EEE 318: Control System I Laboratory

Project Report

Name of the Project : ***Variable-Gain Control for Respiratory Systems.***

Submitted By-

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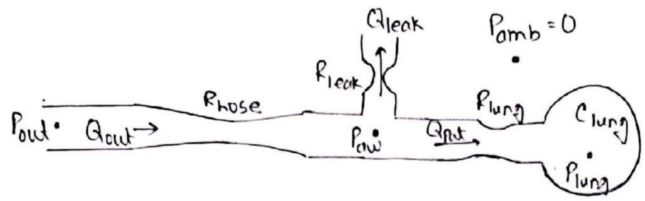
Task 1 :

Determination of transfer function of the respiratory system shown in figure 3 :



$$Q_{out} = Q_{Pat} + Q_{Leak}$$

$$\Rightarrow Q_{Pat} = Q_{out} - Q_{Leak} \quad \text{-----} \; (i)$$

$$\left.\begin{aligned} Q_{out} &= \frac{P_{out} - P_{aw}}{R_{hose}} \\ Q_{Leak} &= \frac{P_{aw} - P_{amb}}{R_{leak}} \\ Q_{pat} &= \frac{P_{aw} - P_{lung}}{R_{lung}} \end{aligned}\right\} \quad \text{----} \; (ii)$$

$$i = C \frac{dV_c}{dt}$$

$$\dot{P}_{lung} = \frac{1}{C} Q_{Pat} \quad \text{-------} \; (iii)$$

$$\Rightarrow \dot{P}_{lung} = \cancel{\frac{1}{C}} \; \frac{P_{aw} - P_{lung}}{R_{lung} C_{lung}} \quad \text{----} \; (iv)$$

$(i) \Rightarrow$

$$Q_{pat} = \frac{P_{out} - P_{aw}}{R_{hose}} - \frac{P_{aw}}{R_{Leak}} \qquad [P_{amb} = 0]$$

$$\Rightarrow \frac{P_{aw} - P_{lung}}{R_{lung}} = \frac{P_{out} - P_{aw}}{R_{hose}} - \frac{P_{aw}}{R_{leak}}$$

$$\Rightarrow \frac{P_{aw}}{R_{lung}} + \frac{P_{aw}}{R_{hose}} + \frac{P_{aw}}{R_{leak}} = \frac{P_{out}}{R_{hose}} + \frac{P_{lung}}{R_{lung}}$$

$$\Rightarrow \boxed{P_{aw} = \frac{B \dfrac{P_{out}}{R_{hose}} + \dfrac{P_{lung}}{R_{lung}}}{\dfrac{1}{R_{lung}} + \dfrac{1}{R_{hose}} + \dfrac{1}{R_{leak}}}} \quad \text{-----} \; (V)$$

$$\dot{P}_{lung} = \frac{P_{aw} - P_{lung}}{C_{lung} R_{lung}}$$

$$= \frac{\dfrac{P_{out}/R_{hose} + P_{lung}/R_{lung}}{1/R_{hose} + 1/R_{leak} + 1/R_{lung}} - P_{lung}}{C_{lung} R_{lung}}$$

$$\boxed{\dot{P}_{lung} = \frac{P_{out}/R_{hose} - \left(\frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right) P_{lung}}{R_{lung} C_{lung} \left(\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}\right)}} \quad \text{.... (vi)}$$

State space representation :

i/p $\longrightarrow P_{out}$

State variable $\longrightarrow P_{lung}$

O/P $\longrightarrow P_{aw}, Q_{pat}$

$$\dot{P}_{lung} = A_h P_{lung} + B_h P_{out} \quad \text{..... (vii)}$$

$$\begin{bmatrix} P_{aw} \\ Q_{pat} \end{bmatrix} = C_h P_{lung} + D_h P_{out} \quad \text{..... (viii)}$$

(v) in (ii) $\Rightarrow$

$$Q_{pat} = \frac{\dfrac{\frac{P_{out}}{R_{hose}} + \frac{P_{lung}}{R_{lung}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}} - P_{lung}}{R_{lung}}$$

$$\Rightarrow Q_{pat} = \frac{\frac{P_{out}}{R_{hose}} - \left(\frac{P_{lung}}{R_{hose}} + \frac{P_{lung}}{R_{leak}}\right)}{R_{lung}\left(\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}\right)}$$

$$A_h = -\frac{\frac{1}{R_{hose}} + \frac{1}{R_{leak}}}{R_{lung}C_{lung}\left(\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}\right)}$$

$$B_h = \frac{\frac{1}{R_{hose}}}{R_{lung}C_{lung}\left(\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}\right)}$$

$$C_h = \begin{bmatrix} \dfrac{\frac{1}{R_{lung}}}{\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}} & \dfrac{-\left(\frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)}{R_{lung}\left(\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}\right)} \end{bmatrix}^T$$

$$D_h = \begin{bmatrix} \dfrac{\frac{1}{R_{hose}}}{\frac{1}{R_{lung}} + \frac{1}{R_{hose}} + \frac{1}{R_{leak}}} & \dfrac{\frac{1}{R_{hose}}}{R_{lung}\left(\frac{1}{R_{hose}} + \frac{1}{R_{leak}} + \frac{1}{R_{lung}}\right)} \end{bmatrix}^T$$

$$\boxed{H(s) = C_h (sI - A_h)^{-1} B_h + D_h}$$

$$A_h = -5.4430$$

$$sI = s$$

$$sI - A_h = s + 5.4430$$

$$C_h (sI - A_h)^{-1} = \begin{bmatrix} 0.4557 \\ -108.86 \end{bmatrix} \left[\frac{1}{s+5.4430} \right]$$

$$C_h(sI - A_h)^{-1} B_h = \begin{bmatrix} \dfrac{0.4557}{s+5.4430} \\ \\ \dfrac{-108.86}{s+5.4430} \end{bmatrix} \begin{bmatrix} 5.0633 \end{bmatrix}$$

$$C_h(sI - A_h)^{-1} B_h + D_h = \begin{bmatrix} \dfrac{2.307}{s+5.4430} \\ \\ \dfrac{-551.19}{s+5.4430} \end{bmatrix} + \begin{bmatrix} 0.5063 \\ \\ 101.265 \end{bmatrix}$$

$$\Rightarrow H(s) = \begin{bmatrix} \dfrac{0.5063s + 5.063}{s+5.443} \\ \\ \dfrac{101.3s}{s+5.443} \end{bmatrix}$$

$$\frac{P_{aw}(s)}{P_{out}(s)} = \frac{0.5063s + 5.063}{s+5.443}$$

$$\frac{Q_{pat}(s)}{P_{out}(s)} = \frac{101.3s}{s+5.443}$$

Code for calculating H and B:

```
clc;
clear all;
close all;

%% Defining of the Parameter of the lung ang hose
R_lung=5/1000;
C_lung =20;
R_leak =60/1000;
R_hose = 4.5/1000;
wn= 2*pi*30;
z=1; % zeta
s=tf('s');

%% Transfer function of the patient hose calculation
Ah=
(-(R_hose^-1+R_leak^-1))/((R_lung*C_lung)*(R_lung^-1+R_hose^-
1+R_leak^-1))
Bh=(R_hose^-1)/((R_lung*C_lung)*(R_lung^-1+R_hose^-1+R_leak^-
1))
Ch1=(R_lung^-1)/((R_lung^-1+R_hose^-1+R_leak^-1))
Ch2=
(-(R_hose^-1+R_leak^-1))/((R_lung)*(R_lung^-1+R_hose^-1+R_lea
k^-1))
Ch=[Ch1 Ch2]'
Dh1= (R_hose^-1)/((R_lung^-1+R_hose^-1+R_leak^-1))
Dh2= ((R_hose^-1))/((R_lung)*(R_lung^-1+R_hose^-1+R_leak^-1))
Dh= [Dh1 Dh2]'

H=ss(Ah,Bh,Ch,Dh);
H=tf(H)


%% Transfer function of the blower calculation
Ab= [-2*z*wn (-wn^2); 1 0]
Bb=[1;0]
Cb=[0 wn^2]

B=ss(Ab,Bb,Cb,0);
B=tf(B)
```

Task 2 :

Determination of the overall transfer function of the closed loop control system shown in figure 4 :

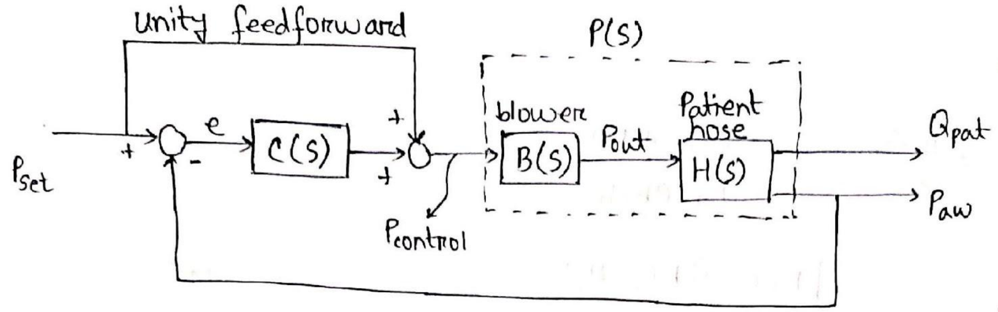
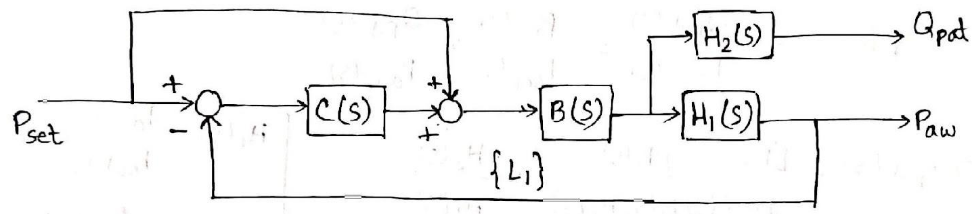


Fig 4. Closed loop control scheme with a linear controller $C(S)$

We re-wrote Fig. 4 to incorporate our $H_1(S)$ and $H_2(S)$ in the above depiction.



We will use Mason's Rule to find out the overall transfer function:

(i) First we figure out $\dfrac{P_{aw}(S)}{P_{set}(S)} = T_{aw}(S)$:

There are two forward paths, $T_1 = CBH_1$ and $T_2 = BH_1$

There is only one feedback loop, $L_1 = -CBH_1$

Determinant of graph $= \Delta = 1 - L_1 = 1 + CBH_1$

Co-factor of k^{th} paths, $\Delta_1 = 1$, $\Delta_2 = 1$

Therefore,

$$T_{aw}(S) = \frac{\sum T_K \Delta_K}{\Delta} = \frac{CBH_1 + BH_1}{1 + CBH_1}$$

$$\Rightarrow T_{aw}(S) = \frac{[1 + C(S)]\, B(S)\, H_1(S)}{1 + C(S)\, B(S)\, H_1(S)} \quad \text{-------- (a)}$$

(ii) Now we find $\frac{Q_{pat}(S)}{P_{set}(S)} = T_{pat}(S)$

We can write the required transfer function as:

$$T_{pat}(S) = \frac{Q_{pat}(S)}{P_{set}(S)} = \frac{P_{aw}(S)}{P_{set}(S)} \times \frac{Q_{pat}(S)}{P_{aw}(S)}$$

$$\Rightarrow T_{pat}(S) = \frac{[1 + C(S)]\, B(S)\, H_1(S)}{1 + C(S)\, B(S)\, H_1(S)} \times \frac{H_2(S)}{H_1(S)}$$

$$\Rightarrow T_{pat}(S) = \frac{[1 + C(S)]\, B(S)\, H_2(S)}{1 + C(S)\, B(S)\, H_1(S)} \quad \text{----- (b)}$$

$$H_1(S) = \frac{P_{aw}(S)}{P_{out}(S)}$$

$$H_2(S) = \frac{Q_{pat}(S)}{P_{out}(S)}$$

Task 3 :

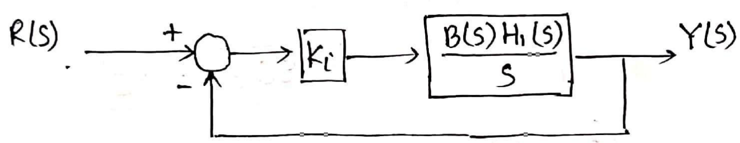
Sketch of the root locus of the control system shown in Fig. 4 for 0<ki<∞ of the integral controller C(s) :

From the previous task, from both equation (a) and (b), we see that, both transfer functions has same denominators $[1 + C(s)B(s)H_1(s)]$. Thus their root locus would be the same.

Making $C(s)$ an integral controller would result in the denominator to be like so:

$$\left[1 + k_i \frac{B(s)H_1(s)}{s}\right]$$

As a result, we code in MATLAB to implement a root locus for such a system below:



We use the functional values found in Task-1 for $H_1(s)$ and the second order system given in page-165 for $B(s)$.

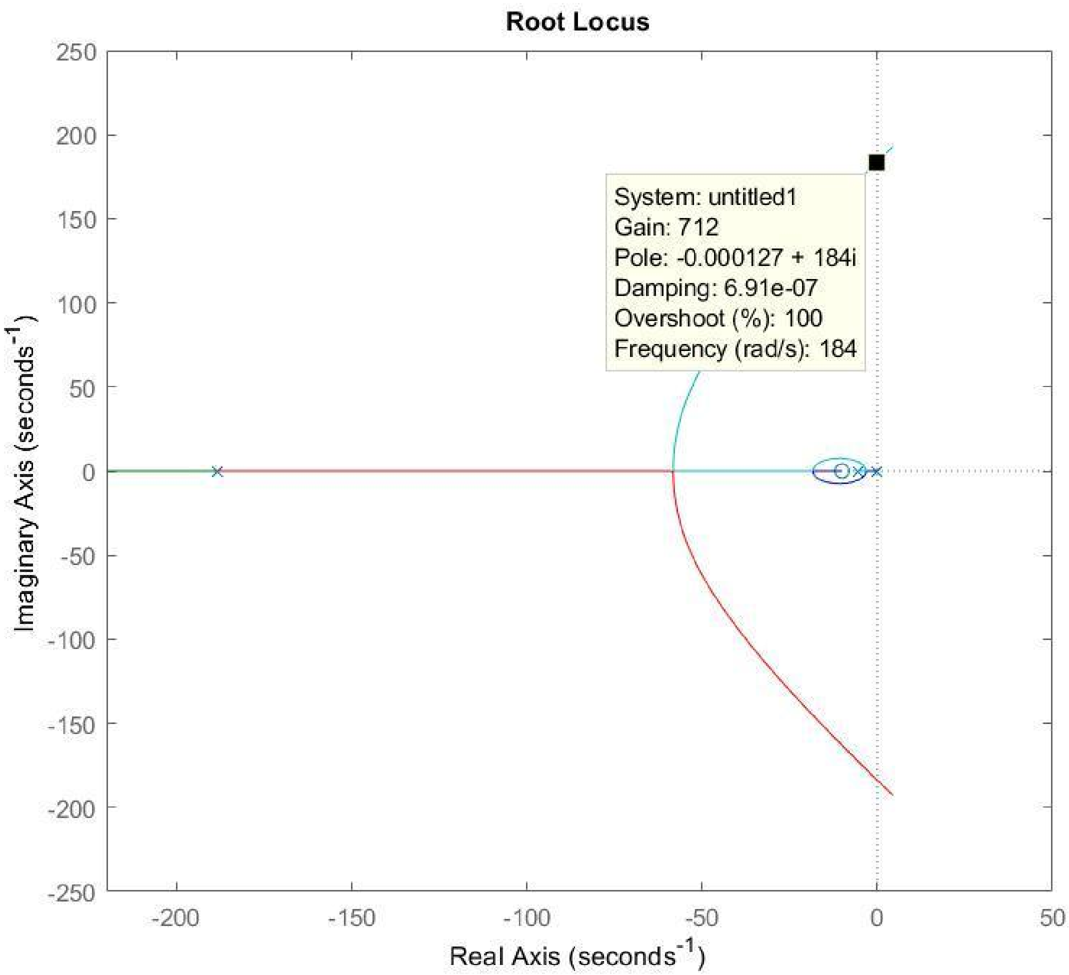
$$B(s) = \frac{P_{out}(s)}{P_{control}(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

Where, $\omega_n = 2\pi(30)$ and $\zeta = 1$

Bottom part of the above code :

```
%% root locus
H1=(0.5036*s+5.063)/(s+5.443) %needed for Paw
H2=(101.3*s)/(s+5.443) % needed for Qpat

rlocus((B*H1)/s, 0:0.01:800)
axis([-220 50 -250 250])
```



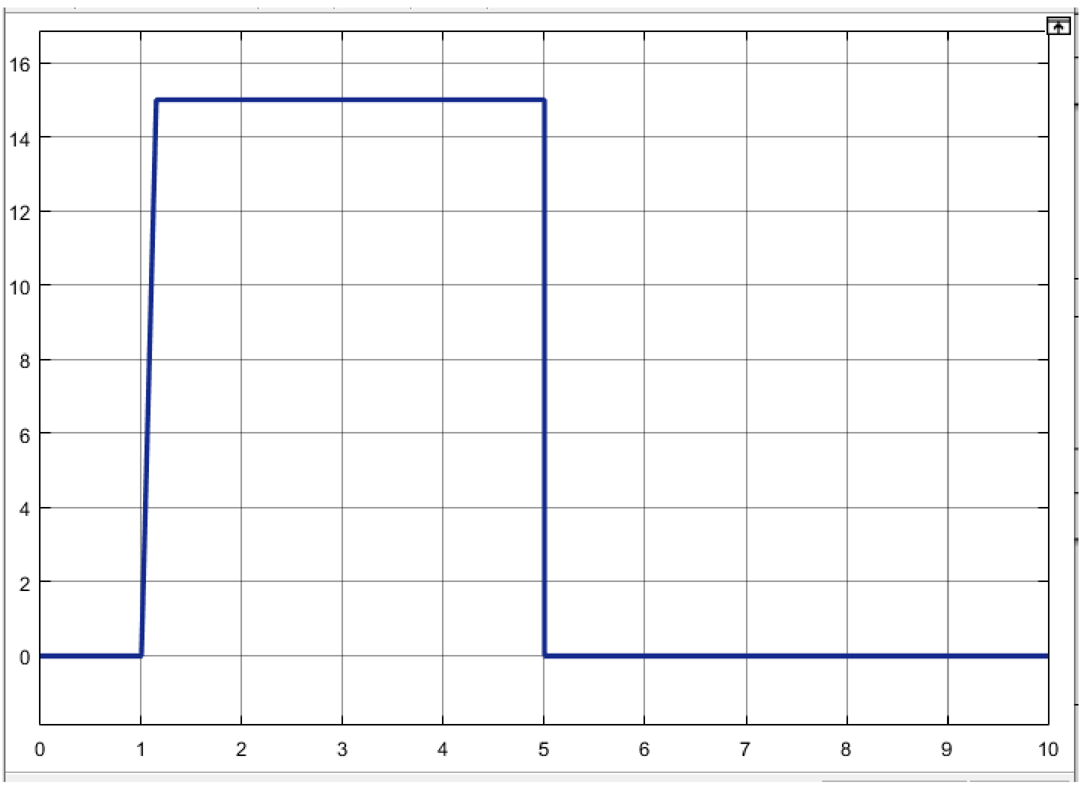
We see system becomes unstable after gain K is more than 712.

Task 4 :

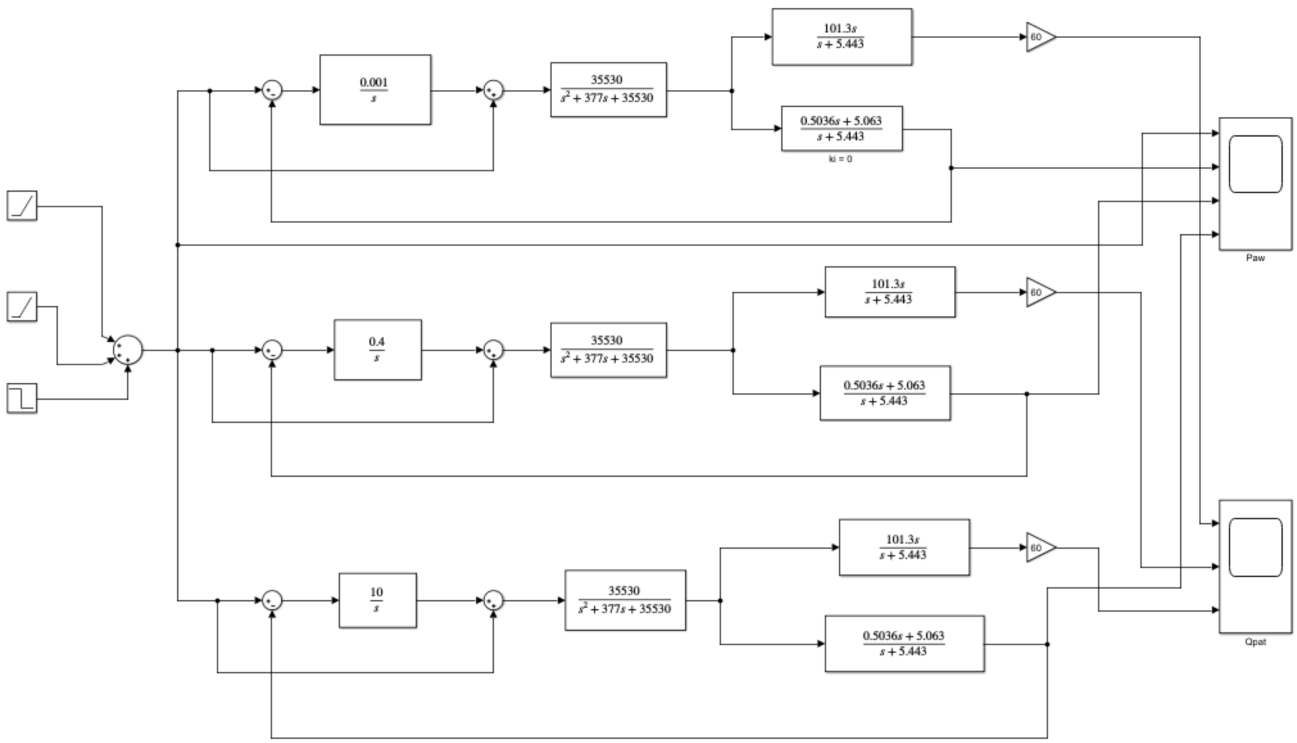
Reproduce the results shown in Fig. 7 for the combined feedback and feedforward control system. Discuss the necessity of both feedback and feedforward control.

P_{set} :

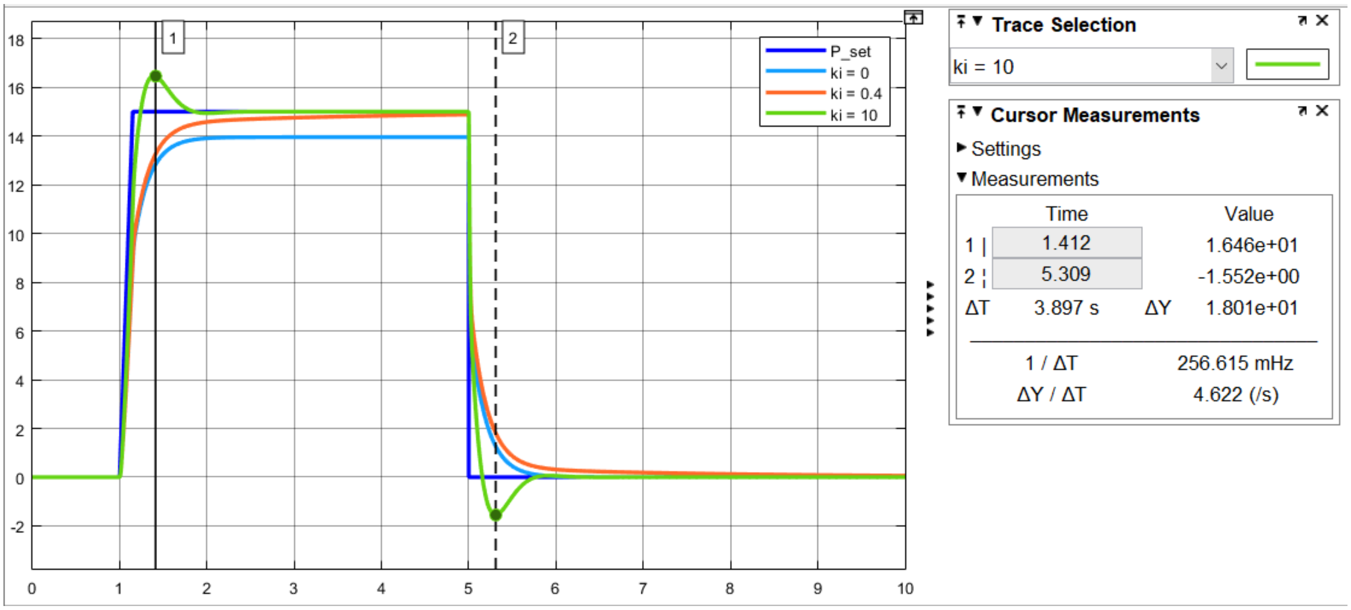
1. Finite slope in the rising part.
2. Infinite slope in the falling part.
3. So, we taken P_{set} to be 100[r(t-1)-r(t-1.15)]-15u(t-5).



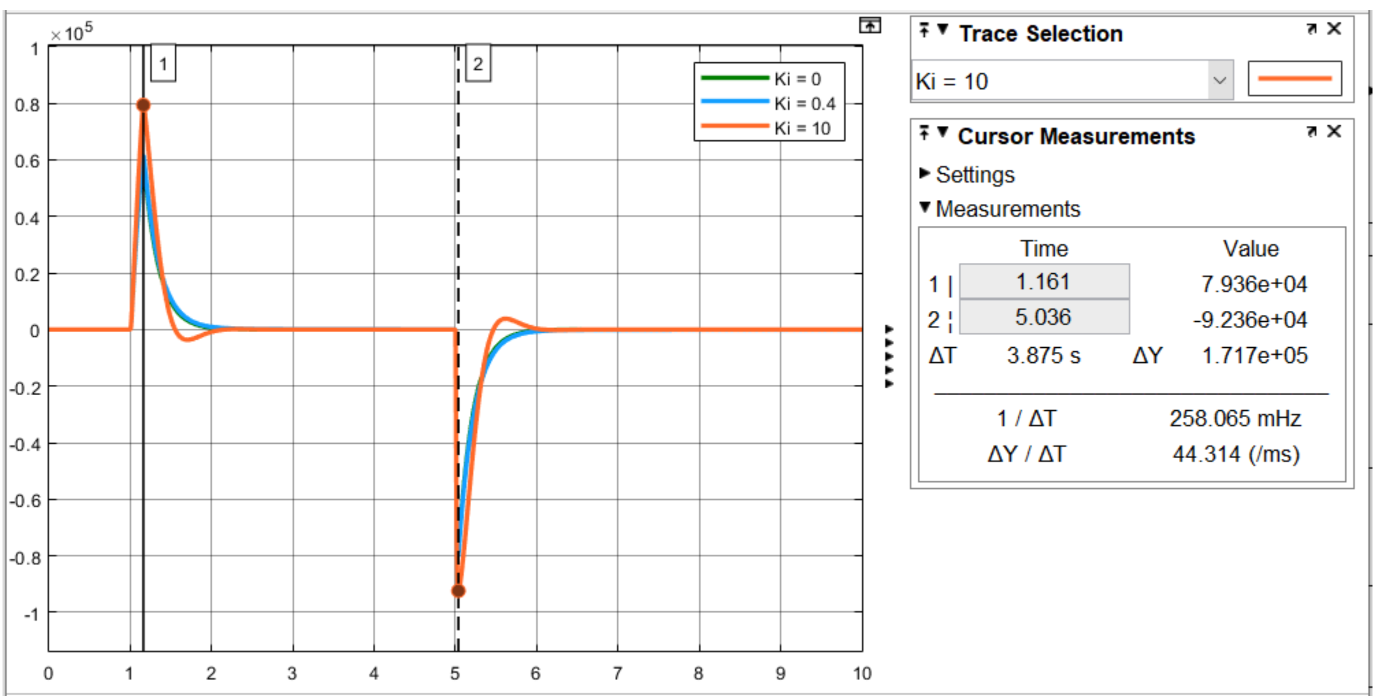
Circuit Diagram :



P_{aw} :



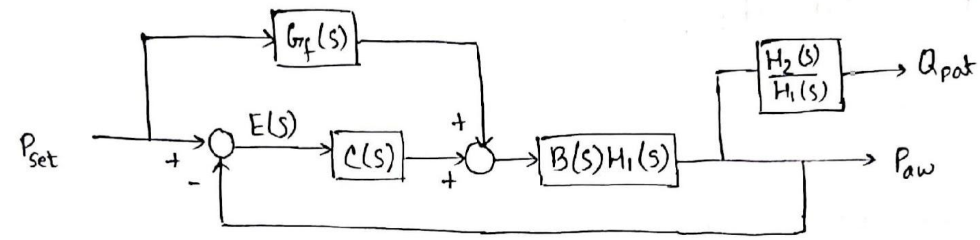
Q_{pat} :



We can observe that results are almost similar to that of the original report.

Necessity of feedback and feedforward system :

We added a feed-forward function $G_f(s)$ to the network to explore the necessity of the feed-forward path:



The transfer function using mason's rule is:

$$T(s) = \frac{P_{aw}(s)}{P_{set}(s)} = \frac{[G_f(s) + C(s)]B(s)H_1(s)}{1 + C(s)B(s)H_1(s)}$$

Therefore, $E(s) = P_{set}(s) - P_{aw}(s) = P_{set}(s)\left[1 - \frac{[G_f(s)+C(s)]B(s)H_1(s)}{1 + C(s)B(s)H_1(s)}\right]$

$$\Rightarrow E(s) = P_{set}(s)\,\frac{1 - G_f(s)B(s)H_1(s)}{1 + C(s)B(s)H_1(s)}$$

We desire systems with steady state errors,

$$e(\infty) = \lim_{s \to 0} sE(s) = 0$$

$$\Rightarrow \lim_{s \to 0} s\,P_{set}(s)\,\frac{1 - G_f(s)B(s)H_1(s)}{1 + C(s)B(s)H_1(s)} = 0$$

$$\Rightarrow [1 - G_f(0)B(0)H_1(0)] \lim_{s \to 0} \frac{sP_{set}(s)}{1 + C(s)B(s)H_1(s)} = 0$$

We assume the plant provides non-zero error (A type-0 system, for which error can never be zero in case of just feedback network).

So, the required condition:

$$1 - G_f(0)\,B(0)\,H_1(0) = 0$$

$$\Rightarrow G_f(0) = \frac{1}{B(0)\,H_1(0)}$$

We can easily find,

$$H_1(0) = \left.\frac{\cancel{22.22}\ 0.5063\,s + 5.063}{s + 5.443}\right|_{s=0}$$

$$= 0.9302$$

and
$$B(0) = \left.\frac{\omega_n^2}{s^2 + \zeta\omega_n s + \omega_n^2}\right|_{s=0}$$

$$= 1$$

$$\therefore G_f(0) = \frac{1}{0.9302 \times 1} = 1.075$$

The utilized $G_f'(0)$ in the paper is 1, which introduces $(1 - 1 \times 0.9302) = 0.0698$ factor reduction of SSE coming from the feedback system, which is very much acceptable

Therefore, a feed-forward system reduces SSE if set properly, no matter the 'Type' of the system.

If we note the necessity of a feedback control, it can be concluded that the feedback introduces an error value between the input and output to the system which can control the response of the system. The system's response moves faster or slower based on the plant and controller, but the error makes the output follow the input set value. Without the feedback, the output may go out of bounds or in other words become unstable as it cannot get any information or correction based on how it is performing.

Task 5 :

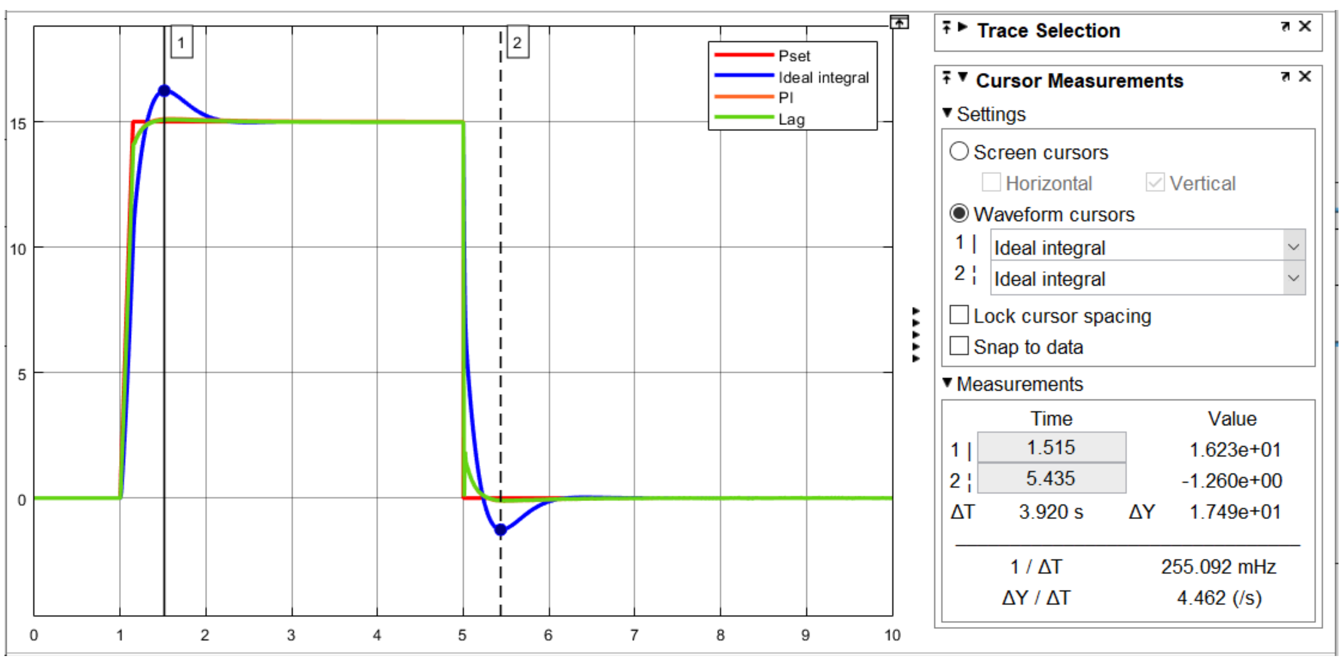
In this configuration, the feedback controller is portrayed as a perfect integrator. Would you lean towards choosing a PI controller or a lag controller? What are your reasons for favoring or not favoring either option?

Ideal integral: 5/s

PI: 5(s+2)/s

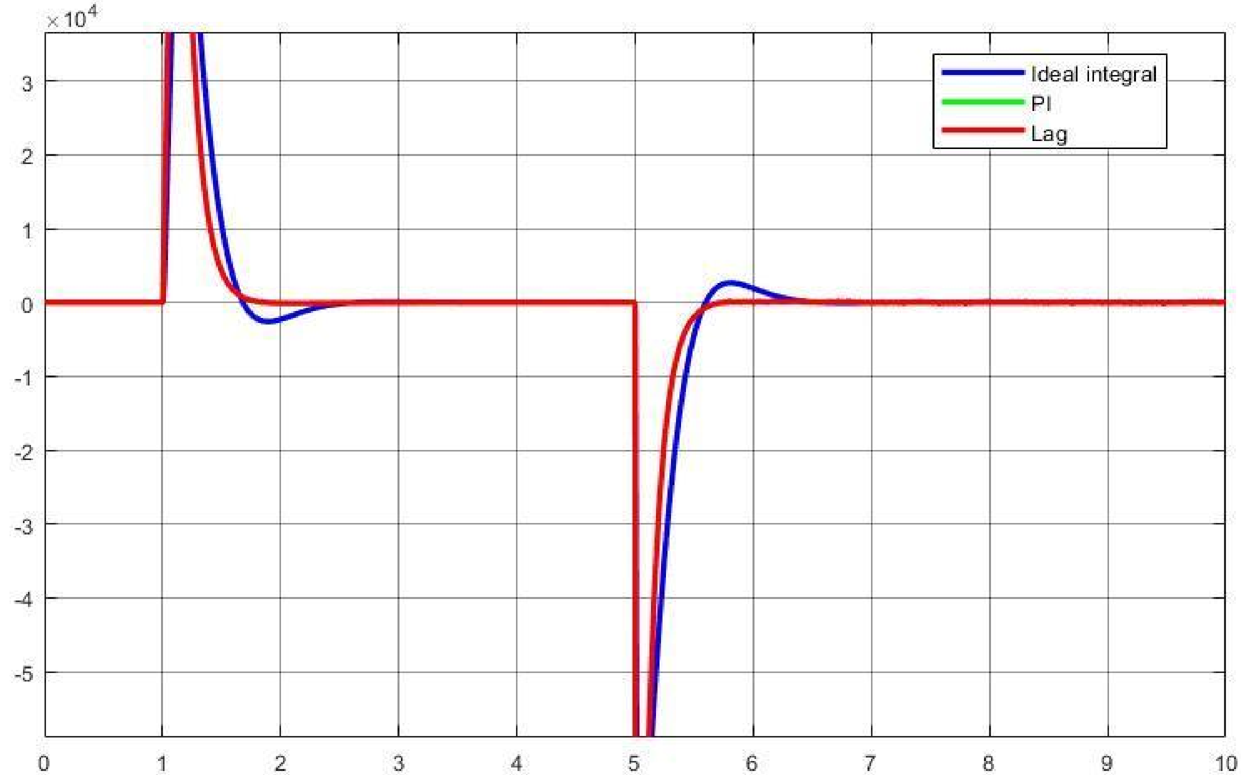
Lag: 5(s+2)/(s+0.2)

P_{aw} :



Overshoot is observed in the case of the ideal integral controller.

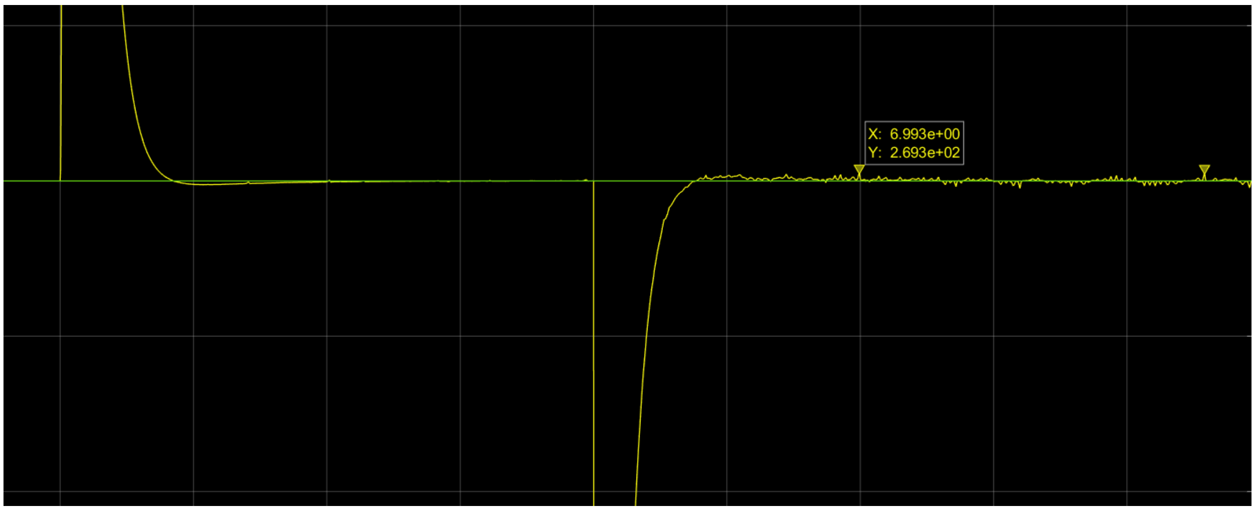
Q_{pat} :



At first glance, opting for a PI controller (as opposed to a pure integral controller) appears to address the issue of false triggering. Additionally, a PI controller eliminates steady-state errors effectively. However, it introduces the downside of providing insufficient roll-off for frequencies beyond the defined bandwidth, which is generally regarded as undesirable.

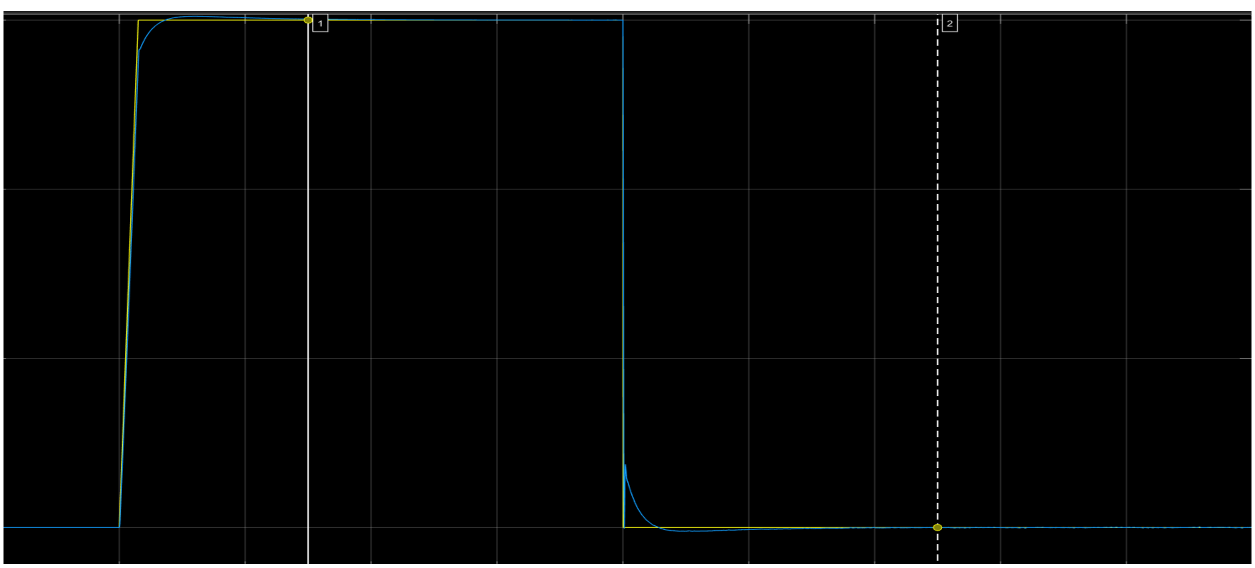
For pi = 5(s+2)/s

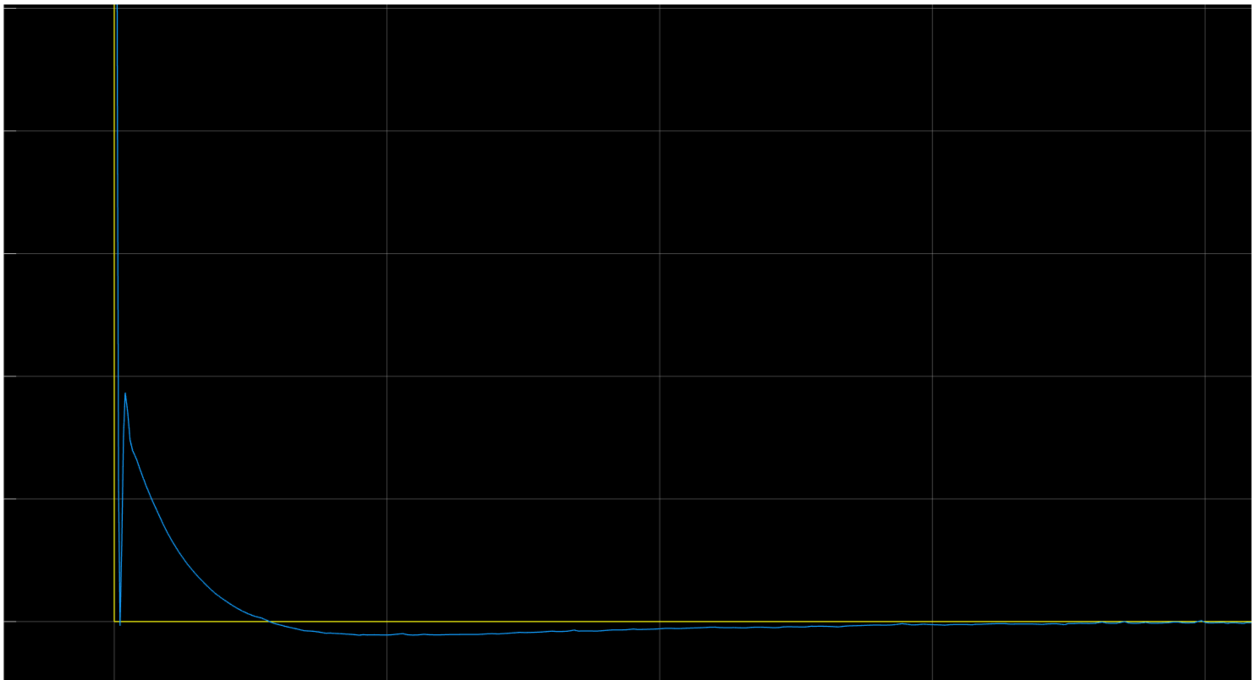
(air flow graph):



(low-frequency disturbance is observed)

AIR PRESSURE GRAPH:

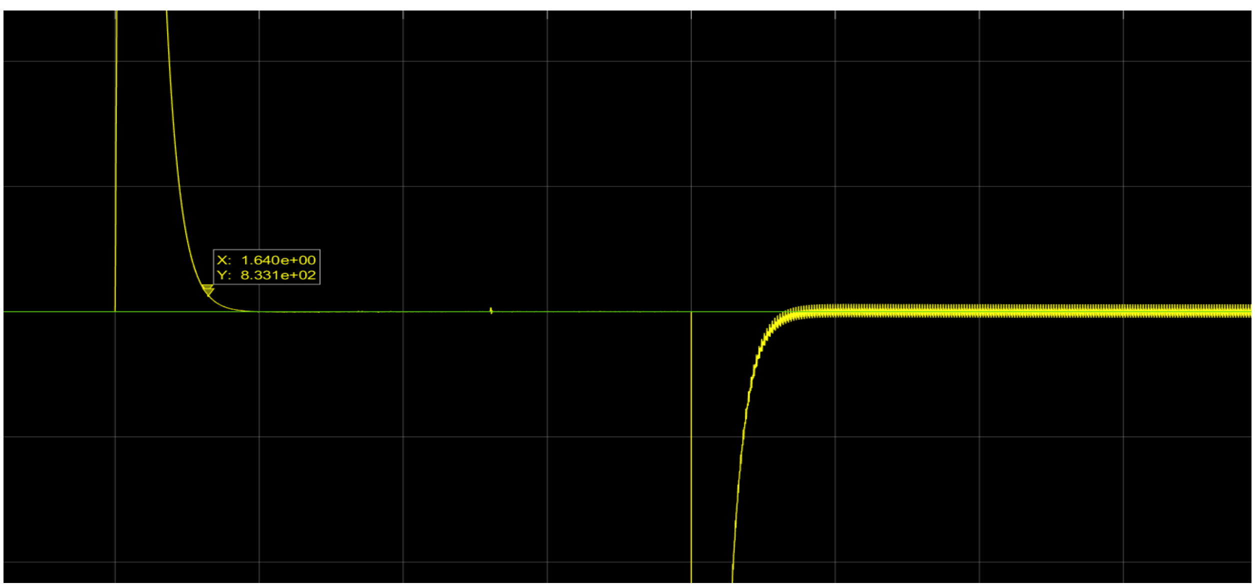




Which is considered undesirable. This implies that as the phase angle decreases, it experiences sudden increments, followed by decreases, possibly leading it to eventually reach zero.

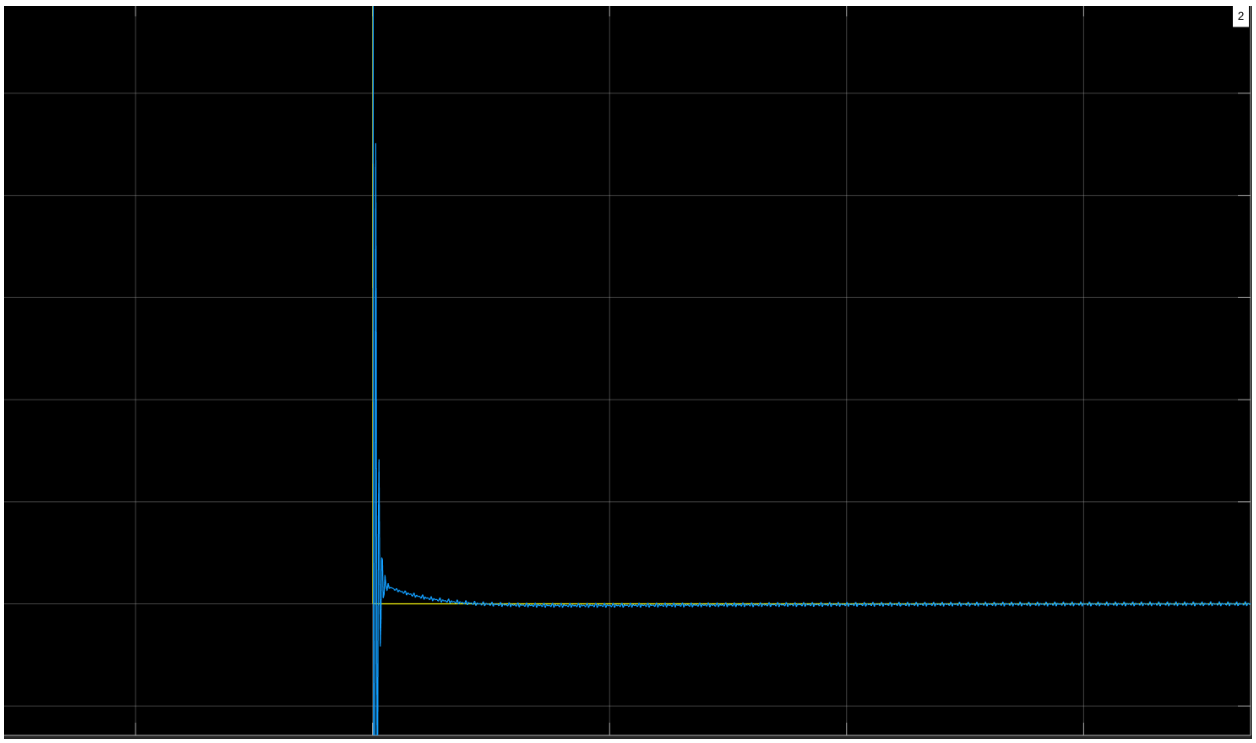
For PI=50(s+2)/s:

FOR AIR FLOW GRAPH:



(low-frequency disturbance is observed)

AIR PRESSURE GRAPH:(DOWN SLOPE)



WHICH IS NOT DESIRABLE.

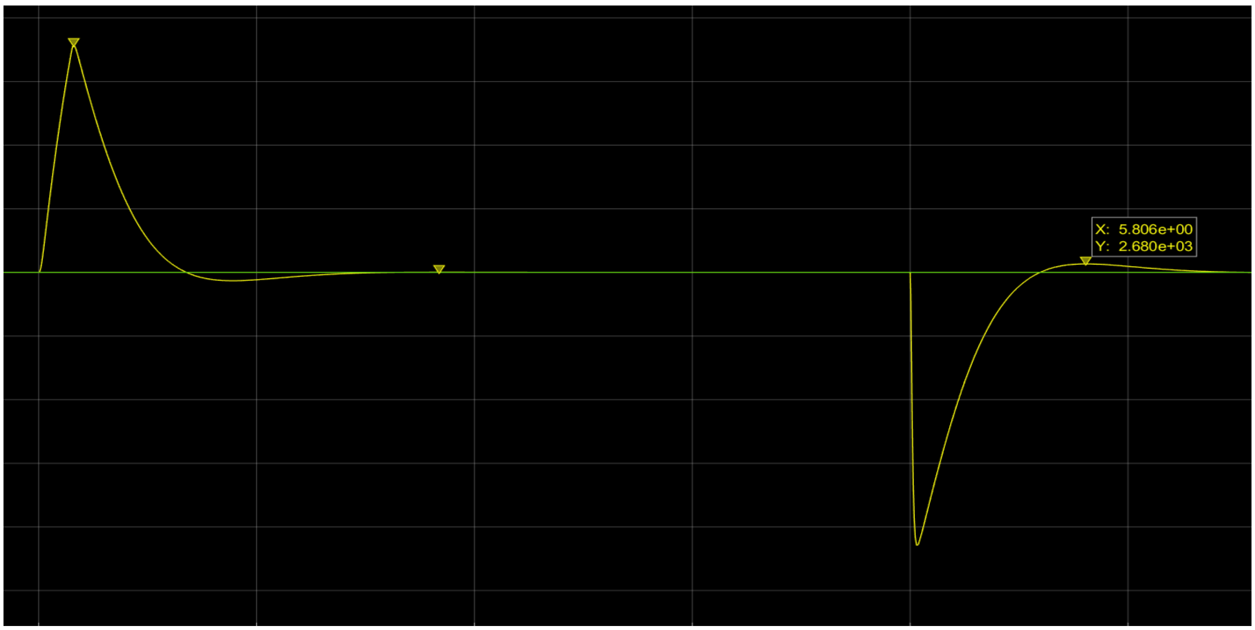
FOR PURE INTEGRAL CONTROLLER:

False triggering is a problem (using variable gain controller this problem can be solved,next part we will see) . PURE INTEGRAL CONTROLLER results in low-frequency disturbance suppression, high frequencyroll-off, and a stabilizing −1 slope across the bandwidth of the system.

For :

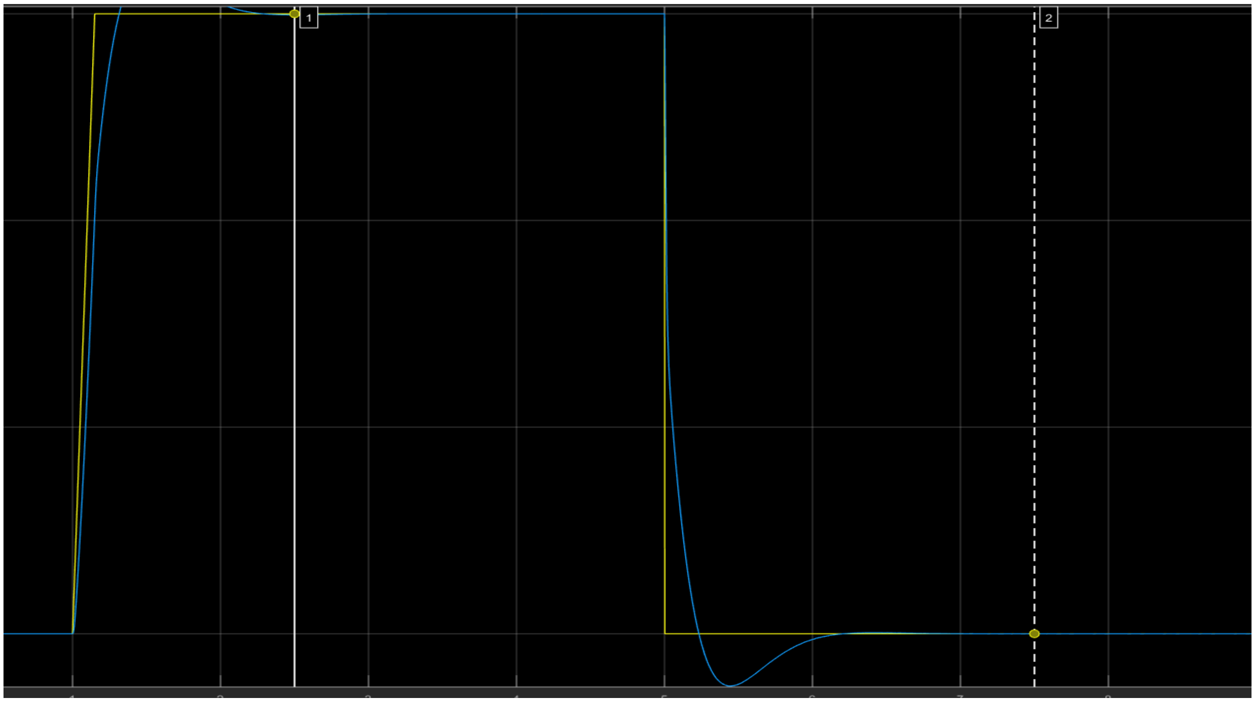
C= 5/s;

Air flow graph:



low-frequency disturbance is suppressed which is a good for the system.

Air Pressure graph:

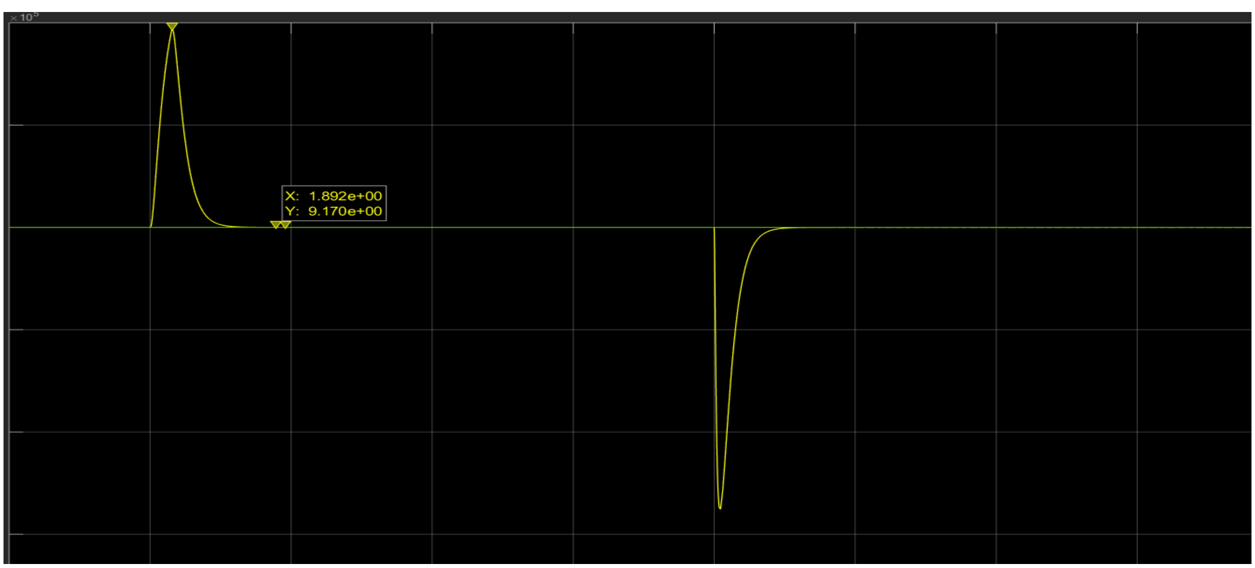


Much better than pi controller .(don't consider overshoot)

For :

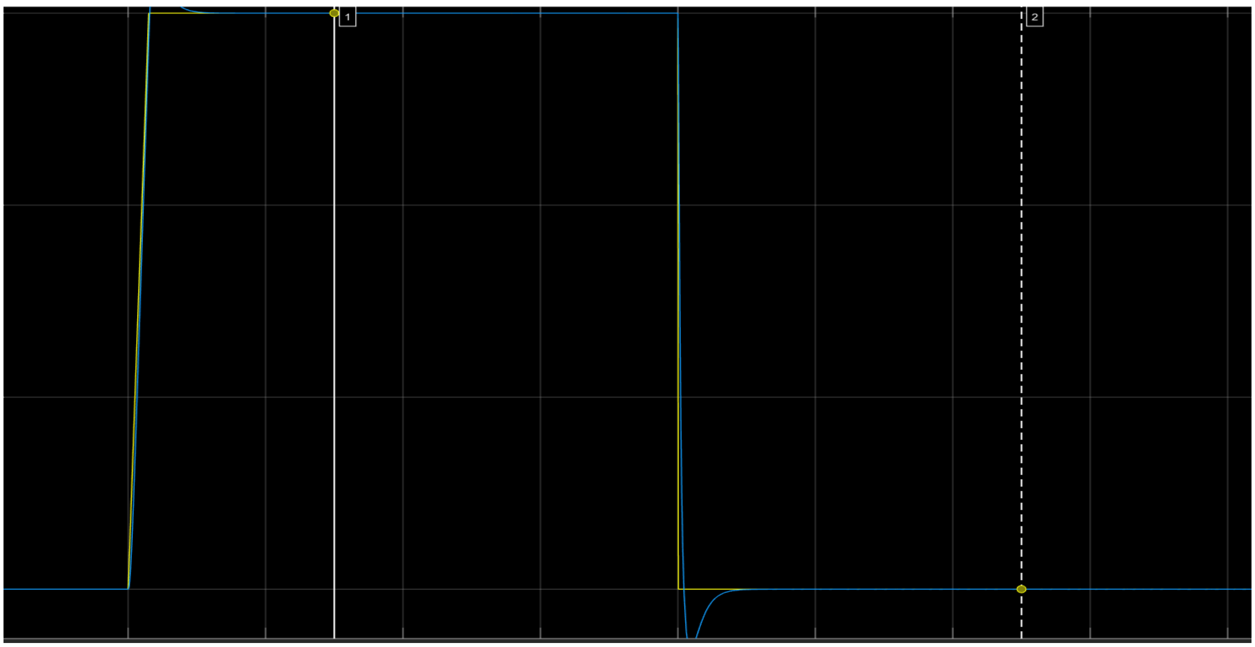
C= 50/s;

Air flow graph:



low-frequency disturbance is suppressed which is a good for the system. As well as ,no overshoot.

Air Pressure graph:



Much better than pi controller .(don’t consider overshoot)

For both pure integral & PI controller steady state error is 0.but in case of lag controller is can’t be 0. So, pure integral is suitable .

Task 6 :

Design your preferred linear controller in order to meet the specifications stated in page 166 between column 1 and 2.

First we will take a system with a linear proportional controller without any feedforward. But we need to find the value of the proportional constant to meet the requirements. The system looks like a Type-0 system. So, it's steady state error can be calculated as,

$$e(\infty) = \frac{1}{1+G(s)} = \frac{1}{1+KBH1} = \frac{1}{1+0.9302K}$$

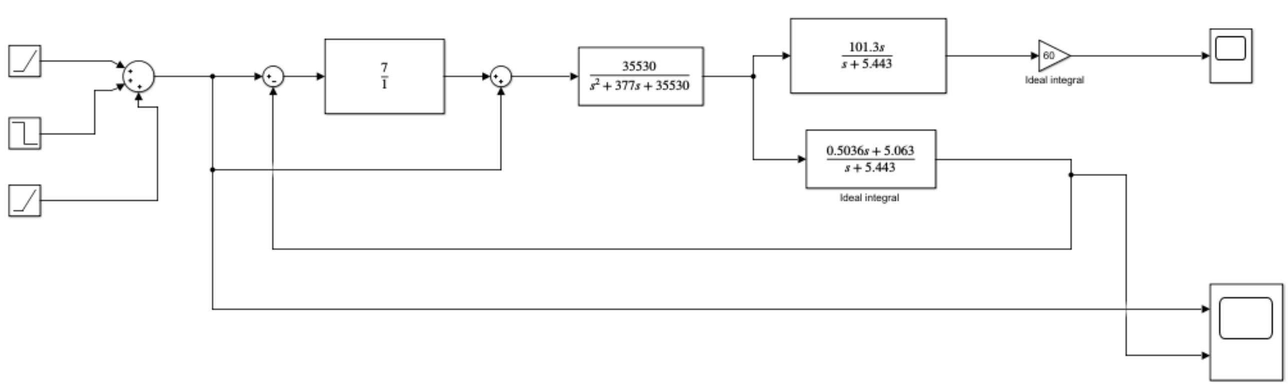
The value has to be within 15±2 mbar.

The system is a Type-0 system and the response will settle the value below 15 mbar. We take the steady state value will be (15-2) = 13 mbar.

$$\text{So, } e(\infty) = \frac{15-13}{15} = \frac{2}{15} = 0.1333$$

$$\frac{1}{1+0.9302K} = \frac{2}{15}\ ; \quad \text{Solving the value of K we get, } K = 6.9877 \approx 7$$

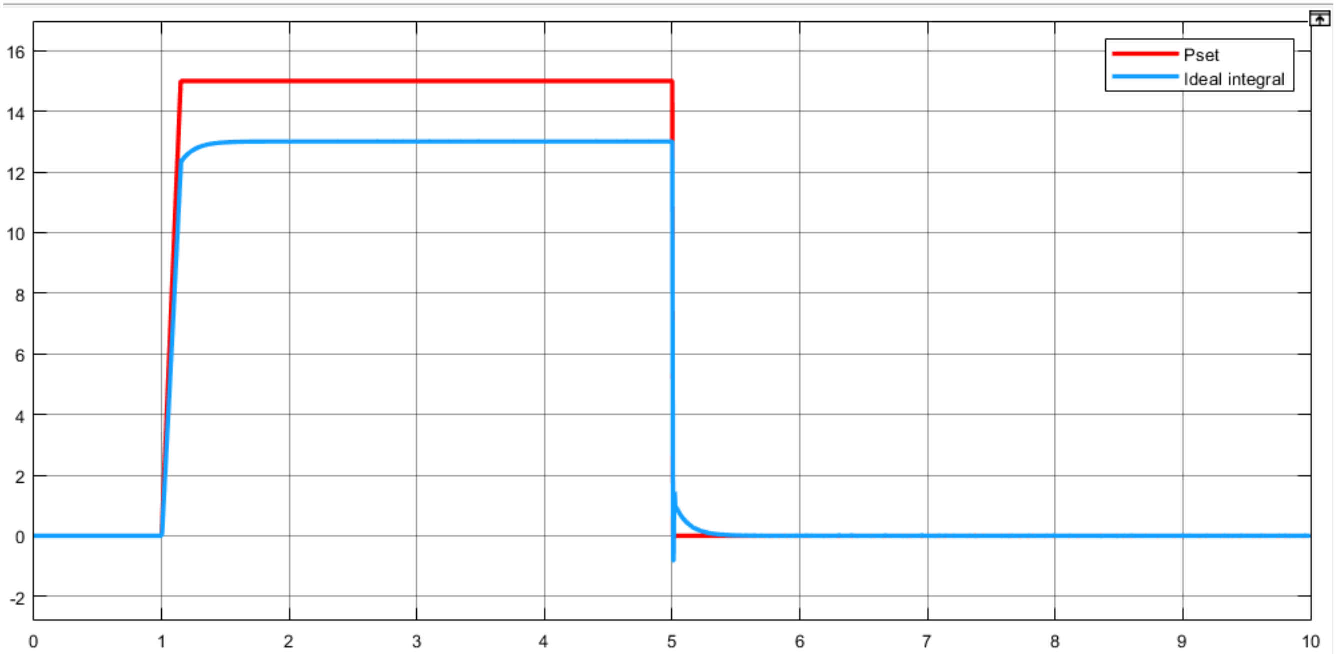
Thus, we get the system as below figure :

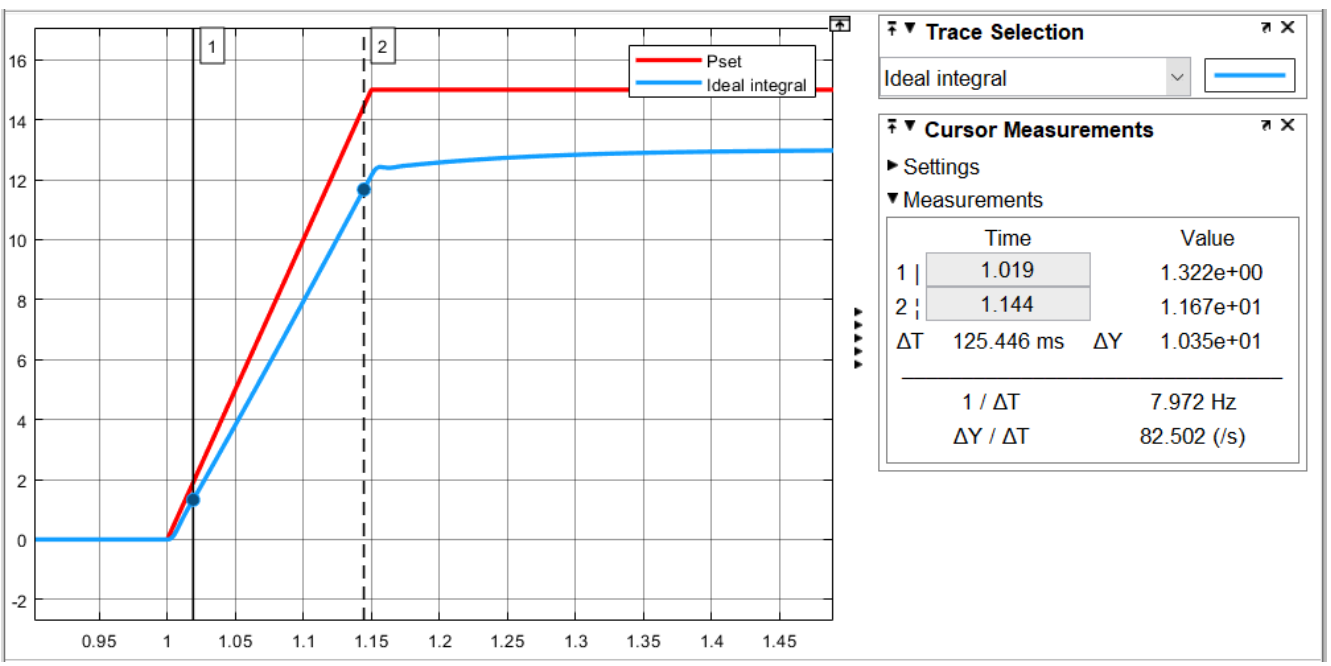


**This is the circuit with the feedforward path.

Output without feedforward :

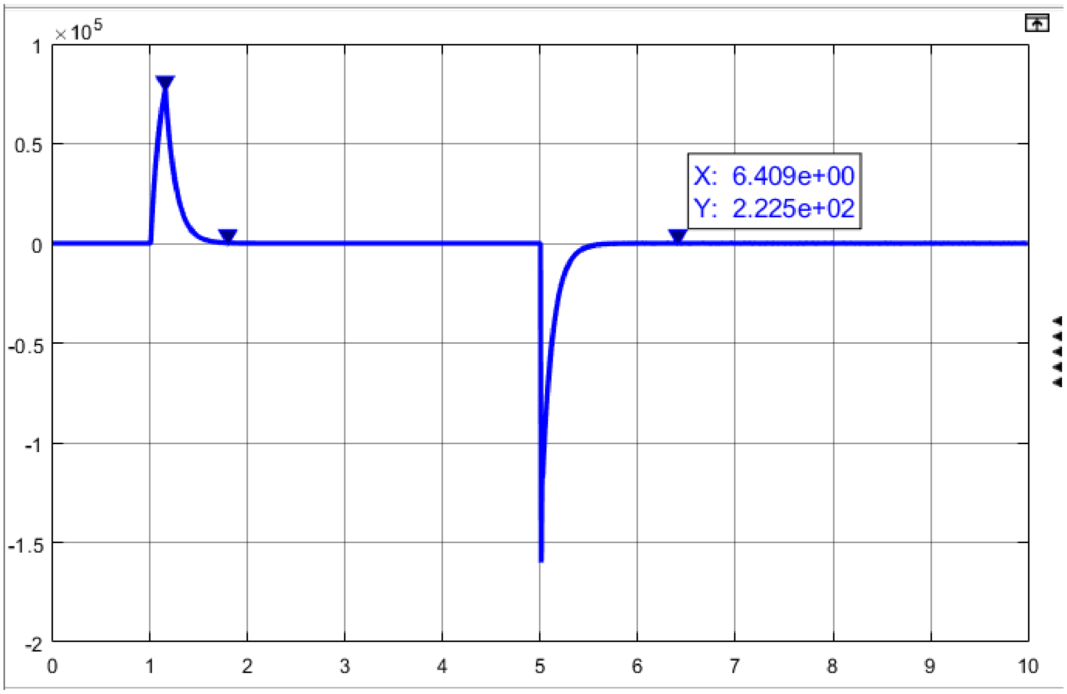
P_{aw} :





Here rise time = 125.446 ms.

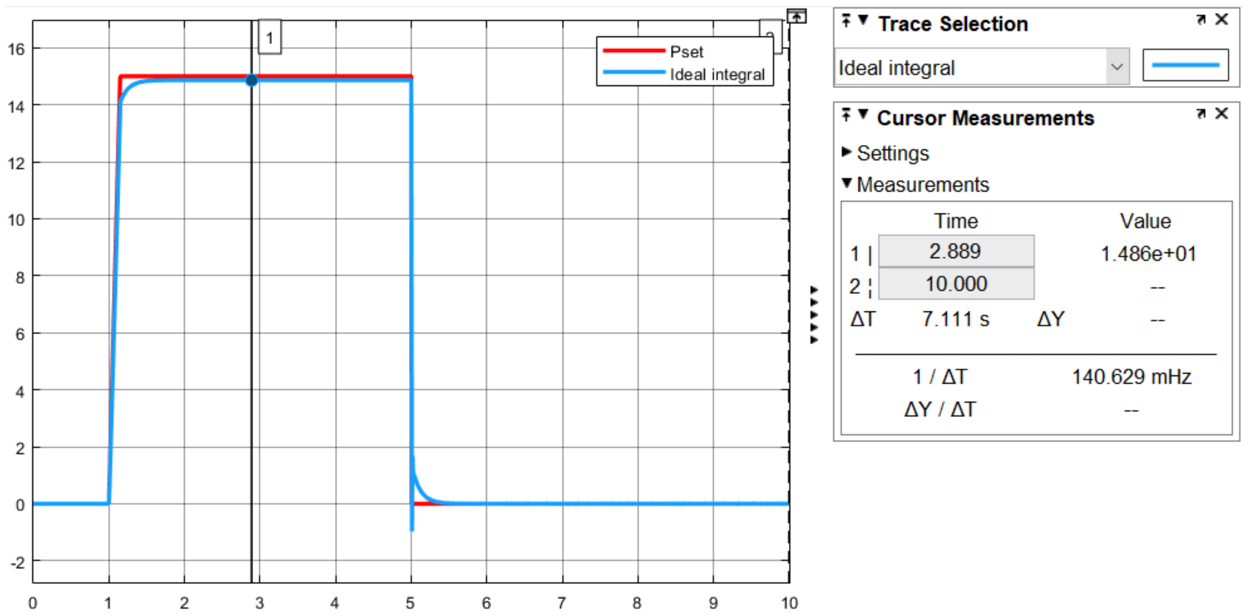
Q_{pat} :



Final value of Q_{pat} is 222.5 mL/min (0.2225 L/min).

This matches the requirment.

Output after adding feedforward :

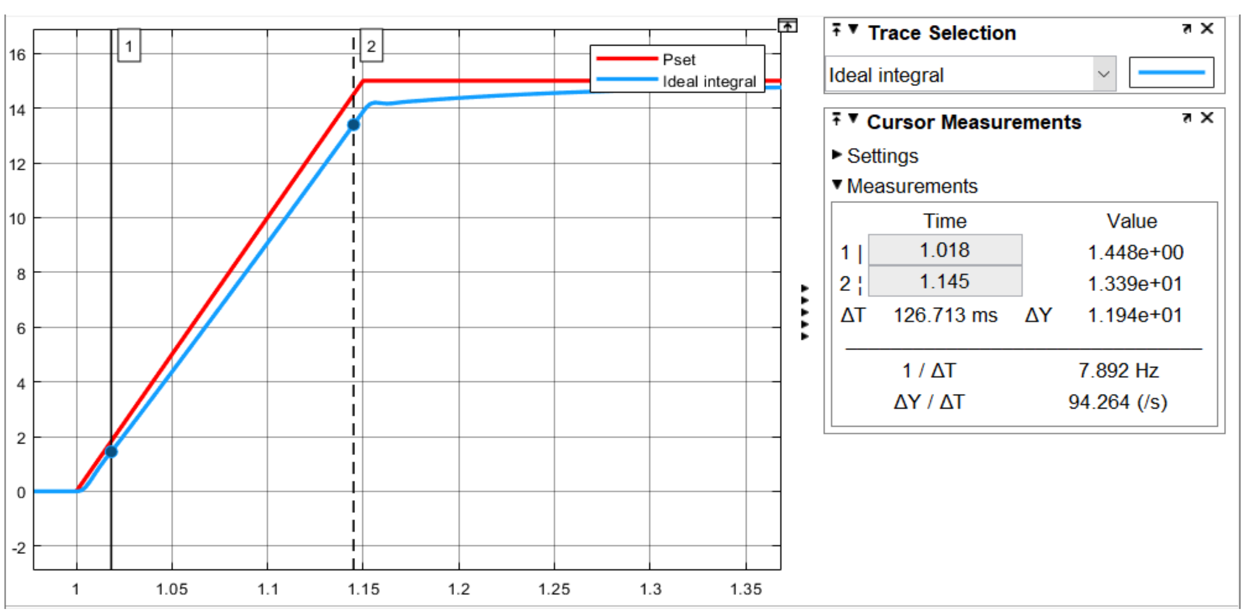


Final value is 14.86.

From "Task-4 (Necessity of feedback and feedforward system)", the error should reduce by 0.0698 factor after adding the feedforward path. The previous error without feed-forward was $\frac{2}{15} = 13.333\%$ and now the error should be $= 0.0698 \times 13.333\% = 0.9306\%$. This means the new steady-state output should be:

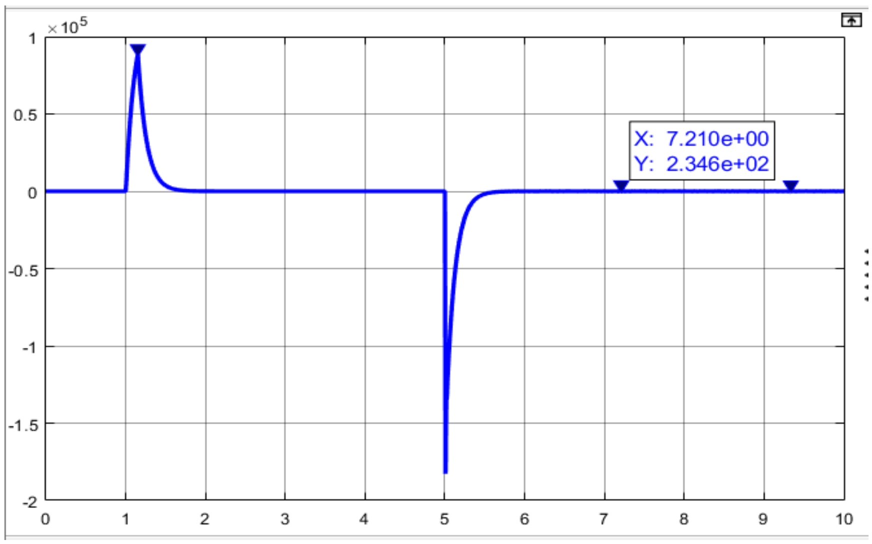
$$15 \times (1 - \frac{0.9306}{15}) = 14.86041$$

This value is obtained from the feedforward system.



Rise time = 126.713 ms.

Q_{pat} :

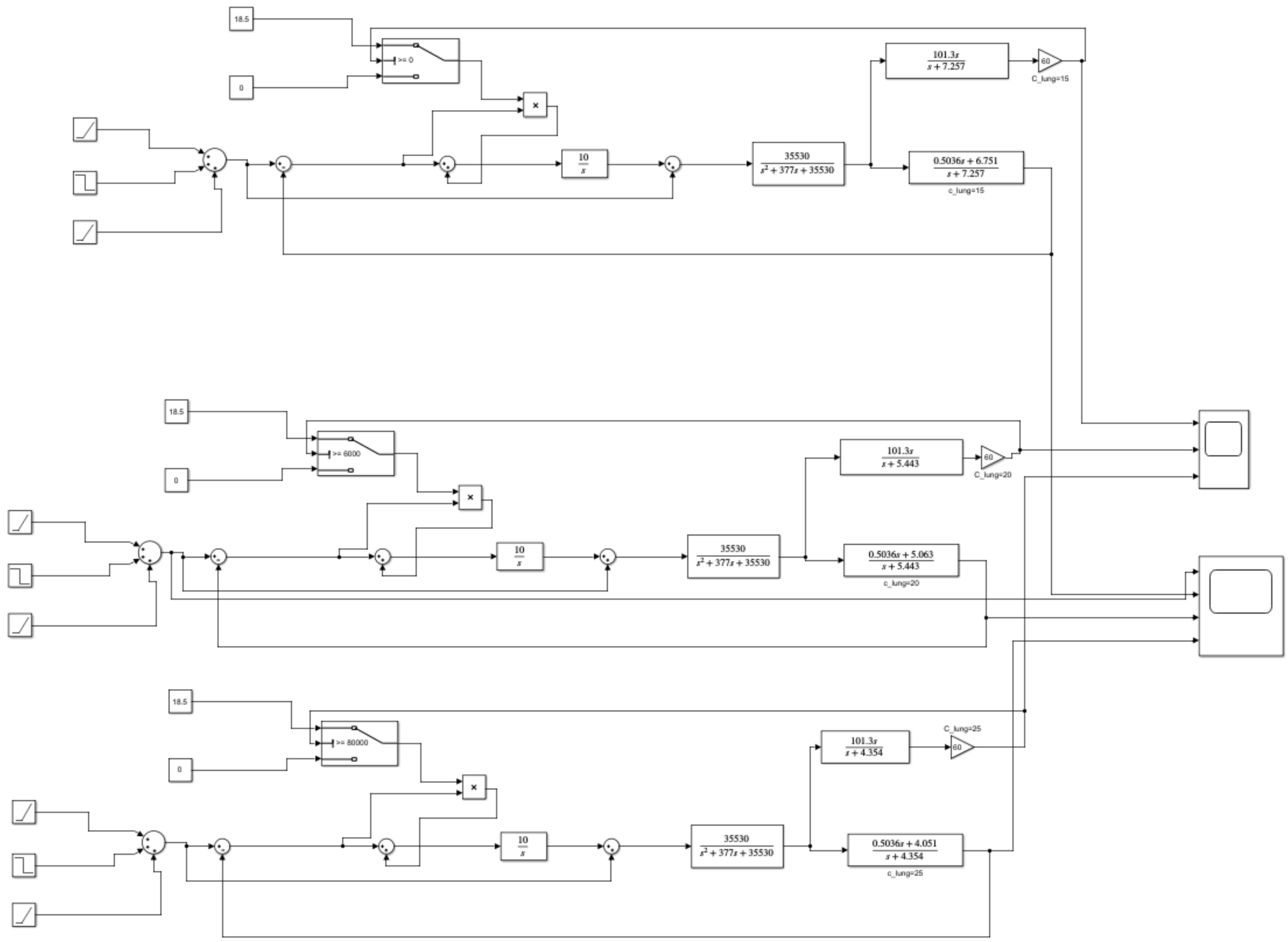


Final value 234.6 mL/min (0.2345 L/min)

Task 7 :

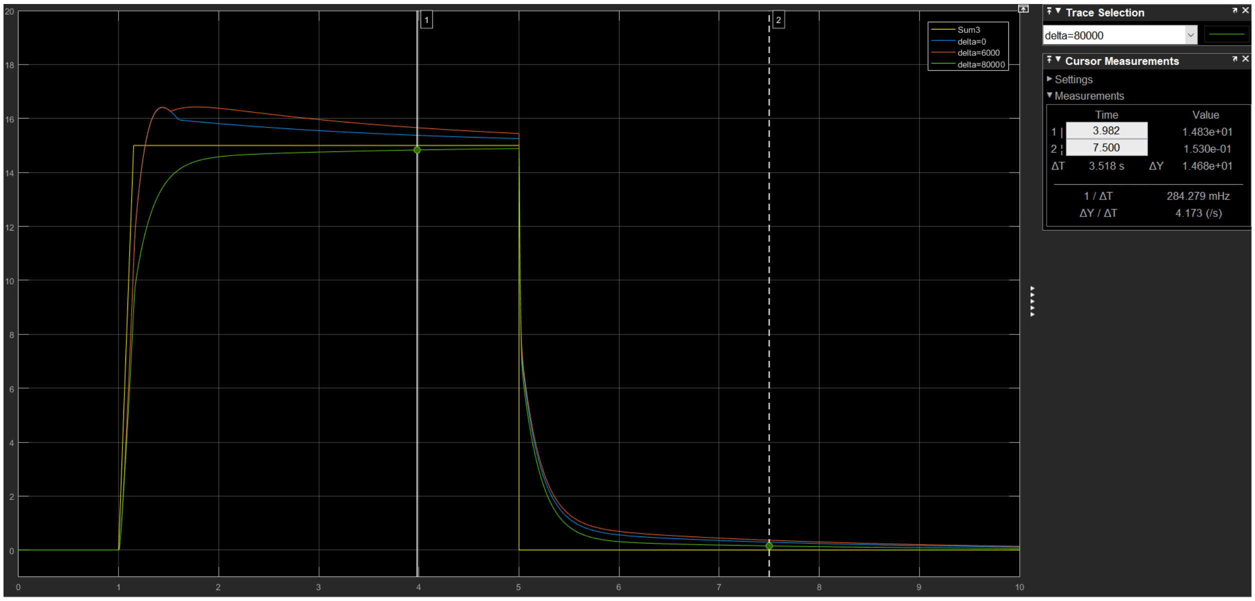
Reproduce the results shown in Fig. 14 for linear and variable gain controllers. What are the pros and cons of nonlinear control over linear control?

Circuit Diagram :

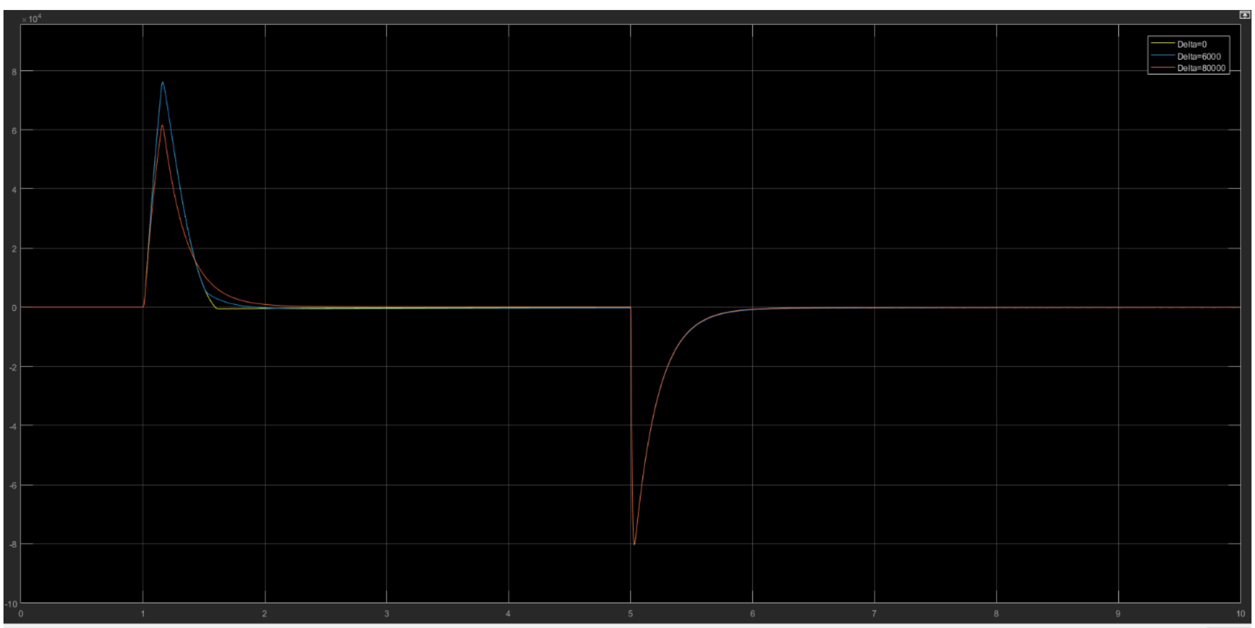

18.5
0
>= 0
101.3s
s + 7.257
60
C_lung=15
10
s
35530
s^2 + 377s + 35530
0.5036s + 6.751
s + 7.257
c_lung=15
18.5
0
>= 6000
101.3s
s + 5.443
60
C_lung=20
10
s
35530
s^2 + 377s + 35530
0.5036s + 5.063
s + 5.443
c_lung=20
18.5
0
>= 80000
101.3s
s + 4.354
C_lung=25
60
10
s
35530
s^2 + 377s + 35530
0.5036s + 4.051
s + 4.354
c_lung=25

Outputs :

P_{aw} :



Q_{pat} :



We see the results are almost similar to that of the report.

Pros & Cons :

Non-Linear Controller Pros:
In case of a linear control system, if the gain is increased, the system becomes faster, but an overshoot occurs. If the gain is decreased to minimize the overshoot the response becomes slower.
This trade-off between high and low gain cannot be adjusted automatically in case of a linear control system. For the compensation of this gain control, non-linear control systems can be very effective. In case of a non-linear control system, this trade-off is handles by a switching device which adds additional gain to the controller when needed based on a threshold value preset in the controller. This type of control system can effectively eliminate the overshoot occurrence and the slow response problem

Non-Linear Controller Cons:
Non-linear control system also comes with its own cons. From the figures above, it can be observed that for the optimal value of delta obtained from the figure of $\boldsymbol{Qpat}$, introduces a high error in the pressure $\boldsymbol{Paw}$. As the feedback to the switch is coming from the $\boldsymbol{Qpat}$, the switch is not effectively optimized for $\boldsymbol{Paw}$. This is not a problem for the paper’s specifications, but can be an undesired outcome for any other systems with different preferences.

Task 8 :

Discuss the performance of both linear and nonlinear control systems in presence of uncertainties such as different lung parameters, pressure drop etc.

Variable gain:

Varying C_lung

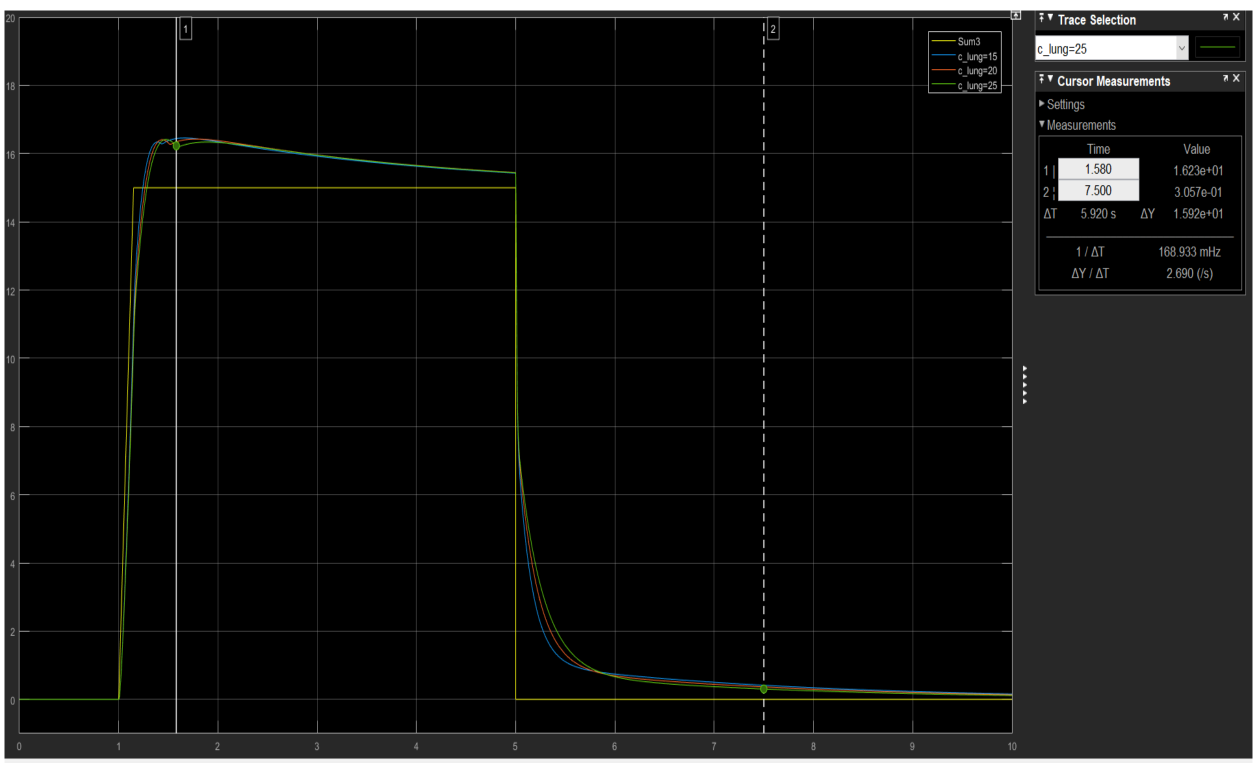
We took 3 values for C_lung .

C_lung=15

C_lung=20 and

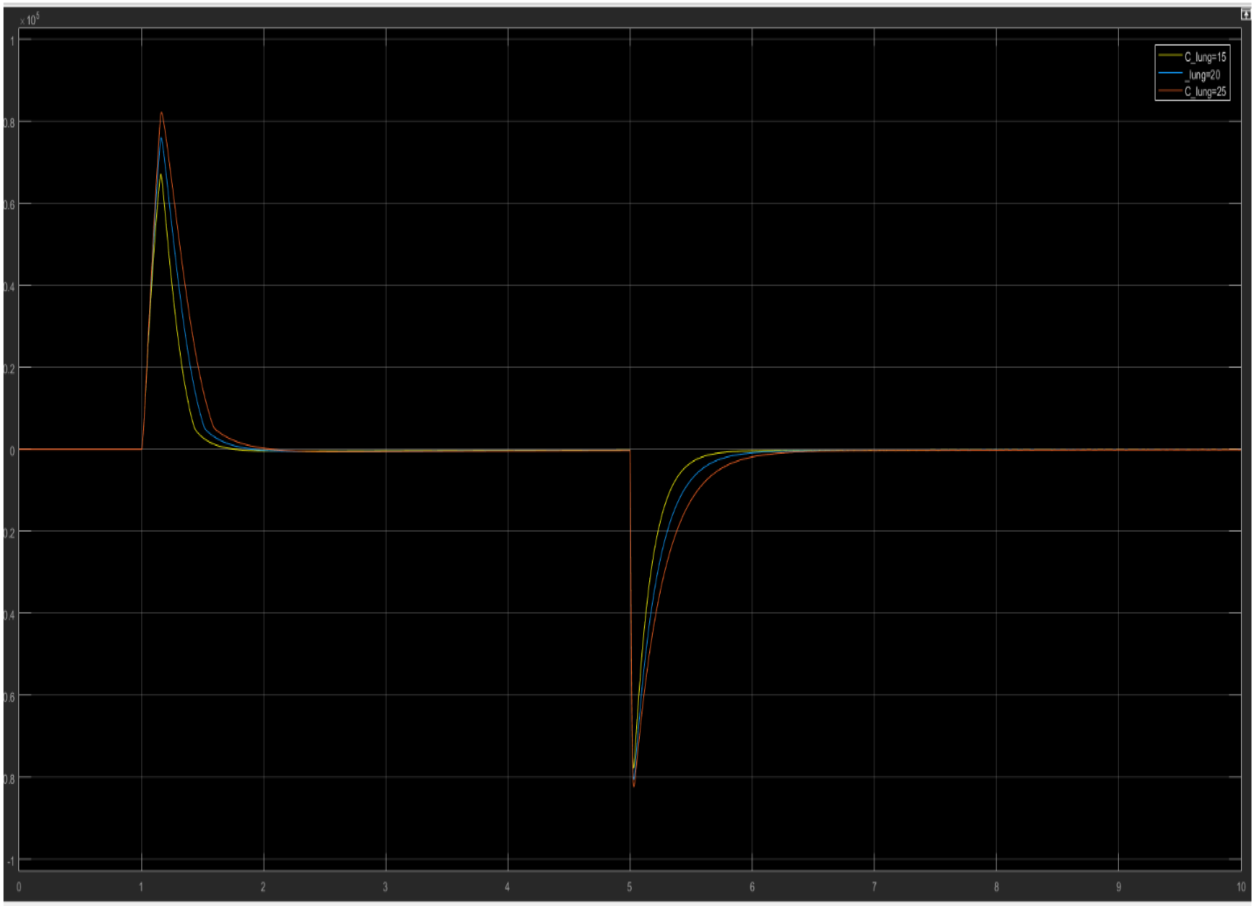
C_lung=25

P_aw:



We can see overshoot in the rising part for all values of C_lung. One thing to notice as the value of C_lung increases, the system becomes a bit slower.

Q_pat:



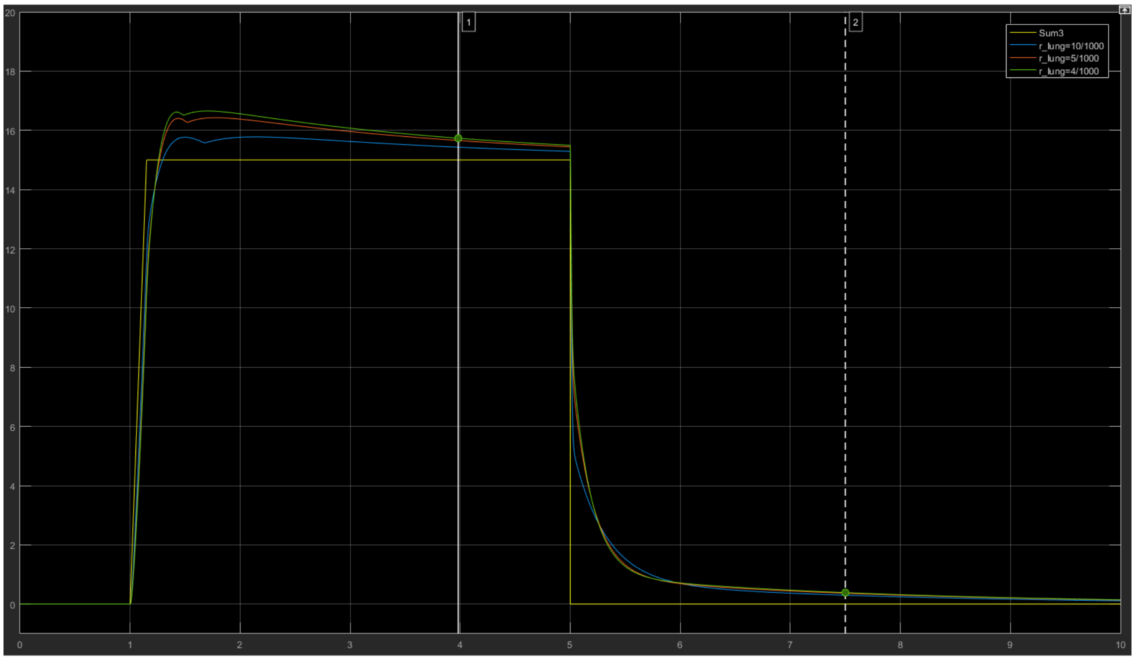
Increasing C_lung makes response slower.

Varying R_lung:

We took 3 different values of R_lung.

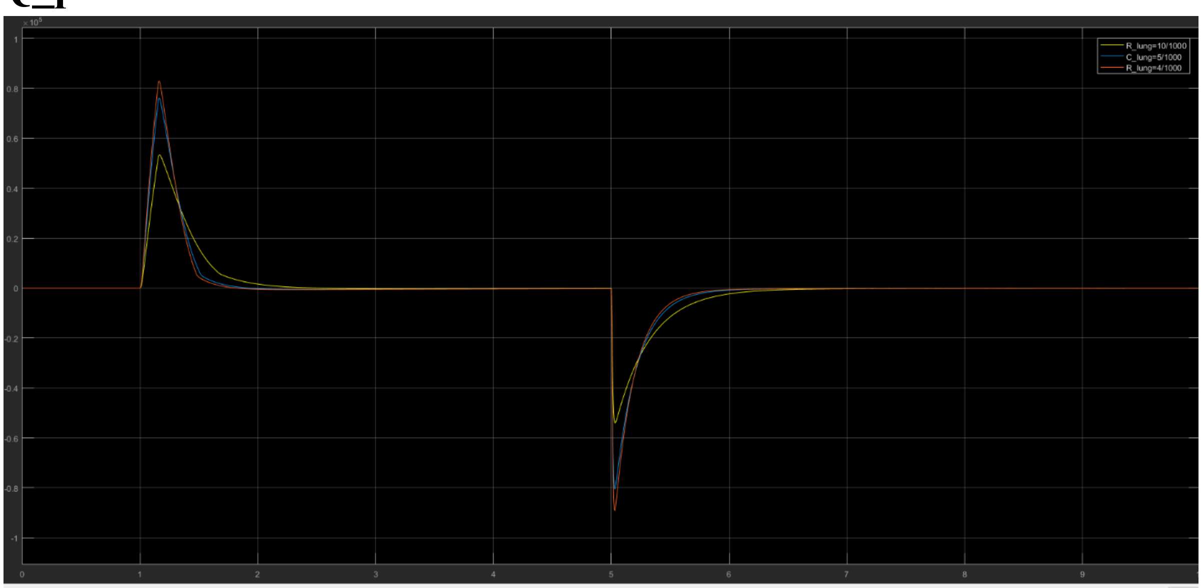
R_lung=10/1000, 5/1000, 4/1000.

P_aw:



As the value of R_lung decreases, we see more overshoot.

Q_pat:



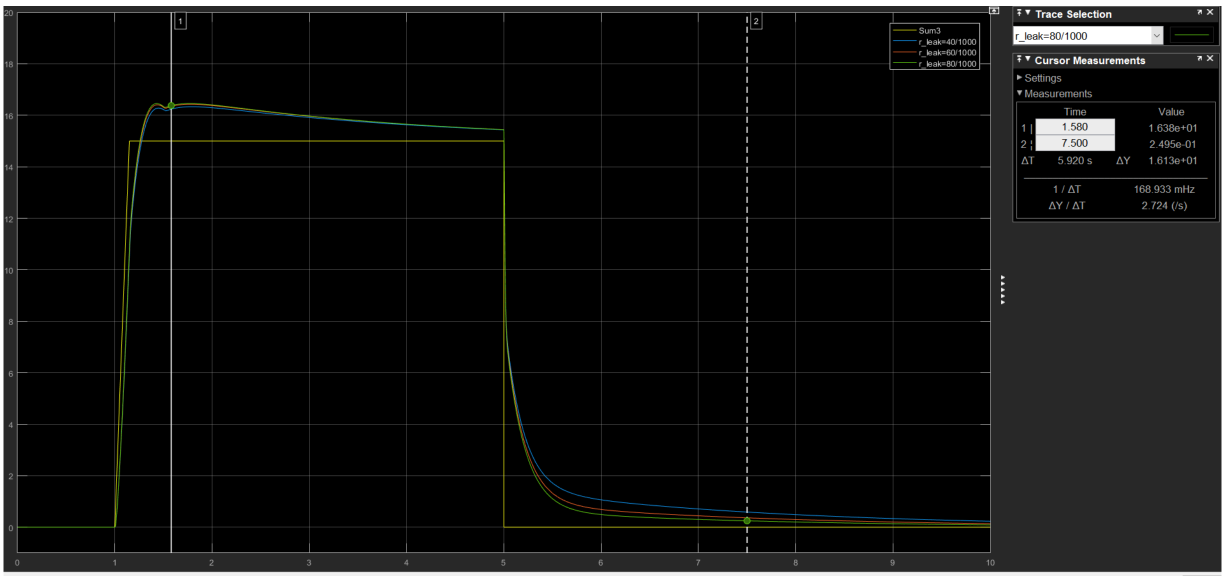
Increasing R_lung makes response slower.

<u>Varying R_leak:</u>

We took 3 values of r_leak.

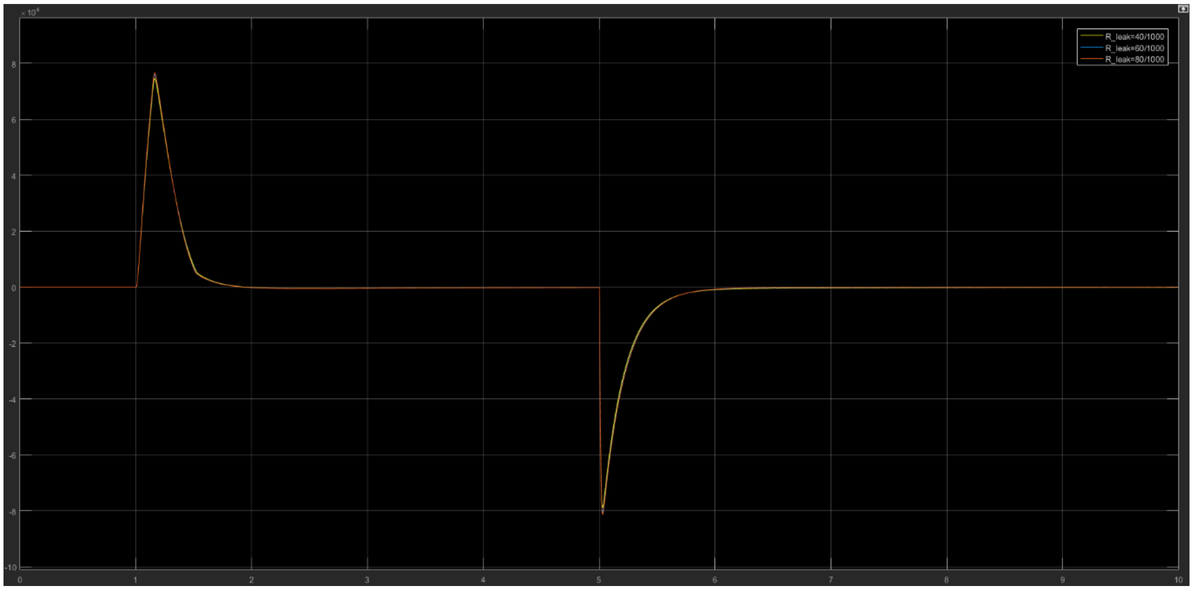
R_leak=40/1000, 60/1000, 80/1000.

<u>P_aw:</u>



In the rising part we don’t see that much of difference but in the falling part we notice as the value of R_leak increases, system becomes faster.

Q_pat:



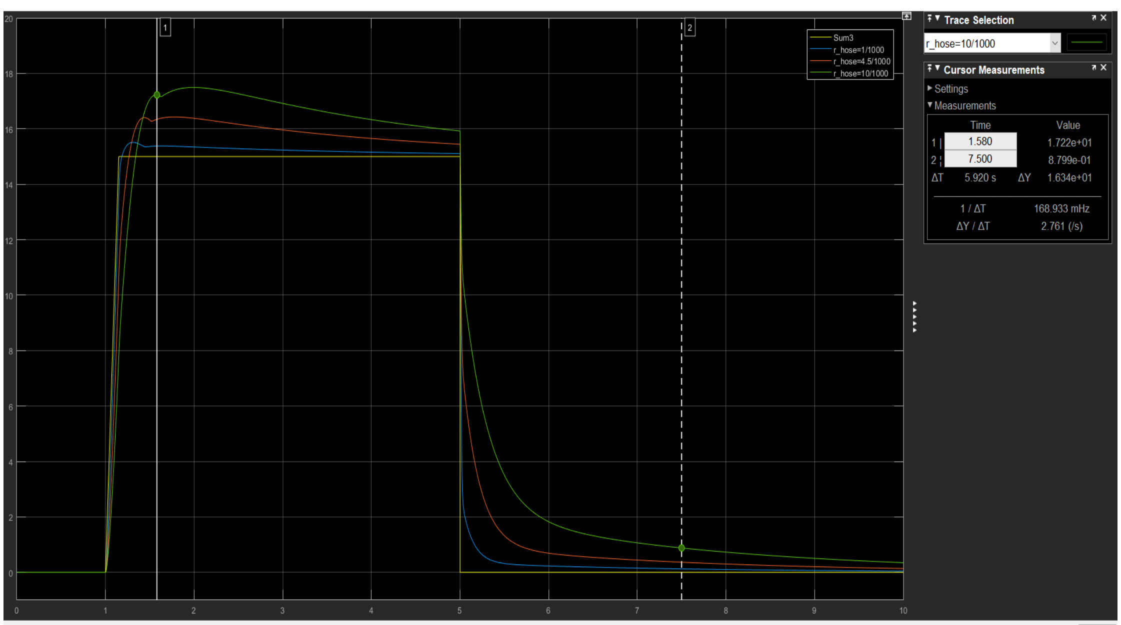
Not that much of difference in the response.

Varying R_hose:

We took 3 values of R_hose.

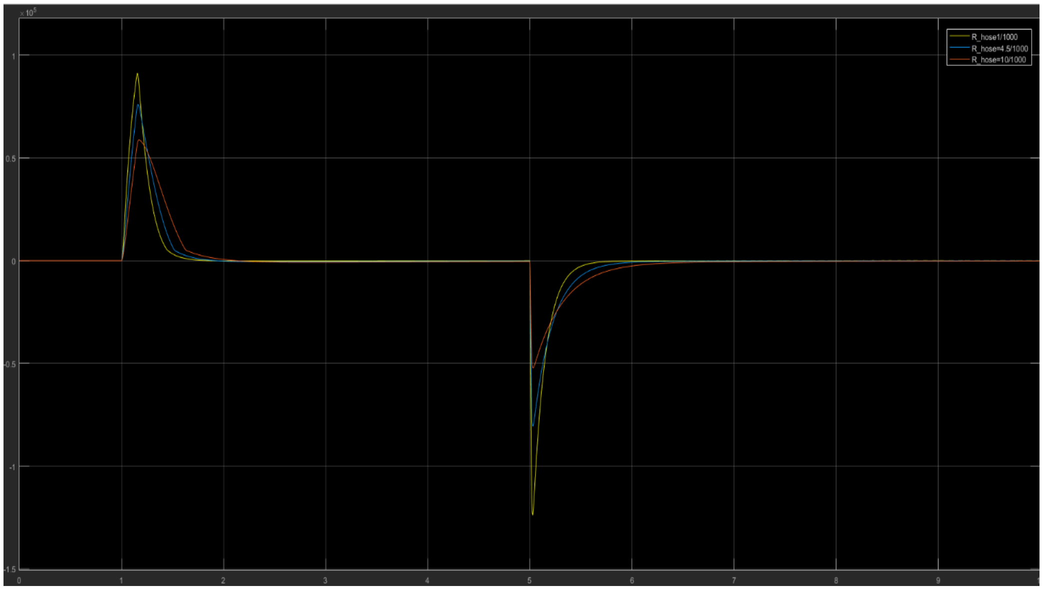
R_leak= 1/1000, 4.5/1000,10/1000

P_aw:



As R_hose increases we see more overshoot.

Q_pat:



As R_hose increases, system becomes slower.

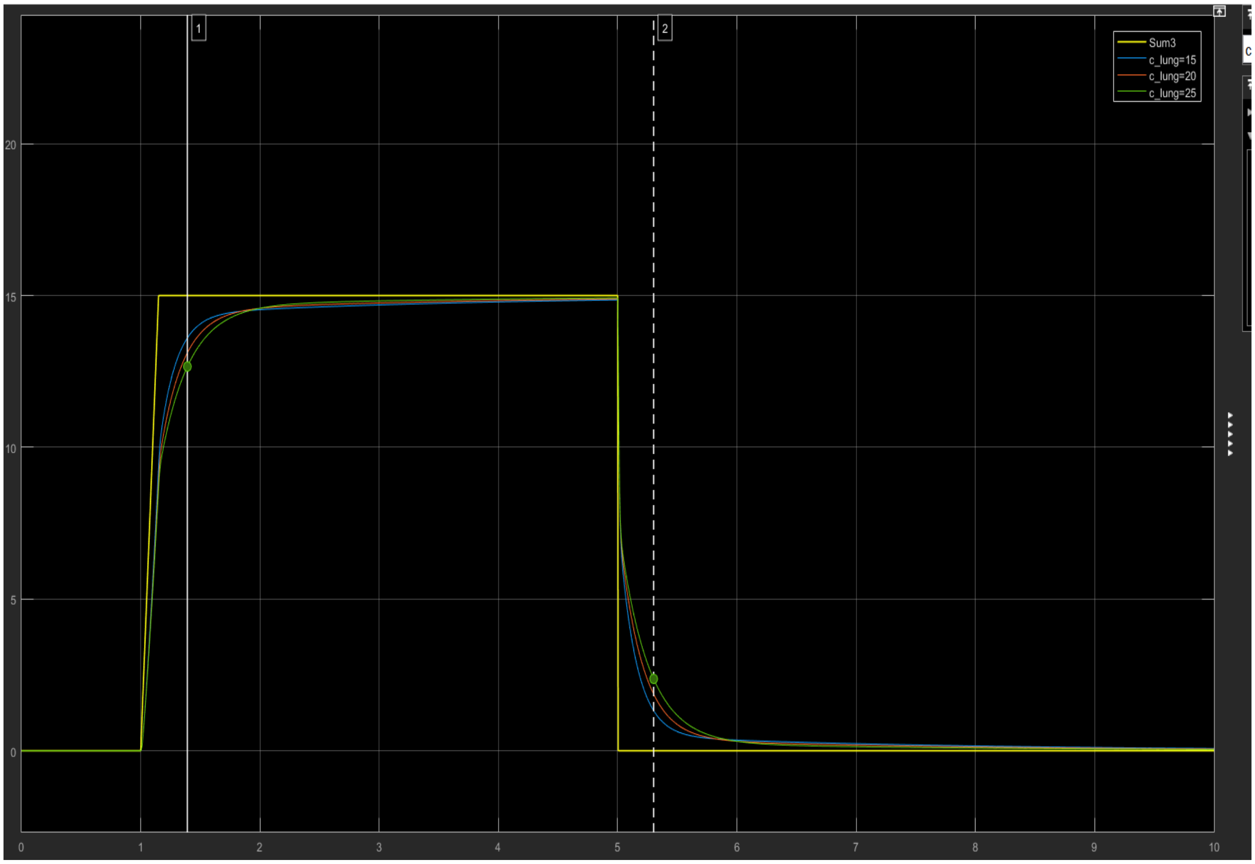
Linear controller:

C_lung:

We took 3 values for C_lung .

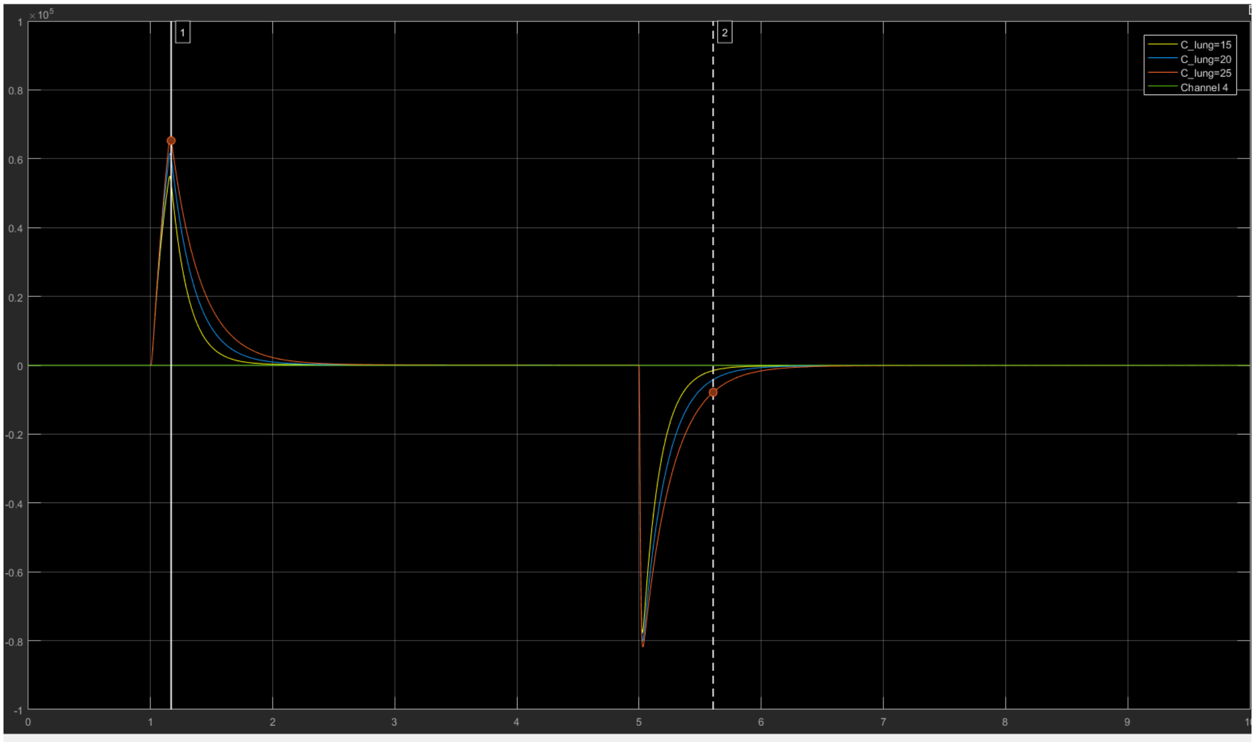
C_lung=15, 20 and 25

P_aw:



As C_lung increases, system becomes slower.

Q_pat:



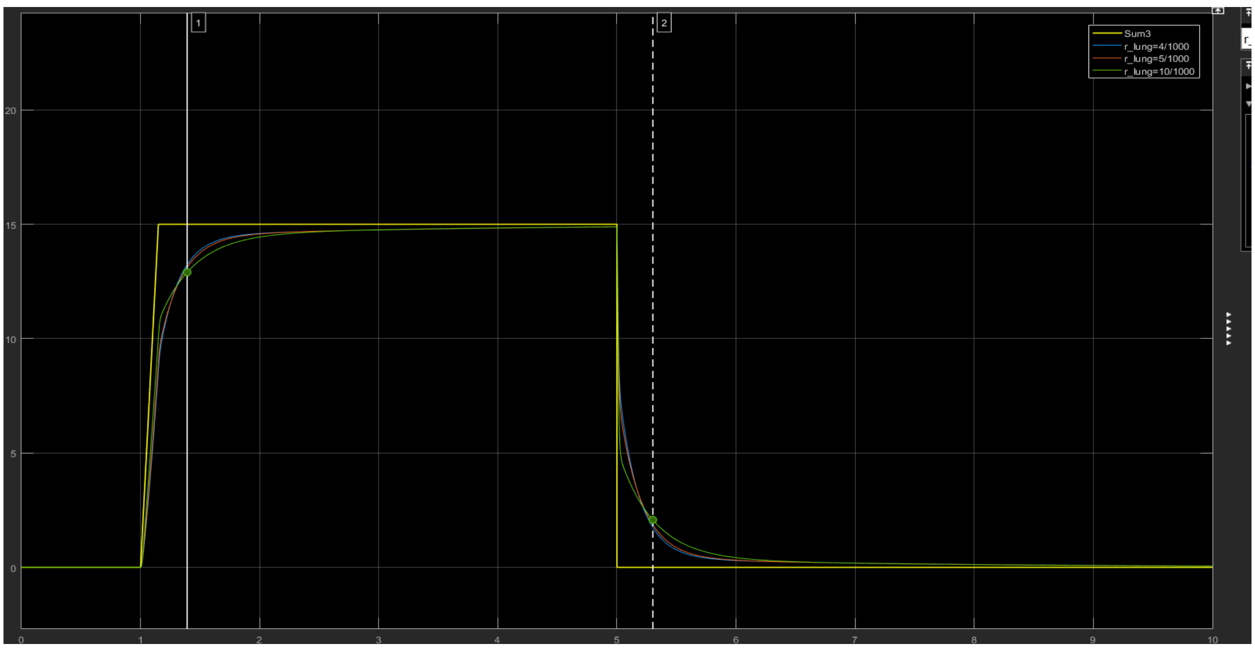
As C_lung increases, system becomes slower.

R_lung:

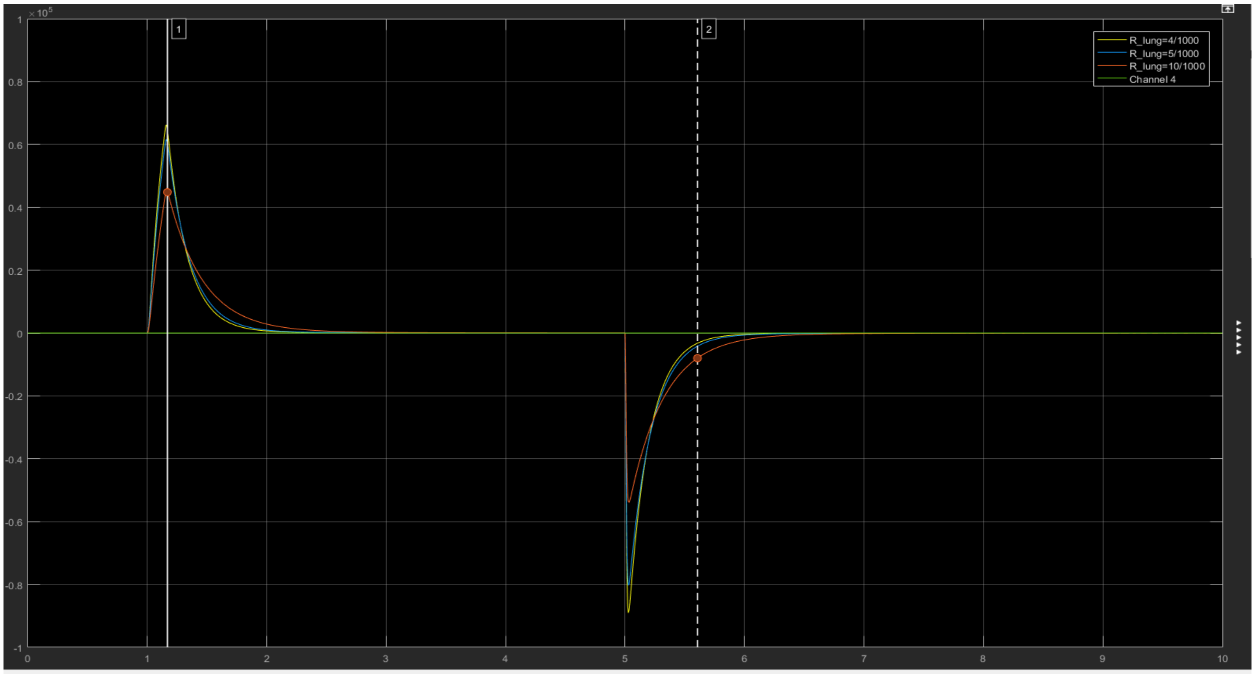
We took 3 different values of R_lung.

R_lung=10/1000, 5/1000, 4/1000.

P_aw:



Q_pat:



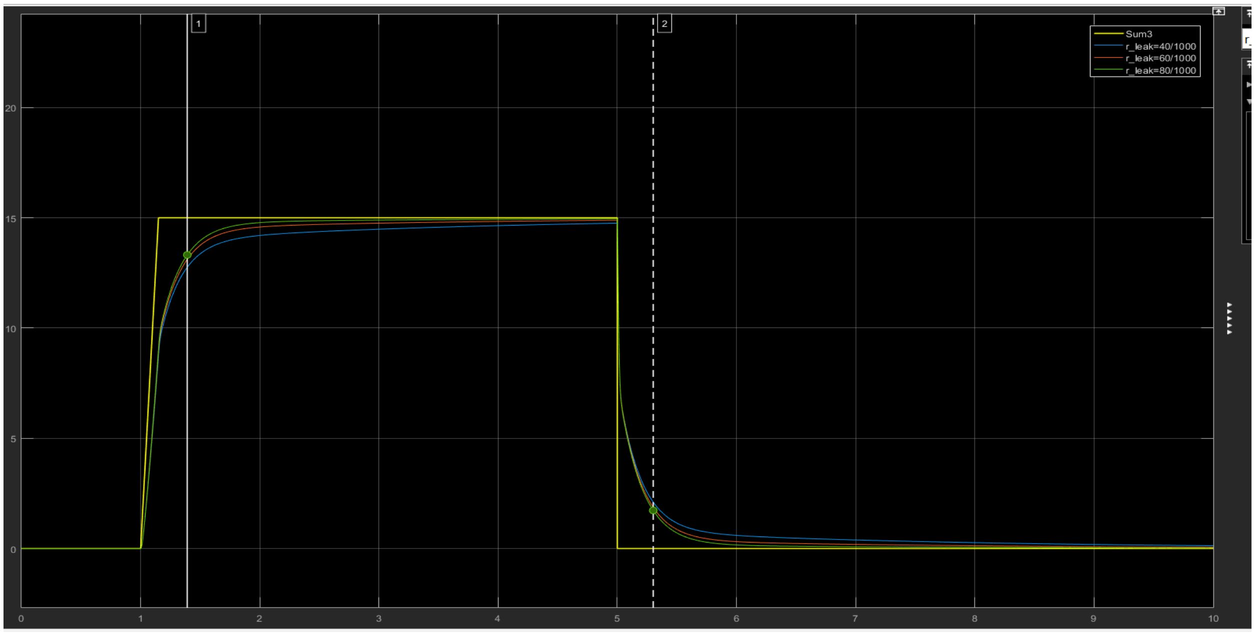
As, R_lung increases, system becomes slower.

Varying R_leak:

We took 3 values of r_leak.

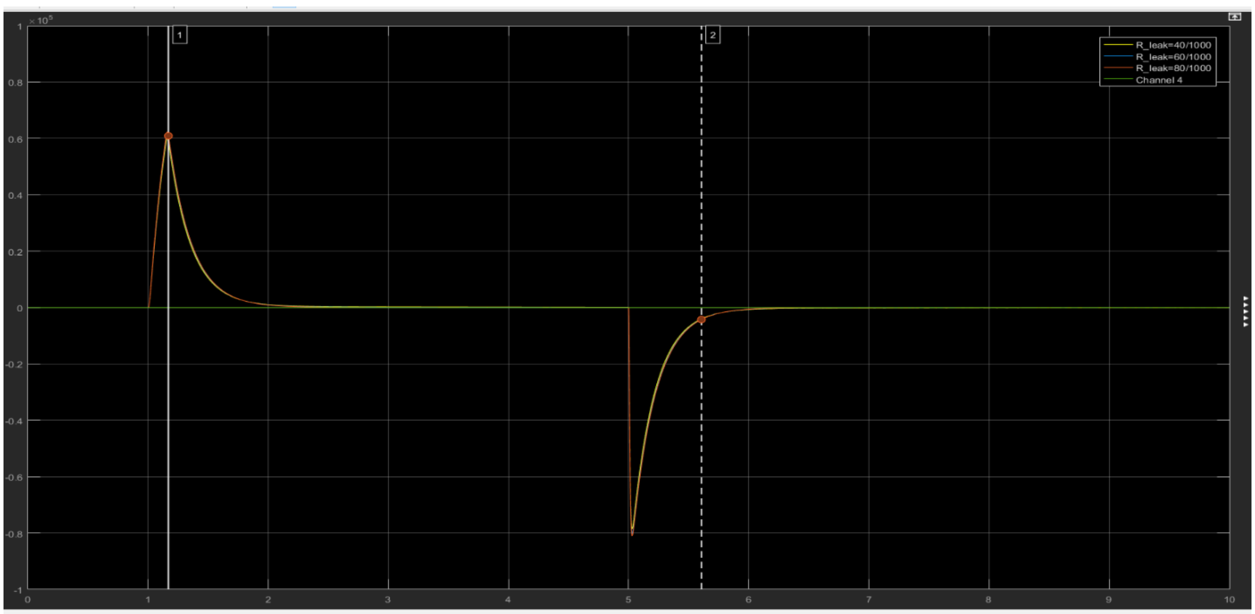
R_leak=40/1000, 60/1000, 80/1000.

P_aw:



Increasing R_leak, makes the system faster.

Q_pat:



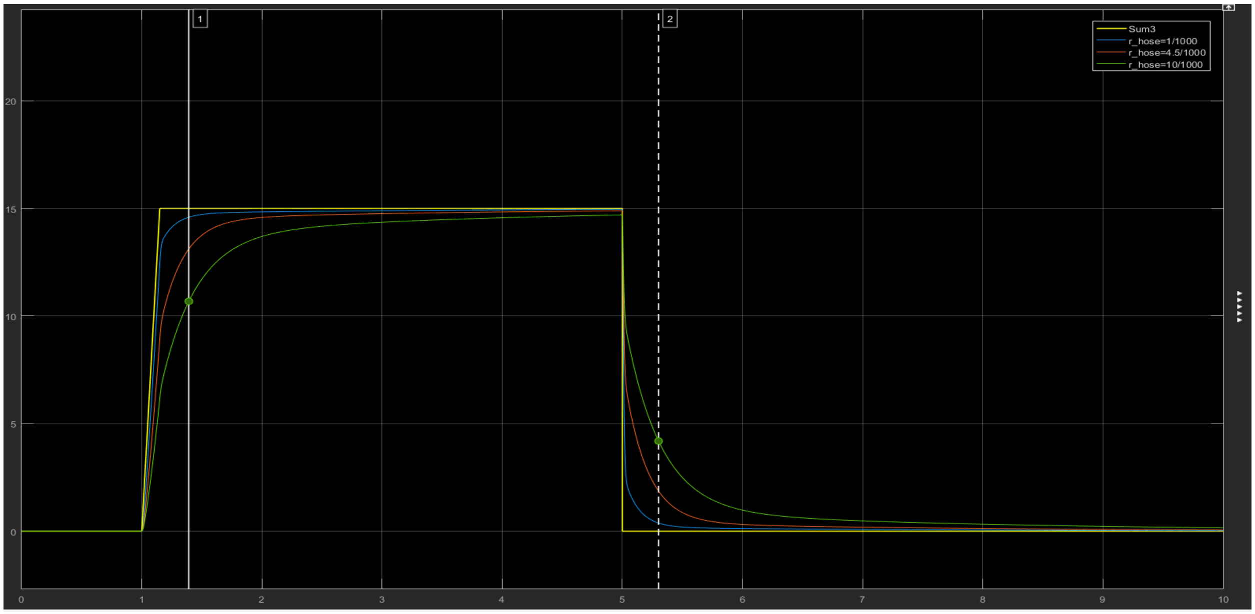
Not much of difference.

<u>Varying R_hose:</u>

We took 3 values of R_hose.

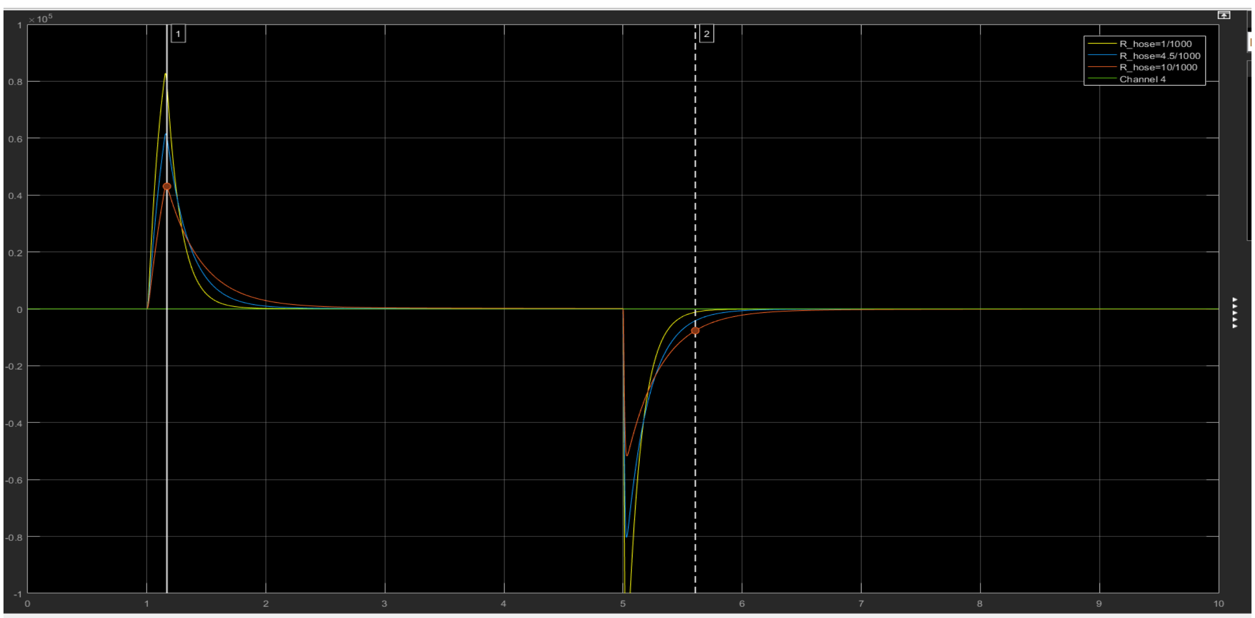
R_leak= 1/1000, 4.5/1000,10/1000

P_aw:



Increasing R_hose makes the system slower.

Q_pat:



Increasing R_hose makes the system slower.