Possible Project Ideas

Here are some possible project ideas, many of which we discussed briefly in class. This is definitely *not* a comprehensive list of possible projects. Indeed, essentially any topic is possible as long as you can relate it to mathematics. If you have an interest that you don't see how to make mathematical, ask me and maybe we can brainstorm something.

- Modeling problems, such as problems from the Mathematical Contest in Modeling.
 Examples include designing and modeling ways to address gerrymandering, board an
 airplane, eradicate ebola, search for a lost airplane, build a traffic roundabout, and
 many others.
- Coding projects in mathematics. See https://projecteuler.net and https://projecteuler.net/archives.
- Sports projects.
 - How do you predict a team's win-loss record from its players' stats?
 - What's the best route to take to see all of a ski mountain as fast as possible?
 - How do you pick a team for a fantasy competition?
 - See for example the Baseball Research Journal for other ideas.

• Gambling

- Model the *Bringing Down The House* strategy for making money in blackjack. How many decks does the casino need to use to protect itself?
- Powerball simulations (not sure what the math question is but there's definitely math going on here)
- Financial projects.
 - How do you predict the future price of a stock?
 - How do you calculate the value of an option?
 - How do you calculate the risk of an investment?
 - How big must a bank or insurance company's pool of money need to be in order to prevent them from going bankrupt with 99.9% certainty?
- Biology projects. How do you model the spread of a disease? What will the ethnic makeup of the human population look like in 1,000 years? Can you predict the symptoms or mechanisms of a virus based on the frequency of codons (such as TGC or ATT)?

- Why were many presidential polls so far off in 2016, and what would have been better methods for prediction?
- Machine learning and Support Vector Machines (SVMs).
- High-dimensional data analysis. We will likely look at a dataset of natural images, but you may have other high-dimensional datasets you'd like to analyze. One tool we will briefly in class is Principal Component Analysis (PCA), but there are many other techniques and machine learning tools to explore.
- Computational topology. This is Henry's area of research. See for example this tutorial I coauthored.
- Planar graph embeddings. Given a non-planar embedding (there are crossing edges) of a planar graph, what's the minimum number of vertices you must move in order to produce a planar embedding?

Here's a simulation.

If you move a vertex "at random", what's the expected number of moves until you reach a planar embedding? How does this expected number of moves depend on properties of the graph?

- Can you write an algorithm to win as often as possible in snake?
- How do you arrange a round-robin tournament where each match consists of three competitors (instead of two)? You could ask that each possible triplet competes exactly once, or you could instead ask that a competitor is matched in a triplet with every other competitor exactly once.
- Random walks in \mathbb{R}^1 , \mathbb{R}^2 , \mathbb{R}^3 ? There's a simple combinatorial argument for showing random walker in \mathbb{R}^1 returns to origin infinitely often. What tweaks can we add?
- Variants on the traveling salesperson problem.
- Voting theory. Arrow's impossibility theorem gives a set of tame axioms that one would want a voting system to satisfy, and then proves that (surprisingly) no such voting system exists. This leaves room for one to consider the pros and cons of various voting systems.
- What is the best way to fit spheres / tetrahedra / dodecahedra / footballs / etc into a box?
- Some project related to math riddles
- Variants on the lion and contamination problem mentioned in class. See this paper or this blog post. If you have more than n lions moving randomly in an $n \times n$ grid, how long do they have to move until you expect them to have cleared the grid?

- Error correcting codes.
- $\bullet\,$ Cryptography, for example RSA.
- Toppling sandpiles. See figures 1-4 in this article, or the figure on the wikipedia page.