Introduction To Linear Algebra

Ferdous Bin Ali

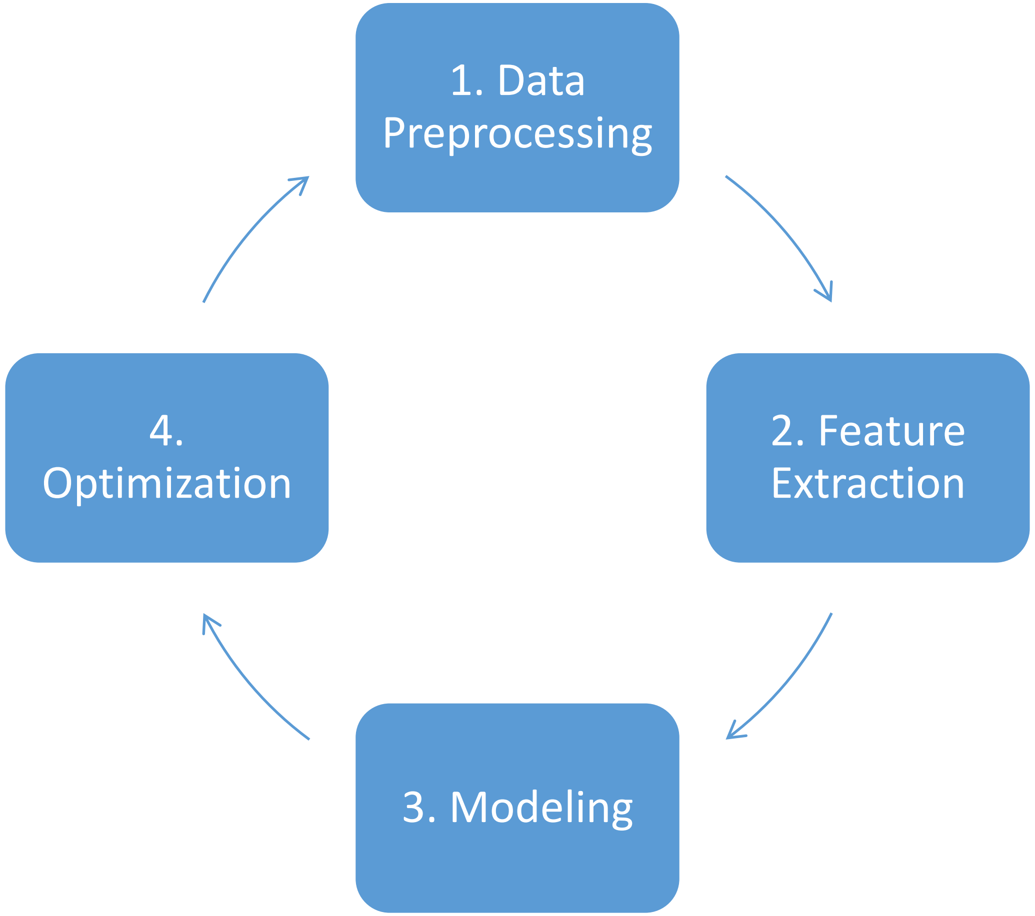
&

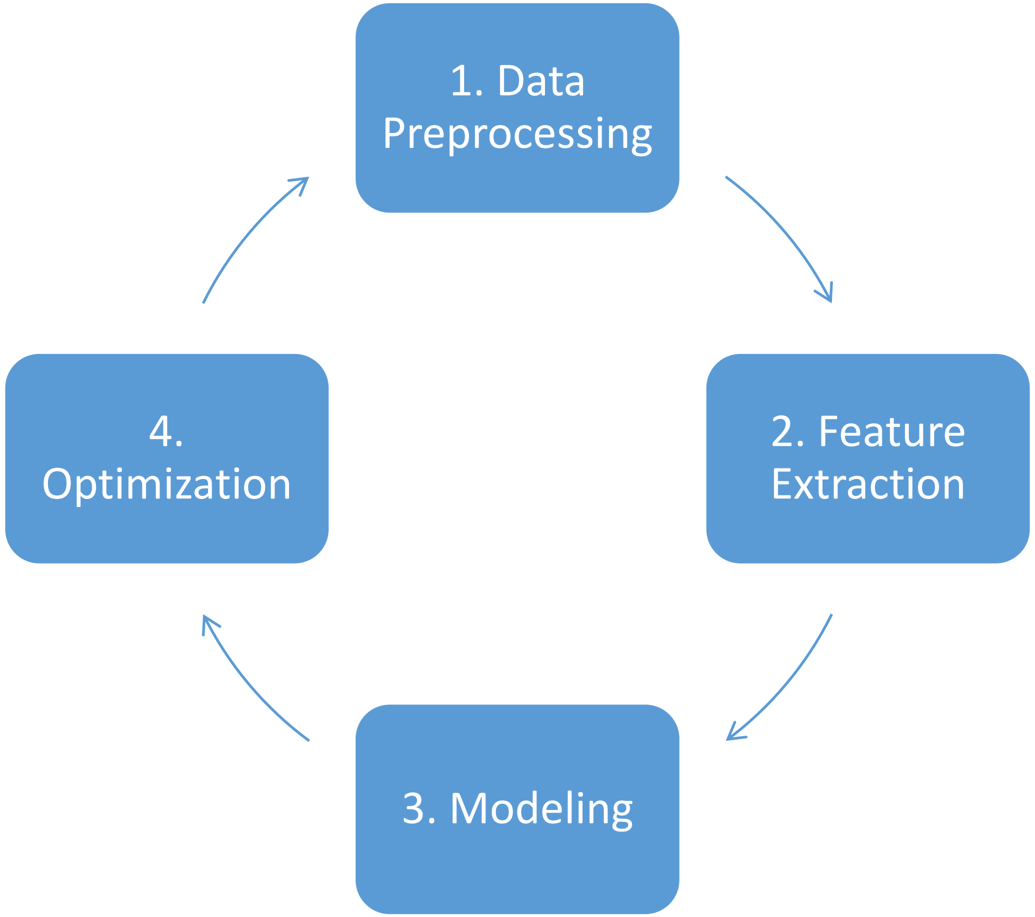
Nurul Akter Towhid

Objective And Rationale

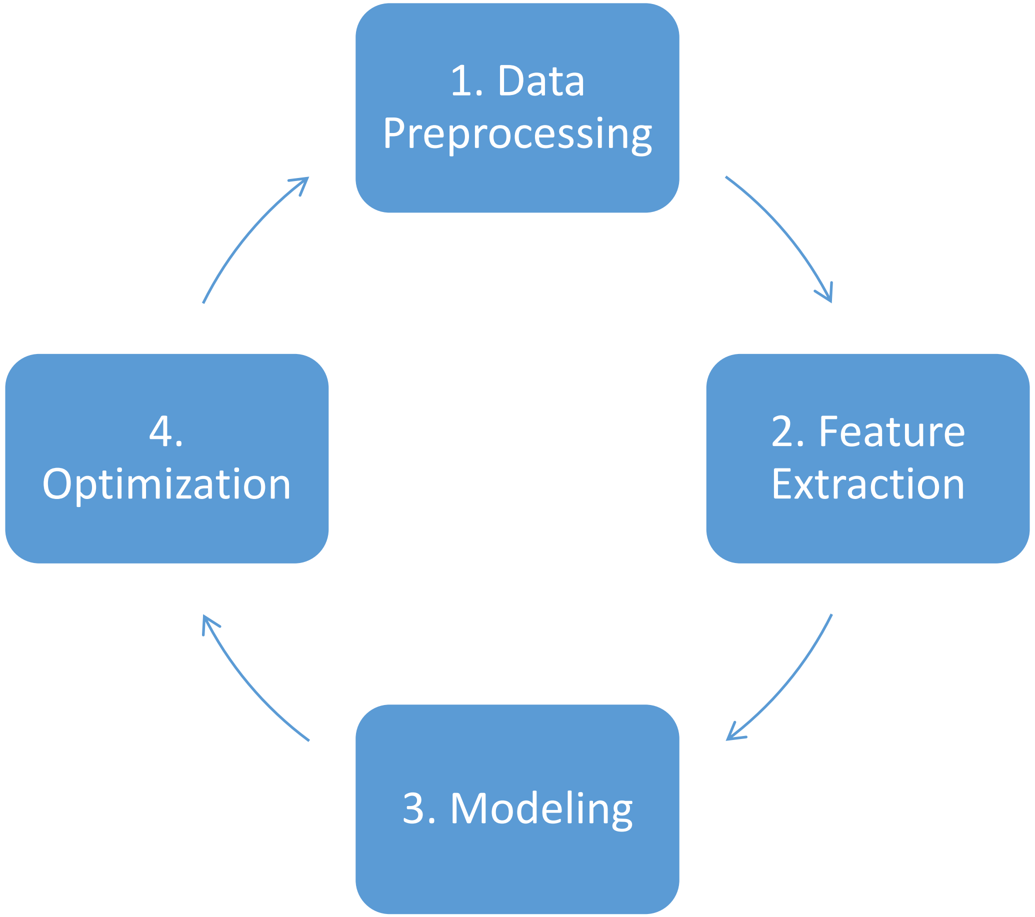
- The objective of this module is to provide a refresher on the mathematical background necessary to progress more advanced Machine Learning topics
- This module includes
  - Linear Algebra
  - Multivariate Calculus
  - Statistics
  - Probability

The Machine Learning Pipeline



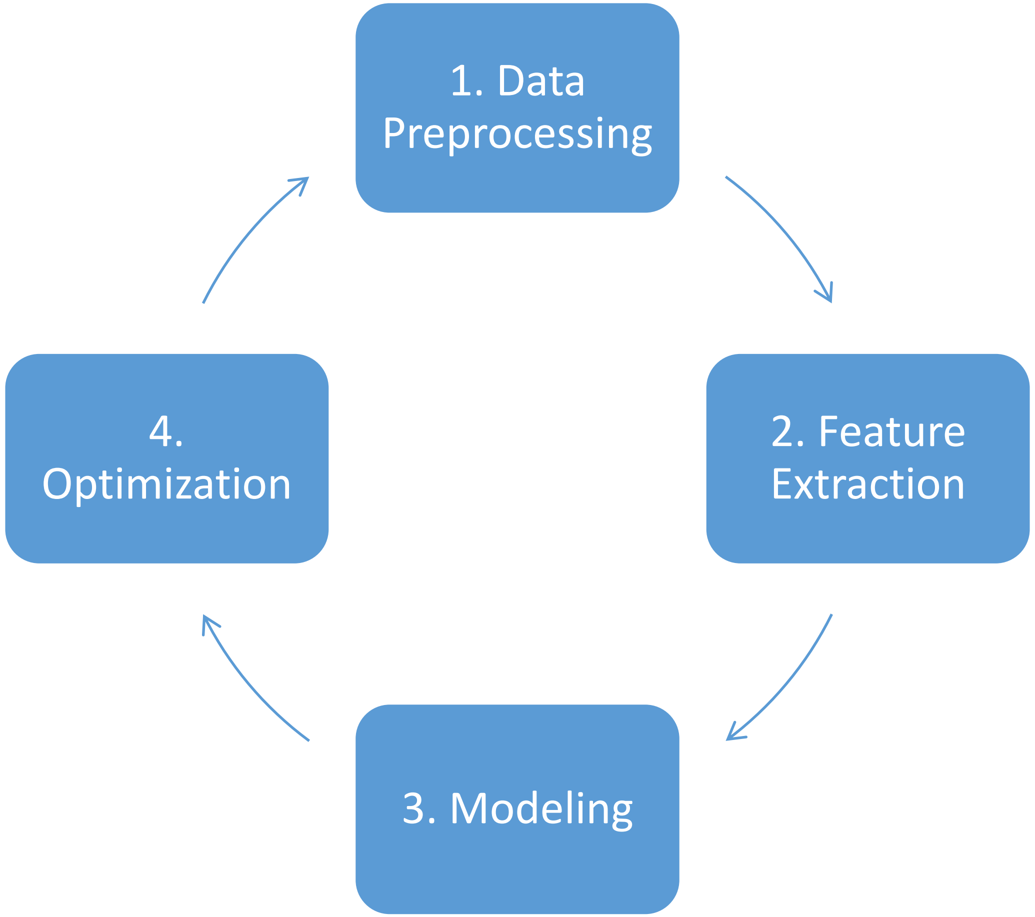


1. Data Preprocessing
    - Collection
    - Formatting
    - Cleaning
    - Labeling



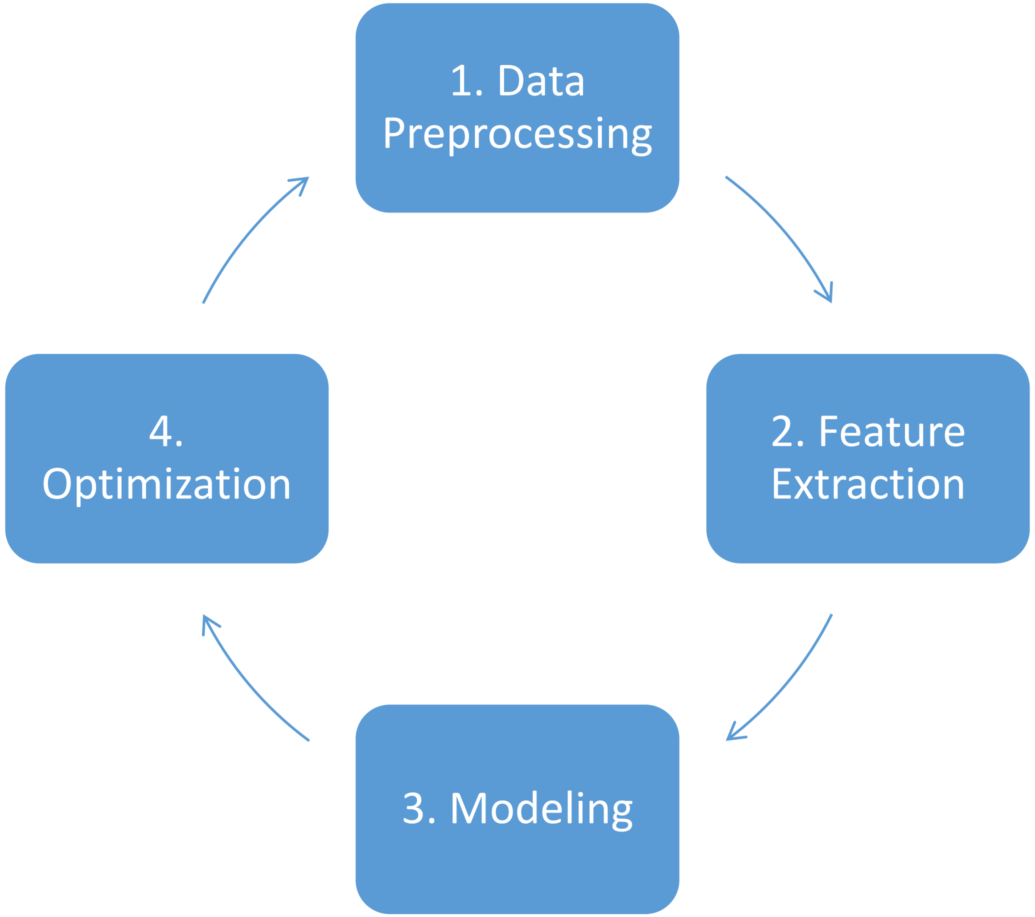
2. Feature Extraction

- Vector
- Matics



3. Modeling

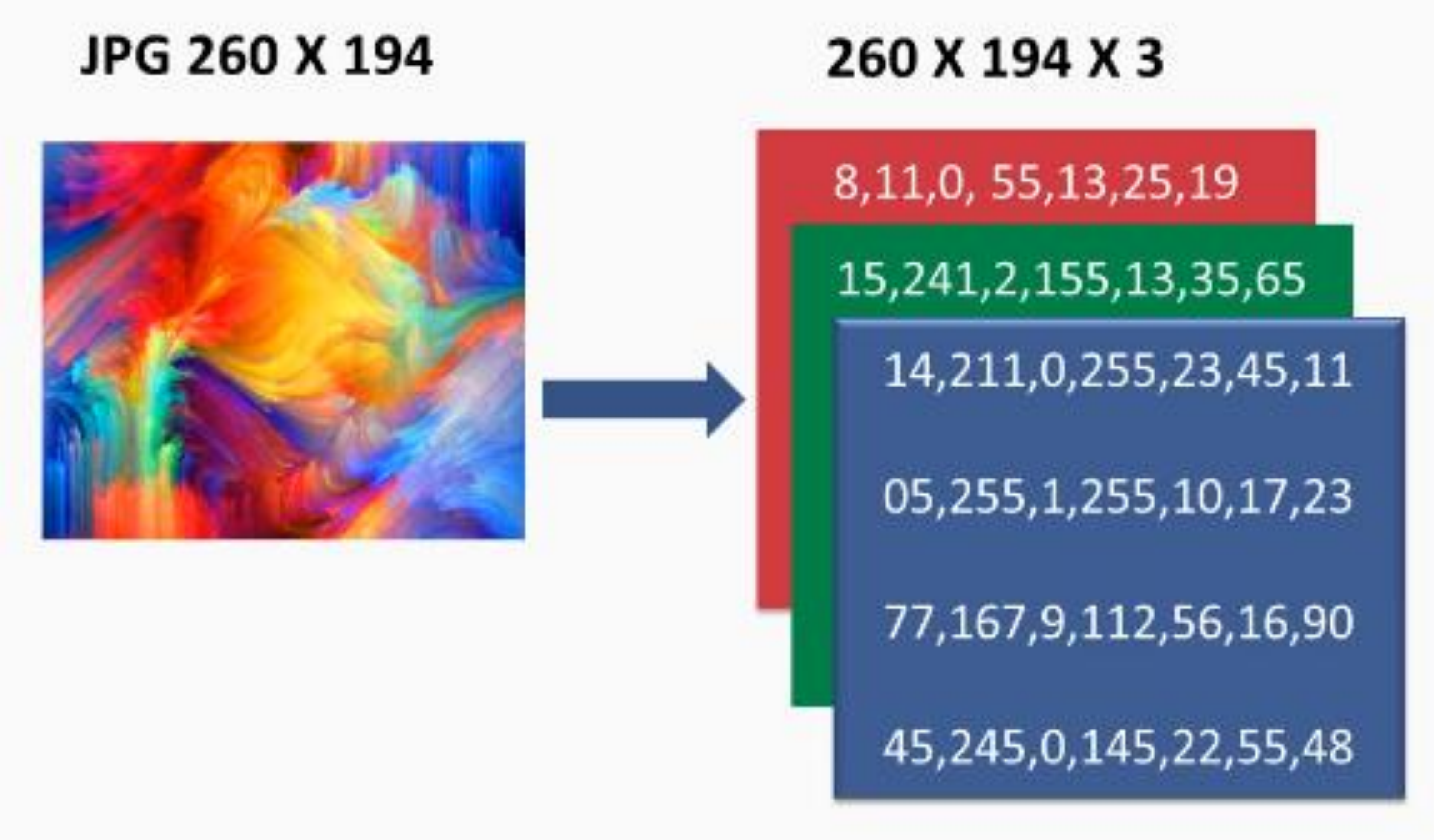
- Geometry
- Probability
- Matrices Operation



4. Optimization

- Training Phase
- Data Evaluation Phase
- Prediction

Image Construction

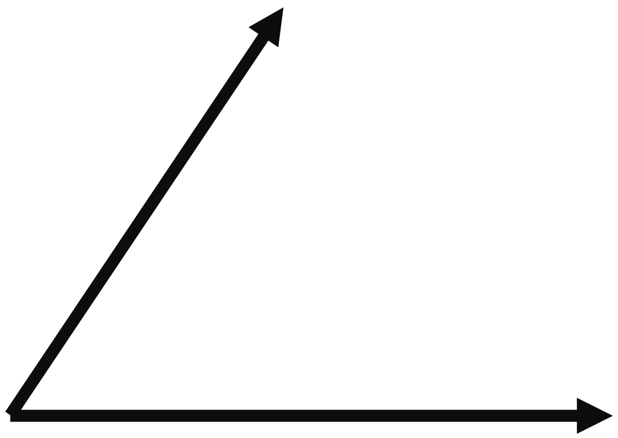


Dataset As a Matrix

39	State-gov	77516	Bachelors	13	Never-married	Adm-clerical	Not-in-family	White	Male	2174	0	40	United-States	<=50K
50	Self-emp-not-inc	83311	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	13	United-States	<=50K
38	Private	215646	HS-grad	9	Divorced	Handlers-cleaners	Not-in-family	White	Male	0	0	40	United-States	<=50K
53	Private	234721	11th	7	Married-civ-spouse	Handlers-cleaners	Husband	Black	Male	0	0	40	United-States	<=50K
28	Private	338409	Bachelors	13	Married-civ-spouse	Prof-specialty	Wife	Black	Female	0	0	40	Cuba	<=50K
37	Private	284582	Masters	14	Married-civ-spouse	Exec-managerial	Wife	White	Female	0	0	40	United-States	<=50K
49	Private	160187	9th	5	Married-spouse-absent	Other-service	Not-in-family	Black	Female	0	0	16	Jamaica	<=50K
52	Self-emp-not-inc	209642	HS-grad	9	Married-civ-spouse	Exec-managerial	Husband	White	Male	0	0	45	United-States	>50K
31	Private	45781	Masters	14	Never-married	Prof-specialty	Not-in-family	White	Female	14084	0	50	United-States	>50K
42	Private	159449	Bachelors	13	Married-civ-spouse	Exec-managerial	Husband	White	Male	5178	0	40	United-States	>50K
37	Private	280464	Some-college	10	Married-civ-spouse	Exec-managerial	Husband	Black	Male	0	0	80	United-States	>50K
30	State-gov	141297	Bachelors	13	Married-civ-spouse	Prof-specialty	Husband	Asian-Pac-Islander	Male	0	0	40	India	>50K
23	Private	122272	Bachelors	13	Never-married	Adm-clerical	Own-child	White	Female	0	0	30	United-States	<=50K
32	Private	205019	Assoc-acdm	12	Never-married	Sales	Not-in-family	Black	Male	0	0	50	United-States	<=50K
40	Private	121772	Assoc-voc	11	Married-civ-spouse	Craft-repair	Husband	Asian-Pac-Islander	Male	0	0	40	?	>50K
34	Private	245487	7th-8th	4	Married-civ-spouse	Transport-moving	Husband	Amer-Indian-Eskimo	Male	0	0	45	Mexico	<=50K
25	Self-emp-not-inc	176756	HS-grad	9	Never-married	Farming-fishing	Own-child	White	Male	0	0	35	United-States	<=50K
32	Private	186824	HS-grad	9	Never-married	Machine-op-inspct	Unmarried	White	Male	0	0	40	United-States	<=50K
38	Private	28887	11th	7	Married-civ-spouse	Sales	Husband	White	Male	0	0	50	United-States	<=50K
43	Self-emp-not-inc	292175	Masters	14	Divorced	Exec-managerial	Unmarried	White	Female	0	0	45	United-States	>50K
40	Private	193524	Doctorate	16	Married-civ-spouse	Prof-specialty	Husband	White	Male	0	0	60	United-States	>50K
54	Private	302146	HS-grad	9	Separated	Other-service	Unmarried	Black	Female	0	0	20	United-States	<=50K
35	Federal-gov	76845	9th	5	Married-civ-spouse	Farming-fishing	Husband	Black	Male	0	0	40	United-States	<=50K
43	Private	117037	11th	7	Married-civ-spouse	Transport-moving	Husband	White	Male	0	2042	40	United-States	<=50K
59	Private	109015	HS-grad	9	Divorced	Tech-support	Unmarried	White	Female	0	0	40	United-States	<=50K

Mathematics in Context

Data is Always Represented as either:

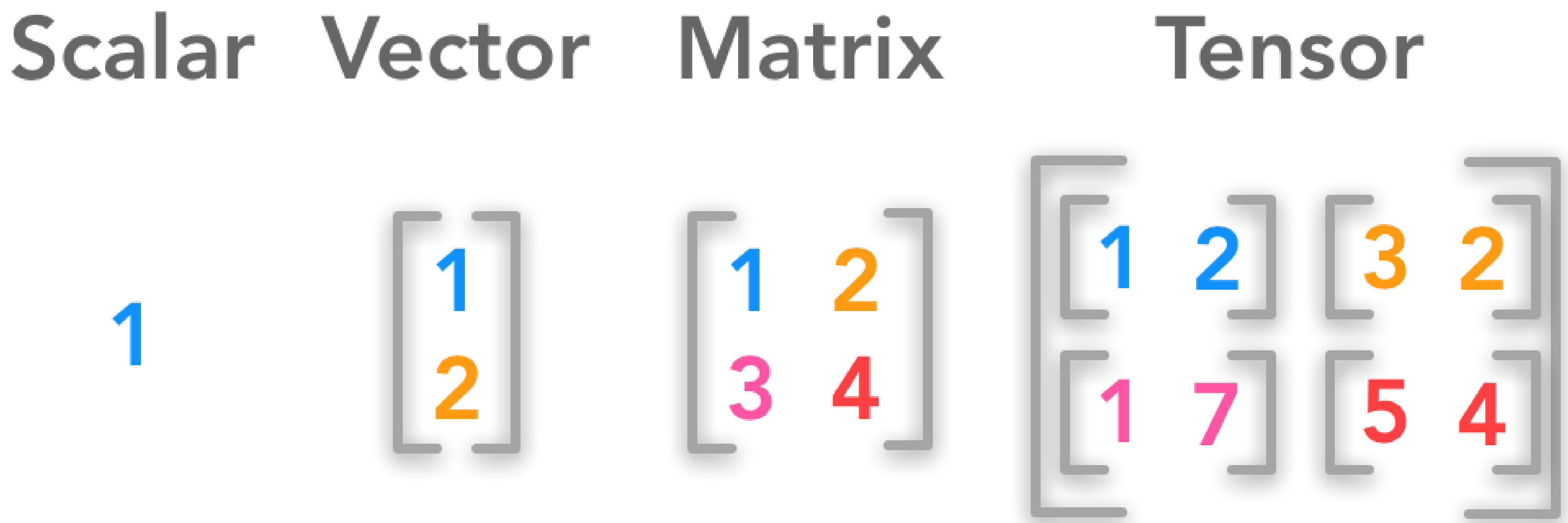


Vector

$$\begin{pmatrix} 1 & 2 & 6 \\ 2 & 8 & 9 \\ 3 & 10 & 11 \\ 4 & 19 & 21 \end{pmatrix}$$

Matrix

Basic Structures in Linear Algebra



Scalar

Scalars are **single numbers** and are an example of a 0*th*-order tensor. The notation x ∈ $\mathbb{R}$ states that x is a scalar belonging to a set of real-values numbers, $\mathbb{R}$.

Few built-in scalar types are **int**, **float**, **complex**, **bytes**, **Unicode** in Python. In In NumPy a python library, there are 24 new fundamental data types to describe different types of scalars. For information regarding datatypes refer documentation here.

Vector

Vectors are ordered arrays of single numbers and are an example of 1st-order tensor. Vectors are fragments of objects known as vector spaces.

$$x = [x_1 \ x_2 \ x_3 \ x_4 \ \dots \ x_n]$$

To identify the necessary component of a vector explicitly, the i*th* scalar element of a vector is written as x[i].

Matrix

Matrices are rectangular arrays consisting of numbers and are an example of 2*nd*-order tensors. If m and n are positive integers, that is m, n ∈ ℕ then the m×n matrix contains m*n numbers, with m rows and n columns.

The full m×n matrix can be written as:

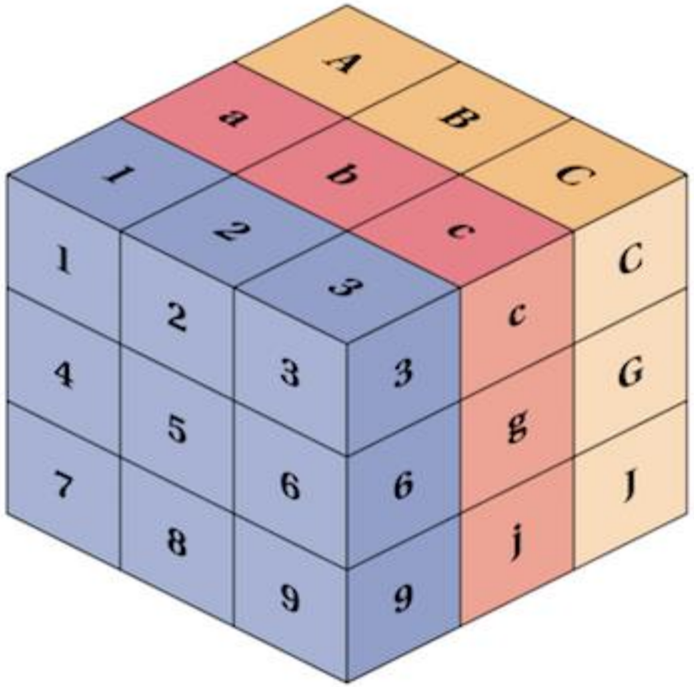
$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

It is often useful to abbreviate the full matrix component display into the following expression

$$A=[a_{ij}]_{m\times n}$$

Tensor

Multidimensional Matrix is called Tensor



TENSOR

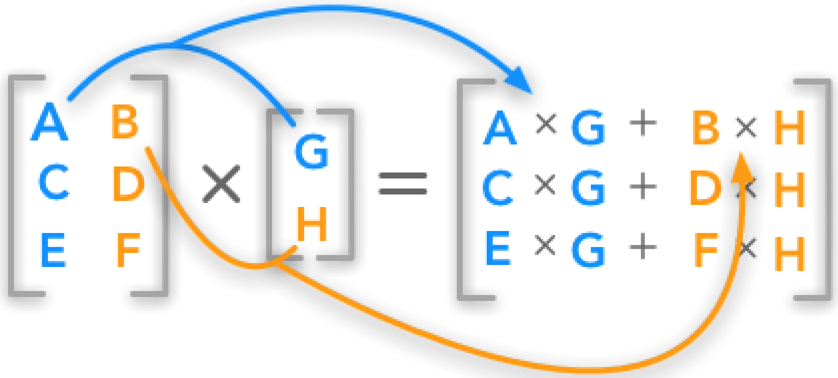
Matrix Scalar Operation: Multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * y = \begin{bmatrix} a*y & b*y \\ c*y & d*y \end{bmatrix}$$

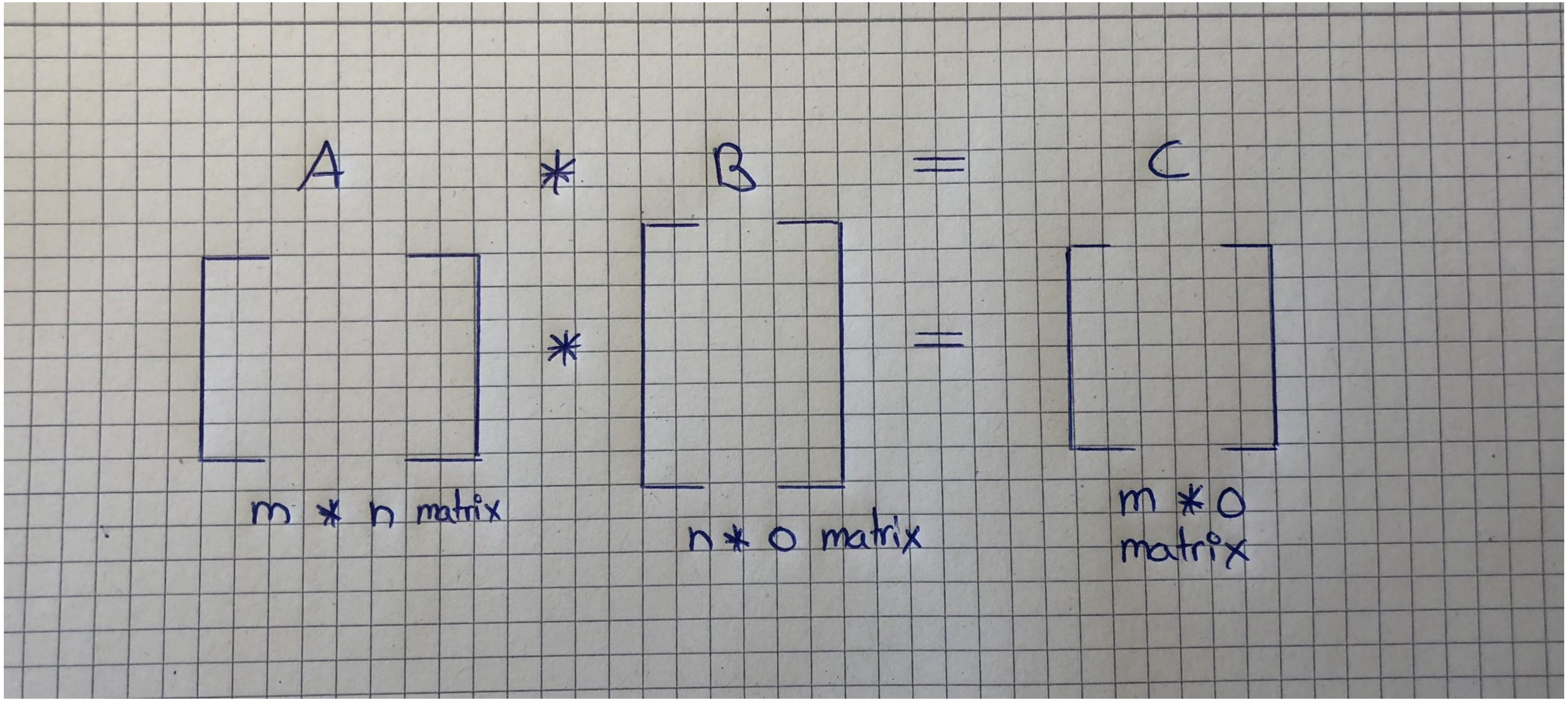
$$\begin{bmatrix} 4 & 8 \\ 3 & 7 \end{bmatrix} * 3 = \begin{bmatrix} 12 & 24 \\ 9 & 21 \end{bmatrix}$$

Matrix Multiplication

A of shape (m x n) and B of shape (n x p) multiplied gives C of shape (m x p)



Matric Multiplication



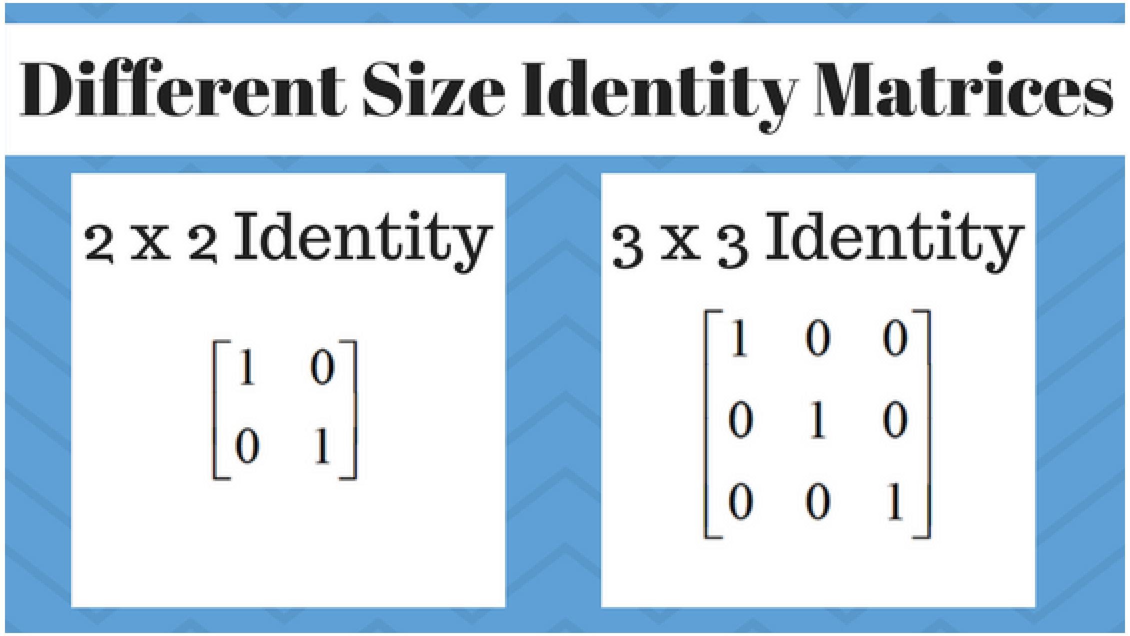
Matrix Transpose

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Identity Matrix

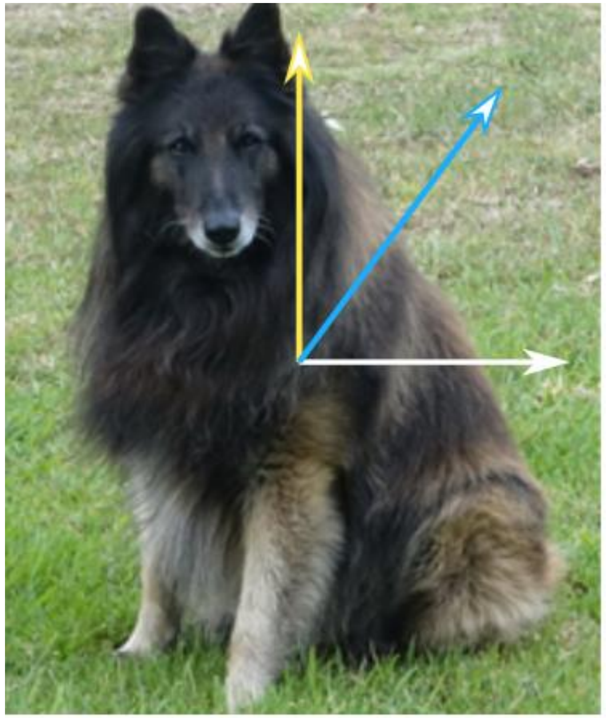
For any whole number n, there is a corresponding n×n identity matrix. These matrices are said to be square since there is always the same number of rows and columns.

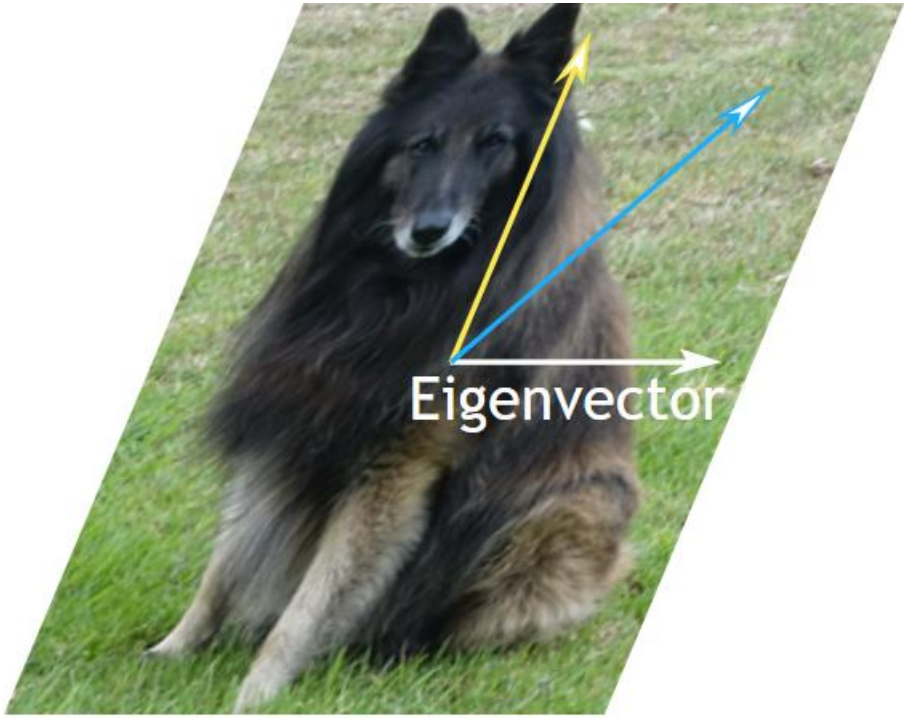
To prevent confusion, a subscript is often used. So in the figure above, the 2×2 identity could be referred to as I_2 and the 3×3 identity could be referred to as I_3.



Eigen Vector and Eigen Value

A simple example is that an eigenvector **does not change direction** in a transformation:





Importance of Eigen Vector

The reason why eigenvalues are so important in mathematics are too many. Here is a short list of the applications that are coming now in mind to me:

- Principal Components Analysis (PCA) in dimensionality reduction and object/image recognition. (See PCA)
- Face recognition by computing eigenvectors of images (See Eigenfaces).
- Physics — stability analysis, the physics of rotating bodies (See Stability Theory).
- Google uses it to rank pages for your search results (See PageRank).

Recap

- Scalar
- Matrix
- Tensor
- Matrix Operations: Sum, Abstraction, Multiplication, Transpose
- Matrix Decomposition
- Identity Matrix
- Eigen Value and Eigen Matrix
- Resource: Video, Book
- Online Course