Assignment 1

Tuesday, September 26, 2023 4:35 PM

Assignment 1

Stefan Tosti
400 367 761

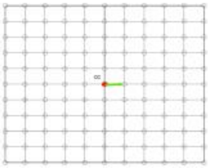
Assignment 1 – ELEC ENG 3TQ3
Due Date: Oct 3rd, 4:30 p.m. 2023
Upload on Avenue

Q1 (10 points): Three players are rolling a dice in a sequential order starting with A i.e. they roll a dice in the following order A – B – C – A – B – C . The winner is a player who rolls 6 first.

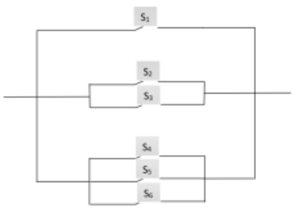
a) (4 pts) Find probability for each one of the players (A,B,C) to win the game.
b) (4 pts) Find the probability that A wins a game if B did not win the game.
c) (2 pts) Assuming that they are using different dice and that you know one of them is not fair. Further assume they repeat the game 1000 times. How would you determine which one of the players uses cheating dice?

Q2 (10 points): Consider AV vehicle moving on a grid given in the Figure below. Assume that the red point has coordinates (0,0) and that the lengths of the squares are 1 in both directions. Green line represents motion towards east from coordinate (0,0) to coordinate (1,0). Consequently for an arbitrary point (i,j) there are 4 possible motions: a) to east (i+1,j) b) to west (i-1,j), c) north (I,j+1) and d) south (I,j-1).

A) Assuming that all 4 motions are equally likely find probability that AV reaches (1,1) in exactly two motions.
B) Find probability mass function of DISTANCE between the location point to coordinate origin after EXACTLY two motions.
C) Repeat part b) for THREE motions.



Q3 (10 pts) Consider a switches diagram in the Figure below. Assuming that all the switches are independent with probabilities of being open 0.5 find the probability that there will be a current flowing through the circuit. What is the probability that switch S1 is closed IF there is a current flowing in the circuit?



Q1

a) $P(6) = \frac{1}{6}$ => Probability of Rolling a 6

the given problem can be solved with a geometric Distribution (Since we Repeat until a desirable outcome has been achieved)

$p = \frac{1}{6}$

$q = \frac{5}{6}$

possible Sequences for A winning? 6, ABC6, ABCABC6, ...

$\Rightarrow P(A\ wins) = \left(\frac{1}{6}\right) + \left(\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}\right) + \left(\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}\right) + \dots$

$= p + p(1-p)^3 + p(1-p)^6 + \dots + p(1-p)^{3k}$

$$\Rightarrow P(A\ wins) = \frac{1}{6}\sum_{k=0}^{\infty}\left(\frac{5}{6}\right)^{3k} = \frac{(1/6)}{1-(5/6)^3} = \frac{36}{91}$$

Possible Sequences for B Winning? A6, ABCA6, ABCABCA6, ...

$P(B\ wins) = \left(\frac{5}{6} \cdot \frac{1}{6}\right) + \left(\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}\right) + \left(\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}\right) + \dots$

$\Rightarrow P(B\ wins) = p(1-p) + p(1-p)^4 + p(1-p)^7 + \dots + p(1-p)^{3k+1}$

$$\Rightarrow P(B\ wins) = \frac{1}{6}\sum_{k=0}^{\infty}\left(\frac{5}{6}\right)^{3k+1} = \frac{1}{6} \cdot \frac{5}{6} \cdot \sum_{k=0}^{\infty}\left(\frac{5}{6}\right)^{3k} = \frac{5}{6} \cdot \frac{36}{91} = \frac{30}{91}$$

Possible Sequences for C Winnings? AB6, ABCAB6, ABCABCAB6, ...

$P(C\ wins) = \left(\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}\right) + \left(\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}\right) + \dots$

$\Rightarrow P(C\ wins) = p(1-p)^2 + p(1-p)^5 + \dots + p(1-p)^{3k+2}$

$$\Rightarrow P(C\ wins) = \frac{1}{6}\sum_{k=0}^{\infty}\left(\frac{5}{6}\right)^{3k+2} = \left(\frac{5}{6}\right)^2 \frac{1}{6}\sum_{k=0}^{\infty}\left(\frac{5}{6}\right)^{3k} = \left(\frac{5}{6}\right)^2 \cdot \frac{36}{91} = \frac{25}{91}$$

Sanity Check: $\frac{36+30+25}{91} = \frac{91}{91} = 100\%$

b) $$P(A|B') = \frac{P(B'|A)\,P(A)}{P(B')} = \frac{(1)\left(\frac{36}{91}\right)}{\left(1-\frac{30}{91}\right)} = \frac{36}{61}$$

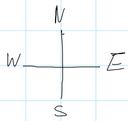
→ logically I would think $P(B'|A)$ is 1 Since if we Assume A wins then we have a 100% chance of B Not winning (?)

c) We can count the # of times each player wins. If the win Rate is larger than the Predicted number of wins, then that player cheated:

If A wins more than 395 Rounds then A cheated
If B wins more than 330 Rounds then B cheated
If C wins more than 275 Rounds then C cheated

Q2

a) We know all 4 motions are equally likely



$P(N) = P(E) = P(S) = P(W) = 0.25$

The AV can Reach (1,1) in only 2 possible ways

Way 1: E, N
Way 2: N, E

$P((1,1)\ in\ 2) = (P(E)P(N)) + (P(N)P(E))$

$= ((0.25)(0.25)) + ((0.25)(0.25))$

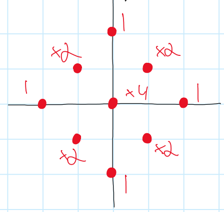
Note that these are mutually exclusive So we do not need to Subtract the intersection

$P((1,1)\ \text{in } 2) = (P(E)P(N)) + (P(N)P(E))$
$= ((0.25)(0.25)) + ((0.25)(0.25))$
$= 0.125$

* Note that these are mutually exclusive so we do not need to subtract the intersection

b) Let X be the discrete Random Variable describing the distance between the location point and coordinate origin

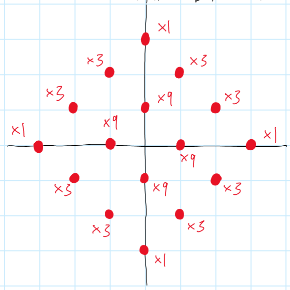
We can plot all possible outcomes of 2 movements Below



There are 16 total destinations
8 destinations have distance $= \sqrt{2}$
4 destinations have distance $= \emptyset$
4 destinations have distance $= 2$
$P(X=\sqrt{2}) = 8/16 = 1/2$
$P(X=2) = 4/16 = 1/4$
$P(X=0) = 4/16 = 1/4$

$$\boxed{P_X(X=x) = \begin{cases} 1/4 \ ; & x=2 \\ 1/2 \ ; & x=\sqrt{2} \\ 1/4 \ ; & x=\emptyset \\ \emptyset \ ; & \text{otherwise} \end{cases}}$$

c) Let X be the discrete Random Variable describing the distance between the location point and coordinate origin



there are 64 Total paths and destinations
24 Paths have distance $= \sqrt{5}$
4 Path have distance $= 3$
36 Paths have distance $= 1$
$P(X=1) = 9/16$
$P(X=\sqrt{5}) = 3/8$
$P(X=3) = 1/16$

$$\boxed{P_X(X=x) = \begin{cases} 9/16 \ ; & x=1 \\ 3/8 \ ; & x=\sqrt{5} \\ 1/16 \ ; & x=3 \\ 0 \ ; & \text{otherwise} \end{cases}}$$

Q3)

a) $P(\text{open}) = 0.5$, $P(\text{closed}) = 0.5$
A Current Requires at least 1 full Path from Start to finish

$P(\text{Current}) = (P(S1)) \cup (P(S2) \cup P(S3)) \cup (P(S4) \cup P(S5) \cup P(S6))$
↳ Recall The Inc-Ex Principle for 3 Sets
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
$P(A) = P(S1) = 0.5$
$P(B) = P(S2) \cup P(S3) = (0.5) + (0.5) - (0.5)(0.5) = 0.75$
$P(C) = P(S4) \cup P(S5) \cup P(S6) = (0.5) + (0.5) + (0.5) - 0.25 - 0.25 - 0.25 + 0.125 = 0.875$

$\Rightarrow P(\text{Current}) = P(A \cup B \cup C) = 0.5 + 0.75 + 0.875 - (0.5)(0.75) - (0.5)(0.875) - (0.75)(0.875) + (0.5)(0.75)(0.875)$
$= 0.984375$

b) $P(S1 \mid C) = \dfrac{P(C \mid S1)\, P(S1)}{P(C)} = \dfrac{(1)(0.5)}{0.984375} = 0.50794$

(4) $P(C|S1) = 1$ Since we must have a Current if S1 is closed