EE 3TR4
Lab 4
-
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Numerical Experiment #1

Section i

The calculation for the output Power Spectrum Density of the system can be found as follows…

$$S_x(f) = Z$$

$$S_y(f) = S_x(f)|H(f)|^2 = Z \cdot |H(f)|^2$$

$$H(f) = rect\left(\frac{f}{2f_b}\right) = rect\left(\frac{f}{2 \cdot 250}\right) = rect\left(\frac{f}{500}\right)$$

$$\therefore The\ output\ PSD\ is;\ S_y(f) = Z \cdot rect\left(\frac{f}{500}\right)$$

The calculation of the Autocorrelation of the system is as follows…

$$Property\ (1)\ -\ sinc(t) \Leftrightarrow rect(f)$$

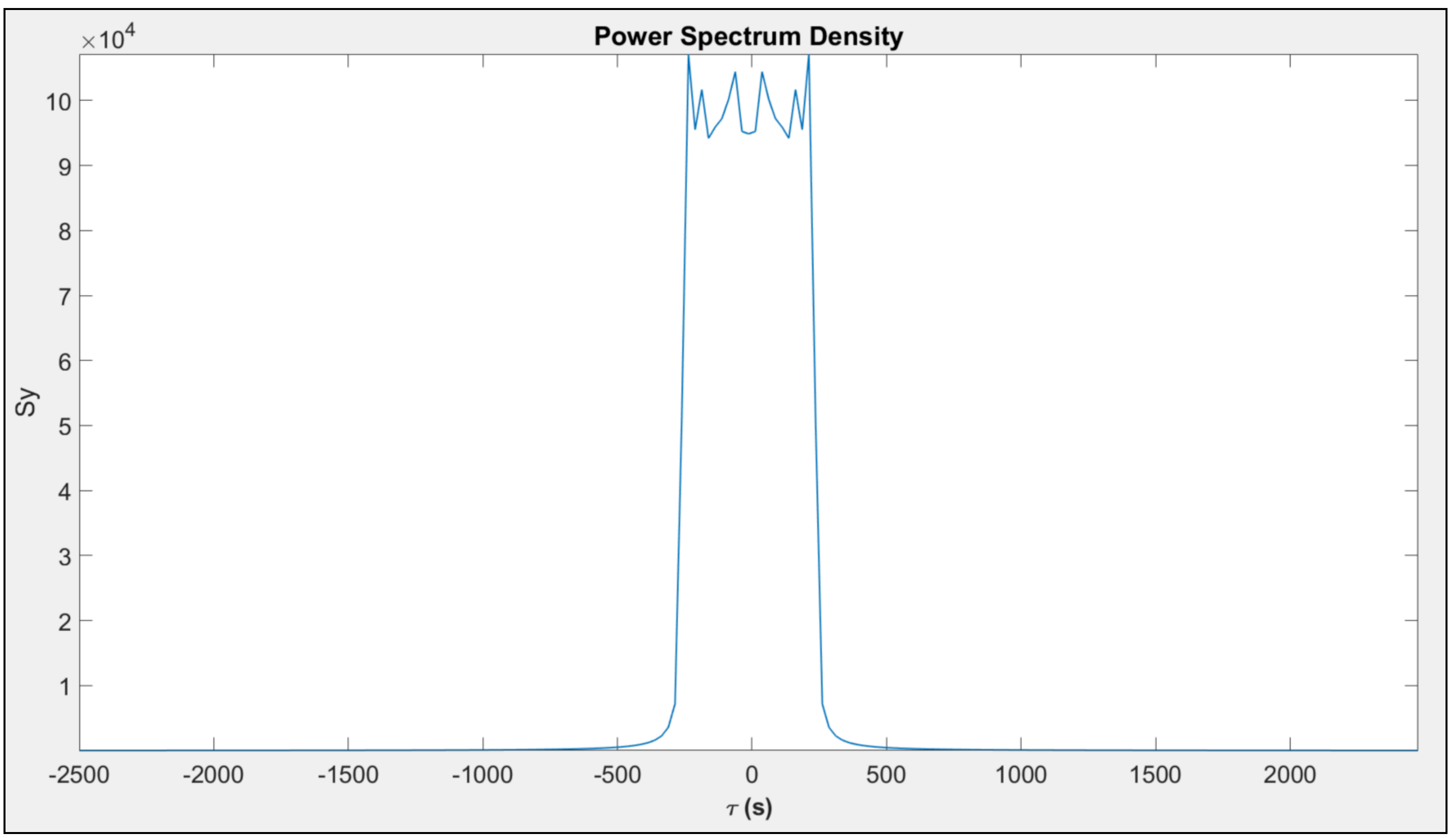
$$Property\ (2)\ -\ sinc(at) \Leftrightarrow \frac{1}{|a|} rect\left(\frac{f}{a}\right)$$

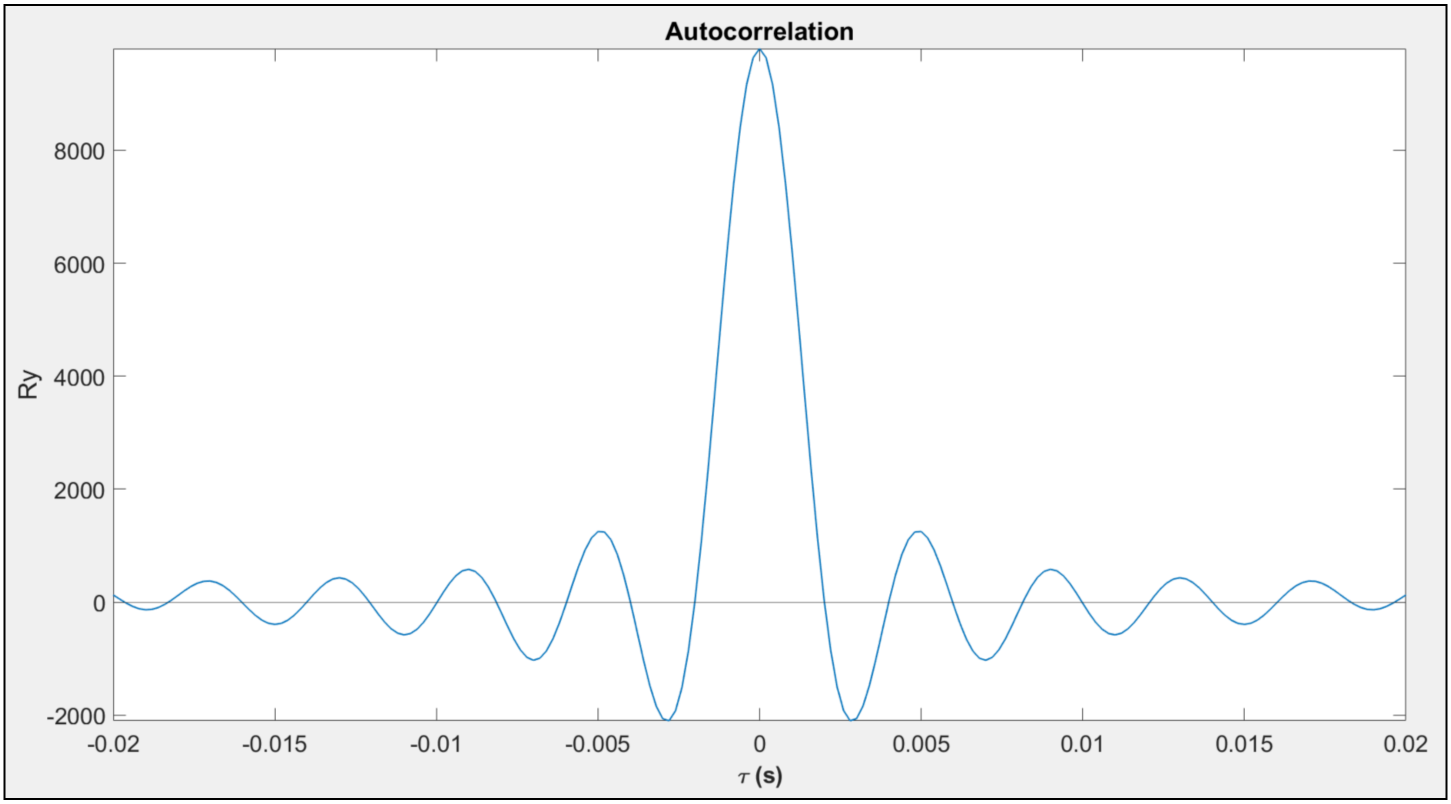
$$sin\left(2f_b t\right) \Leftrightarrow \frac{1}{2f_b} rect\left(\frac{f}{2f_b}\right)$$

$$R_y(z) = Z \cdot 2f_b\, sinc\left(2f_b \tau\right)\ \Leftrightarrow\ Z \cdot rect\left(\frac{f}{500}\right) = S_y(f)$$

$$\therefore The\ autocorrelation\ is;\ R_y(z) = Z \cdot 500 sinc(500\tau)$$

The plot of the autocorrelation and PSD can be found below. Each has been titled accordingly…

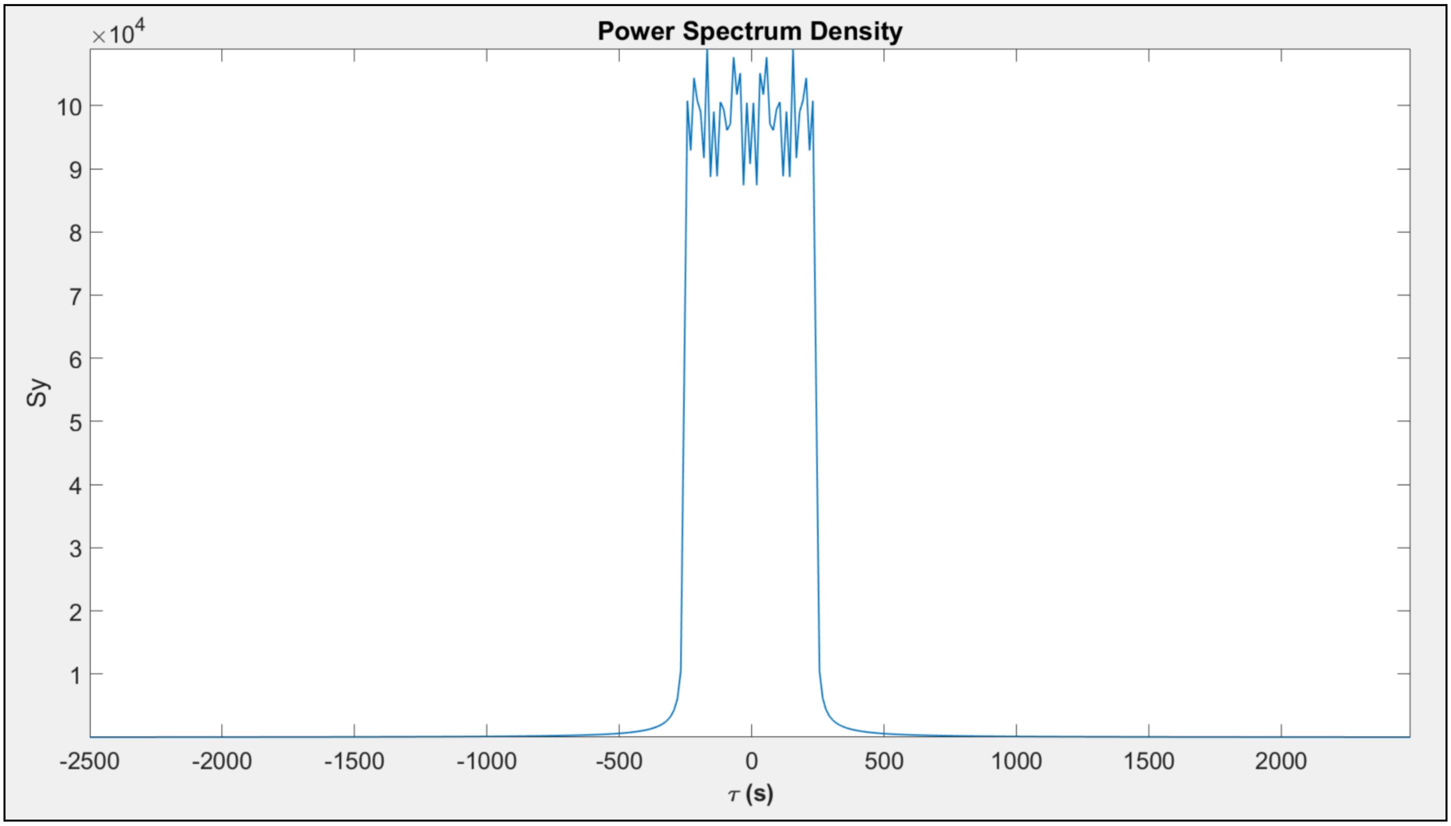


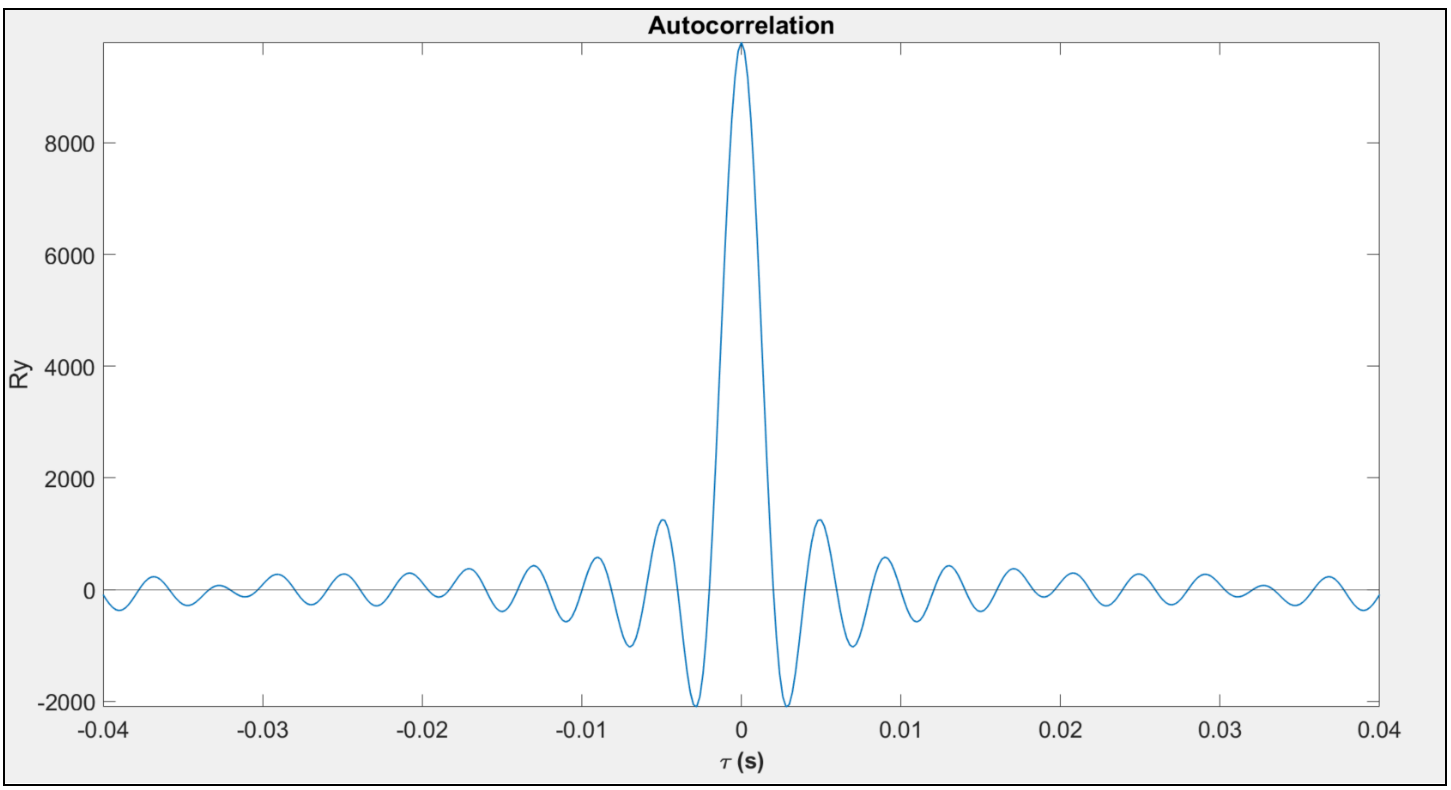


As can be seen from the graphs above, both the autocorrelation and the PSD match our theoretical calculations. Our PSD plot is a rectangular function of bandwidth 250Hz, and we can see that the autocorrelation follows the form of a sinc function. In terms of discrepancies, we can see that there are definitely some non-idealities in the output plot of the PSD. The peak of the PSD is misaligned with what we would expect from our theoretical calculations, this is likely due to the differences between the DFT and the Fourier Transform. It should be noted that the plots above were generated using *maxlag = 100*.

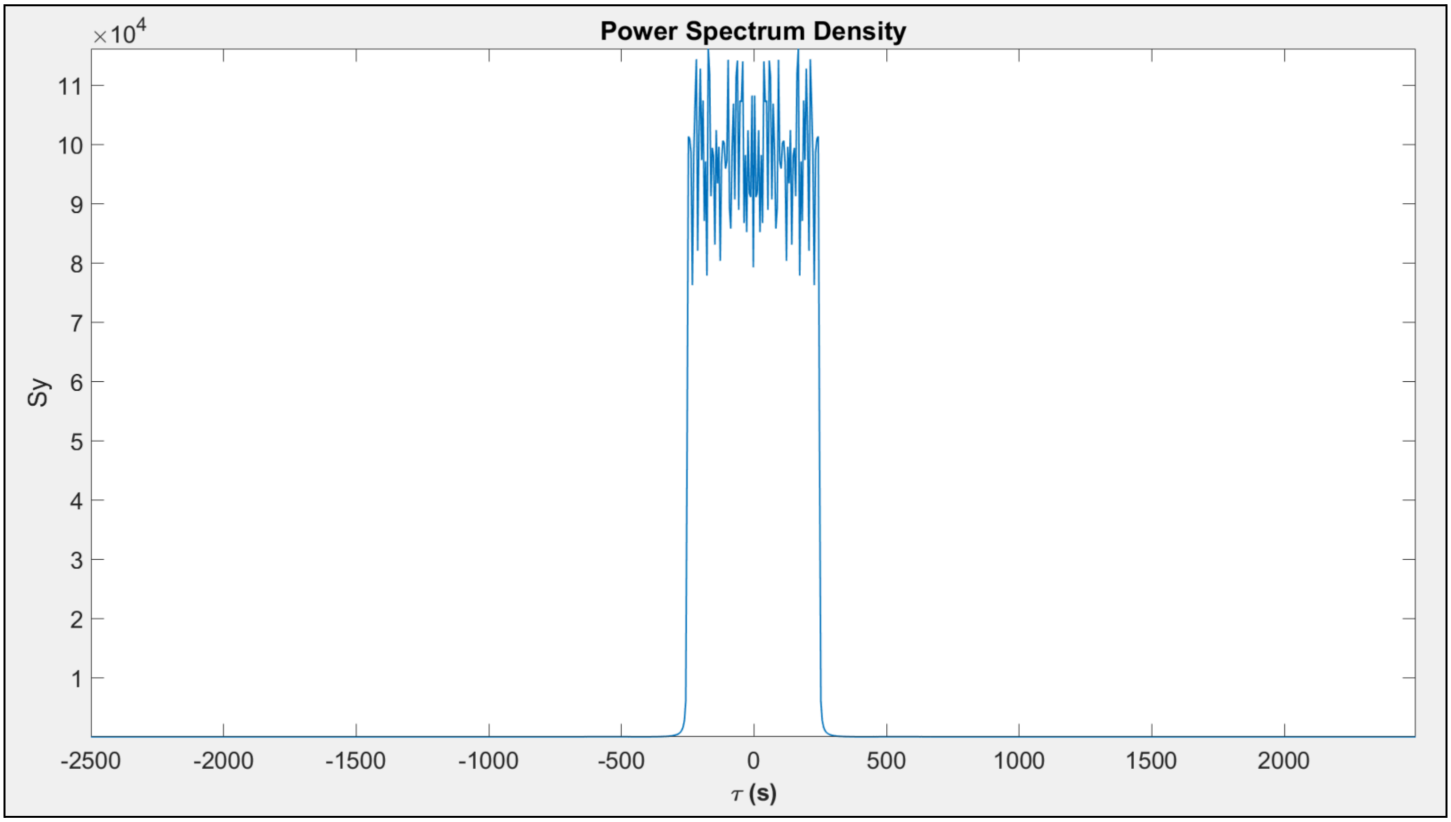
Section ii

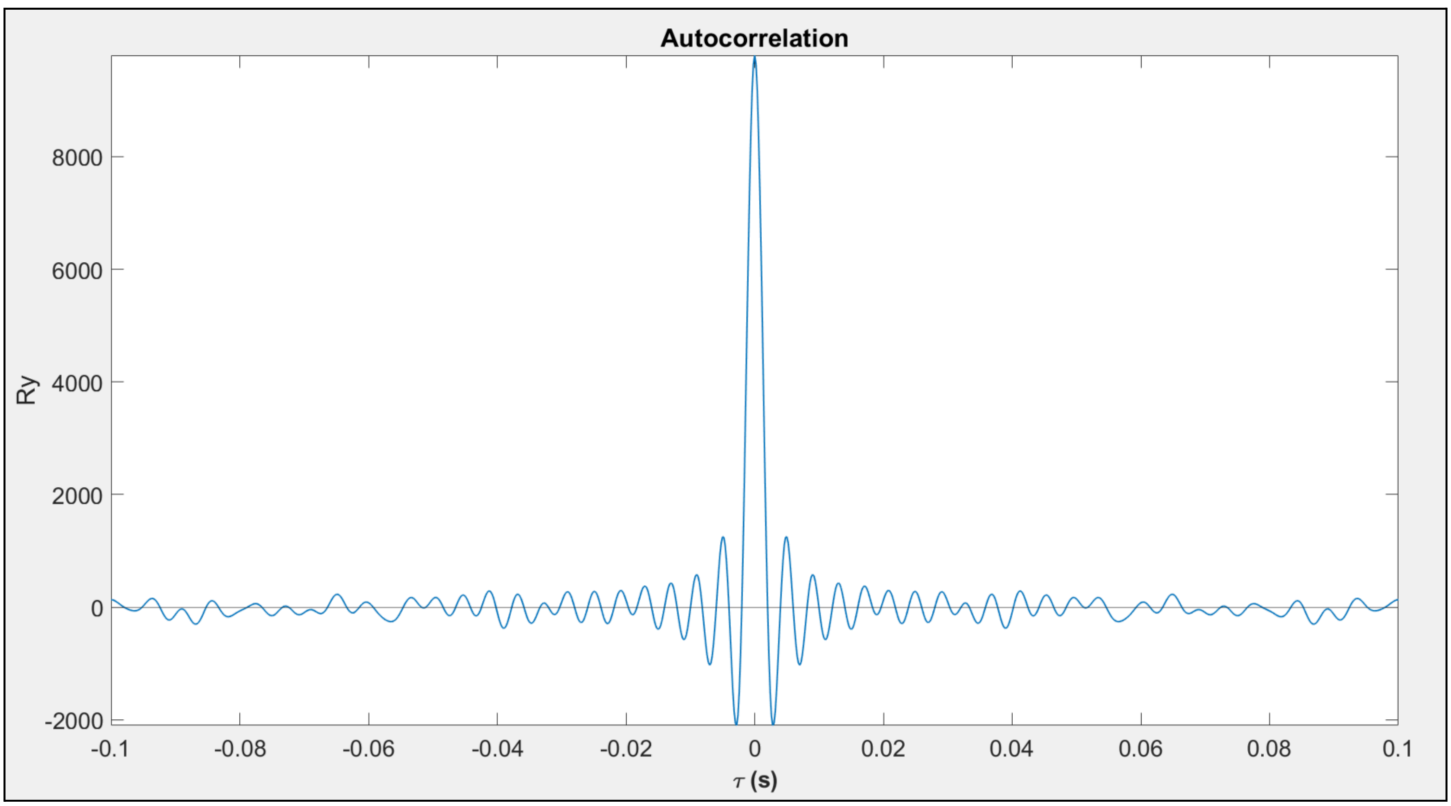
Changing the value of *maxlag = 200* we obtain the plots below…





Changing the value of *maxlag = 500* we obtain the plots below…

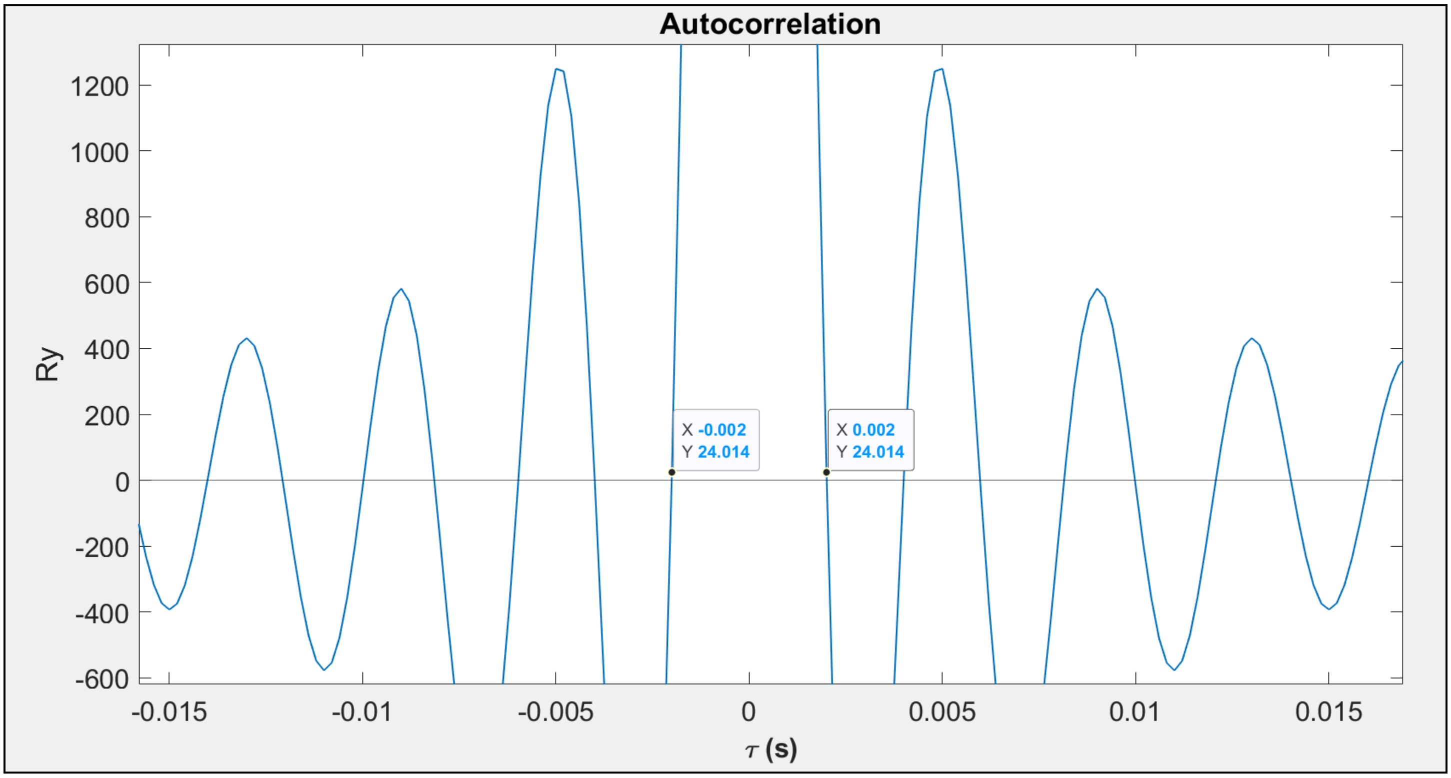




From the above plots, we can see, the more we increase *maxlag* the more the amount of information within the PSD plots increases, as does the variance. When comparing the two previous plots to the plot of *maxlag = 500* we notice a large difference in the frequency of the autocorrelation plots, and subsequently, greater visibility of the encoded information.

Section iii

Using the autocorrelation plot with *maxlag = 500* we can estimate the bandwidth as follows…



$$Bandwidth = \frac{1}{0.02 - (-0.02)} = \frac{1}{0.04} = 250Hz$$

This calculation matches our theoretical calculations above, in which the bandwidth of the autocorrelation was also 250Hz

Numerical Experiment #2

The calculation for the Autocorrelation at the output of the AWGN can be found below…

$$y(t) = A sin\left(2\pi f_c t + \theta\right) + w(t)$$

$$R_y(\tau) = E(y(t)y(t + \tau))$$

$$R_y(\tau) = E\left[\left(A sin\left(2\pi f_c t + \theta\right) + w(t)\right)\left(A sin\left(2\pi f_c (t + \tau) + \theta\right) + w(t + \tau)\right)\right]$$

$$let\ \alpha \Rightarrow 2\pi f_c t + \theta\ \&\ \beta \Rightarrow 2\pi f_c (t + \tau) + \theta$$

$$R_y(\tau) = E\left[A^2 sin(\alpha) sin(\beta) + A sin(\alpha) w(t + \tau) + A sin(\beta) w(t) + w(t) w(t + \tau)\right]$$

$$R_y(\tau) = \frac{A^2}{2} sin\left(2\pi f_c \tau\right)$$

The calculation for the PSD at the output of the AWGN can be found below…

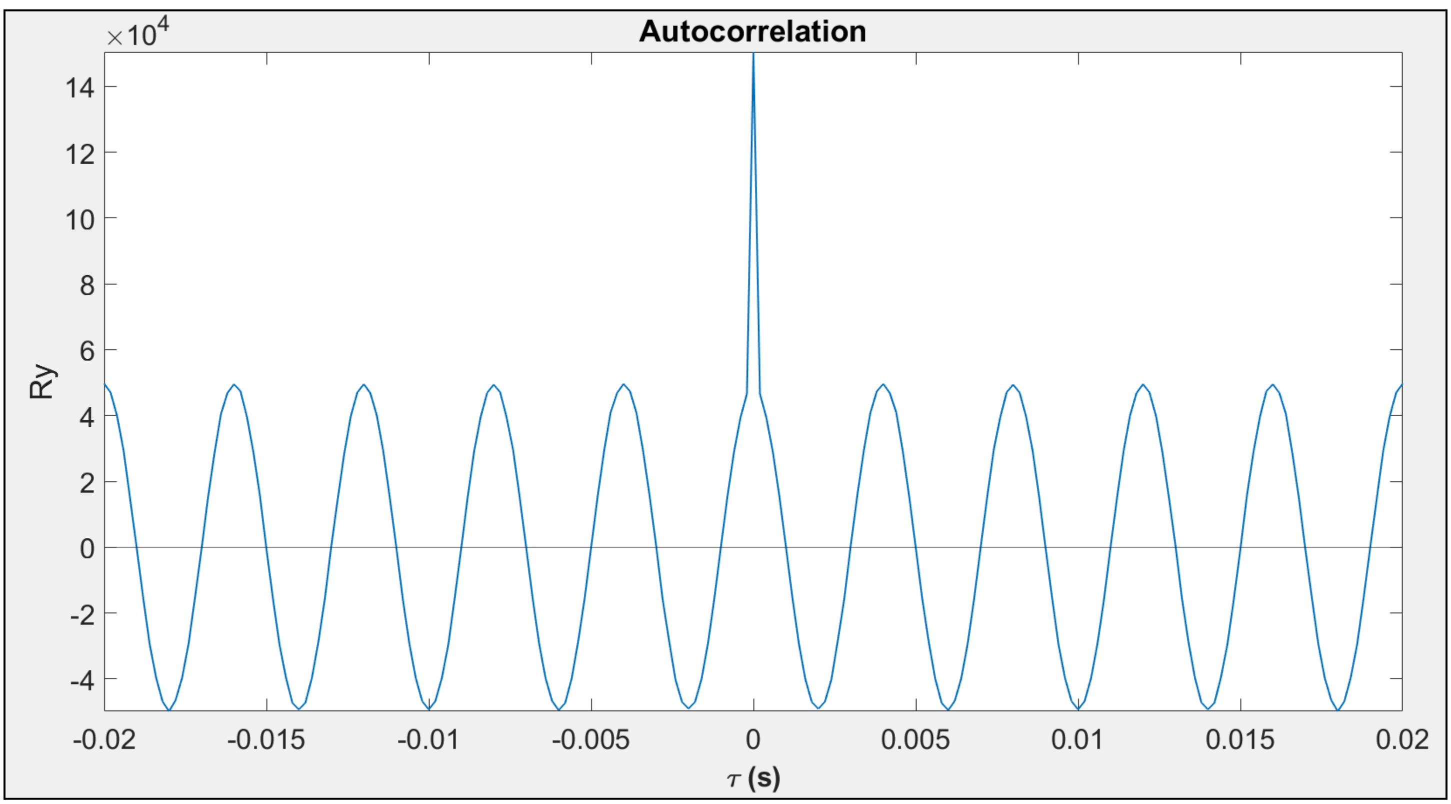
$$S_y(f) \Leftrightarrow R_y(\tau)$$

$$S_y(f) = FT\left[R_y(\tau)\right]$$

$$S_y(f) = FT\left[\frac{A^2}{2} sin\left(2\pi f_c \tau\right)\right]$$

$$S_y(f) = \frac{A^2}{4}\left[\delta\left(f - f_c\right) - \delta\left(f + f_c\right)\right]$$

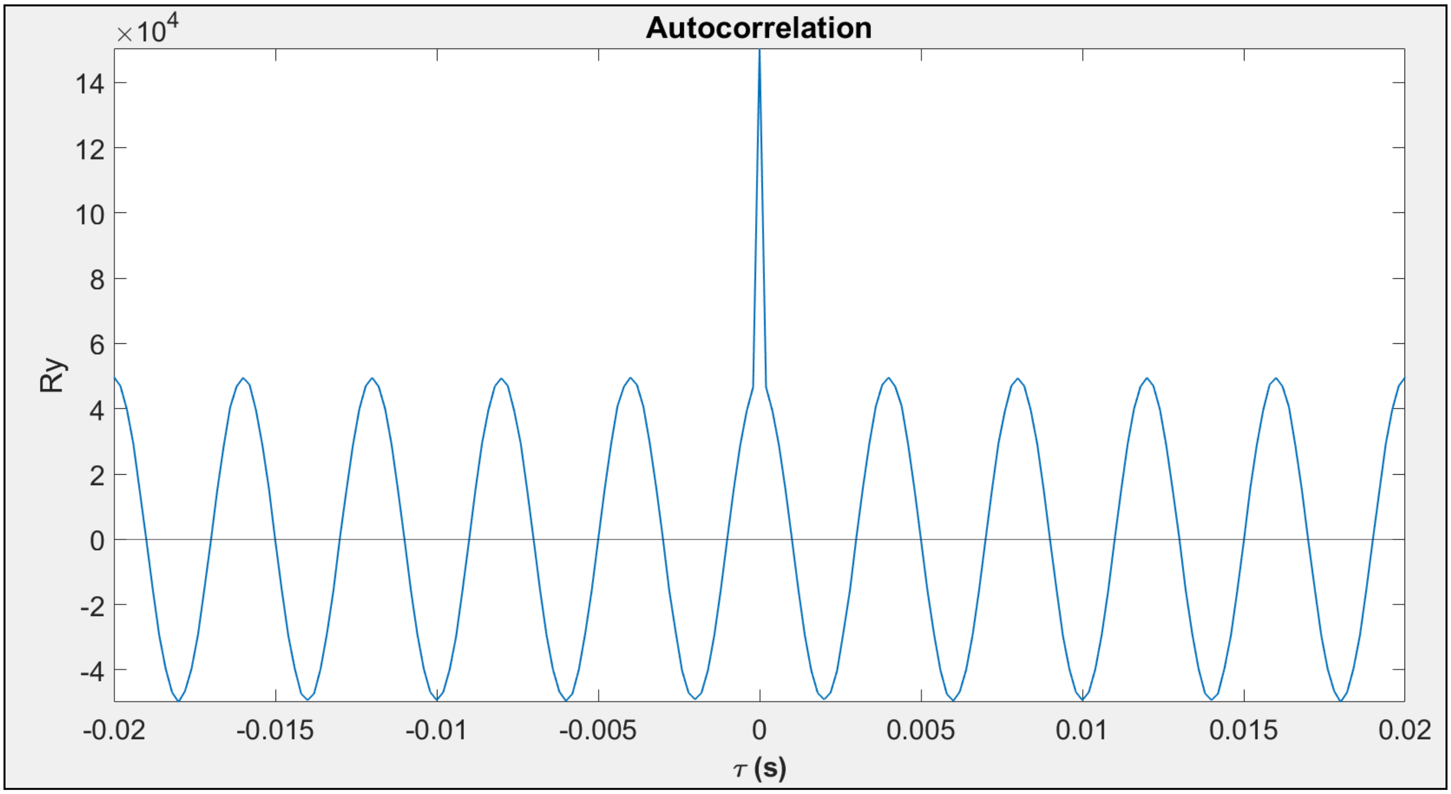
Section i

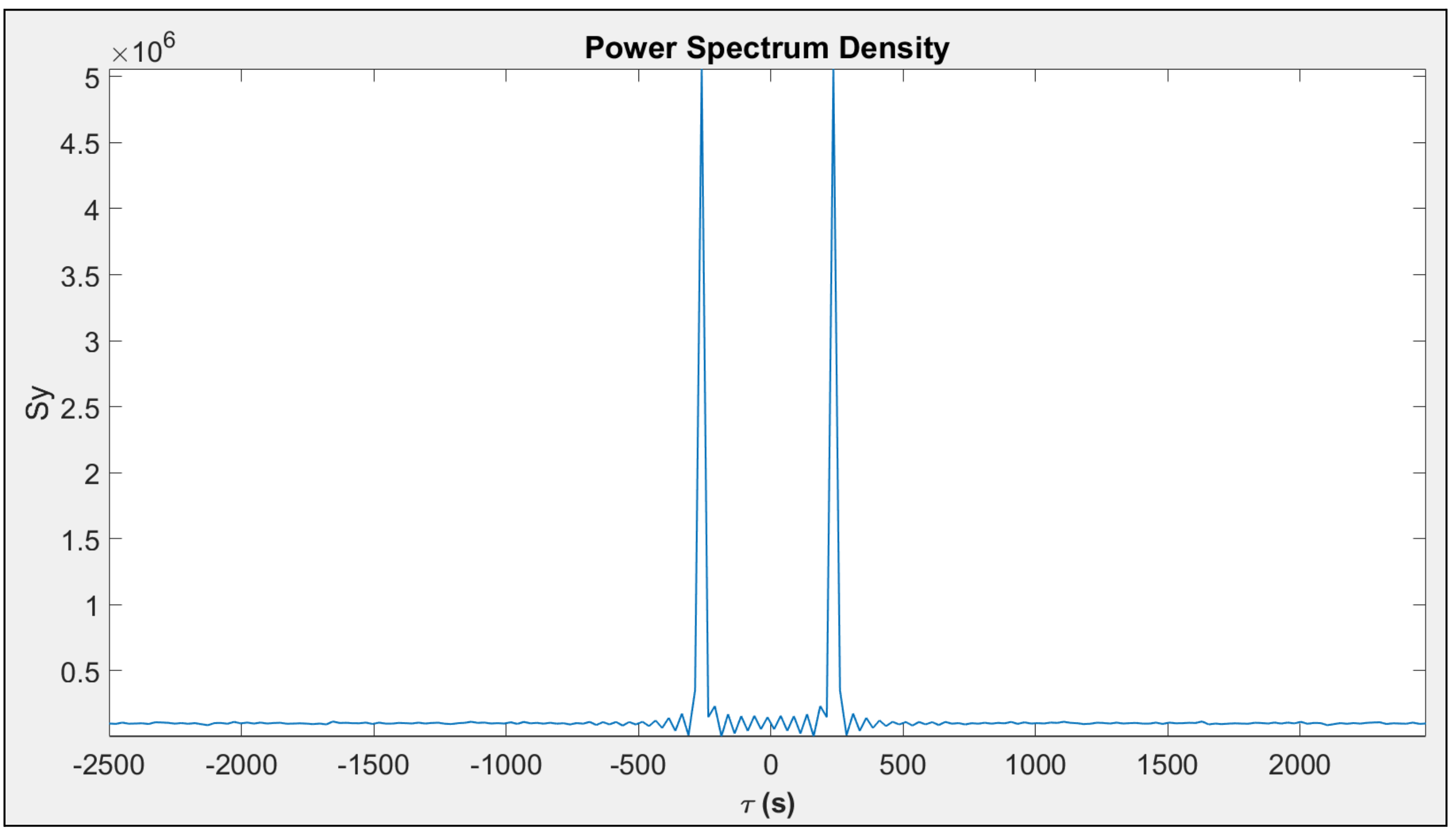


In the autocorrelation plot, we do observe a peak at the zero lag point. This is due to the white noise that we have added. We know that the autocorrelation of white noise is a delta function at the zero lag point. Thus, in our autocorrelation plot, the peak at zero lag is indicative of the white noise in our signal.

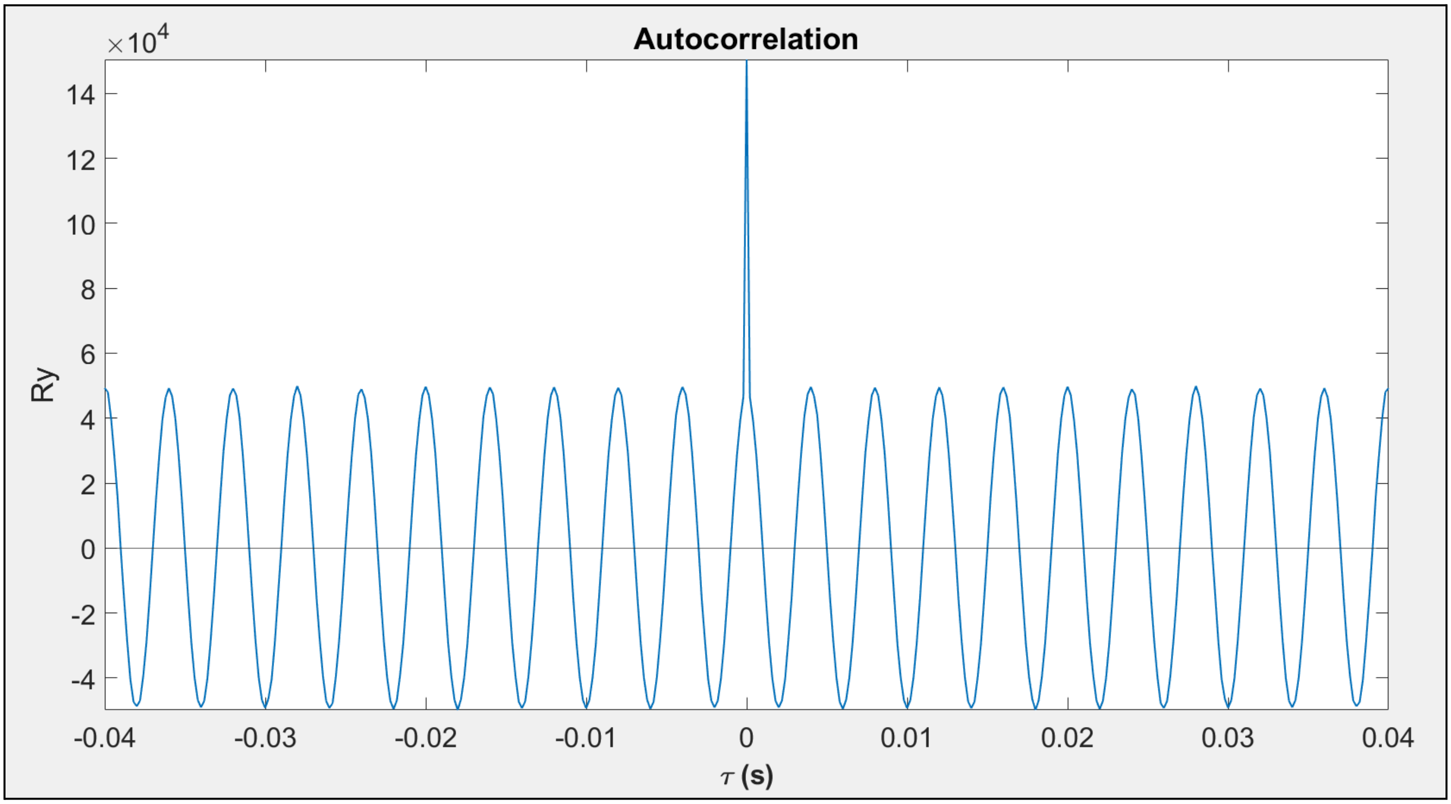
Section ii

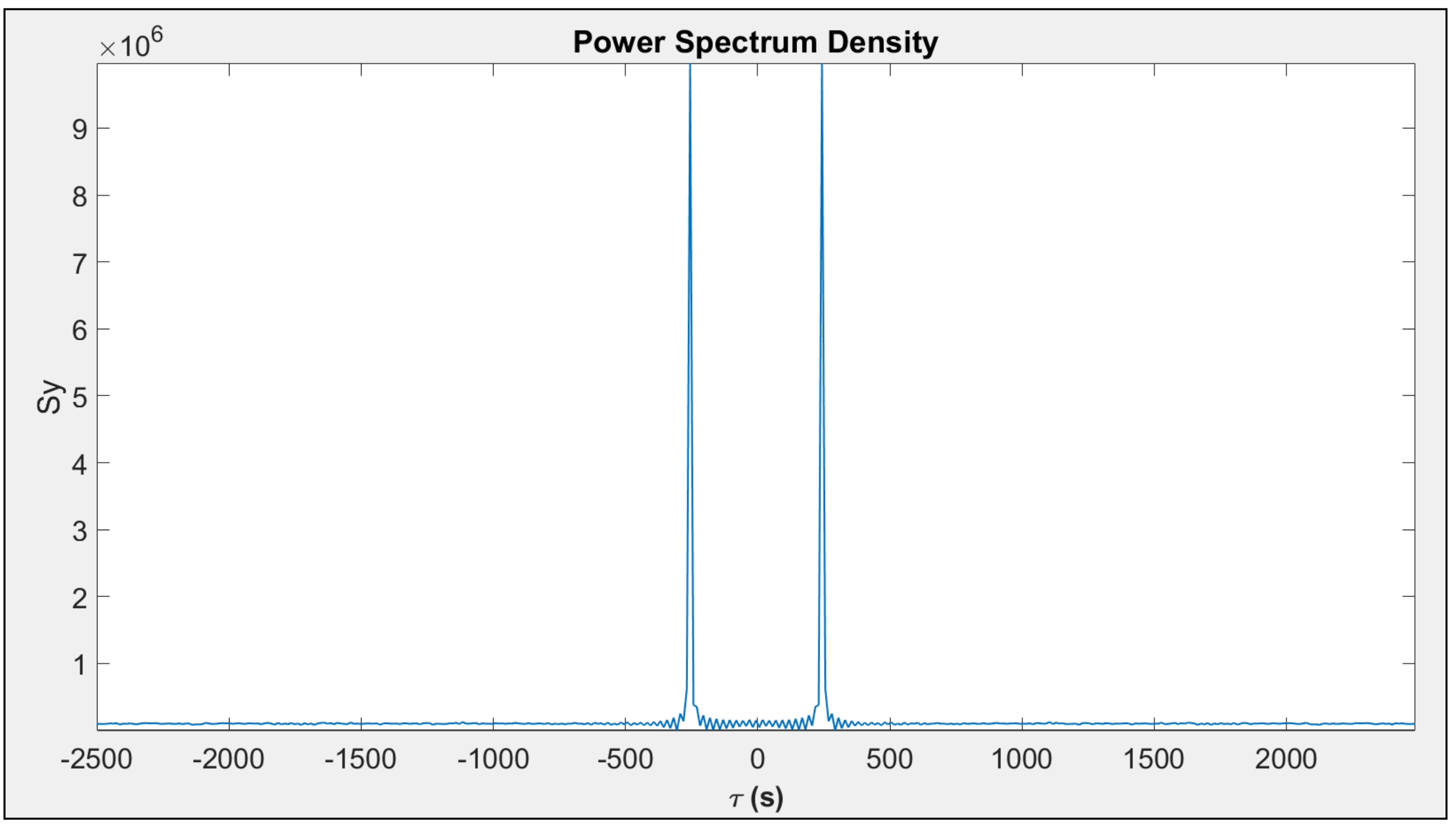
Our plots for *maxlag* = *100* can be found below…



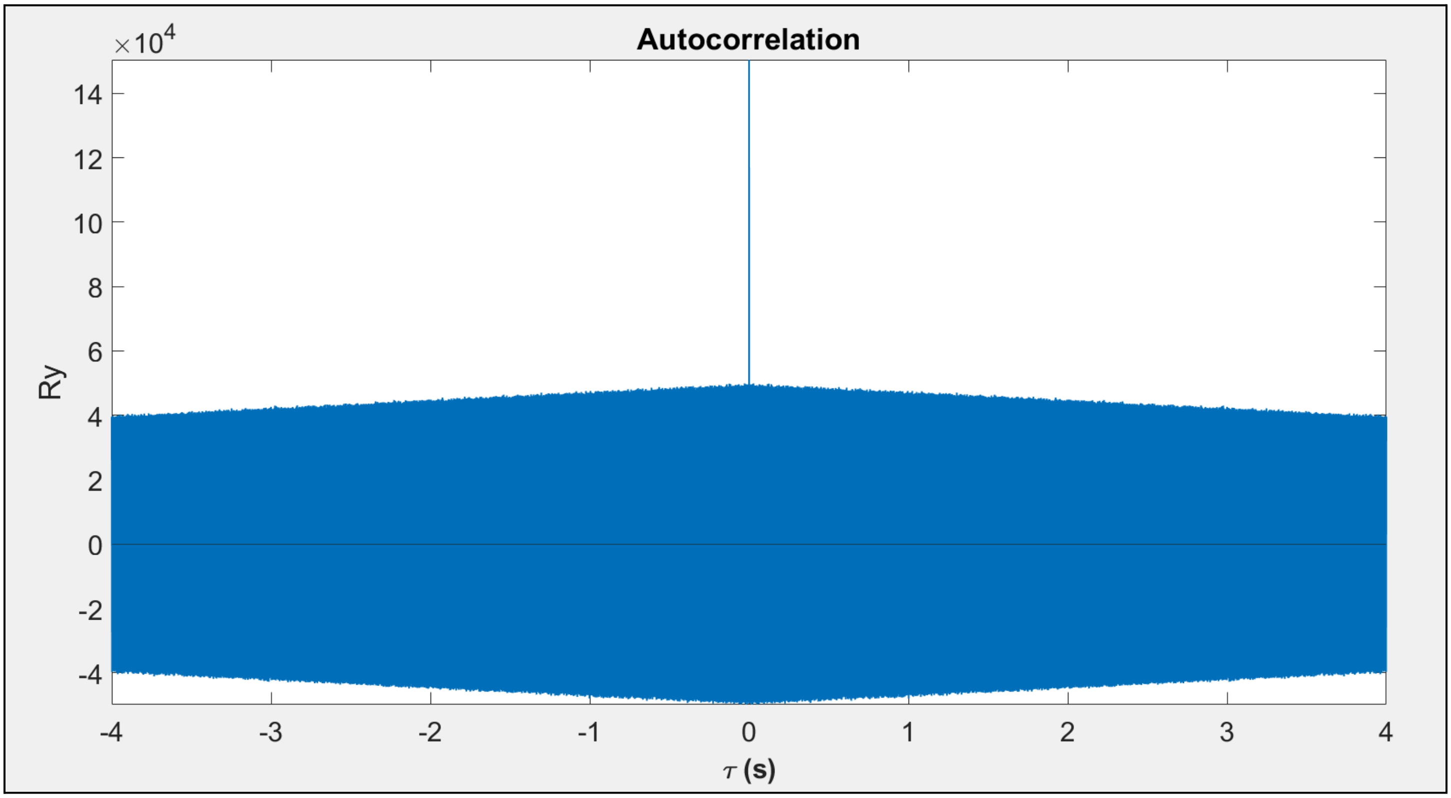


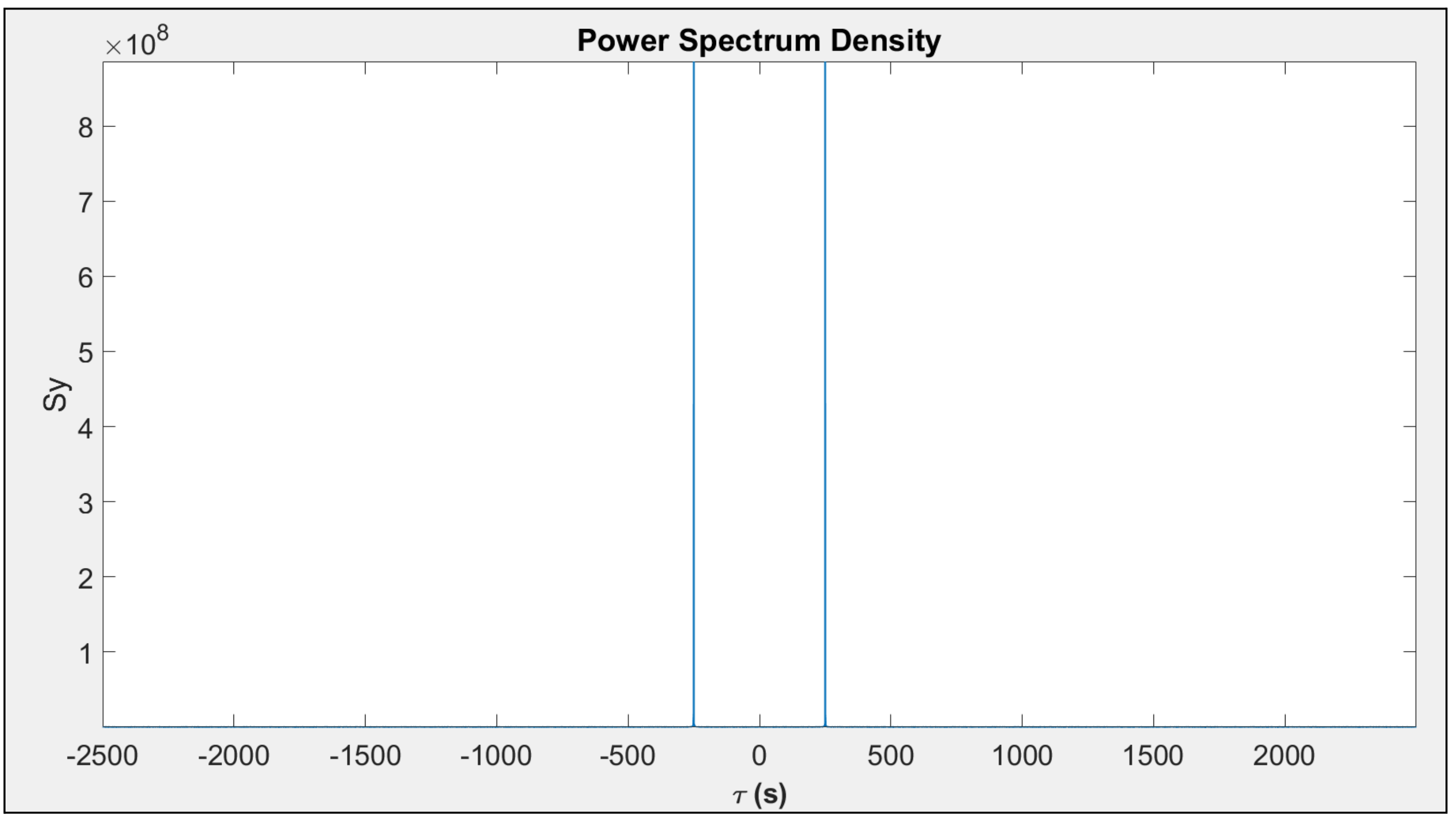
Our plots for *maxlag = 200* can be found below…





Our plots for *maxlag = 20000* can be found below…





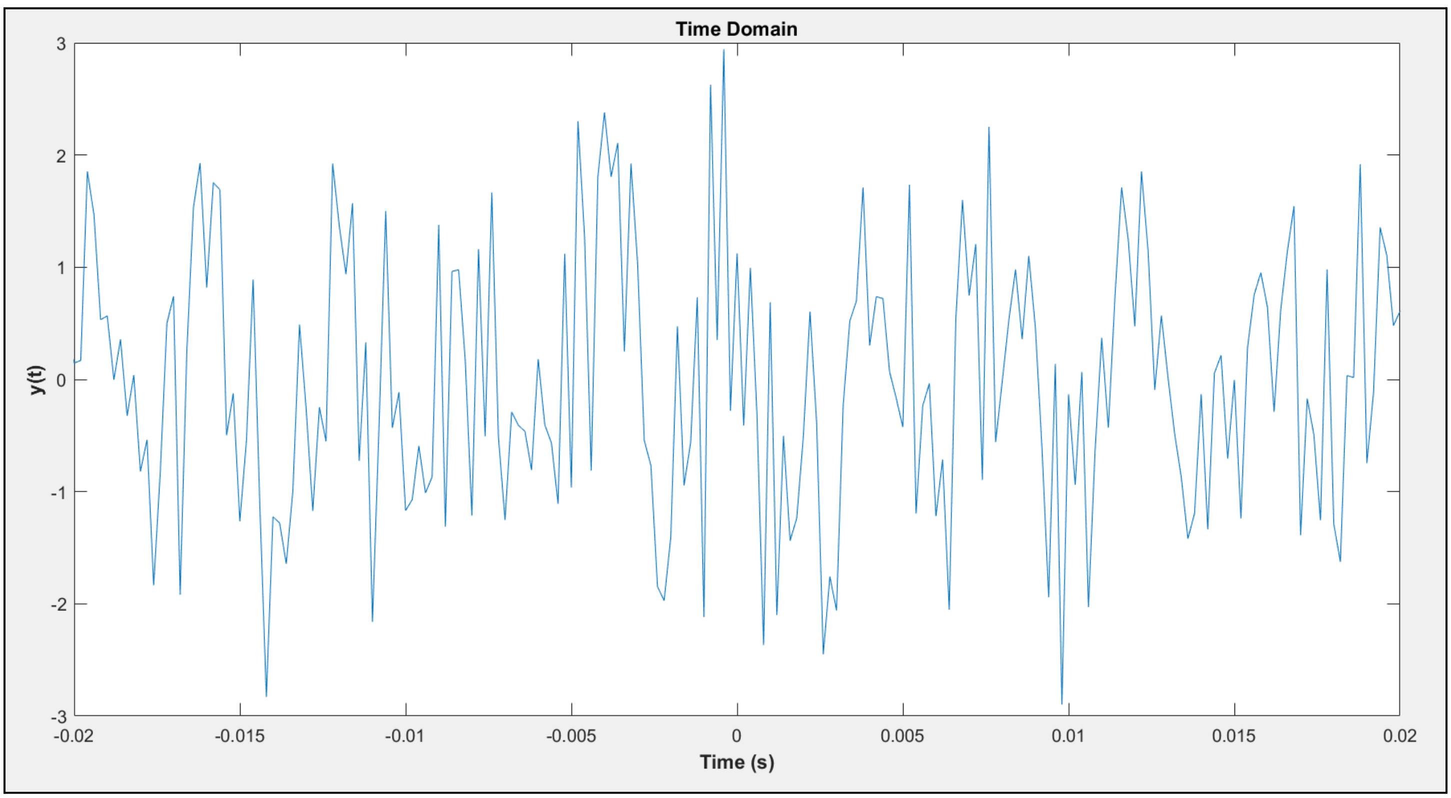
We can estimate the value of f_c from the PSD's above, by examining the location of the impulses…

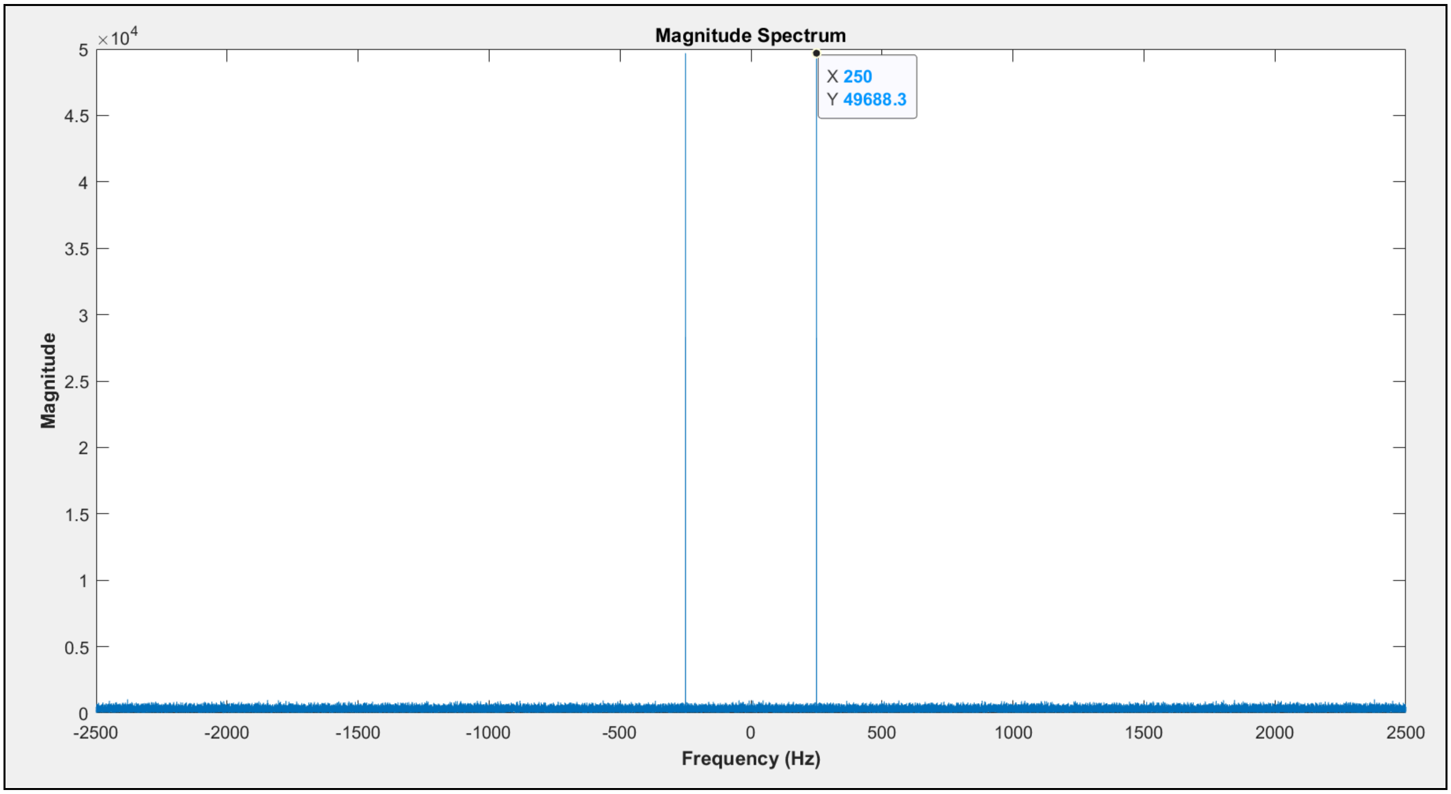
Maxlag	f_c
100	261.194
200	255.611
20000	250.056

The observed relationship between Maxlag and the frequency resolution is a direct proportionality. As we increase the value of *maxlag* the corresponding value of f_c decreases until ultimately converging to what can be approximated as $250Hz$. We also notice that as we increase *maxlag* the plot of the PSD becomes closer to the ideal plot of the theoretically computed values, in that the noise and ripples around the bottom get smaller and smaller, and the impulses become narrower. Ideally, these delta functions would have zero width, and infinite height, which is what is seen in the plot of *maxlag = 2000.* This can be explained by the fact that increasing *maxlag* leads to a longer time window which increases the resolution of the frequency, and thus allows us to more clearly see the frequency makeup of a signal. As was previously mentioned, the frequency measures do converge to the theoretically computed value of *250Hz.*

Section iii

Find below the plots of the signal in the time domain, as well as the magnitude spectrum of the signal in the time domain…





As can be seen from the above figures, we can estimate the value of f_c without calculating the correlation by taking the FFT of the time domain signal. From the Magnitude Spectrum plot above, our impulse functions occur at $\pm\ 250Hz$ which indicate that the dominant frequency components in the time-domain signal (I.e. the frequency of the sinusoid) is $250Hz$. This method would be less accurate if the signal was buried in more noise, since it would make deciphering the dominant frequency more difficult in our Magnitude plot.

MATLAB Code

```
% Stefan Tosti - 400367761
% Rawan Sajid - 400393373

% -------------------------------------------------------------------------
% Below code has been provided from Avenue
% -------------------------------------------------------------------------

%This code sets up the time and frequency vectors for all the numerical
%experiments of Lab3
clear
format long e
tend = 10;
tbeg = -10;
N=100000;
tstep = (tend-tbeg)/N;
sampling_rate = 1/tstep;

%Time window =
tt = tbeg:tstep:tend-tstep;

% Uncomment one at a time for each experiment
%load('lab4_num_expt1')
%load('lab4_num_expt2')
load('lab4_num_expt3')

maxlag = 20000;

%Autocorrelation of yt
Ry  = xcorr(yt,yt,maxlag);

%tau vector
tau_vec = -(maxlag*tstep):tstep:maxlag*tstep;

%Abs. PSD corresponding to yt
Sy = abs(fftshift(fft(fftshift(Ry))));

%define the frequency vector corresponding to tau_vec
Ntau = length(tau_vec);

%Nyquist sampling rate
fmax = sampling_rate/2;
fmin = -fmax;
fstep = (fmax-fmin)/Ntau;

%Frequency window
```

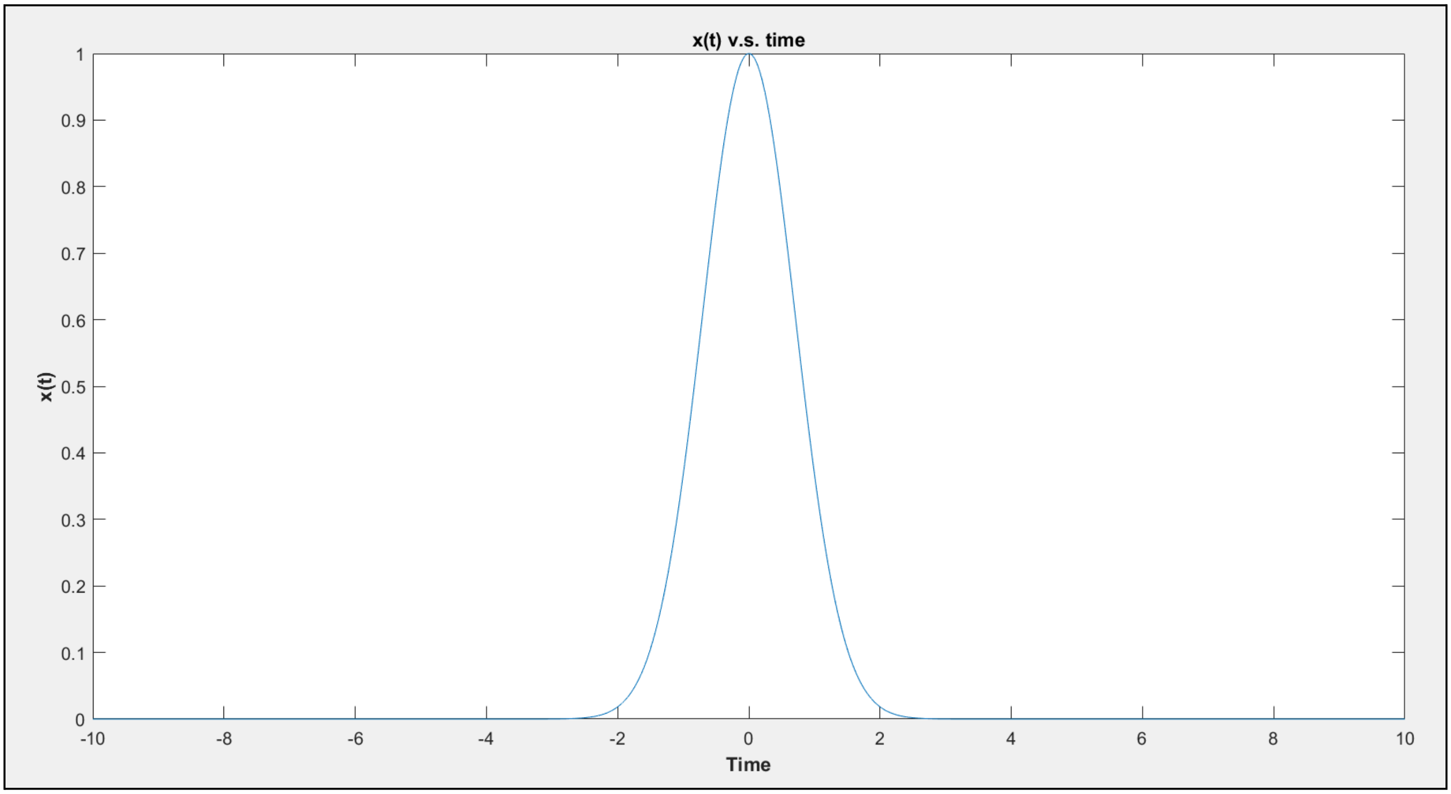
```
freq = fmin:fstep:fmax-fstep;

% ------------------------------------------------------------------------
% ------------------------------------------------------------------------




% ------------------------------------------------------------------------
% Experiment 1
% ------------------------------------------------------------------------

% Plot of autocorrelation
figure(1)
Hp1 = plot(tau_vec,Ry);
set(Hp1,'LineWidth',1)
Ha = gca;
set(Ha,'Fontsize',16)
Hx=xlabel('\tau (s)');
set(Hx,'FontWeight','bold','Fontsize',16)
Hx=ylabel('Ry');
Hx=yline(0);
set(Hx,'FontWeight','bold','Fontsize',16)
title('Autocorrelation');
axis([min(tau_vec) max(tau_vec) min(Ry) max(Ry)])
pause(1)

% Plot of power spectrum density
figure(2)
Hp1 = plot(freq,Sy);
set(Hp1,'LineWidth',1)
Ha = gca;
set(Ha,'Fontsize',16)
Hx=xlabel('\tau (s)');
set(Hx,'FontWeight','bold','Fontsize',16)
Hx=ylabel('Sy');
Hx=yline(0);
set(Hx,'FontWeight','bold','Fontsize',16)
title('Power Spectrum Density');
axis([min(freq) max(freq) min(Sy) max(Sy)])
pause(1)
% ------------------------------------------------------------------------
% ------------------------------------------------------------------------



% ------------------------------------------------------------------------
```

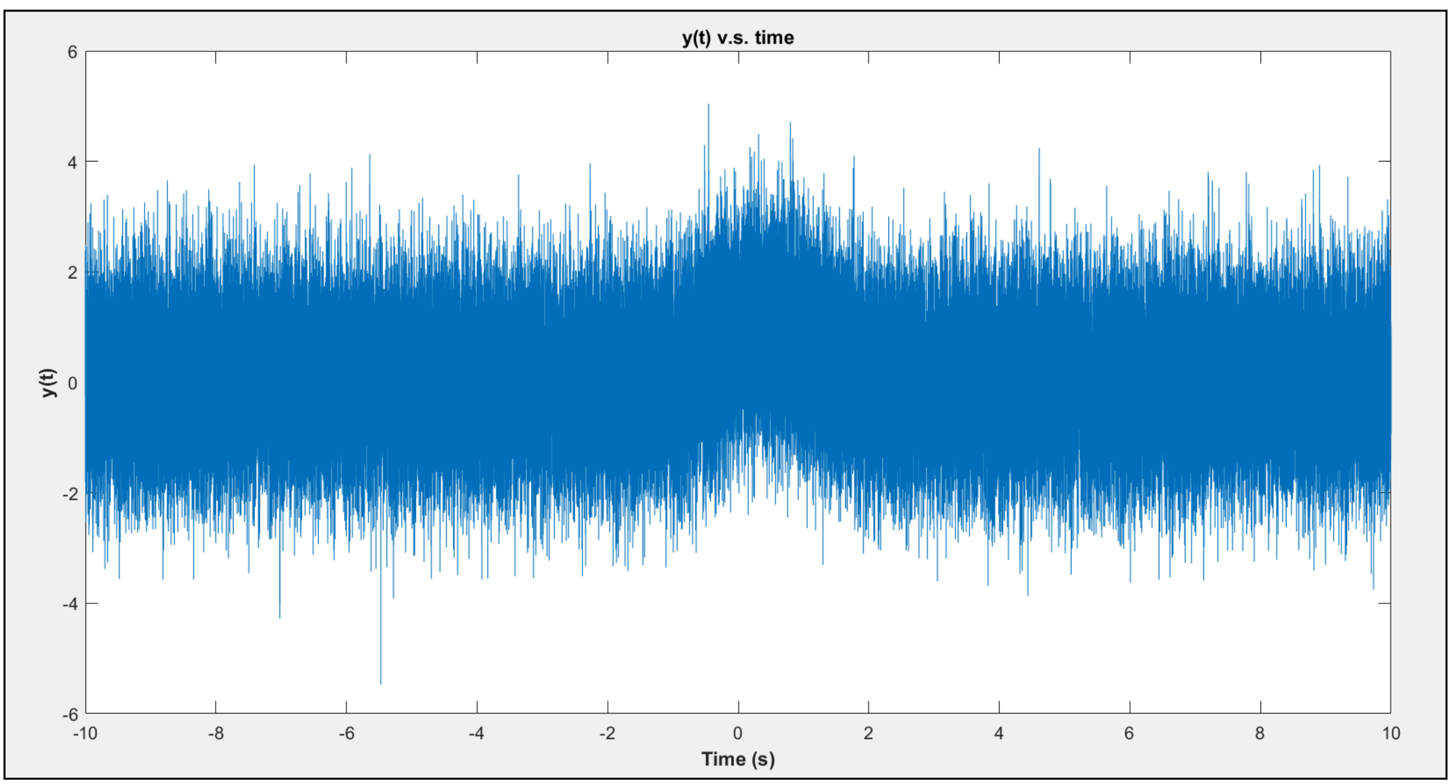
```
% Experiment 2 - part i, ii
% ----------------------------------------------------------------------------
fig = figure(3);
plot(tt, yt);
xlim([-100*tstep 100*tstep]);
title("Signal: Time Domain");
subtitle("y(t)", 'interpreter', 'latex')
xlabel("Time (s)", 'FontWeight', 'bold');
ylabel("y(t)", 'FontWeight', 'bold');
fig = figure(4);
% ----------------------------------------------------------------------------
% ----------------------------------------------------------------------------



% ----------------------------------------------------------------------------
% Experiment 2 - part iii
% ----------------------------------------------------------------------------
Nyt = length(yt);

fmax = sampling_rate/2;
fmin = -fmax;
fstep = (fmax-fmin)/Nyt;

freq = fmin:fstep:fmax-fstep;
plot(freq, abs(fftshift(fft(fftshift(yt)))));
title("Magnitude Spectrum");
xlabel("Frequency (Hz)", 'FontWeight', 'bold');
ylabel("Magnitude", 'FontWeight', 'bold');

fig = figure(5);
plot(tt, yt);
xlim([-100*tstep 100*tstep]);
title("Time Domain");
xlabel("Time (s)", 'FontWeight', 'bold');
ylabel("y(t)", 'FontWeight', 'bold');
fig = figure(4);
% ----------------------------------------------------------------------------
% ----------------------------------------------------------------------------


% ----------------------------------------------------------------------------
% Experiment 3
% ----------------------------------------------------------------------------
maxlag = 20000;
Rxy = xcorr(yt,xt,maxlag);
tau_vec = -(maxlag*tstep):tstep:maxlag*tstep;
```

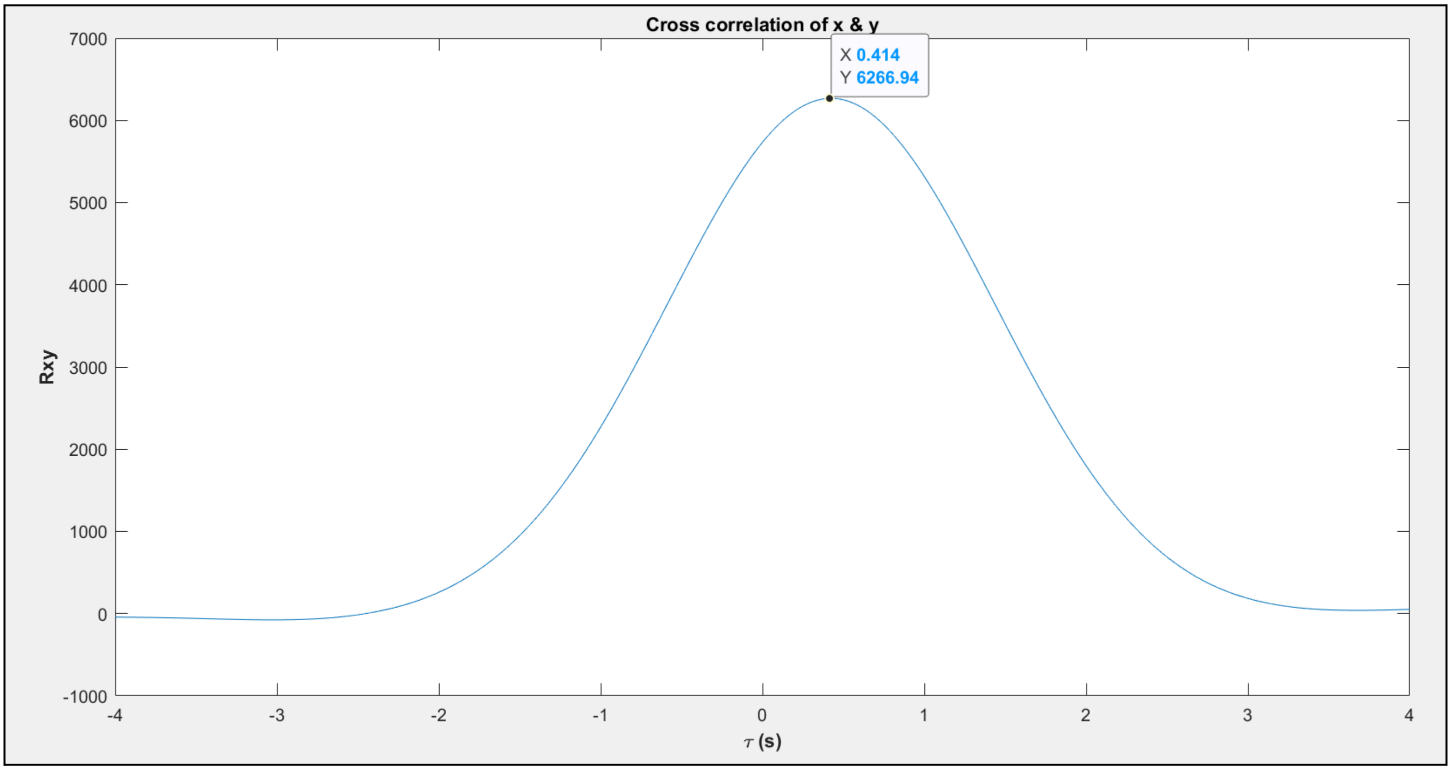
```
Sxy = abs(fftshift(fft(fftshift(Rxy))));

Ntau = length(tau_vec);

fig = figure(5);
plot(tt, xt);
title("x(t) v.s. time");
xlabel("Time", 'FontWeight', 'bold');
ylabel("x(t)", 'FontWeight', 'bold');

fig = figure(6);
plot(tt, yt);
title("y(t) v.s. time");
xlabel("Time (s)", 'FontWeight', 'bold');
ylabel("y(t)", 'FontWeight', 'bold');


fig = figure(7);
plot(tau_vec, Rxy);
title("Cross correlation of x & y");
xlabel("\tau (s)", 'FontWeight', 'bold');
ylabel("Rxy", 'FontWeight', 'bold');
% -------------------------------------------------------------------------
% -------------------------------------------------------------------------
```

Numerical Experiment #3

Below our plots of $x(t)$, $y(t)$ & $cross\ correlation$ can be found. Note that all of these plots were generated using *maxlag = 20000…*







In this case scenario, it is difficult to visually estimate the delay of the signal because of the amount of noise present in *y(t)*. In this section of the lab, we worked around this by finding the cross correlation, R_{xy} and identifying the value of τ for which R_{xy} is maximized. From the above plots, we can roughly estimate $\tau = 0.414$ at the maximum point, and thus we have delay, $T = 0.414$.