$$\text{forward}_{k,i} = \sum_{\substack{\text{all} \\ \text{states} \\ l}} \text{forward}_{l,i-1} \times \text{transition}_{l,k} \times \text{emission}_k(x_i)$$

$$\text{backward}_{k,i} = \sum_{\substack{\text{all} \\ \text{states} \\ l}} \text{backward}_{l,i+1} \times \text{transition}_{k,l} \times \text{emission}_l(x_i)$$

$$\left[\begin{array}{l} \text{forward}_{k,\ \text{1st step}} = \text{stationary probability} \\ \text{backward}_{k,\ \text{last step}} = 1 \end{array}\right]$$

$$\pi^*_{k,i} = \frac{\text{forward}_{k,i} \times \text{backward}_{k,i}}{\text{forward sink}}$$

$$\pi^{**}_{k,l,i} = \frac{\text{forward}_{k,i} \times \text{transition}_{k,l} \times \text{emission}_l(x_{i+1}) \times \text{backward}_{l,i+1}}{\text{forward}_{\text{sink}}}$$

$$\text{forward sink} = \sum_{\substack{\text{all} \\ \text{states} \\ l}} \text{forward}_{l,\ \text{last step}}$$

$$\text{transition}_{k,l} = \sum_{i=1}^{n-1} \pi^{**}_{k,l,i}$$

$$\mu_k = \frac{\text{sum}(\pi^*_k \times \text{data})}{\text{sum}(\pi^*_k)}$$

$$\sigma_k = \sqrt{\frac{\text{sum}(\pi^*_k \times (\text{data} - \mu_k)^2}{\text{sum}(\pi^*_k)}}$$