HPWL_AP_2012

Thursday, July 18, 2024 4:38 PM

ANALYTICAL PLACEMENT FOR HETEROGENEOUS FPGAS

Marcel Gort and Jason H. Anderson

Dept. of Electrical and Computer Engineering
University of Toronto
email: {gortmarc, janders}@eecg.toronto.edu

2.2) ANALYTICAL PLACEMENT

- HPWL over all nets.
- Original netlist is a hypergraph.
  - Nets can have one source and multiple sinks. Mathematically a hyperedge.
- All multi-pin nets (hyperedges) are converted into a set of 2-pin connections (edges).
  - Can use star model, clique model, etc.
- Assume a clique net model.

OBJECTIVE FUNCTION:

$$\Phi(x,y) = \sum_{ij} w_{ij} \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right] \qquad (1)$$

w_{ij} : weight of connection between objects i and j

The objective function can be separated into x and y components and cast into matrix form.

x-component:

$$\Phi(x) = \frac{1}{2} x^T Q_x x + c_x^T x + \text{const} \qquad (2)$$

Q_x : connections between movable objects

c_x : connections between movable objects and fixed objects

This is a 2nd degree polynomial.
Minimize it by taking the partial derivative and setting to zero.

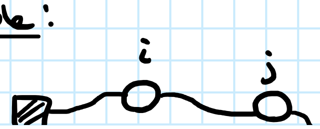
$$0 = \nabla \Phi(x)$$

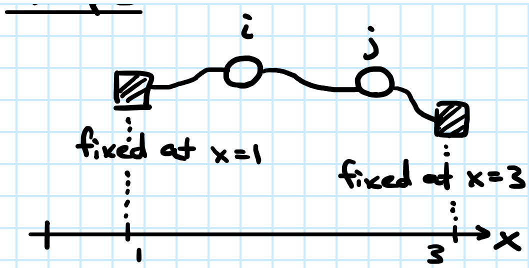
$$0 = Q_x x + c_x^T$$

$$Q_x x = -c_x^T$$

Can be solved with any off-the-shelf lineq solver.
Once solved, x and y hold the x-y coordinates for the movable objects.

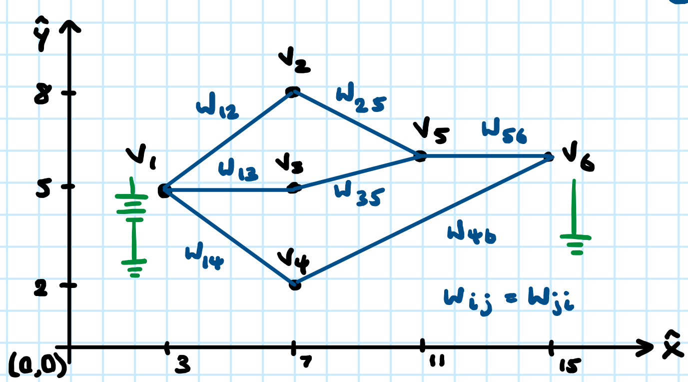
Example:





$$\Phi_x = (x_i - 1)^2 + (x_i - x_j)^2 + (x_j - 3)^2$$

$$\left.\begin{aligned}
0 &= \frac{\partial}{\partial x_i}\Phi_x \\
0 &= 2(x_i - 1) + 2(x_i - x_j) \\
0 &= \frac{\partial}{\partial x_j}\Phi_x \\
0 &= -2(x_i - x_j) + 2(x_j - 3)
\end{aligned}\right\} \underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}}_{Q_x x = c_x}$$



v1 (3, 5) fixed
v2 (7, 8)
v3 (7, 5)
v4 (7, 2)
v5 (11, 6)
v6 (15, 6) fixed

x = [3, 7, 7, 7, 11, 15]
y = [5, 8, 5, 2, 6, 6]

$$W = \begin{array}{c} \\ i=1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} j=1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} \end{array}$$

$$\Phi(x, y) = \sum_{ij} \omega_{ij} \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]$$

$$\begin{aligned}
\Phi(x) &= \sum_{ij} \omega_{ij} (x_i - x_j)^2 \\
&= \sum_{ij} \omega_{ij} (x_i^2 - 2x_i x_j + x_j^2) \\
&= \sum_{ij} \omega_{ij} x_i^2 - 2\sum_{ij} \omega_{ij} x_i x_j + \sum_{ij} \omega_{ij} x_j^2
\end{aligned}$$

The first and third summations are the same because i and j iterate over the same numbers. Summing xi and summing xj are essentially equivalent.

$$x^T Q x = \sum_{i=1}^{n} \sum_{j=1}^{m} x_i Q_{ij} x_j \quad \in \mathbb{R}$$

$$c^T x = \sum_{i=1}^{n} c_i x_i \quad \in \mathbb{R}$$

$$\Phi(x) = \frac{1}{2} x^T Q x + c^T x + \text{const}$$