ELECENG 3TQ3: Assignment 4

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Question 1

For the following parts, we will use the result of the Inspection Paradox with respect to an Exponentially Distributed random variable. For the context of this problem, we will let $\lambda_P = \frac{1}{10}$, $\lambda_M = \frac{1}{10}$, and $\lambda_B = \frac{1}{20}$, for Papa, Mama and Baby bear respectively.

Part A

The probability that Papa Bear is to arrive home first can be determined by taking the ratio of λ_P to the sum of all other λ's

$$\mathbb{P}(\textit{Papa first}) = \frac{\lambda_P}{\lambda_P + \lambda_M + \lambda_B} = \frac{2}{5} = \mathbf{40}\%$$

Part B

The probability that Papa is last to return home is the same as the probability that we have Baby bear first and Mama second, or Mama bear first and Baby second.

$$\begin{aligned}
\mathbb{P}(\textit{Papa last}) &= \mathbb{P}(\textit{Baby first and Mama Second}) \text{ or } \mathbb{P}(\textit{Mama first and Baby Second}) \\
\mathbb{P}(\textit{Papa last}) &= \frac{\lambda_M}{\lambda_P + \lambda_M + \lambda_B} \cdot \frac{\lambda_B}{\lambda_P + \lambda_B} + \frac{\lambda_B}{\lambda_P + \lambda_M + \lambda_B} \cdot \frac{\lambda_M}{\lambda_P + \lambda_M} \\
\mathbb{P}(\textit{Papa last}) &= \frac{7}{30} \approx \mathbf{23.3}\%
\end{aligned}$$

Part C

The probability that Baby Bear is to arrive home first can be determined by taking the ratio of λ_B to the sum of all other λ's

$$\mathbb{P}(\textit{Baby first}) = \frac{\lambda_B}{\lambda_P + \lambda_B + \lambda_M} = \frac{1}{5} = \mathbf{20}\%$$

Part D
With the same reasoning as above, we can determine the probability that Baby bear return before Papa and Mama

$$\mathbb{P}(\textit{Baby first}) = \frac{\lambda_B}{\lambda_P + \lambda_B + \lambda_M} = \frac{1}{5} = \mathbf{20}\%$$

Part E
To determine the probability that an adult bear will return home first, we can determine the probability that Papa arrives first or Mama arrives first.

$$\begin{aligned}
&\mathbb{P}(\textit{Adult First}) = \mathbb{P}(\textit{Mama first}) \text{ or } \mathbb{P}(\textit{Papa first}) \\
&\mathbb{P}(\textit{Adult First}) = \frac{\lambda_M}{\lambda_P + \lambda_M + \lambda_B} + \frac{\lambda_P}{\lambda_P + \lambda_M + \lambda_B} \\
&\mathbb{P}(\textit{Adult first}) = \frac{4}{5} = \mathbf{80}\%
\end{aligned}$$

Part F
Due to the memory-less property of the Exponential Distribution, Papa bear's arrival time does not affect the arrival time of Baby bear and Mama bear, this can simply be computed as the probability that Baby bear arrives before Mama Bear.

$$\mathbb{P}(\textit{Baby before Mama}) = \frac{\lambda_B}{\lambda_B + \lambda_M} = \frac{1}{3} \approx \mathbf{33.3}\%$$

Question 2

We can begin by defining some variables

$$\mathbf{T}_i := i^{th} \text{ arrival time}$$
$$\mathbf{T}^{'} := \mathbf{T}_1 + \mathbf{T}_2 + ... + \mathbf{T}_n$$
$$\mathbf{N} := \text{ Number of available Go Bus seats}$$
$$\mathbf{N_o} := \text{ Number of initially occupied Go Bus seats}$$

The goal is to determine the expected value of the above defined $\mathbf{T}^{'}$ such that $\mathbf{N} + \mathbf{N_o} = 61$. To do this we will make use of the following formula

$$\mathbb{E}(\mathbf{T}^{'}) = \mathbb{E}(\mathbf{N})\mathbb{E}(\mathbf{T})$$

We are given the rate parameter for the interarrival times of, which we can use to determine $\mathbb{E}(\mathbf{T})$

$$\mathbb{E}(\mathbf{T}) = \frac{1}{\lambda} = \frac{1}{2}$$

Rearranging the above for $\mathbf{N}$ we obtain

$$\mathbf{N} = 61 - \mathbf{N_o}$$

We can then obtain the $\mathbb{E}(\mathbf{N})$ as follows

$$\mathbb{E}(\mathbf{N}) = 61 - \mathbb{E}(\mathbf{N_o})$$
$$\mathbb{E}(\mathbf{N}) = 61 - 10 = 51$$

Finally, we can obtain the desired Expected Value of $\mathbf{T}^{'}$

$$\mathbb{E}(\mathbf{T}^{'}) = \mathbb{E}(\mathbf{N})\mathbb{E}(\mathbf{T})$$
$$\mathbb{E}(\mathbf{T}^{'}) = 51 \cdot \frac{1}{2}$$
$$\mathbb{E}(\mathbf{T}^{'}) = 25.5 \text{ minutes}$$