Vectors and Linear Equations

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1   Recall:

Sum of two vectors: $[a, b] + [c, d] = [a + c, b + d]$
Scalar multiplication of a vector and scalar: $a[b, c] = [ab, ac]$

2   Systems of Equations

Solve the following systems of equations for x and y.

1. $x + y = 8$
    $x = 3y$

2. $3x + 4y = 0$
    $x - 2y = 10$

3. $\frac{2}{3}x - \frac{1}{9}y = 4$
    $\frac{x}{y} = 2$

4. $y = x + 1$
    $x^2 + y^2 = 5$

3   Homogeneous Linear Equations

Describe all rational solutions using vectors to the following homogeneous linear equations. For your final answer in the form $q[x, y]$, reduce x and y to lowest terms or convert them both to integers if possible. The order of the negatives does not matter.

1. $x + 2y = 0$

2. $5x - 10y + 4 = 4$

3. $\frac{1}{3}x + \frac{1}{12}y = 0$

4. $\sqrt{2}x + 7y - \sqrt{3} = -\sqrt{3}$

4   Word Problems

1. What is the sum of the two vectors $[3, 8]$ and $[75, 200]$?

2. What is the simplified scalar multiplication of the vector $\sqrt{2}[14, 28\sqrt{2}]$?

3. All vectors $[x, y] = q[51, -68]$ are solutions to the homogeneous equation $ax + by = 0$, where a and b are in lowest terms. Find the value of $3a + 2b$.