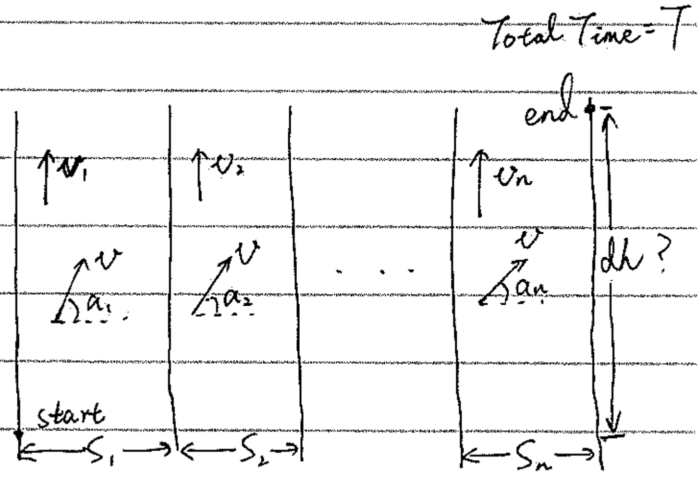
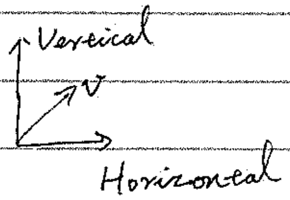
Known: $\begin{cases} \text{Speeds of rivers} : v_1, v_2, \cdots, v_n \\ \text{Person's speed} : v \text{ (constant)} \\ \text{Total time} : T \end{cases}$

Looking for: $\begin{cases} \text{maximize } dh \text{ by determining} \\ a_1, a_2, \cdots, a_n \text{ under } T. \end{cases}$



① For Horizontal direction:



$v_{1H} = v\cos a_1 \quad v_{2H} = v\cdot\cos a_2, \cdots, v_{nH} = v\cdot\cos a_n$

$t_1 = \frac{S_1}{v\cdot\cos a_1}, \; t_2 = \frac{S_2}{v\cdot\cos a_2}, \cdots, t_n = \frac{S_n}{v\cdot\cos a_n},$

$T = t_1 + t_2 + \cdots t_n = \frac{S_1}{v\cdot\cos a_1} + \frac{S_2}{v\cdot\cos a_2} + \cdots + \frac{S_n}{v\cos a_n}$

$= \sum_{i=1}^{n} \frac{S_i}{v\cdot\cos a_i}$

$\therefore \sum_{i=1}^{n} \frac{S_i}{v\cdot\cos a_i} - T = 0$

② For Vertical direction:

$v_{1V} = v_1 + v\cdot\sin a_1, \quad v_{2V} = v_2 + v\cdot\sin a_2, \cdots, v_{nV} = v_n + v\cdot\sin a_n$

$dh_1 = v_{1V}\cdot t_1 = (v_1 + v\cdot\sin a_1)\frac{S_1}{v\cdot\cos a_1}, \cdots, dh_n = v_{nV}\cdot t_n = (v_n + v\cdot\sin a_n)\frac{S_n}{v\cdot\cos a_n}$

$\therefore H = dh_1 + \cdots + dh_n = \sum_{i=1}^{n} (v_i + v\cdot\sin a_i)\frac{S_i}{v\cdot\cos a_i}$

③ $\begin{cases} \text{maximize } H(a) \\ \text{subject to } \sum_{i=1}^{n} \frac{S_i}{v\cdot\cos a_i} - T = 0 \end{cases}$

Introduce a new variable λ (Lagrange multiplier), study the Lagrange function:

$\mathcal{L}(a, \lambda) = \sum_{i=1}^{n} (v_i + v\cdot\sin a_i)\frac{S_i}{v\cdot\cos a_i} - \lambda\left(\sum_{i=1}^{n} \frac{S_i}{v\cdot\cos a_i} - T\right)$