cgm

October 2, 2024

[1]:
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# Assignment 1, Problem 4
# Gradient Method (GM)
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# Import useful libraries
import numpy as np
import scipy.io as sio
import matplotlib.pyplot as plt
```

[2]:
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# Load the dataset
data = sio.loadmat('dataset.mat')
```

[3]:
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# Access data in the dataset
A, b, xtrue = data['A'], data['b'], data['xtrue']
m, n = A.shape
```

[4]:
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# Define the function and the gradient (edit this)
evaluateFunc = lambda x: (1/2)*np.linalg.norm(A.dot(x)-b)**2
evaluateGrad = lambda x: A.T.dot(A.dot(x)-b)
```

[5]:
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# Parameters of the gradient method
xInit = np.zeros((n, 1)) # zero initialization
#stepSize = 1/(2*np.linalg.eigh(A.T@A)[0].max()) # step-size of the gradient␣
 ↪method
maxIter = 200 # maximum number of iterations was 200
tau = 5
```

[6]:
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# Initialize
x = xInit
objVals = np.zeros((maxIter, 1)) # keep track of cost function values
infErrs = np.zeros((maxIter, 1)) # keep track of classification errors

# Run iterations
for iter in range(maxIter):
    # gradient at x
    grad = evaluateGrad(x)
```

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    # update (edit this)
    s = np.zeros((n, 1))
    i = np.argmax(np.abs(grad))
    s[i] = -tau*np.sign(grad[i])

    stepSize = ( (s-x).T @ A.T @ (b - A@x) ) / ( np.linalg.norm(A @ (s-x),␣
↪ord=2)**2 )
    stepSize = stepSize[0,0]
    stepSize = np.min([stepSize, 1])
    stepSize = np.max([stepSize, 0])

    xNext = (1-stepSize) * x + stepSize * s

    # evaluate the objective
    funcNext = evaluateFunc(xNext)

    # store the objective and the classification error
    objVals[iter] = funcNext
    infErrs[iter] = np.linalg.norm(x.ravel()-xtrue.ravel())/np.linalg.
↪norm(xtrue.ravel())

    #print('[%d/%d] [step: %.1e] [objective: %.1e]' % (iter, maxIter, stepSize,␣
↪objVals[iter]))

    # update x
    x = xNext
```

[7]:
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# visualize results
fig = plt.figure(1, figsize=(12, 12))
plt.clf()

# create subplots
gs = fig.add_gridspec(2,2)
ax1 = fig.add_subplot(gs[0, :])
ax2 = fig.add_subplot(gs[1, 0])
ax3 = fig.add_subplot(gs[1, 1])

# plot
ax1.stem(xtrue)
ax1.stem(x, 'r--', markerfmt='r*')
ax1.set_xlim([1, n])

ax2.semilogy(np.arange(maxIter), objVals, 'b-', linewidth=2)
ax2.grid(True)
ax2.axis('tight')
ax2.set_xlabel('iteration')
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ax2.set_ylabel('objective')
ax2.set_title('cost: %.4e' % objVals[-1])

ax2.set_xlim([1, maxIter])
ax3.semilogy(np.arange(maxIter), infErrs, 'r-', linewidth=2)
ax3.grid(True)
ax3.axis('tight')
ax3.set_xlabel('iteration')
ax3.set_ylabel('normalized error')
ax3.set_title('err: %.2e' % infErrs[-1])
ax3.set_xlim([1, maxIter])

plt.draw()
plt.show()
```
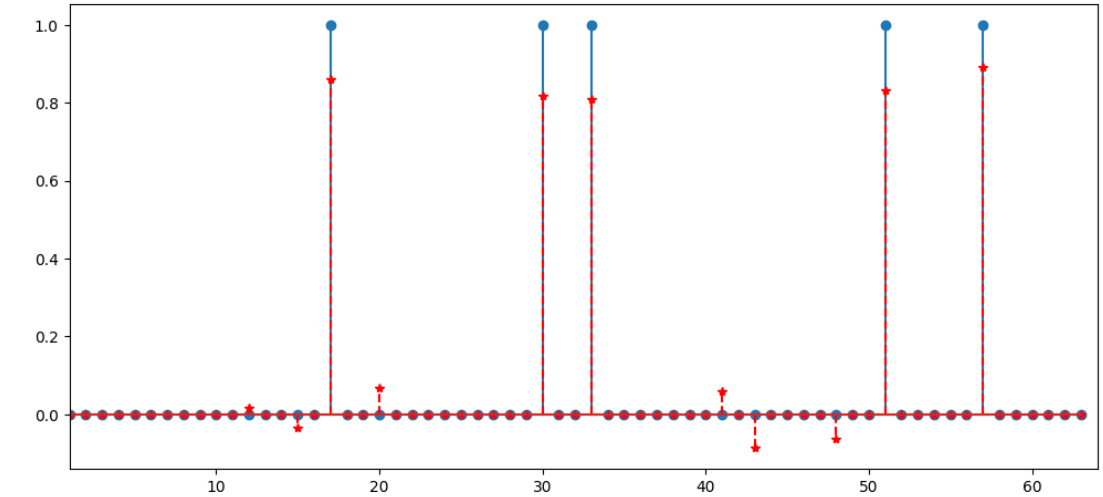
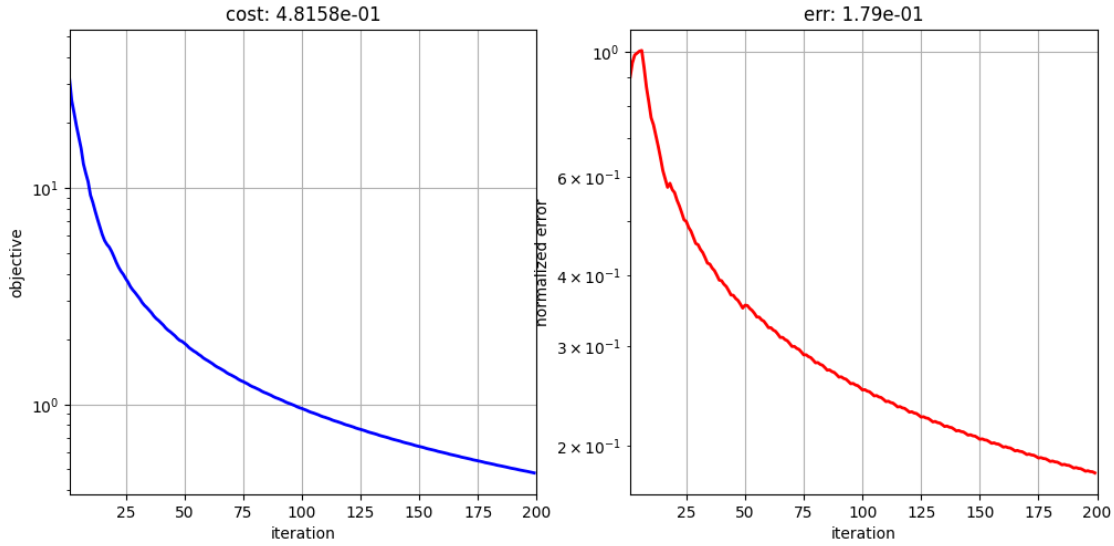

```
/tmp/ipykernel_967/215136572.py:21: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  ax2.set_title('cost: %.4e' % objVals[-1])
/tmp/ipykernel_967/215136572.py:29: DeprecationWarning: Conversion of an array
with ndim > 0 to a scalar is deprecated, and will error in future. Ensure you
extract a single element from your array before performing this operation.
(Deprecated NumPy 1.25.)
  ax3.set_title('err: %.2e' % infErrs[-1])
```


1.0
0.8
0.6
0.4
0.2
0.0
10
20
30
40
50
60


cost: 4.8158e-01
objective
10^1
10^0
25
50
75
100
125
150
175
200
iteration
err: 1.79e-01
normalized error
10^0
6×10^{-1}
4×10^{-1}
3×10^{-1}
2×10^{-1}
25
50
75
100
125
150
175
200
iteration