1 Conclusion

sec:discussion

In this thesis, we have discussed the interrelationships among supersymmetric gauge theory, string theory and integrable lattice model. The answer to the question raised in the introduction "why integrable model exists?" has been addressed to the extra dimensions behind the system. The key concept of the correspondence between supersymmetric gauge theory and integrable lattice model is the underlying two-dimensional TQFT equipped with line operators. We also found that branes in string theory and string dualities provide a natural framework in which such a correspondence may be realized.

Still, our analysis is far from complete. One of the most important question is how to incorporate integrable *field theory* in our framework. As we have seen, integrable lattice models are realized by line operators in a two-dimensional TQFT. A natural direction to proceed is, therefore, to introduce a higher-dimensional defects in a TQFT. Fortunately, there is such a candidate [1]. We know that Costello's four-dimensional Chern-Simons theory can also be embedded into string theory setup [2]. Thus, along these lines, it should be possible to address the problem into an essentially same argument as ours.

Another possible direction is to find *new* integrable models through Gauge/YBE correspondence and to consider additional defects on them. In sections **??** and **??**, we investigated the correspondence between supersymmetric gauge theories and integrable lattice models, and elucidated the counterpart of defect operators in lattice model side. They are, however, already known L-operators, transfer matrices, and quantum algebras in the integrable model literature. Our construction of the correspondence is actually so powerful that we can "define" new integrable lattice models though the correspondence. A seminal work to define such a new integrable lattice model through the Gauge/YBE correspondence is given by Yamazaki [3], and it is really integrable [4–6]. Introducing additional defects in gauge theory side, we may find a new quantum algebra, such as a generalization of Sklyanin algebra.

Lastly, we wish to extend our discussion to the integrability in the AdS/CFT correspondence. Brane tiling techniques, which we considered in section **??**, were originally motivated by the AdS/CFT correspondence. As is well known, $\mathcal{N} = 4$ super Yang-Mills theory in four dimensions admits an integrable structure and so does its gravity dual. We hope our framework may shed light on the aspect of the integrability in the AdS/CFT correspondence, and hopefully provide a new approach to quantum gravity.

We hope to explore problems listed above in near future, but there are many more directions and interesting questions to pursue. In understanding the fundamental questions in physics, we believe it is essential to find more sophisticated viewpoint of the non-perturbative dynamics of quantum field theories, especially from an integrable structure behind the system.

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