

Predictive Distributions via Filtered Historical Simulation for Financial Risk Management

Tyson Clark

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Alexander (2008) Alexander (2009) Barone-Adesi, Giannopoulos, and Vosper (1999) Enders (2008)

Introduction

Investors often fail to account for risks included in their portfolios when things are going well. This was a major issue prior to the 2008 global financial crisis, and the crisis reminded many of the importance of evaluating risk. This was most apparent for investment banks that experienced large losses as a result of some risky assets in their portfolios that were not properly accounted for. Because of the importance of banks in our economy, there are regulations in place that require banks to stress-test their portfolios to evaluate the risk exposure.

Since financial data is not i.i.d. normal, it can be difficult to account for the periods of volatility when something such as the 2008 financial crisis occurs. In this paper, I intend to show that a symmetric GARCH model when bootstrapped in a filtered historical simulation allows for this volatility clustering pattern and will provide more accurate VaR metrics for banks that are seeking to remain compliant with the 10-day, 1% significance VaR levels set by regulations established in Basel II.

In this paper I will show a model that banks have begun to use to meet Basel II requirements. Basel II requires banks using internal VaR models to evaluate their risk at the 99% significance level. It also requires banks to measure potential losses for a 10-day risk horizon. Filtered historical simulation is a semi-parametric approach that combines Monte Carlo simulation based on volatility clustering patterns with the empirical non-normal return distributions from historical data. This overcomes many issues of basic historical simulation by allowing it to find an h-day VaR where h is more than just a few days, and that as it is a non-parametric model, it will not account for the volatility currently prevailing in the market (Alexander (2009)). Filtered historical simulation becomes this semi-parametric model by combining an estimated GARCH model with the i.i.d. bootstrap. This allows the model to account for value correlations without restricting them over time. The GARCH process underlying this filtered historical simulation started as a less complex autoregressive conditional heteroskedasticity (ARCH) model invented by Robert Engle (Engle (1995)). This model as well as the GARCH model I will be using to further discuss filtered historical simulation are explained in detail by Enders (2008). The model was made to account for volatility clustering patterns as is often seen with financial data that will experience market shocks such as bubbles, where a model assuming constant variance is inappropriate. For comparison to the later shown GARCH model, the simplest form of the ARCH model is the ARCH(1) model denoted by $y_t = x_t'\beta + \epsilon_t$ where $\epsilon_t = u_t\sqrt{\alpha_0 + \alpha_1\epsilon_{t-1}^2}$ and $u_t \sim N(0,1)$. This ϵ_t term is what is allowing for the conditional heteroskedasticity.

This ARCH model has some notable weaknesses. It assumes that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks. α_t^2 must remain in certain intervals in certain instances which limits the ability of an ARCH model to allow for excess kurtosis. It also is likely to overpredict the volatility because they respond slowly to large isolated shocks to the series.

Due to some of these limitations, the generalized autoregressive conditional heteroskedasticity (GARCH) model was introduced by Bollerslev (1986). The GARCH model extends Engle's ARCH model by allowing the conditional variance to be an autoregressive moving average (ARMA) process. This allows for both autoregressive and moving average components in the heteroskedastic variance. The basis GARCH(1,1) model can be denoted as $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$. We can see that if this last term in the equation is dropped out, we are left with our ARCH model from before.

It is easy to see here in the introduction to filtered historical simulation that all of these models and

computational techniques are all classical, there is a very Bayesian feel to the overall method, especially concerning the results that are produced. Through the statistical bootstrap of the GARCH process, we produce a predictive distribution that shows a distribution of the VaR for an h-day horizon. As these are all classical techniques, filtered historical simulation is rarely, if ever, looked at from a Bayesian perspective. This is a unique approach that I would like to apply to filtered historical simulation, and the interpretations that can be made thereafter.

As mentioned previously, VaR is a very important metric for banks to consider. As such, many different methods of calculating VaR have appeared. I will highlight three methods mentioned by Alexander (2009) in her book ‘Market Risk Analysis: Volume IV’: normal linear VaR, historical simulation, and Monte Carlo simulation. Again, as stated previously, filtered historical simulation uses aspects of historical simulation and Monte Carlo simulation that allow it to be a semi-parametric model that allows it to be the optimal model for banks to calculate their h-day VaR models.

I will only briefly mention normal linear VaR, as it is only appropriate when a portfolio P&L is a linear function of its risk factors or asset returns. As such, it is not ideal for almost any financial asset.

Historical simulation is the method that is the easiest to conceptualize. Most, if not all, investors have looked at previous return data when looking at their future investment decisions. In fact, I would assume that most novice investors believe this is the best method of estimating future returns. Historical simulation’s biggest issue is that it assumes that all possible future variations have occurred in the past. Looking back at each individual financial crisis, we can see that these models will vastly under-predict the possibility of some new market failure as a result of some issue that has not previously occurred.

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