Proof for – User Cooperation-Enhanced Dynamic Resource Allocation for Throughput Maximization in NOMA-Enabled Wireless Powered MEC

Huaiwen He, Feng Huang, Chenghao Zhou, Hong Shen, Hui Tian

Appendix A
Proof of Lemma 1

We define Υ^{P1} as the optimal policy for **P1**, which maximizes utility $\psi\left(\overline{\boldsymbol{r}^{\mathrm{opt}}}\right)$ by making the optimal decisions $\boldsymbol{r}^{\mathrm{opt}}(t)$ over time slot t. For **P2**, Υ^{P2} is its optimal policy that maximizes utility through the optimal decisions of $\boldsymbol{\lambda}^*(t)$ and $\boldsymbol{r}^*(t)$ in each time slot, while satisfying all the constraints of **P2**. Since $\log(x)$ is a concave and non-decreasing function, constraint (**??**) ensures that $\psi\left(\overline{\boldsymbol{\lambda}^*}\right) \leq \psi\left(\overline{\boldsymbol{r}^*}\right)$, and by Jensen's inequality, $\overline{\psi\left(\boldsymbol{\lambda}^*\right)} \leq \psi\left(\overline{\boldsymbol{\lambda}^*}\right) \leq \psi\left(\overline{\boldsymbol{r}^*}\right)$. Furthermore, we consider a strategy $\Upsilon^{\mathrm{P2}'}$ that always selects $\boldsymbol{\lambda(t)}$ equal to $\overline{\boldsymbol{r}^{\mathrm{opt}}}$, satisfying all constraints of **P2**. Under $\Upsilon^{\mathrm{P2}'}$, we have $\overline{\psi\left(\boldsymbol{\lambda}\right)} = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\psi\left(\overline{\boldsymbol{r}^{\mathrm{opt}}}\right) = \psi\left(\overline{\boldsymbol{r}^{\mathrm{opt}}}\right)$. Since Υ^{P2} is the optimal policy for **P2**, we have $\overline{\psi\left(\boldsymbol{\lambda}^*\right)} \geq \overline{\psi\left(\boldsymbol{\lambda}\right)} = \psi\left(\overline{\boldsymbol{r}^{\mathrm{opt}}}\right)$. So, we can deduce that $\psi\left(\overline{\boldsymbol{r}^*}\right) \geq \overline{\psi\left(\boldsymbol{\lambda}^*\right)} \geq \psi\left(\overline{\boldsymbol{r}^{\mathrm{opt}}}\right)$. The transformed **P2** meets all the constraints of the original **P1** and $\psi\left(\overline{\boldsymbol{r}^{\mathrm{opt}}}\right)$ is the maximum utility of **P1**, we confirm $\psi\left(\overline{\boldsymbol{r}^*}\right) = \psi\left(\overline{\boldsymbol{r}^{\mathrm{opt}}}\right)$, proving **P2** is equivalent to **P1**.

Appendix B
Proof of Optimal Auxiliary Variable.

The first-order derivation of $\Phi_1\left(\lambda_i(t)\right)$ for $\lambda_i(t)$ is derived as

$$\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)} = \frac{V/\ln 2}{1/\omega_i + \lambda_i(t)} - Z_i(t) \tag{1}$$

From (1), $\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)}$ decreases as $\lambda_i(t) \in (0, A_i^{\max})$ rises. If $\left.\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)}\right|_{\lambda_i(t)=0} = \frac{V\omega_i}{\ln 2} - Z_i(t) \leq 0$, we have $\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)} < 0$ over $\lambda_i(t) \in (0, A_i^{\max})$. Therefore, $\Phi_1\left(\lambda_i(t)\right)$ decreases as $\lambda_i(t)$ increases. In this case, the optimal auxiliary variable yields $\lambda_i^*(t) = 0$.

Otherwise, according to

$$\left.\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)}\right|_{\lambda_i(t)=\frac{V}{Z_i(t)\ln 2}-\frac{1}{\omega_i}} = 0 \tag{2}$$

If $\frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i} \geq A_i^{\max}$, we have $\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)} > 0$ over $\lambda_i(t) \in (0, A_i^{\max})$. Therefore, $\Phi_1\left(\lambda_i(t)\right)$ increases as $\lambda_i(t)$ increases. In this case, $\lambda_i^*(t) = A_i^{\max}$.

If $\frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i} \leq A_i^{\max}$ we have either $\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)} > 0$ if $\lambda_i(t) \in \left(0, \frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}\right)$ or $\frac{\partial\left(\Phi_1\left(\lambda_i(t)\right)\right)}{\partial\left(\lambda_i(t)\right)} < 0$ if $\lambda_i(t) \in \left(\frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}, A_i^{\max}\right)$. In this case, the optimal auxiliary variable is $\lambda_i^*(t) = \frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}$.

Thus, the optimal auxiliary variable is

$$\lambda_i^*(t) = \begin{cases} 0, & \text{if } \frac{V\omega_i}{\ln 2} - Z_i(t) \leq 0; \\ \min\left\{\frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}, A^{\max}\right\}, & \text{otherwise.} \end{cases} \tag{3}$$

Appendix C
Proof of Therem 5

Assuming that π represents a viable strategy for **P1**, the minimum upper bound of $\Delta_V\left(\boldsymbol{\Theta}(t)\right)$ can be expressed as

$$\begin{aligned} &\Delta_V\left(\boldsymbol{\Theta}(t)\right) \\ &\leq H - V\mathbb{E}\left\{\sum_{i\in\{1,2\}}\log\left(1+\omega_i\lambda_i\right) \mid \boldsymbol{\Theta}(t), \pi\right\} \\ &+ \sum_{i\in\{1,2\}} Q_i(t)\mathbb{E}\left\{R_i(t) - d_i^l(t) - d_i^f(t) \mid \boldsymbol{\Theta}(t), \pi\right\} \\ &+ \sum_{i\in\{1,2\}} Z_i(t)\mathbb{E}\left\{\lambda_i(t) - R_i(t) \mid \boldsymbol{\Theta}(t), \pi\right\} \end{aligned} \tag{4}$$

Referring to [1], it states that if **P1** is a feasible problem, then for any $\epsilon > 0$, it is guaranteed that there is at least one randomized stationary policy π^* which meets the following specified conditions

$$-\phi\left(\boldsymbol{\lambda(t)} \mid \pi^*\right) \leq -\psi^{\mathrm{opt}} + \epsilon \tag{5a}$$

$$\mathbb{E}\left\{R_i(t) - d_i^l(t) - d_i^f(t) \mid \pi^*\right\} \leq \epsilon \tag{5b}$$

$$\mathbb{E}\left\{\lambda_i(t) - R_i(t) \mid \pi^*\right\} \leq \epsilon \tag{5c}$$

By integrating these results into (4) and taking $\epsilon \to 0$, we obtain

$$\Delta_V\left(\boldsymbol{\Theta}(t)\right) \leq H - V\psi^{\mathrm{opt}} \tag{6}$$

Utilizing the iterated expectation and summing up all the telescoping series of (6) over time $t \in \{0, 1, \ldots, T-1\}$, we have

$$\begin{aligned} \mathbb{E}\left\{L\left(\boldsymbol{\Theta}(t)\right)\right\} - \mathbb{E}\left\{L\left(\boldsymbol{\Theta}\left(0\right)\right)\right\} - V\sum_{t=0}^{T-1}\mathbb{E}\left\{\psi\left(\boldsymbol{\lambda(t)}\right)\right\} \\ \leq HT - VT\psi^{\mathrm{opt}} \end{aligned} \tag{7}$$

By dividing both sides of (7) by VT, applying Jensen's inequality, and considering that the expected value $\mathbb{E}\left\{L\left(\boldsymbol{\Theta}\left(0\right)\right)\right\} \geq 0$ yields

$$\frac{1}{T}\lim_{T\to\infty}\sum_{t=0}^{T-1}\mathbb{E}\left\{\psi\left(\boldsymbol{\lambda}(\boldsymbol{t})\right)\right\} \geq \psi^{\text{opt}} - \frac{H}{V} \tag{8}$$

that is

$$\overline{\psi\left(\boldsymbol{\lambda}(\boldsymbol{t})\right)} \geq \psi^{\text{opt}} - \frac{H}{V} \tag{9}$$

According to $\overline{\psi(\boldsymbol{\lambda^*})} \leq \psi(\overline{\boldsymbol{r}^*})$ proved in Lemma 1, we have $\psi(\overline{\boldsymbol{r}^*}) \geq \overline{\psi(\boldsymbol{\lambda}(t))}$. Thus, $\psi^{\text{opt}} - \psi(\overline{\boldsymbol{r}^*}) \leq \frac{H}{V}$.

Appendix D
Proof of Therem 6

Utilizing the iterated expectations technique and recursively applying telescoping sums over each time slot $t \in \{0, 1, \ldots, T-1\}$ as per equation (31) yields.

$$\begin{aligned}\mathbb{E}\left\{L\left(\boldsymbol{\Theta}(t)\right)\right\} - \mathbb{E}\left\{L\left(\boldsymbol{\Theta}(0)\right)\right\} - V\mathbb{E}\left\{\psi\left(\boldsymbol{\lambda}(\boldsymbol{t})\right) \mid \boldsymbol{\Theta}(t)\right\} \\ \leq TH - \varepsilon\sum_{t=0}^{T-1}\sum_{i\in\{1,2\}}\mathbb{E}\left\{Q_i(t) + Z_i(t)\right\}\end{aligned} \tag{10}$$

By dividing both sides of equation (10) by $T\varepsilon$ and taking the limit as $T \to \infty$, we manipulate the terms to achieve the desired outcome

$$\frac{H}{\varepsilon} - \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{i\in\{1,2\}}\mathbb{E}\left\{Q_i(t) + Z_i(t)\right\} + V\frac{\overline{\psi\left(\boldsymbol{\lambda}(\boldsymbol{t})\right)}}{\varepsilon} \geq 0 \tag{11}$$

Specifically

$$Q^{\Sigma} \leq \frac{H}{\varepsilon} + V\frac{\overline{\psi\left(\boldsymbol{\lambda}(\boldsymbol{t})\right)}}{\varepsilon} \tag{12}$$

References

[1] W. Zhan, C. Luo, G. Min, C. Wang, Q. Zhu, and H. Duan, "Mobility-aware multi-user offloading optimization for mobile edge computing," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 3, pp. 3341–3356, 2020.