Electronic Devices & Circuits II - EE 3EJ4
Lab #5

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2023 - 12 - 04

Questions for Part I

Question 1 - Part 1

We can derive the transfer function as follows…

$T(s) = \frac{V_o(s)}{V_{in}(s)}$, *Recall the ideal op-amp properties;* $V_+ = V_-$ & $i_+ = i_- = 0$

$V_- = V_o(s)\frac{R_3}{R_2+R_3}$

$\frac{V_{in}-V_+}{R_1} = \left(sC_1\right)V_+$

$\frac{V_{in}-V_o(s)\frac{R_3}{R_2+R_3}}{R_1} = \left(sC_1\right)\left(V_o(s)\frac{R_3}{R_2+R_3}\right)$

$$\boxed{T(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{R_3+R_2}{R_3\left(R_1sC_1+1\right)}}$$

To find the low frequency gain, we can evaluate $T(s)$ @ $s = 0$… $T(0) = 1.5 = 3.52dB$

We can compute the $-\ 3dB$ frequency as follows… $f_c = \frac{1}{2\pi R_1C_1} = 1591Hz$

Question 1 - Part 2

For the simulated data in step 1.3…

$A_V = T(0) = \frac{V_o}{V_{in}} = \frac{150mV}{100mV} = 1.5\frac{V}{V} = 3.52dB$

Frequency	M(V(Vout))	P(V(Vout))
1	0.150000077	-0.036069295

To determine the value of V_o that corresponds to f_c …

$20log\left(\frac{V_o}{0.1}\right) = 3.52 - 3 \Rightarrow V_o = 0.106V$

Frequency	M(V(Vout))	P(V(Vout))
1577.683204	0.106318853	-44.74587498

We can correspondingly find the value of f_c …

$f_c = 1577.6832\ Hz$

For the measured data in step 1.8…

$A_V = T(0) = \frac{V_o}{V_{in}} = \frac{149.7mV}{100mV} = 1.497\frac{V}{V} = 3.50dB$

Frequency	M(V(Vout))	P(V(Vout))
1	0.1497483	-0.04507

To determine the value of V_o that corresponds to f_c …

$20log(\frac{V_o}{0.1}) = 3.50 - 3 \Rightarrow V_o = 0.1059V$

Frequency	M(V(Vout))	P(V(Vout))
1513.561	0.1063753	-44.1436

We can correspondingly find the value of f_c …

$f_c = 1513.561\ Hz$

In the above cases, the measured and simulated data match (relatively closely) the data that was obtained from the derived transfer function.

Questions for Part II

Question 2

We can derive the transfer function as follows…

$T(s) = \frac{V_o(s)}{V_{in}(s)}$, *Recall the ideal op-amp properties;* $V_+ = V_-$ & $i_+ = i_- = 0$

Applying KCL at the node between R_1 & R_2 (V_x) …

$$\frac{V_x - V_{in}}{R_1} + \frac{V_x}{R_2 + sC_2} + \frac{V_x - V_{out}}{sC_1} = 0$$

$$V_- = V_+ = V_{out} \frac{R_3}{R_3 + R_4}$$

Rearranging the above, we obtain…

$$T(s) = \frac{V_o}{V_{in}} = \frac{R_3 + R_4}{s^2(C_1C_2R_1R_2R_3) + s(C_2R_2R_3 + C_2R_1R_3 - C_1R_1R_4) + R_3} = \frac{91\times10^6}{s^2 + (15.5\times10^3)s + 45.5\times10^6}$$

We can then find the desired quantities by comparing the above to the standard form equation…

$$T(s) = \frac{a_o}{s^2 + \frac{\omega_o}{Q}s + (\omega_o)^2}$$

$$Low\ Frequency\ gain = \frac{a_o}{{\omega_o}^2} = \frac{91\times10^6}{45.5\times10^6} = 2\frac{V}{V} = 6.02dB$$

Comparing the above to the simulated data in step 2.2…

Frequency	DB(V(Vout))	P(V(Vout))
Hz	dBV	Degrees
1	6.020586918	-0.122644171

Comparing the above to the measured data in step 2.6…

Frequency	Manitude	Phase
Hz	dB	Degrees
1	6.009293758	-0.112854462

As we can see, the measured and simulated data agree with the calculated value from our transfer function

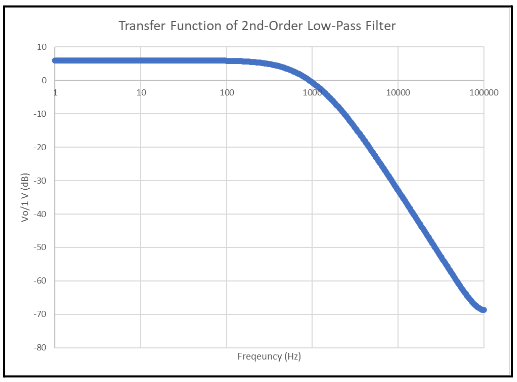
Question 3 - Part 1

We can determine the pole frequency by first determining ω_o and then using $\omega_o = 2\pi f_o$

$$f_o = \frac{\omega_o}{2\pi} = \frac{\sqrt{45.5\times10^6}}{2\pi} = 1.07\ KHz$$

The below graphs are plots from the simulated data 2.2 and 2.6 respectively. We can verify our calculated pole frequency above visually by seeing that the pass-band of the filter ends roughly around 1 KHz in both cases

From step 2.2 …



From step 2.6 …

Question 3 - Part 2

We can determine the cut-off frequency, ω_c, as follows…

$$\left|T\left(j\omega_c\right)\right| = \frac{91\times10^6}{\sqrt{\left(45.5\times10^6-\omega_c^{\ 2}\right)^2+\left(15.5\times10^3\right)^2\omega_c^{\ 2}}} = \sqrt{2}$$

$$\omega_c = 3574.52 \Rightarrow f_c = \frac{3574.52}{2\pi} = 568.9\ Hz$$

We can verify this data from step 2.2 and 2.6 by examining the frequency at which the gain reaches $3.02dB$

For step 2.2 …

Frequency	DB(V(Vout))	P(V(Vout))
565.5555225	3.036726579	-59.17249618

For step 2.6 …

Frequency	Manitude	Phase
588.8436554	3.027947597	-59.54468491

Question 3 - Part 3

We can determine the Pole Quality Factor by isolating for Q in our developed transfer function…

$$\frac{\omega_o}{Q} = 15.5 \times 10^3$$

$$Q = \frac{\omega_o}{15.5\times10^3} = \frac{\sqrt{45.5\times10^6}}{15.5\times10^3} = 0.435$$

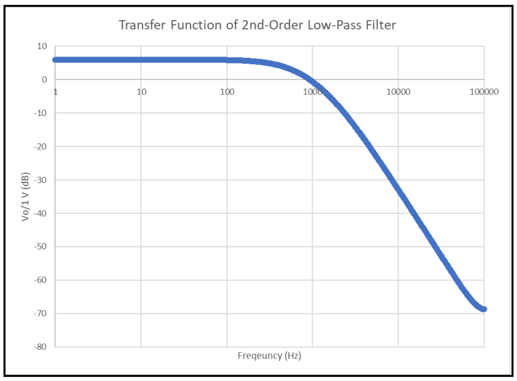
Question 3 - Part 4

The peak value of the magnitude of the transfer function would be $6.02dB$. This occurs because the above Q value is less than 0.707, meaning the system is overdamped and has no “bump” at the pole frequency.

Question 3 - Part 5

The peak value of the transfer function occurs from $0\ Hz$, and then dropping off at our pole frequency of $1.07\ KHz$

We can evaluate the results from ***Part 4*** and ***Part 5*** by again taking a look at the plotted functions from step 2.2 and 2.6 …



From these, we can tell that the magnitude of the gain is maximized at $6.02\ dB$, starting from $0\ Hz$, and then dropping off at our pole frequency of $1.07\ KHz$

Question for Part III

Question 4

We can derive the transfer function as follows…

$T(s) = \frac{V_o(s)}{V_{in}(s)}$, *Recall the ideal op-amp properties;* $V_+ = V_- \ \& \ i_+ = i_- = 0$

Applying KCL at the common node between C_1, C_2, R_2…

$\frac{V_1 - V_{in}}{R_1} + V_1 sC_2 + \frac{V_1 - V_O}{\frac{1}{sC1}} = 0$

Rearranging the above, we obtain…

$$T(s) = \frac{V_O}{V_{in}} = \frac{\frac{-S}{R_1 C_1}}{s^2 + s\left(\frac{1}{C_1 R_2} + \frac{1}{C_2 R_2}\right) + \frac{1}{C_1 C_2 R_1 R_2}}$$

In the above, the center frequency is the same as ω_o …

$$\omega_O^{\ 2} = \frac{1}{C_1 C_2 R_1 R_2} \Rightarrow \omega_O = \sqrt{\frac{1}{C_1 C_2 R_1 R_2}} \approx 4166.7 \frac{rad}{s}$$

$$f_O = \frac{\omega_O}{2\pi} = 663 \ Hz$$

Plugging the above value of ω_O into the derived transfer function, we obtain…

$$Center\ Frequency\ Gain = T\left(4166.7 \frac{rad}{s}\right) = -6.02\ dB, -0.5\ dB$$

We can verify the above using the data in step 3.2 and 3.6…

Frequency	DB(V(Vout))	P(V(Vout))
663.4102514	-6.020589659	-180.0370809

Frequency	Manitude	Phase
660.693448	-6.0078727	178.733222

Question 5 - Part 1

Using the transfer function derived in the previous question, we can obtain the center frequency…

$$\omega_O^{\ 2} = \frac{1}{C_1 C_2 R_1 R_2} \Rightarrow \omega_O = \sqrt{\frac{1}{C_1 C_2 R_1 R_2}} \approx 4166.7 \frac{rad}{s}$$

$$f_O = \frac{\omega_O}{2\pi} = 663 \ Hz$$

Question 5 - Part 2

Using the transfer function derived in the previous question, we can determine the pole quality factor …

$$\frac{\omega_O}{Q} = \frac{1}{C_1 R_2} + \frac{1}{C_2 R_2} \Rightarrow Q = 0.5$$

Question 5 - Part 3

We can determine the 2 pole frequencies as follows…

$$\frac{\frac{-s}{R_1C_1}}{s^2+s(\frac{1}{C_1R_2}+\frac{1}{C_2R_2})+\frac{1}{C_1C_2R_1R_2}} = -9.03dB$$

$\omega_1 = 1725.9\,\frac{rad}{s}$, $\omega_2 = 10,059.2\frac{rad}{s}$

Question 5 - Part 4

We can determine the $3dB$ bandwidth as follows…

$BW = \omega_2 - \omega_1 = 8333.3\frac{rad}{s}$

We can verify the above measurements as follows, using the data from step 3.2 and 3.6…

Frequency	DB(V(Vout))	P(V(Vout))
272.6738966	-9.075976901	-134.708069

Frequency	DB(V(Vout))	P(V(Vout))
1614.064152	-9.081884151	-225.357865

Frequency	Manitude	Phase
269.15348	-8.9916308	-136.16922

Frequency	Manitude	Phase
1621.8101	-9.1015004	132.758679