

Proof for – User Cooperation-Enhanced Dynamic Resource Allocation for Throughput Maximization in NOMA-Enabled Wireless Powered MEC

Huaiwen He, Feng Huang, Chenghao Zhou, Hong Shen, Hui Tian

APPENDIX A PROOF OF LEMMA 1

We define \mathbf{Y}^{P1} as the optimal policy for **P1**, which maximizes utility $\psi(\mathbf{r}^{\text{opt}})$ by making the optimal decisions $\mathbf{r}^{\text{opt}}(t)$ over time slot t . For **P2**, \mathbf{Y}^{P2} is its optimal policy that maximizes utility through the optimal decisions of $\lambda^*(t)$ and $\mathbf{r}^*(t)$ in each time slot, while satisfying all the constraints of **P2**. Since $\log(x)$ is a concave and non-decreasing function, constraint (27b) ensures that $\psi(\lambda^*) \leq \psi(\mathbf{r}^*)$, and by Jensen's inequality, $\psi(\lambda^*) \leq \psi(\lambda^*) \leq \psi(\mathbf{r}^*)$. Furthermore, we consider a strategy $\mathbf{Y}^{\text{P2}'}$ that always selects $\lambda(t)$ equal to \mathbf{r}^{opt} , satisfying all constraints of **P2**. Under $\mathbf{Y}^{\text{P2}'}$, we have $\psi(\lambda) = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \psi(\mathbf{r}^{\text{opt}}) = \psi(\mathbf{r}^{\text{opt}})$. Since \mathbf{Y}^{P2} is the optimal policy for **P2**, we have $\psi(\lambda^*) \geq \psi(\lambda) = \psi(\mathbf{r}^{\text{opt}})$. So, we can deduce that $\psi(\mathbf{r}^*) \geq \psi(\lambda^*) \geq \psi(\mathbf{r}^{\text{opt}})$. The transformed **P2** meets all the constraints of the original **P1** and $\psi(\mathbf{r}^{\text{opt}})$ is the maximum utility of **P1**, we confirm $\psi(\mathbf{r}^*) = \psi(\mathbf{r}^{\text{opt}})$, proving **P2** is equivalent to **P1**.

APPENDIX B PROOF OF OPTIMAL AUXILIARY VARIABLE.

The first-order derivation of $\Phi_1(\lambda_i(t))$ for $\lambda_i(t)$ is derived as

$$\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))} = \frac{V/\ln 2}{1/\omega_i + \lambda_i(t)} - Z_i(t) \quad (1)$$

From (1), $\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))}$ decreases as $\lambda_i(t) \in (0, A_i^{\max})$ rises. If $\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))} \Big|_{\lambda_i(t)=0} = \frac{V\omega_i}{\ln 2} - Z_i(t) \leq 0$, we have $\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))} < 0$ over $\lambda_i(t) \in (0, A_i^{\max})$. Therefore, $\Phi_1(\lambda_i(t))$ decreases as $\lambda_i(t)$ increases. In this case, the optimal auxiliary variable yields $\lambda_i^*(t) = 0$.

Otherwise, according to

$$\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))} \Big|_{\lambda_i(t) = \frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}} = 0 \quad (2)$$

If $\frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i} \geq A_i^{\max}$, we have $\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))} > 0$ over $\lambda_i(t) \in (0, A_i^{\max})$. Therefore, $\Phi_1(\lambda_i(t))$ increases as $\lambda_i(t)$ increases. In this case, $\lambda_i^*(t) = A_i^{\max}$.

If $\frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i} \leq A_i^{\max}$ we have either $\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))} > 0$ if $\lambda_i(t) \in (0, \frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i})$ or $\frac{\partial(\Phi_1(\lambda_i(t)))}{\partial(\lambda_i(t))} < 0$ if $\lambda_i(t) \in$

$(\frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}, A_i^{\max})$. In this case, the optimal auxiliary variable is $\lambda_i^*(t) = \frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}$.

Thus, the optimal auxiliary variable is

$$\lambda_i^*(t) = \begin{cases} 0, & \text{if } \frac{V\omega_i}{\ln 2} - Z_i(t) \leq 0; \\ \min \left\{ \frac{V}{Z_i(t)\ln 2} - \frac{1}{\omega_i}, A_i^{\max} \right\}, & \text{otherwise.} \end{cases} \quad (3)$$

APPENDIX C PROOF OF THEOREM 5

Assuming that π represents a viable strategy for **P1**, the minimum upper bound of $\Delta_V(\Theta(t))$ can be expressed as

$$\begin{aligned} \Delta_V(\Theta(t)) &\leq H - V\mathbb{E} \left\{ \sum_{i \in \{1,2\}} \log(1 + \omega_i \lambda_i) \mid \Theta(t), \pi \right\} \\ &+ \sum_{i \in \{1,2\}} Q_i(t) \mathbb{E} \left\{ R_i(t) - d_i^l(t) - d_i^f(t) \mid \Theta(t), \pi \right\} \\ &+ \sum_{i \in \{1,2\}} Z_i(t) \mathbb{E} \left\{ \lambda_i(t) - R_i(t) \mid \Theta(t), \pi \right\} \end{aligned} \quad (4)$$

Referring to [1], it states that if **P1** is a feasible problem, then for any $\epsilon > 0$, it is guaranteed that there is at least one randomized stationary policy π^* which meets the following specified conditions

$$-\phi(\lambda(t) \mid \pi^*) \leq -\psi^{\text{opt}} + \epsilon \quad (5a)$$

$$\mathbb{E} \left\{ R_i(t) - d_i^l(t) - d_i^f(t) \mid \pi^* \right\} \leq \epsilon \quad (5b)$$

$$\mathbb{E} \left\{ \lambda_i(t) - R_i(t) \mid \pi^* \right\} \leq \epsilon \quad (5c)$$

By integrating these results into (4) and taking $\epsilon \rightarrow 0$, we obtain

$$\Delta_V(\Theta(t)) \leq H - V\psi^{\text{opt}} \quad (6)$$

Utilizing the iterated expectation and summing up all the telescoping series of (6) over time $t \in \{0, 1, \dots, T-1\}$, we have

$$\begin{aligned} \mathbb{E} \{L(\Theta(t))\} - \mathbb{E} \{L(\Theta(0))\} - V \sum_{t=0}^{T-1} \mathbb{E} \{\psi(\lambda(t))\} \\ \leq HT - VT\psi^{\text{opt}} \end{aligned} \quad (7)$$

By dividing both sides of (7) by VT , applying Jensen's inequality, and considering that the expected value $\mathbb{E}\{L(\Theta(0))\} \geq 0$ yields

$$\frac{1}{T} \lim_{T \rightarrow \infty} \sum_{t=0}^{T-1} \mathbb{E}\{\psi(\lambda(t))\} \geq \psi^{\text{opt}} - \frac{H}{V} \quad (8)$$

that is

$$\overline{\psi(\lambda(t))} \geq \psi^{\text{opt}} - \frac{H}{V} \quad (9)$$

According to $\overline{\psi(\lambda^*)} \leq \psi(\mathbf{r}^*)$ proved in Lemma 1, we have $\psi(\mathbf{r}^*) \geq \overline{\psi(\lambda(t))}$. Thus, $\psi^{\text{opt}} - \psi(\mathbf{r}^*) \leq \frac{H}{V}$.

APPENDIX D PROOF OF THEREM 6

Utilizing the iterated expectations technique and recursively applying telescoping sums over each time slot $t \in \{0, 1, \dots, T-1\}$ as per equation (31) yields.

$$\begin{aligned} & \mathbb{E}\{L(\Theta(t))\} - \mathbb{E}\{L(\Theta(0))\} - V\mathbb{E}\{\psi(\lambda(t)) \mid \Theta(t)\} \\ & \leq TH - \varepsilon \sum_{t=0}^{T-1} \sum_{i \in \{1,2\}} \mathbb{E}\{Q_i(t) + Z_i(t)\} \end{aligned} \quad (10)$$

By dividing both sides of equation (10) by $T\varepsilon$ and taking the limit as $T \rightarrow \infty$, we manipulate the terms to achieve the desired outcome

$$\frac{H}{\varepsilon} - \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i \in \{1,2\}} \mathbb{E}\{Q_i(t) + Z_i(t)\} + V \frac{\overline{\psi(\lambda(t))}}{\varepsilon} \geq 0 \quad (11)$$

Specifically

$$Q^\Sigma \leq \frac{H}{\varepsilon} + V \frac{\overline{\psi(\lambda(t))}}{\varepsilon} \quad (12)$$

REFERENCES

- [1] W. Zhan, C. Luo, G. Min, C. Wang, Q. Zhu, and H. Duan, "Mobility-aware multi-user offloading optimization for mobile edge computing," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 3, pp. 3341–3356, 2020.