CH3.1: ADMM

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- PROBLEM STANDARD FORM:

3.1

$$\begin{aligned} &\text{minimize} \quad f(x) + g(z) \\ &\text{subject to} \quad Ax + Bz = c \end{aligned}$$

$x \in \mathbb{R}^n$, $z \in \mathbb{R}^m$,

$A \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{p \times m}$, $c \in \mathbb{R}^p$

Assume f and g are convex.
Compared to the linear equality constrained problem (2.1), x is now split into x and z with objective function f separable across the splitting.

$$p^* = \inf \{ f(x) + g(z) \mid Ax + Bz = c \}$$

$$L_\rho(x, z, y) = f(x) + g(z) + y^T(Ax + Bz - c) + \frac{\rho}{2} \|Ax + Bz - c\|_2^2$$

- ADMM ALGORITHM $\rho > 0$

$$x^{k+1} := \operatorname{argmin}_x L_\rho(x, z^k, y^k) \qquad \text{x-min step}$$

$$z^{k+1} := \operatorname{argmin}_z L_\rho(x^{k+1}, z, y^k) \qquad \text{z-min step}$$

$$y^{k+1} := y^k + \rho(Ax^{k+1} + Bz^{k+1} - c) \qquad \text{dual-var update}$$

Method of Multipliers:

$$(x^{k+1}, z^{k+1}) := \operatorname{argmin}_{x,z} L_\rho(x, z, y^k)$$

$$y^{k+1} := y^k + \rho(Ax^{k+1} + Bz^{k+1} - c)$$

In the method of multipliers, the augmented Lagrangian is minimized jointly with respect to the primal variables x and z.
In ADMM, x and z are updated in alternating (sequential) fashion, hence "alternative direction".
ADMM is a version of the method of multipliers