ELEC4611
Power System Equipment

Tutorial 1 Solutions

Q1.

(i)

$$E(r) = \frac{V}{r\ln(b/a)} \quad \Rightarrow \quad E_{\max} = \frac{V}{a\ln(b/a)}$$

$$b = a \times \exp\left(\frac{V}{aE_{\max}}\right)$$

$$b - a = a\left[\exp\left(\frac{V}{aE_{\max}}\right) - 1\right] = 22 \times \left[\exp\left(\frac{132 \times \sqrt{2}/\sqrt{3}}{22 \times 0.3 \times 21.6}\right) - 1\right] = 24.86 \text{ mm}$$

(ii)

$$E_{\max} = 0.3 \times 21.6 = 6.48 \text{ kV/mm}$$

$$E_{\text{ave}} = \frac{132 \times \sqrt{2}/\sqrt{3}}{24.86} = 4.34 \text{ kV/mm}$$

$$u_f = \frac{E_{\text{ave}}}{E_{\max}} = \frac{4.34}{6.48} = 0.67 \quad \Rightarrow \quad \text{not very efficient use of insulation}$$

Q2.

Electric field stress at a radial distance x:

$$E(x) = \frac{V}{x\ln(R/r)}$$

It is highest at the inner conductor surface:

$$E_{\max} = E(r) = \frac{V}{r\ln(R/r)}$$

$$\frac{dE(r)}{dr} = 0 \quad \Rightarrow \quad r_{\text{opt}} = \frac{R}{e}$$

Also:

$$\frac{d^2E(r)}{dr^2} > 0 \quad \text{when} \quad r = r_{\text{opt}}$$

Thus, the highest field strength is at <u>minimum</u> when $r = r_{\text{opt}}$ and it is:

$$E_{\text{opt}} = E(r_{\text{opt}}) = \frac{V}{r_{\text{opt}}\ln(R/r_{\text{opt}})} = \frac{eV}{R}$$

Deviations from the optimal conductor size:

$$r_1 = 1.1r_{\text{opt}} \quad \Rightarrow \quad E_1 = \frac{V}{1.1r_{\text{opt}}\ln(R/1.1r_{\text{opt}})} = 1.005E_{\text{opt}}$$

$$r_2 = 0.9 r_{\text{opt}} \;\Rightarrow\; E_2 = \frac{V}{0.9 r_{\text{opt}} \ln\left(R/0.9 r_{\text{opt}}\right)} = 1.005 E_{\text{opt}}$$

It can be seen that although the highest stress has gone up, deviation from optimal field strength is very little (<1%).

Q3.

Assign voltages $\left(V_1, V_2, V_3, V_4\right)$ and capacitances $\left(C_1, C_2, C_3, C_4\right)$ to the layers.

For a layer n with inner radius a_n and outer radius b_n, peak electric field is:

$$E_{n(\max)} = \frac{V_n}{a_n \ln\left(b_n/a_n\right)}$$

Thus, for the same peak E in each layer:

$$\frac{V_1}{3\ln(4/3)} = \frac{V_2}{4\ln(5/4)} = \frac{V_3}{5\ln(6/5)} = \frac{V_4}{6\ln(7/6)} \qquad (1)$$

Capacitance of layer n with an axial length l_n:

$$C_n = \frac{2\pi\varepsilon l_n}{\ln\left(b_n/a_n\right)} = const \times \frac{l_n}{\ln\left(b_n/a_n\right)} \qquad (2)$$

But:

$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3 = C_4 V_4 \qquad (3)$$

From (2) and (3):

$$\frac{0.4 V_1}{\ln(4/3)} = \frac{l_2 V_2}{\ln(5/4)} = \frac{l_3 V_3}{\ln(6/5)} = \frac{l_4 V_4}{\ln(7/6)} \qquad (4)$$

Note that the full axial length of the innermost insulation layer is 50cm but the foil between this layer and the next is $l_1 = 40\,\text{cm}$. Thus in (4) above, we used 0.4m. In this way, the foil is completely sandwiched between the 2 insulation layers. It is also OK if the foil extends the full length but it is poor engineering (potential breakup of the foil section at the edge).

(a) From (1) and (4):

$$l_2 = 0.3\,\text{m} \quad ; \quad l_3 = 0.24\,\text{m} \quad ; \quad l_4 = 0.2\,\text{m}$$

(b) Then from (2):

$$C_1 = 0.27\,\text{nF} \quad ; C_2 = 0.261\,\text{nF} \quad ; C_3 = 0.256\,\text{nF} \quad ; C_4 = 0.252\,\text{nF}$$

Total capacitance: $C_T = 1\Big/\left(\sum \frac{1}{C_n}\right) = 65\,\text{pF}$

Capacitive leakage current: $I = \left(2\pi f\right) C_T V = 1.55\,\text{mA}$

(c) $$V_1 = V\frac{X_1}{X_T}$$

$$E_{\max} = \frac{V_1}{3\ln\left(4/3\right)} \Rightarrow E_{\max} = 21.25 \text{ kV/cm (RMS)} = 30.06 \text{ kV/cm (peak)}$$

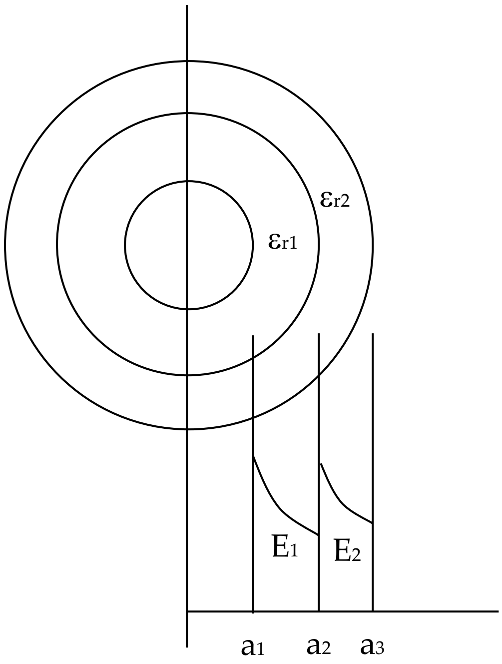
(d) If no grading used:

$$E_{\max} = \frac{76.2}{3\ln\left(7/3\right)} = 29.98 \text{ kV/cm (RMS)} = 42.39 \text{ kV/cm (peak)}$$

The field will be the same irrespective of the insulation (if insulation is homogeneous). Thus it will be 42.39kV/cm (peak) in both cases. Corona onset in air is 30kV/cm. Hence breakdown would occur for the case where insulation is air.

Q4.

(a)



Let $\varepsilon_{r1} = 5$; $\varepsilon_{r2} = 3$; E_{max} = 15 kV/mm, V = 191 kV and a_1 = 20mm.

a_1 = conductor radius.
a_2 = inner tape outer radius.
a_3 = outer tape outer radius.

Assume length of taped joint = 1m.

$$Q = C_1V_1 = C_2V_2$$

$$Q = \frac{2\pi\varepsilon_o\varepsilon_{r1}}{\ln\left(a_2/a_1\right)}\times V_1 = \frac{2\pi\varepsilon_o\varepsilon_{r2}}{\ln\left(a_3/a_2\right)}\times V_2$$

$$E_{\max} = \frac{V_1}{a_1\ln\left(a_2/a_1\right)} = \frac{V_2}{a_2\ln\left(a_3/a_2\right)}$$

Hemce:

$$\frac{Q}{E_{\max}} = \frac{2\pi\varepsilon_o\varepsilon_{r1}V_1}{\ln(a_2/a_1)} \bigg/ \frac{V_1}{a_1\ln(a_2/a_1)} = 2\pi\varepsilon_o\varepsilon_{r1}a_1$$

$$= \frac{2\pi\varepsilon_o\varepsilon_{r2}V_2}{\ln(a_3/a_2)} \bigg/ \frac{V_2}{a_2\ln(a_3/a_2)} = 2\pi\varepsilon_o\varepsilon_{r2}a_2$$

and so: $\varepsilon_{r1}a_1 = \varepsilon_{r2}a_2 \quad \Rightarrow \quad \varepsilon_{r1} = \frac{a_2}{a_1}\varepsilon_{r2} > \varepsilon_{r2}$ (because $a_2 > a_1$)

i.e. the tape with higher permittivity ($\varepsilon_r = 5$) should be used close to the conductor.

(b)

Outer radius of the inner tape:

$$a_2 = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}a_1 = \frac{5}{3}\times 20 = \underline{\mathbf{33.3\ mm}}$$

$$V_1 = E_{\max}a_1\ln\frac{a_2}{a_1} = 15\times 20\times\ln\frac{33.33}{20} = 153.25\ \text{kV(peak)}$$

$$= \frac{153.25}{\sqrt{2}} = \underline{\mathbf{108.36\ kV(rms)}}$$

and:

$$V_2 = V - V_1 = 191 - 108.36 = \underline{\mathbf{82.64\ kV(rms)}} \quad \text{or} \quad 116.87\ \text{kV(peak)}$$

Outer radius of the outer tape, a_3:

$$E_{\max} = \frac{V_2}{a_2\ln(a_3/a_2)} \quad \Rightarrow \quad a_3 = a_2\exp\left(\frac{V_2}{E_{\max}a_2}\right)$$

$$a_3 = 33.33\times\exp\left(\frac{116.87}{15\times 33.33}\right) = \underline{\mathbf{42.11\ mm}}$$

(c) Minimum field strengths in each taped section:

$$E_{\min 1} = \frac{V_1}{a_2\ln(a_2/a_1)} = \frac{153.25}{33.33\times\ln(33.33/20)} = \underline{\mathbf{9\ kV/mm}}$$

$$E_{\min 2} = \frac{V_2}{a_3\ln(a_3/a_2)} = \frac{116.87}{42.11\times\ln(42.11/33.33)} = \underline{\mathbf{11.87\ kV/mm}}$$

(d) The radial distance to middle of outer section:

$$r = a_2 + \frac{a_3 - a_2}{2} = 33.33 + \frac{42.11 - 33.33}{2} = 37.72\text{mm}$$

$$\varepsilon_{r(\text{void})}E_{\text{void}}(r) = \varepsilon_{r(\text{tape})}E_{\text{tape}}(r)$$

$$E_{\text{void}}(r) = \frac{\varepsilon_{r(\text{tape})}}{\varepsilon_{r(\text{void})}} E_{\text{tape}}(r) = \frac{\varepsilon_{r(\text{tape})}}{\varepsilon_{r(\text{void})}} \frac{V_2}{r \ln(a_3/a_2)}$$

$$= \frac{3}{1} \times \frac{116.87}{37.72 \times \ln(42.11/33.33)} = \underline{\mathbf{39.76\ kV/mm}}$$

The breakdown field strength in air at normal pressure is 3 kV/mm. Thus, breakdown will certainly occur in the void.

(e) If only one type of insulation tape is used

$$E_{\max} = \frac{V}{a_1 \ln(a_2/a_1)}$$

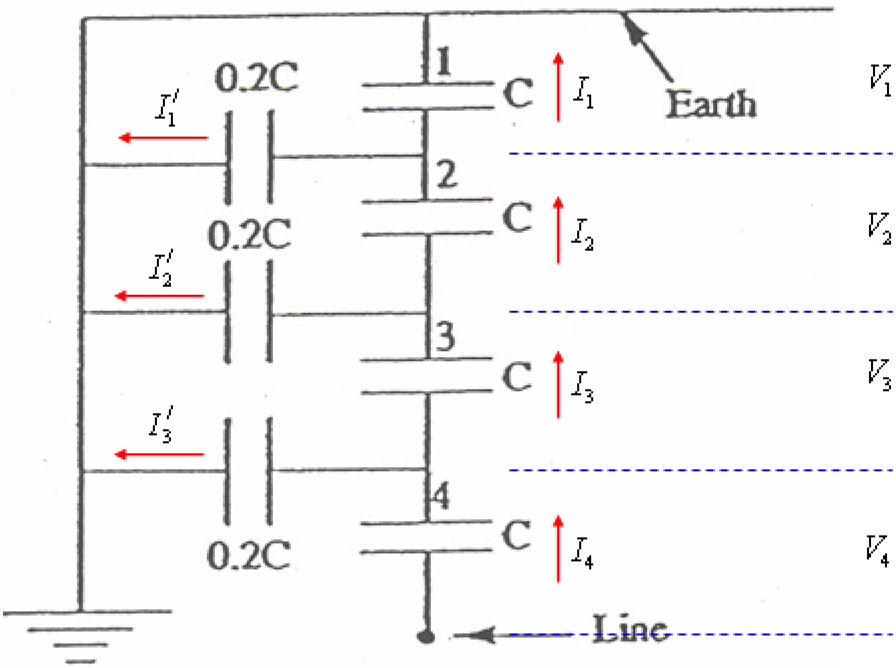
$$\Rightarrow \quad a_2 = a_1 \exp\left(\frac{V}{E_{\max} a_1}\right) = 20 \times \exp\left(\frac{191 \times \sqrt{2}}{15 \times 20}\right) = 49.21 \text{ mm}$$

Hence, the required insulation thickness = 49.21-20 = **<u>29.21 mm</u>**.

In the above analysis, the permittivity values were not used and so the insulation thickness is not affected by the type of tape used.

For comparison, note that for the case above with two different types of tapes the total insulation thickness is 42.11-20 = **21.11 mm**.

Q5.



Use V_1 and I_1 of the insulator unit at the earth end as reference:

$$I_1' = 0.2I_1 \quad \Rightarrow \quad I_2 = I_1 + I_1' = 1.2I_1 \quad \Rightarrow \quad V_2 = 1.2V_1 \;\; \& \;\; V_1 + V_2 = 2.2V_1$$

$$I_2' = 0.2\times(2.2I_1) = 0.44I_1 \quad \Rightarrow \quad I_3 = I_2 + I_2' = 1.64I_1$$
$$\Rightarrow \quad V_3 = 1.64V_1 \;\; \& \;\; V_1 + V_2 + V_3 = 3.84V_1$$

$$I_3' = 0.2\times(3.84I_1) = 0.768I_1 \quad \Rightarrow \quad I_4 = I_3 + I_3' = 2.41I_1$$
$$\Rightarrow \quad V_4 = 2.41V_1$$

$$V_{total} = V_1 + V_2 + V_3 + V_4 = 6.25V_1 = 38.1\,\text{kV}$$

$$\Rightarrow \quad V_1 = 6.08\,\text{kV} \;\; ; \;\; V_2 = 7.30\,\text{kV} \;\; ; \;\; V_3 = 9.97\,\text{kV} \;\; ; \;\; V_4 = 14.65\,\text{kV}$$

$$\text{String efficiency } = \frac{38.1}{4\times 14.65}\times 100\% = 65\%$$

Alternatively, solve the problem by using formula for the voltage of the n-th unit (w.r.t. ground) in a string of n_o units:

$$V_n = V_0\frac{sinh(\alpha n)}{sinh(\alpha n_0)}$$

where $\alpha = \sqrt{C_o/C}$ and V_o is the total voltage on the string.

Here:

$$V_0 = 38.1 \;\; ; \;\; \alpha = \sqrt{0.2} = 0.447 \;\; ; \;\; n_0 = 4$$