Probability Models

Sample Space:

flipping a coin, $S = \{H, T\}$

Rolling a die, $S = \{1,2,3,4,5,6\}$

Flipping 2 coins, $S = \{HH, HT, TH, TT\}$

* **Event:** Any subset E of sample space S is called an Event.
* $E \cap F = EF$
* IF $EF = \phi$, then E an F are said to be mutually exclusive.

$$E \cap E^c = \phi \ , \quad S^c = \phi \ , \quad S = E \cup E^c$$

Probability:

(i) $0 \leq P(E) \leq 1$

(ii) $P(S) = 1$

(iii) $E_1, E_2, \ldots$ mutually exclusive events, $P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$

$$P(S) = 1 = P(E \cup E^c) = P(E) + P(E^c) \qquad [\because EE^c = \phi]$$

$$\therefore P(E^c) = 1 - P(E)$$

↳ complement of event E

* $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Ex 1.3 $S = \{HH, HT, TH, TT\}$

$E = \{HH, HT\}$, $P(E) = \frac{2}{4} = \frac{1}{2}$

$F = \{\cancel{TH, TT}\ HH, TH\}$, $P(F) = \frac{1}{2}$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$$

* $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(EF) - P(FG) - P(EG) + P(EFG)$
* $P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i) - \sum_{i<j} P(E_i E_j) + \sum_{i<j<k} P(E_i E_j E_k) - \cdots + (-1)^{n+1} P(E_1 E_2 \ldots E_n)$