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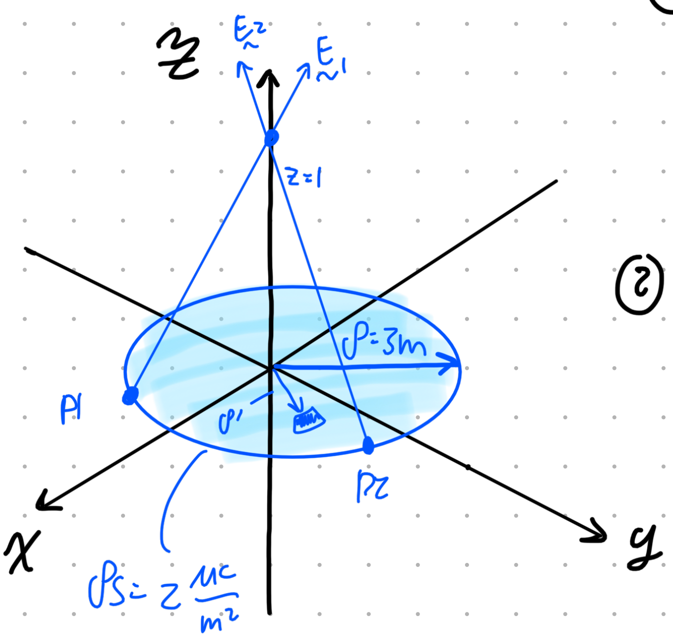
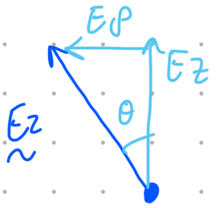


① Notice that due to the symmetry of the problem, the ρ component of all $\underset{\sim}{E}$ will cancel out, leaving only a z component of the field

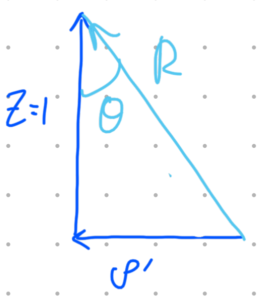
② The charge of an infantesamel charge element, dq, can be written as $dq = \rho_s \cdot ds$, and we know that the given surface changes in the ρ and ϕ directions

$$\Rightarrow dq = \rho_s \cdot \rho' d\rho' d\phi'$$



③ The diagram to the left suggests that the z component of such a force would be $E\cos\theta$

$$\Rightarrow dE_z = dE\cos\theta$$



④ We can represent θ using the diagram to the left, it will be a constant value, since $z=1$, $\rho=\rho'$ for some arbitrary observation point

$$\Rightarrow R = \sqrt{(\rho')^2 + (1)^2}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{(\rho')^2 + (1)^2}}$$

⑤ The $\underset{\sim}{E}$ from some arbitrary point is thus...

$$dE_z = dE\cos\theta = \frac{dq}{4\pi\varepsilon_0 |R|^2} \cdot \cos\theta = \frac{(\rho_s \cdot \rho' d\rho' d\phi')}{4\pi\varepsilon_0 \left(\sqrt{(\rho')^2 + (1)^2}\right)^2} \cdot \frac{1}{\sqrt{(\rho')^2 + (1)^2}}$$

$$\Rightarrow \boxed{dE_z = \frac{\rho_s \rho'}{4\pi\varepsilon_0 \sqrt{(\rho')^2 + (1)^2}^{\,3}} d\rho' d\phi'}$$

⑥ Finally, we can integrate over all ρ and ϕ to determine the Total field ---

$$E_z = \int_0^{2\pi}\int_0^1 \frac{\rho_S \, \rho'}{4\pi\varepsilon_0 \sqrt{(\rho')^2 + (1)^2}^{\,3}} \, d\rho' d\phi'$$

$$= \frac{\rho_S}{4\pi\varepsilon_0} \int_0^{2\pi} d\phi \int_0^1 \frac{\rho'}{\sqrt{(\rho')^2+(1)^2}^{\,3}}$$

$$= \left(\frac{2\times10^{-6}}{4\pi\left(\frac{10^{-9}}{36\pi}\right)}\right)(2\pi)\left(\frac{-1}{\sqrt{2}}+1\right)$$

$$\boxed{= (0, 0, 3.31\times10^{4})\ V/m}$$