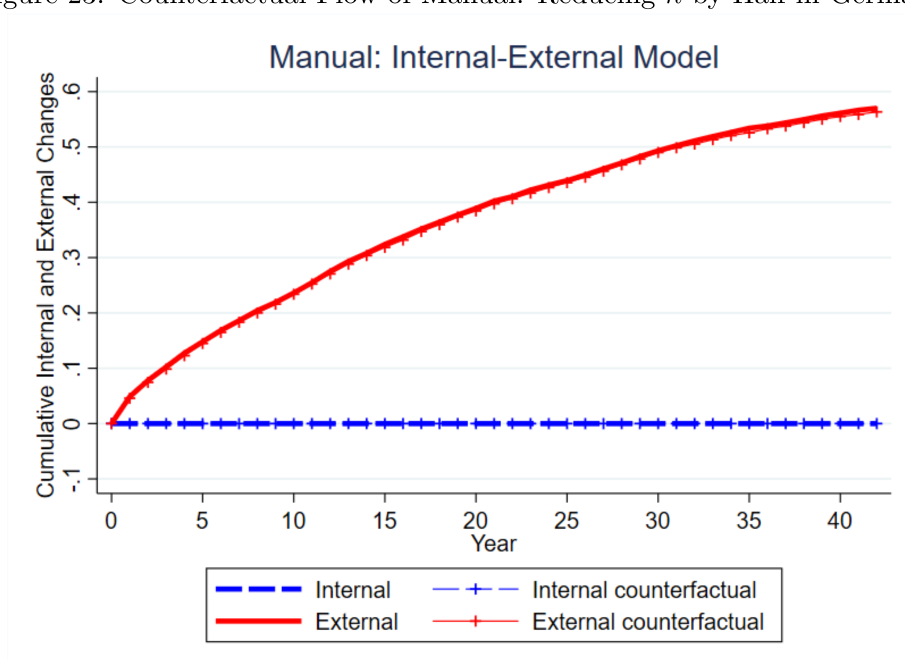
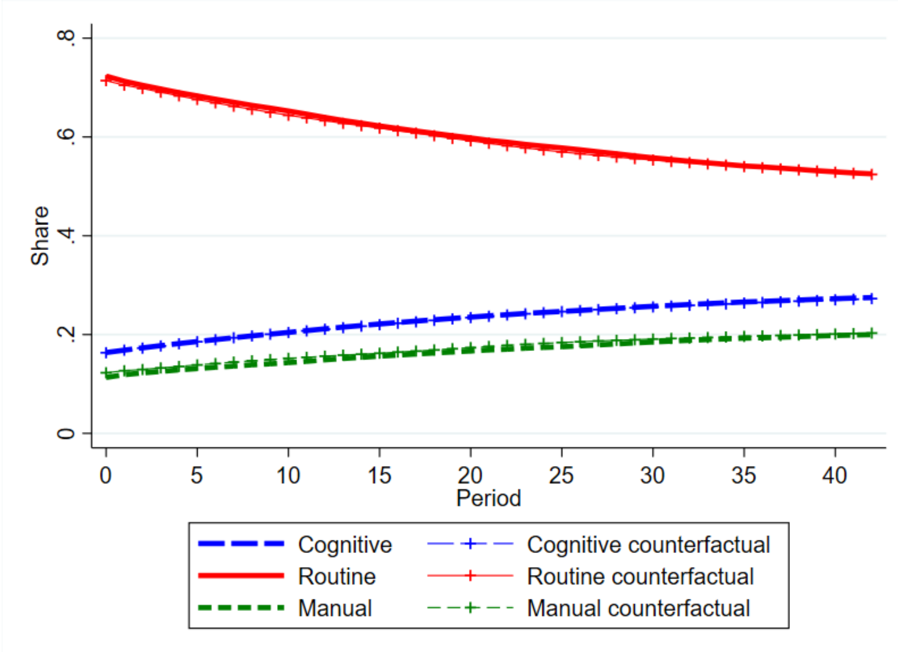


Figure 24: Counterfactual Occupation Share: Reducing τ by Half in Germany



aggregate labor input, and labor productivity are also nearly identical, and we do not report them here.

5.4 Reducing τ in the Germany-calibrated economy

The final experiment reduces the firing tax parameter τ by half ($\tau = 0.208$), keeping κ as in the baseline. Figure 24 shows the path of stocks. The thick lines are the baseline German case, and the thin lines are from the counterfactual economy with $\tau = 0.208$. In contrast to the US case, the firing tax does not have substantial impacts on the path of stock (the lines are not separately visible). Recall that the difference between the US-calibrated economy and the Germany-calibrated economy (other than the values of τ) comes from the process of s_h shock. In the German case, the s_h shock is less volatile than in the US case, and hence, the firm's adjustment of occupational composition with the s_h shock, which was the driving force of the different speed of polarization in the first experiment, is smaller in the current experiment.

Figures 25, 26, and 27 show the impact on the flows. In contrast to the first experiment, the change in internal adjustment is now visibly large in terms of cognitive and routine. The increase in the external adjustment, due to lower τ, is offset by the increase in the internal adjustment.

Table 5 presents the impacts of reducing τ on the productivity. We compare the three variables, aggregate output, aggregate labor, and labor productivity (output divided by labor), in the final

Figure 25: Counterfactual Flow of Cognitive: Reducing τ by Half in Germany

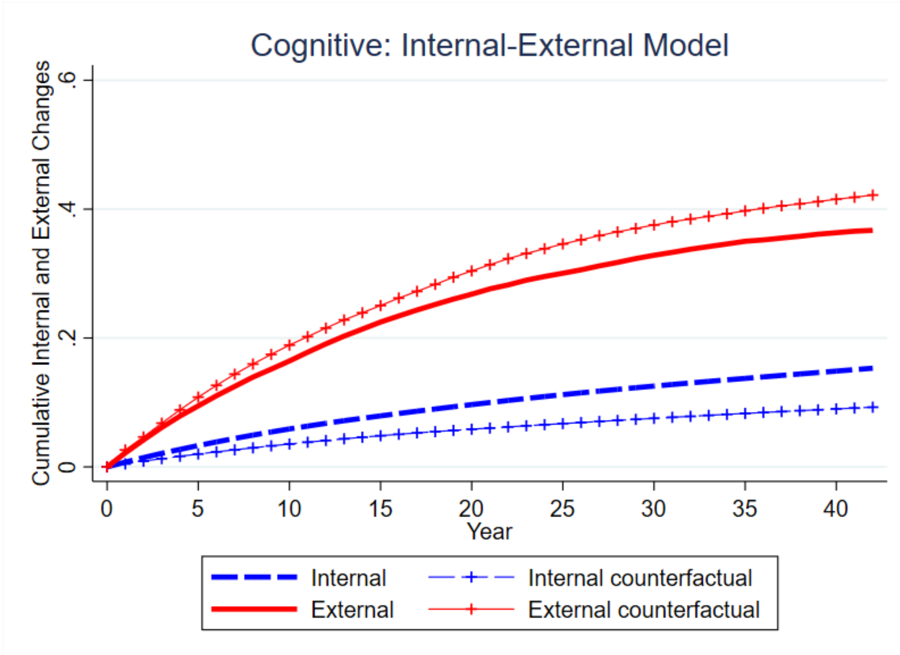


Figure 26: Counterfactual Flow of Routine: Reducing τ by Half in Germany

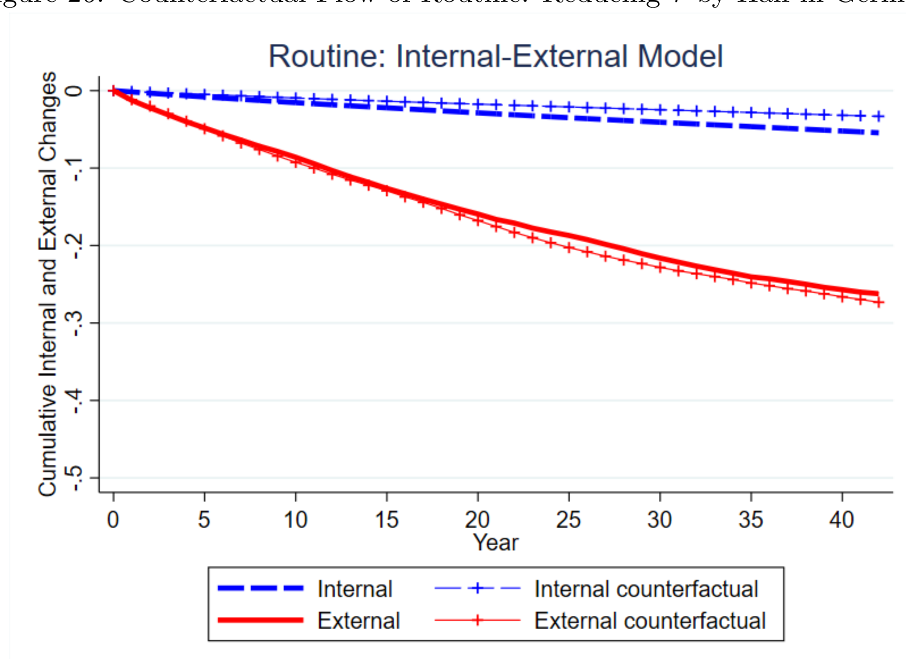
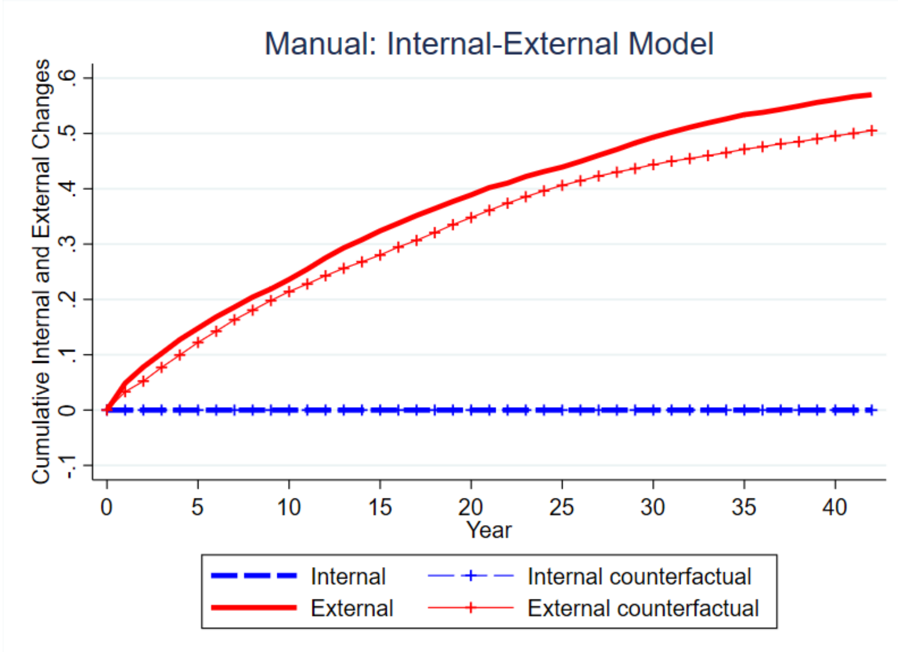


Table 5: Counterfactual Aggregate Variables: Reducing τ by Half

Variable	Baseline	Counterfactual
Aggregate consumption	1.000	1.002
Aggregate output	1.000	1.002
Aggregate labor	1.000	1.003
Labor productivity	1.000	0.999

Figure 27: Counterfactual Flow of Manual: Reducing τ by Half in Germany



steady state. The baseline German results are normalized to one. Noting that the first experiment raises the firing tax rate and this experiment reduces the rate, the results are qualitatively similar to each other, although the magnitudes are substantially smaller here.

6 Conclusion

We analyze how labor-market frictions interact with firms' decisions to reallocate workers across occupations when the economy faces labor-market polarization. Using datasets from the US and Germany, we document that the pattern of occupational adjustments differs between these two countries. Although the aggregate pattern of labor-market polarization is very similar between the two countries, US firms adjust the occupational mix almost entirely through firing and hiring. In Germany, within-firm reallocation plays non-negligible roles in the decline in routine occupations and the increase in cognitive occupations.

We then build a model of firm dynamics with occupational mobility and labor-market frictions. Our model extends the standard firm-dynamics model in the tradition of Hopenhayn (1992) and Hopenhayn and Rogerson (1993) to multiple occupations and automation decisions. We calibrate the model to the German economy and can replicate the different patterns of labor market adjustments during the labor market polarization across the US and Germany.

Using the calibrated model, we conduct two counterfactual experiments for each country. We find that the within-firm reorganization cost has a small impact on the degree of polarization, whereas the firing cost has a significant impact on polarization in the US. In particular, we find that the firing tax makes the labor market *more* polarized in the US-calibrated economy: the level of routine employment is higher without the firing taxes, whereas the level of cognitive employment is lower. Individual firms adjust the composition of occupational employment faster when the firing tax is larger. The reason for this seemingly counterintuitive result is that the firms are forward-looking. In the model, firms are constantly hit by idiosyncratic productivity shocks. Thus, when there is a firing tax, firms that are likely to adopt automation technology will reduce routine hires when they suffer a negative idiosyncratic shock, seeing it as an opportunity to prepare for future automation adoption. On the other hand, without a firing tax, the firm is more likely to keep the routine workers because the firm can easily adjust the occupational composition in the future. This result has important implications for predicting how policies like the firing tax affect the polarizing labor market.

There are several issues that are important to investigate along the lines of our research. First, our model does not feature the entry and exit of firms. How firm entry/exit interacts with worker mobility is an interesting and important question, especially when new technology (such as automation) is embodied in new firms. Second, it is often argued that an illiquid labor market may have the benefit of encouraging firm-specific human capital accumulation. It has long been debated in labor economics how important firm-specific human capital is, and investigating such claims requires further examination of the nature of human capital. Finally, the distinction of within- and across-firm reallocation also matters in the context of aggregate unemployment. One may easily imagine that one of the social costs of across-firm labor adjustments can be the unemployment of routine workers. Our model framework does not feature unemployment, although it is possible to extend the model by adding friction in hiring workers. We leave these topics to future research.

References

Acemoglu, D., and D. Autor (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," *Handbook of Labor Economics*, 4, 1043–1171.

Antoni, M., T. Graf, S. Griessemer, S. Kaimer, M. Köhler, C. Lehnert, M. Oertel, A. Schmucker, S. Seth, C. Seysen, and P. vom Berge (2019): "Weakly Anonymous Version of the Sample of Integrated Labour Market Biographies (SIAB) – Version 7517 v1," Research Data Centre of the Federal Employment Agency (BA) at the Institute for Employment Research (IAB). DOI: 10.5164/IAB.SIAB7517.de.en.v1 (The data access was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and subsequently remote data access.).

Autor, D. H., and D. Dorn (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," *American Economic Review*, pp. 1553–1597.

Autor, D. H., L. F. Katz, and M. S. Kearney (2006): "The Polarization of the US Labor Market," *American Economic Review*, 96, 189–194.

Bachmann, R., and C. Bayer (2013): "'Wait-and-See' Business Cycles?," *Journal of Monetary Economics*, 60, 704–719.

Battisti, M., C. Dustmann, and U. Schönberg (2017): "Technological and Organizational Change and the Careers of Workers," Working paper.

Behaghel, L., E. Caroli, and E. Walkowiak (2012): "Information and Communication Technologies and Skill Upgrading: the Role of Internal vs External Labour Markets," *Oxford Economic Papers*, pp. 490–517.

Blanchard, O., and J. Wolfers (2000): "The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence," *The Economic Journal*, pp. C1–C33.

Böhm, M. J., H.-M. von Gaudecker, and F. Schran (2024): "Occupation Growth, Skill Prices, and Wage Inequality," *Journal of Labor Economics*, 42(1), 201–243.

Carrillo-Tudela, C., L. Visschers, and D. Wiczer (2022): "Cyclical Earnings, Career and Employment Transitions," Working paper.

Cortes, G. M., N. Jaimovich, and H. Siu (2017): “Disappearing Routine Jobs: Who, How, and Why?,” *Journal of Monetary Economics*, 91, 69–87.

Dauth, W., S. Findeisen, J. Suedekum, and N. Woessner (2018): “Adjusting to Robots: Worker-Level Evidence,” Working paper.

Drew, J. A. R., S. Flood, and J. R. Warren (2014): “Making Full Use of the Longitudinal Design of the Current Population Survey: Methods for Linking Records across 16 Months,” *Journal of Economic and Social Measurement*, 39, 121–144.

Eden, M., and P. Gaggl (2018): “On the Welfare Implications of Automation,” *Review of Economic Dynamics*, 29, 15–43.

Garibaldi, P., and G. L. Violante (2005): “The Employment Effects of Severance Payments with Wage Rigidities,” *Economic Journal*, 115, 799–832.

Goerkea, L., and M. Pannenberg (2010): “An Economic Analysis of Dismissal Legislation: Determinants of Severance Pay in West Germany,” *International Review of Law and Economics*, 30, 71–85.

Goos, M., and A. Manning (2007): “Lousy and Lovely Jobs: The Rising Polarization of Work in Britain,” *Review of Economics and Statistics*, 89, 118–133.

Goos, M., A. Manning, and A. Salomons (2009): “Job Polarization in Europe,” *American Economic Review*, 99, 58–63.

Hopenhayn, H., and R. Rogerson (1993): “Job Turnover and Policy Evaluation: A General Equilibrium Analysis,” *Journal of Political Economy*, 101, 915–938.

Hopenhayn, H. A. (1992): “Entry, Exit, and Firm Dynamics in Long Run Equilibrium,” *Econometrica*, 60, 1127–1150.

Hornstein, A., P. Krusell, and G. L. Violante (2007): “Technology-Policy Interaction in Frictional Labour-Markets,” *Review of Economic Studies*, pp. 1089–1124.

Humlum, A. (2021): “Robot Adoption and Labor Market Dynamics,” Working paper.

Jaimovich, N., I. Saporta-Eksten, H. Siu, and Y. Yedid-Levi (2021): “The Macroeconomics of Automation: Data, Theory, and Policy Analysis,” *Journal of Monetary Economics*, 122, 1–16.

Kambourov, G., and I. Manovskii (2009): “Occupational Specificity of Human Capital,” *International Economic Review*, 50(1), 63–115.

——— (2013): “A Cautionary Note on Using (March) CPS and PSID Data to Study Worker Mobility,” *Macroeconomic Dynamics*, 17, 172–194.

Lee, S. Y. T., and Y. Shin (2017): “Horizontal and Vertical Polarization: Task-specific technological change in a multi-sector economy,” Working paper.

Lee, Y., and T. Mukoyama (2015): “Productivity and Employment Dynamics of US Manufacturing Plants,” *Economics Letters*, 136, 190–193.

Ljungqvist, L., and T. J. Sargent (1998): “The European Unemployment Dilemma,” *Journal of Political Economy*, pp. 514–550.

Mortensen, D. T., and C. A. Pissarides (1999): “Unemployment Responses to’Skill-Biased’Technology Shocks: The Role of Labour Market Policy,” *Economic Journal*, 109(455), 242–265.

Moscarini, G., and K. Thomsson (2007): “Occupational and Job Mobility in the US,” *Scandinavian Journal of Economics*, 109, 807–836.

Mukoyama, T., and S. Osotimehin (2019): “Barriers to Reallocation and Economic Growth: The Effects of Firing Costs,” *American Economic Journal: Macroeconomics*, 11, 235–270.

Restrepo, P. (2023): “Automation: Theory, Evidence, and Outlook,” Nber working paper 31910.

Rodrigo, R. (2021): “Robot Adoption, Organizational Capital, and the Productivity Paradox,” Working paper.

Stinson, M. H. (2003): “Technical Description of SIPP Job Identification Number Editing in the 1990-1993 SIPP Panels,” Working paper.

Veracierto, M. (2001): “Employment Flows, Capital Mobility, and Policy Analysis,” *International Economic Review*, 42, 571–596.

vom Lehn, C. (2020): “Labor Market Polarization, the Decline of Routine Work, and Technological Change: A Quantitative Analysis,” *Journal of Monetary Economics*, 110, 62–80.

Xiong, H. (2008): “The US Occupational Mobility from 1988 to 2003: Evidence from SIPP,” Working paper.

A Data details

A.1 Survey of Income and Program Participation (SIPP)

A.1.1 Data description

SIPP is a dataset of household-based panel survey, administrated by the US Census Bureau. We use the following seven panels from the SIPP for our analysis: 1990, 1991, 1992, 1993, 1996, 2001, and 2004. These panels have a sample of 14,000 to 52,000 individuals. Each panel is a nationally representative sample of households interviewed every four months. Individuals are asked to provide their employment information as detailed as in a weekly basis. With these SIPP panels, we identify the workers' job and occupation switches on an annual basis.

As noted in Stinson (2003), the 1990–1993 panels had substantial miscoding in their job IDs. Thus, we use the revised job IDs provided by the US Census Bureau. We do not use the panels before 1990, as no revised job IDs are provided. We are not able to use the 2008 panel because the US Census Bureau's data cleaning procedure has made occupational switches within firms unidentifiable for that panel.

A.1.2 Sample selection

We select observations where an individual is between ages 23 and 55. We drop observations where an individual works in the public sector or is self-employed. We also drop observations where no occupation information is available. We only focus on individuals who report valid employment status.

A.1.3 Data cleaning

In the SIPP, workers are asked to list up to two employers for each week. When a worker has two occupations at the same time, we select the occupation for the greater number of hours worked. We drop the observations with managerial occupations to eliminate the flows due to promotions. Those managerial occupations include:

- Legislators,
- Chief executives and general administrators, public administration
- Administrators and officials, public administration
- Administrators, protective services

- Financial managers
- Personnel and labor relations managers
- Purchasing managers
- Managers, marketing, advertising, and public relations
- Administrators, education, and related fields
- Managers, medicine and health
- Postmasters and mail superintendents
- Managers, food serving and lodging establishments
- Managers, properties and real estate, funeral directors
- Managers, service organizations, n.e.c.
- Managers and administrators, n.e.c.

A.1.4 Attrition

One of the major problems in longitudinal survey data is that individuals can drop from the sample over time. The SIPP is not exempt from this problem as well, which creates biases in the decomposition results. Therefore, we run a robustness check by running the decomposition with the balanced panels of the SIPP in Appendix C.4.

A.1.5 Identifying job and occupational switches

We follow Xiong (2008) to identify occupational switches of workers. We first define the three broad occupational groups as listed in Appendix A.4. When a person reports multiple occupations, we use the one for the job that reports the largest number of hours worked in the month. Keeping the monthly frequency of the SIPP panel, we then identify the occupational switches by comparing the occupation of the worker in the current month and the 12 months ago. The identified switches are then aggregated to the annual frequency.

The identified occupational switches are classified into within-firm and across-firm switches by using the Job ID. The within-firm switches are the switches where the worker stays in the same firm. The across-firm switches are the switches where the worker moves to a different firm.

The literature widely acknowledges that measurement errors in occupational codes can lead to spurious transitions, as highlighted in studies such Kambourov and Manovskii (2009) and Moscarini and Thomsson (2007). Our approach here is similar to that in Carrillo-Tudela, Visschers, and Wiczer (2022): We use a high degree of aggregation (i.e., three broad occupational groups) to minimize the coding errors. Additionally, since 1986, the SIPP interviewing process has incorporated a practice known as "dependent interviewing," wherein if a worker confirms no change in job type or employer from the previous interview, the occupational code from the prior interview is retained. This method significantly reduces erroneous occupational transitions, particularly among those switching jobs within the same firm.

A.2 Sample of Integrated Labor Market Biographies (SIAB)

A.2.1 Data description

We utilize the Sample of Integrated Labour Market Biographies (SIAB) spanning from 1975 to 2017 for our analysis of German labor markets. This dataset is provided by the Institute for Employment Research (IAB) in Germany. It constitutes a 2% sample of the Integrated Employment Biographies (IEB) population, encompassing employees covered by social security, individuals engaged in marginal part-time employment (since 1999), recipients of unemployment insurance benefits, and those officially registered as job-seeking or participating in active labor market policy programs. Excluded from this dataset are the self-employed, civil servants, individuals in military service, and those not actively participating in the labor force.

A.2.2 Sample selection

We select individuals who have German citizenship and have never worked or resided in East Germany. We then select observations where the individual is between ages 23 and 55. We drop observations where no occupation information is available.

A.2.3 Data cleaning

We look at a worker's labor market information at the beginning of each calendar year. If a worker has multiple jobs, to identify the main occupation, we select an occupation that is associated with the highest wage per day. We drop the observations of managerial occupations to eliminate the flows due to promotions. Those managerial occupations include:

- Foremen, master mechanics

- Entrepreneurs, managing directors
- Members of Parliament, ministers
- Senior government officials
- Association leaders, officials

A.2.4 Attrition

Workers may disappear from the social security records for various reasons (leave the labor force, migrate abroad, become a public servant or self-employed, or pass away). The IAB is adding new individuals to the sample every year to keep it as 2% of the entire population in Germany.

A.2.5 Identifying job and occupational switches

To identify occupational switches of workers, we first define the three broad occupational groups as listed in Appendix A.4 following follow Böhm, von Gaudecker, and Schran (2024). We then look at the worker's labor market status and information at the beginning of a calendar year. When a worker reports multiple occupations, we use the one for the job that reports the highest wage per day. Keeping the annual frequency of the SIAB panel, we then identify the occupational switches by comparing the occupation of the worker in the current year and the previous year. We classify within-firm and across-firm switches by using the establishment IDs.

A.3 Current Population Survey (CPS)

A.3.1 Data description

CPS, administered by the US Census Bureau, is conducted with a sample of around 60,000 households and consists of the basic monthly questions focusing on labor force participation and supplemental questions, such as the annual March income supplement. Each individual shows up in the records at most eight times: respondents are contacted monthly for the first four consecutive months, followed by eight months of gap, and then the monthly interview resumes for the last four months. We use the Public Use Microdata File of the Basic Monthly CPS files from January 1994 to October 2019, which are obtained from the DataWeb FTP of the US Census Bureau. The respondents are matched based on Drew, Flood, and Warren (2014).

A.3.2 Sample selection

For comparability with the SIPP estimates, we restrict our focus to males between the ages of 23 and 55. We drop observations where an individual works in the public sector or is self-employed. We also drop observations where no occupation information is available.

A.4 Occupational groups

A.4.1 US

We classify occupations into the three broad groups, as defined by Acemoglu and Autor (2011). For the SIPP and CPS, we aggregate the US Census' 1990/2000 Occupational Classification codes into these three broader categories:

1. Nonroutine cognitive: professional, technical, management, business , and financial occupations.

2. Routine: clerical, administrative support, sales workers, craftsmen, foremen, operatives, installation, maintenance and repair occupations, production and transportation occupations, laborers.

3. Nonroutine manual: service workers.

A.4.2 Germany

For the SIAB, we follow Böhm, von Gaudecker, and Schran (2024) to group three-digit occupations (120 occupations according to the KLDB1988 classification) into nine categories and define the three groups, which correspond to those in Acemoglu and Autor (2011), as follows:

1. Nonroutine cognitive: managers, professionals, and technicians.

2. Routine: craftspeople, sales personnel, office workers, production workers, operations, and laborers.

3. Nonroutine manual: service personnel.

B Decomposition method

Let ℓ_{it} be the stock of employment of occupation i at time t. Further, let

$$E_t \equiv \sum_{i=c,r,m} \ell_{it}$$

be the employment. The employment share at time t for occupation i is

$$\frac{\ell_{it}}{E_t}.$$

We want to decompose

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right)$$

into net flows.

$$\log(\ell_{it}) = \log\left(\sum_{j=c,r,m,k=s,d} f^{ji,k}_{t-1,t} + f^{Ui}_{t-1,t}\right) = \log\left(\sum_{j=c,r,m,k=s,d} f^{ij,k}_{t,t+1} + f^{iU}_{t,t+1}\right).$$

Here, U includes unemployment, out-of-labor force, and dropped/added sample. s is for the same firm, and d is for the different firm. Thus,

$$\begin{aligned}
\log(\ell_{i,t+1}) - \log(\ell_{it}) &= \log\left(\frac{\sum_{j=c,r,m,k=s,d} f^{ji,k}_{t,t+1} + f^{Ui}_{t,t+1}}{\sum_{j=c,r,m,k=s,d} f^{ij,k}_{t,t+1} + f^{iU}_{t,t+1}}\right) \\
&= \log\left(1 + \frac{\sum_{j\neq i,k=s,d}(f^{ji,k}_{t,t+1} - f^{ij,k}_{t,t+1}) + (f^{Ui}_{t,t+1} - f^{iU}_{t,t+1})}{\sum_{j=c,r,m,k=s,d} f^{ij,k}_{t,t+1} + f^{iU}_{t,t+1}}\right) \\
&\approx \frac{\sum_{j\neq i,k=s,d}(f^{ji,k}_{t,t+1} - f^{ij,k}_{t,t+1}) + (f^{Ui}_{t,t+1} - f^{iU}_{t,t+1})}{\ell_{it}}
\end{aligned}$$

Note also that

$$\begin{aligned}
\log(E_{t+1}) - \log(E_t) &\approx \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \ell_{it} \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \left(\sum_{j=c,r,m,k=s,d} f^{ij,k}_{t,t+1} + f^{iU}_{t,t+1}\right) \frac{E_{t+1} - E_t}{E_t}
\end{aligned}$$

Let

$$\Delta^E_{t,t+1} \equiv \frac{E_{t+1} - E_t}{E_t}.$$

Combining the above, we have

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right)-\log\left(\frac{\ell_{it}}{E_t}\right)=\frac{1}{\ell_{it}}\left[\sum_{j\neq i}(f^{ji,s}_{t,t+1}-f^{ij,s}_{t,t+1})+\sum_{j\neq i}(f^{ji,d}_{t,t+1}-f^{ij,d}_{t,t+1})+(f^{Ui}_{t,t+1}-f^{iU}_{t,t+1})-\ell_{it}\Delta^E_{t,t+1}\right].$$

To calculate the cumulative changes from period t to period $t+T$, note that

$$\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right)-\log\left(\frac{\ell_{it}}{E_t}\right)=\sum_{\tau=0}^{T-1}\left[\log\left(\frac{\ell_{i,t+\tau+1}}{E_{t+\tau+1}}\right)-\log\left(\frac{\ell_{i,t+\tau}}{E_{t+\tau}}\right)\right].$$

Then, we can apply the decomposition formula to obtain

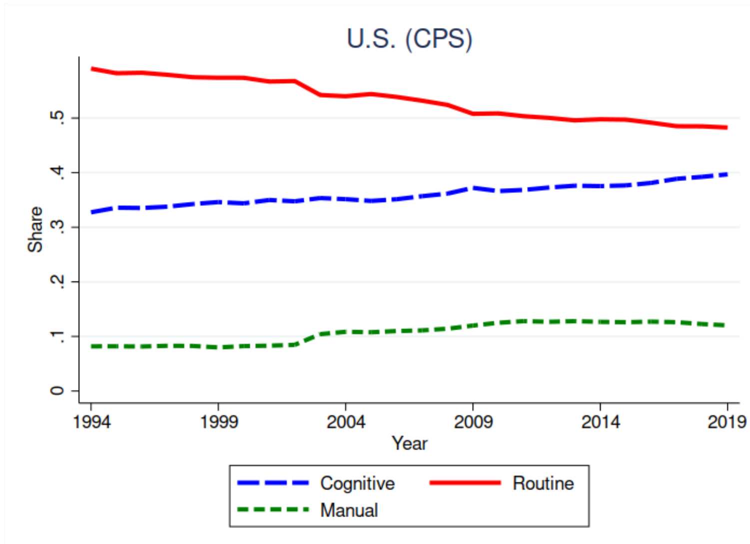
$$\begin{aligned}
&\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right)-\log\left(\frac{\ell_{it}}{E_t}\right)\\
&=\Bigg[\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,s}_{t+\tau,t+\tau+1}-f^{ij,s}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}+\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,d}_{t+\tau,t+\tau+1}-f^{ij,d}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}\\
&\quad+\sum_{\tau=0}^{T-1}\frac{f^{Ui}_{t+\tau,t+\tau+1}-f^{iU}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}-\sum_{\tau=0}^{T-1}\Delta^E_{t+\tau,t+\tau+1}\Bigg].
\end{aligned}$$

C Robustness of the empirical results

C.1 Occupational Employment Shares from the CPS

In this subsection, we show the results of the changes in occupational employment share using the Current Population Survey (CPS) data. Figure 28 show the pattern. The results are consistent with the SIPP results in Figure 1 in the main text.

Figure 28: Occupational Employment Shares in the US, CPS, 1994–2019



Data Source: CPS

C.2 Decomposition results with detailed external flows

In this section, we provide detailed decomposition results for the US and Germany. Table 6 corresponds to Table 1 in the main text, but with the breakdowns of the external reallocation to the job-to-job (EE) flows, the flows into/exit from unemployment (U), the flows into/exit from out of labor force (OLF). We include the effect from the size of employment Δ^E to the last term. Due to the low frequency of observations out of the unemployment state in the earlier period, we start in 1977 for the German data to comply with the disclosure policy of the SIAB.

We found that flows into/exit from out of labor force (OLF) are the most important component of the external reallocation both in the US and Germany. The second largest component is the job-to-job (EE) flows for the US, while in Germany, the flows into unemployment (U) play a significant role in external reallocation for all occupations. On the other hand, the flows into/exit from unemployment (U) are the smallest component of the external reallocation in the US.

Table 6: Decompositions of Occupational Employment Share Changes for the US and Germany

Occupational employment share					
	(3)	(4)		(5)	
US (SIPP)	log (Δshare)	Internal		External	
1989–2007			EE	U	OLF
Cognitive	0.17	0.01	0.05	0.01	0.11
Routine	−0.14	0.00	−0.01	0.02	−0.15
Manual	0.23	−0.01	−0.05	0.02	0.27
Germany (SIAB)	log (Δshare)	Internal		External	
1977–2017			EE	U	OLF
Cognitive	0.66	0.17	0.13	−0.15	0.51
Routine	−0.29	−0.04	−0.02	−0.28	0.05
Manual	0.31	−0.04	−0.04	−0.21	0.59

Data Source: SIPP (US); SIAB (Germany)

C.3 Effects of demographics and industry composition

To see the extent to which the differences in the demographic composition (age, education, and industry) can explain the differences in the reallocation patterns between the US and Germany, we conduct the following experiments. We first calculate the stock and the flow variables in the decomposition formula (1) for the US by age, education, and industry. We use four groups for age (23-29, 30-39, 40-49, and 50-55), two groups for education (university graduates and others), and three groups for industry (agriculture and mining, manufacturing, services). We then take the weighted average of the stock and flow variables so that the age, education, or industry characteristics of the US become the same as that of Germany for each year during the period 1989–2007.

Table 7 summarizes the results of the experiments. We found that the differences in age, education, and industry composition can not entirely explain the differences in the internal-external reallocation patterns between the US and Germany.

Table 7: Age, Skill, and Industry Composition for the US

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
US	1989	2007	$\log(\Delta\text{share})$	Internal	External
Cognitive	0.25	0.30	0.173	0.006	0.167
Routine	0.62	0.54	−0.140	0.001	−0.140
Manual	0.13	0.16	0.230	−0.013	0.243
US: Age	1989	2007	$\log(\Delta\text{share})$	Internal	External
Cognitive	0.25	0.30	0.166	0.008	0.157
Routine	0.62	0.54	−0.137	−0.001	−0.137
Manual	0.13	0.16	0.234	−0.016	0.250
US: Education	1989	2007	$\log(\Delta\text{share})$	Internal	External
Cognitive	0.14	0.18	0.247	0.042	0.205
Routine	0.71	0.63	−0.128	−0.006	−0.121
Manual	0.15	0.19	0.267	−0.017	0.305
US: Industry	1989	2007	$\log(\Delta\text{share})$	Internal	External
Cognitive	0.28	0.34	0.197	0.011	0.186
Routine	0.57	0.47	0.189	−0.008	−0.181
Manual	0.16	0.19	0.217	0.003	0.214
Germany	1975	2017	$\log(\Delta\text{share})$	Internal	External
Cognitive	0.15	0.30	0.711	0.171	0.541
Routine	0.71	0.52	−0.324	−0.052	−0.271
Manual	0.14	0.18	0.263	−0.042	0.305

Data Source: SIPP (US); SIAB (Germany)

One exception is the differences in educational composition, which could increase the internal inflow for cognitive occupations significantly. In the counterfactual experiment, the internal net inflow for cognitive occupations in the US is 0.042 (17% of the total cognitive reallocation), while the same number is 0.171 for Germany (25% of the total cognitive reallocation). On the other hand, we find even educational composition can not explain the differences in the internal net inflow for routine occupations. In the same experiment, the internal net inflow for routine occupations in the US is −0.006 (4% of the total routine reallocation), while the same number is -0.052 for Germany (19% of the total routine reallocation).

C.4 Balanced panel for SIPP

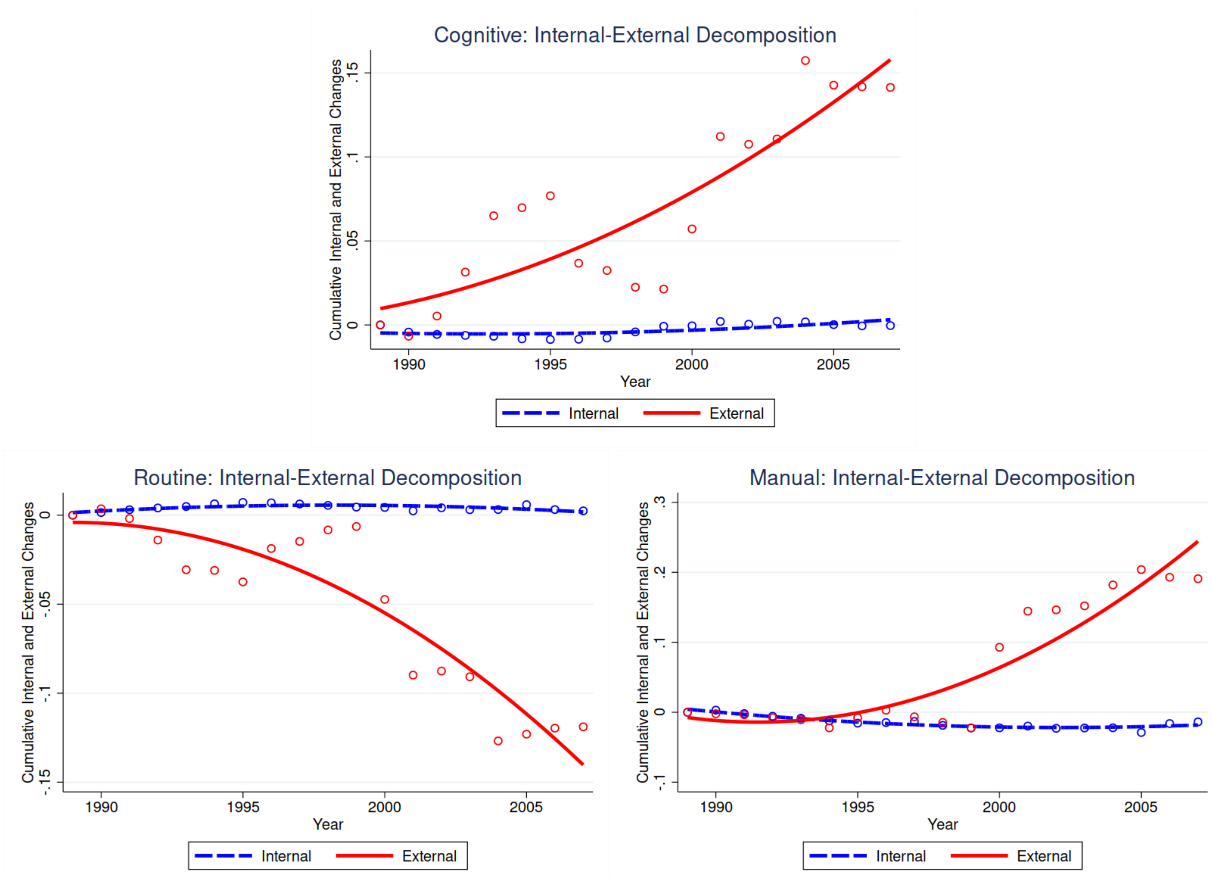
To check the robustness of our results in Table 1, and Figures 2 and 3 for the sample attrition issue of the SIPP sample, we create a balanced panel for the SIPP and run the decomposition again. That is, we select individuals who report their labor market status without any missing observations over the sample period of each SIPP panel and use the created balanced panel data for our analysis. Our internal-external decomposition results do not change the patterns even for the balanced panel case, as seen in Table 8 and Figure 29.

Table 8: Decompositions of Occupational Employment Share Changes for the US, Balanced Panel

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
US	1989	2007	log (Δshare)	Internal	External
Cognitive	0.29	0.33	0.14	0.00	0.14
Routine	0.60	0.53	−0.12	0.00	−0.12
Manual	0.11	0.13	0.18	−0.01	0.19

Data Source: SIPP; 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels.

Figure 29: Cumulative Changes in Occupational Employment in the US, SIPP, 1989–2007, Balanced Panel



Data Source: SIPP; 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels.

D A simple model of heterogeneous labor

There are two periods. There is a measure N workers (thus the total labor supply is fixed) and measure one homogeneous firms. The firm operates both periods. For simplicity, assume that the discount rate is zero for both workers and firms.

The firm's production function at each period is

$$f(n_c, n_r, s_a) = n_c^{\mu}(n_r + s_a)^{1-\mu},$$

where n_c is the number of workers engaging in the cognitive task, and n_r is the number of workers engaging in the routine task. We abstract from manual tasks for simplicity. The labor market is competitive.

Initially, both the firm and the worker believe that s_a will be zero for both periods. Between periods 1 and 2, there is an unexpected shock (an "MIT shock") that makes all firms' s_a to be $\bar{s}_a > 0$. After observing this event, both firms and households reoptimize.

Now, deviating from the baseline model, imagine a model where labor is heterogeneous before tasks are assigned. In particular, only skilled workers (indexed by s) can perform cognitive tasks, and all routine tasks are performed by unskilled workers (indexed by u). One can think of this situation as a corner solution where (in the equilibrium we look at) the wages of cognitive occupation are strictly higher than the wages of routine occupation so that all skilled workers choose to be in cognitive occupation even though they can perform routine tasks as well.

On the labor supply side, each worker has to pay training costs at the beginning of each period to be qualified as a skilled worker. The cost is xw_c, where w_c is the wage as a cognitive worker, and x is idiosyncratic and distributed following the distribution function $F(x) = \Pr[X \leq x]$. For each worker, the value of x is the same for both periods. Thus, at the beginning of period 1 (note that the MIT shock is not anticipated), a worker decides to become skilled if

$$2w_c(1-x) \geq 2w_r,$$

which means if $x \leq x^*$, where

$$x^* \equiv 1 - \frac{w_r}{w_c}.$$

Here, w_r is the wage for routine tasks. Suppose that the distribution for x is uniform: $F(x) = x$. Then, the skilled labor supply is

$$N_s = N\left(1 - \frac{1}{p}\right) \tag{2}$$

and unskilled labor supply is

$$N_u = N\frac{1}{p}, \tag{3}$$

where

$$p \equiv \frac{w_c}{w_r} \geq 1$$

is the skill premium. The relative supply curve is

$$\frac{N_s}{N_u} = p - 1. \tag{4}$$

On the demand side, the firm's first-order conditions for the first period are

$$w_r = (1-\mu)\left(\frac{n_r}{n_c} + \frac{s_a}{n_c}\right)^{-\mu} \tag{5}$$

and

$$w_c = \mu\left(\frac{n_r}{n_c} + \frac{s_a}{n_c}\right)^{1-\mu}. \tag{6}$$

Therefore,

$$p = \frac{\mu}{1-\mu}\left(\frac{n_r}{n_c} + \frac{s_a}{n_c}\right).$$

Because $s_a = 0$ in the first period, the first-period relative demand function is

$$p = \frac{\mu}{1-\mu}\frac{n_r}{n_c}. \tag{7}$$

In equilibrium, $N_s = n_c$ and $N_u = n_r$ and thus from (4) and (7), the equilibrium price satisfies

$$p = \frac{\mu}{1-\mu}\frac{1}{p-1}.$$

From this equation, we can solve for p as

$$p = \frac{1+\sqrt{1+4\mu/(1-\mu)}}{2}$$

and then we can solve for (N_s, N_u, w_c, w_r) from other conditions.

In the second period, after the MIT shock, the firm and the consumers reoptimize. First, consider the US economy, where there are no firing taxes. In this case, all firms can fire all workers at the end of period 1, let them make the skill decision, and rehire with the optimal choice. Supply decisions are not affected by changes in s_a. The relative demand curve is now

$$p = \frac{\mu}{1-\mu}\left(\frac{N_u}{N_s} + \frac{\bar{s}_a}{N_s}\right). \tag{8}$$

One can easily see that because $\bar{s}_a/N_s > 0$, the relative demand curve shifts up, and thus both p and N_s/N_u go up in equilibrium. The four unknowns (N_s, N_u, w_c, w_r) can be solved from the equations (2), (3), (5), and (6). Inspecting these equations, one can easily see the following:

Proposition 1 *When the firing cost is not present, with automation, $N_s = n_c$ and w_c go up, and $N_u = n_r$ and w_u go down.*

Thus, this model generates the labor market polarization based on automation, as in our baseline model (with homogeneous workers). The homogeneous-skills version of the model can easily be obtained by setting $w_r = w_c = w$ in (5) and (6) and add the (unified) labor market equilibrium condition $n_c + n_r = N$ (three equations with three unknowns w, n_c, and n_r). The difference is that the movement of wages is now ambiguous.

Note that, in this framework, polarization may occur by supply factors, such as the shift in the $F(\cdot)$ function. If it becomes cheaper to become skilled, for example, the equilibrium N_s goes up, and N_u goes down. However, in this case, the wage implications are different. When the supply factor is dominant, the skill premium would fall as N_s/N_u goes up. For example, when $F(x) = mx$ for $x \in [0, 1/m]$ and $m \geq 1$, when m goes up it becomes cheaper to become skilled. The relative labor supply curve can be rewritten as

$$\frac{N_s}{N_u} = \frac{p-1}{1-p(1-1/m)}.$$

Combining with the relative demand curve (7), one can see that N_s/N_u is increasing in m and p is decreasing in m. Empirically, p has been going up in the US since the 1970s, although we have observed some decline in the last few years. The evidence seems to support the shift in demand (such as automation), although the supply factor may have played some role.

Now, let us consider the case with firing taxes. Suppose that the firm has to pay $\tau > 0$ firing tax per worker fired. If the firm wants to fire a worker and rehire after training, the firm has to pay τ in addition to w_c per switched worker. The worker still has to pay the training cost, and the total training cost is $\kappa[I(n_c' - n_c)]^2$, where $\kappa > 0$ is the parameter, n_c is the period 1 cognitive workers, n_c' is the period 2 cognitive workers, and $I \in [0, 1]$ is the fraction of internally reallocated workers (which is determined by the firm). Using this notation, the firing tax can be written as

$(1-I)\tau(n_c'-n_c) = (1-I)\tau(n_r-n_r')$.

The firm decides n_c', n_r', and I given w_c' and w_r', where prime ($'$) indicate the period 2 variable. The firm cannot force the workers to train, thus the worker decides to obtain skills by the rule

$$w_c'(1-x) \geq w_r'.$$

The labor supply rules analogous to (2) and (3) hold.

The firm's first-order conditions are now

$$\begin{aligned}
w_c' &= \mu\left(\frac{n_r'}{n_c'}+\frac{s_a}{n_c'}\right)^{1-\mu} - (1-I)\tau - 2\kappa I^2(n_c'-n_c), \\
w_r' &= (1-\mu)\left(\frac{n_r'}{n_c'}+\frac{s_a}{n_c'}\right)^{-\mu}, \\
\tau(n_c'-n_c) &= 2\kappa I(n_c'-n_c)^2.
\end{aligned}$$

Then, in the equilibrium,

$$\begin{aligned}
p &= \frac{\mu}{1-\mu}\left(\frac{N_u'}{N_s'}+\frac{\bar{s}_a}{N_s'}\right) - \frac{\tau}{1-\mu}\left(\frac{N_u'}{N_s'}+\frac{\bar{s}_a}{N_s'}\right)^{\mu}, \\
I &= \frac{\tau}{2\kappa(N_c'-N_c)}.
\end{aligned}$$

It follows that the relative demand curve shifts down with $\tau > 0$, and hence both p and N_s/N_u is lower with the higher firing tax. From the condition on I, the following holds.

Proposition 2 *When $\tau > 0$, some workers switch from routine occupation to cognitive occupation by going through reassignment within the firm when automation occurs. The fraction of within-firm reallocation, I, is increasing in τ and decreasing in κ.*

This proposition shows that, even when the tasks are tied to different skill types of workers, when the endogenous choice of skills is taken into account, a qualitatively similar outcome is obtained as in the homogeneous-skills case.

E Alternative specification of automation cost

In this section, we present a variant of the baseline model in which we set the adoption cost to decline at a constant rate to describe the diffusion process of technology. Specifically, the adoption cost is replaced by

$$\Gamma(\underline{s}_a, \overline{s}_a; t) = \rho_a^t \bar{c}_a,$$

which is now time variant. On the transition path, firms decide whether they adopt or not, depending on the current Γ. The value functions for the firms not yet automated are modified as

$$\begin{aligned}
& V_t(\mathbf{n}, s_h; \underline{s}_a) \\
= & \max_{\mathbf{n}' \geq \mathbf{0}, d \in \{0,1\}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \qquad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' - d\Gamma(\underline{s}_a, \overline{s}_a; t) \\
& \qquad +\beta \mathbb{E}_{s'_h} [d W_{t+1}(\mathbf{n}', s'_h; \overline{s}_a) + (1-d) V_{t+1}(\mathbf{n}', s'_h; \underline{s}_a) | s_h],
\end{aligned}$$

where $d = 1$ if firms plan to adopt and $d = 0$ otherwise. Other model ingredients are similar to the main text. We set $\bar{c}_a = 0.190$ and $\rho_a = 0.990$.

E.1 Model fit

Figures 30-37 present model fit for the alternative specification. Overall, the results are similar to the baseline model, while the graphs are not as smooth as in the main text.

E.2 Counterfactual on the reorganization cost parameter κ

This subsection repeats the counterfactual exercise for reducing κ by half with the alternative specification in Figures 38-41. The results with the alternative specification are also similar to those in the main text. Once again, the values of aggregate output, aggregate labor, and labor productivity are almost identical between the baseline and the counterfactual.

E.3 Counterfactual on firing tax parameter τ

Counterfactual results repeated with the alternative specification for reducing τ by half in Figures 42-45 and Table 9 are similar again to those in the main text.

Figure 30: Occupation Share in Data versus Model: US

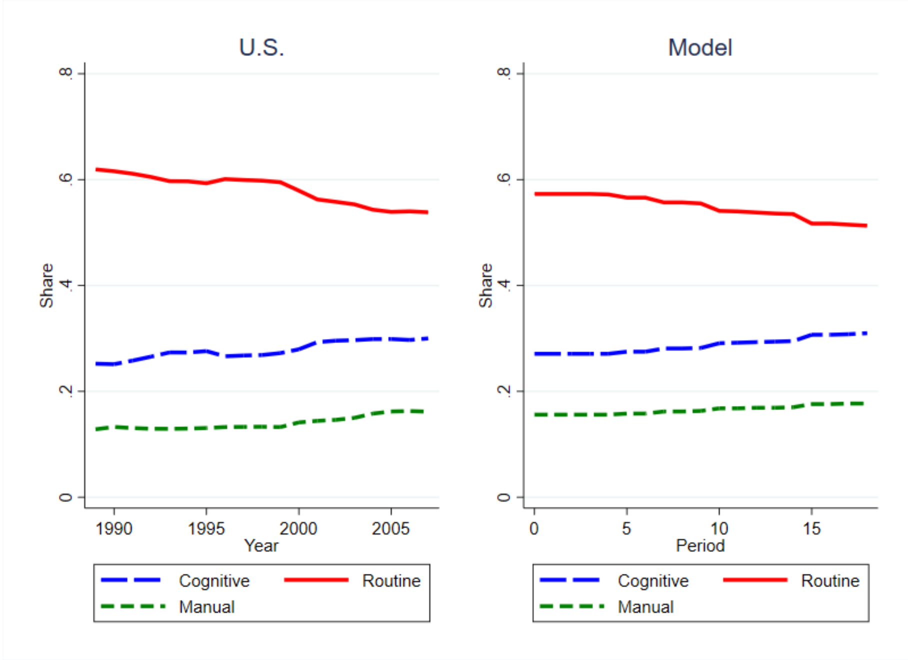


Figure 31: Occupation Share in Data versus Model: Germany

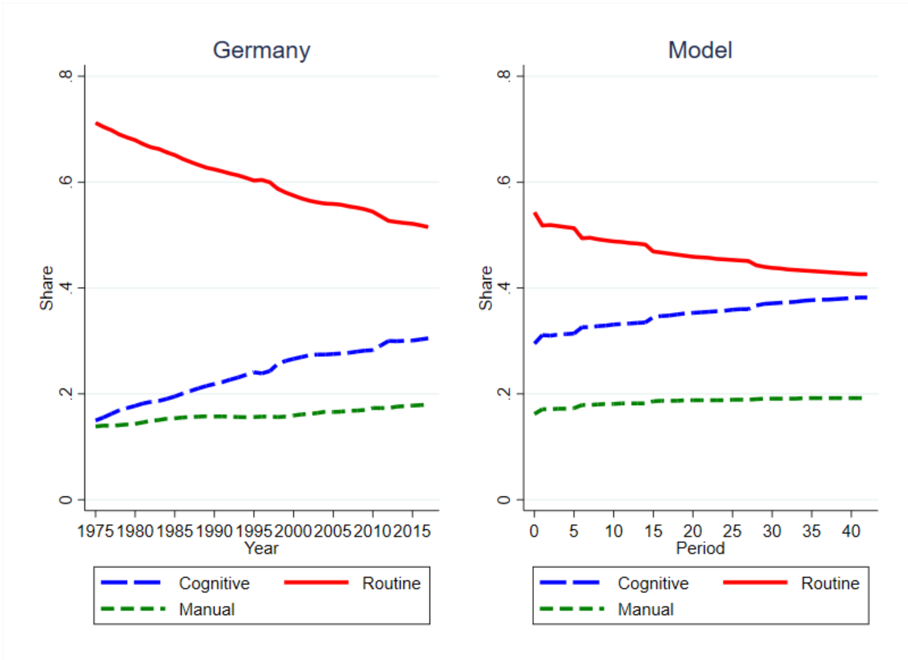


Table 9: Counterfactual Productivity: Reducing τ by Half

Variable	Baseline	Counter-Factual
Aggregate Output	1.000	1.100
Aggregate Labor	1.000	1.168
Labor Productivity	1.000	0.950

Figure 32: Cumulative Share Changes of Cognitive in Data versus Model: US

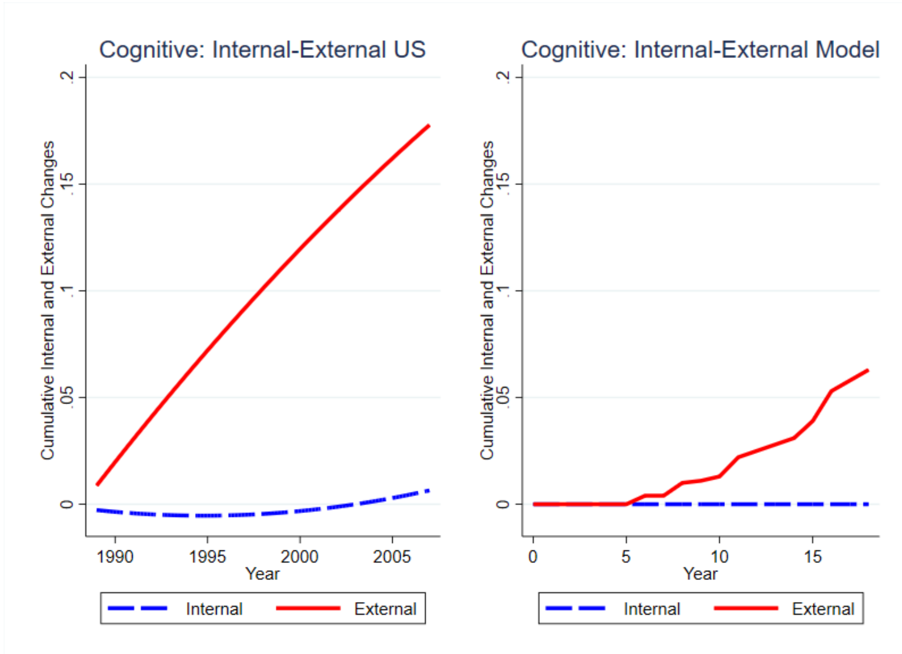


Figure 33: Cumulative Share Changes of Cognitive in Data versus Model: Germany

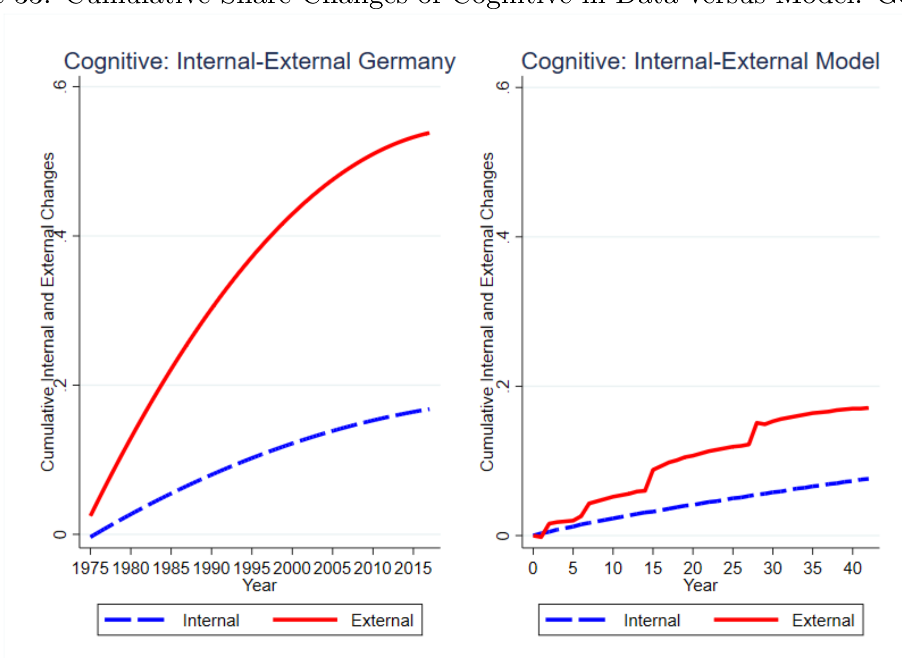


Figure 34: Cumulative Share Changes of Routine in Data versus Model: US

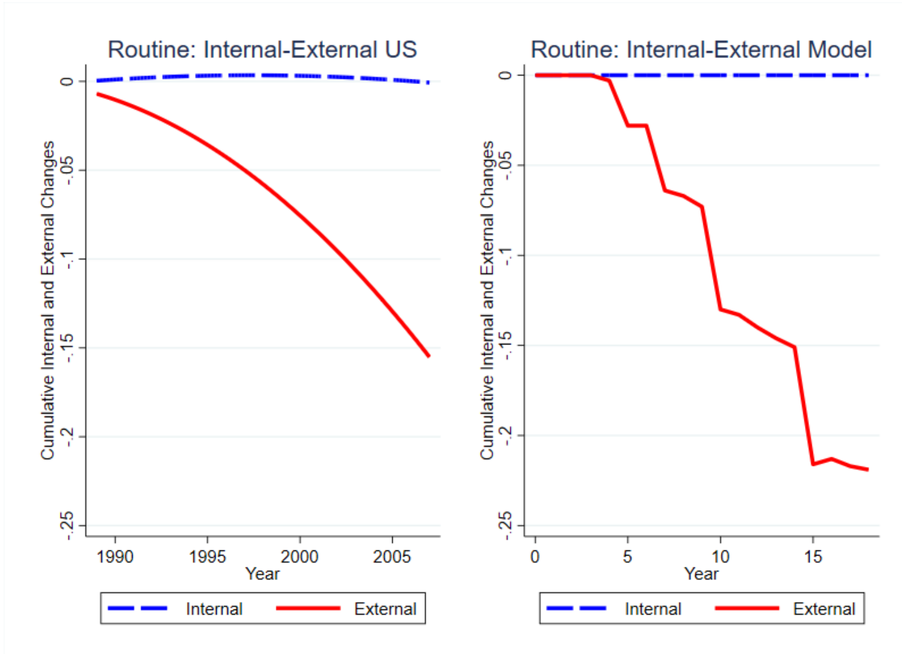


Figure 35: Cumulative Share Changes of Routine in Data versus Model: Germany

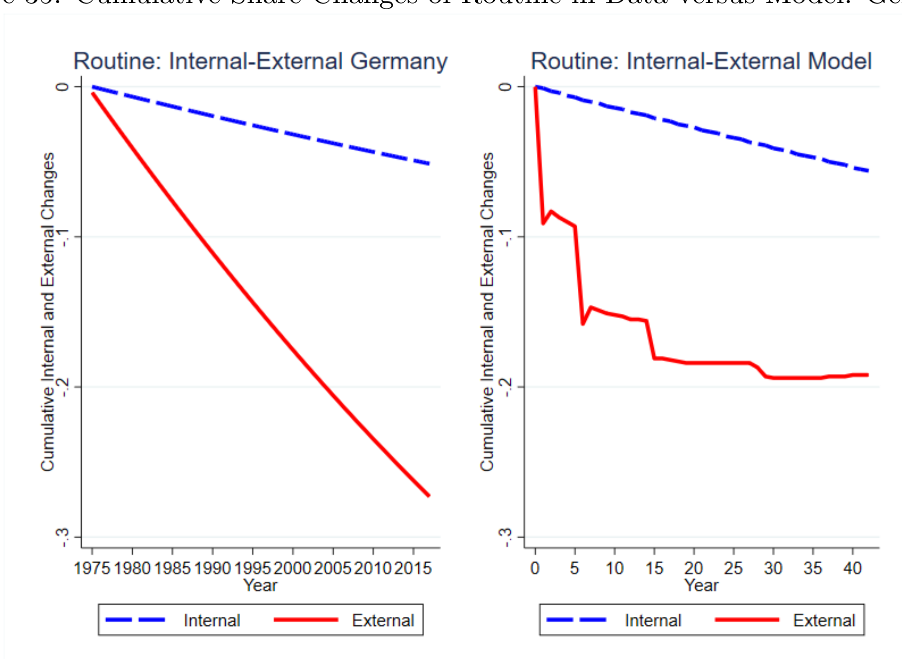


Figure 36: Cumulative Share Changes of Manual in Data versus Model: US

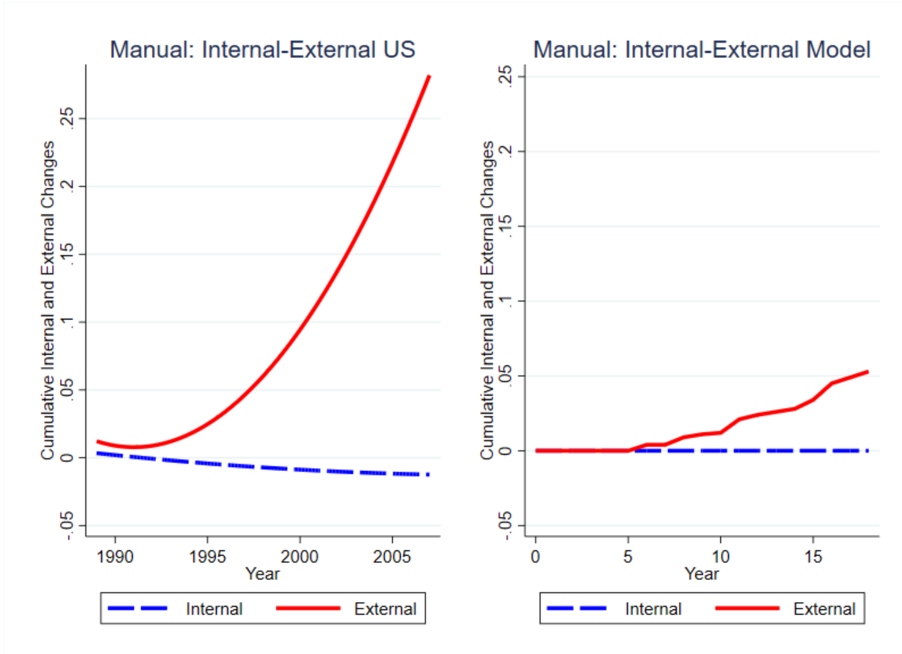


Figure 37: Cumulative Share Changes of Manual in Data versus Model: Germany

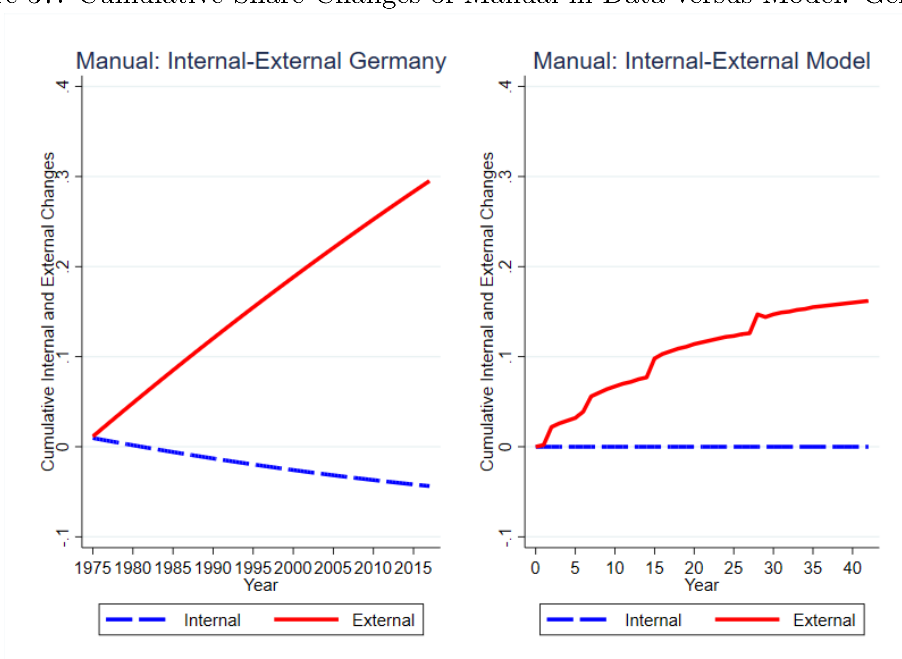


Figure 38: Counterfactual Occupation Share: Reducing κ by Half

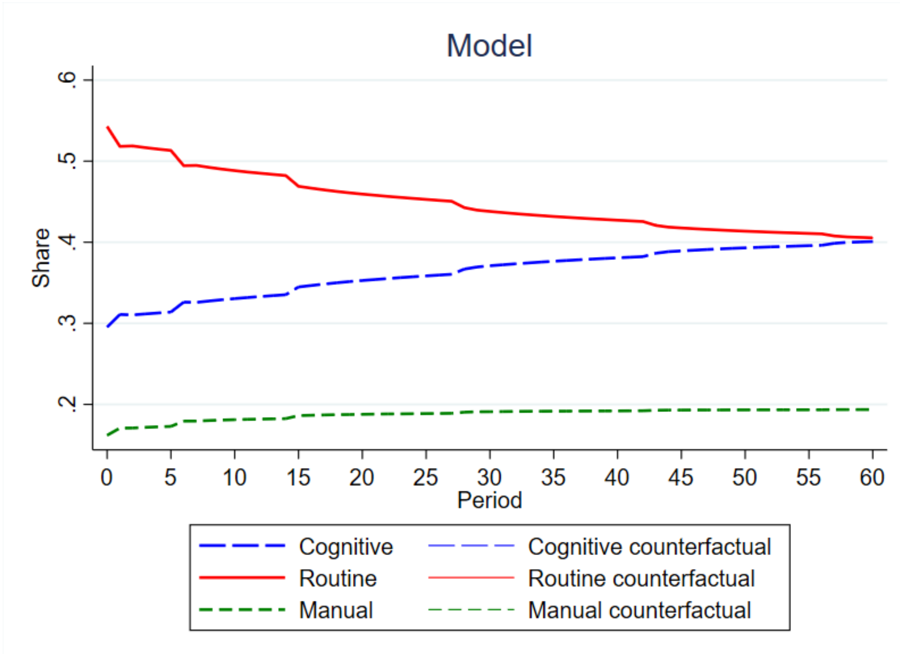


Figure 39: Counterfactual Flow of Cognitive: Reducing κ by Half

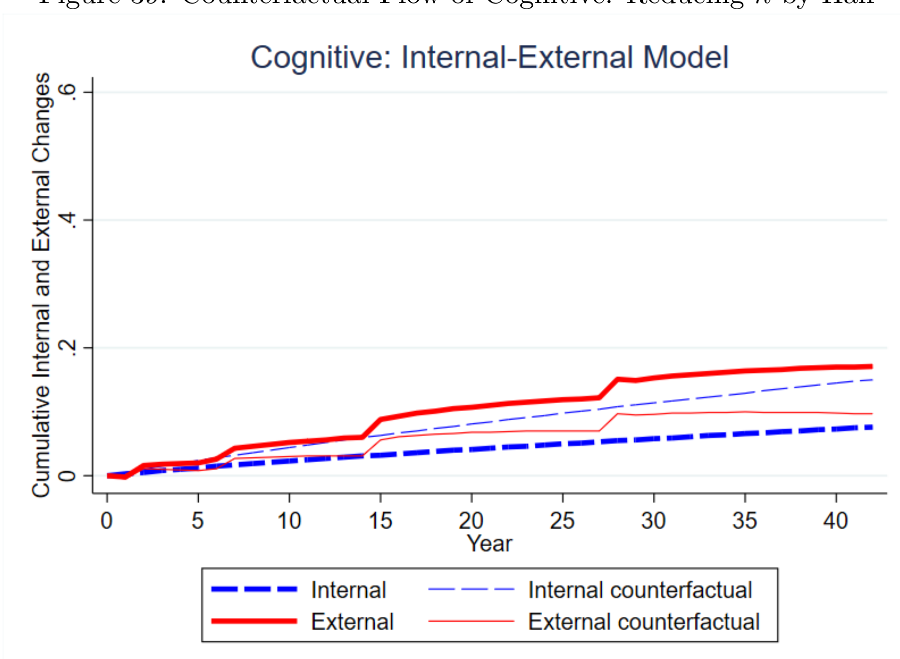


Figure 40: Counterfactual Flow of Routine: Reducing κ by Half

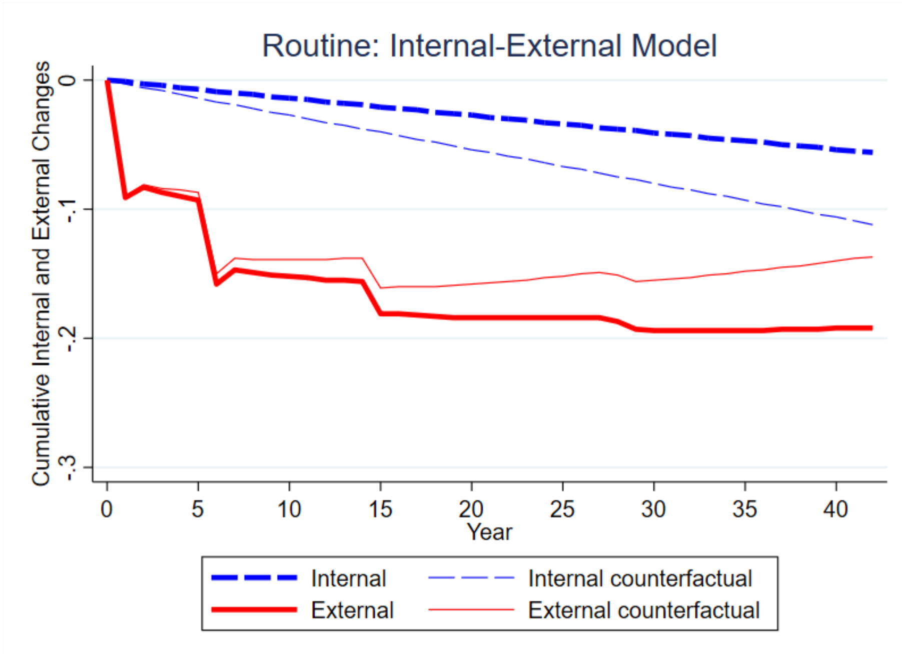


Figure 41: Counterfactual Flow of Manual: Reducing κ by Half

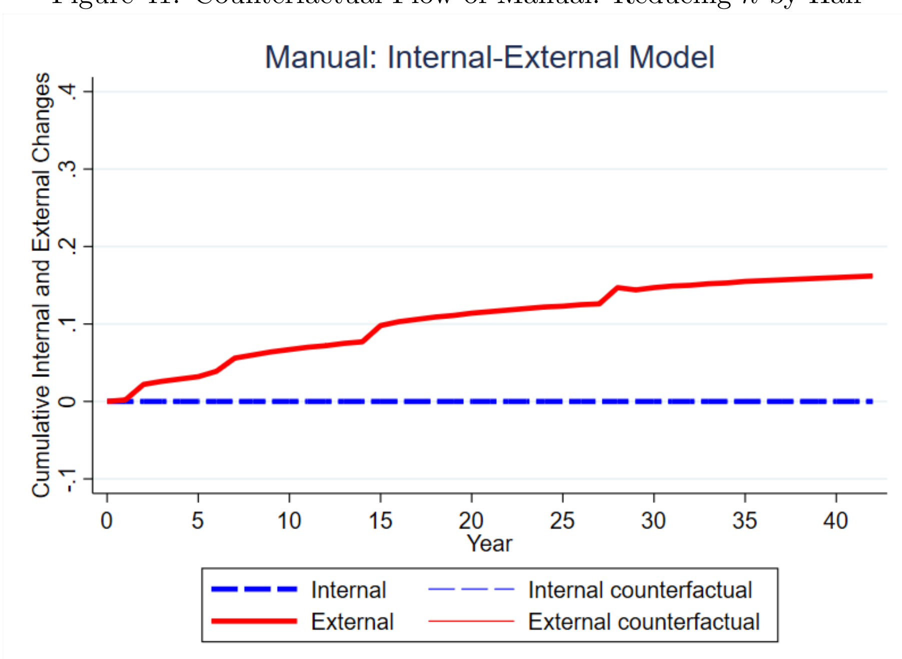


Figure 42: Counterfactual Occupation Share: Reducing τ by Half

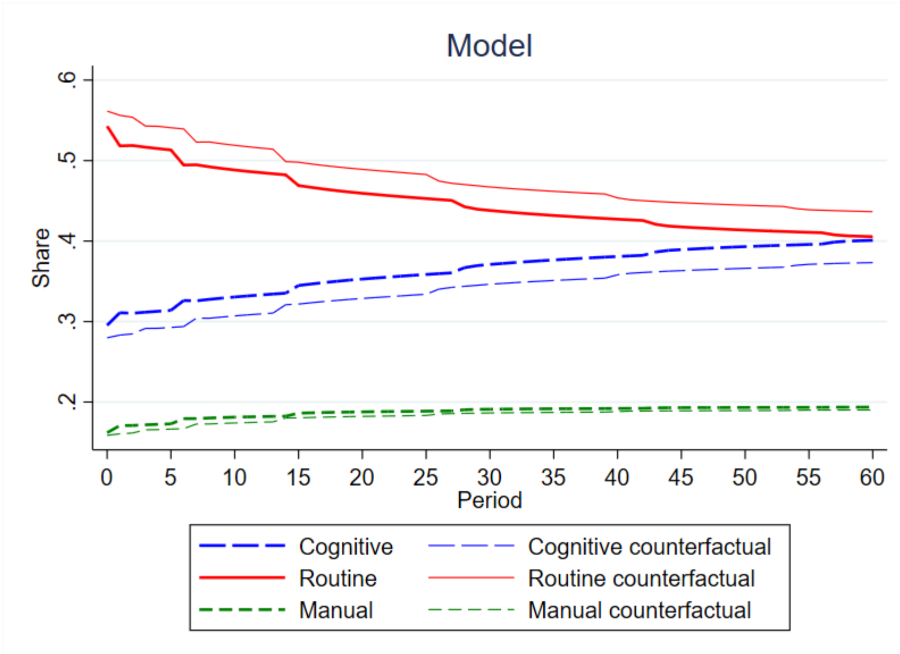


Figure 43: Counterfactual Flow of Cognitive: Reducing τ by Half

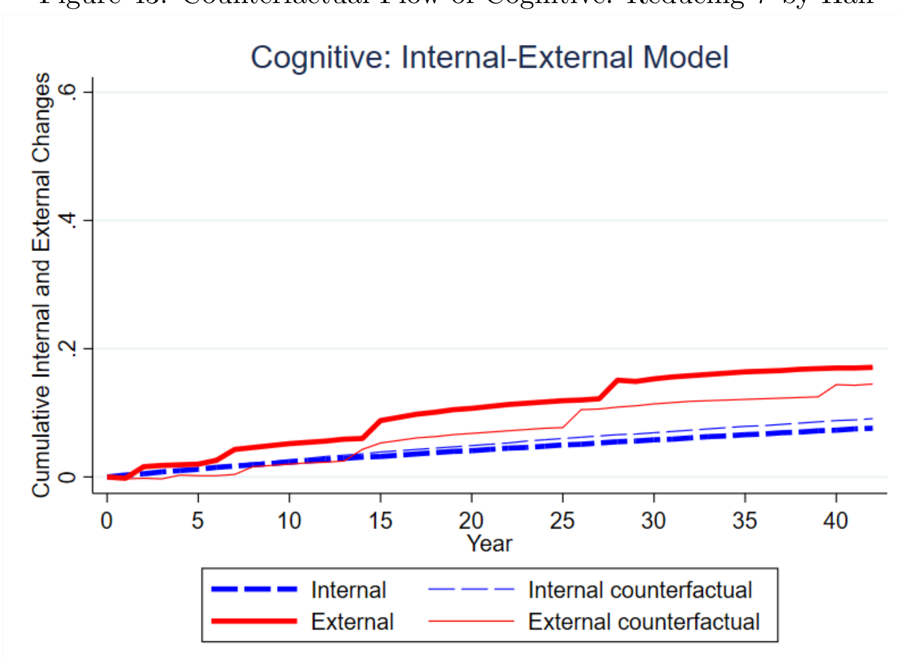


Figure 44: Counterfactual Flow of Routine: Reducing τ by Half

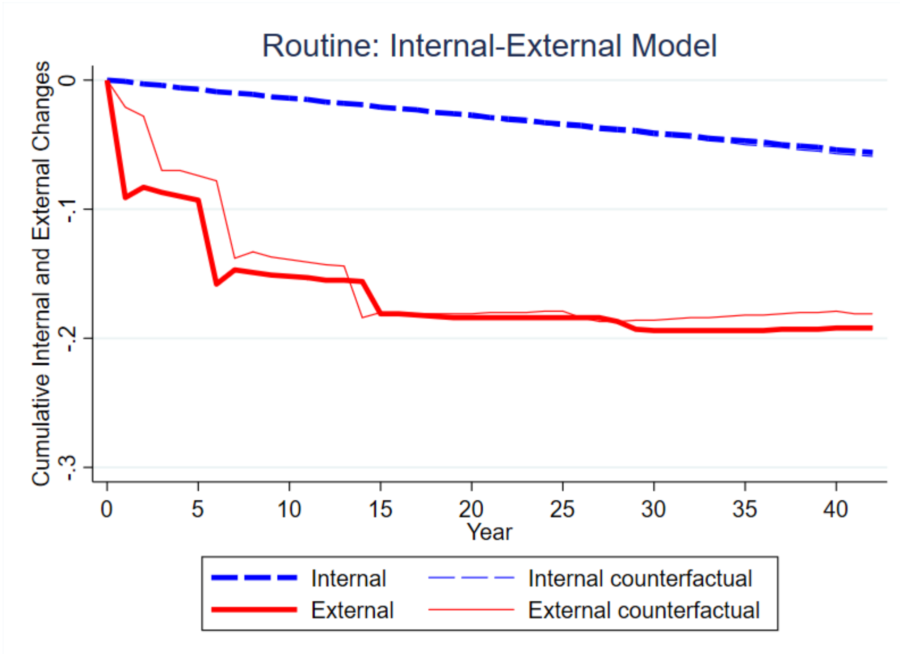
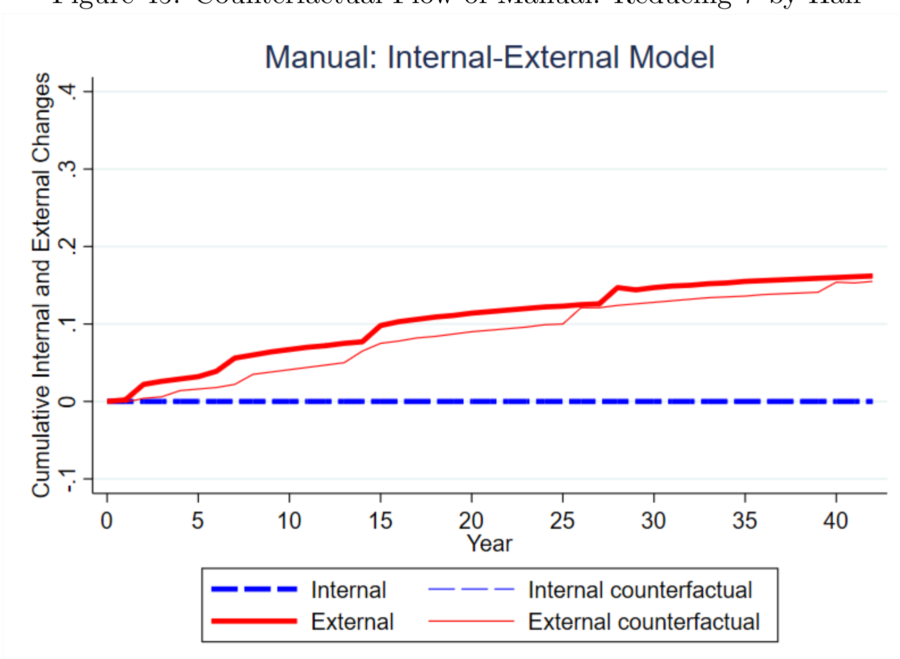


Figure 45: Counterfactual Flow of Manual: Reducing τ by Half



F Computing the transition dynamics

We initially compute the steady state where $s_a = \underline{s}_a$ for all firms. In that steady state, no firm has a possibility of "automating" and moving to $s_a = \overline{s}_a$.

We assume that the economy is initially in a steady state where all firms have $s_a = \underline{s}_a$ and expect it to stay constant forever. Then, at a point in time (call time 0), the economy unexpectedly shifts to a new regime where a firm can endogenously switch to $s_a = \overline{s}_a$ when it is profitable. In particular, after time 0, with probability p, the firm (at any point in time) can decide whether it automates with adoption cost $\Gamma(\underline{s}_a, \overline{s}_a)$. The regime switch is permanent, and all economic agents understand the nature of the switch. At the firm level, the transition from $\underline{s}_a$ to $\overline{s}_a$ is one time and permanent: once they change s_a to $\overline{s}_a$, it stays at that value. The aggregate economy experiences a gradual transition from the steady state where all firms have $s_a = \underline{s}_a$ to another steady state where all firms have $s_a = \overline{s}_a$. We interpret this transition dynamics as the process of labor-market polarization, driven by automation at each firm.

To analyze the macroeconomic dynamics of this transition, we first compute the initial and final steady states. As in the previous section, let $\mathbf{n} = (n_m, n_c, n_r)$ be the previous period's occupational employment and $\mathbf{n}' = (n'_m, n'_c, n'_r)$ be the current period's employment decision. In the initial steady state where no firms automate, a firm's dynamic programming problem is

$$\begin{aligned}
\underline{V}(\mathbf{n}, s_h; \underline{s}_a) \quad = \quad & \max_{\mathbf{n}', x'} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'), 0\} + \max\{n_r - (n'_r + x'), 0\}) \\
& -\kappa x'^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w\mathbf{1} \cdot \mathbf{n}' \\
& +\beta \mathbb{E}_{s'_h}[\underline{V}(\mathbf{n}', s'_h; \underline{s}_a)|s_h]],
\end{aligned}$$

subject to

$$\begin{aligned}
n'_m &\geq 0, \\
n'_c &\geq x', \\
n'_r &\geq 0, \\
0 &\leq x' \leq n_r.
\end{aligned}$$

Note that the time notation is not included because the only element of the model that is affected by calendar time is the automation decision (which is absent here). Here, we have already eliminated the notation of $\hat{n}'_i$ and $\tilde{n}'_i$ using the new notation of x'.

x' can be solved analytically once $\mathbf{n}$ and $\mathbf{n}'$ are given. Denote the solution as $x'(\mathbf{n}, \mathbf{n}')$. Then the problem can be rewritten as:

$$\begin{aligned}
& \underline{V}(\mathbf{n}, s_h; \underline{s}_a) \\
= \ & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - \underline{w}\mathbf{1} \cdot \mathbf{n}' + \beta \mathbb{E}_{s_h'}[\underline{V}(\mathbf{n}', s_h'; \underline{s}_a) | s_h]].
\end{aligned}$$

At the final state where all firms have completed the automation, the Bellman equation is

$$\begin{aligned}
& \overline{W}(\mathbf{n}, s_h; \overline{s}_a) \\
= \ & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s}_a) - \overline{w}\mathbf{1} \cdot \mathbf{n}' + \beta \mathbb{E}_{s_h'}[\overline{W}(\mathbf{n}', s_h'; \overline{s}_a) | s_h]].
\end{aligned}$$

After computing the initial and final steady states, we compute the transition dynamics. Let $d = 1$ if firms plan to adopt and $d = 0$ otherwise. The value functions for the firms not yet automated are written as

$$\begin{aligned}
& V_t(\mathbf{n}, s_h; \underline{s}_a) \\
= \ & \max_{\mathbf{n}' \geq \mathbf{0}, d \in \{0,1\}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' \\
& \quad +\beta \mathbb{E}_{s_h'}[p\{d(W_{t+1}(\mathbf{n}', s_h'; \overline{s}_a) - \Gamma(\underline{s}_a, \overline{s}_a)) + (1-d)V_{t+1}(\mathbf{n}', s_h'; \underline{s}_a)\} + (1-p)V_{t+1}(\mathbf{n}', s_h'; \underline{s}_a) | s_h],
\end{aligned}$$

and the firms that are already automated solve the Bellman equation

$$\begin{aligned}
& W_t(\mathbf{n}, s_h; \overline{s}_a) \\
= \ & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s}_a) - w_t \mathbf{1} \cdot \mathbf{n}' \\
& \quad +\beta \mathbb{E}_{s_h'}[W_{t+1}(\mathbf{n}', s_h'; \overline{s}_a) | s_h].
\end{aligned}$$

In addition, the distributions of firms are defined as below. Let $m_t^V(\mathbf{n}, s_h; \underline{s}_a)$ and $m_t^W(\mathbf{n}, s_h; \overline{s}_a)$ be the measures of non-automated and automated firms in the period t, and M_t^V and M_t^W be the total mass of the corresponding firms. The mass is defined as

$$\begin{aligned} M_t^V &= \sum_{\mathbf{g_n}} \sum_{g_h} m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \\ M_t^W &= \sum_{\mathbf{g_n}} \sum_{g_h} m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a). \end{aligned}$$

The counterparts at the initial steady state are denoted by $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ and $\underline{M}^V$. At the final steady state, they are $\overline{m}^W(\mathbf{n}, s_h; \overline{s}_a)$ and $\overline{M}^W$. We assume $M_t^V = M_t^W = \underline{M}^V = \overline{M}^W = 1$, as we shut down entry-exit.

We compute these objects with the following steps.

F.1 Preparation

We discretize the labor and shock, and the grid points are denoted by $(n_m^{g_m}, n_c^{g_c}, n_r^{g_r}) = \mathbf{n}^{\mathbf{g_n}}$, respectively, and $s_h^{g_h}$ where integer $g_\cdot \in \{1, ..., g_\cdot^{max}\}$. Later, we redistribute the weight of an off-grid point $\mathbf{n}$ to the neighboring grid points, such as $\mathbf{n}^{\mathbf{g_n}}$, by the following discrete measure G such that

$$G(\mathbf{n}, \mathbf{n}^{\mathbf{g_n}}) = \begin{cases} \frac{\prod |n_j^{g_j'} - n_j|}{\prod_j |n_j^{g_j'} - n_j^{g_j}|} & \text{if } n_j \text{ is between } n_j^{g_j} \text{ and } n_j^{g_j'} \text{ including endpoint for all } j = m, c, r, \\ 0 & \text{otherwise,} \end{cases}$$

where g_j' is either $g_j - 1$ or $g_j + 1$. The transition probability from $s_h^{g_h}$ to $s_h^{g_h'}$ is denoted by $P(s_h^{g_h'} | s_h^{g_h})$.

While $(\beta, \eta, \phi, \tau, \kappa)$ are given from outside model, ξ is pinned down within the model. First, assuming $\tau = 0$ and $\underline{w}=1$, we solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s}_a)$ by value function iteration. Next, simulating the above firms' decision rule repeatedly as

$$\underline{m}^{V,new}(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g_h'}; \underline{s}_a) = \sum_{\mathbf{g_n}} \sum_{g_h} G(\underline{\mathbf{n}}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g_h'} | s_h^{g_h}) \underline{m}^{V,old}(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a),$$

we can obtain an invariant distribution of firms $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$. Then, the labor demand is computed as $\underline{N} = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) \underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)$. Then, by the intra-temporal optimality,

$$\xi = \frac{\underline{w}}{\underline{N}^{\frac{1}{\eta}}}.$$

F.2 Computing the initial and final steady states

Setting $\tau > 0$, we guess the GE wage $\underline{w}$, solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s}_a)$ by value function iteration, and compute the invariant distribution $\underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ by using the obtained decision rule similarly to the previous subsection. Then, we check if $\underline{w}$ equates the demand and supply of labor as

$$\left(\frac{\underline{w}}{\xi}\right)^\eta = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) \underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a).$$

If there is excess demand, we increase $\underline{w}$ and *vice versa.* Then, we repeat it until $\underline{w}$ equates the demand and supply of labor. We apply the same steps for $\overline{w}$ and $\overline{W}$.

F.3 Backward induction

First, we guess the path of w_t on the transition. Given w_t, we solve for V_t and W_t, and corresponding decision rules $\mathbf{n}_t'(\mathbf{n}, s_h; \underline{s}_a)$ and $\mathbf{n}_t'(\mathbf{n}, s_h; \overline{s}_a)$ by backward induction from T to 1, while we set $W_{T+1} = \overline{W}$ and $V_{T+1} = \overline{V}$. The latter is a hypothetical non-automated value function at the final steady state and obtained by solving

$$\begin{aligned}
&\overline{V}(\mathbf{n}, s_h; \underline{s}_a) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}, d \in \{0,1\}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}_j'(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}_j'(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \underline{s}_a) - \overline{w} \mathbf{1} \cdot \mathbf{n}' \\
& + \beta \mathbb{E}_{s_h'} [p\{d(\overline{W}(\mathbf{n}', s_h'; \overline{s}_a) - \Gamma(\underline{s}_a, \overline{s}_a)) + (1-d)\overline{V}(\mathbf{n}', s_h'; \underline{s}_a)\} + (1-p)\overline{V}(\mathbf{n}', s_h'; \underline{s}_a) | s_h] \Bigg],
\end{aligned}$$

At each t, we solve for V_t and W_t and the decision rules, and proceed to $t-1$.

F.4 Simulating forward

Using the decision rules obtained above for $t = 1, ..., T$, we can compute $m_t^V(\mathbf{n}, s_h; \underline{s}_a)$, $m_t^W(\mathbf{n}, s_h; \overline{s}_a)$ as follows. Let $\phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)$ be the indicator of firms adopting at the grid point $(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h})$. First,

$$m_t^V(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g_h'}; \underline{s}_a) = \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g_h'} | s_h^{g_h})(1 - \phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a)) m_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a).$$

Second,

$$
\begin{aligned}
m_t^W(\mathbf{n}^{\mathbf{g_n}\prime}, s_h^{g_h'}; \overline{s}_a) &= \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) \phi_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) m_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) \\
&+ \sum_{\mathbf{g_n}} \sum_{g_h} G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) m_{t-1}^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a)
\end{aligned}
$$

where the first term on the right-hand side represents the non-automated firms which become automated at the end of period t, and the second is the automated firms from the last period. As for the period 1 measure, we set $m_1^V(\mathbf{n}, s_h; \underline{s}_a) = \underline{m}^V(\mathbf{n}, s_h; \underline{s}_a)$ and $m_1^W(\mathbf{n}, s_h; \overline{s}_a) = 0$.

F.5 Updating the guess

We check if w_t for each t equates the demand and supply of labor as

$$
\left(\frac{w_t}{\xi}\right)^\eta = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) m_t^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s}_a) + \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a) m_t^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s}_a)
$$

where the first term on the right-hand side is the labor demand from non-automated firms and the second term is the demand from automated firms. If there is excess demand, increase w_t and *vice versa.* Then, we go back to the backward induction until w_t equates the demand and supply of labor for $t = 1, ..., T$.