Online Appendix

A Incomplete market: Bond economy

Here, consider another incomplete-market EU economy. Specifically, the consumers are only allowed to trade noncontingent bonds. For the ease of computation, I assume the utility function is quadratic:

$$u(c) = \alpha c - \frac{\gamma}{2}c^2, \tag{2}$$

where $\alpha > 0$ and $\gamma > 0$. The value of α is assumed to be sufficiently large so that utility is increasing in c over the relevant range.

A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, b} \; u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + qb = 1,$$

$$c_2 = 1 + b,$$

and

$$\tilde{c}_2 = 1 - \tau + b,$$

where q is the bond price and b is the bond holding. After solving for the first-order condition, the bond demand of Type-I consumers is

$$b = \frac{q(\gamma - \alpha) + \alpha - \gamma(1 - \pi\tau)}{\gamma(q^2 + 1)}.$$

Similarly, the Type-II consumer's problem is

$$\max_{c'_1, c'_2, \tilde{c}'_2, b'} \; u(c'_1) + (1-\pi)u(c'_2) + \pi u(\tilde{c}'_2),$$

subject to

$$c'_1 + qb' = 1,$$

$$c'_2 = 1 + b',$$

and

$$\tilde{c}'_2 = 1 + \tau + b',$$

The bond demand of the Type-II consumers is

$$b = \frac{q(\gamma - \alpha) + \alpha - \gamma(1 + \pi\tau)}{\gamma(q^2 + 1)}.$$

The bond price q is set so that the excess bond demand is zero:

$$b + b' = 0.$$

It is straightforward to derive that, in equilibrium,

$$q = 1$$

and

$$(b, b') = \left(\frac{\pi}{2}\tau, -\frac{\pi}{2}\tau\right)$$

hold. The resulting consumptions are:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \left(\frac{\pi}{2} - 1\right)\tau, 1 + \left(1 - \frac{\pi}{2}\right)\tau\right),$$

which achieves some but not perfect consumption smoothing.

In the limit of $\pi \to 0$, the consumption profile when the irregular state takes place in period 2 would approach

$$\lim_{\pi \to 0}(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau),$$

which is identical to (1). In the current example, the incomplete-market structure that delivers the equivalence to the MIT-shock outcome is not unique. The following section looks at the economy with one Arrow security.

B When the type of asset structures matters

For simplicity, in the main text (Section 3), I considered a setting where in both the IU economy (Section 3.1) and the EU economies (Sections 3.3 and Appendix A), the equilibrium security holdings are zero. This assumption has two consequences: (i) For the IU economy, the outcome is identical regardless of whether the assumption is that the existing security is (a) an Arrow security for the regular state or (b) a state-noncontingent bond. (ii) The limiting outcome of the EU economies in the Arrow-security economy (Section 3.3) and the bond economy (Appendix A) are both consistent with the IU outcome. Here, I consider a setting where the equilibrium security holdings in these situations are nonzero, and show that the type of asset structures in both the IU economies and the EU economies matters.

Consider the same setting as in Section 3, except for the endowments. A Type-I consumer receives the endowment of 0 in period 1 and 2 in the period-2 regular state. In the period-2 irregular state, a Type-I consumer receives $(2 - \tau)$, where $\tau \in (0, 1)$. A Type-II consumer receives 2 in period 1 and 0 in the period-2 regular state. In the period-2 irregular state, a Type-II consumer receives τ.

Below, I show several IU outcomes can exist depending on the asset structure, because the level of asset that can be carried into the MIT-shock state can differ depending on the assumed asset structure. I show several (incomplete-market) EU allocations can also exist depending on the asset structure, and one can map an EU allocation to a corresponding IU allocation.

The lesson of this section is that, when an MIT-shock experiment is conducted, one has to use the underlying asset structure that is consistent with a particular background EU economy. If the background EU economy is an incomplete-market economy with bonds, the consumers have to be allowed to carry over the bond holdings after the MIT shock. If the background EU economy features state-by-state Arrow security, one has to think carefully about whether to allow for a similar ex-ante arrangement for the MIT-shock state.

B.1 MIT shock: Arrow security economy

First, consider an MIT shock (IU) economy. Suppose that, before the shock, the economy permits an Arrow security contingent on the regular state. The problem for a Type-I consumer is

$$\max_{c_1, c_2, a} u(c_1) + u(c_2),$$

subject to

$$c_1 + pa = 0$$

and

$$c_2 = 2 + a,$$

where p is the price of the Arrow security that pays out one unit of the consumption good in the regular state period 2. Because the consumers perceive only the regular state, the IU economy has a complete asset market with one Arrow security. The consumption in period 1 and 2 is denoted as c_1 and c_2, respectively, and the security holding is represented by a.

The Type-II consumer faces the problem:

$$\max_{c'_1, c'_2, s'} u(c'_1) + u(c'_2),$$

subject to

$$c'_1 + pa' = 2$$

and

$$c'_2 = a',$$

where prime ($'$) denotes variables for the Type-II consumers.

The unique competitive equilibrium is with $p = 1$, $a = -1$, $a' = 1$, and $c_1 = c'_1 = c_2 = c'_2 = 1$. Now suppose the MIT shock hits the economy. Because the irregular state is not spanned by the Arrow security, the ex-post allocation will be $\tilde{c}_2 = 2 - \tau$ and $\tilde{c}'_2 = \tau$. (As in the main text, a tilde ($\tilde{\ }$) denotes the irregular state.) Thus, the entire ex-post allocation ends up with

$$(c_1, c'_1, \tilde{c}_2, \tilde{c}'_2) = (1, 1, 2 - \tau, \tau). \tag{3}$$

B.2 MIT shock: Bond economy

Second, alternatively, consider another MIT shock (IU) economy with a different asset structure. Suppose, instead, the asset in the economy is a noncontingent bond. The problem for a Type-I consumer is

$$\max_{c_1,c_2,a} \ u(c_1) + u(c_2),$$

subject to

$$c_1 + qb = 0$$

and

$$c_2 = 2 + b,$$

where q is the price of the bond that pays out one unit of the consumption good in the regular state period 2. Once again, because the consumers perceive only the regular state, the IU economy's asset market is complete. The consumption in period 1 and 2 is denoted as c_1 and c_2, respectively, and the bond holding is represented by b.

The Type-II consumer faces the identical problem:

$$\max_{c_1',c_2',s'} \ u(c_1') + u(c_2'),$$

subject to

$$c_1' + pb' = 2$$

and

$$c_2' = b',$$

where prime ($'$) denotes variables for the Type-II consumers.

The unique competitive equilibrium is with $q = 1$, $b = -1$, $b' = 1$, and $c_1 = c_1' = c_2 = c_2' = 1$. Now suppose the MIT shock hits the economy. Now, in contrast to the previous case, the noncontingent bond remains in the economy, and the ex-post allocation will be $\tilde{c}_2 = 1-\tau$ and $\tilde{c}_2' = 1+\tau$. A tilde ($\tilde{}$) denotes the irregular state. Thus, the entire ex-post allocation ends up with

$$(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1-\tau, 1+\tau). \tag{4}$$

Thus, comparing (3) and (4), one can see that the allocation after the MIT shock is different depending on the asset structure.

B.3 Complete market

The next three sections consider EU economies. That is, I consider a situation where the irregular state occurs with probability π, and the consumers perceive that event. First, suppose the existence of a full set of Arrow securities. The notations are the same as in Section 3.2. A Type-I consumer's problem is

$$\max_{c_1,c_2,\tilde{c}_2,a,\tilde{a}} \ u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + pa + \tilde{p}\tilde{a} = 0,$$

$$c_2 = 2 + a,$$

and

$$\tilde{c}_2 = 2 - \tau + \tilde{a},$$

where a and $\tilde{a}$ denote the holding of Arrow securities.

A Type-II consumer's problem is (with the same notation convention as in the last section)

$$\max_{c_1', c_2', \tilde{c}_2', a', \tilde{a}'} \quad u(c_1') + (1-\pi)u(c_2') + \pi u(\tilde{c}_2'),$$

subject to

$$c_1' + pa' + \tilde{p}\tilde{a}' = 2,$$

$$c_2' = a',$$

and

$$\tilde{c}_2' = \tau + \tilde{a}'.$$

One can confirm the competitive equilibrium, where the consumer's first-order conditions are satisfied and the market-clearing conditions

$$a + a' = 0$$

and

$$\tilde{a} + \tilde{a}' = 0$$

have the following solution:

$$p = 1 - \pi$$

and

$$\tilde{p} = \pi$$

with

$$(a, a', \tilde{a}, \tilde{a}') = \left(-1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, -1 + \frac{2-\pi}{2}\tau, 1 - \frac{2-\pi}{2}\tau\right).$$

The resulting consumptions is:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau\right);$$

that is, each consumer can smooth consumption across time and state.

To see whether this allocation corresponds to one of the IU allocations, take $\pi \to 0$ and look at the consumption when the irregular shock hits. The result is

$$\lim_{\pi \to 0}(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1, 1).$$

The outcome is different from both (3) and (4).

B.4 Incomplete market: Arrow security economy

Now, consider another EU economy. Suppose, as in Section 3.3, the Arrow security does not exist for the irregular state although the consumers recognize the possibility of the irregular state in the future. A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, a} \ u(c_1) + (1 - \pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + pa = 0,$$

$$c_2 = 2 + a,$$

and

$$\tilde{c}_2 = 2 - \tau.$$

For Type-II consumers,

$$\max_{c_1', c_2', \tilde{c}_2', a'} \ u(c_1') + (1 - \pi)u(c_2') + \pi u(\tilde{c}_2'),$$

subject to

$$c_1' + pa' = 2,$$

$$c_2' = a',$$

and

$$\tilde{c}_2' = \tau.$$

The competitive equilibrium is $p = 1 - \pi$, $a = -1$, $a' = 1$. The resulting consumption is:

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1, 1, 2 - \tau, \tau).$$

Thus, in the limit of $\pi \to 0$,

$$\lim_{\pi \to 0}(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 2 - \tau, \tau),$$

which is identical to (3).

B.5 Incomplete market: Bond economy

For yet another EU economy, consider an economy with only a state-noncontingent bond. Similarly to Appendix A, I assume the utility function is quadratic:

$$u(c) = \alpha c - \frac{\gamma}{2}c^2, \tag{5}$$

where $\alpha > 0$ and $\gamma > 0$. The value of α is assumed to be sufficiently large so that utility is increasing in c over the relevant range.

A Type-I consumer's problem is

$$\max_{c_1, c_2, \tilde{c}_2, b} \; u(c_1) + (1-\pi)u(c_2) + \pi u(\tilde{c}_2),$$

subject to

$$c_1 + qb = 0,$$

$$c_2 = 2 + b,$$

and

$$\tilde{c}_2 = 2 - \tau + b,$$

where q is the bond price and b is the bond holding. After solving for the first-order condition, the bond demand of Type-I consumers is

$$b = \frac{-q\alpha + \alpha - \gamma(2 - \pi\tau)}{\gamma(q^2+1)}.$$

Similarly, the Type-II consumer's problem is

$$\max_{c_1', c_2', \tilde{c}_2', b'} \; u(c_1') + (1-\pi)u(c_2') + \pi u(\tilde{c}_2'),$$

subject to

$$c_1' + qb' = 2,$$

$$c_2' = b',$$

and

$$\tilde{c}_2' = \tau + b',$$

The bond demand of the Type-II consumers is

$$b = \frac{q(2\gamma - \alpha) + \alpha - \gamma\pi\tau}{\gamma(q^2+1)}.$$

The bond price q is set so that

$$b + b' = 0,$$

and therefore, in equilibrium,

$$q = 1$$

and

$$(b, b') = \left(\frac{\pi}{2}\tau - 1, 1 - \frac{\pi}{2}\tau\right)$$

hold. The resulting consumptions are

$$(c_1, c_1', c_2, c_2', \tilde{c}_2, \tilde{c}_2') = \left(1 - \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 + \frac{\pi}{2}\tau, 1 - \frac{\pi}{2}\tau, 1 + \left(\frac{\pi}{2} - 1\right)\tau, 1 + \left(1 - \frac{\pi}{2}\right)\tau\right).$$

In the limit of $\pi \to 0$, the consumption profile when the irregular state takes place in period 2 would approach

$$\lim_{\pi \to 0}(c_1, c_1', \tilde{c}_2, \tilde{c}_2') = (1, 1, 1 - \tau, 1 + \tau),$$

which is identical to (4).

The analysis above first confirms the conclusion of the main text: the probability-zero limit of an EU economy allocation converges to an IU economy allocation only when the asset market is missing for the MIT-shock state. A new point this section makes is that the type of incompleteness matters. Here, the IU outcome in an Arrow-security economy (Section B.1) can be approximated by an appropriate EU economy with an Arrow security (Section B.4), and the outcome in an IU economy with a bond (Section B.2) can be approximated by an incomplete-market EU economy with a bond (Section B.5).

The principle here is that the asset holding that can be carried into the irregular state has to be consistent between the EU economy and the IU economy. The value of the assets also have to be reevaluated carefully when computing each consumer's asset holding upon the occurrence of the MIT shock.

This principle may sound obvious. In some situations, however, re-evaluating the asset value coming into the MIT-shock economy requires careful examination. For example, suppose the asset value of a consumer, a, is a sum of a stock px, where p is the stock price and x is the quantity of the stock holding, and a bond b; therefore,

$$a = px + b.$$

When the realization of the irregular state moves the stock price from p to $\hat{p}$, the asset has to be reevaluated as

$$\hat{a} \equiv \hat{p}x + b$$

when starting the MIT-shock state, even though both the stock and bond can be carried into the MIT-shock state. The reevaluation would not be a big issue in an economy with a simple asset structure as in Huggett (1993) and Aiyagari (1994), but would matter in a more complex economy such as Krusell et al.'s (2010) (for which Mukoyama (2013) is an example of an MIT-shock analysis), as discussed in the main text.