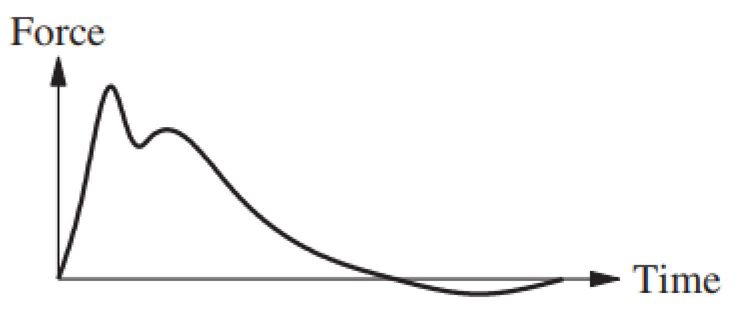
Estructuras Aeronáuticas

Análisis Dinámico de Cargas Impulsivas



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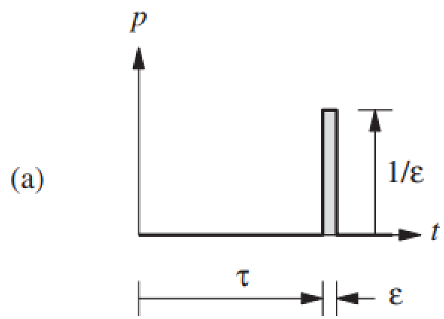
1. Response to Arbitrarily Time-varying Forces

Response to Unit Pulse $\frac{d}{dt}(m\dot{u}) = p$

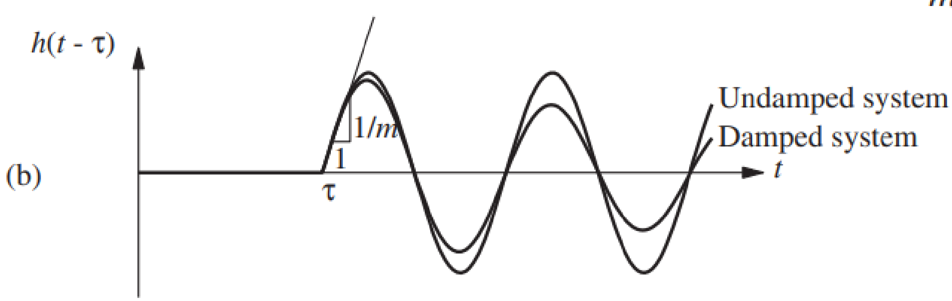
$$p = m\ddot{u}$$



$$\int_{t_1}^{t_2} p\,dt = m(\dot{u}_2 - \dot{u}_1) = m\,\Delta\dot{u}$$

$$\dot{u}(\tau) = \frac{1}{m} \qquad u(\tau) = 0$$

$$h(t-\tau) \equiv u(t) = \frac{1}{m\omega_D} e^{-\zeta\omega_n(t-\tau)} \sin[\omega_D(t-\tau)] \qquad t \geq \tau$$



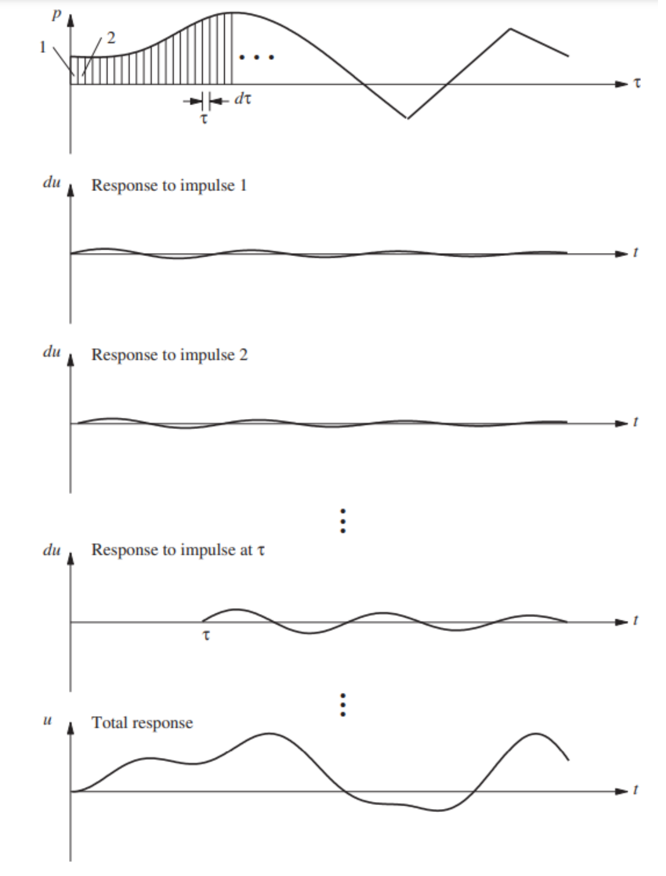
1. Response to Arbitrarily Time-varying Forces

Response to Arbitrary Force

$$du(t) = [p(\tau)\,d\tau]h(t-\tau) \qquad t > \tau$$

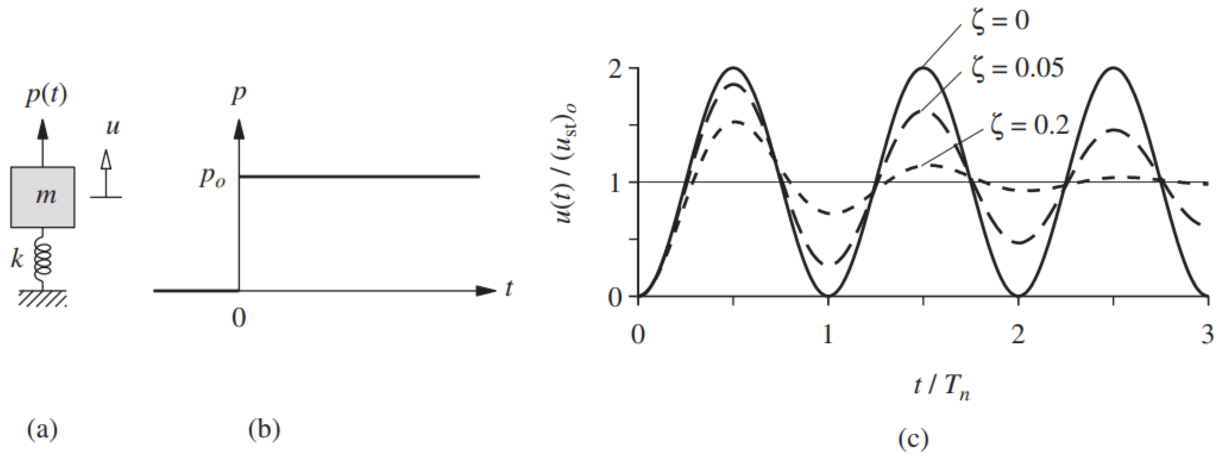
$$u(t) = \int_0^t p(\tau)h(t-\tau)\,d\tau$$

$$u(t) = \frac{1}{m\omega_D}\int_0^t p(\tau)e^{-\zeta\omega_n(t-\tau)}\sin[\omega_D(t-\tau)]\,d\tau$$



2. Response to Step and Ramp Forces

Response to Step Force



$$u_o = 2(u_{st})_o$$

$$p(t) = p_o \qquad (u_{st})_o = p_o/k,$$

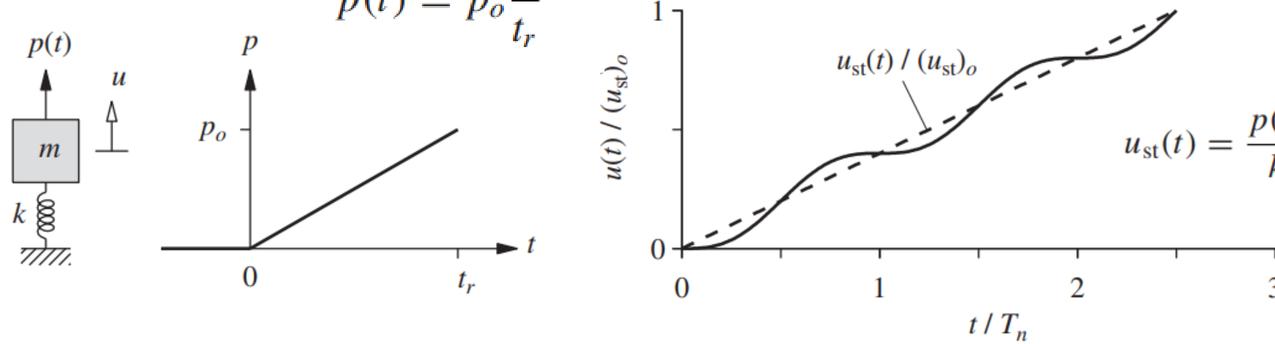
$$u(t) = (u_{st})_o(1 - \cos\omega_n t) = (u_{st})_o\left(1 - \cos\frac{2\pi t}{T_n}\right)$$

$$u(t) = (u_{st})_o\left[1 - e^{-\zeta\omega_n t}\left(\cos\omega_D t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_D t\right)\right]$$

2. Response to Step and Ramp Forces

Response to Ramp Force

$$p(t) = p_o \frac{t}{t_r}$$



$$u_{st}(t) = \frac{p(t)}{k} = (u_{st})_o \frac{t}{t_r}$$

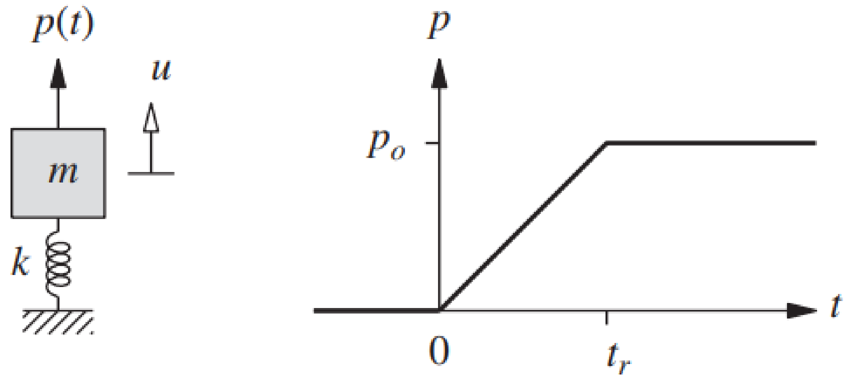
(a) (b) (c)

$$u(t) = \frac{1}{m\omega_n} \int_0^t \frac{p_o}{t_r} \tau \sin \omega_n (t - \tau)\, d\tau$$

$$u(t) = (u_{st})_o \left(\frac{t}{t_r} - \frac{\sin \omega_n t}{\omega_n t_r} \right) = (u_{st})_o \left(\frac{t}{T_n} \frac{T_n}{t_r} - \frac{\sin 2\pi t / T_n}{2\pi t_r / T_n} \right)$$

2. Response to Step and Ramp Forces

Response to Step Force with Finite Rise Time

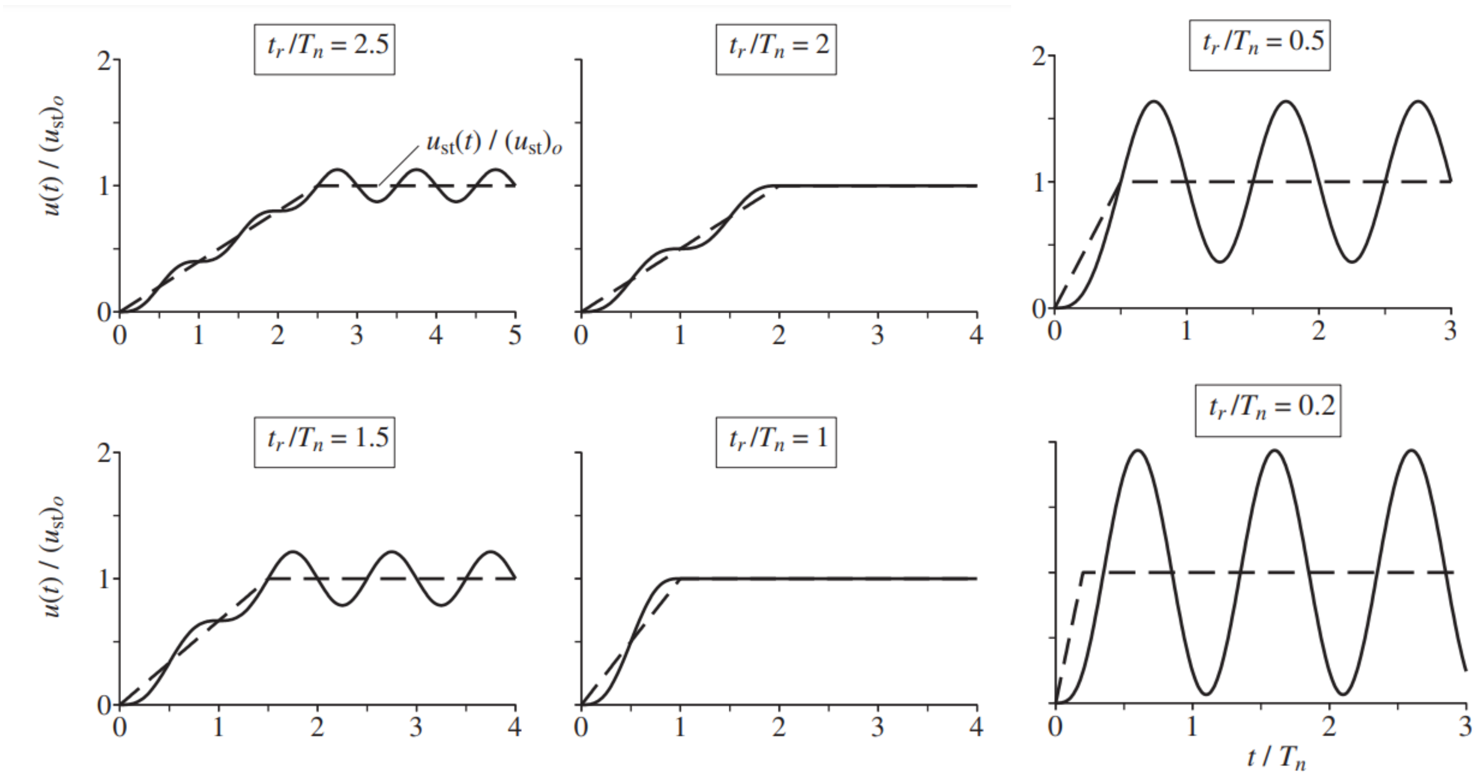


$$u(t) = (u_{\mathrm{st}})_o \left(\frac{t}{t_r} - \frac{\sin \omega_n t}{\omega_n t_r} \right) \qquad t \leq t_r$$

$$u(t) = u(t_r) \cos \omega_n (t - t_r) + \frac{\dot{u}(t_r)}{\omega_n} \sin \omega_n (t - t_r) + (u_{\mathrm{st}})_o [1 - \cos \omega_n (t - t_r)]$$

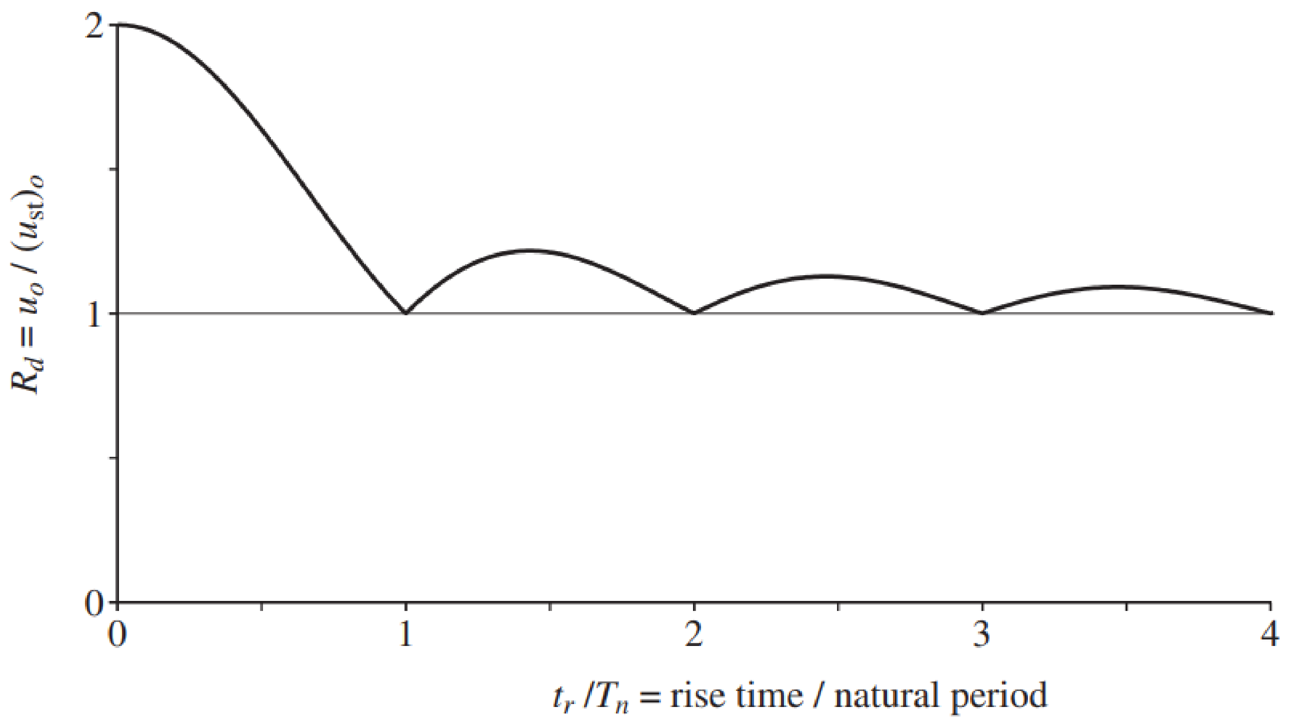
2. Response to Step and Ramp Forces

Response to Step Force with Finite Rise Time



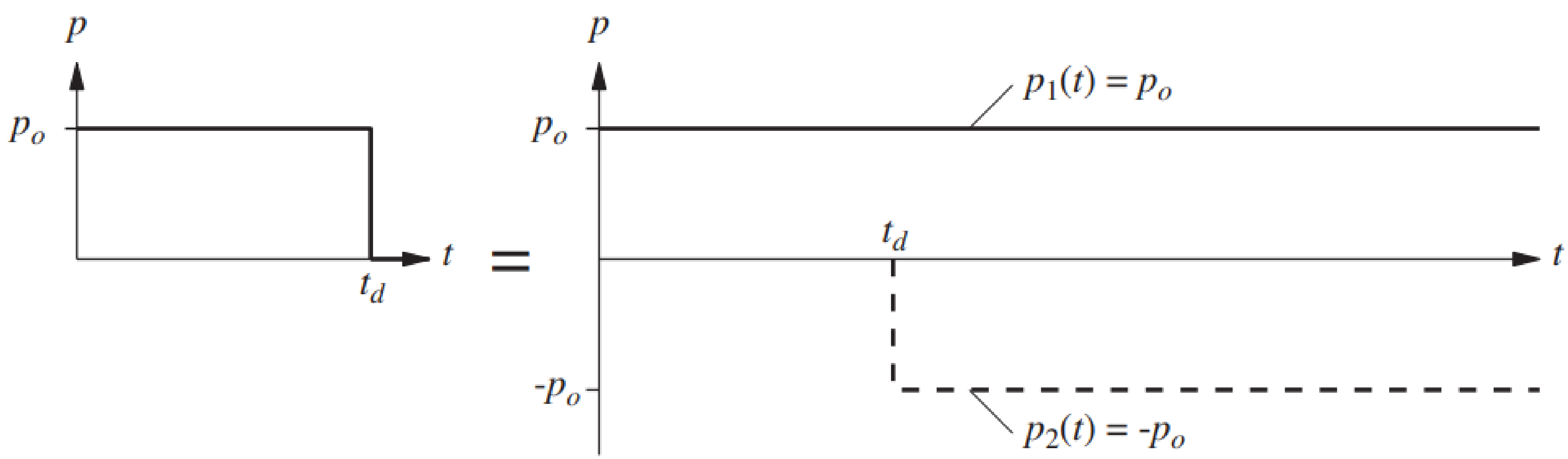
2. Response to Step and Ramp Forces

Response Spectrum



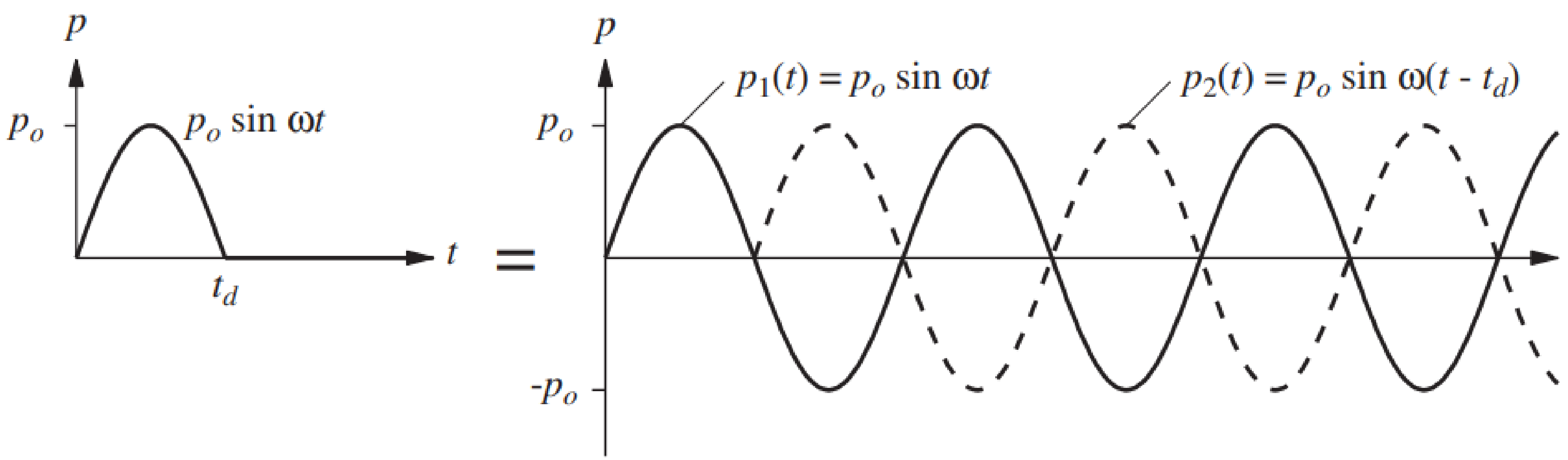
3. Response to Pulse Excitations

Solutions Methods



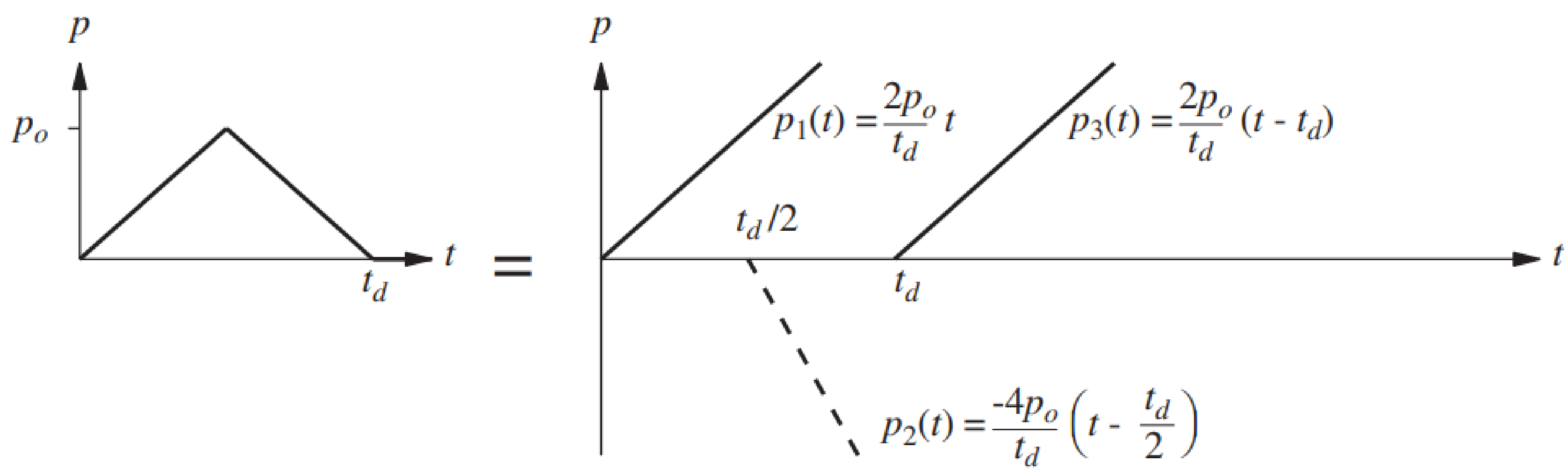
3. Response to Pulse Excitations

Solutions Methods



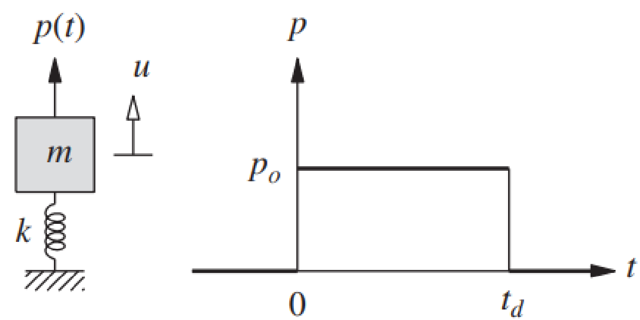
3. Response to Pulse Excitations

Solutions Methods



3. Response to Pulse Excitations

Rectangular Pulse Force



$$m\ddot{u} + ku = p(t) = \begin{cases} p_o & t \le t_d \\ 0 & t \ge t_d \end{cases}$$

$$u(0) = \dot{u}(0) = 0.$$

1. *Forced vibration phase.* During this phase, the system is subjected to a step force. The response of the system is given by Eq. (4.3.2), repeated for convenience:

$$\frac{u(t)}{(u_{\mathrm{st}})_o} = 1 - \cos\omega_n t = 1 - \cos\frac{2\pi t}{T_n} \qquad t \le t_d$$

2. *Free vibration phase.* After the force ends at t_d, the system undergoes free vibration, defined by modifying Eq. (2.1.3) appropriately:

$$u(t) = u(t_d)\cos\omega_n(t - t_d) + \frac{\dot{u}(t_d)}{\omega_n}\sin\omega_n(t - t_d)$$

3. Response to Pulse Excitations

Rectangular Pulse Force

$$u(t_d) = (u_{st})_o[1 - \cos\omega_n t_d] \qquad \dot{u}(t_d) = (u_{st})_o\omega_n \sin\omega_n t_d$$

Substituting these in Eq. (4.7.3) gives

$$\frac{u(t)}{(u_{st})_o} = (1 - \cos\omega_n t_d)\cos\omega_n(t - t_d) + \sin\omega_n t_d \sin\omega_n(t - t_d) \qquad t \geq t_d$$

which can be simplified, using a trigonometric identity, to

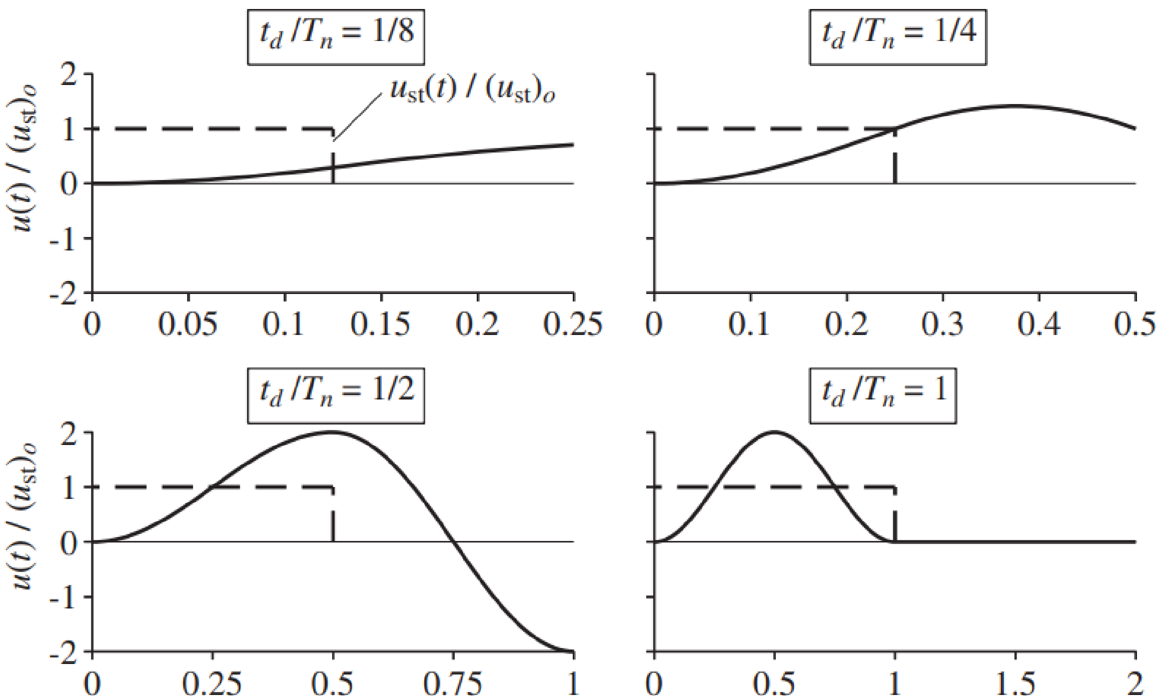
$$\frac{u(t)}{(u_{st})_o} = \cos\omega_n(t - t_d) - \cos\omega_n t \qquad t \geq t_d$$

Expressing $\omega_n = 2\pi/T_n$ and using trigonometric identities enables us to rewrite these equations as

$$\frac{u(t)}{(u_{st})_o} = \left(2\sin\frac{\pi t_d}{T_n}\right)\sin\left[2\pi\left(\frac{t}{T_n} - \frac{1}{2}\frac{t_d}{T_n}\right)\right] \qquad t \geq t_d$$

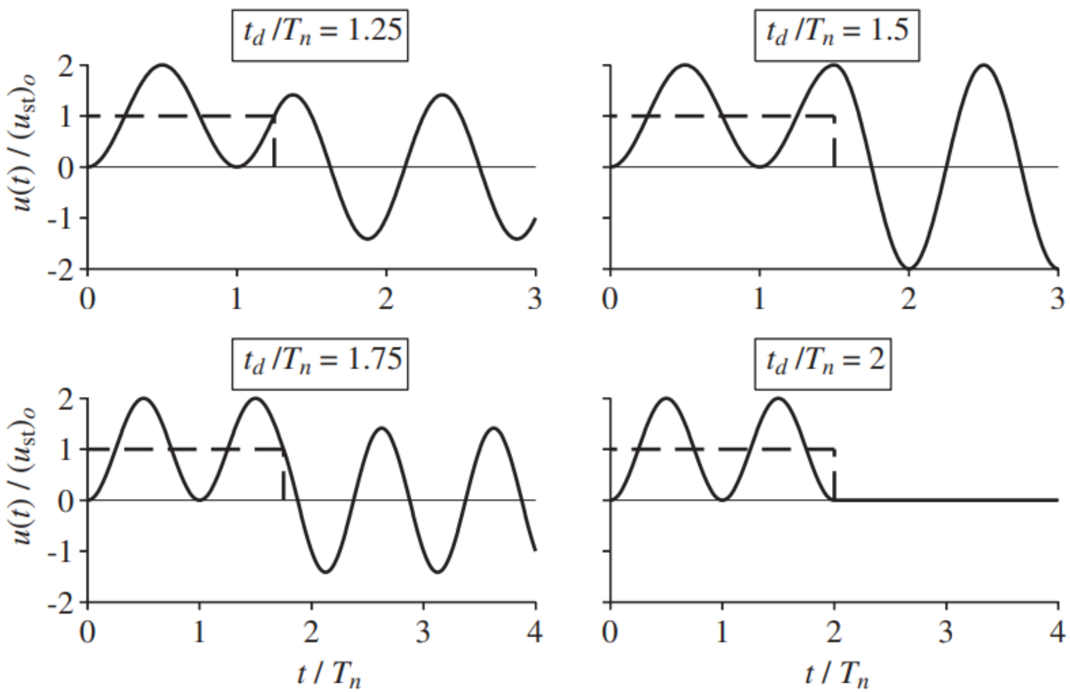
3. Response to Pulse Excitations

Rectangular Pulse Force



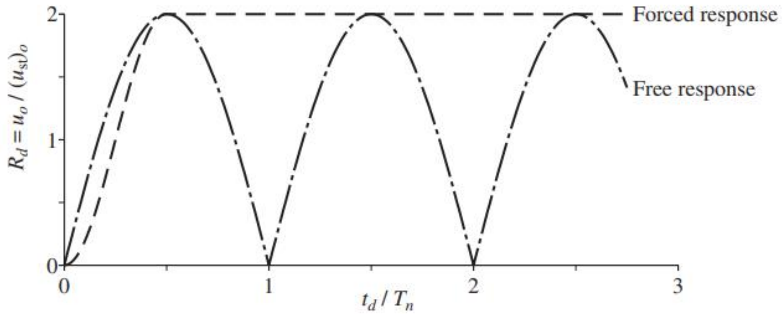
3. Response to Pulse Excitations

Rectangular Pulse Force



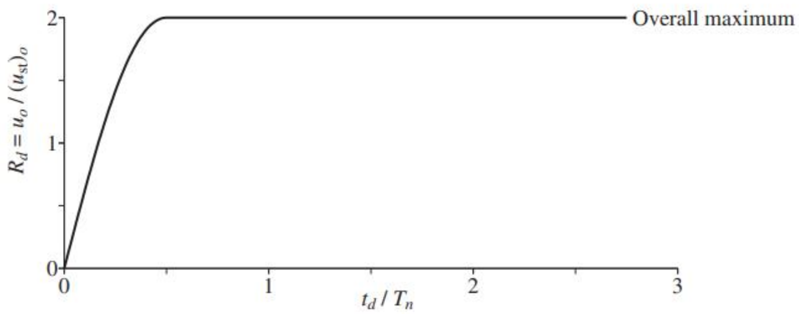
3. Response to Pulse Excitations

Rectangular Pulse Force



$$R_d = \frac{u_o}{(u_{st})_o} = \begin{cases} 1 - \cos(2\pi t_d/T_n) & t_d/T_n \leq \frac{1}{2} \\ 2 & t_d/T_n \geq \frac{1}{2} \end{cases}$$

$$u_o = \sqrt{[u(t_d)]^2 + \left[\frac{\dot{u}(t_d)}{\omega_n}\right]^2}$$

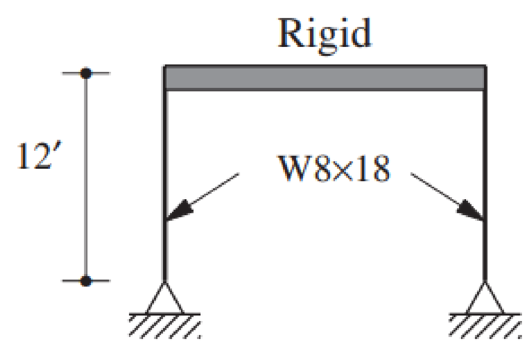


$$R_d = \frac{u_o}{(u_{st})_o} = \begin{cases} 2 \sin \pi t_d/T_n & t_d/T_n \leq \frac{1}{2} \\ 2 & t_d/T_n \geq \frac{1}{2} \end{cases}$$

3. Response to Pulse Excitations

Example

A one-story building, idealized as a 12-ft-high frame with two columns hinged at the base and a rigid beam, has a natural period of 0.5 sec. Each column is an American standard wide-flange steel section W8 × 18. Its properties for bending about its major axis are $I_x = 61.9\ \text{in}^4$, $S = I_x/c = 15.2\ \text{in}^3$; $E = 30{,}000$ ksi. Neglecting damping, determine the maximum response of this frame due to a rectangular pulse force of amplitude 4 kips and duration $t_d = 0.2$ sec. The response quantities of interest are displacement at the top of the frame and maximum bending stress in the columns.



3. Response to Pulse Excitations

Example

1. *Determine R_d.*

$$\frac{t_d}{T_n} = \frac{0.2}{0.5} = 0.4$$

$$R_d = \frac{u_o}{(u_{st})_o} = 2\sin\frac{\pi t_d}{T_n} = 2\sin(0.4\pi) = 1.902$$

2. *Determine the lateral stiffness of the frame.*

$$k_{col} = \frac{3EI}{L^3} = \frac{3(30{,}000)61.9}{(12\times 12)^3} = 1.865 \text{ kips/in.}$$

$$k = 2 \times 1.865 = 3.73 \text{ kips/in.}$$

3. *Determine $(u_{st})_o$.*

$$(u_{st})_o = \frac{p_o}{k} = \frac{4}{3.73} = 1.07 \text{ in.}$$

3. Response to Pulse Excitations

Example

4. *Determine the maximum dynamic deformation.*

$$u_o = (u_{\mathrm{st}})_o R_d = (1.07)(1.902) = 2.04 \text{ in.}$$

5. *Determine the bending stress.* The resulting bending moments in each column are shown in Fig. E4.1c. At the top of the column the bending moment is largest and is given by

$$M = \frac{3EI}{L^2} u_o = \left[\frac{3(30{,}000)61.9}{(12 \times 12)^2} \right] 2.04 = 547.8 \text{ kip-in.}$$

Alternatively, we can find the bending moment from the equivalent static force:

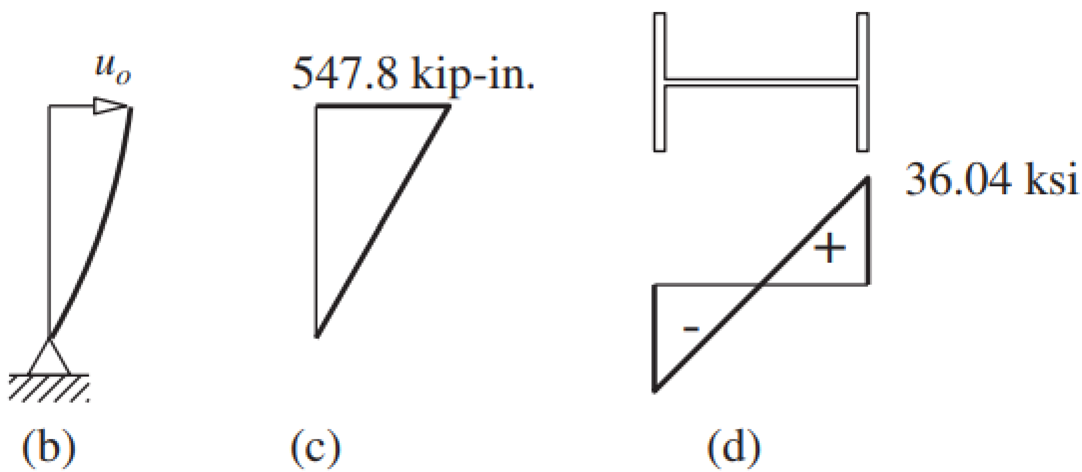
$$f_{So} = p_o R_d = 4(1.902) = 7.61 \text{ kips}$$

$$M = \frac{f_{So}}{2} h = \left(\frac{7.61}{2} \right) 12 \times 12 = 547.8 \text{ kip-in.}$$

3. Response to Pulse Excitations

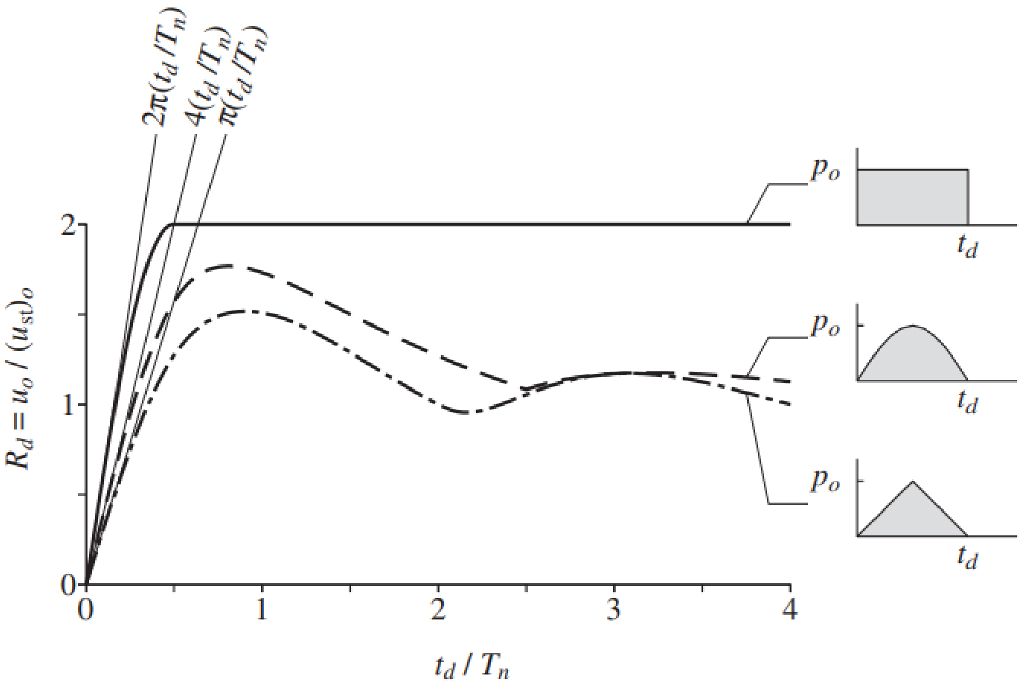
Example

$$\sigma = \frac{M}{S} = \frac{547.8}{15.2} = 36.04 \text{ ksi}$$



3. Response to Pulse Excitations

Effect Pulse Shape



$$\mathcal{I} = \int_0^{t_d} p(t)\,dt$$

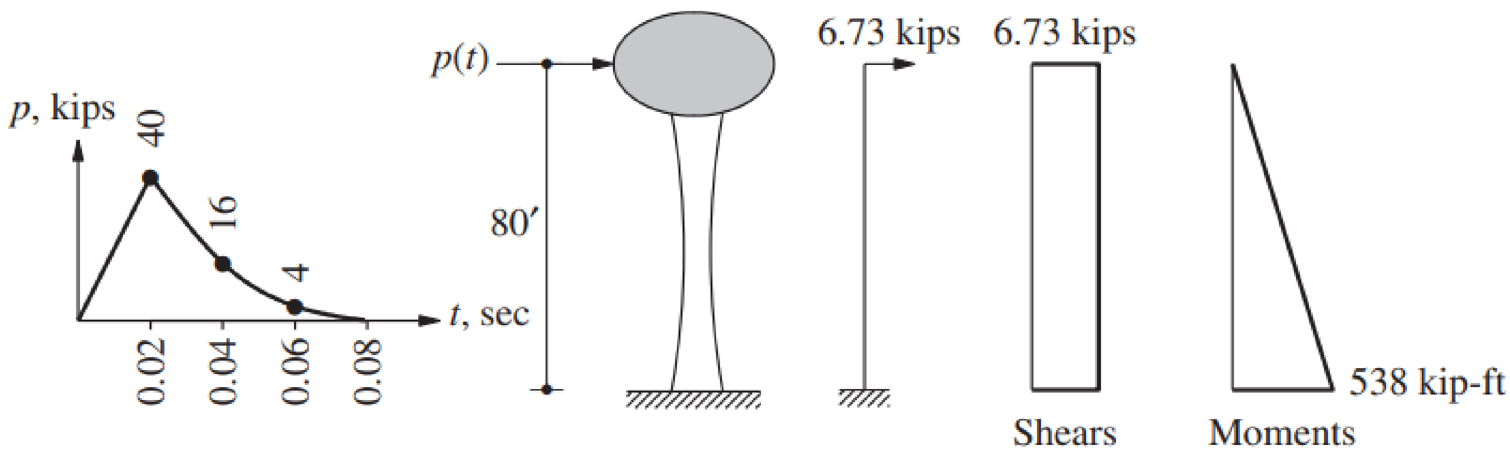
$$u(t) = \mathcal{I}\left(\frac{1}{m\omega_n}\sin\omega_n t\right)$$

$$u_o = \frac{\mathcal{I}}{m\omega_n} = \frac{\mathcal{I}}{k}\frac{2\pi}{T_n}$$

3. Response to Pulse Excitations

Example

The 80-ft-high full water tank of Example 2.7 is subjected to the force $p(t)$ shown in Fig. E4.2, caused by an aboveground explosion. Determine the maximum base shear and bending moment at the base of the tower supporting the tank.



3. Response to Pulse Excitations

Example

For this water tank, from Example 2.7, weight $w = 100.03$ kips, $k = 8.2$ kips/in. $T_n = 1.12$ sec, and $\zeta = 1.23\%$. The ratio $t_d/T_n = 0.08/1.12 = 0.071$. Because $t_d/T_n < 0.25$, the forcing function may be treated as a pure impulse of magnitude

$$\mathcal{I} = \int_0^{0.08} p(t)\,dt = \frac{0.02}{2}[0 + 2(40) + 2(16) + 2(4) + 0] = 1.2 \text{ kip-sec}$$

where the integral is calculated by the trapezoidal rule. Neglecting the effect of damping, the maximum displacement is

$$u_o = \frac{\mathcal{I}}{k}\frac{2\pi}{T_n} = \frac{(1.2)2\pi}{(8.2)(1.12)} = 0.821 \text{ in.}$$

The equivalent static force f_{So} associated with this displacement is [from Eq. (1.8.1)]

$$f_{So} = ku_o = (8.2)0.821 = 6.73 \text{ kips}$$

The resulting shearing forces and bending moments over the height of the tower are shown in Fig. E4.2. The base shear and moment are $V_b = 6.73$ kips and $M_b = 538$ kip-ft.

3. Response to Pulse Excitations

Effect Damping