Introduction to Probability

Class 07

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Probability

Probability is simply how likely something is to happen.

$$P(E) = \frac{the\ number\ of\ outcomes\ in\ E}{the\ total\ number\ of\ outcomes\ in\ S}$$

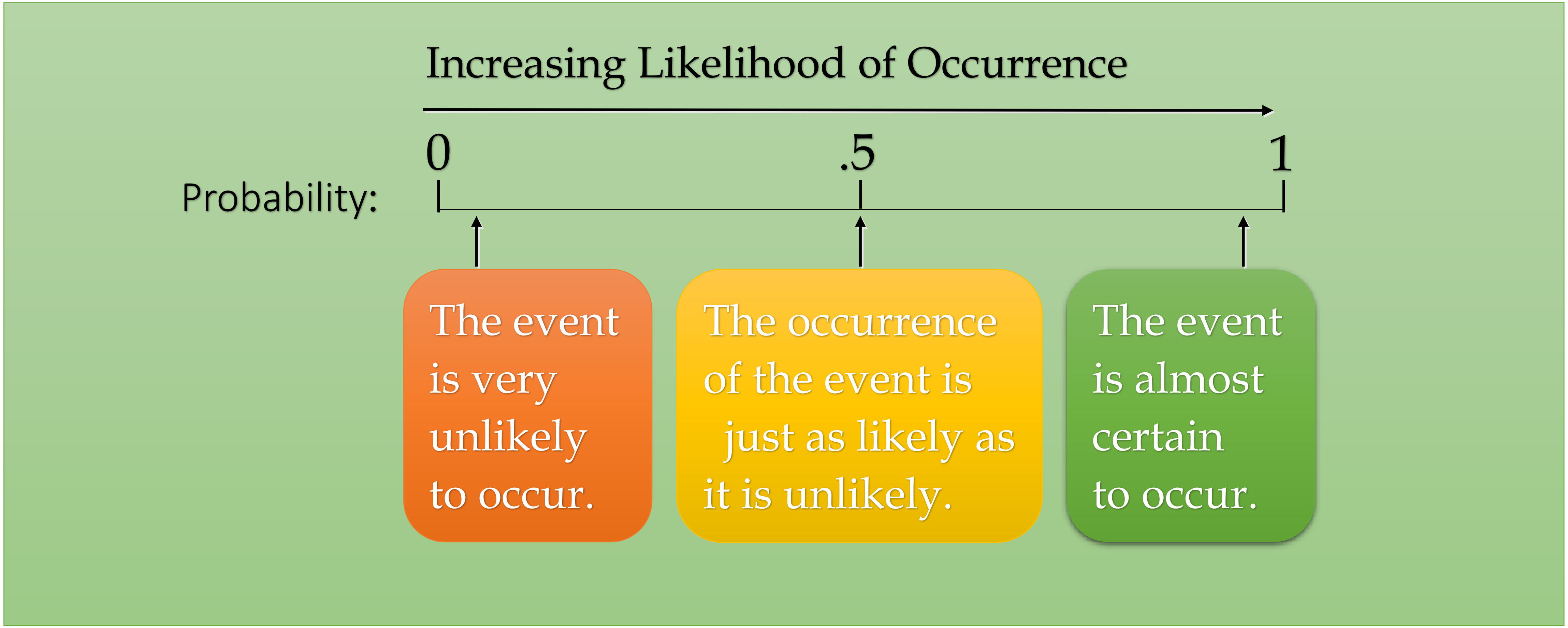
Dice example:

Number 6 on the die

$$\frac{1}{6}$$

Number of possible sides on the die

Probability as a Numerical Measure of the Likelihood of Occurrence



An Experiment and Its Sample Space

An experiment is any process that generates well-defined outcomes.

The sample space for an experiment is the set of all experimental outcomes.

An experimental outcome is also called a sample point.

Axioms of Probability

- **Axiom 1:** The probability of an event is a real number greater than or equal to 0.
- **Axiom 2:** The probability that at least one of all the possible outcomes of a process (such as rolling a die) will occur is 1.
- **Axiom 3:** If two events *A* and *B* are mutually exclusive, then the probability of either *A* or *B* occurring is the probability of *A* occurring plus the probability of *B* occurring.

Some Basic Relationships of Probability

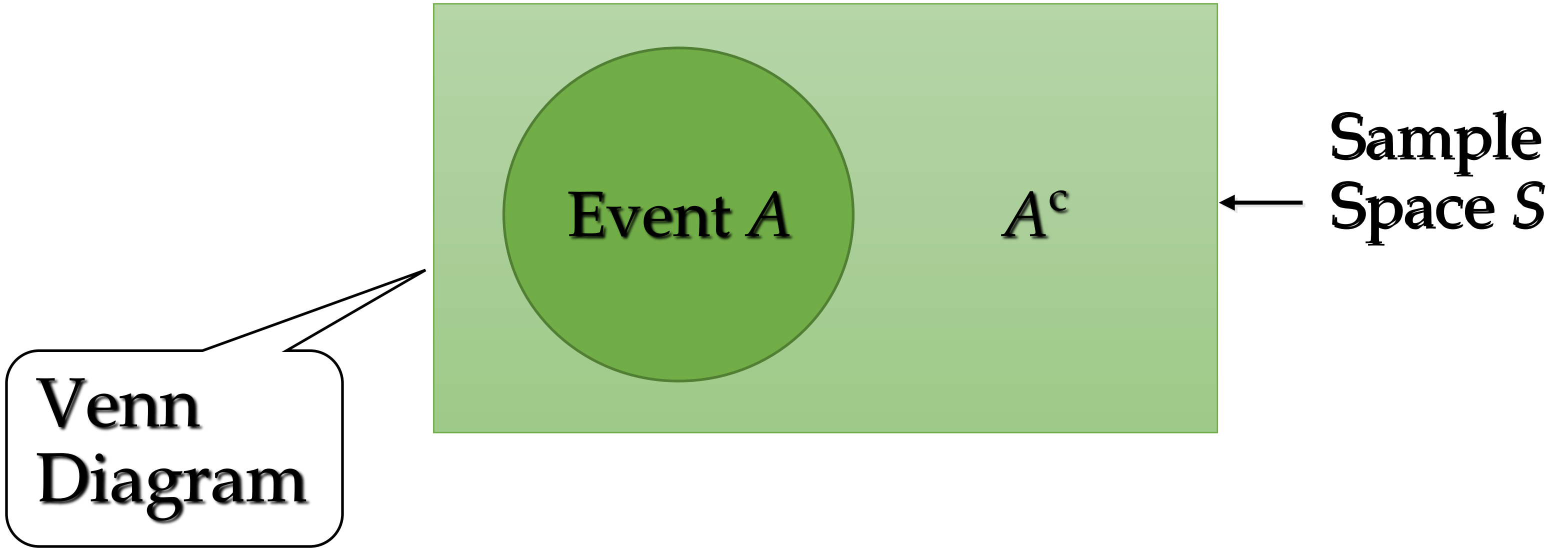
There are some basic probability relationships that can be used to compute the probability of an event without knowledge of all the sample point probabilities

- Complement of an Event
- Union of Two Events
- Intersection of Two Events
- Mutually Exclusive Events

Complement of an Event

The complement of event A is defined to be the event consisting of all sample points that are not in A.

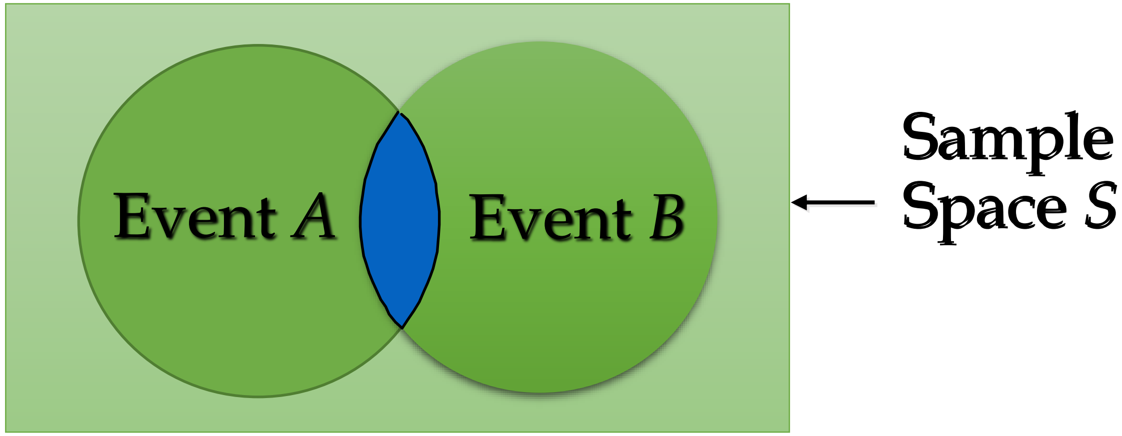
The complement of A is denoted by A^c.



Union of Two Events

The union of events A and B is the event containing all sample points that are in A or B or both.

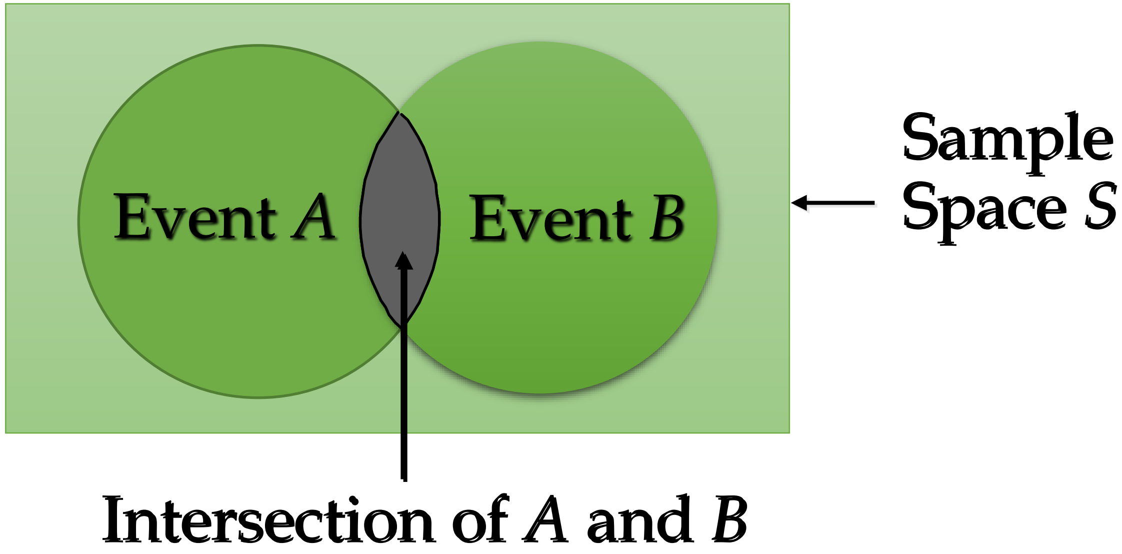
The union of events A and B is denoted by $A \cup B$.



Intersection of Two Events

The intersection of events A and B is the set of all sample points that are in both A and B.

The intersection of events A and B is denoted by $A \cap B$.

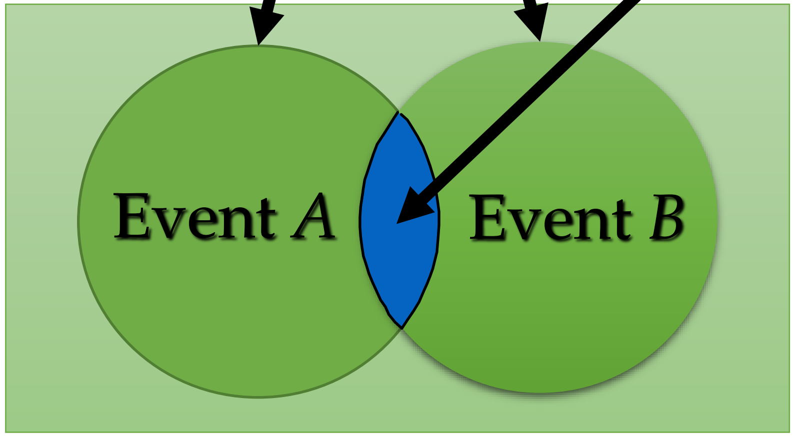


Addition Law

The addition law provides a way to compute the probability of event A, or B, or both A and B occurring.

The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



A student goes to the library. The probability that she checks out

(a) a work of fiction is 0.40,

(b) a work of non-fiction is 0.30, and

(c) both fiction and non-fiction is 0.20.

What is the probability that the student checks out a work of fiction, non-fiction, or both?

Solution:

Let F = the event that the student checks out fiction

N = the event that the student checks out non-fiction.

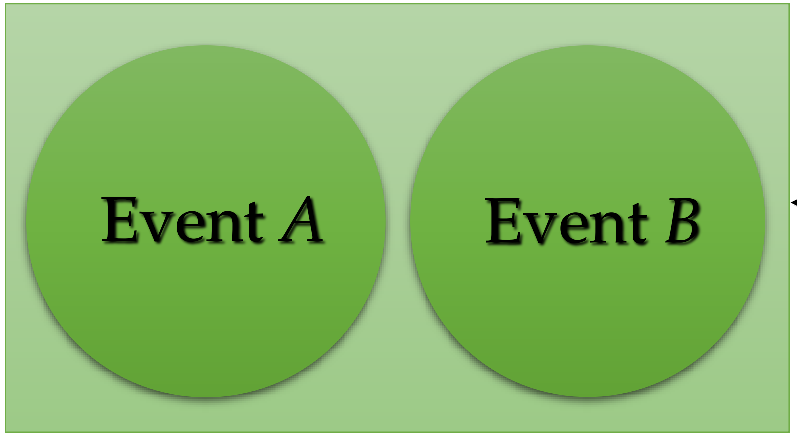
$P(F \cup N) = P(F) + P(N) - P(F \cap N)$

$P(F \cup N) = 0.40 + 0.30 - 0.20 = 0.50$

Mutually Exclusive Events

Two events are said to be <u>mutually exclusive</u> if the events have no sample points in common.

Two events are mutually exclusive if, when one event occurs, the other cannot occur.



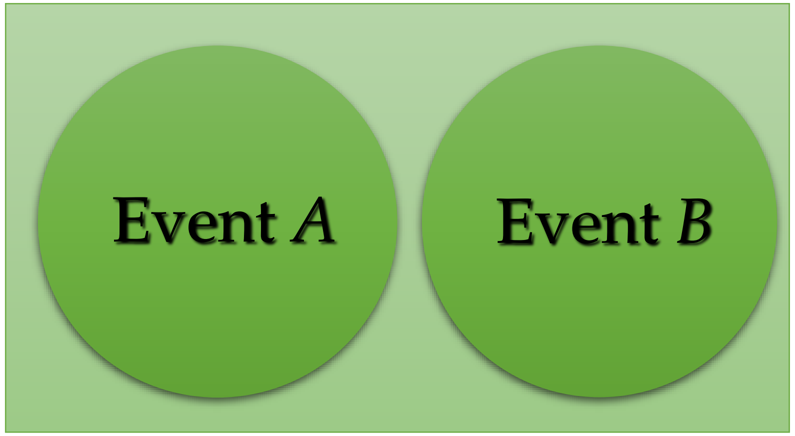
Sample Space S

Mutually Exclusive Events

If events A and B are mutually exclusive, $P(A \cap B) = 0$.

The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$



there's no need to include "– $P(A \cap B)$"

Conditional Probability

The probability of an event given that another event has occurred is called a conditional probability.

The conditional probability of A given B is denoted by $P(A \mid B)$.

A conditional probability is computed as follows :

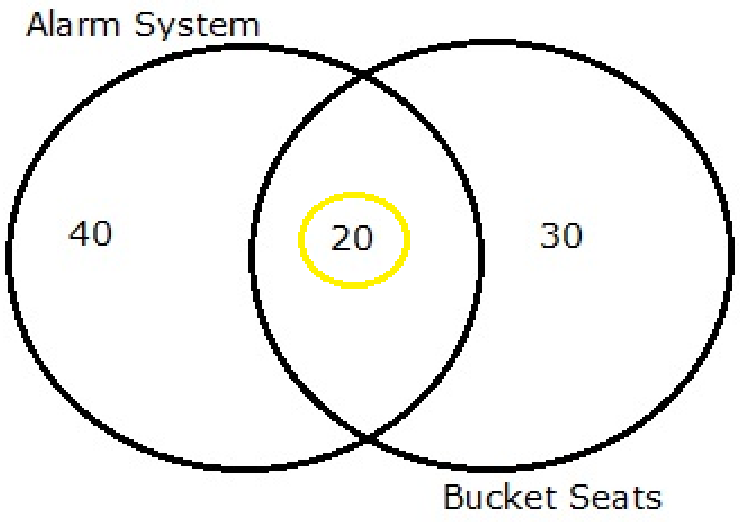
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

In a group of 100 sports car buyers, 40 bought alarm systems, 30 purchased bucket seats, and 20 purchased an alarm system and bucket seats. If a car buyer chosen at random bought an alarm system, what is the probability they also bought bucket seats?

Step 1: Figure out P(A). It's given in the question as 40%, or 0.4.

Step 2: Figure out P(A∩B). This is the intersection of A and B: both happening together. It's given in the question 20 out of 100 buyers, or 0.2.

Step 3: Insert your answers into the formula:
$P(B|A) = P(A \cap B) / P(A) = 0.2 / 0.4 = 0.5$.



Bayesian Theorem

Bayes' theorem is a formula that describes how to update the probabilities of hypotheses when given evidence. It follows simply from the axioms of conditional probability, but can be used to powerfully reason about a wide range of problems involving belief updates.

LIKELIHOOD

The probability of "B" being True, given "A" is True

PRIOR

The probability "A" being True. This is the knowledge.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

POSTERIOR

The probability of "A" being True, given "B" is True

MARGINALIZATION

The probability "B" being True.

In a particular pain clinic, 10% of patients are prescribed narcotic pain killers. Overall, five percent of the clinic's patients are addicted to narcotics (including pain killers and illegal substances). Out of all the people prescribed pain pills, 8% are addicts. If a patient is an addict, what is the probability that they will be prescribed pain pills?

Step 1: Figure out what your event "A" is from the question. That information is in the italicized part of this particular question. The event that happens first (A) is being prescribed pain pills. That's given as 10%.

Step 2: Figure out what your event "B" is from the question. That information is also in the italicized part of this particular question. Event B is being an addict. That's given as 5%.

Step 3: Figure out what the probability of event B (Step 2) given event A (Step 1). In other words, find what (B|A) is. We want to know "Given that people are prescribed pain pills, what's the probability they are an addict?" That is given in the question as 8%, or .08.

Step 4: Insert your answers from Steps 1, 2 and 3 into the formula and solve.
$P(A|B) = P(B|A) * P(A) / P(B) = (0.08 * 0.1)/0.05 = 0.16$

Reference

- Interesting Visualization
- Probability Course
- Probability For Dummies