CH7/8: Global Consensus ADMM Lasso

Friday, April 26, 2024 8:56 PM

GCP STANDARD FORM:

minimize : $\sum_{i=1}^{N} f_i(x_i) \rightarrow x_1 = x_2 = \dots = x_N$

subject to: $x_i - z = 0, \quad i = 1, \dots, N$

Each xi is local to each node

x_i = local variable

All of the local variables should agree after convergence.

Dual problem

- convex equality constrained optimization problem

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & Ax = b \end{array}$$

- Lagrangian: $L(x, y) = f(x) + y^T(Ax - b)$
- dual function: $g(y) = \inf_x L(x, y)$
- dual problem: maximize $g(y)$
- recover $x^\star = \operatorname{argmin}_x L(x, y^\star)$

ADMM:

$$L_\rho(x_1, \dots, x_N, z, y) = \sum_{i=1}^{N}\Big(f_i(x_i) + y^T(x_i - z) + \underbrace{\frac{\rho}{2}\|x_i - z\|_2^2}_{\text{Aug Lagrange}}\Big)$$

$$x_i^{k+1} = \operatorname*{argmin}_{x_i} L_\rho(x_i, z, y_i)$$

$$= \operatorname*{argmin}_{x_i}\Big(f_i(x_i) + y^T(x_i - z) + \frac{\rho}{2}\|x_i - z\|_2^2\Big)$$

$$z^{k+1} = \frac{1}{N}\sum_{i=1}^{N}\Big(x_i^{k+1} + \frac{1}{\rho}y_i^k\Big)$$

$$y_i^{k+1} = y_i^k + \rho\big(x_i^{k+1} - z^{k+1}\big)$$

SIMPLIFIED USING AVERAGES:

$$z^{k+1} = \bar{x}^{k+1} + \Big(\frac{1}{\rho}\Big)\bar{y}^k$$

$$\bar{y}^{k+1} = \bar{y}^k + \rho\big(\bar{x}^{k+1} - z^{k+1}\big)$$

$\Big\} \quad \underline{\bar{y}^{k+1} = 0}$

$z^k = \bar{x}^k$

Meaning, the dual variables have average zero after the first iteration.

$$\Rightarrow x_i^{k+1} = \operatorname{argmin}\Big(f_i(x_i) + y^T(x_i - z) + \frac{\rho}{2}\|x_i - z\|^2\Big)$$

$$y = y + \rho(x - z) \qquad z = \bar{x}$$ iteration.

$$x_i^{k+1} = \arg\min_{x_i}\left(f_i(x_i) + y^T(x_i - z) + \frac{\rho}{2}\|x_i - z\|_2^2\right)$$

$$= \arg\min_{x_i}\left(f_i(x_i) + y^T(x_i - \bar{x}^k) + \frac{\rho}{2}\|x_i - \bar{x}^k\|_2^2\right)$$

$$y_i^{k+1} = y_i^k + \rho\left(x_i^{k+1} - \bar{x}^{k+1}\right)$$