EE 3CL4
Lab 3

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Stefan Tosti - Tostis - 400367761 - L06 - C02
Connor Usaty - Usatyc - 400409624 - L06 - C01

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Section 1 - Member Contributions

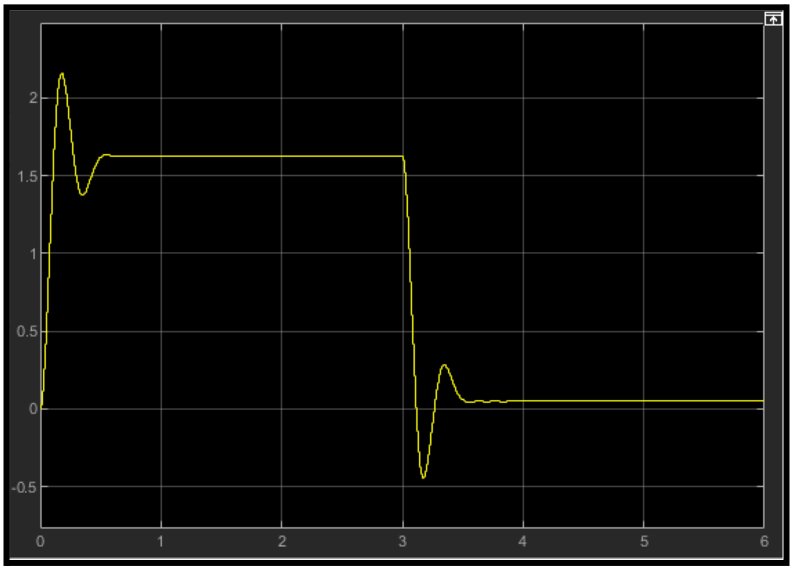
Both lab group members contributed an equal amount to the final product of this lab. Pre-lab calculations were done collaboratively, and were completed before the designated lab section. In the lab, the experiments were carried out by both group members, with each member contributing to building the required simulink model and acquiring the experimental data. Once we completed the in-lab portion, the report, calculations, and final analyses were completed by both members.

Section 2 - Objectives

The objective of this lab was to design a proportional controller with velocity feedback for the DC motor servomechanism. We also wanted to explore the trafeoff's involved in the selection process of the controller parameters, and how those parameters impact the transient and steady state responses.

Experiment 1 - Trade-offs in Rise time, Steady state error & Percent Overshoot

The response of our motor after running for a 6 second interval can be found below with $K_p = 1$…



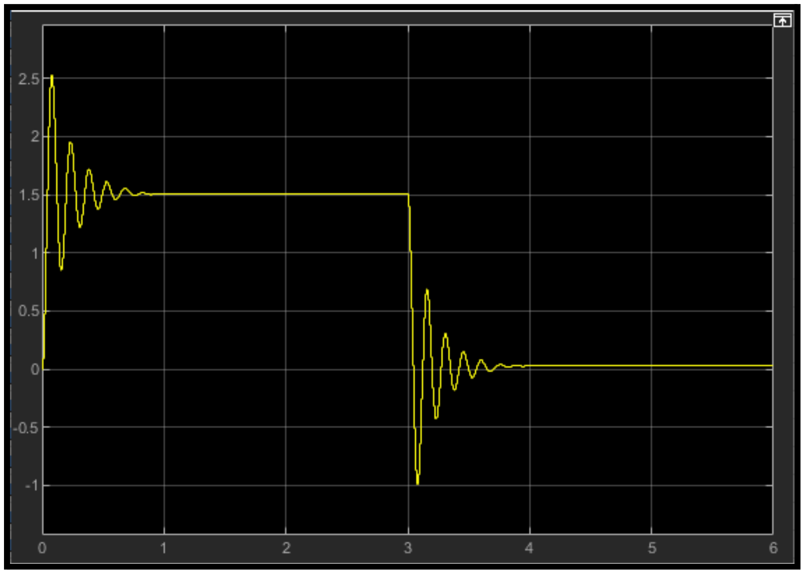
In order to measure the required values, we exported our data to Excel and performed the following computations…

$$T_{r,\,10\% - 90\%} = T_{90\%} - T_{10\%} = 102ms - 26ms = 76ms$$

$$PO = \frac{MPT - y_{ss}}{y_{ss}} \cdot 100\% = \frac{2.15875 - 1.625195}{1.622129} \cdot 100\% = 32.8\%$$

$$e_{ss} = ss_{actual} - ss_{expected} = 1.625196 - 1.5 = 0.125195\, rad$$

The response of our motor after running for a 6 second interval can be found below with $K_p = 5...$



In order to measure the required values, we exported our data to Excel and performed the following computations…

$$T_{r,\,10\% - 90\%} = T_{90\%} - T_{10\%} = 40ms - 12ms = 28ms$$

$$PO = \frac{MPT - y_{ss}}{y_{ss}} \cdot 100\% = \frac{2.526719 - 1.505605}{1.505605} \cdot 100\% = 67.8\%$$

$$e_{ss} = ss_{actual} - ss_{expected} = 1.505605 - 1.5 = 0.0505605\, rad$$

From the above results, we can see that the value for k_p has significant effects on the rise time, percent overshoot, and steady state error of our system. In addition to this, we can also see that there are trade-offs between achieving a small steady state error and rise time, and a small percent overshoot. We can further examine this by referring to the equations derived in our pre-lab…

$$T_r = \frac{2.16 + 1.2\sqrt{k_p A \tau_m}}{2k_p A} \qquad P.O = 100e^{-\frac{\pi}{\sqrt{4k_p A\tau_m - 1}}} \qquad e_{ss} = \frac{-\tau_d}{k_p}$$

We can see from the above equations that a larger value of k_p will result in a smaller rise time and steady state error, but a larger percent overshoot. Conversely, a smaller value for k_p will result in a larger rise time and steady state error, but a smaller percent overshoot. These mathematical relations verify the trends seen in our experimental data.

To theoretically calculate the value of k_p that satisfies the specified system requirements, we can use equation 11 from the lab manual, as well as the values for A & τ_m that we found in Lab 2…

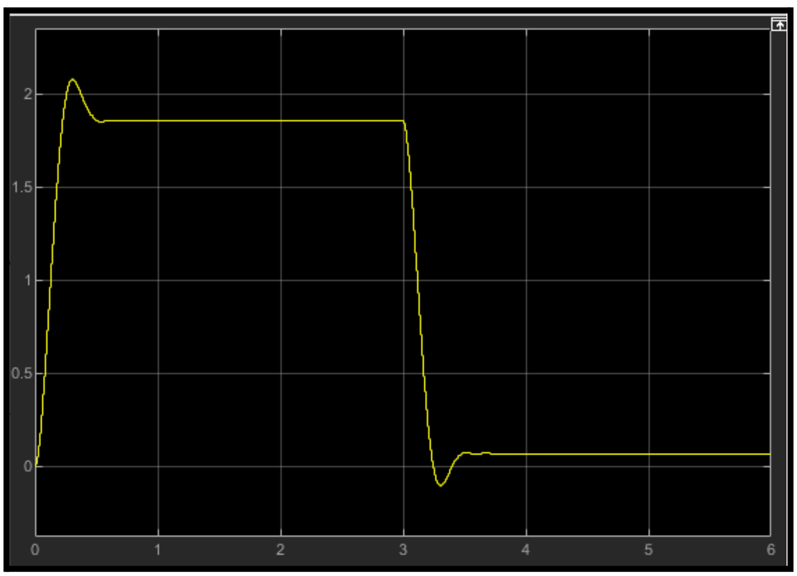
$A = 26.07$
$\tau_m = 0.079$
$P.O = 0.2$

We thus obtain the following result for k_p…

$$\frac{\left(\left(\frac{-\pi}{\ln\left(\frac{20}{100}\right)}\right)^2+1\right)}{(4)(26.07)(0.079)}$$

$$= 0.583899101027$$

Using the above obtained value, we adjusted the proportional gain controller in our system and obtained the output result…



In order to measure the required values, we exported this data to Excel and performed the following computations…

$$PO = \frac{MPT - y_{ss}}{y_{ss}} \cdot 100\% = \frac{2.079023 - 1.84291}{1.84291} = 12.8\%$$

$$e_{ss} = \frac{steady\ state - expected}{expected} \cdot 100\% = \frac{1.84291 - 1.5}{1.5} * 100 = 22.86\%$$

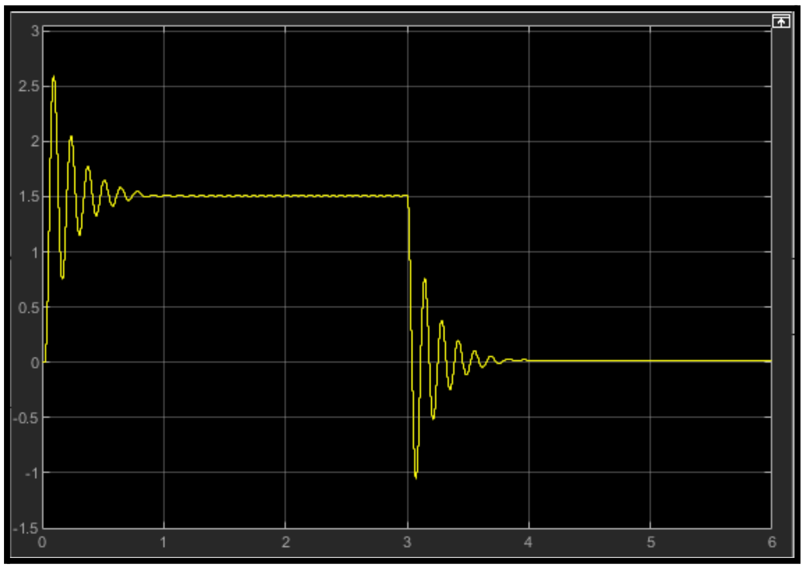
As can be seen, the above response achieved a percent overshoot that is less than 20%
To obtain a steady state error of $< 1\%$ our group used $eq.$ 16 from the lab manual…

$$k_p = \frac{\tau_d}{e_{ss}} = \frac{0.079}{0.015} = 5.267$$

Using the theoretical value of $k_p = 5.267$ our group achieved the following steady state error…

$$e_{ss} = \frac{steady\ state - expected}{expected} \cdot 100\% = \frac{1.517871 - 1.5}{1.5} \cdot 100\% = 1.19\%$$

Likely due to a combination of roundoff error, and the intricacies of the physical motor, the theoretically calculated value of k_p was slightly too high and did not allow us to achieve a steady state error of less than 1%. As a result, our group used trial and error to simulate the responses for various difference values of k_p until we minimized our steady state error. By setting $k_p = 6$ our group was able to achieve the following response and corresponding steady state error…



$$e_{ss} = \frac{steady\ state - expected}{expected} \cdot 100\% = \frac{1.505605 - 1.5}{1.5} \cdot 100\% = 0.37\%$$

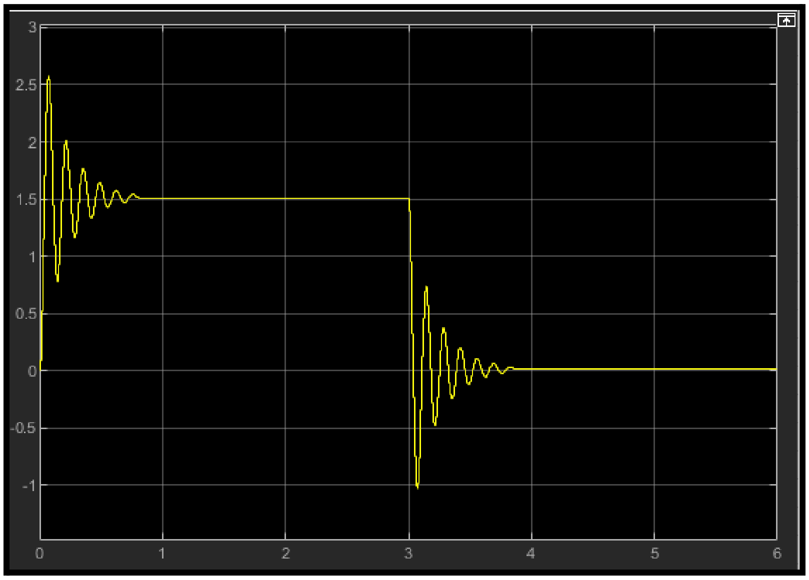
As can be seen from the above, we have achieved a steady state error of 0.37% which is substantially lower than the required 1% error.

From the above outcomes, we feel that it is not possible to satisfy both the steady state error as well as the percent overshoot constraints with just one k_p value. Our results point to the fact that optimizing one parameter leads to a substantial increase in the remaining parameter. When we achieved a percent overshoot that was less than 20%, our steady state error was quite large. When we achieved a steady state error of less than 1% we had a much larger percent overshoot. Thus, our group has come to the conclusion that having a single k_p value leads to an inverse relationship between steady state error and percent overshoot.

Experiment 3 - Proportional Controller with Velocity Feedback

In this experiment, we introduced a velocity feedback block into our system in an attempt to satisfy both the percent overshoot and steady state error conditions. The desired conditions required that we have a steady state error of less than 1% and a percent overshoot of less than 10%

Our first test of the system involved using the same k_p value that we used in experiment 2, that allowed us to minimize our steady state error $\left(k_p = 6\right)$, and setting $k_v = 0$. In this case, the response of the system is as found below…



In order to measure the required values, we exported this data to Excel and performed the following computations…

$$PO = \frac{MPT - y_{ss}}{y_{ss}} \cdot 100\% = \frac{2.578848 - 1.508627}{1.508627} \cdot 100\% = 70.9\%$$

$$e_{ss} = \frac{steady\ state - expected}{expected} \cdot 100\% = 0.57\%$$

As can be seen from the above calculations, this setup gives us the required steady state error, however the percent overshoot is far too high.

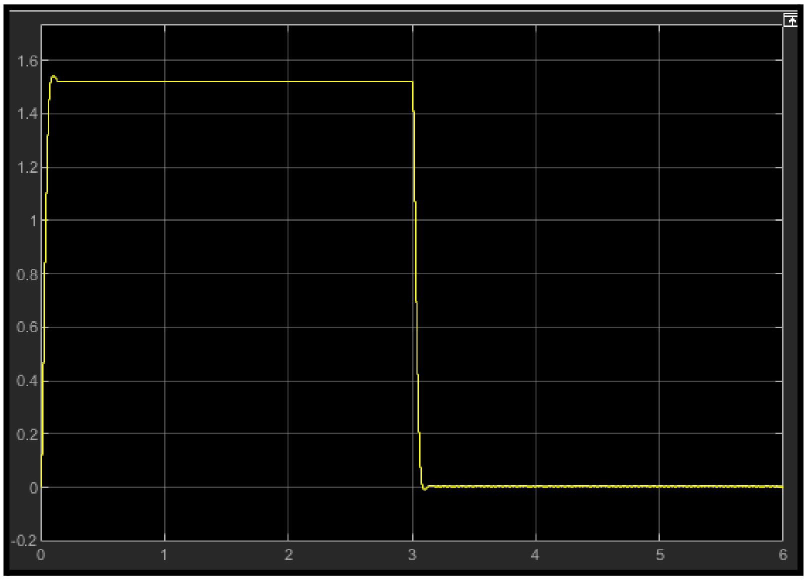
To simultaneously achieve both requirements we first found the value of the damping coefficient, ξ, that would allow us to have a 10% percent overshoot…

$$\zeta = \frac{-ln^2\left(\frac{P.O.}{100}\right)}{\sqrt{\pi^2 + ln^2\left(\frac{P.O.}{100}\right)}} = 0.59116$$

Using this calculated value of zeta, we can then isolate for k_v in equation 17 in the lab manual…

$$\zeta = \frac{1+k_v A}{2\sqrt{k_p A \tau_m}} \Rightarrow k_v = \frac{2\zeta\sqrt{k_p A \tau_m} - 1}{A} = 0.121$$

We then generated the below response using $k_v = 0.121$ & $k_p = 2$ (k_p was chosen through trial and error since our initial value of 6 did not satisfy the requirements)



In order to measure the required values, we exported this data to Excel and performed the following computations…

$$PO = \frac{MPT - y_{ss}}{y_{ss}} \cdot 100\% = \frac{1.502539 - 1.5}{1.5} \cdot 100\% = 0.17\%$$

$$e_{ss} = \frac{steady\ state - expected}{expected} \cdot 100\% = \frac{1.547871 - 1.5}{1.5} = 3.19\%$$

As can be seen, both requirements for our controller were satisfied by choosing $k_v = 0.121$ & $k_p = 2$

Comparison Between Experiment 2 & 3 Approaches

In Experiment 2, employing proportional control alone revealed limitations in meeting both the $e_{ss} <= 1\%$ and $PO <= 20\%$ constraints simultaneously. The singular control variable, k_p, posed a trade-off between minimizing steady-state error (e_{ss}) and percent overshoot (P.O). As $P.O = 100e^{-\frac{\pi}{\sqrt{4k_p A\tau_m - 1}}}$ $and\ e_{ss} = \frac{-\tau_d}{k_p}$, increasing kp for better e_{ss} led to an undesired rise in P.O. This inherent relationship hindered the ability to optimize both parameters concurrently.

In Experiment 3, integrating proportional control with velocity feedback addressed the limitations of Experiment 2. The introduction of a new control variable, k_v, alongside k_p, provided flexibility in optimization. Leveraging both k_v and k_p facilitated the achievement of prescribed constraints, allowing for simultaneous minimization of e_{ss} through k_p and P.O through k_v. This dual-variable control approach demonstrated superior performance, overcoming the trade-off observed with sole proportional control in Experiment 2.