The Short-Run and Long-Run Component of Idiosyncratic Volatility and Stock Returns

Yunting Liu*

June 19, 2019

*Yunting Liu: Assistant Professor, Department of Finance, School of Economics, Peking University, Beijing, China, 100871; yuntingliu@pku.edu.cn. I'm grateful to comments from Federico Bandi, Gregory Duffee and Jon Faust. Special thanks to Yixuan Di for providing excellent research assistance. The author acknowledges research support from School of Economics, Peking University. I have no conflicting interests to declare.

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Abstract

To capture the dynamics of idiosyncratic volatility of stock returns over different horizons and investigate the relationship between idiosyncratic volatility and expected stock returns, this paper develops and estimates a parsimonious model for idiosyncratic volatility consisting of a short-run and long-run component. The conditional long-run (short-run) component is found to be negatively (positively) related to expected stock returns. The negative relationship between the long-run component and expected stock returns may be reflecting stocks with high long-run idiosyncratic volatility are less exposed to systematic risk. The positive relationship between the conditional short-run component and expected returns could be due to investors require compensation for bearing idiosyncratic volatility risk, when they face trading frictions and hold under-diversified portfolios. My findings highlight the importance of distinguishing the short-run and long-run component of idiosyncratic volatility in investigating mechanisms behind the relation between idiosyncratic volatility and the cross-section of stock returns.

Keywords: idiosyncratic volatility, short-run, long-run, cross-sectional stock returns, risk factors, trading friction, under-diversification

JEL Code: G12, G17

1 Introduction

Whether a stock's expected return depends on idiosyncratic volatility has been a central question in the asset pricing literature. Ang et al. (2006) (hereafter AHXZ) in a recent influential paper, present evidence that stocks with high realized idiosyncratic volatility have anomalously low returns in the subsequent month. This phenomenon is challenging because traditional asset pricing theories predict no relation between idiosyncratic volatility and expected returns when investors are well diversified and markets are complete and frictionless, or a positive relation when investors don't hold well diversified portfolios and face trading frictions (for example, Merton (1987)).

There is, however, contradictory evidence to the findings of Ang et al. (2006). Ang et al. (2006) define the idiosyncratic volatility as the standard deviation of the residuals from the Fama and French (1993) (hereafter FF-3) model estimated with daily returns in the previous month. Because idiosyncratic volatility is time-varying, lagged idiosyncratic volatility might not be a good proxy for the conditional idiosyncratic volatility. Fu (2009) constructs measures of conditional idiosyncratic volatility using exponential generalized autoregressive processes (EGARCH) and instead finds a strong positive relationship between conditional idiosyncratic volatility and average returns. This result casts doubt on the finding by Ang et al. (2006) and has first-order consequences on the interpretation of the relationship between idiosyncratic volatility and expected returns.

This paper argues that accounting for the dynamics of idiosyncratic volatility over short and long horizons is crucial to measure conditional idiosyncratic volatility and understand the relationship between idiosyncratic volatility and expected stock returns. I document empirical evidence that idiosyncratic volatility displays long memory properties: it decays quickly between one to three months but it persists at longer frequencies. To capture the dynamics of idiosyncratic volatility over short and long horizons, I develops a parsimonious model of idiosyncratic volatility featuring two components differing in persistence. This modeling approach is in line with Adrian and Rosenberg (2008), Corsi (2009), and Christoffersen et al. (2008). The more persistent component is called the long-run component and could be modeled as containing a unit root. The other is a short-run one and is less persistent. I study two types of models for idiosyncratic volatility. In one type of the model, the long-run component is constrained to have a unit root and the short-run component is a white noise. The other type of the model doesn't impose these restrictions.

Parameter estimates from the model suggest that these restrictions could be plausible assumptions about the idiosyncratic volatility process. The short-run component is found to be short-lived and the long-run component is persistent and close to random walk. To investigate the cross-sectional relationship between these two components and stock returns, I use both portfolio analysis and Fama and MacBeth (1973) regressions. Results from both type of methods indicate that the conditional long-run idiosyncratic volatility is significantly negatively related to expected stock returns, while the conditional short-run component is significantly positively related to expected returns.

To investigate the economic mechanism driving the cross-sectional relationship between idiosyncratic volatility and stock returns, I also study the relation over longer horizons besides one month. I include three different predictive horizons (1, 6, 12 months) in the portfolio analysis. I find that the predictive relationship between conditional long-run idiosyncratic volatility and expected returns holds for returns measured at longer periods. However, the predictive relationship of the conditional short-run idiosyncratic volatility only holds for 1-month horizon. The finding highlights that there are persistent variations in expected returns which are negatively related to the conditional long-run idiosyncratic volatility. As Cochrane (1999) explains, if predictability reflects risk, it is likely to persist. Therefore, a risk-based explanation may be desirable to explain the persistent negative relationship between the conditional long-run volatility and expected returns. But the positive relationship between the conditional short-run idiosyncratic volatility and expected stock returns may not be driven by exposure to systematic risk factors.

In terms of the dynamics of conditional idiosyncratic volatility over the short-run, Fu (2009) argues that the conditional idiosyncratic volatility is time-varying and exponential generalized autoregressive conditional heteroscedasicity (EGARCH) model could capture the short-run variation in conditional idiosyncratic volatility. I conduct Monte-Carlo simulations to investigate the cross-sectional relationship between conditional idiosyncratic volatility and stock returns using the EGARCH approach. Simulated data from the two components idiosyncratic model could match the key features of the autocorrelations of realized volatility. However, I find that the EGARCH model doesn't capture the short-run variation in idiosyncratic volatility and expected returns. Simulation results highlight the importance of the two components model to jointly capture the short-run and long-run variation of conditional idiosyncratic volatility.

To further study the cross-section relationship between the conditional short-run and long-run idiosyncratic volatility and stock returns, I investigate whether the difference in portfolio returns sorted by the short-run and long-run components of idiosyncratic volatility could be explained by different exposure to systematic risk factors. The return difference in portfolios sorted by the conditional short-run components is not found to be correlated to common systematic risk factors, such as Fama and French (2015) five factors, momentum factors (Carhart (1997)). The lack of correlation with systematic risk factors is direct evidence against risk-based explanations for the predictability of the short-run idiosyncratic volatility. The positive relationship between conditional short-run idiosyncratic volatility could be due to investors require compensation for bearing idiosyncratic volatility risk when they face trading frictions in short horizons and hence hold under-diversified portfolios. (Merton (1987))

In contrast, the difference in returns of portfolios sorted by the conditional long-run idiosyncratic volatility comoves with systematic risk factors. In particular, I find that portfolios with high idiosyncratic volatility are less exposed to the profitability factors RMW defined by Fama and French (2015). I also define a factor IVFL which is the difference of the return of high long-run idiosyncratic volatility portfolio and low long-run idiosyncratic volatility one. This 'slope' factor is also useful in capturing the negative relationship between the conditional long-run components and expected stock returns. Moreover, the IVFL factor encompasses information in the profitability factor as the profitability factor becomes insignificant after the IVFL factor is added.

This paper deepens the understanding of fluctuations of idiosyncratic volatility and relevant mechanism behind the cross-section relationship between idiosyncratic volatility and stock returns. Ang et al. (2009) show that lagged realized idiosyncratic volatility has strong explanatory power for 1-month-ahead realized idiosyncratic volatility, they don't provide a structural model for the dynamics of idiosyncratic volatility. Fu (2009) argues short-run variation of conditional idiosyncratic volatility has important cross-section implication. However, Monte-Carlo simulations from the two components idiosyncratic volatility model indicate that jointly capturing the short-run and long-run dynamics of idiosyncratic volatility is important to reconcile conflicting evidence on the relationship of idiosyncratic volatility and stock returns. In addition, separating the idiosyncratic volatility into short-run and long-run horizons is useful to evaluate different mechanisms behind the cross-section relationship between idiosyncratic volatility and stock returns. Mechanisms such as trading frictions, may act more on the short-run variation in idiosyncratic volatility, whereas risk-based mechanisms tend to matter more for persistent volatility fluctuations in the long-run.

2 Estimating Idiosyncratic Volatilities

In this section, I describe the data and methods used to estimate idiosyncratic volatilities.

2.1 Data

My dataset includes monthly and daily return data on stocks traded in the NYSE, AMEX, and NASDAQ return files from the Center for Research in Security Prices (CRSP). The accounting variables are from Compustat annual industrial files of income-statement and balance-sheet data.

The CRSP returns cover NYSE and AMEX stocks until 1973 when NASDAQ returns also come on line. The COMPUSTAT data are from 1963 to 2017. The 1963 start date reflects the fact that the book value of common equity (COMPUSTAT item 60), is not generally available prior to 1962. More importantly, COMPUSTAT data from earlier years have a serious selection bias; the pre-1962 data are tilted toward big historically successful firms.

The procedures below are standard in the literature following Fama and French (1992). To ensure that the accounting variables are known before the returns they are used to explain, I match the accounting data for all fiscal year ends in calendar year $t-1$ with the returns for July of year t to June of year $t+1$. The 6-month (minimum) gap between fiscal year end and the return tests is conservative. I use a firm's market equity at the end of December of year $t-1$ to compute its book-to-market ratio for year $t-1$.

2.2 Idiosyncratic Volatility Definition

Following Ang et al. (2006) and Bali and Cakici (2008), I concentrate on idiosyncratic volatility defined and measured relative to the Fama and French (1993) (FF-3) model.[1] Specifically, I consider the following specification for each firm at each month

$$r^i_{t,d} = \alpha^i_t + \beta^i_{MKT} MKT_{t,d} + \beta^i_{SMB} SMB_{t,d} + \beta^i_{HML} HML_{t,d} + \sigma^i_t \epsilon^i_{t,d} \tag{1}$$

where for day d in month t, $r^i_{t,d}$ is stock i's excess return, $MKT_{t,d}$ is the market excess returns, $SMB_{t,d}$ and $HML_{t,d}$ capture size and book-to-market effects, respectively. The residuals $\eta^i_{t,d} \equiv \sigma^i_{t,d}\epsilon^i_{t,d}$ is the idiosyncratic risk for month t. I define the idiosyncratic volatility of

[1] I also consider defining the idiosyncratic volatility relative to the Carhart (1997) four-factor model and Fama and French (2015) five-factor model. The negative relationship between lagged realized volatility and average stock returns also holds when defining idiosyncratic volatility relative to these factors. The results are available upon request.

stock returns for firm i in month t as

$$v_t^i = \sigma_t^i \sqrt{N_m} \tag{2}$$

where N_m is the number of trading days in month t for firm i. It is useful to note that the idiosyncratic volatility v_t^i is the daily standard deviation of residuals times the square root of the number trading days in that month. The inclusion of N_m transforms the daily return residuals into monthly residuals. This procedure can be can be seen at French et al. (1987) and Fu (2009).

Since the latent conditional volatility v_t^i cannot be directly observed, I use realized volatility: squared daily return residuals in month t obtained through the cross-sectional regression of equation (1) to measure the individual stock's idiosyncratic volatility for month t. Specifically,

$$IV_t^i \equiv \sqrt{\sum_{d=1}^{N_m} (\eta_{t,d}^i)^2} \tag{3}$$

When I refer to idiosyncratic volatility in this paper, I mean idiosyncratic volatility relative to the FF-3 model.

2.3 Time Series Properties of Realized Idiosyncratic Volatility

Table 1 presents the time-series properties of the realized idiosyncratic volatility (IV). I first compute the time-series statistics of idiosyncratic volatility for each firm and then summarize the mean statistics across about 22,000 firms. The mean of idiosyncratic volatility is on average 15.54% across stocks and the mean standard deviation for IV is 9.21%. The skewness is 2.00 and kurtosis is 8.23 which suggests that the idiosyncratic volatility is positively skewed and fat-tailed. The autocorrelation for the realized idiosyncratic volatility is 0.39 at 1-month lag, 0.31 at 2-month lag, 0.21 at 5-month lag, 0.12 at lag 10, and 0.12 at one-year lag. The autocorrelation of 0.39 at one-month lag and 0.31 at two-month lag suggests that shocks to idiosyncratic volatility are not very persistent at short horizons (within a quarter). However, the autocorrelations decay slowly over longer periods, for example over a year after shocks. The autocorrelation of realized idiosyncratic with a lag period of 12 months is still more than 0.1. In comparison, the autocorrelation of an AR(1) process with first-order autocorrelation of 0.39 would predict that the autocorrelation with 12 months lag is less than 0.2 basis point (One basis point equals to one hundredth of one percentage point).

[**Insert Table 1 about here**]

This pattern of a relatively quick initial decline in the autocorrelation function followed by a slower decay is sufficient evidence to dismiss the simple ARMA model to model idiosyncratic volatility (for example, Barndorff-Nielsen and Shephard (2002)). Corsi (2009) proposes that the aggregate stock return volatility could be captured by a additive cascade model of volatility defined over different time periods. The empirical evidence in the section suggests that the idiosyncratic volatility of stock returns could also be better captured by a process with components of different persistence. I model the log of idiosyncratic volatility as the sum of a short-run and long-run component. The short-run component is less persistent and has a large impact on the autocorrelations of idiosyncratic volatility at short horizons (over a quarter). There also exists a persistent long-run component, which dominates the autocorrelations at longer horizons (over a year and further).

2.4 Decomposing Idiosyncratic Volatility

To decompose the idiosyncratic volatility into short-run and long-run components, I model the idiosyncratic volatility v_t^i follows

$$\begin{aligned} \textbf{Idiosyncratic Volatility} : & \log v_t^i = s_t^i + l_t^i \\ \textbf{Short-run Component} : & s_{t+1}^i = \rho_s^i s_t^i + \sigma_s^i \epsilon_{s,t}^i \\ \textbf{Long-run component} : & l_{t+1}^i = \phi_i + \rho_l^i l_t^i + \sigma_l^i \epsilon_{l,t}^i \end{aligned} \tag{4}$$

I refer this model as the short and long-run (SL) model hereafter. In equation (4), the log-volatility is the sum of two components, s_t and l_t. The short-run component s_t is a mean-reverting process with mean zero and shocks to the short-run component die off quickly over time. The long-run component l_t is a persistent component. I normalize the process such that $\rho_l > \rho_s$ and σ_s and σ_l are the volatility of shocks to the short-run and long-run components. This restriction identifies the model as otherwise the two components can be interchangable. Adrian and Rosenberg (2008) consider a two components model for aggregate stock market volatility and find that the prices of risk are different for the short-run and long-run component. My paper differs from theirs in the focus on idiosyncratic volatility of stock returns.

For each firm, equation (4) is readily in the state space form and the unobserved short-run and long-run components can be directly estimated via Kalman filter. I consider $\hat{s}_t \equiv \mathbb{E}_{t-1}(s_t|y_1, y_2, ...y_{t-1})$ and $\hat{l}_t \equiv \mathbb{E}_{t-1}(l_{t+1}|y_1, y_2, ...y_t)$ as the expectation for the short-run and long-run components at time t based on information available at time $t-1$. The smoothed estimates $\tilde{s}_t = \mathbb{E}(s_t|y_1, y_2, ...y_T)$ and $\tilde{l}_t = \mathbb{E}(l_t|y_1, y_2, ...y_T)$, which use the whole sample information, may produce more precise estimates for the expectation of unobserved components s_t and l_t at each point in time. Therefore, full information set estimates are appropriate for asset pricing tests because of the gain in accuracy. However, in terms of the evaluation of trading strategies, incorporating future observations in the forecast leads to using substantial information beyond what investors are aware of. Therefore, I use both the filter estimates and smoothed estimates for evaluating trading strategies and asset pricing models based on idiosyncratic volatility, respectively.

2.5 A Permanent and Transitory Special Case

In my empirical work, I also investigate a special case of equation (4) where the long-run component is permanent and follows a random walk and the short-run component is purely transitory and follows a white noise. I refer this model as the permanent and transitory (PT) model.

$$\begin{aligned} \textbf{Idiosyncratic Volatility} &: \log v_t^i = s_t^i + l_t^i \\ \textbf{Short-run Component} &: s_{t+1}^i = \sigma_s^i \epsilon_{s,t}^i \\ \textbf{Long-run component} &: l_{t+1}^i = l_t^i + \sigma_l^i \epsilon_{l,t}^i \end{aligned} \tag{5}$$

Equation (5) may be viewed as a special case of (4) with the restriction that $\rho_s = 0$ and $\rho_l = 1$. The log-volatility is the sum of two components, s_t and l_t. The short-run component s_t is a white noise process and the long-run component l_t follows a random walk. Thus, changes to the long-run volatility could be permanent and are persistent over time. For each firm, equation (5) can also be estimated using Kalman filter. Since the long-run component l_t follows a random walk, the one-step-ahead conditional expectation of the long-run component $\mathbb{E}_t(l_{t+1}|y_1, y_2, ..., y_t) = \mathbb{E}_t(l_t|y_1, y_2, ..., y_t)$ and that of the short-run component $\mathbb{E}_t(s_{t+1}|y_1, y_2, ..., y_{t-1})$ trivially equals to zero. For the permanent and transitory (PT) model, I only consider the expectation of the long-run component: $\hat{l}_t = \mathbb{E}_{t-1}(l_t|y_1, y_2, ..., y_{t-1})$ and

$\tilde{l}_t = \mathbb{E}(l_t|y_1, y_2, ..., y_T)$ and investigate their relationship with expected returns.

2.6 Parameter Estimates of the Idiosyncratic Volatility Model

In practice, the true conditional idiosyncratic volatility v_t cannot be directly observed. Consequently, the realized volatility IV_t is a proxy to the latent volatility subject to measurement errors. Because measurement errors are largely identically and independent distributed over time, it has little forecasting power for forming conditional expectations. In empirical studies, realized volatilities are usually treated as measuring latent volatilities without errors. (for example, Bollerslev and Zhou (2002), Chua et al. (2010)). In this paper, I take this approach as well. One reason for this treatment is it massively simplifies subsequent estimation and analysis.

Besides, the errors-in-variable problem in realized volatility tends to weaken the predictability of estimated conditional volatility on stock returns in this paper, if not having no effect. Taking the measurement error into full consideration may be useful for future research. Nevertheless, I don't think accounting for it would impact the conclusion of the paper that there is a negative (positive) relationship between the conditional volatility of the long-run (short-run) component and expected stock returns.

Table 2 summarizes the parameter estimates for the short-run and long-run volatility (SL) model with equation (4) and the permanent transitory volatility (PT) model with equation (5). Both the SL and PT model are estimated using the maximum likelihood method. For the SL model, the mean AR(1) parameter for the short-run component is -0.06 while the median is -0.003. The long-run component is more persistent with mean AR(1) coefficient of 0.79 and median of 0.94. The mean volatility of shocks to the short-run component is 0.11 and the median is 0.10 . For the long-run component, the mean volatility is 0.07 and the median is 0.02. Therefore, the long-run component is very persistent but shocks to it tend to be small. The short-run component doesn't persist long but shocks to it are relatively bigger.

[**Insert Table 2 about here**]

The permanent and transitory (PT) model can be viewed as a special case of the SL model with $\rho_s = 0$ and $\rho_l = 1$. Given that the median estimate of ρ_s is 0.003 and ρ_l is 0.94 from the SL model, the PT model can be a plausible model to capture the dynamics of idiosyncratic volatility. For the PT model, the only parameters to be estimated are the volatility of shocks to the short-run and long-run component. The mean volatility of shocks to the short-run component is 0.14 and the median is 0.11. As for the volatility of shocks to the long-run component, the mean is 0.05 and the median is 0.01. The magnitude of shocks is also largely similar to estimates from the SL model.

3 Portfolio Sorts of Idiosyncratic Volatility in the Cross-Section

This section considers the performance of portfolios formed by different measures of idiosyncratic volatilities and asks whether exposures to different volatilities are systematically important for expected stock returns. To examine trading strategies based on idiosyncratic volatility, I consider the standard portfolio formation strategies following Jegadeesh and

Titman (1993) with a holding period of $N = 1, 6, 12$ months. For strategies with multi-month holding period, I average across the N subdeciles formed at the beginning of month $t - s$, for $s = 0, 1, 2, 3, ...N - 1$ as the return for a given decile.

3.1 Patterns in Average Returns for Idiosyncratic Volatility

I first consider value-weighted quintile portfolios formed every month by sorting stocks based on realized idiosyncratic volatility relative to the Fama and French (1993) model. Portfolios are formed every month, based on realized volatility computed using daily data over the previous month. Similar to findings of Ang et al. (2006), portfolios with high realized idiosyncratic volatility have low returns. Table 3 shows that the average return increases slightly from 0.91% per month to 0.95% going from the quintile 1 (low idiosyncratic volatility stock) to quintile 3. Then portfolios returns drop tremendously going to quintile 5. The portfolio with the highest idiosyncratic volatility (quintile 5) has a surprisingly low average return with 0.16% per month. The difference of returns between the highest and lowest portfolio is as large as 0.75% per month, which is statistically significant with robust t-statistic of -2.70. The FF3-alpha for quintile 5, is -1.14% per month with a robust t-statistics of -6.91. Therefore, the difference in quintile portfolio returns can not be explained by our standard asset pricing model such as Fama and French (1993). The difference in average returns in Table 3 indicates a significant negative relationship between expected return and idiosyncratic volatility. Panel A in Table 4 also reports portfolio performances with a holding period of $N = 6, 12$ months. For holding period of $N = 6$, the average return decreases from 1.07% per month to 0.42% going from the quintile 1 (low idiosyncratic volatility stocks) to quintile 5 (high idiosyncratic volatility stocks). The difference in returns is -0.65% with a robust t-statistics of -3.59. Similarly, the average return decreases from 0.98% per month to 0.56% going from the quintile 1 to quintile 5 when the holding period is $N = 12$. The difference in returns is -0.42% with a robust t-statistics of -3.31. Therefore, the negative relation between lagged idiosyncratic volatility and stocks returns holds for longer holding periods, suggesting the cross-sectional relationship is persistent and expected returns have a persistent component.

[**Insert Table 3 and 4 about here**]

3.2 Portfolios Sorted by Short-Run and Long-Run Volatilities

Estimating the state-space model of (4) and (5) produces estimates of the conditional short-run and long-run volatilities at the monthly frequency. I form value-weighted quintile portfolios sorting by the short-run volatility $\hat{s}_t$ or $\tilde{s}_t$ and long-run volatility $\hat{l}_t$ or $\tilde{l}_t$.

I first consider the performance of portfolios sorted by the filtered estimates of the conditional long-run component with a holding period of one month. Table 5 reveals substantial spread in average returns across quintile portfolios. For both the SL model and PT model, the portfolio with the highest long-run volatilities earns particularly low average returns. The average return for quintile 5 portfolio with the highest long-run idiosyncratic volatility is as low as 0.14% per month for the SL model and 0.15% for the PT model. The spread in the average return between the portfolio with the lowest and highest long-run volatility is −0.78% per month for the SL model and −0.76% for the PT model. The difference in returns also can not be explained by the Fama and French (1993) model. The highest long-run volatility portfolio has FF3-alpha of −1.18% for the SL model and −1.25% for the PT model. The alphas are statistically significant with robust t-statistics of -6.27 and -6.06 respectively. Therefore, there is a strong negative relation between conditional long-run volatility and stock returns.

Table 6 reports the portfolio performance by sorting stocks based on the smoothed conditional long-run component. Similar to the findings in Table 5, high long-run idiosyncratic volatility portfolios earn low returns. The quintile 5 portfolio with highest smoothed long-run idiosyncratic volatility earns a return of -0.23% for the SL model and as low as -0.64% for the PT model. The return spread between quintile 1 and 5 portfolio for the PT model is -1.66% with a t-statistic of -3.86. Compared to the spread of about -1.24% for the high minus low long-run volatility portfolios using SL model, the larger spread found in the PT model may be because allowing for unit root better captures the dynamics of idiosyncratic volatility. The potential existence of unit root further supports that there could be some highly persistent variation in idiosyncratic volatility.

Panel B and C in Table 4 also report portfolio performance of sorting on long-run idiosyncratic volatility for holding period of $N = 6, 12$ on the smoothed estimates of long-run idiosyncratic volatility. The statistical significances actually increase for longer holding periods. For the SL model, the return spread is $-1.24\%, -1.19\%, -0.82\%$ respectively for $N = 1, 6, 12$. The corresponding t-statistic is -2.85, -4.44 and -4.65. For comparison, the spread and corresponding t-statistic is $-0.75\%, -0.65\%, -0.42\%$ and $-2.70, -3.59, -3.31$ for sorting on realized idiosyncratic volatility.

Therefore, the more significant statistics from the model demonstrates that the conditional long-run idiosyncratic volatility is an important measure for the conditional idiosyncratic volatility and could better reveal economic mechanisms behind the cross-section relationship between idiosyncratic volatility and stock returns. And the return spread of sorting on conditional long-run idiosyncratic volatility is valid for multiple holding periods, suggesting a persistent relationship between idiosyncratic volatility and expected stock returns.

As for the conditional short-run component, Table 7 indicates there is a significant positive relation between conditional short-run idiosyncratic volatility and expected stock returns. For the SL model, portfolios sorted by filtered estimates $\hat{s}_t$ and smoothed estimates $\tilde{s}_t$ both reveal a positive relationship between the conditional short-run component and expected stock returns.

The spread in the high minus low short-run volatility portfolio earns an average monthly return of 0.16% for the filtered estimates and 0.56% for the smoothed estimates. The statistical significance is also stronger for smoothed estimates with a statistical significance of 4.09 over 2.36 of the filtered estimates, which indicates that the model is relatively successful in capturing the dynamics and underlying relationship between idiosyncratic volatility and cross-section of stock returns. Extending the strategy for multiple holding periods of $N = 6$ and $N = 12$ doesn't reveal any significant relationship between conditional short-run volatility and expected. For brevity, the numbers are not reported here. The result suggests that when the short-run conditional volatility is higher, stocks prices fall thus expected stock return becomes higher. Thus, there is a positive relationship between the conditional short-run idiosyncratic volatility and expected stock returns. However, stock prices tend to revert back over longer periods, thus holding portfolios over longer period doesn't create any significant return spread. This is consistent with the economic story that the cross-section relationship between the short-run idiosyncratic volatility and stock returns is not due to risk but mispricing. I would further investigate this issue in the later section.

4 Cross-Sectional Regressions

My empirical analysis thus far is based on portfolio sorts. In this section, I investigate the cross-sectional relationship between average stock returns and estimated conditional idiosyncratic volatilities. I employ Fama and MacBeth (1973) regressions of the cross-section of stock returns on idiosyncratic volatilities and other firm characteristics on a monthly

[Insert Table 5, 6 and 7 about here]

basis and calculate the time-series averages of the coefficients. My goal is to test whether the coefficient on idiosyncratic volatility is significantly different from zero in explaining cross-sectional stock returns.

Specifically, I run the following cross-sectional regressions each month for the SL and PT model:

$$R^e_{i,t+1} = \gamma_{l,t}\hat{l}_t + \gamma_{s,t}\hat{s}_t + \epsilon_{i,t+1} \tag{6}$$

$$R^e_{i,t+1} = \gamma_{l,t}\tilde{l}_t + \gamma_{s,t}\tilde{s}_t + \epsilon_{i,t+1} \tag{7}$$

where $r^e_{t+1,i}$ is stock i's excess return in month $t+1$ minus its Fama and French (1993) factor adjustments, the volatility with tilde signs means that they are smoothed estimates. And if there are hat signs, they are with filtered estimates.

Table 8 shows time-series averages of the coefficients from the month-by-month Fama-Macbeth (FM) regressions of the cross-section of stock returns on different measures of idiosyncratic volatility. The average coefficient on variables used to explain expected returns provides standard FM tests for determining which variables on average have explanatory power during the July 1963 to December 2017 period.

The average coefficient on the log of realized idiosyncratic volatility (IV) is -0.52 with a t-statistic of -5.67. The finding confirms a negative relationship between idiosyncratic volatility and expected return found by Ang et al. (2006).

[**Insert Table 8 about here**]

The next important regressions are to put measures of conditional short-run and long-run idiosyncratic volatility into Fama-Macbeth regressions. The regression using SL model is reported in Table 8. The regression results indicate there exists a negative (positive) relationship between conditional long-run (short-run) volatility and expected returns. The average coefficient is -0.49 (2.38) with a significant t-statistic of -4.03 (6.01) for the filter long-run (short-run) component. The statistical significance is also higher for the smoothed estimates. The Fama-Macbeth regression results lend support to the finding of constructing portfolios by sorting on $\hat{s}_t$ and $\hat{l}_t$ in Section 3.2.

It is also be useful to explain the finding using the PT model to measure conditional long-run component. Table 9 shows that the average coefficient on the filtered estimates of conditional long-run volatility $\hat{l}_t$ has a coefficient of -0.49 with a t-statistic of -4.12.

4.1 Additional Robustness Check

As robustness checks for the finding in the previous section, I include two additional controls: returns reversals and unexpected idiosyncratic volatility in this section. The findings in the previous section remain robust considering these two channels.

4.1.1 Controlling for Return Reversals

Stock returns display short-term reversals (Jegadeesh (1990) and Lehmann (1990)). Return reversal describes a phenomenon that if a stock's previous month return is too high (low), it will tend to reverse the following month and earn a low (high) return. Following Huang et al. (2010), I use the returns of individual stocks in the prior month to control for return reversals. Therefore, the equation (7) is modified to allow for previous month's stock return

$$r^i_{t,d} = \gamma_{l,t}\hat{l}_t + \gamma_{s,t}\hat{s}_t + \beta_{r,t-1}r^i_{t-1} + v^i_t\epsilon^i_{t,d} \tag{8}$$

Without previous month's stock return r^i_{t-1}, the relationship between idiosyncratic risk and expected stock returns may be negatively biased because the coefficient incorporates part of the return reversal that should have been captured by the stock returns of the previous month. Including return reversals into the Fama-Macbeth regression Table 8 shows that the coefficient on the log of realized volatility is reduced to a coefficient of -0.41 with a t-statistic of -4.37. This finding is in line with Huang et al. (2010) that part of the finding by Ang et al. (2006) can be explained by return reversals. And the coefficient on the lagged month return is statistically significant with a statistic of -10.62.

However, accounting for return reversals doesn't quite diminish the coefficients of the conditional short-run and long-run component. Some of the coefficients become even more significant adding the lagged month return control. Still, the coefficient on lagged month return is significant for both the SL and PT model with t-statistics of -11.36 and -12.23. Therefore, returns reversals are not the key driver of the relationship between short-run and long-run conditional idiosyncratic volatility and expected returns.

4.1.2 Controlling for Unexpected Idiosyncratic Volatility

In line with the argument of French et al. (1987), the relationship between expected idiosyncratic volatility and expected returns in the above regression may be clouded by the relationship between unexpected idiosyncratic volatility and unexpected stock returns. To control for this effect, I add unexpected idiosyncratic volatility to the Fama-Macbeth regression. I define the unexpected idiosyncratic volatility as

$$\mu_t = \log v_t - \hat{s}_t - \hat{l}_t$$

where $\log v_t$ is the log of realized volatility at time t, $\hat{s}_t$ and $\hat{l}_t$ are volatility forecasts made at time $t-1$ for the short-run and long-run component at time t. When the unexpected idiosyncratic volatility is added to the regression, the relationship between conditional short-run and long-run idiosyncratic volatility and expected stock returns is robust.

Table 8 includes results for the SL model. The coefficient on the filtered short-run (long-run) idiosyncratic volatility is 3.36 (-0.77) with a robust t-statistic of -8.48 (-5.68), which further supports that there is a strong negative (positive) relationship between conditional long-run (short-run) component and average stock returns. When the unexpected idiosyncratic volatility is added to the regression besides lagged month return, the coefficient on the long-run component actually becomes more significant. The coefficient on the unexpected idiosyncratic volatility is also significant. For the filtered idiosyncratic volatility estimates group, the coefficient of unexpected idiosyncratic volatility is 4.89 with high t-statistic of 23.60. The positive relationship between unexpected idiosyncratic volatility and stock returns is consistent with the positive contemporaneous relationship between stock returns and firm-level idiosyncratic volatility found by Duffee (1995) and Grullon et al. (2012). Table 9 reports similar results of adding unexpected idiosyncratic volatility for the PT model.

5 Monte Carlo Simulations

Because the realized idiosyncratic volatility is strongly time-varying and has an average first-order auto-correlation of about 0.39, Fu (2009) argues that exponential generalized autoregressive conditional heteroscedasicity (EGARCH) model could be used to capture the short-run variation in conditional idiosyncratic volatility. Can EGARCH model successfully capture variation in the short-run component of idiosyncratic volatility highlighted in this paper and hence produce a positive relationship between conditional idiosyncratic volatility and cross-section of stock returns? This section attempts to answer the question using Monte-Carlo simulations.

5.1 A Discrete Time Model for Monte Carlo Simulations

Consider a simple discrete time model in which the daily returns of an individual stock is characterized as

$$r^i_{t,d} = \sigma^i_t \epsilon^i_{t,d} + \gamma_s s^i_t + \gamma_l l^i_t \tag{9}$$

where for day d in month t, $r^i_{t,d}$ is stock i's excess return. The residuals $\eta^i_{t,d} \equiv \sigma^i_t \epsilon^i_{t,d}$ is the idiosyncratic risk for month t whereas σ^i_t is the standard deviation of the residual. The following definition is consistent with the notation in Section 2. I define the idiosyncratic volatility of stock returns for firm i in month t as v^i_t

$$v^i_t = \sigma^i_t \sqrt{N_m}$$

where N_m is the number of trading days in month t for firm i. The volatility dynamics for v_t^i follows the specification in the previous section as

$$\begin{aligned}
\textbf{Idiosyncratic Volatility} &: \log v_t^i = s_t^i + l_t^i \\
\textbf{Short-run Component} &: s_{t+1}^i = \rho_s^i s_t^i + \sigma_s^i \epsilon_{s,t}^i \\
\textbf{Long-run component} &: l_{t+1}^i = \phi_i + \rho_l^i l_t^i + \sigma_l^i \epsilon_{l,t}^i
\end{aligned}$$

The log-volatility is the sum of two components, s_t and l_t. The parameter γ_s and γ_l capture the relationship between short-run and long-run component of idiosyncratic volatility and cross-section of stock returns. For example, when $\gamma_s = 0$, there is no relationship between conditional short-run idiosyncratic volatility and expected stock returns.

Without loss of generality, I assume the risk-free rate to be zero. For each stock, the monthly excess return then takes the following form

$$R_t^i = \prod_{d=1}^{N_m} (1 + r_{t,d}^i) - 1$$

The parameters are set as follows in the simulation based on empirical estimates using the CRSP data. I set γ_s to be 4.12 and γ_l to be -0.74, which are the average regression coefficients in the Fama-Macbeth regressions in Table 8. The parameter of the process is set as as $\rho_l = 0.94$ which is the median persistence parameter of the long-run component. The persistence of short-run component is set to be $\rho_s = -0.07$, the median persistence parameter of the short-run component. And σ_s and σ_l are the volatility of shocks to the short-run and long-run components, which can be set to 0.11 and 0.07 respectively based on the empirical estimates. The constant ϕ_i is set to be uniformly distributed with a mean of 2.43 with a standard deviation of 0.62. The numbers correspond to the empirical estimates using CRSP data. In simulated data, each stock has 230 months of daily stock returns observations, which is the median number of daily stock observations for CRSP common stocks. For illustration, I investigate cross-sectional implications using 120 stocks; more stocks should not affect the results in any quantitative manner.

In the simulated data, the autocorrelations of realized idiosyncratic volatility matches the empirical pattern. The first order autocorrelation of realized volatility is 0.26, second order to be 0.24, third order to be 0.22 and the autocorrelation of 12 months lag is 0.09. Thus, the realized volatility decay quickly in one month and slowly in longer periods.

To evaluate the quantitative relationship between associated with EGARCH model, I consider the Fama and Macbeth regressions by regressing stock returns on conditional EGARCH volatility obtained using the full-sample of simulated data. The EGARCH(p,q) model is modeled as [2]

$$R_t^i = \alpha_t^i + \epsilon_t^i \tag{10}$$

where ϵ_t^i is assumed to have a serially independent normal distribution $\epsilon_t^i \sim N(0, {\sigma_t^i}^2)$. The conditional variance ${\sigma_t^i}^2$ follows

$$\log {\sigma_t^i}^2 = a_i + \sum_{l=1}^{p} b_{i,l} \log {\sigma_{t-l}^i}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left(\frac{\epsilon_{t-k}^i}{\sigma_{t-k}^i} \right) + \kappa \left[\left| \frac{\epsilon_{t-k}^i}{\sigma_{t-k}^i} \right| - \left(\frac{2}{\pi} \right)^{1/2} \right] \right\} \tag{11}$$

Estimating EGARCH model produces an average coefficient of b_1 as 0.50 and the coefficient of c_1 is 0.07. Therefore, the persistence parameter of idiosyncratic volatility b_1 in EGARCH model is indeed influenced by the short-run dynamics of idiosyncratic volatility. However, the average regression coefficient of stock returns on the conditional idiosyncratic volatility obtained by EGARCH model is -0.35 with a t-statistic of -4.15. This indicates that the EGARCH estimate of conditional volatility largely captures the conditional volatility of

[2] I report the EGARCH(1,1) estimates while EGARCH with different lags produce similar results. I also consider specifications of estimating the time t EGARCH volatility recursively using the information available up to time $t-1$. Since stock returns are not skewed in my simulation, the look-ahead bias documented by Guo et al. (2014) doesn't exist. Hence, similar results are produced using full sample and information up to $t-1$.

the long-run component instead of the short-run component. This is intuitively plausible as the conditional volatility is a positive function of lagged volatility, which largely consists of the persistent long-run component. Hence, we would expect a negative relationship between the conditional volatility estimated by EGARCH and expected stock returns. As Fink et al. (2012), Guo et al. (2014) document, the positive relation found by Fu (2009) could be due to using future return information and the look-ahead bias is quantitatively important for observed skewness in stock returns. Using information up to time $t-1$ to predict conditional volatility at time t could produce a negative relationship between conditional volatility and expected stock returns.

However, existing literature doesn't provide an structural model on the dynamics of idiosyncratic volatility and an unified setting to reconcile evidence on the opposite sign of the relationship between idiosyncratic volatility and stock returns. This paper provides an underlying process for idiosyncratic volatility with a short-run and long-run component, which have distinctive relationship with stock returns. The simulation results also demonstrate the necessity of separating the conditional volatility into short-run and long-run components to better capture variations of the conditional volatility over different horizons. Neither the lagged realized volatility used by Ang et al. (2006) nor the conditional volatility estimated by the EGARCH model can ully capture the dynamics and relevant information in the conditional idiosyncratic volatility.

6 The Cross-Section Relation of the Short-Run and Long-Run Component and Expected Stock Returns

Since the short-run and long-run component has opposite relation with the cross-section of stock returns, this phenomenon highlights different economic mechanisms in generating the cross-section relationship. Besides, distinctive mechanisms may act on short-run and long-run components differently. Therefore, the decomposition could also be useful in evaluating underlying economic mechanisms in generating the cross-section relationship. To interpret the cross-section relationship between the short-run and long-run component and expected returns, it is thus useful to take a look at possible channels that idiosyncratic volatility matters.

6.1 Why idiosyncratic volatility and returns are related?

Traditional asset pricing theories with rational agents with full information, frictionless and complete market predict no relationship between idiosyncratic volatility and expected returns. For various reasons, either real world financial markets deviate from the perfect ones or the idiosyncratic volatility defined is not perfectly measured. Thus idiosyncratic volatility could be related to the cross-section of stock returns.

First, in reality, investors may not hold perfectly diversified portfolios. Various theories assuming under-diversification predict that idiosyncratic risk is positively related to the expected stocks returns in the cross-section, for example, informational friction (Merton (1987)) and transaction costs (Hirshleifer (1988)). Under-diversified investors demand return compensations for bearing idiosyncratic risk. There will then be a positive relationship between idiosyncratic volatility and the cross-section of stock returns. However, frictions that prevent investors from adjusting portfolios to perfectly diversify risk tend to matter more in short-horizons. For returns measured at longer terms, frictions such transaction cost, limited attention diminish. This is consistent with the finding of the portfolio analysis of Section 3.2. The return spread sorted by the conditional short-run idiosyncratic volatility exists only with a holding period of $N = 1$. Besides, the difficulty of perfect diversification is more evident in short-horizons when shocks to idiosyncratic volatility has a relatively non-persistent component. In the SL model, the mean estimates of the volatility of shocks to the short-run component θ_s is more than 50% larger than that of the long-run component θ_l.

Therefore, the positive relationship between the conditional short-run idiosyncratic volatility and cross-section of stock returns could be due to investors are facing frictions and hold under-diversified portfolios.

Ang et al. (2006) find that stocks with high realized idiosyncratic volatility in one month earn abysmally low average returns in the next month. One reason that they find the opposite result is earlier studies don't sort stocks or examine idiosyncratic volatility at the stock level. Explanations of the negative relationship between idiosyncratic volatility and stock returns may be categorized into two types. The first type of explanation is risk-based: high idiosyncratic volatility stocks earn lower returns because they have less systematic risk. The other is non-risk-based and stock prices deviates from what are predicted by the perfect market.

On the risk-based side, imperfect measurement of risk plays a key role in explaining the negative relationship between the idiosyncratic volatility and cross-section of stocks. Babenko et al. (2016) provides a dynamic asset pricing framework in which idiosyncratic volatility is negatively correlated with systematic risk and can therefore predict returns. The crucial assumption is an increase in idiosyncratic part of firm value decreases the sensitivity of firm value to priced risk factors. Similarly, Chen et al. (2018) study a risk-shifting problem of equity householders who take on more investments with high idiosyncratic risk when firms are in distress and when the aggregate economy is in a bad state. Thus, the negative covariance between the equity beta and market risk premium in the conditional CAPM may explain the negative excess returns and negative CAPM alphas in the high-idiosyncratic-volatility firms. Recent studies, for example, Grullon et al. (2012) find firms with high idiosyncratic volatility are usually the firms with abundant growth options. Guo and Savickas (2010) hypothesize that firm with high idiosyncratic stocks are firms with abundant growth options, and could be more sensitive to discount factor news. And they find a factor structure among portfolios that are sorted by idiosyncratic volatility. Bhamra and Shim (2017) introduce stochastic cash flow risks into an equity evaluation model with growth options to simultaneously explain the positive contemporaneous relationship of IVOL and stock returns and the negative relationship of IVOL and expected stock return.

As for the non-risked based explanation, a number of explanations have been proposed. The long list of explanations could include lottery preferences (Bali et al. (2011)), limits to arbitrage (Stambaugh et al. (2015)), etc. As argued by Cochrane (1999), to distinguish between risk and non-risk based explanation, we could examine whether the relationship is persistent or not. In a recent paper, Stambaugh et al. (2015) argues that costly arbitrage leads to the pricing of idiosyncratic risk. However, the cost is higher for overpriced stocks than or underpriced ones. The negative relation among overpriced stocks is stronger, especially for stocks less easily shorted, so the overall IVOL-return relation is negative. Since mispricing tends to be corrected in the long-run, it is unclear whether their story could generate a persistent negative IVOL-return relationship. It is worth investigating whether their findings hold under longer return horizons. This paper finds that the return spread sorted by conditional long-run idiosyncratic volatility persists with multiple months holding horizons. Therefore, the underlying mechanism behind the negative relationship between the conditional long-run idiosyncratic volatility and stock returns may be risk-driven and the next section would quantitatively investigate the risk exposure for portfolios sorted by idiosyncratic volatility.

6.2 Investigating Risk Exposures

To investigate whether the cross-section relationship between the short-run or long-run idiosyncratic volatility and stock returns can be explained by risk exposures or not, I first study the return difference between portfolios sorted by the short-run and long-run idiosyncratic volatility. If these return differences are driven by risk, they should comove with systematic risk factors. If no insignificant correlation is found, the relation could then be non-risk-based and driven by forces such as market frictions.

To this end, I first compute the correlation between the return of the high minus low (5-1) portfolio sorted by the conditional short-run idiosyncratic volatility, long-run idiosyncratic volatility, lagged realized volatility and Fama and French (2015) five factors. Because the evidence of Novy-Marx (2013) and Titman et al. (2004), profitability RMW and investment factors CMA are added in addition to the Fama and French (1993) three factors. [3] As for the short-run and long-run component, I use the filter estimates here which use the information at $t-1$ to predict conditional volatility at time t.

Table 10 reports the correlation between portfolio spreads sorted by idiosyncratic volatility and the five factors. First, it is found that the return spread sorted by the conditional short-run component , denoted as IVFS, is not correlated with the spread sorted by the conditional long-run idiosyncratic volatility IVFL, realized idiosyncratic volatility IVFR and also the five factors. The correlation is -0.07 with the IVFL portfolio, 0 with the excess market return, 0.04 to the SMB factor, 0.07 with the HML factor, 0.05 with the RMW factor and 0.09 with the CMA factor. The lack of correlation with systematic risk factors further suggests that the cross-sectional relationship between the conditional short-run component and stock returns is likely not driven by risk.

In the meantime, the return spread sorted by the conditional long-run component, denoted as IVFL, is strongly correlated with the return spread sorted by the realized idiosyncratic volatility, denoted as IVFR, with a correlation of 0.95. Therefore, the cross-sectional relation of realized idiosyncratic volatility found by Ang et al. (2006) is mostly captured by the long-run component idiosyncratic volatility. Besides, the return spread IVFL correlates with book-to-market factor HML, profitability factor RMW and investment factor CMA factors with a negative sign. Especially, the correlation with the profitability factor is strong with a coefficient of -0.61. Given these factors may earn positive risk premiums, negative exposure to them could help explain the negative relationship between the conditional long-run

[3] The question investigated here is not related to momentum. Including the momentum factor of Carhart (1997) doesn't impact the main results of this paper.

idiosyncratic volatility and cross-sectional of stock returns.

[**Insert Table 10 about here**]

Table 11 systematically examine how these five factors are useful in explaining the portfolio returns sorted by the realized idiosyncratic volatility beyond the FF-3 model. The test assets used here are the 25 portfolios formed monthly on size and realized idiosyncratic volatility provided on Kenneth French's website. [4] Similar to Fama (2016), the five-factor model provides a better description of average returns on the Size-IVOL portfolio. Table 13 reports the Gibbons et al. (1989)'s GRS statistic, which is reduced from 6.78 to 5.57 from the three-factor to five-factor model. The average absolute alpha is also reduced significantly from 0.23 to 0.13. In particular, Table 11 shows that stocks with high idiosyncratic volatility have significant negative exposure to the profitability factor RMW.

While the profitability is useful in explaining the negative relationship between idiosyncratic volatility and cross-section of stock returns, it does not fully explain the relationship. Strong exposure to RMW still misses part of the low average returns of high realized idiosyncratic small stocks. Therefore, I investigate whether there are additional factor structure behind the negative relationship between idiosyncratic volatility and the cross-section

[4] According to information on Kennth French's website,, the portfolios, which are constructed monthly, are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the variance of the residuals from the FF three-factor model IVOL. The monthly size breakpoints are the NYSE market equity quintiles. IVOL is estimated using 60 days (minimum 20) of lagged returns. The IVOL breakpoints are NYSE quintiles.

of stock returns. In particular, I investigate whether IVFL is factor in explaining the cross-sectional relationship between idiosyncratic volatility and the cross-section of stock returns. This is testing whether there is a slope structure between portfolios sorted by idiosyncratic volatility.

Table 12 and 13 report the results of adding the IVFL factor to the five-factor model. As Table 13 shows, the gain of adding the IVFL factor is not particularly evident beyond the five-factor model. The GRS statistic is reduced from 5.57 to 5.03 and the average absolute alpha decreases from 0.13 to 0.11. However, Table 12 also shows that after adding the IVFL factor, the significance of the RMW factor is largely to be almost insignificant. This pattern suggests that the information of RMW is largely captured by the $IVFL$ factor. In other words, the economic interpretation of the slope factor may be largely captured by the profitability RMW factor.

Even though the profitability factor may be an important element driving the cross-sectional relationship between the conditional idiosyncratic volatility and stock returns, existing literature still lacks a well-grounded economic explanation in regard to the empirical finding in Table 12. We could develop an economic model to rationalize the profitability factor in explaining the negative relationship between idiosyncratic volatility and stock returns based on the framework of Babenko et al. (2016) and Bhamra and Shim (2017). [5] The key element is the value of firms is comprised of assets in place and growth options. Holding other thing fixed, high idiosyncratic volatility increases the value of idiosyncratic growth options. The proportion of firm value that is systematic become smaller and hence firm value is less sensitive to priced profitability factors. Nevertheless, Table 12 shows that high idiosyncratic stocks are not just less sensitive to profitability factors. Low idiosyncratic volatility stocks have positive exposure to the idiosyncratic volatility factor, but the high idiosyncratic volatility stocks have smaller yet negative exposure to the profitability

[5] Babenko et al. (2016) provide an explanation for the negative relation between the idiosyncratic volatility and stock returns. But the relationship doesn't hold under double sorts of size and idiosyncratic volatility.

factors. This pattern is at odds with the prediction of Babenko et al. (2016). They don't accommodate a negative exposure to systematic risk. Therefore, future research could be devoted to understanding the distinctive risk exposures of high idiosyncratic volatility stock to the profitability factor. Also, the profitability factor is mostly capturing the slope structure among portfolios sorted by the conditional long-run idiosyncratic volatility. Additional risk structure may be also important for explaining the negative relationship between the conditional long-run idiosyncratic volatility and stock returns.

7 Conclusion

This paper contributes to the empirical literature on the relation between conditional idiosyncratic volatility and cross-section of stock returns. First, the paper develops and estimates a model for idiosyncratic volatility to better capture the dynamics of idiosyncratic volatility and decomposes the volatility of idiosyncratic stock return into short-run and long-run components. I find that there is a significant negative (positive) relationship between conditional long-run (short-run) idiosyncratic volatility and expected stock returns. The result highlights that idiosyncratic variations over short-run and long-run horizons have important implications for the cross-section of stock returns. It is also useful to reconcile conflicting evidence on the relation between idiosyncratic volatility and the cross-sectional of stock returns.

The cross-section relationship with opposite signs also demonstrates that separating the idiosyncratic volatility into short-run and long-run components reveal different economic mechanisms behind the cross-sectional relationship between idiosyncratic volatility and stock returns. The paper finds the return spread sorted by the short-run idiosyncratic volatility comoves little with systematic risk factors, suggesting that the positive relation between conditional short-run volatility and stock returns are not risk-driven. This is consistent with the case when investors face trading frictions in the short-horizon and hold under-diversified portfolios, they require compensation for bearing idiosyncratic risk.

In the meantime, the return spread sorted by the conditional long-run idiosyncratic volatility is significantly related to important systematic risk factors. Further tests suggest that the negative relationship between conditional long-run idiosyncratic risk can be partly explained by exposure to the profitability factor. The profitability factor RMW mostly captures the slope factor of portfolios sorted by the conditional long-run idiosyncratic volatility. One avenue for further research is to find more systematic factors, explore additional risk structures in capturing the relationship between idiosyncratic volatility and stock returns.

References

Adrian, Tobias and Joshua Rosenberg, "Stock Returns and Volatility: Pricing the Short-Run and Long-Run Components of Market Risk," *The Journal of Finance*, 2008, *63* (6), 2997–3030.

Ang, Andrew, Robert J Hodrick, Yuhang Xing, and Xiaoyan Zhang, "The Cross-Section of Volatility and Expected Returns," *The Journal of Finance*, 2006, *61* (1), 259–299.

_ , _ , _ , and _ , "High Idiosyncratic Volatility and Low Returns: International and Further US Evidence," *Journal of Financial Economics*, 2009, *91* (1), 1–23.

Babenko, Ilona, Oliver Boguth, and Yuri Tserlukevich, "Idiosyncratic cash flows and systematic risk," *The Journal of Finance*, 2016, *71* (1), 425–456.

Bali, Turan G and Nusret Cakici, "Idiosyncratic Volatility and the Cross Section of Expected Returns," *Journal of Financial and Quantitative Analysis*, 2008, *43* (01), 29–58.

Bali, Turan G., Nusret Cakici, and Robert F. Whitelaw, "Maxing out: Stocks as lotteries and the cross-section of expected returns," *Journal of Financial Economics*, 2011, *99* (2), 427 – 446.

Barndorff-Nielsen, Ole E and Neil Shephard, "Econometric Analysis of Realized Volatility and Its Use in Estimating Stochastic Volatility Models," *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 2002, *64* (2), 253–280.

Bhamra, Harjoat S. and Kyung Hwan Shim, "Stochastic idiosyncratic cash flow risk and real options: Implications for stock returns," *Journal of Economic Theory*, 2017, *168*, 400 – 431.

Bollerslev, Tim and Hao Zhou, "Estimating stochastic volatility diffusion using conditional moments of integrated volatility," *Journal of Econometrics*, 2002, *109* (1), 33–65.

Carhart, Mark M, "On persistence in mutual fund performance," *The Journal of Finance*, 1997, *52* (1), 57–82.

Chen, Zhiyao, Ilya A Strebulaev, Yuhang Xing, and Xiaoyan Zhang, "Strategic risk shifting and the idiosyncratic volatility puzzle," *Rock Center for Corporate Governance at Stanford University Working Paper*, 2018, (182).

Christoffersen, Peter, Kris Jacobs, Chayawat Ornthanalai, and Yintian Wang, "Option Valuation with Long-Run and Short-Run Volatility Components," *Journal of Financial Economics*, 2008, *90* (3), 272–297.

Chua, Choong Tze, Jeremy Goh, and Zhe Zhang, "Expected Volatility, Unexpected Volatility, and the Cross-Section of Stock Returns," *Journal of Financial Research*, 2010, *33* (2), 103–123.

Cochrane, John H, "Portfolio Advice for a Multifactor World," *Economic Perspectives*, 1999, *23* (3), 59.

Corsi, Fulvio, "A Simple Approximate Long-Memory Model of Realized Volatility," *Journal of Financial Econometrics*, 2009, *7* (2), 174–196.

Duffee, Gregory R., "Stock returns and volatility a firm-level analysis," *Journal of Financial Economics*, 1995, *37* (3), 399–420.

Fama, Eugene F and James D MacBeth, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy*, 1973, *81* (3), 607–636.

_ **and Kenneth R French**, "The Cross-Section of Expected Stock Returns," *the Journal of Finance*, 1992, *47* (2), 427–465.

— **and** — , “Common Risk Factors in the Returns on Stocks and Bonds,” *Journal of Financial Economics*, 1993, *33* (1), 3–56.

Fama, Eugene F. and Kenneth R. French, “A five-factor asset pricing model,” *Journal of Financial Economics*, 2015, *116* (1), 1 – 22.

Fama, Kenneth R. French Eugene F., “Dissecting Anomalies with a Five-Factor Model,” *Review of Financial Studies*, 2016, *29* (1), 69–103.

Fink, Jason D, Kristin E Fink, and Hui He, “Expected idiosyncratic volatility measures and expected returns,” *Financial Management*, 2012, *41* (3), 519–553.

French, Kenneth R, G William Schwert, and Robert F Stambaugh, “Expected Stock Returns and Volatility,” *Journal of Financial Economics*, 1987, *19* (1), 3–29.

Fu, Fangjian, “Idiosyncratic Risk and the Cross-Section of Expected Stock Returns,” *Journal of Financial Economics*, 2009, *91* (1), 24–37.

Gibbons, R M., Ross, A S., and Shanken, “A test of the efficiency of a given portfolio,” *Econometrica*, 1989, *57* (5), 1121–1152.

Grullon, Gustavo, Evgeny Lyandres, and Alexei Zhdanov, “Real Options, Volatility, and Stock Returns,” *Journal of Finance*, 2012, *67* (4), 1499–1537.

Guo, Hui and Robert Savickas, “Relation between time-series and cross-sectional effects of idiosyncratic variance on stock returns,” *Journal of Banking & Finance*, 2010, *34* (7), 1637–1649.

— **, Haimanot Kassa, and Michael F Ferguson**, “On the relation between EGARCH idiosyncratic volatility and expected stock returns,” *Journal of Financial and Quantitative Analysis*, 2014, *49* (1), 271–296.

Hirshleifer, David, "Residual risk, trading costs, and commodity futures risk premia," *The Review of Financial Studies*, 1988, *1* (2), 173–193.

Huang, Wei, Qianqiu Liu, S Ghon Rhee, and Liang Zhang, "Return Reversals, Idiosyncratic Risk, and Expected Returns," *The Review of Financial Studies*, 2010, *23* (1), 147–168.

Jegadeesh, Narasimhan, "Evidence of Predictable Behavior of Security Returns," *The Journal of Finance*, 1990, *45* (3), 881–898.

_ **and Sheridan Titman**, "Returns to buying winners and selling losers: Implications for stock market efficiency," *The Journal of finance*, 1993, *48* (1), 65–91.

Lehmann, Bruce N, "Fads, Martingales, and Market Efficiency," *The Quarterly Journal of Economics*, 1990, *105* (1), 1–28.

Merton, Robert C, "A Simple Model of Capital Market Equilibrium with Incomplete Information," *The Journal of Finance*, 1987, *42* (3), 483–510.

Novy-Marx, Robert, "The other side of value: The gross profitability premium," *Journal of Financial Economics*, 2013, *108* (1), 1–28.

Stambaugh, Robert F, Jianfeng Yu, and Yu Yuan, "Arbitrage asymmetry and the idiosyncratic volatility puzzle," *The Journal of Finance*, 2015, *70* (5), 1903–1948.

Titman, Sheridan, KC John Wei, and Feixue Xie, "Capital investments and stock returns," *Journal of financial and Quantitative Analysis*, 2004, *39* (4), 677–700.

Tables and Figures

Table 1: Time Series Properties of Idiosyncratic Volatility

Panel A: Some Summary Statistics of Idiosyncratic Volatility						
Mean	Std.Dev.	Skewness	Kurtosis			
15.54	9.21	2.00	8.23			
Panel B: Autocorrelations of Idiosyncratic Volatility						
ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)	ACF(10)	ACF(12)
0.39	0.31	0.28	0.23	0.21	0.12	0.12

This table summarizes the time-series statistics for idiosyncratic volatility. I first compute the statistics for each stock and the average the statistics across all stocks. The sample period is July 1963 to December 2017. The ACF stands for estimated autocorrelations at different lags. The unit of the mean and standard deviation is percentage point.

Table 2: Parameter Estimates for Idiosyncratic Volatility Model

Panel A: The Short and Long Run Volatility (SL) Model				
Variables	ρ_s	ρ_l	σ_s	σ_l
Mean	-0.76	0.79	0.11	0.07
Median	-0.003	0.94	0.10	0.02
Panel B: The Permanent and Transitory Volatility Model				
Variables	σ_s	σ_l		
Mean	0.14	0.05		
Median	0.11	0.01		

This table summarizes the properties of parameter estimates for the short-run and long-run idiosyncratic volatility process. I first compute the parameter estimates for each stock and then construct the mean and median statistics across all stocks. The sample period is July 1963 to December 2017.

Table 3: Portfolios Sorted by Idiosyncratic Volatility

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.91	3.66	41.3%	0.07
				[1.78]
2	0.92	4.52	31.8%	-0.02
				[-0.45]
3	0.95	5.62	16.3%	-0.09
				[-1.48]
4	0.66	6.93	7.8%	-0.45
				[-4.51]
5 (high)	0.16	8.26	2.7%	-1.07
				[-7.55]
5-1	-0.75	6.59		-1.14
	[-2.70]			[-6.91]

I form value-weighted quintile portfolios every month by sorting stocks based on idiosyncratic volatility relative to the Fama and French (1993) model. Portfolios are formed every month, based on volatility computed using daily data over the previous month. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std.Dev. are measured in monthly percentage terms and apply to total, not excess returns. The row $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column report Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is July 1963 to December 2017.

Table 4: Portfolios Sorted by Idiosyncratic Volatility with Multiple Holding Period

Panel A: Ranking on Realized Idiosyncratic Volatility						
Period	1 Low	2	3	4	5 High	5-1
$N = 1$	0.91	0.92	0.95	0.66	0.16	-0.75
						[-2.70]
$N = 6$	1.07	1.07	1.07	0.91	0.42	-0.65
						[-3.59]
$N = 12$	0.98	1.00	1.02	0.93	0.56	-0.42
						[-3.31]
Panel B: Ranking on Conditional Idiosyncratic Volatility: PT						
Period	1 Low	2	3	4	5 High	5-1
$N = 1$	1.02	1.01	0.94	0.54	-0.64	-1.66
						[-3.89]
$N = 6$	1.13	1.15	1.12	0.72	-0.28	-1.41
						[-5.12]
$N = 12$	1.02	1.07	1.06	0.82	0.06	-0.95
						[-5.19]
Panel C: Ranking on Conditional Idiosyncratic Volatility: SL						
Period	1 Low	2	3	4	5 High	5-1
$N = 1$	1.01	1.01	0.95	0.56	-0.23	-1.24
						[-2.85]
$N = 6$	1.12	1.14	1.13	0.72	-0.07	-1.19
						[-4.44]
$N = 12$	1.01	1.07	1.06	0.82	0.18	-0.82
						[-4.65]

I form value-weighted quintile portfolios every month by sorting stocks based on idiosyncratic volatility relative to the Fama and French (1993) model. Holding period is one month, six months or twelve months. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std.Dev. are measured in monthly percentage terms and apply to total, not excess returns. The row $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column report Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is July 1963 to December 2017.

Table 5: Portfolios Sorted by the Filtered Estimates of Expected Long-Run Volatility

Panel A: Short and Long-Run Volatility (SL) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.92	3.66	47.1%	0.09
				[2.23]
2	0.97	4.72	31.9%	-0.02
				[-0.45]
3	1.02	6.19	13.9%	-0.05
				[-0.61]
4	0.82	7.92	5.6%	-0.35
				[-3.06]
5 (high)	0.14	9.72	1.6%	-1.18
				[-6.57]
5-1	-0.78	8.13		-1.28
	[-2.23]			[-6.27]
Panel B: Permanent and Transitory Volatility (PT) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.92	3.65	46.9%	0.09
				[2.42]
2	0.97	4.70	31.9%	-0.02
				[-0.49]
3	1.01	6.12	14.1%	-0.06
				[-0.73]
4	0.82	7.88	5.7%	-0.35
				[-3.08]
5 (high)	0.15	9.69	1.6%	-1.16
				[-6.27]
5-1	-0.76	8.10		-1.25
	[-2.20]			[-6.06]

Portfolios are formed every month based on the filtered estimates of the conditional short-run idiosyncratic volatility $\hat{l}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std.Dev. are measured in monthly percentage terms and apply to total, not excess returns. The row $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column report Jensen's alpha with respect to the Fama-French (1993) three factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is July 1963 to December 2017.

Table 6: Portfolios Sorted by the Smoothed Estimates of Expected Long-Run Volatility

Panel A: Short and Long-Run Volatility (SL) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	1.01	3.53	46.6%	0.21
				[4.94]
2	1.01	4.67	31.9%	0.03
				[0.63]
3	0.95	6.22	14.1%	-0.13
				[-1.75]
4	0.56	8.25	5.7%	-0.66
				[-5.45]
5 (high)	-0.23	11.12	1.6%	-1.69
				[-7.00]
5-1	-1.24	9.65		-1.9
	[-2.85]			[-7.11]

Panel B: Permanent and Transitory Volatility (PT) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	1.02	3.54	47.4%	0.21
				[5.13]
2	1.01	4.73	32.0%	0.03
				[0.57]
3	0.94	6.25	8.35%	-0.14
				[-1.92]
4	0.54	8.35	5.5%	-0.7
				[-5.58]
5 (high)	-0.64	10.97	1.5%	-2.09
				[-8.58]
5-1	-1.66	9.51		-2.3
	[-3.89]			[-8.59]

Portfolios are formed every month based on the smoothed estimates of the long-run idiosyncratic volatility $\tilde{l}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std.Dev. are measured in monthly percentage terms and apply to total, not excess returns. The row $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column report Jensen's alpha with respect to the Fama-French (1993) three factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is July 1963 to December 2017.

Table 7: Portfolios Sorted by Estimates of Expected Short-Run Volatility

Panel A: The Filtered Estimates of Expected Short-Run Volatility

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.79	4.62	12.4%	-0.14
				[-2.55]
2	0.89	4.43	23.6%	-0.02
				[-0.46]
3	0.93	4.48	27.8%	0.02
				[0.37]
4	0.92	4.46	23.4%	0.01
				[0.30]
5 (high)	0.95	4.64	12.8%	-0.00
				[-0.02]
5-1	0.16	1.72		0.13
	[2.36]			[1.93]

Panel B: The Smoothed Estimates of Expected Short-Run Volatility

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.62	3.91	23.5%	-0.23
				[-4.18]
2	0.84	4.15	21.4%	-0.06
				[-1.29]
3	0.97	4.37	19.5%	0.07
				[1.85]
4	1.13	4.74	19.1%	0.20
				[4.42]
5 (high)	1.18	5.65	16.5%	0.11
				[1.30]
5-1	0.56	3.17		0.35
	[4.09]			[2.83]

Portfolios are formed every month based on conditional short-run idiosyncratic volatility of $\hat{s}_t$ or $\tilde{s}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std.Dev. are measured in monthly percentage terms and apply to total, not excess returns. The row $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column report Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics are reported in square brackets. The sample period is July 1963 to December 2017.

Table 8: Relationship between Idiosyncratic Risk and Expected Returns: Cross-Sectional Evidence

Short and Long-Run Volatility (SL) Model						
$\log v_t$	$\hat{s}_t$	$\hat{l}_t$	$\tilde{s}_t$	$\tilde{l}_t$	$Ret(-1)$	μ_t
-0.52						
[-5.67]						
	2.38	-0.49				
	[6.01]	[-4.03]				
			4.12	-0.74		
			[35.99]	[-5.73]		
-0.41					-4.78	
[-4.37]					[-10.62]	
	2.71	-0.47			-5.11	
	[6.84]	[-3.69]			[-11.36]	
			4.04	-0.78	-5.46	
			[35.72]	[-5.76]	[-12.23]	
	3.36	-0.77			-3.35	4.89
	[8.48]	[-5.68]			[-7.92]	[23.60]
			1.18	-1.05	-4.35	4.60
			[17.56]	[-8.16]	[-10.23]	[22.43]

The average coefficient is the time-series average of monthly regression coefficients for July 1963 to December 2017, and the t-statistics is the average coefficient divided by its time-series standard error. The t-statistic is reported in brackets.

Table 9: Relationship between Idiosyncratic Risk and Expected Returns: Cross-Sectional Evidence

Permanent and Transitory Volatility (PT) Model				
$\log v_t$	$\hat{l}_t$	$\tilde{l}_t$	$Ret(-1)$	μ_t
-0.53				
[-5.79]				
	-0.49			
	[-4.12]			
		-0.49		
		[-3.56]		
-0.43			-4.78	
[-4.61]			[-10.57]	
	-0.48		-5.07	
	[-3.90]		[-11.20]	
		-0.55	-5.65	
		[-3.83]	[-12.46]	
	-0.26			4.89
	[-2.13]			[23.35]
		-1.05		5.11
		[-8.18]		[26.07]
	-0.26		-4.19	4.75
	[-2.08]		[-9.71]	[22.91]
		-1.08	-4.50	4.96
		[-8.16]	[-10.41]	[25.70]

The average coefficient is the time-series average of monthly regression coefficients for July 1963 to December 2017, and the t-statistics is the average coefficient divided by its time-series standard error. The t-statistic is reported in brackets.

Table 10: Correlation of the Return Spread with Fama and French (2015) Five Factors

	IVFS	IVFL	IVFR	$R_m - R_F$	SMB	HML	RMW	CMA
IVFS	1.00	-0.07	-0.03	-0.00	0.04	0.07	0.05	0.09
IVFL	-0.07	1.00	0.95	0.52	0.68	-0.34	-0.61	-0.38
IVFR	-0.03	0.95	1.00	0.50	0.65	-0.33	-0.59	-0.37
$R_m - R_F$	-0.00	0.52	0.50	1.00	0.27	-0.26	-0.23	-0.38
SMB	0.04	0.68	0.65	0.27	1.00	-0.07	-0.35	-0.10
HML	0.07	-0.34	-0.33	-0.26	-0.07	1.00	0.06	0.70
RMW	0.05	-0.61	-0.59	-0.23	-0.35	0.06	1.00	-0.04
CMA	0.09	-0.38	-0.37	-0.38	-0.10	0.70	-0.04	1.00

The table reports pairwise correlation between the return spread of the high minus low portfolio. The variable IVFS denotes the return spread of sorting stocks by the conditional short-run idiosyncratic volatility, IVFL denotes the return spread sorted by the conditional long-run idiosyncratic volatility and IVFR the return spread sorted by lagged realized idiosyncratic volatility. And $R_m - R_F$ is the excess market return, SMB is the size factor, HML is the book-to-market factor, HML is the profitability factor and CMA is the investment factor.

Table 11: Regressions for the 25 Size-IVOL portfolios, July 1963 to December 2017

IVOL→	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor $R_{i,t} - R_{F,t} = a_i + b_i(R_{M,t} - R_{F,t}) + s_i SMB_t + h_i HML_i + \epsilon_{i,t}$										
			a					t		
Small	0.38	0.32	0.10	-0.19	-1.24	5.27	4.50	1.39	-2.21	-8.38
2	0.28	0.24	0.19	0.03	-0.73	4.21	3.45	2.60	0.39	-7.59
3	0.17	0.20	0.12	0.08	-0.46	2.54	2.86	1.72	1.09	-4.92
4	0.19	0.15	0.07	0.04	-0.33	2.42	1.95	0.95	0.55	-3.23
Big	0.09	0.10	0.02	-0.06	-0.09	1.56	1.91	0.38	-0.99	-0.98
Panel B: Five-factor $R_{i,t} - R_{F,t} = a_i + b_i(R_{M,t} - R_{F,t}) + s_i SMB_t + h_i HML_i + r_i RMW_i + c_i CMA_i \epsilon_{i,t}$										
			a					$t(a)$		
Small	0.27	0.18	0.04	-0.10	-0.87	3.75	2.51	0.51	-0.92	-5.58
2	0.14	0.06	0.02	-0.08	-0.47	2.28	0.97	0.34	-1.11	-5.47
3	0.02	0.04	-0.05	-0.05	-0.24	0.39	0.61	-0.75	-0.64	-2.82
4	0.03	-0.03	-0.09	-0.07	-0.10	0.45	-0.37	-1.30	-0.93	-1.03
Big	0.01	-0.03	-0.09	-0.05	0.13	0.12	-0.70	-1.73	-0.85	1.47
			b					$t(b)$		
Small	0.71	0.98	1.09	1.15	1.13	35.33	45.14	38.44	29.94	21.23
2	0.78	1.01	1.12	1.24	1.27	45.81	55.71	54.43	49.49	34.67
3	0.79	0.99	1.10	1.19	1.25	46.98	45.47	51.64	47.36	39.90
4	0.82	0.99	1.13	1.20	1.26	36.32	42.72	52.48	49.01	42.67
Big	0.83	0.97	1.05	1.11	1.18	56.81	66.15	75.36	70.02	46.35
			s					$t(s)$		
Small	0.67	0.93	1.04	1.19	1.36	25.23	26.08	22.93	18.91	16.49
2	0.56	0.75	0.83	0.94	1.12	23.05	27.38	24.25	22.17	24.06
3	0.31	0.47	0.57	0.69	0.82	12.60	13.93	16.44	16.63	19.65
4	0.10	0.18	0.23	0.32	0.51	3.16	4.99	6.16	8.30	13.90
Big	-0.28	-0.25	-0.17	-0.12	0.03	-13.10	-12.68	-7.30	-4.84	0.74
			h					$t(h)$		
Small	0.40	0.35	0.34	0.28	0.22	9.69	6.74	4.49	2.82	1.92
2	0.34	0.33	0.26	0.18	-0.07	10.51	7.18	4.92	2.86	-0.88
3	0.33	0.35	0.28	0.17	-0.16	8.77	6.67	5.24	2.72	-2.50
4	0.30	0.26	0.21	0.12	-0.18	5.63	4.57	3.92	2.27	-3.23
Big	0.12	-0.04	0.05	0.06	-0.18	3.90	-1.51	1.40	1.69	-3.28
			r					$t(r)$		
Small	0.28	0.35	0.17	-0.15	-0.83	4.93	5.21	1.97	-1.31	-6.31
2	0.32	0.42	0.41	0.32	-0.53	8.90	7.80	6.36	4.17	-7.79
3	0.31	0.41	0.43	0.35	-0.45	7.57	6.54	7.29	5.43	-8.00
4	0.31	0.40	0.37	0.27	-0.51	6.50	6.62	6.27	4.72	-8.81
Big	0.18	0.26	0.25	0.00	-0.42	4.59	7.23	5.55	0.11	-7.72
			c					$t(c)$		
Small	0.07	0.08	-0.02	-0.22	-0.42	1.27	1.32	-0.23	-1.71	-2.05
2	0.14	0.15	0.10	-0.03	-0.39	3.10	3.04	1.88	-0.50	-4.12
3	0.19	0.05	0.10	0.02	-0.32	3.66	0.96	1.94	0.33	-3.68
4	0.21	0.15	0.12	0.07	-0.26	2.97	2.51	1.96	1.00	-2.81
Big	0.08	0.22	0.12	-0.05	-0.35	1.55	4.10	2.46	-1.03	-4.13

The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 Size-IVOL portfolios, The RHS variable are the excess market return $R_m - R_F$, the size factor, SMB, the value factor, HML, the profitability factor, RMW, the investment factor, CMA and IVFL: the return spread in univariate sort on conditional long-run idiosyncratic volatility. Panel A shows intercepts from the FF-three factor model and Panel B shows five-factor intercepts and slopes. The sample period is July 1963 to December 2017.

Table 12: Regressions for the 25 Size-IVOL portfolios, July 1963 to December 2017

Five-factor plus IVFL: $R_{i,t} - R_{F,t} = a_i + b_i(R_{M,t} - R_{F,t}) + s_i SMB_t + h_i HML_i + r_i RMW_i + c_i CMA_i + v_i IVFL_i \epsilon_{i,t}$										
			a					$t(a)$		
Small	0.22	0.17	0.08	0.01	-0.61	3.16	2.22	0.94	0.07	-4.16
2	0.06	0.00	-0.01	-0.07	-0.31	1.03	0.01	-0.17	-0.90	-3.78
3	-0.06	-0.01	-0.07	-0.05	-0.11	-0.94	-0.19	-1.01	-0.66	-1.33
4	-0.05	-0.08	-0.11	-0.06	0.06	-0.69	-1.15	-1.64	-0.77	0.70
Big	-0.06	-0.08	-0.10	-0.04	0.27	-1.09	-1.52	-1.87	-0.64	3.59
			b					$t(b)$		
Small	0.74	0.99	1.07	1.08	0.95	37.13	50.05	43.07	33.55	20.11
2	0.83	1.05	1.14	1.23	1.16	51.98	62.35	60.54	53.09	45.71
3	0.84	1.02	1.11	1.20	1.16	53.03	54.07	51.63	46.18	46.55
4	0.88	1.03	1.15	1.19	1.16	39.58	48.14	55.68	48.67	46.25
Big	0.88	1.00	1.06	1.10	1.08	60.09	66.34	74.71	70.90	53.37
			s					$t(s)$		
Small	0.77	0.97	0.96	0.96	0.77	15.14	16.24	14.29	11.83	9.14
2	0.74	0.88	0.91	0.91	0.77	17.39	18.04	16.40	15.46	12.99
3	0.49	0.59	0.61	0.70	0.51	11.45	9.88	11.74	13.69	10.86
4	0.29	0.30	0.28	0.30	0.17	5 6.09	5.52	5.54	6.42	3.01
Big	-0.13	-0.15	-0.14	-0.15	-0.30	-4.57	-5.93	-3.92	-3.80	-5.09
			h					$t(h)$		
Small	0.38	0.34	0.35	0.33	0.34	8.10	6.03	4.67	3.45	3.52
2	0.30	0.30	0.25	0.19	0.00	8.36	5.88	4.23	2.90	0.06
3	0.29	0.32	0.27	0.17	-0.10	7.39	5.58	4.86	2.71	-1.89
4	0.26	0.23	0.20	0.12	-0.11	4.89	3.92	3.62	2.39	-2.13
Big	0.09	-0.06	0.04	0.07	-0.12	3.23	-2.35	1.24	1.85	-2.10
			r					$t(r)$		
Small	0.15	0.30	0.28	0.15	-0.09	1.68	3.08	2.52	1.20	-0.75
2	0.10	0.26	0.31	0.36	-0.08	1.38	2.88	3.24	3.77	-0.91
3	0.08	0.26	0.38	0.35	-0.06	1.09	2.58	4.78	4.91	-0.90
4	0.07	0.25	0.31	0.30	-0.08	0.97	2.81	4.14	4.93	-0.97
Big	-0.00	0.14	0.23	0.04	-0.01	-0.09	3.86	4.47	0.83	-0.06
			c					$t(c)$		
Small	-0.00	0.05	0.04	-0.06	-0.01	-0.04	0.86	0.42	-0.50	-0.04
2	0.01	0.06	0.05	-0.01	-0.14	0.30	1.21	0.94	-0.14	-1.94
3	0.06	-0.03	0.07	0.02	-0.10	1.36	-0.54	1.36	0.28	-1.41
4	0.08	0.07	0.08	0.09	-0.01	1.16	1.20	1.41	1.18	-0.20
Big	-0.02	0.15	0.10	-0.03	-0.11	-0.53	3.12	2.21	-0.62	-1.74
			v					$t(v)$		
Small	-0.08	-0.03	0.07	0.19	0.48	-3.08	-1.15	1.91	4.07	9.89
2	-0.15	-0.11	-0.06	0.03	0.29	-6.62	-4.27	-2.52	0.83	6.11
3	-0.15	-0.10	-0.04	-0.00	0.26	-7.09	-3.56	-1.48	-0.12	8.20
4	-0.16	-0.10	-0.04	0.02	0.28	-6.89	-4.11	-2.02	0.82	9.72
Big	-0.12	-0.08	-0.02	0.02	0.27	-8.04	-5.91	-0.96	1.28	9.12

The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 Size-IVOL portfolios, The RHS variable are the excess market return $R_m - R_F$, the size factor, SMB, the value factor, HML, the profitability factor, RMW, the investment factor, CMA and IVFL: the return spread in univariate sort on conditional long-run idiosyncratic volatility. The sample period is July 1963 to December 2017.

Table 13: Summary statistics for tests of three, five and five-plus-IVFL factor models, July 1963-December 2017

Model Factors	GRS	$p(GRS)$	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{Aa_i^2}{A\bar{r}_i^2}$	$\frac{As^2(a_i)}{A\|\bar{r}_i\|}$	$A(R^2)$
Mkt SMB HML	6.78	0.0	0.23	0.77	0.92	0.05	0.88
Mkt SMB HML RMW CMA	5.57	0.0	0.13	0.43	0.37	0.12	0.9
Mkt SMB HML RMW CMA IVFL	5.03	0.0	0.11	0.36	0.21	0.2	0.91

This table tests how well three, five, five plus IVFL factors explain monthly excess returns on the 25 Size-$IVOL$ portfolios. The table shows (1) the GRS statistics testing whether the expected values of all 25 intercept estimates are zero; (2) $p(GRS)$, the p-value for the GRS statistic (3) the average absolute value of the intercepts, $A|a_i|$ (4) the average absolute value of the intercepts over the average absolute value of $\bar{r}_i$, which is the average excess returns on portfolio i minus the average market portfolio excess returns, (5)$Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$ (6)$As^2(a_i)/A|\bar{r}_i|$, the average of the estimates of the variances of the sampling errors of the estimated intercepts over $A\bar{r}_i^2$ and (7)$A(R^2)$, the average value of the regression R^2 corrected for degrees of freedom. The sample period is July 1963 to December 2017.