Electronic Devices and Circuits I - EE2CJ4
Lab #5

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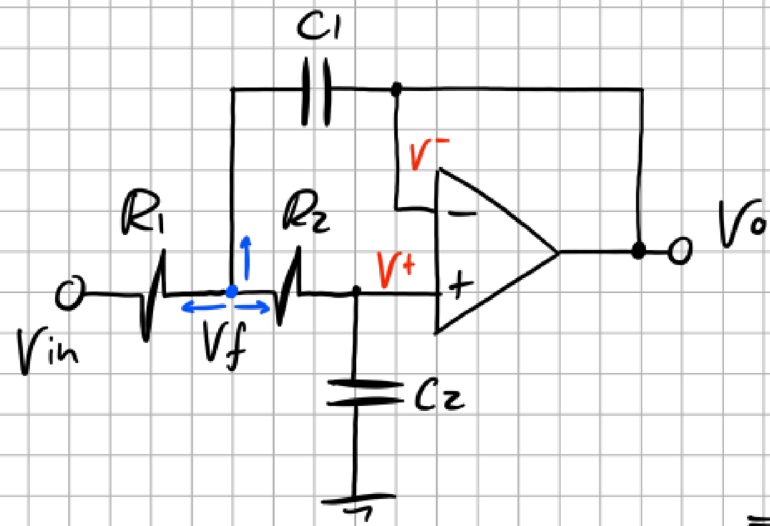
a) Derive an expression for the transfer function of the filter.



$Z_1 = R_1$, $Z_2 = R_2$ $\Big\}$ $R_1 = R_2 = R$

$Z_3 = 1/sC_1$, $Z_4 = 1/sC_2$ $\Big\}$ $C_1 = C_2 = C$

$V^+ = V^-$

$$V_o = V_f \cdot \frac{Z_4}{Z_2 + Z_4}$$

$$\Rightarrow V_f = V_o \cdot \frac{Z_2 + Z_4}{Z_4} \quad - ①$$

KCL @ Vf...

$$\frac{V_f - V_i}{Z_1} + \frac{V_f - V_o}{Z_3} + \frac{V_f}{Z_2 + Z_4} = 0 \qquad \text{Subbing in ①}$$

$$\frac{V_o \cdot \frac{Z_2 + Z_4}{Z_4} - V_i}{Z_1} + \frac{V_o \cdot \frac{Z_2 + Z_4}{Z_4} - V_o}{Z_3} + \frac{V_o \cdot \frac{Z_2 + Z_4}{Z_4}}{Z_2 + Z_4} = 0$$

$$V_o \left(\frac{Z_2 + Z_4}{Z_4 Z_1} + \frac{Z_2 + Z_4}{Z_4 Z_3} - \frac{1}{Z_3} + \frac{1}{Z_4} \right) = \frac{V_i}{Z_1}$$

$$\frac{V_o}{V_i} = \frac{1}{\frac{Z_2 + Z_4}{Z_4} + \frac{Z_1 (Z_2 + Z_4)}{Z_4 Z_3} - \frac{Z_1}{Z_3} + \frac{Z_1}{Z_4}}$$

$$\frac{V_o}{V_i} = \frac{1}{sC_2\left(R_2 + \frac{1}{sC_2}\right) + s^2 C_1 C_2 R_1 \left(R_2 + \frac{1}{sC_2}\right) - sC_1 R_1 + sC_2 R_2}$$

$$\boxed{H(s) = \frac{1}{s^2 C^2 R^2 + 2sCR + 1}}$$

b) Evaluate the filter transfer function abs(Vo/V_i) using the transfer function derived in part (a) for the frequencies shown in the table

Subbing in $S = j\omega$, *taking the magnitude, and using the fact that* $\omega = 2\pi f$, *we get...*

$$\boxed{|H(j\omega)| = \frac{1}{\sqrt{(-4\pi^2 f^2 C^2 R^2 + 1)^2 + (4\pi f C R)^2}}}$$

$Frequency$	$\left\|\frac{V_o}{Vi}\right\| - Analytical$	$\left\|\frac{V_o}{Vi}\right\| - Measured$
$50\ Hz$	0.91	
$100\ Hz$	0.72	
$200\ Hz$	0.39	
$500\ Hz$	0.092	
$1\ KHz$	0.025	
$1.1\ KHz$	0.021	
$1.2\ KHz$	0.017	
$1.3\ Hz$	0.015	
$1.4\ KHz$	0.013	
$1.5\ KHz$	0.011	
$1.6\ KHz$	0.0098	
$1.7\ KHz$	0.0087	
$1.8\ KHz$	0.0078	
$1.9\ KHz$	0.0070	
$2\ KHz$	0.0063	
$5\ KHz$	0.0010	

c) Measure the transfer function using the AD2 board and fill the corresponding components of the table below. Use a sine wave with an amplitude of 2V and offset of 0V ($V_{cc} = \pm 5V$).

$Frequency$	$\left\|\frac{V_o}{Vi}\right\| - Analytical$	$\left\|\frac{V_o}{Vi}\right\| - Measured$
$50\ Hz$	0.91	0.899
$100\ Hz$	0.72	0.717
$200\ Hz$	0.39	0.403
$500\ Hz$	0.092	0.110
$1\ KHz$	0.025	0.0410
$1.1\ KHz$	0.021	0.039
$1.2\ KHz$	0.017	0.035
$1.3\ KHz$	0.015	0.032
$1.4\ KHz$	0.013	0.0319
$1.5\ KHz$	0.011	0.0265
$1.6\ KHz$	0.0098	0.0248
$1.7\ KHz$	0.0087	0.0247
$1.8\ KHz$	0.0078	0.0229
$1.9\ KHz$	0.0070	0.0228
$2\ KHz$	0.0063	0.0211
$5\ KHz$	0.0010	0.0200

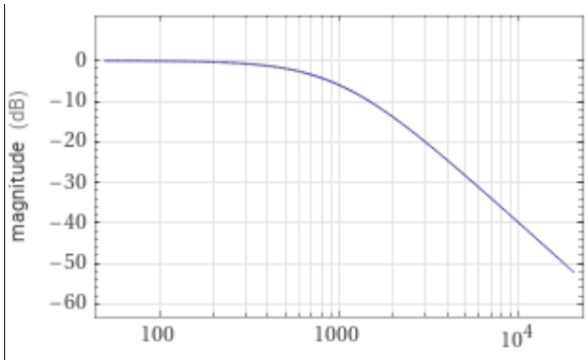
Refer to the bottom of the lab for some AD2 oscilloscope output waveforms, and the built circuit…

d) What is the cut-off frequency of this filter?

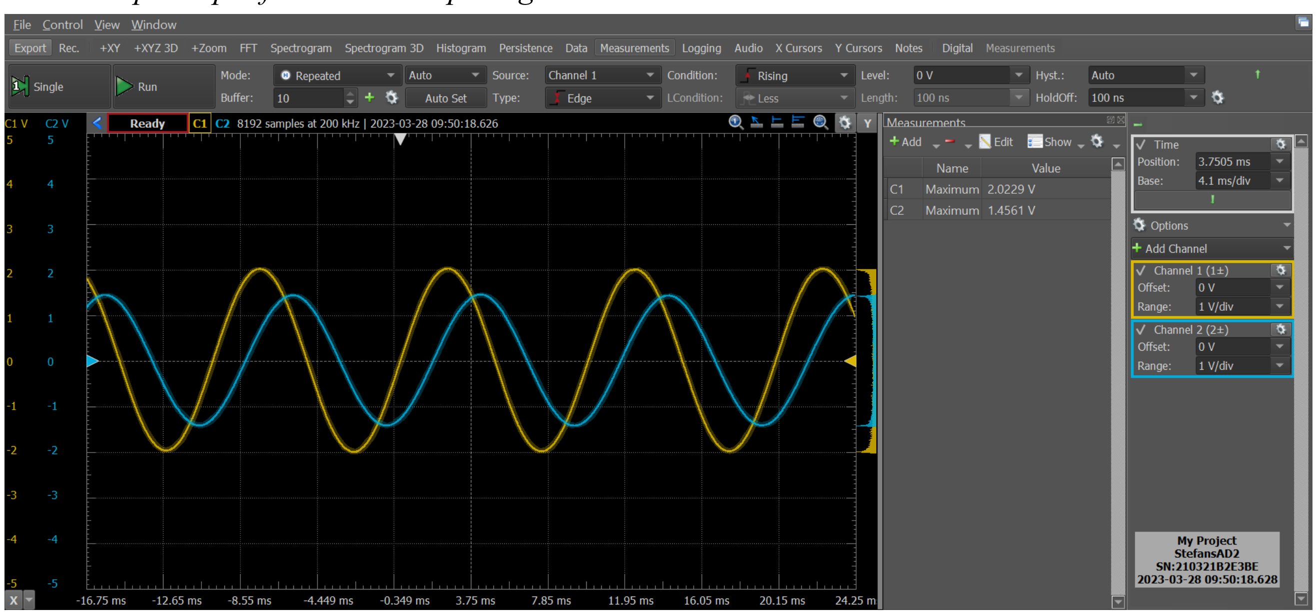
$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(10\cdot10^{3})(100\cdot10^{-9})} = 159.15\ Hz$$

d) How do the theoretical and measured results compare? Comment on your results.

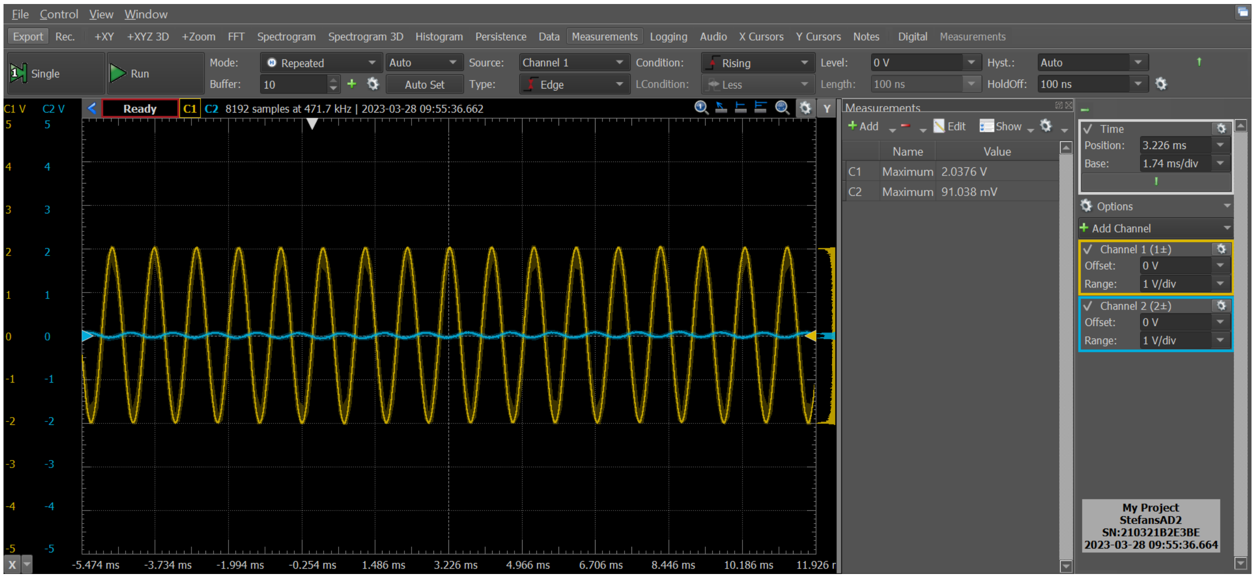
The table above shows that the theoretical and experimental values are very similar for lower frequencies, and then get less and less accurate as we move into higher frequencies. This is because as we move further and further away from the cutoff frequency, the gain of the circuit diminishes further and further. This explains why our theoretical and experimental values get further and further apart as the frequency of the input waveform, the experimental errors get higher and higher. From the generated Bode Plot on the right, we can see that the cutoff frequency is about 1000 rad/s or 160Hz. This verifies our above calculations for the cutoff frequency, and also shows how the gain diminishes after this frequency.



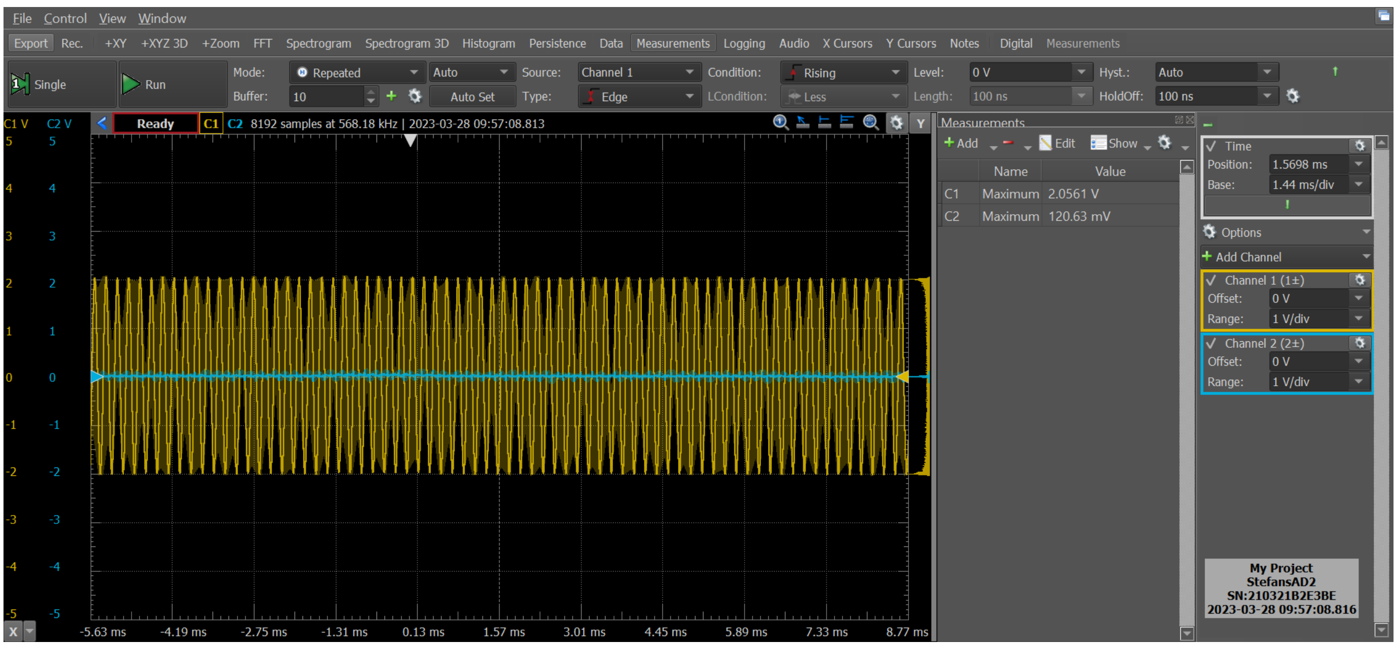
Oscilloscope output for a 100Hz input signal…



Oscilloscope output for a 1KHz input signal…



Oscilloscope output for a 5KHz input signal…



Physically built circuit…