

$\frac{\partial \psi}{\partial t} = \mp v \frac{\partial \psi}{\partial x}$ ,  $E = hf = \hbar \omega$ ,  $E = hf = \frac{hc}{\lambda} = \hbar c \kappa$ , 波数  $\kappa \equiv \frac{1}{\lambda}$ , 行波平面波函数  $\psi(\vec{r}) = A e^{i(\vec{k} \cdot \vec{r} - \omega t)}$ . 波速:  $\frac{dr_k}{dt} = \pm \frac{\omega}{k} = \pm v$ ,  $\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$

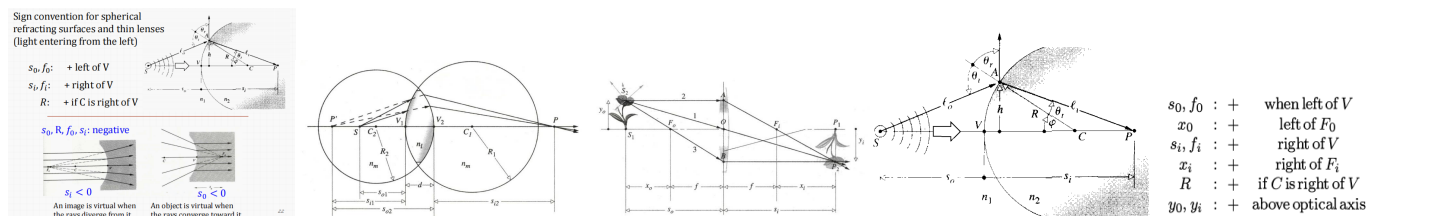
球坐标求导  $\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2}$  球谐波函数  $\psi(r, t) = \left( \frac{A}{r} \right) \cos[k(r \mp vt)]$ ,  $u_{\text{total}} = u_E + u_B = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2$ , 平行板电容器:  $E \cdot S = \frac{\sigma \cdot S}{\varepsilon_0}$ ,  $E = \frac{\sigma}{\varepsilon_0}$ ,  $\mu E = \frac{1}{2} \varepsilon_0 E^2 = \frac{\sigma^2}{2\varepsilon_0}$ ; 脉冲 **wavetrain** 的空间长度  $l = ct$ ; 对应能量  $\varepsilon = \frac{E}{IS(\text{面积})} = \frac{E}{\pi \frac{d^2}{4} l}$ ;  $S = \frac{uc \Delta t A}{\Delta t A} = uc = \frac{1}{\mu_0} EB$ ,  $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ ,  $\vec{S} = c^2 \varepsilon_0 \vec{E} \times \vec{B}$ ,  $I = \langle S \rangle_T = c^2 \varepsilon_0 |\vec{E}_0 \times \vec{B}_0| \langle \cos^2(\vec{k} \cdot \vec{r} - \omega t) \rangle = \frac{c^2 \varepsilon_0}{2} |\vec{E}_0 \times \vec{B}_0| = \frac{c \varepsilon_0}{2} E_0^2$ , 辐射功率:  $P = IS$  (单位瓦特, 这里的  $S$  为截面积), 电磁球面波径向衰减  $E(r, t) = \left( \frac{A}{r} \right) \cos(kr \mp \omega t)$  光子通量:  $\Phi = \frac{AI}{h\nu_0}$ . 辐射功率  $S = uc$ , 压强  $P_r = \frac{S(t)}{c}$ ,  $\langle P_r \rangle = \frac{\langle S(t) \rangle}{c} = \frac{I}{c}$ , 光压力为  $AP_r = \frac{\Delta p}{\Delta t}$  ( $p$ : momentum per unit volume)  $AP_r = \frac{p_v c \Delta t A}{\Delta t} = A \frac{S}{c}$ , 动量  $p_v = \frac{S}{c^2}$ ,  $p = \frac{E}{c} = \frac{h}{\lambda} = \hbar k$ . 电偶极辐射  $\mu = \mu_0 \cos \omega t$ ,  $\mu_0 = qd$ ,  $E = \frac{\mu_0 k^2 \sin \theta}{4\pi \varepsilon_0} \frac{\cos(kr - \omega t)}{r}$  Irradiance is:  $I(\theta) = \frac{\mu_0^2 \omega^4}{32\pi^2 c^3 \varepsilon_0} \frac{\sin^2 \theta}{r^2} \Rightarrow I \propto \omega^4$  介质中的光速  $v = \frac{1}{\sqrt{\varepsilon \mu}}$ ,  $n \equiv \frac{c}{v} = \frac{\sqrt{\varepsilon \mu}}{\sqrt{\varepsilon_0 \mu_0}}$ , 非磁性材料中有  $\mu \approx \mu_0$  相应  $n = \sqrt{\frac{\varepsilon}{\varepsilon_0}} = K_E \varepsilon_0$ ,  $n(\omega) = \sqrt{K_E(\omega)}$   $K_E$  是介电常数 (dielectric constant) 感生电场:  $\text{emf} = -\frac{d\Phi}{dt} = \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_A \vec{B} \cdot d\vec{S}$ , 高斯定律  $\oint_A \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \iiint_V \rho dV$  极化 Dipole moment  $\vec{\mu} = q_e \vec{r}$ ,  $\vec{P} = N \vec{\mu} = N q_e \vec{r}$ ,  $P$ : the resultant dipole moment per unit volume, 外部存在电场时, 受迫震荡  $q_e E_0 \cos \omega t - m_e \omega_0^2 x = m_e \frac{d^2 x}{dt^2}$ ,  $\omega_0 = \sqrt{\frac{k_E}{m_e}}$ , 设  $x(t) = x_0 \cos \omega t$ , 有  $x(t) = \frac{q_e / m_e}{\omega_0^2 - \omega^2} E_0 \cos \omega t$ , 因为  $\vec{P} = N q_e \vec{x}$ ,  $P = \frac{N q_e^2 E / m_e}{\omega_0^2 - \omega^2}$ ,  $\varepsilon = \varepsilon_0 + \frac{P(t)}{E(t)} = \varepsilon_0 + \frac{q_e^2 N / m_e}{(\omega_0^2 - \omega^2)}$ , 色散关系  $n^2(\omega) = \frac{\varepsilon}{\varepsilon_0} = 1 + \frac{N q_e^2}{\varepsilon_0 m_e} \sum_j \left( \frac{1}{\omega_{0j}^2 - \omega^2} \right)$ ,  $\omega \approx \omega_0$  时, 发生共振.  $\sum_j$  指一个节点材料中可能有多个  $\omega_0$ , 只有一个  $\omega_0$ , 可以去  $\sum_j$ . 存在阻尼  $m_e \gamma \frac{dx}{dt}$  时,  $m[\ddot{x} + \gamma \dot{x} + \omega_0^2 x] = q_e E(x, t)$ . 设  $x = x_0 e^{-i\omega t}$ ,  $E = E_0 e^{-i\omega t} \Rightarrow P = N q_e x = \frac{N q_e^2}{m(-\omega^2 - i\omega\gamma + \omega_0^2)} E$ ,  $n^2(\omega) = 1 + \frac{N q_e^2}{\varepsilon_0 m_e} \sum_j \left( \frac{f_j}{\omega_{0j}^2 - \omega^2 - i\gamma_j \omega} \right)$  群速度/相速度:  $V_{\text{phase}} = \frac{dz}{dt} = \frac{\omega}{k_z} = v$ ,  $V_{\text{group}} = \frac{d\omega}{dk} \Big|_{k=k_0} = \frac{c}{n(\omega) + \omega \frac{dn(\omega)}{d\omega}}$ ,

**CH3 光传播瑞利散射**: 偶极子的在远处产生的辐射电场为  $E = \frac{\mu_0 k^2 \sin \theta}{4\pi \varepsilon_0} \frac{\cos(kr - \omega t)}{r}$ , 辐射度 Irradiance  $I(\theta) = \frac{\mu_0^2 \omega^4}{32\pi^2 c^3 \varepsilon_0} \frac{\sin^2 \theta}{r^2}$   $I \propto \omega^4$ , 当粒子尺度显著大于  $\lambda$ , 该散射称为 Mie scattering (对各个波段均匀散射). 稀薄气体横向散射不受影响. 稠密介质/晶体 (Dense media) 侧向散射相消干涉.

**Internal - external reflection**: External: 从光疏介质到光密介质 ( $n_i < n_t$ ); Internal: 密 - 疏 ( $n_i > n_t$ ) 发生半波损失, 相位相差  $\pi$ , 折射定律  $n_i \sin \theta_i = n_t \sin \theta_t$ . Snell's law  $\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_t}{n_i} = n_{ti}$ . 费马定理 (修正) 光从  $S$  点到  $P$  点, 穿过光学长度最小的路线. (相对于该路径的变化是静止的)  $\frac{df}{dx} = 0$

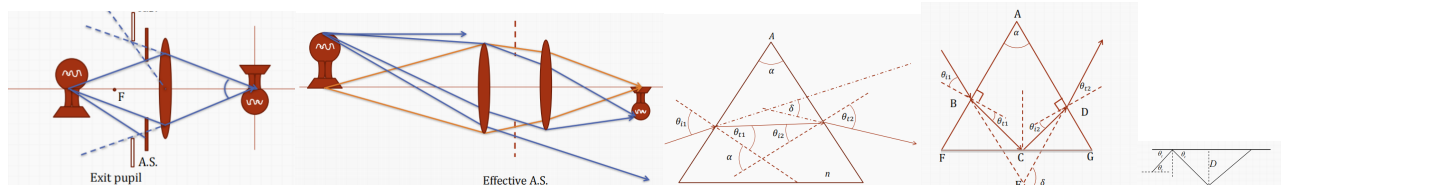
反射折射系数:  $r_{\perp} \equiv \left( \frac{E_{0r}}{E_{0i}} \right)_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$ ,  $r_{\parallel} \equiv \left( \frac{E_{0r}}{E_{0i}} \right)_{\parallel} = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_t + n_t \cos \theta_i}$ ,  $t_{\perp} \equiv \left( \frac{E_{0t}}{E_{0i}} \right)_{\perp} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}$ ,  $t_{\parallel} \equiv \left( \frac{E_{0t}}{E_{0i}} \right)_{\parallel} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}$ . 布鲁斯特角 (极化角  $\theta_p$ )  $(\theta_i + \theta_t) = 90^\circ$ ,  $r_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} = 0$  全反射:  $\theta_i = 90^\circ$ , 此时入射角  $\theta'_p = 90^\circ - \theta_p$ , Reflectance (R):  $R \equiv \frac{I_r A \cos \theta_r}{I_i A \cos \theta_i} = \frac{I_r}{I_i} = \left( \frac{E_{0r}}{E_{0i}} \right)^2 = r^2$ , Transmittance (T):  $T \equiv \frac{I_t A \cos \theta_t}{I_i A \cos \theta_i} = \left( \frac{n_t \cos \theta_t}{n_i \cos \theta_i} \right) t^2$ ,  $R + T = 1$ ,  $R_{\perp} + T_{\perp} = 1$ ,  $R_{\parallel} + T_{\parallel} = 1$ .  $n_{ti} = \frac{\sin^2 \theta_i}{\sin^2 \theta_t}$ ,  $\left( \frac{i}{t} \right)$  消逝波:  $n_{\parallel} = \left( \frac{E_{0r}}{E_{0i}} \right) = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_t + n_t \cos \theta_i}$ ,  $\vec{E}_t = \vec{E}_{0t} \exp i(\vec{k}_t \cdot \vec{r} - \omega t)$ ,  $\vec{k}_t \cdot \vec{r} = k_{tx}x + k_{ty}y$ ,  $k_{ty} = i k_t \left( \frac{\sin^2 \theta_i}{n_{ti}^2} - 1 \right)^{\frac{1}{2}} = i\beta$ ,  $k_{tx} = \frac{k_t}{n_{ti}} \sin \theta_i \Rightarrow \vec{E}_t = \exp(-\beta y) \exp i \left( \frac{k_{tx} \sin \theta_i}{n_{ti}} - \omega t \right)$ ,  $\beta = k_t \left( \frac{\sin^2 \theta_i}{n_{ti}^2} - 1 \right)^{\frac{1}{2}}$ ; 金属中的电磁波  $\vec{E} = \vec{E}_0 e^{i(\vec{k}z - \omega t)}$ ,  $\vec{k} = k + i\kappa$ ,  $\vec{E} = \vec{E}_0 e^{-i(kz - \omega t)}$ ,  $k \equiv \omega \sqrt{\frac{\varepsilon \mu}{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\varepsilon \omega} \right)^2} + 1 \right]^{\frac{1}{2}}$ ,  $\kappa = \sqrt{\frac{\varepsilon \mu}{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\varepsilon \omega} \right)^2} - 1 \right]^{\frac{1}{2}}$ , 趋肤深度  $d = \frac{1}{\kappa}$ , 色散: 金属除了吸收自由电子, 还会吸收特定频率的束缚电子  $n^2(\omega) = 1 - \frac{N q_e^2}{\varepsilon_0 m_e \omega^2} \Rightarrow n^2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$ ,  $\omega_p = \sqrt{\frac{N q_e^2}{\varepsilon_0 m_e}}$   $n^2(\omega) = 1 + \frac{N q_e^2}{\varepsilon_0 m_e} \left[ \frac{f_e}{-\omega^2 + i\gamma_e \omega} + \sum_j \frac{f_j}{\omega_{0j}^2 - \omega^2 + i\gamma_j \omega} \right]$   
free electrons bound electrons

**CH4 几何光学** 进入球透明介质 OPL  $= n_1 \sqrt{R^2 + (s_0 + R)^2 - 2R(s_0 + R) \cos \varphi} + n_2 \sqrt{R^2 + (s_i - R)^2 + 2R(s_i - R) \cos \varphi}$ ,  $\frac{d(\text{OPL})}{d\varphi} = 0$ .  $\frac{n_1 R(s_0 + R) \sin \varphi}{2l_0} - \frac{n_2 R(s_i - R) \sin \varphi}{2l_i} = 0$ ,  $\frac{n_1}{l_0} + \frac{n_2}{s_i} = \frac{1}{R} \left( \frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_0} \right)$ ; 旁轴近似:  $s_i = l_i$ ,  $s_0 = l_0 \Rightarrow \frac{n_1}{l_0} + \frac{n_2}{l_i} = \frac{1}{R} (n_2 - n_1)$  注意, 如果光线已经出了球体, 需要按照超出值代入  $s_0$  再计算一遍  $\left( \frac{n}{s_0} + \frac{1}{s_i} \right) = \frac{1}{-R} (1 - n)$ .  $\frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} s_i = \infty \Rightarrow \frac{n_1}{s_0} + \frac{n_2}{\infty} = \frac{n_2 - n_1}{R}$  此时物象在焦点处,  $s_0 \equiv f_0$ ,  $f_0 = \frac{n_1}{n_2 - n_1} R$ , 当  $s_0 = \infty$ ,  $s_i = f_i = \frac{n_2}{n_2 - n_1} R$ , 圆锥曲线上一点  $A$ , 作垂线与准线于  $D$ , 有  $\frac{L_{AD}}{L_{AF}} = e$ , 当  $\frac{n_{\text{空气}}}{n_{\text{介质}}} = e$  时, 焦点出发的点光源会变成平行光。



Gaussina lens formula  $\frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f}$  where  $\frac{1}{f} = (n_l - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$ , Newtonian form of the lens equation:  $x_0 x_i = f^2$  过有平行平面边界的介质的光线会发生横向位移, 但与穿入传出的光传播方向平行. 偏移量 (垂直于传播方向)  $a = \frac{d \sin(\theta_i - \theta_t)}{\cos \theta_i}$ , 薄薄膜而言,  $d \approx 0$

横向放大量  $M_T$  (垂直于光轴的放大倍数)  $M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o} = -\frac{x_i}{f} = -\frac{f}{x_o}$ ; 纵向  $M_L \equiv \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$ ,  $x_o x_i = f^2$ . 多个凸透镜:  $s_{i2} = \frac{f_2 d - f_2 s_{o1} f_1 / (s_{o1} - f_1)}{d - f_2 - s_{o1} f_1 / (s_{o1} - f_1)}$ ,  $M_T = M_{T1} M_{T2} = \frac{f_1 s_{i2}}{d(s_{o1} - f_1) - s_{o1} f_1}$ ,  $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$



孔径光阑 (Aperture stops) Vignetting 渐晕 Relative Aperture 相对孔径: 辐照度或通量密度 (单位时间内单位面积的能量) 与入口瞳孔面积成正比, 与图像面积成反比  $I \propto \left( \frac{D}{f} \right)^2$ ,  $\frac{D}{f}$  被称为相对孔径. 孔径的大小可以通过仰角判断:  $\theta = \arctan \left( \frac{h}{d} \right)$ ,  $h$  为成像的高度,  $d$  从物体到障碍的距离如果前面有透镜需要考虑  $M_T$  透镜-光圈-透镜, 入射考虑第一个透镜, 出射光瞳考虑第二个, 光圈的像位置为光瞳位置, 光瞳直径为像的大小.

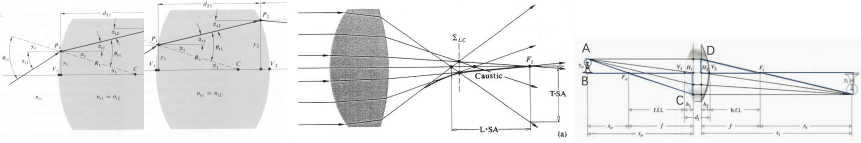
非平面镜: 球面镜  $\frac{1}{s_0} + \frac{1}{s_i} = -\frac{2}{R}$ ,  $s_o, s_i$  在镜子一侧都为正;  $M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o}$ ; 色散棱镜:  $\delta = \theta_{i1} + \sin^{-1} [\sin \alpha (n^2 - \sin^2 \theta_{i1})^{1/2} - \sin \theta_{i1} \cos \alpha] - \alpha$ ,  $n = \frac{\sin \theta_{i1}}{\sin \theta_{t1}} = \frac{\sin \frac{\theta}{2}}{\sin \frac{\alpha}{2}}$

Quantity	Sign
$s_o$	Left of V, real object
$s_i$	Left of V, real image
$f$	Concave mirror
$R$	C right of V, convex
$s_o$	Above axis, erect object
$s_i$	Above axis, erect image
$s_o$	Right of V, virtual object
$s_i$	Right of V, virtual image
$f$	Convex mirror
$R$	C left of V, concave
$s_o$	Below axis, inverted object
$s_i$	Below axis, inverted image

反射棱镜: Reflecting prisms.  $\delta = \theta_{i1} + \theta_{t2} + \alpha$ ; 光纤Fiberoptics:  $\frac{\sin \theta_i}{\sin \theta_t} = \frac{n_f}{n_a}$   $n_a = 1, l = n_f L (n_f^2 - \sin^2 \theta_i)^{-\frac{1}{2}}$  反射次数  $N = \frac{l}{D/\sin \theta_t} \pm 1 = \frac{L \sin \theta_i}{D(n_f^2 - \sin^2 \theta_i)^{1/2}} \pm 1$

光纤允许的最大角度  $\sin \theta_{\max} = \frac{1}{n_i} (n_f^2 - n_c^2)^{\frac{1}{2}}$  为纤芯的折射率,  $n_c$  为外覆层折射率 Numer aperture (NA) :  $n_i \sin \theta_{\max}$ ; 多模态色散 Lowest order mode  $t_{\min} = \frac{L}{v_f} = \frac{L}{c/n_f} = \frac{Ln_f}{c}, t_{\max} = \frac{l}{v_f} = \frac{L/\cos \theta_t}{c/n_f} = \frac{Ln_f/n_c}{c/n_f} = \frac{Ln_f^2}{cn_c}$ , intermodal delay  $\Delta t = \frac{Ln_f}{c} \left( \frac{n_f}{n_c} - 1 \right)$  Attenuation 衰减  $\text{dB} = -10 \log_{10} (P_o/P_{in})$

CH5 厚透镜  $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}, M_T = \frac{y_i}{y_o} = -\frac{x_i}{f} = -\frac{f}{x_o}, \frac{1}{f} = (n_l - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l - 1)d_l}{n_l R_1 R_2} \right], h_1 = -\frac{f(n_l - 1)d_l}{n_l R_2}, h_2 = -\frac{f(n_l - 1)d_l}{n_l R_1}$  屈光度 (dioptric power)  $D \equiv \frac{1}{f}$ , 两个距离很近的一对透镜:  $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} D = D_1 + D_2$ . 棱镜组



在第一个表面上有  $n_{t1}\alpha_{t1} = n_{i1}\alpha_{i1} - D_1 y_{i1} \Rightarrow$  Matrix form  $\begin{bmatrix} n_{t1}\alpha_{t1} \\ y_{t1} \end{bmatrix} = \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_{i1}\alpha_{i1} \\ y_{i1} \end{bmatrix} n_{t1} \equiv \begin{bmatrix} n_{t1}\alpha_{t1} \\ y_{t1} \end{bmatrix} n_{i1} = \begin{bmatrix} n_{i1}\alpha_{i1} \\ y_{i1} \end{bmatrix}$ , Refraction matrix  $\mathcal{R}_1 \equiv \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix}$ , 同理:  $n_{i2}\alpha_{i2} = n_{t1}\alpha_{t1} + 0, y_{i2} = d_{21}\alpha_{t1} + y_{c1} \Rightarrow \mathcal{T}_{21} \equiv \begin{bmatrix} 1 & 0 \\ d_{21} & 1 \end{bmatrix} n_{i2} = \mathcal{T}_{21} n_{t1} = \mathcal{T}_{21} \mathcal{R}_1 n_{i1}$

相差 Aberrations  $\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} + h^2 \left[ \frac{n_1}{2s_o} \left( \frac{1}{s_o} + \frac{1}{R} \right)^2 + \frac{n_2}{2s_i} \left( \frac{1}{R} - \frac{1}{s_i} \right)^2 \right]$  边缘光线汇聚点距离旁轴近似的光线汇聚点的前面, SA is positive, 反之亦然。

CH6 波的叠加 同频率波的叠加  $E_1 = E_{01} \sin(\omega t + \alpha_1), E_2 = E_{02} \sin(\omega t + \alpha_2) \Rightarrow E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} \cos(\alpha_2 - \alpha_1), \tan \alpha = \frac{E_{01} \sin \alpha_1 + E_{02} \sin \alpha_2}{E_{01} \cos \alpha_1 + E_{02} \cos \alpha_2}$ , 其中  $\alpha_1(x_1, \epsilon_1) = -(kx_1 + \epsilon_1), \alpha_2(x_2, \epsilon_2) = -(kx_2 + \epsilon_2)$  多波的叠加  $E = \sum_{i=1}^N E_{0i} \cos(\alpha_i \pm \omega t), E_0^2 = \sum_{i=1}^N E_{0i}^2 + 2 \sum_{j>i}^N \sum_{i=1}^N E_{0i}E_{0j} \cos(\alpha_i - \alpha_j)$ ,  $\tan \alpha = \frac{\sum_{i=1}^N E_{0i} \sin \alpha_i}{\sum_{i=1}^N E_{0i} \cos \alpha_i}$ , 相干光 ( $\alpha_i = \alpha_j$ )  $E_0^2 = N^2 E_{01}^2$ . 非相干光  $I = \langle E_0^2 \rangle = \sum_{i=1}^N \langle E_{0i}^2 \rangle = N E_{01}^2$

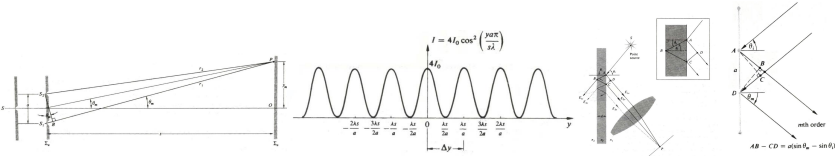
拍:  $E_1 = E_{01} \cos(k_1 x - \omega_1 t), E_2 = E_{01} \cos(k_2 x - \omega_2 t) \Rightarrow E = 2E_{01} \cos \frac{[(k_1 + k_2)x - (\omega_1 + \omega_2)t]}{2} \times \cos \frac{[(k_1 - k_2)x - (\omega_1 - \omega_2)t]}{2}$ , irradiance:  $E_0^2(x, t) = 4E_{01}^2(x, t) \cos^2(k_m x - \omega t) = 2E_{01}^2(x, t) [1 + \cos(2k_m x - 2\omega_m t)]$  平均频率/波数  $\bar{\omega} \equiv \frac{1}{2}(\omega_1 + \omega_2), \bar{k} \equiv \frac{1}{2}(k_1 + k_2)$ , 调制频率/波数  $\omega_m \equiv \frac{1}{2}(\omega_1 - \omega_2), k_m \equiv \frac{1}{2}(k_1 - k_2)$ , 拍频  $2\omega_m$  or  $(\omega_1 - \omega_2)$  相/群速度:  $v = -\frac{(\partial \varphi / \partial t)_x}{(\partial \varphi / \partial x)_t} = \frac{\bar{\omega}}{\bar{k}}, v_g = \frac{d\omega}{dk}$

傅里叶变换 (周期)  $f(x) = \frac{A_0}{2} + \sum_{m=1}^{\infty} A_m \cos mkx + \sum_{m=1}^{\infty} B_m \sin mkx, A_m = \frac{2}{\lambda} \int_0^{\lambda} f(x) \cos mkx dx, B_m = \frac{2}{\lambda} \int_0^{\lambda} f(x) \sin mkx dx$ ,

非周期性波  $f(x) = \frac{2}{a} + \int_{-\infty}^{\infty} A(k) \cos kx dk, A(k) = \int_{-\infty}^{\infty} f(x) \cos kx dx, B(k) = \int_{-\infty}^{\infty} f(x) \sin kx dx$

相干距离 Coherence time  $\Delta v \sim \frac{1}{\Delta t}$  Coherence length  $\Delta x_c = c \Delta t_c$

CH7 干涉衍射  $I = \langle \vec{E}_1^2 + \vec{E}_2^2 + 2\vec{E}_1 \cdot \vec{E}_2 \rangle_T = I_1 + I_2 + I_{12}, I_{12} = \vec{E}_{01} \cdot \vec{E}_{02} \cos \delta; \vec{E}_{01}$  and  $\vec{E}_{02}$  互相垂直, 则  $I_{12} = 0, \vec{E}_{01}$  and  $\vec{E}_{02}$  平行  $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta, I_1 + I_2 < I < I_{\max}$  干涉条件: 1 频率相同, 2 相干; 菲涅尔-阿拉戈定律: E 方向正交不会发生干涉。



双缝衍射第  $m$  条干涉条纹的位置为  $y_m \approx \frac{s \lambda}{a} m \lambda$ , 角度为  $\theta_m \approx \frac{y_m}{s} = \frac{m \lambda}{a}$ , (两激光辐照度相同时) Phase difference:  $\delta = k(r_1 - r_2), I = 4I_0 \cos^2 \frac{\gamma a \pi}{s \lambda}$

等倾干涉:  $\Lambda = 2n_f d \cos \theta_t; n_1 = n_2 = n$  (肥皂泡薄膜), 出现半波损失 ( $n_1 > n_f > n_2$  不会)  $d \cos \theta_t = \frac{\lambda}{4} 2m, \theta_t \approx 0$  薄膜干涉,  $(m + \frac{1}{2}) \lambda_0 = 2n_f d_m \Rightarrow d_m = (m + \frac{1}{2}) \frac{\lambda_f}{2}$  (亮条纹), Haidinger's Fringes/牛顿环第  $m$  阶干涉相消发生在:  $2n_f d_m = m \lambda_0 \Rightarrow x_m = [m \lambda_f R]^{\frac{1}{2}}$  第  $m$  阶干涉相长  $x_m = [(m + \frac{1}{2}) \lambda_f R]^{\frac{1}{2}}$

等倾干涉时, 条纹宽度随倾角变化  $\alpha \propto \frac{\lambda_f}{2 \Delta x} = \frac{\lambda_0}{2 \Delta x}, \Delta x$  为两相邻亮条纹水平距离。一定要注意在介质中时, 光程与折射率, 波长之间的关系

CH8 衍射  $\vec{E} = E_0(r) e^{i(kR - \omega t)} \left[ \frac{\sin(N\delta/2)}{\sin(\delta/2)} \right], R = \frac{1}{2}(N - 1)d \sin \theta + r_1, I = I_0 \left[ \frac{\sin^2(N\delta/2)}{\sin^2(\delta/2)} \right] = I_0 \frac{\sin^2[N(\frac{k d}{2}) \sin \theta]}{\sin^2[(\frac{k d}{2}) \sin \theta]}$ ; 最小值 0 在  $\sin \frac{N\delta}{2} = 0$ , 但  $\sin \frac{\delta}{2} \neq 0$  处取得  $\frac{\delta}{2} = \pm \frac{\pi}{N}, \pm \frac{2\pi}{N}, \pm \frac{3\pi}{N}, \dots, \pm \frac{(N-1)\pi}{N}, \pm \frac{(N+1)\pi}{N}$  次大的峰值在  $\sin \frac{N\delta}{2} = \pm 1$  处取得, 此时  $\frac{\delta}{2} = \pm \frac{\pi}{2N}, \pm \frac{3\pi}{2N}, \pm \frac{5\pi}{2N}, \dots; \frac{dI}{d\beta} = 0$  处,  $\frac{dI}{d\beta} = I(0) \frac{2 \sin \beta (\beta \cos \beta - \sin \beta)}{\beta^3} = 0; \beta = \pm 1.43\pi, \pm 2.46\pi, \pm 3.47\pi; N$  缝光栅:  $I(\theta) = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin N\alpha}{\sin \alpha} \right)^2$ , 双缝  $I(\theta) = I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \left( \frac{\sin 2\alpha}{\sin \alpha} \right)^2 = 4I_0 \left( \frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha; \alpha = \left( \frac{k a}{2} \right) \sin \theta; \beta \equiv \left( \frac{k b}{2} \right) \sin \theta$

Grating:  $a(\sin \theta_m - \sin \theta_i) = m \lambda$   $a$  是相邻狭缝的距离, Airy disk:  $\Delta \theta \approx \sin \Delta \theta = \frac{q_1}{f} = 1.22 \frac{\lambda}{D}; q_1 = \frac{3.83 R}{ka} = 1.22 \frac{R \lambda}{2a} = 1.22 \frac{f \lambda}{D}$

CH9 极化/偏振 两个线偏振光  $\vec{E}_x(z, t) = i E_{0x} \cos(kz - \omega t), \vec{E}_y(z, t) = j E_{0y} \cos(kz - \omega t + \epsilon), \epsilon = n\pi$  时  $E$  为线偏振,  $\epsilon = -\frac{\pi}{2} + 2m\pi, E_{0x} = E_{0y}$ , 右旋 (面向光传来的方向, 电场矢量顺时针旋转);  $\epsilon = -\frac{\pi}{2} + 2m\pi \dots$  左旋:  $E_{0x} \neq E_{0y}$ , 且  $\epsilon$  随机, 会合成椭圆偏光。

双折射:  $n_o = \frac{c}{v_{\perp}}, n_e = \frac{c}{v_{\parallel}}, \Delta n = (n_o - n_e)$  方解石的双折射系数是正的, 石英的是负值; o 光与光轴确定的平面为 o 主平面; 主截面 (principal section): 「光轴」和晶体「表面法线」共同确定的平面; 极化程度:  $V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$ . 玻片:  $\Lambda = d(|n_o - n_e|), \Delta \varphi = \frac{2\pi}{\lambda_0} d(|n_o - n_e|)$ , 全波片  $\Delta \varphi = 2\pi$  半波片  $d(|n_o - n_e|) = \frac{(2m+1)\lambda_0}{4}$  玻片  $d(|n_o - n_e|) = \frac{(4m+1)\lambda_0}{4}, m = 0, 1, 2, \dots$  补偿器  $\beta = \frac{\pi d}{\lambda_0} (n_L - n_R); \beta = V B d$ ; 马吕斯定律:  $I(\theta) = I(0) \cos^2 \theta$ ,

第一个滤波器: 各向同性, 同样通过所有状态; 第二滤光片: 线性偏振片, 透射轴水平; 第三个滤波器: 线性偏振镜, 透射轴在 45; 第四滤光片: 对 L(右旋)态不透明的圆偏振片.  $S_0 = 2I_0; S_1 = 2(I_1 - I_0); S_3 = 2(I_2 - I_0); S_3 = 2(I_3 - I_0) I_0, I_1, I_2, I_3$  are transmitted irradiances of these four filters.  $S_0^2 = S_1^2 + S_2^2 + S_3^2$ . 极化程度  $V = \sqrt{(S_1^2 + S_2^2 + S_3^2) / S_0^2}$ ; Jones vector:  $E = \begin{bmatrix} E_{0x}(t) e^{i\varphi_x} \\ E_{0y}(t) e^{i\varphi_y} \end{bmatrix}$