From Zeno's Paradox to the Structure of Space-Time

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Abstract and Background

- **Visualization** refers to the use of visual representations, such as charts, graphs, maps, diagrams, animations, and videos, to communicate information, data, or ideas in a more accessible and understandable format. The goal of visualization is to present complex or abstract concepts in a visual form, making it easier for people to grasp and interpret the information. It allows individuals to see patterns, trends, and relationships that might not be apparent in raw data or text. In scientific research, the primary medium for visualization is video, so developing skills in documentary video is critical in the communication and dissemination of engineering and research accomplishments.
- In this work, I employed a visual approach to study the well-known paradox proposed by Zeno: **the Achilles and Tortoise paradox.** Through the creation of videos, I not only explained the origins of the Zeno's paradox and its resolutions but also revealed the underlying spacetime transformation principles embedded within the paradox.

Research Questions

The main challenges in creating this video arise from technical aspects of video production and conceptual issues stemming from the Zeno's paradox itself.

- How to strategically **structure the video layout** to captivate viewers' attention?
- What kind of **video editing tools** should be used to make the visuals as clean and appealing as possible?
- How to add **voiceover** to a video? How to integrate audio with visuals?
- Where to **search for relevant information** about Zeno's paradox?

The above mentioned are the issues I encounter before engaging in research projects.

Methods and Tools

- **Manim** (a package of Python): Generating video clips
- **Microsoft Azure**: Generating voiceovers
- **Microsoft Clipchamp**: merging audio with visuals
- LaTeX : editing texts

Zeno's Paradox

In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.

—-as recounted by Aristotle, Physics VI:9, 239b15

In the paradox of Achilles and the tortoise, Achilles is in a footrace with the tortoise. Achilles allows the tortoise a head start of 100 meters, for example. Suppose that each racer starts running at some constant speed, one faster than the other. After some finite time, Achilles will have run 100 meters, bringing him to the tortoise's starting point. During this time, the tortoise has run a much shorter distance, say 2 meters. It will then take Achilles some further time to run that distance, by which time the tortoise will have advanced farther; and then more time still to reach this third point, while the tortoise moves ahead. Thus, whenever Achilles arrives somewhere the tortoise has been, he still has some distance to go before he can even reach the tortoise.

Result: Structure Arrangement

1. Introduction of Basic Information
   If we denote that:
   - x_α: the position of Achilles
   - v_α: the speed of Achilles
   - x_τ: the position of the tortoise
   - v_τ: the speed of the tortoise
   - ℓ : The original distance between them

   Now let's mark their positions on the coordinate axis. The blue color represents Achilles, and the yellow color represents the turtle

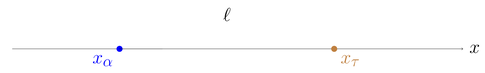

Figure 1. Setting up the configurations

By separately writing out their equations of motion,

$$v_\alpha t = v_\tau t + \ell$$

we have the time point when Achilles catches up the tortoise

$$t = \frac{\ell}{v_\alpha - v_\tau}$$

2. Zeno's Perspective
   However, Zeno offered a unique insight into this seemingly simple problem. He argued that

   Achilles will never overpass the tortoise, if the process is viewed from different STEPS (so called Zeno's time).

   Now, we are linking the time dilation in the Zeno Paradox with real-world time using a table format

t' (Zeno's time)	x_α	x_τ
1	ℓ	$\ell + \frac{\ell}{v_\alpha}\cdot v_\tau$
2	$\ell + \frac{\ell}{v_\alpha}\cdot v_\tau$	$\ell + \frac{\ell}{v_\alpha}\cdot v_\tau + \frac{\ell}{v_\alpha}\cdot\frac{v_\tau}{v_\alpha}\cdot v_\tau$
...	...	...
n	$\ell\cdot\sum_{k=0}^{n-1}\left(\frac{v_\tau}{v_\alpha}\right)^k$	$\ell\cdot\sum_{k=0}^{n}\left(\frac{v_\tau}{v_\alpha}\right)^k$

Table 1. the relation of zeno's time with the positions of Achilles and tortoise

So, in Zeno's viewpoint, only when $n \to \infty$,

$$\lim_{n\to\infty}(x_\tau - x_\alpha) = \lim_{n\to\infty}\left(\frac{v_\tau}{v_\alpha}\right)^n = 0$$

Achilles can "truly" catch up to the tortoise.

3. Zeno Transformation

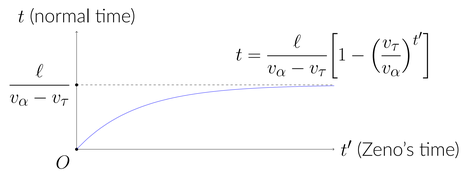

Figure 2. Zeno spacetime transformation

Result: Comparison of Three Types of Spacetime

Zeno's Spacetime

$$t = \frac{\ell}{v_\alpha - v_\tau}\left[1 - \left(\frac{v_\tau}{v_\alpha}\right)^{t'}\right]$$

Zeno's paradox prompted contemplation about the nature of the universe in the world of that time, leading people to question: ***What is the essence of time?***

Galileo's Spacetime

This question was not resolved until two thousand years later by an Italian named Galileo.

$$\begin{aligned} t &= t' \\ x &= x' + vt \\ y &= y' \\ z &= z' \end{aligned}$$

His classical view of spacetime considered time to flow uniformly, unaffected by the velocity of an object's motion. which effectively debunked the Zeno paradox.

Einstein's Spacetime

However, the story doesn't end here. Four hundred years later, Albert Einstein introduced the theory of relativity and its spacetime concept.

$$\begin{aligned} t &= \frac{t' + vx/c^2}{\sqrt{1 - v^2/c^2}} \\ x &= \frac{x' + vt}{\sqrt{1 - v^2/c^2}} \\ y &= y' \\ z &= z' \end{aligned}$$

Although not in the form proposed by Zeno, he demonstrated that time and space are indeed closely interconnected.

Outputs and Conclusions

- the full video clip can be found at
  `https://www.bilibili.com/video/BV1Yr4y1o7rN/?share_source=copy_web&vd_source=995be1`
  or
  `https://youtu.be/7HK2Nvd1jns`
- the source code for this program can be found at
  `https://github.com/Touiku/Code_for_Manim-.git`
- Through this project, I have come to realise that just like Zeno's Paradox, the study of any scientific problem is gradual and requires the combined efforts of many people. It also taught me how to use video software and how to make academic posters, which improved my academic communication skills.

References

[1] Manim documentry, https://docs.manim.community/en/stable.

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