Markowitz’s Portfolio Optimization Analysis: Harnessing AI Tech Growth and Diversifying Risk

Name: **Tang Chen Tin**

Executive Summary:

In an era defined by rapid advancements in artificial intelligence and its significant impact on the technology sector, this report delves into constructing a portfolio that leverages Markowitz's Portfolio Optimization Model to strategically emphasize tech stocks, while also maintaining a diversified approach with allocations in other sectors. The goal is to deliver a superior performance in comparison to both the S&P 500 benchmark and an equally weighted portfolio. This analysis encompasses a 36-month performance period, divided into training and test phases, to assess and validate the robustness of the portfolio. Moreover, the report explores a combined portfolio approach, integrating the strengths of both the optimal portfolio and the equally weighted portfolio (1/N) strategy. This synthesis aims to harness the growth prospects of tech stocks while safeguarding against the inherent risks associated with a solely optimal portfolio. The executive summary encapsulates the methodologies, significant insights, and essential performance indicators that demonstrate the advantage of an optimized, yet cautiously diversified investment strategy in a volatile market environment.

1. BACKGROUND

The backdrop of this project is the remarkable surge in AI technologies in 2023 and 2024, leading to significant growth in tech industry stocks and contributing to an overall 23% increase in S&P500 indices in 2023. This scenario underscores the market's bullish sentiment towards innovative tech firms and the broader implications for investment strategies.

1.1. MARKOWITZ MODEL

The Markowitz Model, pioneered by Harry Markowitz in his seminal 1952 work, "Portfolio Selection," provides a quantitative framework for portfolio diversification, emphasizing the balance between risk and return (Gajendrakar, 2024). By considering the covariance among stocks, the model identifies the efficient frontier—portfolios offering the highest expected return for a given level of risk (Investopedia, 2024).

1.2. OBJECTIVE

The objective of this project is to discover a strategic investment portfolio that offers a favourable risk-reward ratio, targeting investors seeking slightly higher risk for potential higher returns without venturing into ultimate high-risk investments through strategic allocation. This is done by leveraging Markowitz's Portfolio Optimization Model to construct an optimal portfolio predominantly concentrated on technology stocks, while ensuring diversification across other sectors, then compare its performance with an equally weighted portfolio (1/N strategy) and the market benchmark, SPDR S&P 500 ETF Trust (NYSEARCA: SPY) to evaluate performance. Additionally, this project will explore the performance of a portfolio that is equally allocated between the optimal portfolio (50%) and the 1/N strategy (50%), to assess how hybrid allocation strategies can further optimize returns and manage risks.

2. FIRM DESCRIPTION

Company Name	Ticker	Sector	Rationale of Selection
Apple Inc.	AAPL	Technology	A leading innovator in consumer electronics, Apple's expansive product ecosystem and brand loyalty underscore its growth potential, aligning with technological advancement trends (Yahoo Finance, 2024).
Microsoft Corporation	MSFT	Technology	Dominating the software industry with a broad portfolio, Microsoft's ventures into cloud computing and AI technologies position it as a growth-centric investment (Yahoo Finance, 2024).
NVIDIA Corporation	NVDA	Technology	At the forefront of the semiconductor industry, NVIDIA's pivotal role in gaming, data centres, and AI innovation drives its growth prospects (Yahoo Finance, 2024).
Super Micro Computer	SMCI	Technology	Specializing in high-performance server technology, SMCI's contributions to data centres and cloud infrastructure mark it as a key player in tech hardware growth (Yahoo Finance, 2024).
Visa Inc.	V	Technology	As a global leader in digital payments, Visa's expansive network and adoption of payment innovations reflect its growth trajectory in fintech (Yahoo Finance, 2024).
Mastercard Incorporated	MA	Technology	Parallel to Visa, Mastercard's continual expansion in digital payment technologies and services underscores its position as a growth entity in the evolving financial sector (Yahoo Finance, 2024).
Procter & Gamble	PG	Consumer Goods	A staple in consumer goods, Procter & Gamble's diverse portfolio and brand strength offer defensive stability amidst market volatility (Yahoo Finance, 2024).
Johnson & Johnson	JNJ	Healthcare	In healthcare, JNJ's blend of pharmaceuticals, medical devices, and consumer health products provides defensive resilience (Yahoo Finance, 2024).
Berkshire Hathaway Inc Class B	BRK-B	Financial Services	Warren Buffett's conglomerate, with its vast portfolio of financial services, insurance, and other investments, offers a defensive hedge with potential for steady growth (Yahoo Finance, 2024).
JPMorgan Chase & Co.	JPM	Financial Services	A leading global financial services firm, JPMorgan's extensive operations in banking, asset management, and financial services fortify its defensive position in financial markets (Yahoo Finance, 2024).

3. METHODOLOGY

3.1. Data Collection

3.1.1. **Data Acquisition**

Historical adjusted price data for the selected stocks and SPY, an ETF tracking the S&P 500 index, along with the risk-free rate as indicated by the 13-week Treasury bill rate (referred to as "^IRX"), are obtained for a period of approximately 36 months from **22 April 2021 to 5 April 2024**. These data are sourced from financial databases and accessed using the 'quantmod' package in R.

3.1.2. **Data Segmentation**

The collected dataset is divided into two parts. The initial 24 months of data, from 22 April 2021 to 21 April 2023, serve as the training set for developing the investment strategy. The subsequent 12 months, from 24 April 2023 to 5 April 2024 form the test set used to evaluate the strategy's performance. Using the training set, the Markowitz

Portfolio Optimization Model is applied to determine the strategic allocation of assets in the portfolio, aiming to maximize expected returns for a given level of risk. The test set is then used to assess the effectiveness of the optimized portfolio compared to a simple 1/N strategy and the performance of the S&P 500 index.

3.2. Estimation

3.2.1. Estimating Key Parameters

By calculating the daily returns, we estimate two crucial statistical measures for portfolio construction:

The **expected return** (denoted as μ), which represents the mean daily return for each stock over the training set period. Stocks like AAPL, MSFT, NVDA, and SMCI show positive expected daily returns, indicating an average increase in their value during the training period. Stocks like MA and JPM show a slight negative expected return, suggesting a slight average decrease in their value during the training period.

The **covariance matrix** Σ provides a measure of how returns on two stocks move together. A positive number indicates that the stocks tend to move in the same direction, while a negative number means they move in opposite directions. For example, the highest covariance among the pairs is between NVDA and MSFT, which is 0.00046752, implying that these stocks may move strongly in the same direction and may not provide as much diversification benefit when combined in a portfolio.

3.3. Construction of Portfolio

3.3.1. Minimum-Risk Portfolio and Zero-Covariance Portfolio

The process begins by determining the minimum-risk portfolio (w_m), which aims for the lowest volatility based on historical data. Concurrently, we compute the zero-covariance portfolio (w_z), which is designed to have the least possible co-movement with other assets in the portfolio.

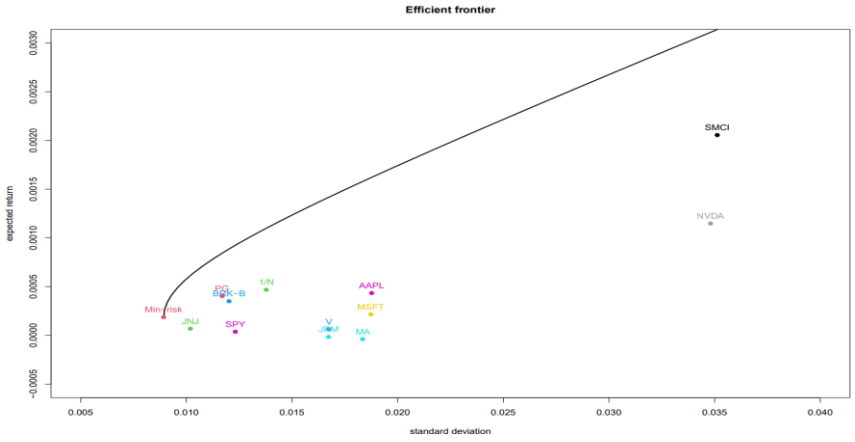
3.3.2. Optimal Portfolios

Based on Markowitz's optimization theory, we calculate optimal portfolios (w_{opt}) by mixing the minimum-risk portfolio (w_m) with the zero-covariance portfolio (w_z), adjusted by varying levels of risk aversion through the parameter gamma (γ).

$$w_{opt} = \frac{1}{\gamma} \cdot w_z + w_m$$

Since actual market parameters, expected returns (μ), and the covariance matrix (Σ) are not precisely known, we substitute these with our best estimates from historical data, known as "plug-in" estimates. The portfolios thus derived offer a spectrum of risk-reward balances, shaped by the range of γ, aiming to construct a series of portfolios from highly risk-averse (higher γ) to less risk-averse (lower γ). This technique allows us to explore the efficient frontier practically, providing insights into how different risk preferences could have shaped portfolio choices.

3.4. In-Sample Efficient Frontier



We plot the efficient frontier by illustrating all possible combinations of assets in our chosen set that provide the maximum expected return for a given level of risk in the (σ, μ) plane. The minimum-risk portfolio, represents the combination of assets with the lowest possible risk for a given level of return, is distinctly marked on this graph.

Additionally, we analyse the SPY's historical performance as it serves as our market benchmark. We can observe the SPY has lower σ and μ when compared to technology stocks. Growth stocks like "NVDA" and "SMCI" have higher expected returns but also higher risks are located on the right side of the efficient frontier while Defensive stocks like "JNJ" and "PG" generally have lower expected returns but also lower risk are located on the left side.

Furthermore, we calculate the 1/N strategy, an equally weighted portfolio. In our case, with 10 stocks, each stock constitutes 10% of the total portfolio. The performance of this strategy is also highlighted in the (σ, μ) plane. We can see that this strategy

falls below the efficient frontier, it suggests that even though it is diversified, it is not risk optimized. For the same level of risk, Optimal Portfolio could offer a higher expected return. The optimal portfolio, according to Markowitz's theory, is located on the efficient frontier. It is the set of portfolios that offer the highest expected return for a given level of risk.

4. RESULTS

For the optimal portfolio, a risk aversion coefficient (γ) of 2 is utilized, reflecting a moderate risk aversion stance that is willing to take on some risk for a reasonable expected return but is not seeking highly risky investments. Below is the optimal portfolio by illustrating individual stocks' weightage.

Ticker	V	MA	AAPL	MSFT	NVDA	SMCI	PG	JNJ	BRK-B	JPM
Weight	0.3705	-1.4864	0.3202	-1.1685	0.5904	0.8422	2.0387	-1.2408	2.3363	-1.6026

Next, we derive key performance indicators (KPIs) for each portfolio and the S&P 500 benchmark. These KPIs include mean daily return, standard deviation, cumulative return, Alpha, Beta, Sharpe ratio and Treynor's ratio. Alpha (α) measures the active return on an investment compared to the market benchmark index (SPY) while Beta (β) reflects the sensitivity of a portfolio's returns to the returns of the market benchmark index (SPY). The Sharpe ratio compares the excess return per unit of deviation, providing insight into the risk-adjusted return. Treynor's ratio uses beta to measure volatility against the market.

4.1. In-Sample Performance Evaluation

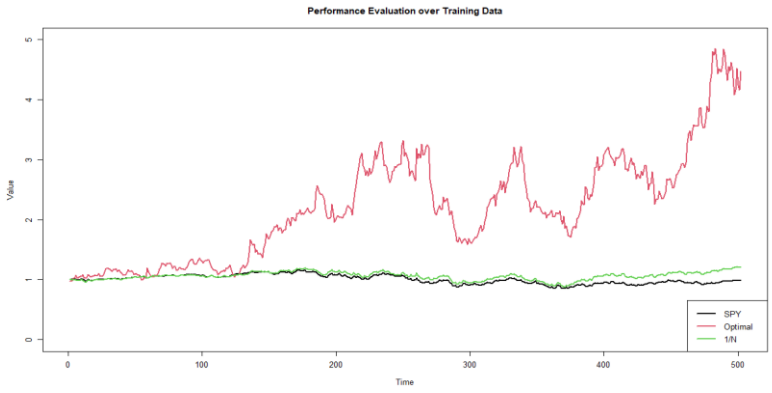
In evaluating the in-sample performance of constructed portfolios, we are doing a comparison of KPIs between SPY, Optimal Portfolio and 1/N Portfolio as shown in the table below.

Portfolio	Mean Daily Return	Standard Deviation	Cumulative Return	Alpha	Beta	Sharpe Ratio	Treynor Ratio
SPY	0.00003769	0.0123195	-1.897%	0.00000000	1.000000	-0.00245741	-0.00003027
Optimal	0.00396007	0.0443412	347.062%	0.00391994	0.919240	0.08777634	0.00423405
1/N	0.00046817	0.0137815	20.617%	0.00043210	1.053326	0.02903935	0.00037995

Interpretation of performance comparison:

Metrics	Comment
Mean Daily Return	Optimal portfolio has a significantly higher mean return compared to SPY and 1/N. The significant outperformance could be due to a concentration of stocks that benefited greatly from the post-COVID-19 recovery and technology boom.
Standard Deviation	Optimal portfolio is significantly riskier than SPY and 1/N.
Cumulative Return	Optimal portfolio has outperformed both SPY and 1/N by a large margin.
Alpha	Optimal portfolio has a positive alpha, it has outperformed the market on a risk-adjusted basis.
Beta	Optimal portfolio's beta is less than 1, indicating it is less volatile than the market, whereas 1/N is slightly more volatile. It might be the result of specific risk exposures of the Optimal portfolio (tech-focused) that are different from those driving the market index. In this case, SPY exposed to the risk of market's recovery post-COVID-19 and the emergence of AI technologies.
Sharpe Ratio	Optimal portfolio is positive and higher than SPY and 1/N, indicating better risk-adjusted returns.
Treynor Ratio	Optimal portfolio has higher Treynor ratio, shows better portfolio's return when considering systematic risk.

Besides, we graphically represent the performance evaluation over the training data period, allowing for a visual comparison of the optimal portfolio against both the market index and the equally weighted portfolio using cumulative returns which reflects the compounding effect of gains or losses over time.



According to the graph, both Optimal and 1/N Portfolios are providing returns that beat the market average. We can observe that the optimal portfolio has achieved higher returns, due to better risk-adjusted allocations, but also with higher volatility over time as tech sector having gone through interest rates hike in 2022 which is highly sensitive to tech sector. There is a noticeable surge in the performance of our optimal portfolio around early 2023. This aligns with the initial excitement around AI advancements when ChatGPT first released, suggesting our portfolio capitalized on this momentum.

4.2. Out-of-Sample Performance Evaluation

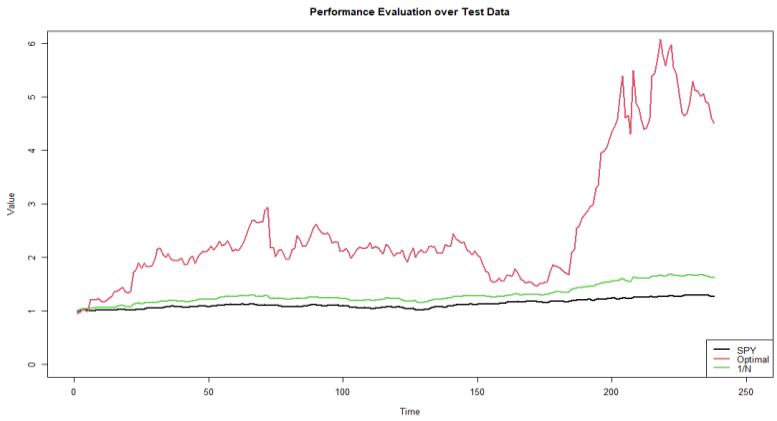
Below is a table of comparison of KPIs between SPY, Optimal Portfolio and 1/N Portfolio.

Portfolio	Mean Daily Return	Standard Deviation	Cumulative Return	Alpha	Beta	Sharpe Ratio	Treynor Ratio
SPY	0.00109	0.00729	28.69%	0.00000	1.0000	0.1207	0.00088
Optimal	0.00806	0.05945	351.71%	0.00560	2.5580	0.1321	0.00307
1/N	0.00213	0.01061	63.92%	0.00094	1.1245	0.1816	0.00171

Interpretation of performance comparison:

Metrics	Comment
Mean Daily Return	Optimal portfolio has a significantly higher mean return compared to SPY and 1/N. Given the period was marked by significant advances in AI, tech stocks saw substantial gains which the Optimal portfolio was positioned to capture.
Standard Deviation	Optimal portfolio is significantly riskier than SPY and 1/N.
Cumulative Return	Optimal portfolio has outperformed both SPY and 1/N by a large margin due to tech boom.
Alpha	Optimal portfolio has a positive alpha, indicating it has outperformed the market on a risk-adjusted basis.
Beta	Optimal portfolio's beta is more than 1, indicating it's much more volatile than the market. Test data includes a period where tech stocks experienced significant growth. This would result in a higher beta for a portfolio focused on tech stocks during that time, reflecting that increased risk and sensitivity to market movements compared to the training period.
Sharpe Ratio	1/N is higher than both SPY and Optimal, indicating better risk-adjusted returns.
Treynor Ratio	Optimal portfolio has higher Treynor ratio, shows better portfolio's return when considering systematic risk. This period was characterized by significant technological advancements (such as the introduction of ChatGPT 4.0), it is possible that the broader tech sector saw higher returns, benefiting the 1/N portfolio if it was more heavily weighted towards tech stocks.

Below is the graph represent the performance evaluation over the training data period, a visual comparison of the optimal portfolio against both the market index and the equally weighted portfolio using cumulative returns.



According to the graph, we can observe that the optimal portfolio has achieved higher returns, due to better risk-adjusted allocations that captured growth opportunities more effectively. Both Optimal and 1/N Portfolios are providing returns that beat the market average. The test data reveals a significant uptick starting in October 2023. It is reasonable to infer that our optimal portfolio benefited from key developments in AI technology, such as enhanced GPU capabilities and the roll-out of GPT-4.0, demonstrating the portfolio's responsiveness to technological innovation and market trends.

5. FURTHER EVALUATION

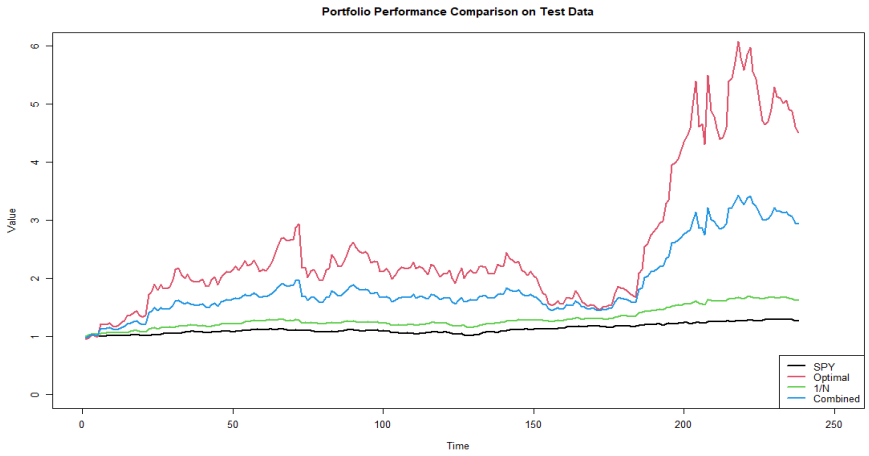
5.1. Combined Portfolio

The methodology of creating a Combined Portfolio with 50% allocation to an optimal portfolio and 50% to a 1/N portfolio is a blend strategy. It aims to balance the theoretical optimization of the Markowitz model with the practical simplicity and diversification of an equally weighted approach. Below is the table of comparison of KPIs.

Portfolio	Mean Daily Return	Standard Deviation	Cumulative Return	Alpha	Beta	Sharpe Ratio	Treynor Ratio
SPY	0.00109	0.00729	28.69%	0.00000	1.0000	0.1207	0.00088
Optimal	0.00806	0.05945	351.71%	0.00560	2.5580	0.1321	0.00307
Combined	0.00510	0.03370	194.20%	0.00327	1.8413	0.1451	0.00266
1/N	0.00213	0.01061	63.92%	0.00094	1.1245	0.1816	0.00171

By combining the two, we can capture a portion of the higher returns and sharper profile of the optimal portfolio, while potentially mitigating risk and volatility through the equal diversification of the 1/N portfolio. The reason that all the performance metrics for this blended portfolio fall between the optimal and 1/N portfolios is due to averaging the characteristics of both strategies.

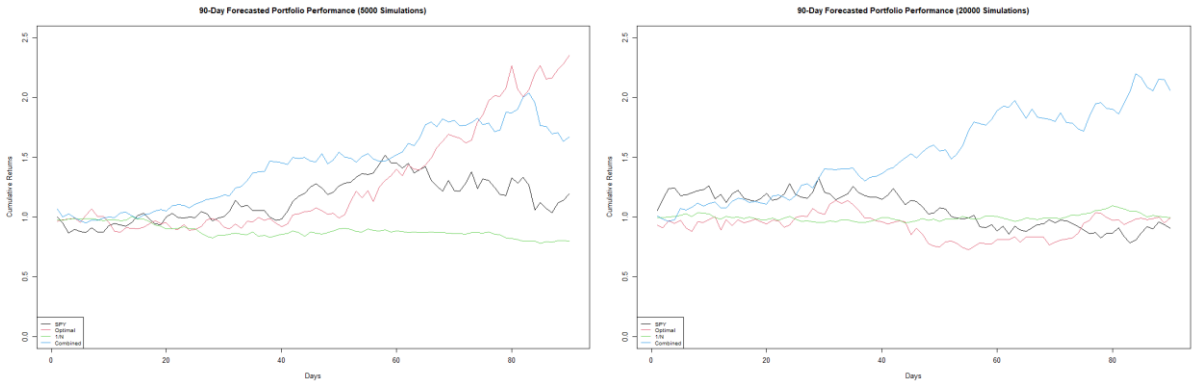
Below is the graph represent the performance evaluation over the test data period.



The combined portfolio offer a middle ground between the typically higher risk and return of the optimal portfolio and the more diversified and possibly lower risk of the 1/N strategy. This offers a moderate risk-return profile that seeks to leverage the strengths and hedge against the weaknesses of both strategies.

5.2. Monte Carlo Simulation

A Monte Carlo simulation is applied to evaluate the performance of SPY, Optimal, 1/N and Combine portfolios due to its ability to model the uncertainty inherent in financial markets. This method allows for a comprehensive view of potential risks and returns. In the context of Markowitz model backtesting, Monte Carlo simulations offer an additional layer of analysis, providing a forward-looking perspective that complements historical data, helping to inform better risk management and investment decision-making. The assumption made here are 1) stock returns are normally distributed, 2) expected returns are constant over time, and 3) return parameters are known (Lee, 2014). Below are the outcomes of Monte Carlo Simulation.



The "Combined" portfolio, which is a mix of the "Optimal" and "1/N" strategies, tends to demonstrate a middle-ground performance—exhibiting lower volatility than the "Optimal" but with potentially higher returns than the “Optimal” and "1/N". The "Optimal" portfolio displays higher peaks and troughs, indicating greater volatility and potential for higher returns. When running 20,000 simulations, "Optimal" portfolio may appear to perform the worst with lower cumulative returns and higher volatility in these simulations as it has a wider range of possible returns—both positive and negative—due to its higher risk profile, while the "Combined" portfolio performs better due to diversification benefits resulting in higher cumulative returns with higher volatility. Meanwhile, the SPY, representing the market, shows steady performance, and the "1/N" strategy maintains the most consistent and least volatile path, highlighting its role as a stabilizing element in the combined portfolio.

6. CONCLUSION

The combined portfolio emerges as a compelling strategy for investors aiming to strike a balance between risk and reward. By blending the optimal and equally weighted approaches, the combined portfolio harnesses the growth potential of high-performing assets while maintaining a cushion against market volatility. The Monte Carlo simulations suggests that the optimal portfolio may not always perform the best, as higher historical returns may come with increased risk and potential drawdowns. Consequently, the combined portfolio represents a prudent choice for investors seeking to participate in the market’s upside, yet are cautious about steep downturns, aiming for a moderate risk profile with a robust potential for appreciation.

REFERENCES

Gajendrakar, P. (2024, April 4). Markowitz model. WallStreetMojo. Retrieved from https://www.wallstreetmojo.com/markowitz-model/

Lee, M. I. (2014). Stress Testing Monte Carlo Assumptions. In O. S. Mitchell, R. Maurer, & P. B. Hammond (Eds.), Recreating Sustainable Retirement: Resilience, Solvency, and Tail Risk. Pension Research Council, Oxford University Press. https://doi.org/10.1093/acprof:oso/9780198719243.003.0004

The Investopedia Team. (2023, August 29). Modern portfolio theory (MPT). Investopedia. Reviewed by Scott, G. Fact checked by Kvilhaug, S. Retrieved from https://www.investopedia.com/terms/m/modernportfoliotheory.asp

Yahoo Finance. (n.d.). Stock Price, News, Quote & History. Retrieved April 14, 2024, from https://finance.yahoo.com/